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Simultaneous Allocation of Multi-Type Distributed Generations and Capacitors Using Generic Analytical Expressions

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ABSTRACT This paper proposes a method for determining the optimal sites and sizes of multi-type distributed generations (DG) and capacitors for minimizing reactive power losses (RPL) in distribution systems. The proposed method is developed based on generic closed-form analytical expressions for calculating optimal sizes of DG units and capacitors at their candidate sites. The reduction in RPL with DG and capacitors is evaluated using another analytical expression that relates power injections of DG and capacitors with RPL. An optimal power flow algorithm (OPF) is incorporated in the proposed method to consider the constraints of the distribution systems, DG, and capacitors. Various types of DG are considered, and their optimal power factors can be accurately computed while optimizing the sizes of capacitors in a simultaneous manner to reduce RPL. The 69-bus distribution system is used to test the proposed method. An exact search method is employed to verify the accuracy of the proposed method. The effectiveness of the proposed method is demonstrated for solving the optimal allocation problem with different combinations of multi-type DG units and capacitors.

INDEX TERMS Distribution systems, distributed generations, capacitors, optimal power flow, reactive power.

I. INTRODUCTION

Sufficient reactive power supply in electrical power systems plays an important role in maintaining proper reliability and security. Voltage stability is greatly affected by the ability of power systems to efficiently supply reactive power from the allocated reactive power sources. Indeed, reactive power shortage (RPS) can cause several technical problems and lead to system blackout [1]–[4]. High reactive power demand and excessive reactive power losses are the key reasons for voltage collapse. The locations, capacities, and control schemes of the different sources of reactive powers significantly affect the ability of the distribution system to effectively respond to the critical conditions.

A proper distribution strategy of reactive power sources in power systems can greatly help to compensate RPS during heavy loading conditions. It is the responsibility of system operators and planners to ensure an adequate supply of reactive power by effective placement of reactive power sources in power systems. In the distribution system level, the reactive power sources can include capacitors and different types of distributed generation (DG). These units have great impacts on several indices of distribution systems, such as voltage profile, power flow, losses, and voltage stability. DG can be classified into conventional sources (e.g., diesel engines) and renewable energy resources (e.g., photovoltaic, wind, etc.). According to their output power characteristics, DG can be sources or sinks of reactive power. For instance, induction-based wind DG units need reactive power [5], [6] while photovoltaic units have the ability to absorb/release reactive power for controlling voltages [7]–[9]. Capacitors are also common devices for reactive power compensation which are distributed in distribution systems. A coordinated control strategy of the various reactive power sources can greatly maximize the benefits to distribution systems while alleviating operational problems with high DG penetrations [10]–[13].

Driven by increasing the penetration and types of DG technologies in distribution systems, the optimal placement of these units as well as capacitors becomes a significant

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subject to be studied. The optimal sizes and proper locations, and the best types of DG are required to be computed while optimizing capacitors. Since the optimal placement of these units has numerous alternative solutions due to various discrete and continuous variables, solving this optimization problem requires enormous computational efforts to determine the global optimal solution. In the literature, different methods are presented for the optimal placement of DG [14]–[17], and others are proposed for optimal placement of capacitors [18], [19]. It will be more beneficial for the distribution system to simultaneously find the optimal mix of the different reactive power sources, i.e., DG and capacitor units, for a sufficient reactive power supply. Recently, great interest is directed to the simultaneous placement of DG and capacitors in distribution systems [20]–[23]. The minimization of the reactive power losses is considered in the placement of such units in [19], [22]–[24] due to its positive impacts on distribution systems. Several existing approaches, e.g. [23], [25]–[27], allocates multiple DG units with equal or unity power factors. However, the reactive power of DG units can greatly improve the performance of distribution systems. Based on this fact, the recent revised IEEE 1547 Standard [28] regulates the use of DG reactive power capability for voltage support. In this work, the proposed method is directed to calculate the optimal power factors of multi-type DG while investigating its positive impacts on distribution systems.

In this work, a new method for simultaneously determining the optimal mix of various multi-type DG technologies and capacitors is proposed. Unlike the methods in the literature, this paper provides generic closed-form analytical expressions for the optimal sizing of multiple DG and capacitors to minimize reactive power losses (RPL). The optimal sizing of any combination of DG and capacitors can be calculated directly by the proposed analytical expressions. These analytical expressions are formulated in matrix forms whereas the dimensions of the matrices depend on the number of the units to be placed and the eligible buses for the installation. These analytical expressions can be effectively employed for determining the optimal sizing for all combinations of sites, thereby assign the optimal locations. Furthermore, in the literature, many papers assume that power factors of DG units are equal and specified, while the proposed method can calculate the optimal power factors of the DG units with capacitors, and thus, it effectively compensates RPS in distribution systems. An optimal power flow (OPF) algorithm is combined with the analytical expressions to allocate various units without violating the constraints.

The major contributions of this paper can be itemized as follows:

- Generic analytical expressions are proposed for optimally allocating DGs and capacitors for RPL minimization.
- Three DG types are modeled and their active and reactive power generations are optimized.
- The proposed allocation method can accurately determine the optimal mix of multi-type DGs and capacitors.
- Intensive simulations are performed, including 1) investigating the impacts of the DG power factor on loss minimization at different levels of RPS, and 2) quantifying the impacts of DGs and capacitors on active power losses, voltage profiles, and maximum system load-ability.

The remainder of this paper contains five sections, organized as follows. The placement problem of DG and capacitors is described in section 2. In section 3, a new set of analytical expressions is formulated for the optimal sizing of the units. The proposed method is given in Section 4. The results and conclusions are presented in Sections 5 and 6, respectively.

II. PROBLEM DESCRIPTION

A. RPS IN DISTRIBUTION SYSTEMS

The management of reactive power is a key subject in transmission and distribution levels to avoid RPS and support voltage. In the distribution system level, the reactive power must be properly compensated locally in the downstream distribution networks. RPS causes voltage fall which can lead to equipment malfunctions while rising reactive power generation rises voltage level. RPS can be the reason for the blackout of the entire system as has happened in some nations [29]. Since there is a specified allowed range of the operating voltage for consumer loads in distribution systems, usually within ±5% of the rated voltage [30], voltage rise/drop cause equipment malfunctions. The contribution of capacitors in the case of voltage rise/drop does not have a valuable effect, as the output from them is proportional to terminal voltage square. Some DG technologies with their ability to produce reactive power can have a positive impact on RPS in distribution systems. These technologies could be interfaced by power electronics (e.g., PV, micro turbines) or natural sources of reactive power (e.g., synchronous machines, doubly fed induction generators) and thereby can contribute to supply reactive power for supporting voltages. The optimal placement (i.e., the effective locations, the optimal capacities, and even the best mix) of these different sources of reactive powers can obviously enhance system stability.

B. DG AND CAPACITOR MODELS

DGs can be classified into three different models: 1) Unspecified active power (UAP), 2) Unspecified reactive power (URP), and 3) Unspecified active power (UARP) DG models. The active generated power of the UAP DG model is not specified, and it is needed to be optimally calculated. Unlike UAP, the variable in the URP DG model is the reactive power generation, not the active power generation. The UARP DG model has two variables required to be computed (active and reactive power generation). On the other hand, the capacitor model involves one variable, which is reactive power generation, while the active power generation is equal to

TABLE 1. DG and capacitor models.

Unit type	Active Power Generation	Reactive Power Generation
UAP DG	$P_g^{Min} \le P_g \le P_g^{Max}$	$Q_g = Q_g^{Spec}$
URP DG	$P_g = P_g^{Spec}$	$Q_g^{Min} \le Q_g \le Q_g^{Max}$
UARP DG	$P_g^{Min} \le P_g \le P_g^{Max}$	$Q_g^{Min} \le Q_g \le Q_g^{Max}$
Capacitor	$P_g = 0$	$Q_g^{Min} \le Q_g \le Q_g^{Max}$

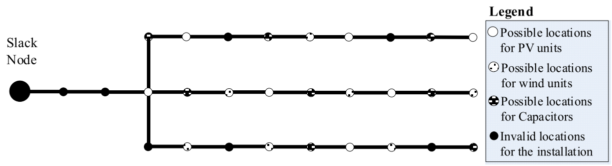

FIGURE 1. Example of a distribution system.

zero. The mathematical representations of these units are summarized in Table I.

C. POSSIBLE COMBINATIONS OF VALID SITES

The optimal allocation of DGs and capacitors is a complex optimization problem due to the nonlinearity of distribution systems and a large number of possible solutions (i.e., locations and sizes of DG and capacitors). Fig. 1 shows an example of a radial distribution system in which different components (e.g., PV unit, wind unit, capacitors) are required to be allocated at their recommended locations, i.e., buses. These recommended locations for a specified type of these units can be defined based on several factors, such as the distribution of fuel sources, strategies of investors, and meteorological conditions (for renewable DG). The aim of the optimal placement of DGs and capacitors is to determine the best set of locations for these units among the recommendation locations. For example, consider that it is required to install a number of N_{DG} DG units and a number of N_C capacitors to a distribution system. This distribution system includes N_B buses that are eligible locations for installing DGs and capacitors. Each unit of type i can be installed only in their recommended buses N_{Bi}, and so the following general formulae can be written:

$$N_{DG} = \sum_{i=1}^{N_{DGT}} N_{DGi}; \quad N_B = \sum_{i=1}^{N_{DGT}} N_{Bi} \tag{1}$$

where N_{DGT} represents the total number of multi-type DG units that are required to be allocated, and N_{DGi} is required DG number of type i to be installed in the distribution system. The number of all possible combinations for locations of multi-type units can be calculated by:

$$N_{Com} = \left(\prod_{i=1}^{N_{DGT}} C_{N_{DGi}}^{N_{Bi}}\right)\left(C_{N_{Cab}}^{N_{Capi}}\right)((N_{DG} + N_{Cab})!) \tag{2}$$

It is worth to mention that the number of combinations is huge, especially in the case of allocating multi-type DGs in a large distribution system. Therefore, a fast and accurate method is needed to assess all of these combinations, thereby determining the best combination.

III. PROPOSED ANALYTICAL EXPRESSIONS WITH DG AND CAPACITORS

A. REACTIVE LOSSES WITH DG AND CAPACITORS

To effectively compensate RPS in distribution systems, the total reactive power losses through the distribution lines must be minimized. The total reactive power losses Q_{loss} can be expressed as follows:

$$Q_{loss} = \sum_{j\in\varphi} \frac{X_j}{V_j^2}\left(P_j^2 + Q_j^2\right) \tag{3}$$

where P_j and Q_j are the active and reactive power flows in the distribution branch j, respectively, at the base case (without DG and capacitor). φ isa set of system branches, V_j is voltage magnitude of the receiving bus of the branch j, and X_j is the reactance of the branch j.

In the case of adding a DG unit or a capacitor, which injects P_g and/or Q_g, at particular buses, the variation in the total reactive losses can be directly calculated by (4). Where α represents a set of the branches in which power flows are not affected after adding the units, and β represents a set of the branches in which power flows are affected after adding the units.

$$Q_{loss,DG} = \sum_{j\in\alpha} \frac{X_j}{V_j^2}\left(P_j^2 + Q_j^2\right) + \sum_{j\in\beta} \frac{X_j}{V_j^2}\left((P_j - P_g)^2 + (Q_j - Q_g)^2\right), \quad \alpha \cup \beta = \varphi \tag{4}$$

The latter equation can be rewritten in a general form to express the impact of installing multiple DG units or capacitors at a set of locations ψ on the total reactive power losses, as follows

$$Q_{loss,DG} = \sum_{j\in\alpha} \frac{X_j}{V_j^2}\left(P_j^2 + Q_j^2\right) + \sum_{j\in\beta} \frac{X_j}{V_j^2}\left(\left(P_j - \sum_{i\in\psi}\Omega_{ij}P_{gi}\right)^2 + \left(Q_j - \sum_{i\in\psi}\Omega_{ij}Q_{gi}\right)^2\right) \tag{5}$$

The Ω matrix is constructed based on the fact that each bus has only one direct path to the reference bus in radial

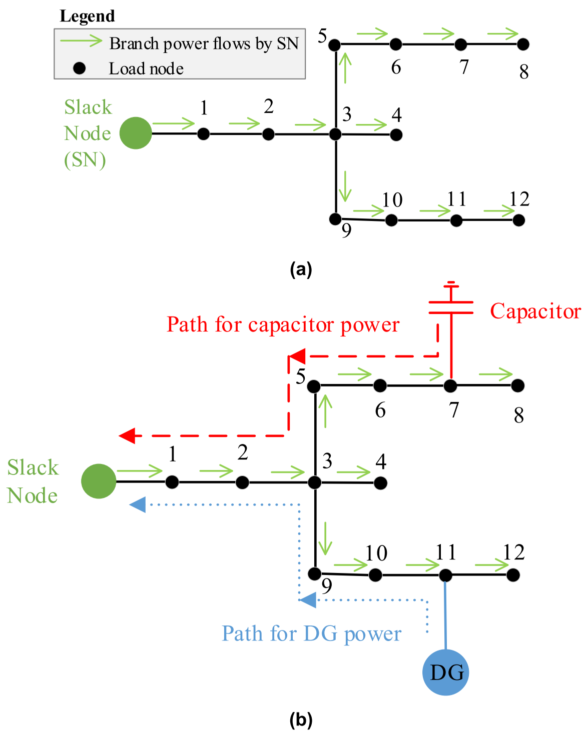

FIGURE 2. Power flow variation when adding DG and capacitor units to a distribution system. a) Power flows before adding DG and capacitor units, and b) Power flows after adding DG and capacitor units.

distribution systems. To illustrate this concept, we describe the power flow variation when adding DG and capacitor units to a distribution system shown in Fig. 2. Fig. 2(a) shows the power flow at the base condition, i.e. without DG/capacitor, whereas the power is flowing from the slack node (SN) through the branches to loads. When adding DG and capacitor units (Fig. 2(b)), as the load powers are constant, all additional generated power afforded by DG or capacitor must flow to the slack node which is the only flexible node to inject/absorb power in the system. For instance, the load at node 4 is constant, and so the power flow through the 3-4 line is constant, and it is not a function of DG or capacitor powers. Therefore, the power flows in only upper-stream branches will be affected by the installed unit, complying with the superposition theorem. Note that the Ω matrix has N_B columns and $(N_{DG} + N_C)$ rows. For instance, the matrix Ω for the distribution system shown in Fig. 2 in the case of adding DG and capacitor units at buses 11 and 7, respectively, is expressed as follows

$$\Omega = \overbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}}^{\text{System buses: } ①②③④⑤⑥⑦⑧⑨⑩⑪⑫} \begin{matrix} ⑦ \text{ Cap. bus} \\ ⑪ \text{ DG bus} \end{matrix} \tag{6}$$

Equations (7) and (8) are introduced in order to incorporate the power factor of DG (PF_g) in expressing the reactive power losses with DG.

$$Q_{gi} = M_{gi} P_{gi} \tag{7}$$

where

$$M_{gi} = \frac{\sqrt{1 - PF_{gi}^2}}{PF_{gi}} \tag{8}$$

B. ANALYTICAL EXPRESSIONS FOR SIZING DG AND CAPACITORS

The optimal placement of DG and capacitors aims at effectively compensating RPS in distribution systems. This objective can be achieved by determining the optimal locations and sizes of the units to minimize the total reactive power losses. Since the reactive power losses can be represented by the proposed equation (5), the objective function of the placement problem is expressed as the minimization of $Q_{loss,DG}$. The state variables in this optimization problem are the active and reactive powers of the units (P_g, Q_g). Equations (9) and (10) can be written by considering the fact that the variations of $Q_{loss,DG}$ with P_g and Q_g are zero at the minimum point.

$$\frac{\partial Q_{loss,DG}}{\partial P_{gm}} = 0, \quad \forall m \in \Psi \tag{9}$$

$$\frac{\partial Q_{loss,DG}}{\partial Q_{gm}} = \frac{\partial Q_{loss,DG}}{\partial P_{gm}}, \quad \forall m \in \Psi \tag{10}$$

The two latter equations are expressed for each DG/capacitor unit; therefore, the number of equations is twice the number of units. Equations (9) and (10) can be rearranged in matrix forms expressed by (11) and (12), respectively, as follows

$$\begin{bmatrix} P_{g\,\Psi_1} \\ P_{g\,\Psi_2} \\ \vdots \\ P_{g\,\Psi_N} \end{bmatrix} = \begin{bmatrix} A_{\Psi_1,\Psi_1} & A_{\Psi_1,\Psi_2} & \cdots & A_{\Psi_1,\Psi_3} \\ A_{\Psi_2,\Psi_1} & A_{\Psi_2,\Psi_2} & \cdots & A_{\Psi_2,\Psi_N} \\ \vdots & \vdots & \vdots & \vdots \\ A_{\Psi_N,\Psi_1} & A_{\Psi_N,\Psi_2} & \cdots & A_{\Psi_N,\Psi_N} \end{bmatrix}^{-1} \begin{bmatrix} B_{\Psi_1} \\ B_{\Psi_2} \\ \vdots \\ B_{\Psi_N} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} Q_{g\,\Psi_1} \\ Q_{g\,\Psi_2} \\ \vdots \\ Q_{g\,\Psi_N} \end{bmatrix} = \begin{bmatrix} P_{g\,\Psi_1} \\ P_{g\,\Psi_2} \\ \vdots \\ P_{g\,\Psi_N} \end{bmatrix} - \begin{bmatrix} C_{\Psi_1,\Psi_1} & C_{\Psi_1,\Psi_2} & \cdots & C_{\Psi_1,\Psi_3} \\ C_{\Psi_2,\Psi_1} & C_{\Psi_2,\Psi_2} & \cdots & C_{\Psi_2,\Psi_N} \\ \vdots & \vdots & \vdots & \vdots \\ C_{\Psi_N,\Psi_1} & C_{\Psi_N,\Psi_2} & \cdots & C_{\Psi_N,\Psi_N} \end{bmatrix}^{-1} \times \begin{bmatrix} D_{\Psi_1} \\ D_{\Psi_2} \\ \vdots \\ D_{\Psi_N} \end{bmatrix} \tag{12}$$

in which

$$A_{n,m} = \sum_{j \in \beta} \Omega_{nj} \Omega_{mj} \left(1 + M_{DGm} M_{DGn}\right) \frac{X_j}{V_j^2},$$

$$B_m = \sum_{j \in \beta} \Omega_{mj} \left(P_j + M_{DGm} Q_j\right) \frac{X_j}{V_j^2},$$

$$C_{n,m} = \sum_{j \in \beta} \Omega_{nj} \Omega_{mj} \frac{X_j}{V_j^2},$$

$$D_m = \sum_{j \in \beta} \Omega_{mj} \left(P_j - Q_j\right) \frac{X_j}{V_j^2}$$

where the dimensions of **A** and **C** matrices are $(N_{DG}+N_C)\times(N_{DG}+N_C)$, and the lengths of **D** and **B** are $(N_{DG}+N_C)$. These four matrices can be calculated directly from the steady-state results of the base case condition computed by power flow methods. For each combination of valid sites to install DG and capacitors, equations (11) and/or (12) are used to calculate their corresponding optimal sizes (P_g and Q_g). Note that the model of each unit type illustrated in Table I must be considered for computing P_g and/or Q_g. For instance, the optimal sizing of capacitors requires computing only Q_g using (12) while P_g is equal to zero. Regarding the UAP DG type, its optimal active power generation is calculated by equations (11), and its reactive power generation is equal to a specified value (Q^{Spec}). Unlike the UAP DG type, the URP DG type requires the calculation of reactive power generation only by (12), and its active power generation is equal to a specified value (P^{Spec}). The only unit type for which both (11) and (12) are utilized is UARP DG since it allows optimizing both active and reactive power generations.

IV. THE PROPOSED METHOD

The proposed method combines the proposed analytical expressions and an OPF algorithm to accurately solve the placement problem of DG and capacitors. The main reason for employing OPF is to incorporate equality and inequality constraints in the optimization model. As the proposed analytical expressions are expressed in general forms, and can optimally solve the placement problem with any combination of DG/capacitor sites, these expressions are helpful to assess all combinations of sites. This assessment process is important to define the best combination of sites of DG and capacitors as well as their optimal sizes in a short time. To do so, equations (11) and (12) are used to determine the optimal value of P_g and Q_g for all possible combinations of sites, and the corresponding reactive power losses are evaluated using (5).

It is worth to mention that the computation burden of the evaluation process is reduced when using the proposed analytical expressions compared with the search-based methods. The OPF formulation applies the constraints of the system, DG and capacitors and slightly corrects the unit size calculated by the analytical expressions to the exact optimal solution. Fig. 3 shows the solution steps of the proposed method. As illustrated in the figure, the analytical expressions are required to calculate optimal sizes and locations of the units for all possible sites, whereas OPF is performed once to apply the constraints. This hybrid formulation is efficient as the better combination of sites among all the possible combinations is determined by the proposed analytical expressions while considering the constraints by OPF.

V. RESULT AND DISCUSSIONS

The 69-bus radial distribution system (Fig. 4) is used to evaluate the performance of the proposed method for solving the placement problem of DG and capacitors [31]. This system contains 68 load buses and a reference bus. The total

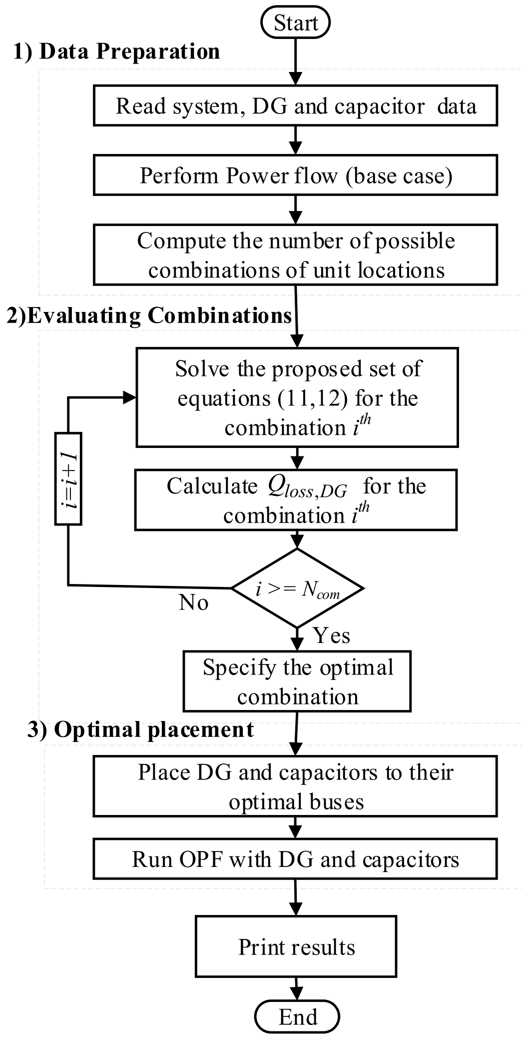

FIGURE 3. Flowchart of the proposed method.

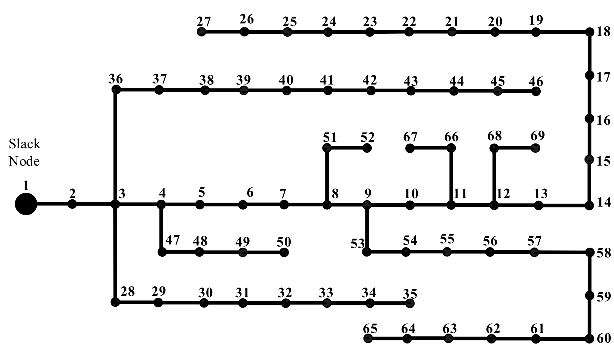

FIGURE 4. 69-bus distribution system.

active and reactive power losses without DG and capacitors are 0.225 MW and 0.102 Mvar, respectively. The proposed method is programmed in the C++ programming language.

A. ASSUMPTIONS

- Only one unit is allowed to be placed in each bus;
- The power factors of UAP and UARP DG units are 1.0 and 0.9 lagging, respectively;

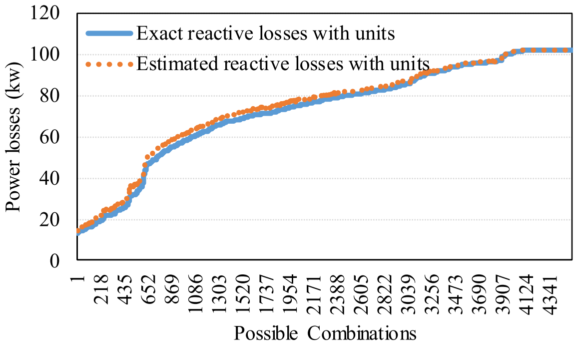

FIGURE 5. The computed reactive losses for installing one DG and one capacitor.

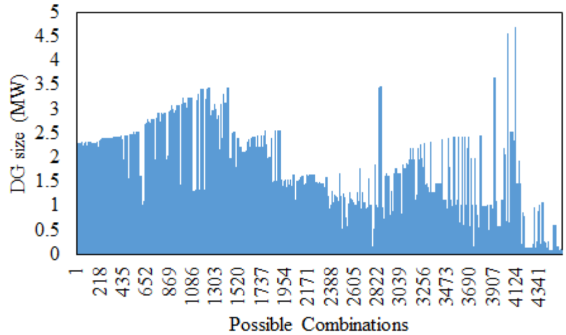

FIGURE 6. The calculated optimal DG size for all possible combinations of DG and capacitor sites.

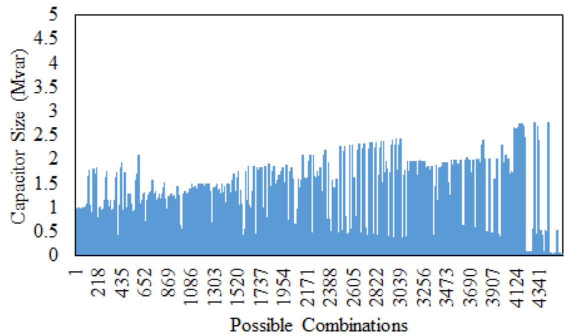

FIGURE 7. The calculated optimal capacitor size for all possible combinations of DG and capacitor sites.

- Bus 1 represents the distribution substation where no unit is required to be placed;
- The total penetration of the units to be placed is set to be less than or equal 100% of the total load;
- The minimum and maximum allowed values for voltage is 0.95 and 1.05, respectively;
- The size of DG and capacitors are assumed to be continuous variables, if not, their sizes can be corrected after being optimally calculated to the closest available commercial size of each unit.

B. VALIDATION

The proposed method is validated by comparing with an exhaustive approach. Two units (one DG with 0.90 power factor lagging and one capacitor) are needed to be allocated in the test system for minimizing reactive losses. Since there are 68 possible sites for the two different units, the number of possible combinations of locations N_{com} is 4556.

Fig. 5 compares the exact reactive losses computed by the exhaustive approach and the proposed estimated reactive losses formulated in (5) for all combinations of locations of the two units. To clarify this figure, the data are presented so that the x-axis starts with the best combination and ends with the worst combination in terms of the reactive loss reduction. It is clear that the minimum values of the exact and estimated losses are located in the same combination, which is the best combination. Therefore, the proposed formulation for estimating reactive losses can be accurately employed for determining the best combination even when placing different units. According to the exact and the estimated values, the best combination of locations is bus 12 (capacitor bus) and bus 61 (DG bus). The corresponding reactive losses are reduced to be only 13.3 kvar which considered significantly smaller compared with 102 kvar at the base case. However, for the last combinations in the x-axis of Fig. 5, the values of reactive losses after installing the units are very high and almost equal to the base case. Therefore, these combinations can be considered non-recommended sites for DG/capacitor installations. This analysis is helpful to quantify the benefits of all possible combination sites of DGs and capacitors, and so assigning the best combination.

Fig. 6 and Fig. 7 show the calculated optimal sizes of all possible combinations of sites for DG and capacitor placement. These optimal sizes are directly computed for each combination with employing (11) and (12). The optimal DG and capacitor sizes are 2.3 MVA and 0.7 Mvar, respectively. It is obvious that the values of DG and capacitor sizes vary significantly with respect to the combinations of sites. This trend demonstrates the importance of determining the optimal sites and sizes of multiple DGs and capacitors. It is important to note that the existing methods solve a complex optimization model for each combination which takes a very long time. However, the proposed method solves all these combinations proposed formulation in a direct way, allowing to rapidly assess all these combinations.

C. PLACEMENT OF DG AND CAPACITORS

In this subsection, the effect of unit type to be placed in the distribution system is examined. For this purpose, three units of the same type (i.e., three UAP DG units, three UARP DG units, or three capacitors) are required to be placed in the distribution system. To practically simulate the placement of DG and capacitors, we assume that the recommended list of buses for each type of the units, as shown in Fig. 8. The terms of comparison between the three scenarios include reactive losses, active losses, maximum load-ability (ML) of the distribution system, and voltage deviation (VD) after installing the units. VD is the summation of the squares of

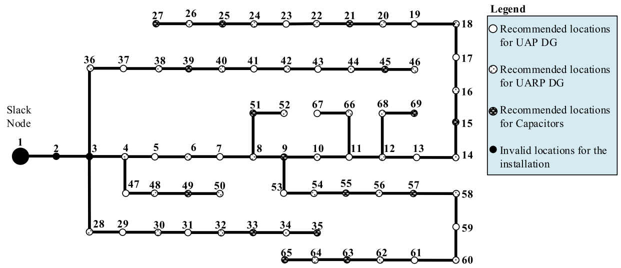

FIGURE 8. 69-bus distribution system with recommended locations for each unit.

TABLE 2. Optimal placement of three units for minimizing reactive losses.

Scenario	Proper Buses	Optimal Sizes (MVA)	Reactive losses (kvar)	Active losses (kW)	VD	ML
UAP DG	11 17 61	0.59 0.38 1.74	34.9709	69.5084	0.00449	2.92
Cap.	15 49 63	0.41 0.57 1.21	67.3618	147.793	0.05762	2.46
UARP DG	18 50 62	0.69 0.87 2.15	5.71037	12.0926	0.00029	3.22
Without unit	-	-	102.17	225	0.09934	2.19

voltage deviation from the nominal value for all system buses, and ML is the highest value of the factor to be multiplied with loads while keeping the distribution system in the stable region.

Table II compares the results of the optimal placement for the units by using the proposed method for three scenarios (Scenarios 1, 2, 3). Three UAP DG units, three capacitors, and three UARP DG units are considered to be installed in scenarios 1, 2, and 3, respectively. It is interesting to note that the optimal sizes and proper locations of the units are completely different for the three scenarios, even though the number of units to be placed is equal, i.e. 3 units. For example, the optimal set of sites are (11, 17, 61), (15, 49, 63), and (18, 50, 62) for scenarios 1, 2, and 3, respectively. In addition, the corresponding optimal sizes of the units greatly vary in each scenario. Another notice is that in all terms of comparison, placing capacitors is the worst scenario in which the values of active losses, reactive losses, VD and ML are 147.8 kW, 67.4 kvar, 0.05762, and 2.46, respectively. For scenario 1, the figures are improved to be 69.5 kW, 35.0 kvar, 0.00449, and 2.92, respectively. The best scenario is the placement of UARP DG units (i.e. scenario 3) in all terms of comparisons since they can generate both active and reactive powers. The corresponding figures are, respectively, 12.1 kW, 5.7 kvar, 0.00029, and 3.22. Another benefit of the best scenario is that it yields a higher total capacity of the units compared to the other two scenarios. For example, the total capacity of the installed units for scenario 3 is 3.71 MVA which is higher than those of the other two scenarios (only 2.71 and 2.19 MVA).

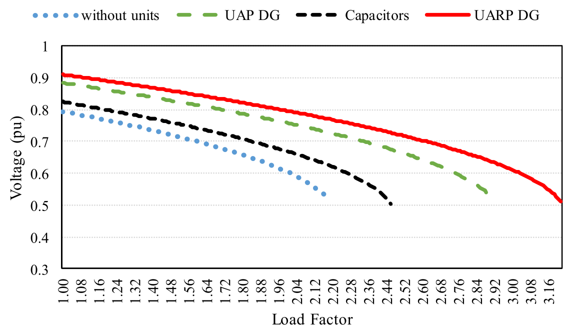

FIGURE 9. System load-ability for the different scenarios (bus 69).

Consequently, the ability of the UARP DG type to generate both active and reactive powers can contribute positively to increase the hosting capacity of these units.

Fig. 9 compares the ML of the distribution system for the three scenarios with the base case. Specifically, the voltage at bus 69 is plotted with changing the load factor from 1.00 to 3.24. It is obvious that the voltage level and the ML values for the three scenarios are much higher than the base case i.e. without the units, in which the voltage level decreases rapidly with the load factor. However, scenario 2 (installing capacitors) yields the lowest voltage level and the ML value compared with scenarios 1 and 3. Scenario 3 in which the DGs have the reactive power capability achieves the higher voltage level while maximizing the ML of the distribution system.

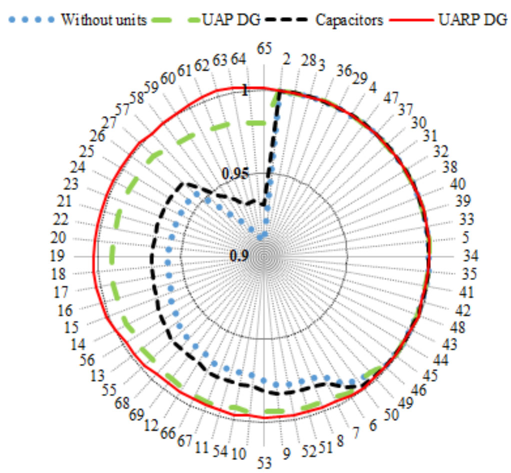

FIGURE 10. Voltage profiles for the different scenarios.

Fig. 10 shows the voltage profile at the system buses for the three scenarios and the base case. The voltage profiles for the three scenarios are better than that of the base case. Nevertheless, the installation DG units in scenarios 1 and 3 have better voltage profiles compared to that of the capacitors. The voltage profiles for scenario 3 is the best profile as it is almost constant and equal to the nominal voltage (1.0 pu) at all buses.

This analysis demonstrates that the type of units to be installed in the distribution system has a great impact on

TABLE 3. DG and capacitor numbers for all cases.

Unit Type	C1	C2	C3	C4	C5	C6	C7
UAP DG	2	2	1	-	-	1	1
UARP DG	1	-	2	2	1	-	1
Cap.	-	1	-	1	2	2	1

distribution systems. Therefore, the DG type must be carefully selected. The reactive power capability of DG can have pronounced positive impacts in terms of active losses, reactive losses, voltage profiles, ML, and the total DG hosting capacity in distribution systems. However, in the case that it is not available to install only this DG type, it will be required to find the optimal mix of different types of available units to maximize their technical benefits to the system, which is studied in the next subsection.

D. OPTIMAL MIX OF DG AND CAPACITOR

Here, the optimal mix of three units is computed using the proposed method in the 69-bus distribution system. Seven cases (C1 to C7) with different mixtures of three different units are considered, as shown in Table III. Each unit type is placed according to the recommended list of buses, as given in Fig. 8. The determined optimal locations and sizes of the units for all cases are compared in Table IV. It is worth to note that the optimal solution (i.e., locations and sizes) for each case varies significantly from case to case. This variation verifies the importance of computing the optimal combination of the available units to assign the optimal mix (i.e., the best case from the optimal seven cases). Table V compares the seven cases in terms of active and reactive losses, VD, and ML in the distribution system after installing the units. In general, all cases improve the distribution system compared to the base condition where active and reactive losses, VD, and ML are reduced, but with different rates. For instance, C3 and C7 are the best cases for minimizing reactive losses to be 8.15 kvar and 8.99 kvar, respectively, while the lowest active losses occur in C2 (14.14 kW) and C4 (10.76 kW). The best two cases when considering the VD and ML values are (C2, C7) and (C2, C6), respectively. It is important to notice that these cases are studied to illustrate that the determination of the optimal mix can have various benefits to the distribution system. Since the proposed method is based on generic formulations, different cases with various units to be placed and simulated, thereby selecting the optimal mix. The proposed method can be employed by the utilities to quantify the different benefits of diverse combinations of available multi-type units, and so assign the best combination.

E. LOSS MINIMIZATION AT HIGH RPS

In this subsection, the impact of calculating optimal power factors of DG units to be allocated on the loss minimization is investigated at different RPS levels (RPSL). RPSL is incremented by increasing the reactive power level (RPL) of

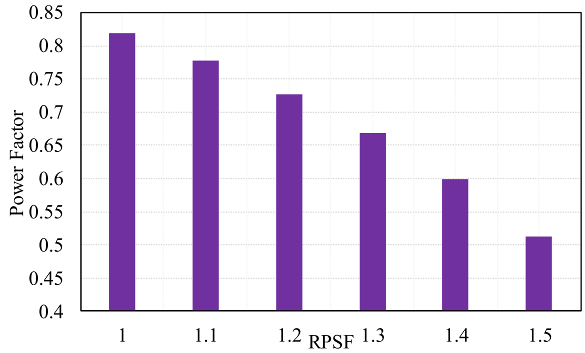

FIGURE 11. Calculated unified power factor of the three units for different RPSL using proposed approach.

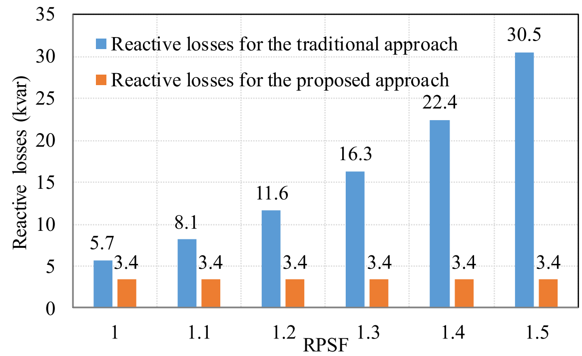

FIGURE 12. Reactive losses for different RPSL.

loads while decreasing the active power level (APL) with a precise amount for keeping the apparent power of the load constant. The mathematical relation between RPL and APL is given in the appendix. Here, two approaches are compared: 1) traditional approach and 2) proposed approach. The traditional approach involves placing DG units with equal specified power factors while the optimal power factors are computed in the proposed approach with employing the presented formulae (11), and (12). In this test, three units are simulated to be allocated with different RPL values (from 1.0 to 1.5 with 0.1 steps). Fig. 11 shows the calculated optimal power factor of DG units using the proposed approach at different RPSL values. It is clear that with increasing RPSL (i.e., increase RPSL), the calculated power factor of units is reduced, which means that more reactive power generation is required at high RPSL. For example, the optimal DG power factor is 0.82 (lagging) when RPSL is 1.0, and it is decreased to 0.51 (lagging) for the case that RPSL equals 1.5. Therefore, RPSL of a distribution system greatly affects the planning of DGs with respect to their reactive power capability.

Figs. 12 and 13 compare the reactive power losses and active power losses, respectively, at different RPSL for the traditional and proposed approaches. It is clear that the reactive and active power losses can be greatly reduced by the proposed approach compared with those of the traditional approach. For instance, the values of the reactive power losses and active power losses are only 3.4 kvar 7.0 kW,

TABLE 4. **Optimal sizes and locations of the three units for the seven cases.**

Unit Type	C1	C2	C3	C4	C5	C6	C7
UAP DG	0.7@61 0.57@17	1.82@61 0.57@17	0.54@17	-	-	1.94@61	1.81@61
UARP DG	2.8@62	-	2.19@62 0.87@50	1.06@12 1.90@62	2.2@62	-	0.69@18
Cap.	-	1.29@63	-	0.4@63	1.00@9 0.56@49	0.41@15 1.21@63	1.23@63

*Unit size (MVA) @ DG bus

TABLE 5. **Reactive losses, active losses, VD values, and ML values for the seven cases.**

Cases	C1	C2	C3	C4	C5	C6	C7
Reactive losses (kvar)	11.96	10.90	**8.15**	9.23	12.80	13.11	**8.99**
Active losses (kW)	16.11	**14.14**	17.70	**10.76**	23.93	20.07	19.44
VD	0.0007 6	**0.0007 0**	0.0009 3	0.0010 3	0.0089 1	0.0072 5	**0.0003 3**
ML	3.23	**3.25**	3.23	3.22	2.23	**3.26**	3.24

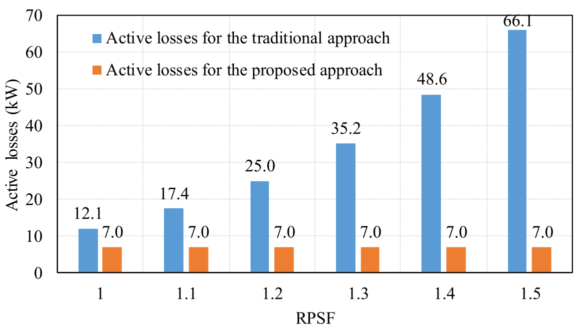

FIGURE 13. **Active losses for different RPSL.**

respectively, for all RPSL values. On the contrary, the reactive and active power losses using the traditional approach excessively increase with rising RPSL (e.g. 30.5 kvar and 66.1 kW at 1.5 RPL). The advantages feature of the proposed approach is achieved by computing the optimal power factor, thanks to the proposed formulation. The active loss follows the same trend, as shown in Fig. 13. This indicates that the power factors of the units play an important role in loss minimization, and they are needed to be optimally calculated.

VI. CONCLUSION

The proper placement of reactive power sources increases system strength during critical conditions. This paper has proposed a new method for determining the optimal mix of different DG types and capacitors in distribution systems for reactive power minimization. A general set of new analytical expressions is combined with OPF for solving the optimization problem. The accuracy of the proposed method is verified with an exhaustive method, and the impact of DGs and capacitors on the distribution system is studied with considering losses, ML and VD. The proposed method is applied to solve different combinations of different types of DG and capacitor units, and the optimal mix for maximizing the benefits of units is accurately determined. Since the optimal power factors of DG units can be accurately calculated with the proposed formulations, reactive powers can be effectively compensated.

In the future, this research work will be expanded in several directions. First, other reactive power sources (e.g. DSTATCOM) and voltage control devices (e.g. on-load tap changer transformers and step voltage regulators) will be considered when allocating multi-type units. Second, various energy storage devices, such as electric vehicles, will be allocated in a simultaneous manner with DGs and capacitors. Finally, the costs of DG and capacitors will be incorporated into the planning model.

APPENDIX

In this appendix, the mathematical relation between RPL and APL for keeping the apparent power of the load constant is presented. At each bus *i* of a distribution system, the following equation is satisfied:

$$\left|S_{load,i}\right|^2 - P_{load,i}^2 - Q_{load,i}^2 = 0 \quad (13)$$

where $S_{load,i}$, $P_{load,i}$, and $Q_{load,i}$ represent the apparent power, active power, and reactive power of the load at bus *i*, respectively. If we change the active power level with multiplying by APL and the reactive power level with multiplying by RPL, the apparent power will be consequently changed by SL, as follows

$$\left|S_{load,i} \times SL_{load,i}\right|^2 - \left(P_{load,i} \times APL_{load,i}\right)^2 - \left(Q_{load,i} \times RPL_{load,i}\right)^2 = 0 \quad (14)$$

In order to keep the apparent power constant with changing APL or RPL factors, the SL is required to be equal to 1 in (14). Therefore, to keep the apparent power constant with changing RPL, APL must be calculated from (15).

$$APL_{load,i} = \sqrt{\frac{\left|S_{load,i}\right|^2 - \left(Q_{load,i} \times RPL_{load,i}\right)^2}{\left(P_{load,i}\right)^2}} \quad (15)$$

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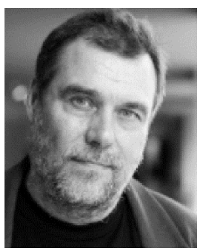

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