IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received November 26, 2019, accepted December 12, 2019, date of publication December 17, 2019, date of current version December 26, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2960348

A Risk-Averse Newsvendor Model Under the Framework of Uncertainty Theory

SHENGZHONG ZHANG[1], YINGMIN YU[1], JIDONG LI[2], AND QIHONG ZHU[3]
[1]School of Economics and Management, Chang'an University, Xi'an 710064, China
[2]Shenzhen Huawei Technologies Company, Ltd., Shenzhen 518129, China
[3]School of Business Administration, Shanghai Lixin University of Accounting and Finance, Shanghai 201620, China

Corresponding author: Qihong Zhu (zhuqihong0430@gmail.com)

ABSTRACT Due to the ever-changing and complex market environment, companies frequently face highly uncertain demand where data are so insufficient that the use of random or fuzzy variables, which are typically assumed in the literature, is impractical. Furthermore, companies are often risk-averse when making decisions. To address these two challenges, in this paper, we present the first study on a risk-averse newsvendor problem using the framework of uncertainty theory. To measure risk aversion, we adopt the measure of tail value-at-risk redefined based on uncertainty theory. We are able to analytically derive the optimal order quantity that maximizes the newsvendor's expected utility. We find that the optimal order quantity of a risk-averse newsvendor is less than that of a risk-neutral newsvendor. Furthermore, as the degree of risk aversion increases, the optimal order quantity decreases. Also, we show that the optimal order quantity may be independent of the risk confidence level when the degree of risk aversion is below a threshold. Moreover, we use numerical examples to illustrate how various parameters, such as the degree of risk aversion, salvage value, and unit ordering cost, affect the optimal order quantity.

INDEX TERMS Newsvendor problem, uncertainty theory, uncertain variable, risk aversion.

I. INTRODUCTION

Due to fierce market competition and rapid product upgrades, the selling season of many products (such as fashion, electronic products, and toys) has become increasingly shorter. In addition, the demand uncertainty faced by retailers when making inventory decisions is increasing due to various factors, such as uncertain market sizes and consumers' ever-changing appetites (e.g., consumers' low-carbon preference). The newsvendor model is commonly used as a basic model for ordering under uncertainties because it provides an effective analytical framework [1]. Several researchers have also extended the classical newsvendor model in different directions (see Khouja [2] and Qin *et al.* [3] for two excellent reviews).

Most researchers treat uncertain market demand as a random variable based on probability theory. For example, one method to address uncertain demand is to assume that demand is subject to a specific distribution. Arcelus *et al.* [4] explored the bicriteria decision newsvendor problem under the assumption that demand follows a uniform distribution. Dominey and Hill [5] studied the effectiveness of several approaches for approximating a compound Poisson distribution in a single-period setting. Similarly, Kitaeva *et al.* [6] considered demand a compound Poisson process with price-dependent intensity and a continuous batch size distribution; equations for retail price maximization of an expected profit with the optimal order quantity are obtained, and an approximate solution is proposed. Yu and Zhu [7] proved that when the market demand obeys a uniform distribution, a capital-constrained retailer could gradually increase his order quantity when he possesses more collateral assets. Furthermore, some researchers assume that insufficient information is available to obtain the specific distribution of the demand except for its mean and variance. Kamburowski [8] studied a distribution-free newsvendor problem under the worst-case and best-case demand scenarios. He *et al.* [9] studied the impact of the intelligence level of decision-makers on their expected profit using a distribution-free newsvendor model. Godfrey and Powell [10] directly estimated the value function using the Concave, Adaptive Value Estimation (CAVE) algorithm to solve the newsvendor problem rather than estimate the demand distribution. Helena [11] used a hybrid of Hurwicz and Bayes decision rules to design a novel approach

The associate editor coordinating the review of this manuscript and approving it for publication was Pengcheng Liu.

for the sale of new, innovative products under complete uncertainty. Sarkar *et al.* [12] considered no specific probability distribution for customers' demand, except for a known mean and standard deviation, in a single-period newsvendor problem with a consignment policy. Some researchers regard uncertain demand as a fuzzy variable. Chen and Yo [13] investigated the optimal order quantity of newsvendors when quantity discounts are available in a fuzzy environment. Xu and Hu [14] focused on the newsvendor problem by assuming that the uncertain market demand is a random fuzzy variable and presented a hybrid algorithm to obtain the optimal order quantity. Yu *et al.* [15] modeled the uncertain demand as a fuzzy variable because probability theory is not applicable in some cases; the optimal pricing and inventory decisions that could maximize the expected profit were obtained.

However, due to various issues, it can be difficult to collect sufficient data of some products or the available historical data may be too noisy and have limited reference value. These problems can occur for innovative products with no historical data or green products that face unknown market preference. Consequently, the specific distribution, even summary measures such as the mean and variance, or the membership function of market demand can be difficult to estimate. Consequently, probability theory and fuzzy theory are not applicable in these increasingly common situations. Therefore, we must consider these variables as uncertain variables and address them by inviting domain experts to give their corresponding belief degrees [16]. However, the variance of belief degree is often larger than the actual frequency, and the belief degree cannot be calculated via probability theory or fuzzy set theory. Liu [17] formally established uncertainty theory to rationally address personal belief degrees. Subsequently, uncertainty theory has been applied in various fields. For example, Zhang and Chen [18] studied the project scheduling problem using an uncertain programming model. Gao [19] applied uncertainty theory to address facility location problems. Qin *et al.* [20] proposed uncertain mean-semi-absolute deviation adjusting models for portfolio optimization problems. Uncertainty theory has also been applied to study the single-period inventory problem. Qin and Kar [21] first introduced uncertainty theory to the newsvendor problem by assuming demand to be an uncertain variable. They obtained the optimal order quantity that could maximize the newsvendor's expected profit. Ding [22] analyzed the multi-item newsvendor problem with uncertain demand and uncertain storage space under a warehousing chance constraint. Ding and Gao [23] derived an optimal replenishment policy of an uncertain multi-item newsvendor problem using uncertain programming. Ding [24] also explored how to make optimal order when uncertain market demand and random demand coexist based on uncertainty theory and chance theory. Wang *et al.* [25] developed both single-item and multi-item single-period inventory models with a budget constraint when market demands are assumed to be uncertain random variables.

However, the above papers using uncertainty theory to study newsvendor models assume that the newsvendor is risk-neutral, implying that the objective is to maximize the expected profit or minimize the expected cost. However, due to the complex market environment and uncertain demand, a newsvendor is more likely to be risk averse, and the objectives are no longer consistent with profit maximization or cost minimization [26]. Researchers have increasingly realized the significant impact of risk preference on newsvendors' decision making. Wang *et al.* [28] showed that a risk-averse newsvendor will order less than an arbitrarily small quantity as the selling price increases if the price is higher than a threshold value. Wu *et al.* [29] studied the effect of capacity uncertainty on inventory decisions in both risk-neutral and risk-averse scenarios; they discovered that the result obtained in the former scenario becomes invalid in the latter scenario. Murarka *et al.* [30] incorporated coherent risk measures into the classical newsvendor problem to capture risk when the order quantity decision is made.

Next, we review the different measurements of risk aversion. Expected utility, mean-variance, and value-at-risk (*VaR*) are the three traditional methods used to model risk aversion in inventory problems. However, these three methods may be unable to accurately describe the risk characteristics of decision makers [31]. Many scholars (e.g., see [32], [33]) have used conditional value-at-risk (*CVaR*) to measure risk attitude since it was first proposed by Rockafller and Uryasev [34]. *CVaR* is a coherent risk measure exhibiting subadditivity that can address the shortcomings of *VaR* by accounting for both expected profit and risk. However, a disadvantage of $CVaR_\alpha$ is that it measures the average income below the α-quantile and ignores the portion of income above the α-quantile. This limitation can lead to overly conservative decisions [35]. To overcome this disadvantage, scholars have adopted the $mean - CVaR$, which maximizes the expected profit while minimizing the downside risk of the profit to balance expected profit and risk. Xie *et al.* [36] considered a single-period supply chain with a newsvendor retailer by applying the $mean - CVaR$ criterion; they investigated three contracts to coordinate the supply chain in the case of risk neutrality and risk aversion. Xu and Li [37] studied the optimal ordering problem of the newsvendor model based on $mean-CVaR$ in the presence of a shortage cost. Gao *et al.* [38] explored a joint decision problem using the $mean - CVaR$ criterion when the newsvendor not only decides the order quantity but also adopts a weather hedging strategy. Similar to the approaches of measuring risk attitude mentioned above, we consider not only the expected profit but also the risk of loss. The difference is that we use tail value-at-risk (*TVaR*) to replace *CVaR*. *TVaR* is redefined by Peng [39] based on uncertainty theory as one of the risk metrics in uncertain risk analysis. *TVaR* is a good risk measurement that not only measures the extent of the loss suffered but also the probability of loss. As explained by Peng [39], *CVaR* is based on the probability theory framework and, therefore, is not applicable to the uncertainty theory framework.

Therefore, we adopt the risk aversion measure *mean − TVaR* in this paper.

Briefly, we aim to investigate the optimal order strategy for the single-period risk-averse newsvendor model with uncertain demand. We apply uncertain statistics to estimate the empirical distribution of the demand to solve the newsvendor model with an uncertain variable. Furthermore, we analyze how various parameters, such as the degree of risk aversion, affect the optimal order policy.

Our study represents a major extension of Qin and Kar [21], who applied uncertainty theory to address the newsvendor problem but assuming risk neutrality. Specifically, in this research, we make the following major contributions. First, we are the first to analyze a risk-averse newsvendor using the framework of uncertainty theory. To measure risk aversion, we adopt the measure of tail value-at-risk redefined based on uncertainty theory. Second, we are able to analytically derive the optimal order quantity. We show that as the degree of risk aversion increases, the optimal order quantity decreases. Also, the optimal order quantity may be independent of the risk confidence level when the degree of risk aversion is below a threshold. Third, we conduct numerical examples to further investigate how product characteristics, such as the salvage value, can impact the optimal order quantity. In short, this research contributes to the academic literature as well as industry practice by solving the risk-averse newsvendor model under uncertain theory.

The remainder of this paper is organized as follows. Section II provides some preliminaries regarding uncertainty theory. In Section III, a risk-averse newsvendor model is formulated under uncertain demand using the risk metric of *mean − TVaR*. In Section IV, the optimal order policy is analytically derived. Moreover, how various parameters affect the optimal policy is investigated. In Section V, numerical examples are given to verify the analytical results and present sensitivity analysis of the parameters. Section VI presents the conclusions and possible research directions.

II. PRELIMINARIES

Probability theory and uncertainty theory are two mathematical systems that rationally address indeterminacy. The former is used for modeling frequencies and is suitable when there are sufficient data. The latter is used for modeling belief degrees that are given by domain experts when there are no samples to estimate the probability distribution. Some fundamental concepts and properties of uncertainty theory that are used throughout this paper are as follows.

If nonempty collection L is a $\sigma-$ algebra over Γ, the element Λ of L is called an event. M is a set function from L to interval (0, 1) if M satisfies the following four axioms:

Axiom 1 (Normality Axiom): $M\{\Gamma\} = 1$ for the universal set Γ.

Axiom 2 (Duality Axiom): $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event.

Axiom 3 (Subadditivity Axiom): For each countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\} \tag{1}$$

Then, we call M an uncertain measure, and (Γ, L, M) is an uncertain space [17].

Axiom 4 [17]: Let (Γ_k, L_k, M_k) be the uncertainty spaces for $k = 1, 2, \ldots$. The product uncertain measure M is an uncertain measure satisfying

$$M\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\} \tag{2}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1, 2, \ldots$.

Definition 1 [40]: An uncertain variable ξ is a measurable function from the uncertain space (Γ, L, M) to the real number set; thus, for any Borel set over the real number set, the collection $\{\xi \in B\} = \{\gamma \in \Gamma \,|\xi(\gamma) \in B\}$ is an event. The belief degree represents the strength with which we believe that an event will occur. The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = M\{\xi \le x\}$ for any real number x.

Definition 2 [40]: A function Φ^{-1} is an inverse uncertainty distribution of an uncertain variable ξ if and only if

$$M\left\{\xi \le \Phi^{-1}(\alpha)\right\} = \alpha \tag{3}$$

for all $\alpha \in (0, 1)$.

Definition 3 [40]: Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(\xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \ldots, \xi_n$, the variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an inverse uncertainty distribution as follows:

$$\psi^{-1}(\alpha) = f\left(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\right) \tag{4}$$

Definition 4 [40]: An uncertain variable ξ is normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1} x, \quad \in R \tag{5}$$

denoted by $\xi \sim N(e, \sigma)$, where $e, \sigma \in R, \sigma > 0$.

The inverse uncertainty distribution of the normal uncertain variable $\xi \sim N(e, \sigma)$ is

$$\Phi^{-1}(\alpha) = e + \frac{\sigma\sqrt{3}}{\pi}\ln\frac{\alpha}{1-\alpha} \tag{6}$$

Definition 5 [40]: Let ξ be an uncertain variable. Provided that at least one of the two integrals are finite, the expected value of ξ is defined as

$$E[\xi] = \int_0^{\infty} M\{\xi \ge r\}dr - \int_{-\infty}^{0} M\{\xi \le r\}dr \tag{7}$$

TABLE 1. Notations of the parameters and variables.

Decision Variables	
y	Ordering quantity
Parameters	
p	Per unit selling price
h	Per unit salvage value
q	Per unit ordering cost
ξ	Market demand
$f(\xi, y)$	Profit function for order quantity y and demand ξ
y^*	Optimal order quantity
$F(y)$	Utility function of newsvendor
α , β	Risk confidence level ($\alpha, \beta \in (0,1]$)
$\Psi(x)$	Belief degree distribution function of $f(\xi, y)$
λ	Degree of risk aversion ($\lambda \in [0,1]$)

Theorem 1 [41]: Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(\xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \ldots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_{m+2}, \ldots, \xi_n, \xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an expected value

$$E[\xi] = \int_0^1 f\Big(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots \Phi_n^{-1}(1-\alpha)\Big) d\alpha \quad (8)$$

Definition 6 [39]: Assume that a system contains uncertain factors $\xi_1, \xi_2, \ldots, \xi_n$ and a loss function f. Let $\alpha \in (0, 1]$ be the risk confidence level. Then, the *VaR* of loss function f is the function $f_{VaR} : (0, 1] \to R$ such that

$$f_{VaR}(\alpha) = \inf\{x \,|M\{f(\xi_1, \xi_2, \ldots, \xi_n) \geq x\} < 1-\alpha\} \quad (9)$$

The *TVaR* of the loss function f is the function f_{VaR} : $(0, 1] \to R$ such that

$$f_{TVaR}(\alpha) = \frac{1}{\alpha}\int_0^{\alpha} f_{VaR}(\beta) d\beta \quad (10)$$

Theorem 2 [39]: Assume that a system contains uncertain factors $\xi_1, \xi_2, \ldots, \xi_n$ and has a loss function f. If $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively, and if the function $f(x_1, x_2, \ldots x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots x_n$, we have

$$f_{TVaR}(\alpha) = \frac{1}{\alpha} f\Big(\Phi_1^{-1}(1-\beta), \ldots, \Phi_m^{-1}(1-\beta), \Phi_{m+1}^{-1}(\beta), \ldots, \Phi_n^{-1}(\beta)\Big) d\beta \quad (11)$$

III. MODEL

In our model, we consider a risk-averse newsvendor with uncertain demand based on uncertainty theory. Faced with a continuous and uncertain market demand ξ, the newsvendor orders y unit products from a supplier at unit ordering cost q before the selling season begins and sells the products at unit selling price p. At the end of the regular selling season, the excess is disposed at unit salvage value h. Without loss of generality, we set $p > q > h > 0$. For clarity, the relevant notations used in our model are summarized in Table 1.

On the basis of the above model setup, the profit function can be written as

$$f(\xi, y) = \begin{cases} (p-q)\, y & y < \xi \\ (h-q)\, y + (p-h)\, \xi & y \geq \xi \end{cases} \quad (12)$$

Figure 1 plots $f(\xi, y)$ as a function of ξ.

Since market demand ξ is an uncertain variable, the profit function $f(\xi, y)$ is also an uncertain variable. Under risk-neutral conditions, the newsvendor's objective is to maximize the expected profit; thus, the optimal order quantity can be stated as $y^* = \arg\max\{E[f(\xi, y)]\}$, where $E[\cdot]$ is the expected operator. However, the objective of a risk-averse newsvendor is to maximize the expected utility, which can be expressed as $F(y) = \lambda TVaR_\alpha(f(\xi, y)) + (1-\lambda)E[f(\xi, y)]$ at a given level of risk confidence $\alpha(\alpha \in (0, 1])$ under the criterion of *mean* − *TVaR*, where $\lambda(\lambda \in [0, 1])$ is the degree of risk aversion. A larger λ indicates a higher degree of risk aversion for the newsvendor. α is the risk confidence level, which represents the newsvendor's preference for downside risk. When α equals 1, $TVaR_\alpha$ is equal to the expected profit.

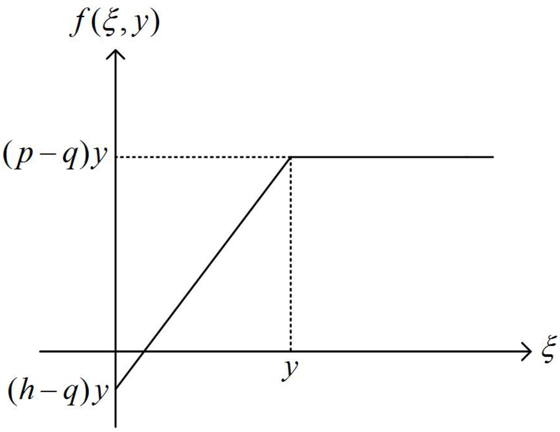

FIGURE 1. Profit as a function of uncertain variable ξ.

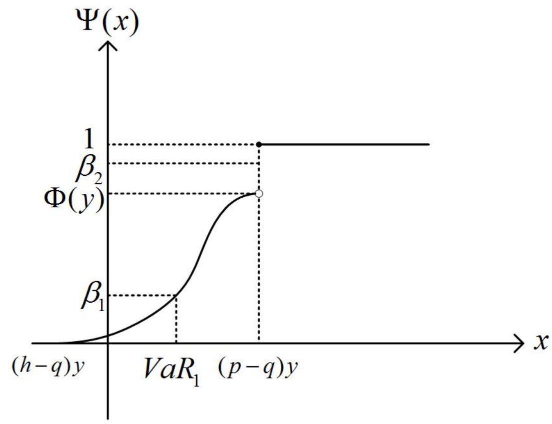

FIGURE 2. Belief degree distribution function of $f(\xi, y)$.

The objective is to find the optimal order quantity y^* to maximize $F(y)$; thus, $F(y^*) = \max_y F(y)$. We obtain the following results.

Theorem 3: Let $\Phi(\xi)$ and $\Phi^{1}(\xi)$ be the uncertainty distribution and corresponding inverse uncertainty distribution of ξ, where $\Phi(\xi)$ is continuous and invertible. At a given level of risk confidence $\alpha(\alpha \in (0, 1])$, we have

$$TVaR_\alpha(f(\xi, y)) = \begin{cases} \frac{1}{\alpha}\int_0^\alpha \left[(p-h)\Phi^{-1}(\beta) + (h-q)y\right] d\beta & \alpha < \Phi(y) \\ \frac{1}{\alpha}\left(\int_0^{\Phi(y)} \left[(p-h)\Phi^{-1}(\beta) + (h-q)y\right] d\beta \right. \\ \left. \quad + \int_{\Phi(y)}^{\alpha} (p-q)y d\beta \right) & \alpha \geq \Phi(y) \end{cases} \tag{13}$$

Theorem 4: Let $E[f(\xi, y)]$ be the expected profit of the newsvendor. Given a certain level of risk confidence $\alpha(\alpha \in (0, 1])$, we have

$$E[f(\xi, y)] = (p-h)\left(\int_{\frac{(q-h)y}{p-h}}^{y} (1-\Phi(x))dx - \int_0^{\frac{(q-h)y}{p-h}} \Phi(x)dx\right] \tag{14}$$

Hence, $TVaR_\alpha(f(\xi, y))$ and $E[f(\xi, y)]$ can be derived based on Theorem 3 and Theorem 4, respectively. Then, we can obtain the computational formula of $F(y)$ according to its definition as follows: $F(y) = \lambda TVaR_\alpha(f(\xi, y)) + (1-\lambda)E[f(\xi, y)]$. Then, we can derive the optimal order quantity y^*. The result is given below.

Theorem 5: At a given level of risk confidence $\alpha(\alpha \in (0, 1])$, the optimal order quantity y^* is (15), as shown at the bottom of this page.

Based on Theorem 5, we can conclude that the optimal order quantity varies according to the relationship between the degree of risk aversion λ of the newsvendor and the critical value λ^*, which is determined by the selling price, unit ordering cost, salvage value and risk confidence level.

IV. SENSITIVITY ANALYSIS

By further analyzing the mathematical expression of the optimal order quantity obtained in the above section, we can obtain the following sensitivity analysis of the influence of the parameters on the optimal ordering decision.

Corollary 1: Compared with a risk-neutral newsvendor, a risk-averse newsvendor has a lower optimal order quantity. Furthermore, the optimal order quantity decreases in the degree of risk aversion.

We can also obtain the following results from Corollary 1. A degree of risk aversion less than λ^* indicates that the newsvendor is more optimistic. In this case, the optimal order quantity y^* is independent of the selected risk confidence level α. Therefore, newsvendors with the same risk aversion attitude can have the same optimal ordering decision despite having different risk confidence levels. When the degree of risk aversion is greater than λ^*, the newsvendor is more pessimistic. Here, different risk confidence levels lead to different optimal order decisions, even though the degrees of risk aversion are identical. When the risk confidence level is

$$y^* = \begin{cases} \Phi^{-1}\left(\frac{(p-q)\alpha}{(p-h)\left[(1-\alpha)\lambda + \alpha\right]}\right) & \lambda \geq \lambda^* \\ \Phi^{-1}\left(\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}\right) & \lambda < \lambda^* \end{cases} \quad \left(\lambda^* = \frac{\alpha(p-h) + (q-p)}{(\alpha-1)(p-h)}\right) \tag{15}$$

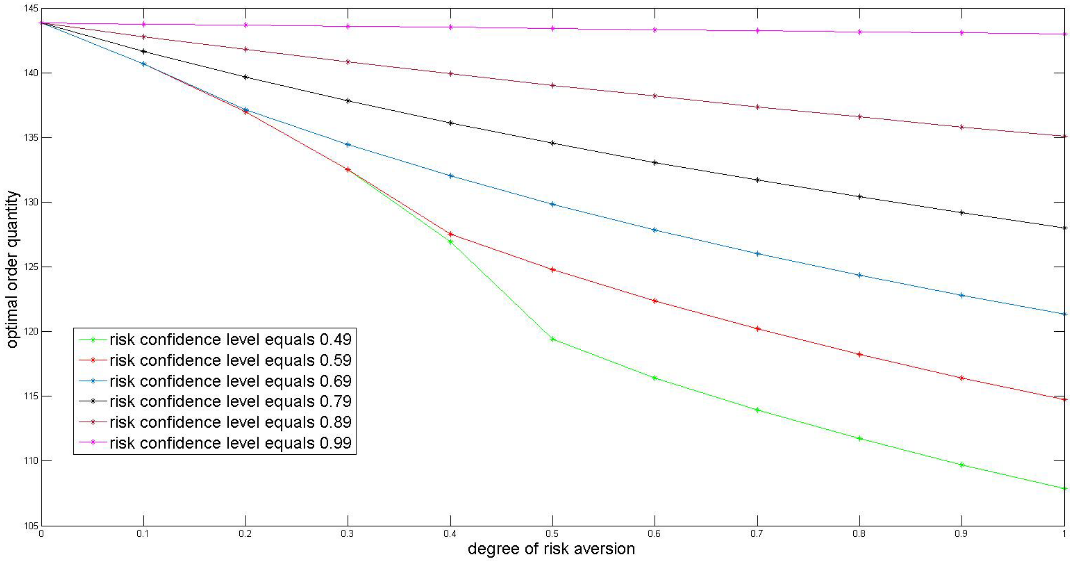

FIGURE 3. The optimal order quantities with different values of λ and α.

higher, i.e., the tolerance for risk is higher, the newsvendor is more optimistic and orders a greater quantity of product.

The reasons of Corollary 1 are as follows. When a newsvendor is risk averse, different degrees of risk aversion will lead to different ordering decisions. Both the expected profits and the downside risk of the profit are considered when the newsvendor makes ordering decisions. When the order quantity is small, the probability of overstocking is low; thus, all products can be sold with profit. In contrast, when the order quantity is large, a significant portion of the order quantity may be unsold and has to be salvaged at a lower price, resulting in a direct financial loss to the newsvendor. Hence, when a risk-averse newsvendor makes an ordering decision, s/he must predict the market demand and pay more attention to the direct losses caused by over-ordering. A risk-averse newsvendor tends to order fewer products to maximize the expected utility. In addition, the higher the degree of risk aversion, the lower the newsvendor's expectation of market demand, and the more negative impact of overstocking. As a result, the newsvendor will order less.

Corollary 2: At given levels of risk confidence and risk aversion, the optimal order quantity increases in the unit selling price and unit salvage value but decreases in the unit ordering cost.

Corollary 2 can help us decide the optimal order quantities of different products. When a newsvendor is risk averse, the ordering decision will be affected by the expected profit and risk simultaneously. The characteristics of the product can greatly influence the ordering decision. For example, a product with a high salvage value and low cost will result in greater profit and reduce the potential economic loss caused by inventory leftover.

V. NUMERICAL EXAMPLES

In this section, we conduct numerical examples to illustrate Corollary 1 and Corollary 2. We mainly discuss how the risk aversion coefficient, risk confidence level, unit selling price, unit ordering cost, and salvage value affect the ordering decision of the newsvendor. Suppose the market demand is $\xi \sim N\,(120, 40)$ and the uncertainty distribution and inverse uncertainty distribution are

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(120 - x)}{5\sqrt{3}}\right)\right)^{-1},$$
$$\Phi^{-1}(x) = 120 + \frac{5\sqrt{3}}{\pi}\ln\frac{x}{1-x} \quad (16)$$

VI. IMPACTS OF RISK AVERSION AND THE RISK CONFIDENCE LEVEL

By employing MATLAB2014a, we can obtain the value of the optimal order quantity y^* under different degrees of risk aversion λ and risk confidence levels α. The parameters we employ are $p = 23$, $q = 11.5$, and $h = 7.6$. The results by varying the values of λ and α are plotted in Figure 3.

The followings can be observed from Figure 3. First, the optimal order quantity decreases as λ increases, which is reasonable because as risk aversion increases, the newsvendor will be more concerned with to the downside risk of profit rather than the expected profit and choose to order less. The sensitivity of the optimal order quantity to the degree of risk aversion also varies as the risk confidence level changes. The higher the risk confidence level, the less risk aversion will affect the optimal order quantity. However, changes in λ do not always affect the newsvendor's optimal order quantity. When $\alpha \geq 0.99$, the optimal order quantity is rarely affected

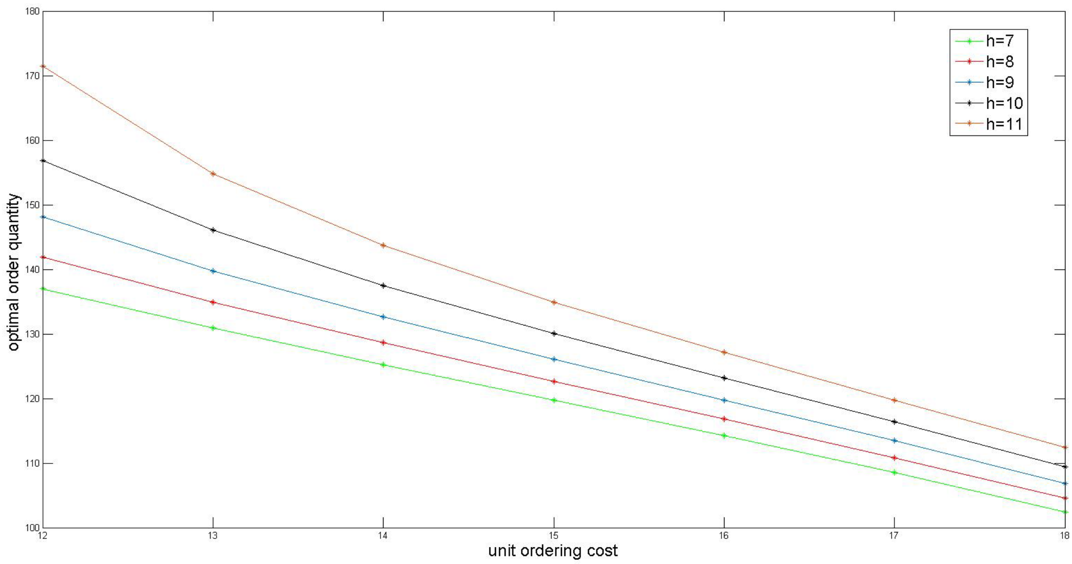

FIGURE 4. The optimal order quantities with different values of q and h.

by λ because as the risk confidence level approaches 1, $TVaR_\alpha$ approaches the expected profit. In this case, the impact of λ on the ordering decision is negligible. Second, the optimal order quantity will increase as α increases. The higher the risk confidence level is, indicating a higher newsvendor's acceptance of risk, the higher the expected profit. Thus, the newsvendor tends to order more. The degree of risk aversion also affects the sensitivity of the optimal order quantity to the risk confidence level. With a higher degree of risk aversion, the risk confidence level will have a stronger effect on the optimal order quantity. Stated differently, with a higher λ, α will have a stronger impact on the optimal order quantity. However, α does not always affect the newsvendor's optimal order quantity. When $\lambda < 0.3$, the risk confidence level has a limited impact on the optimal order quantity because when λ is low, the newsvendor pays more attention to the expected profit. Therefore, $TVaR_\alpha$ has minimal influence on decision making. When $\lambda = 0$, i.e., the newsvendor is risk-neutral, as shown in Figure 3, the optimal order quantity of a risk-averse newsvendor is always smaller than that of a risk-neutral newsvendor.

VII. IMPACTS OF THE UNIT ORDERING COST, UNIT SELLING PRICE, AND UNIT SALVAGE VALUE

In this subsection, we focus on how unit ordering cost q, unit selling price p, and unit salvage value h affect the optimal order quantity. Since the selling price affects the order quantity by influencing the profit margin of a risk-averse newsvendor as does the ordering cost, we focus only on the impact of q and h on the optimal order quantity here.

We assume that $\lambda = 0.55$, $\alpha = 0.99$, and $p = 23$. Figure 4 shows the changes in the optimal order quantity with different values of q and h. We can make the following observations from Figure 4. First, the optimal order quantity increases as h increases. This is because the higher the unit salvage value, the less the economic loss caused by overstocking the same amount of product. Therefore, the newsvendor will tend to order more items with a higher salvage value. Second, the optimal order quantity will decrease as q increases because the cost increase will reduce profit profitability; therefore, the expected profit from the sales of the same quantity of products will also decrease. In this case, the newsvendor will reduce the optimal order quantity in light of inventory risk concerns. Third, the change caused by q is greater than that caused by h, implying that the newsvendor is more sensitive to the unit ordering cost than the salvage value. This result holds because newsvendors are more concerned with the loss of product margins due to higher unit ordering cost than less overstock loss due to higher salvage values. In short, the lower the unit ordering cost and the higher the salvage value of the product, the higher the optimal order quantity.

VIII. DISCUSSION AND CONCLUSION

In this paper, we study a risk-averse newsvendor model based on uncertainty theory. Stated differently, we incorporate a newsvendor's risk aversion under the assumption that demand is an uncertain variable and analyze its optimal ordering decision. We formulate a utility function considering both risk and expected profit. We then obtain the optimal order quantity by maximizing the utility function. Numerical examples are further provided to verify our analytic results and illustrate the impacts of various parameters on the optimal order quantity.

In our study, we treat demand as an uncertain variable, which is a main contribution because existing studies usually assume that demand is a random or fuzzy variable. We are able to solve the newsvendor model in an uncertain environment, further broadening the applicability of this model in business practice where insufficient data is available. In our model, the estimation of the uncertain variable is given by experts with knowledge of uncertain statistics because insufficient observational data or historical data are available. In contrast, the use of random or fuzzy variables, which are typically assumed in the literature, is more appropriate for addressing uncertainty when related data are accessible. In short, our main contribution is that our model and results are more practical for inventory management with insufficient data.

Our main research results are as follows. First, compared with a risk-neutral newsvendor, a risk-averse newsvendor has a lower optimal order quantity. Furthermore, as the degree of risk aversion increases, the optimal order quantity decreases. Second, the sensitivity of the optimal order quantity to the degree of risk aversion depends on the risk confidence level. In particular, the optimal order quantity may be independent of the risk confidence level when the degree of risk aversion is below a threshold. Third, the optimal order quantity decreases in the unit ordering cost but increases in the salvage value and unit selling price.

In future research, this paper can be extended in the following directions. First, we could expand the single-item risk-averse newsvendor problem in the framework of uncertainty theory to scenarios involving multi-item newsvendors. Second, we could extend our research to study supply chain coordination with contracts, such as buy back contracts and revenue sharing contracts, in an uncertain environment. Finally, a multi-period dynamic inventory problem in an uncertain environment could be investigated.

APPENDIX

PROOF OF THEOREM 3

The profit function shows that when $\xi \le y, f(\xi,y) = (h-q)y + (p-h)\xi \le (p-q)y$, and $\xi > y$, we have $f(\xi,y) = (p-q)y$. Therefore, for any ξ, we have $f(\xi,y) \le (p-q)y$; thus, the belief degree of $f(\xi,y) \le (p-q)y$ is equal to 1 (i.e., $M\{f(\xi,y) \le (p-q)y\} = 1$). In addition, we have the following:

$$M\{f(\xi,y)=(p-q)y\}=M\{\xi > y\}=1-M\{\xi \le y\}=1-\Phi(y)$$

The belief degree distribution function $\Psi(x)$ of the profit function $f(\xi,y)$ is a step function and is continuous at the break point. Figure 2 plots $\Psi(x)$.

According to (9) and Figure 2, given a certain level of risk confidence $\beta(\beta \in (0,1])$, we have

$$\begin{aligned} VaR_\beta &= \inf\{x|M\{f(\xi,y) \ge x\} < 1-\beta\} \\ &= \inf\{x|M\{f(\xi,y) \le x\} > \beta\} \\ &= Sup\{x|M\{\{\xi \le y, (h-q)y + (p-h)\xi \le x\} \\ &\quad \cup \{\xi > y, (p-q)y \le x\}\} \le \beta\} \end{aligned}$$

(I) When $\beta < \Phi(y)$ (such as β_1 in Figure 2), the VaR corresponding to β is always less than $(p-q)y$, which corresponds to $\Phi(y)$, and VaR_β can be calculated as

$$\begin{aligned} VaR_\beta &= \inf\left\{x|M\left\{\left\{\xi \le y, \xi \le \frac{x+(q-h)y}{p-h}\right\}\cup\{\emptyset\}\right\} > \beta\right\} \\ &= \inf\left\{x|M\left\{\xi \le \frac{x+(q-h)y}{p-h}\right\} > \beta\right\} \end{aligned}$$

As $\Phi^1(\xi)$ is the inverse uncertainty distribution function of ξ, the equation above is equivalent to $\frac{VaR_\beta+(q-h)y}{p-h} = \Phi^1(\beta)$. Hence, $VaR_\beta = (p-h)\Phi^{-1}(\beta) + (h-q)y$.

(II) When $1 \ge \beta \ge \Phi(y)$ (such as β_2 in Figure 2), we obtain $VaR_\beta = (p-q)y$.

We substitute VaR_β into (10), yielding $TVaR_\alpha(f(\xi,y))$. Because VaR_β is a piecewise function, $TVaR_\alpha(f(\xi,y))$ is also a piecewise function that can be expressed as

$$TVaR_\alpha(f(\xi,y)) = \begin{cases} \frac{1}{\alpha}\int_0^\alpha \left[(p-h)\Phi^{-1}(\beta) + (h-q)y\right]d\beta & \alpha < \Phi(y) \\ \frac{1}{\alpha}\left(\int_0^{\Phi(y)} \left[(p-h)\Phi^{-1}(\beta) + (h-q)y\right]d\beta + \int_{\Phi(y)}^{\alpha} (p-q)y\,d\beta\right) & \alpha \ge \Phi(y) \end{cases}$$

Theorem 3 is proved. □

PROOF OF THEOREM 4

According to Qin and Kar [21], we have

$$E[f(\xi,y)] = (p-h)\left(\int_{\frac{(q-h)y}{p-h}}^{y} M\{\xi \ge x\}dx - \int_0^{\frac{(q-h)y}{p-h}} M\{\xi \le x\}dx\right)$$

Considering that $\Phi(x)$ is a continuous function that satisfies $\Phi(x) = M\{\xi \le x\}$ and $M\{\xi > x\} = 1-\Phi(x)$, we can obtain

$$E[f(\xi,y)] = (p-h)\left(\int_{\frac{(q-h)y}{p-h}}^{y} (1-\Phi(x))dx - \int_0^{\frac{(q-h)y}{p-h}} \Phi(x)dx]\right)$$

Theorem 4 is proved. □

PROOF OF THEOREM 5

Proof: We first take the derivative of $F(y)$ as follows:

$$\frac{dF(y)}{dy} = \lambda\frac{dCVaR}{dy} + (1-\lambda)\frac{d(E[f(\xi,y)])}{dy}.$$

Because $F(y)$ is a piecewise function, it must be discussed in segments. In addition, notably, $\lim_{y\to\Phi^{-1}(\alpha)} F(y) = F(\Phi^{-1}(\alpha))$; thus, $F(y)$ is continuous at its breakpoints.

Next, we discuss different situations according to the relationship between α and $\Phi(y)$.

Case 1: $\alpha < \Phi(y)$; in this case, $y > \Phi^{-1}(\alpha)$, and we have

$$\frac{dF(y)}{dy} = \lambda(h-q) + (1-\lambda)\left[(p-q)-(p-h)\Phi(y)\right],$$
$$\frac{d^2F(y)}{dy^2} = (1-\lambda)(h-p)\frac{d\Phi(y)}{dy}$$

Since $\Phi(y)$ is a continuously and monotonically increasing function, we can infer that $\frac{d\Phi(y)}{dy} \geq 0$. By combining $p > h$ and $\lambda \in [0,1]$, it can be verified that $\frac{d^2F(y)}{dy^2} \leq 0$, implying that $F(y)$ has a maximum value.

Letting $\frac{dF(y)}{dy} = 0$, we obtain y_1, which satisfies

$$\Phi(y_1) = \frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h},$$
$$y_1 = \Phi^{-1}\left(\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}\right)$$

Because the domain of $F(y)$ is $y > \Phi^{-1}(\alpha)$ in Case 1, we must consider the values of $F(y)$ at the boundaries of the domain to determine its maximum. Let

$$\lambda_1 = \frac{\alpha(p-h)+(q-p)}{(\alpha-1)(p-h)}.$$

(I) When $y_1 > \Phi^{-1}(\alpha)$, $\lambda < \lambda_1$. Thus, the domain of $F(y)$ contains y_1. $F(y)$ has a maximum at y_1; therefore, $y_1^* = y_1$ is the optimal solution.

(II) Conversely, when $\lambda \geq \lambda_1$, because $\frac{d\Phi(y)}{dy}$<0 and $F(y)$ is continuous at point $y = \Phi^1(\alpha)$, the maximum of $F(y)$ is at the left boundary of the domain. Hence, the optimal solution is $y_1^* = \Phi^1(\alpha)$.

Case 2: $\alpha \geq \Phi(y)$; in this case, $y \leq \Phi^{-1}(\alpha)$, and we have

$$\frac{dF(y)}{dy} = \frac{\lambda}{\alpha}(h-p)\Phi(y) + \lambda(p-q) + (1-\lambda)\left[(p-q)-(p-h)\Phi(y)\right],$$
$$\frac{d^2F(y)}{dy^2} = \left[\frac{h-p}{\alpha} * (\lambda+\alpha-\alpha\lambda)\right]\frac{d\Phi(y)}{dy}$$

Similarly, since $\frac{d^2F(y)}{dy^2} \leq 0$, $F(y)$ has a maximum in its domain. Letting $\frac{dF(y)}{dy} = 0$, we obtain y_2, which satisfies

$$\Phi(y_2) = \frac{(p-q)\alpha}{(p-h)(\alpha+\lambda-\alpha\lambda)},$$
$$y_2 = \Phi^{-1}\left(\frac{(p-q)\alpha}{(p-h)(\alpha+\lambda-\alpha\lambda)}\right)$$

Similar to Case 1, we consider the boundaries of the domain to determine the maximum of $F(y)$.

The domain of $F(y)$ is $y \leq \Phi^{-1}(\alpha)$ in Case 2. Let

$$\lambda_2 = \frac{p-q}{(p-h)(1-\alpha)} - \frac{\alpha}{1-\alpha}.$$

(I) When $y_2 \leq \Phi^{-1}(\alpha)$, $\lambda \geq \lambda_2$. Thus, the domain of $F(y)$ contains y_2, and $F(y)$ has its maximum at y_2. Thus, $y_2^* = y_2$.

(II) When $\lambda < \lambda_2$, we have $\frac{dF(y)}{dy} > 0$. In this case, the maximum of $F(y)$ occurs at the right boundary of the domain. Thus, the optimal solution is $y_2^* = \Phi^1(\alpha)$. Notably, $\lambda_1 = \lambda_2$ in this situation.

To obtain the optimal solution that satisfies $F(y^*) = \max_y F(y)$, we compare $F(y_1^*)$ and $F(y_2^*)$, which are derived from Cases 1 and 2.

(I) When $\lambda < \lambda_1$, we have $F(y_1^*) = F(y_1)$ and $F(y_2^*) = F(\Phi^{-1}(\alpha))$. Since $F(y)$ is continuous at $y = \Phi^1(\alpha)$, $F(y_1^*) = F(y_1) > F(\Phi^{-1}(\alpha)) = F(y_2^*)$ is true.

Thus, the optimal order quantity is $y^* = y_1^* = y_1$.

(II) When $\lambda \geq \lambda_1$, we obtain $F(y_2^*) = F(y_2) > F(\Phi^{-1}(\alpha)) = F(y_1^*)$, indicating that the optimal order quantity is $y^* = y_2^* = y_2$.

Theorem 5 is proved. □

PROOF OF COROLLARY 1

Proof: Based on Theorem 5, the optimal order quantity y^* can be described as

$$y^* = \begin{cases} \Phi^{-1}\left(\frac{(p-q)\alpha}{(p-h)\left[(1-\alpha)\lambda+\alpha\right]}\right) & \lambda \geq \lambda^* \\ \Phi^{-1}\left(\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}\right) & \lambda < \lambda^* \end{cases}$$

where $\lambda^* = \frac{\alpha(p-h)+(q-p)}{(\alpha-1)(p-h)}$. Setting $\lambda = 0$ and $\alpha = 1$, we can obtain the expected profit function of a risk-neutral newsvendor as follows:

$$E\left[f(\xi,y)\right] = (p-h)\left(\int_{\frac{(q-h)y}{p-h}}^{y}(1-\Phi(x))dx - \int_{0}^{\frac{(q-h)y}{p-h}}\Phi(x)dx]\right)$$

Considering the derivative of the equation above, we obtain the corresponding optimal order quantity $y' = \Phi^{-1}(p-q/p-h)$. Since we have $\lambda \in [0,1], \alpha \in (0,1]$, and $p > q > h > 0$, we can infer that $\frac{\lambda(h-q)}{(1-\lambda)(p-h)} \leq 0$. As $\alpha/[(1-\alpha)\lambda+\alpha] \leq 1$, we can obtain

$$\frac{\lambda}{1-\lambda} \times \frac{h-q}{p-h} + \frac{p-q}{p-h} \leq \frac{p-q}{p-h},$$
$$\frac{(p-q)\alpha}{(p-h)[(1-\alpha)\lambda+\alpha]} \leq \frac{p-q}{p-h}$$

Since $\Phi(x)$ and its inverse function $\Phi^1(x)$ are both monotonically increasing, we can obtain $y^* \leq y'$ for any $\lambda \in [0,1]$; thus, the optimal order quantity of a risk-averse newsvendor is less than that of a risk-neutral newsvendor. Next, we further analyze the impact of the degree of risk aversion λ on the optimal order quantity y^*.

(I) When $\lambda < \lambda^*$, we can obtain

$$y^* = \Phi^{-1}\left(\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}\right).$$

As $\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}$ is a decreasing function of λ and $\Phi^1(x)$ is an increasing function, y^* is decreasing with respect to λ; Therefore, an increase in λ will lead to a decrease in y^*.

(II) When $\lambda \geq \lambda^*$,

$$y^* = \Phi^{-1}\left(\frac{(p-q)\,\alpha}{(p-h)\,[(1-\alpha)\,\lambda+\alpha]}\right).$$

As $\frac{(p-q)\alpha}{(p-h)[(1-\alpha)\lambda+\alpha]}$ is a decreasing function of λ, $\Phi^1(x)$ is an increasing function; thus, y^* is decreasing with respect to λ. Hence, y^* decreases as λ increases.

In summary, a risk-averse newsvendor has a smaller optimal order quantity, and the optimal order quantity decreases as the degree of newsvendor risk aversion increases.

Corollary 1 is proved. □

PROOF OF COROLLARY 2

$\frac{\lambda}{1-\lambda} * \frac{h-q}{p-h} + \frac{p-q}{p-h}$ is an increasing function of p and h but a decreasing function of q as is $\frac{(p-q)\alpha}{(p-h)[(1-\alpha)\lambda+\alpha]}$. Additionally, $\Phi^1(x)$ is an increasing function. Therefore, y^* is always increasing with respect to p and h but is decreasing with respect to q.

Corollary 2 is proved. □

ACKNOWLEDGMENT

The authors are grateful for the valuable advice provided by Prof. B. D. Liu of Tsinghua University during the writing of the paper. Also, they would like to thank the anonymous reviewers and editor for their precious time and constructive comments for improvement of the original paper.

REFERENCES

[1] G. Gallego, "A minmax distribution free procedure for the (Q,R) inventory model," *Oper. Res. Lett.*, vol. 11, pp. 55–60, 1992.
[2] M. Khouja, "The single-period (newsvendor) problem: Literature review and suggestions for future research," *Omega*, vol. 27, pp. 537–553, 1999.
[3] Y. Qin, R. Wang, A. J. Vakharia, Y. Chen, and M. M. H. Seref, "The newsvendor problem: Review and directions for future research," *Eur. J. Oper. Res.*, vol. 213, pp. 361–374, Sep. 2011.
[4] F. J. Arcelus, S. Kumar, and G. Srinivasan, "Bicriteria optimization in the newsvendor problem with uniformly distributed demand," *4OR*, vol. 10, pp. 267–286, 2012.
[5] M. Dominey and R. Hill, "Performance of approximations for compound Poisson distributed demand in the newsvendor problem," *Int. J. Prod. Econ.*, vol. 92, pp. 145–155, 2004.
[6] A. Kitaeva, A. Zhukovskaya, and O. Zmeev, "Compound Poisson demand with price-dependent intensity for fast moving items: Price optimisation and parameters estimation," *Int. J. Prod. Res.*, vol. 55, pp. 4153–4163, 2017.
[7] J. Yu and D. Zhu, "Study on the selection strategy of supply chain financing modes based on the retailer's trade grade," *Sustainability.*, vol. 10, p. 3045, 2018.
[8] J. Kamburowski, "The distribution-free newsboy problem under the worst-case and best-case scenarios," *Eur. J. Oper. Res.*, vol. 237, pp. 106–112, Aug. 2014.
[9] H. Dayi and C. Danfeng, "Does decision-maker's intelligence matter? Evidences from distribution-free newsvendor problem," *IEEE Access*, vol. 7, pp. 18101–18112, 2019, doi: 10.1109/ACCESS.2019.2896589.
[10] G. Godfrey and W. Powell, "An adaptive distribution-free algorithm for the newsvendor problem with censored demands with applications to inventory and distribution," *Manage. Sci.*, vol. 47, pp. 1101–1112, 2001.
[11] G. W. Helena, "Newsvendor problem under complete uncertainty: A case of innovative products," *Central Eur. J. Oper. Res.*, vol. 25, no. 3, pp. 561–585, 2017.
[12] B. Sarkar, C. Zhang, and A. Majumder, "A distribution free newsvendor model with consignment policy and retailer's royalty reduction," *Int. J. Prod. Res.*, vol. 56, pp. 5025–5044, 2018.
[13] S. Chen and Y. Ho, "Analysis of the newsboy problem with fuzzy demands and incremental discounts," *Int. J. Prod. Econ.*, vol. 129, pp. 169–177, 2011.
[14] Y. Xu and J. Hu, "Random fuzzy demand newsboy problem," *Phys. Procedia*, vol. 25, pp. 924–931, 2012, doi: 10.1016/j.phpro.2012.03.179.
[15] Y. Yu, J. Zhu, and C. Wang, "A newsvendor model with fuzzy price-dependent demand," *Appl. Math. Model.*, vol. 37, pp. 2644–2661, 2013.
[16] B. Liu, "Why is there a need for uncertainty theory," *J. Uncertain Syst.*, vol. 6, no. 1, pp. 3–10, Feb. 2012.
[17] B. Liu, *Uncertainty Theory*, vol. 154, 2nd ed. Berlin, Germany: Springer-Verlag, 2007, pp. 1–79.
[18] X. Zhang and X. Chen, "A new uncertain programming model for project scheduling problem," *Inf., Int. Interdiscipl. J.*, vol. 15, pp. 3901–3910, 2012.
[19] Y. Gao, "Uncertain models for single facility location problems on networks," *Appl. Math. Model.*, vol. 36, pp. 2592–2599, 2012.
[20] Z. Qin, S. Kar, and H. Zheng, "Uncertain portfolio adjusting model using semiabsolute deviation," *Soft Comput.*, vol. 20, pp. 717–725, 2016.
[21] Z. Qin and S. Kar, "Single-period inventory problem under uncertain environment," *Appl. Math. Comput.*, vol. 219, pp. 9630–9638, 2013.
[22] S. Ding, "Uncertain multi-product newsboy problem with chance constraint," *Appl. Math. Comput.*, vol. 223, pp. 139–146, 2013, doi: 10.1016/j.amc.2013.07.083.
[23] S. Ding and Y. Gao, "The (σ, S) policy for uncertain multi-product newsvendor problem," *Expert Syst. Appl.*, vol. 41, pp. 3769–3776, 2014.
[24] S. Ding, "Uncertain random newsvendor problem," *J. Intell. Fuzzy Syst.*, vol. 26, pp. 483–490, 2014.
[25] D. Wang, Z. Qin, and S. Kar, "A novel single-period inventory problem with uncertain random demand and its application," *Appl. Math. Comput.*, vol. 269, pp. 133–145, 2015, doi: 10.1016/j.amc.2015.06.102.
[26] M. E. Schweitzer and G. P. Cachon, "Decision bias in the newsvendor problem with a known demand distribution: Experimental evidence," *Manage. Sci.*, vol. 46, no. 3, pp. 404–420, 2000.
[27] C. Tang and A. Brown, "The impact of alternative performance measures on single-period inventory policy," *J. Ind. Manage. Optim.*, vol. 2, pp. 297–318, 2017.
[28] C. X. Wang, S. Webster, and N. C. Suresh, "Would a risk-averse newsvendor order less at a higher selling price?" *Eur. J. Oper. Res.*, vol. 196, pp. 544–553, Jul. 2009.
[29] M. Wu, S. Zhu, and R. Teunter, "The risk-averse newsvendor problem with random capacity," *Eur. J. Oper. Res.*, vol. 231, pp. 328–336, 2013.
[30] U. Murarka, V. Sinha, L. Thakur, and M. Tiwari, "Multiple criteria risk averse model for multi-product newsvendor problem using conditional value at risk constraints," *Inf. Sci.*, vol. 478, pp. 595–605, 2019.
[31] M. Wu, S. X. Zhu, and R. H. Teunter, "A risk-averse competitive newsvendor problem under the CVaR criterion," *Int. J. Prod. Econ.*, vol. 156, no. 5, pp. 13–23, 2014.
[32] M. A. M. Abdel-Aal and S. Z. Selim, "Risk-averse multi-product selective newsvendor problem with different market entry scenarios under CVaR criterion," *Comput. Ind. Eng.*, vol. 103, pp. 250–261, 2017.
[33] X. Xu, Z. Meng, P. Ji, C. Dang, and H. Wang, "On the newsvendor model with conditional Value-at-Risk of opportunity loss," *Int. J. Prod. Res.*, vol. 54, pp. 2449–2458, 2016.
[34] R. Rockafellar and S. Uryasev, "Conditional value-at-risk for general loss distributions," *J. Banking Finance*, vol. 26, no. 7, pp. 1443–1471, Jul. 2002.
[35] W. Yang, M. Clutter, B. Mei, and J. Siry, "Assessing the role of U.S. timberland assets in a mixed portfolio under the mean-conditional value at risk framework," *Forest Policy Econ.*, vol. 50, pp. 118–126, 2015.
[36] Y. Xie, H. Wang, and H. Lu, "Coordination of supply chains with a retailer under the mean-CVaR criterion," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 7, pp. 1039–1053, Jul. 2018.
[37] M. Xu and J. Li, "Optimal decisions when balancing expected profit and conditional value-at-risk in newsvendor models," *J. Syst. Sci. Complex.*, vol. 23, no. 6, pp. 1054–1070, 2010.
[38] F. Gao, F. Chen, and X. Chao, "Joint optimal ordering and weather hedging decisions: Mean-CVaR model," *Flexible Services Manuf. J.*, vol. 23, no. 1, pp. 1–25, 2011.
[39] J. Peng, "Risk metrics of loss function for uncertain system," *Fuzzy Optim. Decis. Making*, vol. 12, no. 1, pp. 53–64, Mar. 2013.
[40] B. Liu, *Uncertainty Theory—A Branch of Mathematics for Modeling Human Uncertainty*. Berlin, Germany: Springer-Verlag, 2010, doi: 10.1007/978-3-642-13959-8.
[41] Y. H. Liu and M. Ha, "Expected value of function of uncertain variables," *J. Uncertain Syst.*, vol. 4, no. 3, pp. 181–186, Sep. 2010.

SHENGZHONG ZHANG received the B.A. degree in accounting, and the M.S. and Ph.D. degrees in transportation planning and management from Chang'an University, Xi'an, China, in 2001, 2003, and 2006, respectively.

He is currently the Dean of the China-Korea Institute of Logistics, and also a Professor with the School of Economics and Management, Chang'an University. His research interests include supply chain management, risk management and decision-making behavior under uncertainty, and so on. He has published more than 40 articles. His research has been supported by the National Natural Science Foundation of China and the Program for New Century Excellent Talents (NCET) supported by the Ministry of Education of China.

Dr. Zhang serves as the Vice-President of the Shaanxi Logistics Society and the Deputy Secretary-General of Transportation and Logistics Branch of CHTS.

YINGMIN YU received the B.A. degree in logistics management from Chang'an University, Xi'an, in 2017, where she is currently pursuing the M.S. degree in logistics engineering and management with the School of Economics and Management. Her research interests include decision-making behavior under uncertainty and green supply chain management.

JIDONG LI received the B.A. degree in information management from Chang'an University, Xi'an, in 2015, and the M.S. degree in logistics engineering and management with the School of Economics and Management, Chang'an University, in 2018. He is currently a Supply Chain Engineer with Huawei. His research interests include decision-making behavior under uncertainty and inventory management.

QIHONG ZHU received the Ph.D. degree from the Antai College of Economics and Management, Shanghai Jiao Tong University, China, in 2009. She is currently a Lecturer with the School of Business Administration, Shanghai Lixin University of Accounting and Finance, and currently a Visiting Scholar with the Anderson Management School, University of California, Riverside, CA, USA. Her research interests include differentiation, technological innovation, and decision-making behavior under uncertainty.