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# Robust Distributed Fault Diagnosis for Large-Scale Interconnected Multi-Motor Web-Winding Systems

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**ABSTRACT** Web-winding systems are generally large-scale interconnected systems with many motor-driven subsystems. If the centralized fault diagnosis methods are adopted, the information exchanges among the motor driven subsystems will be required. However, the information exchanges are not always available due to the communication restriction, high system cost and so on. Moreover, it will lead to the high computational cost. To solve this problem well, the web-winding system is considered as a synthetic system with several dynamic subsystems subjected to multiple disturbances and actuator faults. Then, the methods of disturbance attenuation based distributed fault diagnosis method and disturbance compensation based distributed fault diagnosis method are developed to estimate the actuator faults. The objective of fault detection, fault isolation and fault estimation can also be realized via these methods. Meanwhile, sufficient conditions of asymptotic stability of the estimation error system are derived based on the Lyapunov theory. Observer gain matrices are obtained by solving the linear matrix inequalities (LMIs). Finally, simulations and analysis are performed on the three-motor web-winding system to verify the effectiveness of the proposed two fault diagnosis methods.

**INDEX TERMS** Distributed fault diagnosis, nonlinear disturbance observer, large-scale multi-motor web-winding system, LMI, asymptotic stability.

## I. INTRODUCTION

Large-scale multi-motor web-winding system, which is actually one of mechanical and electrical systems, is widely used, such as paper machine, cool rolling system, coating machine [1]. In practical applications, the uneven wear or fracture of the bearing and linkages will lead to the web-winding system actuator intermittent faults [2], [3]. Additionally, the bearing wear or circuit aging will also result in the changes of output torque of the driven motor, which can also be considered as the actuator incipient faults of web-winding system [4]. That is, the complexity of large-scale multi-motor web-winding systems will result in a high fault rate. No matter what faults occur, these subsystems will be in abnormal situations due to the existence of interaction between various subsystems. Moreover, it will

reduce the product quality, prevent the continuous operation or even bring about huge property loss and casualties [5], [6]. In order to detect the system faults in time and conduct the condition-based maintenance (CBM), the fault diagnosis, which contains fault detection, fault isolation and estimation, is required.

Currently, the researches about fault diagnosis for the web-winding systems are mainly carried out based on linear time-invariant (LTI) model [7]–[9], linear time variant (LTV) model [10] and linear parameter varying (LPV) model [11], respectively. However, nonlinearity is one of the important characteristics for web-winding systems. These models are established without considering the coupling relationship between velocity and tension, and they don't express all the real dynamic behavior of the process of unwinding and rewinding well. To overcome these disadvantages, the authors of [12], [13] develop a dynamic model for web-winding machines according to the system operation mechanism.

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Then, many studies are carried out based on this model, such as the robust control [14], [15], observer design [16], [17], fault detection and fault isolation [18], [19]. Additionally, the research of fault estimation theory has also received considerable attention. In [21], the system faults are firstly modeled as additive and multiplicative unknown dynamics, and then they are estimated by using one-step previous state information. In [22], [23] the adaptive observer is designed to simultaneously estimate the actuator and sensor faults of Markovian jump systems. In [24], a robust state-space observer is proposed to simultaneously estimate the descriptor states, actuator faults and their finite times derivatives and attenuate input disturbances in any desired accuracy. In [25], a method of real-time fault compensation is proposed based on state estimation and compensation technique for the sensor subjecting to the noisy and drifting. In [26], the design and analysis of fault tolerant system based on real-time fault-identification technique and state-estimation techniques is studied. In [27], the fault diagnosis and fault tolerant in variable speed drives are reviewed. The authors of [28] make a bibliographical review on fault diagnosis and fault tolerant.

However, the web-winding systems are strong nonlinear and coupled, and the fault estimation cannot be easily achieved so that it is difficult to provide a good decision-making for CBM. Moreover, these works are based on the centralized structure. Actually, the web-winding systems are generally large-scale interconnected systems with many motor driven subsystems. The classical centralized methods rely on the information sharing and the transmission among all the subsystems [20]. But the information exchanges are not always available due to communication restriction, high system cost and so on. Moreover, it will lead to the high computational cost. Nevertheless, the distributed fault diagnosis is carried out based on local states information and information from neighboring subsystems. It can decrease the amount of information transmission and the computational cost. It is a good choice for the fault diagnosis of large-scale multi-motor driven web-winding systems.

Currently, many works about distributed fault diagnosis have also received a large amount of attention. In [29], [30], the problem of distributed fault diagnosis is studied by regarding the effects of various faults on the subsystem as a lumped fault. In [31], the effects of interconnection on subsystems are regarded as the disturbances of the corresponding subsystem, and then the distributed fault detection and isolation strategies are designed in [32], [33]. In [34], the distributed continuous-time fault estimation for multiple devices is investigated. In [35], [36], the unknown inputs and fault estimation algorithms are developed for multi-agent systems. By diffeomorphism transformation, the distributed interconnected nonlinear systems are transformed into a system with block triangular structure, and then a high gain observer with sliding mode is developed for states and faults estimation in [37]. However, in these previous literatures, the coupling between the subsystems are assumed to be linear. Actually, it is inconsistent with the actual situations of web-winding systems.

Then, a robust distributed high gain observer is designed for multi-motor web-winding systems in [16]. But the researches of fault diagnosis for web-winding systems are few. Hence, it motivates us to conduct the distributed fault diagnosis for large-scale multi-motor web-winding systems.

On the other hand, there exist multiple disturbances including the external disturbances, unmodeled dynamics and parametric variations in practical web-winding process. The problem of fault diagnosis becomes complicated due to the existence of faults and disturbances simultaneously. According to [38]–[41], these uncertainties can be described by different mathematic formulations including exogenous systems, harmonic signals and norm bounded variables. To improve the robustness for the fault diagnosis system, the problems of disturbance attenuation or disturbance compensation have been widely studied and many methods have been proposed [42], such as  $H_\infty$  method [43], sliding-mode observer (SMO) method [44], [45] and disturbance observer method [46]–[48]. It should be noted that the  $H_\infty$  method belongs to one of the disturbance attenuation approaches. For the SMO method, the upper limits of the uncertainties or disturbances need to be known in priority. The disturbance observer technique, which is a method that the disturbances are estimated and eliminated by adding a compensation term in the system, is widely used in many fields of industry. It is suitable to conduct the fault diagnosis for interconnected web-winding systems. Moreover, to the best of the authors' knowledge, the issue of disturbance observer based distributed robust fault diagnosis for web-winding system with multiple disturbances has not been fully investigated up to now, which motivates us for the current study again.

Based the above-mentioned investigation, the methods of disturbance attenuation and disturbance accommodation based distributed fault diagnosis are put forward in this study. The main contributions of this paper can be summarized as follows: (1) The web-winding system model with multiple disturbances and faults is converted into the large-scale interconnected system model. Therefore, in this study the practical problem of fault diagnosis for web-winding system is firstly converted into the fault diagnosis for large-scale interconnected systems, which is helpful for the current study. (2) Two distributed robust fault diagnosis methods are designed to detect, isolate and estimate the system actuator faults. The existence of the designed observers is derived by using the Lyapunov theory, and the designed fault diagnosis observers can be easily achieved by solving LMIs with few adjustable parameters. Moreover, with the designed fault diagnosis methods, the fault diagnosis system can not only achieve the goal of fault diagnosis, but also has the advantage of simple design and easy implementation. The simulation results show that the disturbance accommodation based method can deal with the incipient faults well when comparing with the disturbance attenuation based method.

The remainder of this paper is organized as follows. In Section II, the distributed web-winding system model with actuator faults and multiple disturbances is presented.

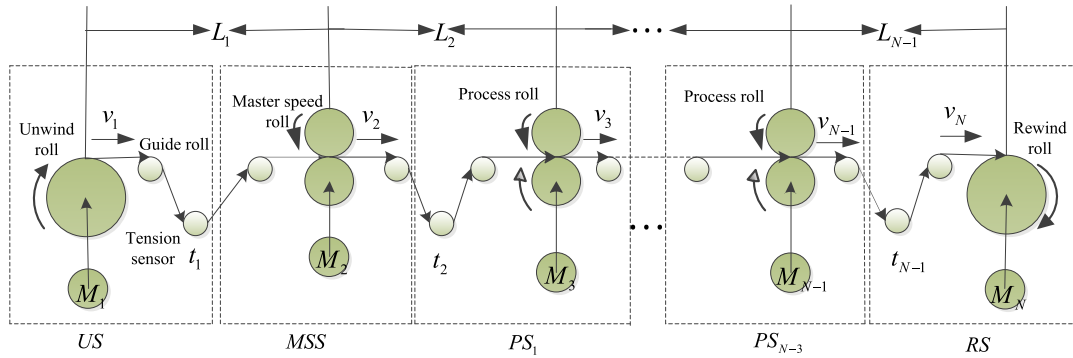


FIGURE 1. Sketch of large-scale interconnected multi-motor web-winding subsystem division.

Section III describes the design of distributed robust fault diagnosis observers, and the convergence analysis for estimation error dynamics is performed. In Section IV, the observer gain matrices are obtained by solving LMIs, and the simulations are conducted on the three-motor web-winding system to verify the effectiveness of the proposed methods. Section V concludes this paper.

**II. MODEL FOR LARGE-SCALE INTERCONNECTED MULTI-MOTOR WEB-WINDING SYSTEMS**

A large-scale interconnected multi-motor web-winding system is generally utilized to provide the transport of the web from the unwind roll to the rewind roll. Figure 1 is a sketch of interconnected multi-motor web-winding subsystems division with  $N$  reels driven by motor  $M_1, M_2, \dots, M_N$ , respectively.  $L_j$  and  $t_j$  are the web length and tension for the  $j$ th span, and  $v_j$  is the web velocity on the  $j$ th roll. There are  $N$  subsystems shown in Figure 1, containing unwind subsystem (US), master speed subsystem (MSS), rewind subsystem (RS), and  $N - 3$  process subsystems (PSs). Every subsystem is a motor driven subsystem with the dynamics of driving roll and web tension except for MSS. The MSS, which is generally the second subsystem downstream of the US in almost all winding systems, is utilized to set the transport velocity of the overall web line. Due to that the unwind/rewind roll releases/accumulates material to/from the processing section of the web line, their radii and moments of inertia are time-varying. Then, according to the [12]–[14], the dynamics of the web tension  $t_j$  and web velocity  $v_j$  can be expressed as

$$\begin{cases} \dot{t}_j = \frac{ES}{L_j}(v_{j+1} - v_j) + \frac{1}{L_j}t_{j-1}v_j - \frac{1}{L_j}t_jv_{j+1}, \\ \dot{v}_j = \frac{R_j^2}{J_j}(t_j - t_{j-1}) - \frac{b_{ff}}{J_j}v_j + \frac{R_j n_j}{J_j}u_j - \frac{\dot{J}_j}{J_j}v_j + \frac{\dot{R}_j}{R_j}v_j. \end{cases} \quad (1)$$

and the corresponding dynamics of radii and moments of inertia can be expressed as

$$\begin{cases} J_j = J_{cj} + n_j^2 J_{mj} + \frac{\pi}{2} b \rho (R_j^4 - R_{cj}^4), \quad j = 1, N, \\ \dot{R}_1 \approx -\frac{h}{2\pi} \frac{v_1}{R_1}; \quad \dot{R}_N \approx \frac{h}{2\pi} \frac{v_N}{R_N}, \\ J_j = J_{cj} + n_j^2 J_{mj}, \quad \dot{R}_j = 0, \quad j = 2, 3, \dots, N - 1. \end{cases}$$

where,  $t_0$  is the wound-in web tension in the unwind roll;  $E$  and  $S$  are Young’s modulus and cross-sectional area of the web, respectively;  $b_{ff}$  is the friction coefficient of the  $j$ th roll shaft;  $n_j$  is the gearing ratio between the motor shaft and the corresponding roll shaft;  $\rho$ ,  $b$  and  $h$  are density, width and thickness of the web, respectively;  $R_j$  and  $J_j$  are the radius and effective inertia of the  $j$ th roll, respectively;  $J_{mj}$  is the inertia of all the rotating elements on the motor side;  $J_{cj}$  and  $R_{cj}$  are the inertia and radius of the  $j$ th empty roll, respectively. Obviously, the inertia  $J_1$  and radius  $R_1$  decrease with the releasing of web material, and simultaneously the inertia  $J_N$  and radius  $R_N$  increase due to the collected web material.

By considering the real situations of the multi-motor winding system, there exist uncertainties in  $R_j$  and  $J_j$ . Moreover, there also exist uncertainties from the environment acting on the Young’s modulus  $E$  and friction coefficient  $b_{ff}$ . Then,  $v_j(t)$  is introduced to denote the lumped effects of parametric variations, uncertain disturbances and unmodeled dynamics on the  $j$ th subsystem. Additionally, the external periodic disturbances caused by rotating motors are inevitable and the vector  $d_j(t)$  is introduced to denote them. That is to say, there exist multiple disturbances in the practical operational process of web winding system. According to [38]–[41], the external periodic disturbances can be generated by a neutrally stable system described by

$$\begin{cases} \dot{\varpi}_j(t) = W_j \varpi(t) + R_j \xi_j(t), \\ d_j(t) = G_j \varpi_j(t), \end{cases} \quad (2)$$

where,  $\varpi_j(t)$  is the state variable,  $W_j$ ,  $R_j$  and  $G_j$  are the known parameter matrices with suitable dimensions, and  $\xi_j(t)$  is the additional disturbance which are resulted from the perturbations and uncertainties in the above subsystem and supposed to have the bounded  $H_2$  norm.

On the other hand, due to that the multi-motor web-winding system is actually one of electro-mechanical systems, then according to [2], [3], the uneven wear or fracture of the bearing and linkages will lead to the web-winding system actuator intermittent faults. Additionally, the bearing wear or circuit aging will also result in the changes of output torque of the motor, which can also be considered as the web-winding

system actuator incipient faults [4]. In this study, only the actuator faults are considered. By the [49], when the system suffers from the actuator faults, the web-winding system input will be replaced by  $u_{jf}(t) = u_j(t) + [(\theta_j - 1)u_j(t) + u_{f0j}(t)]$ . Let  $\kappa_j(t) = (\theta_j - 1)u_j(t) + u_{f0j}(t)$ . In order to simplify the design of fault diagnosis observer, the equivalent fault of the  $j$ th subsystem is defined as  $f_j(t) = \frac{R_j n_j}{J_j} \kappa_j(t)$  for  $j = 2, 3, \dots, N$ , and  $f_1(t) = -\frac{R_1 n_1}{J_1} \kappa_1(t)$ . Meanwhile, it is assumed that the sensors are fault-free and there is no measurement noise. Then, according to the dynamic model of [12]–[14], system (1) and (2), the dynamic model of the  $j$ th subsystem with multiple disturbances and actuator fault  $f_j(t)$  can be presented as

$$\begin{cases} \dot{x}_j = A_j x_j + B_j u_j + \Phi_j(x_j) + \delta_j(x_j, \bar{x}_j) \\ \quad + D_j d_j(t) + F_j f_j(t) + H_j v_j(t), \\ y_j = C_j x_j, \end{cases} \quad (3)$$

where,  $x_1 = [t_1, v_1]^T$ ,  $x_2 = v_2$ ,  $x_j = [t_{j-1}, v_j]^T$  ( $j \geq 3$ ),  $\bar{x}_j = [x_1^T, x_2^T, \dots, x_{j-1}^T, x_{j+1}^T, \dots, x_N^T]^T$ ,  $B_j = F_j \lambda_j$  ( $\lambda_j \neq 0$ ),  $\Phi_j(x_j)$  is the nonlinear function vector,  $\delta_j(x_j, \bar{x}_j)$  is the known coupling function representing the effects of the other subsystems on the  $j$ th subsystem, and  $y_j(t)$  is the measurable output vector. Then, the dynamic models of US, MSS, PS and RS can be presented as follows.

#### Unwind subsystem

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u_1 + \Phi_1(x_1) + \delta_1(x_1, \bar{x}_1) \\ \quad + D_1 d_1(t) + F_1 f_1(t) + H_1 v_1(t), \\ y_1 = C_1 x_1, \end{cases} \quad (4)$$

where,

$$\begin{aligned} \bar{x}_1 &= x_2^T, \quad A_1 = \begin{bmatrix} 0 & \frac{t_0 - ES}{L_1} \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{R_1 n_1}{J_1} \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \Phi_1(x_1) &= \left[ 0, \frac{R_1^2}{J_1} t_1 - \frac{b_{f1}}{J_1} v_1 - \frac{h}{2\pi J_1} \left( \frac{J_1}{R_1^2} - 2\pi b \rho R_1^2 \right) v_1^2 \right]^T, \\ \delta_1(x_1, \bar{x}_1) &= \left[ \frac{(ES - t_1) V_2}{L_1} \quad 0 \right]^T, \end{aligned}$$

and  $u_1$  denotes the input torque from the driving motor  $M_1$ .

#### Master speed subsystem

$$\begin{cases} \dot{x}_2 = A_2 x_2 + B_2 u_2 + \delta_2(x_2, \bar{x}_2) + D_2 d_2(t) \\ \quad + F_2 f_2(t) + H_2 v_2(t), \\ y_2 = C_2 x_2, \end{cases} \quad (5)$$

where,

$$\begin{aligned} \bar{x}_2 &= [x_1^T, x_3^T]^T, \quad A_2 = -\frac{b_{f2}}{J_2}, \quad B_2 = \frac{R_2 n_2}{J_2}, \\ D_2 &= 1, \quad F_2 = 1, \quad \delta_2(x_2, x_1, x_3) = \frac{R_2^2 (t_2 - t_1)}{J_2}, \quad H_2 = 0.1. \end{aligned}$$

#### The $j$ th process subsystem

$$\begin{cases} \dot{x}_j = A_j x_j + B_j u_j + \Phi_j(x_j) + \delta_j(x_j, \bar{x}_j) \\ \quad + D_j d_j(t) + F_j f_j(t) + H_j v_j(t), \\ y_j = C_j x_j, \end{cases} \quad (6)$$

where,

$$\begin{aligned} \bar{x}_3 &= [x_1^T, x_2^T, x_4^T]^T, \quad \bar{x}_j = [x_{j-1}, x_{j+1}]^T \quad (4 \leq j \leq N-1), \\ A_j &= \begin{bmatrix} 0 & \frac{ES}{L_{j-1}} \\ -\frac{R_j^2}{J_j} & \frac{b_{fj}}{J_j} \end{bmatrix}, \quad B_j = \begin{bmatrix} 0 \\ \frac{R_j n_j}{J_j} \end{bmatrix}, \quad D_j = \begin{bmatrix} 0.10 \\ 0 & 1 \end{bmatrix}, \\ F_j &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad H_j = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \Phi_j(x_j) = \left[ -\frac{t_{j-1} v_j}{L_{j-1}}, 0 \right]^T, \\ \delta_j(x_j, x_{j-1}, x_{j+1}) &= \left[ \frac{(t_{j-2} - ES) V_{j-1}}{L_{j-1}}, \frac{R_j^2 t_j}{L_j} \right]^T, \end{aligned}$$

and  $u_j$  denotes the input torque from the driving motor  $M_j$ .

#### Rewind subsystem

$$\begin{cases} \dot{x}_N = A_N x_N + B_N u_N + \Phi_N(x_N) + \delta_N(x_N, \bar{x}_N) \\ \quad + D_N d_N(t) + F_N f_N(t) + H_N v_N(t), \\ y_N = C_N x_N, \end{cases} \quad (7)$$

where,

$$\begin{aligned} \bar{x}_N &= x_{N-1}, \quad A_N = \begin{bmatrix} 0 & \frac{ES}{L_{N-1}} \\ 0 & 0 \end{bmatrix}, \quad B_N = \begin{bmatrix} 0 \\ \frac{R_N n_N}{J_N} \end{bmatrix}, \\ D_N &= \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}, \quad F_N = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad H_N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ \delta_N(x_N, x_{N-1}) &= \left[ \frac{(t_{N-2} - ES) V_{N-1}}{L_{N-1}}, 0 \right]^T, \\ \Phi_N(x_N) &= \left[ -\frac{t_{N-1} v_N}{L_{N-1}}, -\frac{R_N^2}{J_N} t_{N-1} - \frac{b_{fN}}{J_N} v_N \right. \\ &\quad \left. + \frac{h}{2\pi J_N} \left( \frac{J_N}{R_N^2} - 2\pi b \rho R_N^2 \right) v_N^2 \right]^T, \end{aligned}$$

and  $u_N$  denotes the input torque from the driving motor  $M_N$ .

In accordance with the relationship between subsystems, the interconnected topology of web-winding systems with  $N$  motor driven subsystems can be shown as Figure 2. For simplicity,  $\delta_j(x_j, x_k, x_l, x_m)$  is chosen to denote the interconnection function with  $j \neq k, l, m$ . Then, the  $j$ th subsystem dynamic model can be summarized as

$$\begin{cases} \dot{x}_j = A_j x_j + B_j u_j + \Phi_j(x_j) + \delta_j(x_j, x_k, x_l, x_m) \\ \quad + D_j d_j(t) + F_j f_j(t) + H_j v_j(t), \\ y_j = C_j x_j, \quad j = 1, 2, \dots, N. \end{cases} \quad (8)$$

Obviously, these nonlinear function  $\Phi_j(x_j)$  and interconnected  $\delta_j(x_j, x_k, x_l, x_m)$  are smooth and satisfy the partial derivative theorem of multivariate function. Therefore, the following inequalities can be achieved.

$$\|\Phi_j(x_j) - \Phi_j(\hat{x}_j)\| \leq \kappa_j \|x_j - \hat{x}_j\|, \quad (9)$$

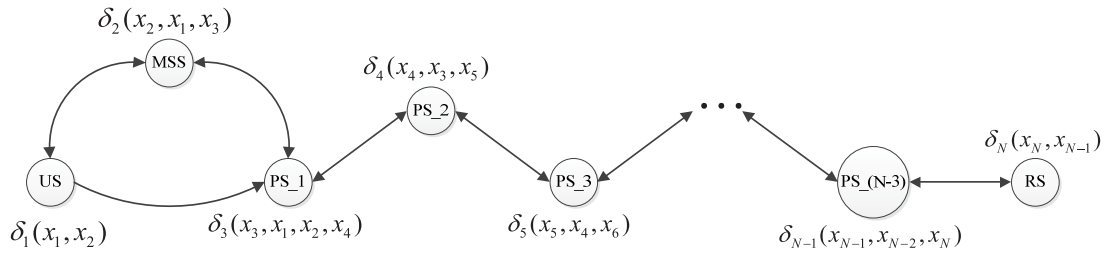


FIGURE 2. Interconnection topology for N motor driven web-winding systems.

$$\|\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)\| \leq \gamma_j \|x_j - \hat{x}_j\| + \gamma_k \|x_k - \hat{x}_k\| + \gamma_l \|x_l - \hat{x}_l\| + \gamma_m \|x_m - \hat{x}_m\|, \quad (10)$$

where,  $\kappa_j$  is the Lipschitz constant,  $\gamma_j = \max_{x \in \Theta} \|\frac{\partial \delta_j}{\partial x_j}\|$ ,  $\gamma_k = \max_{x \in \Theta} \|\frac{\partial \delta_j}{\partial x_k}\|$ ,  $\gamma_l = \max_{x \in \Theta} \|\frac{\partial \delta_j}{\partial x_l}\|$ ,  $\gamma_m = \max_{x \in \Theta} \|\frac{\partial \delta_j}{\partial x_m}\|$ , and they are continuously differentiable strictly positive constants with  $\Theta$  being the bounded set of state vector  $x(t) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T$ .

In the next section, the objective is to design robust distributed fault diagnosis observers, with which the system faults can be estimated and the external disturbances can be compensated or rejected at the same time. To proceed with the design of fault diagnosis observers, the following assumptions are required.

*Assumption 1:* It is assumed that fault  $f_j(t)$  satisfies that  $\|\dot{f}_j(t)\| < \sigma_j$ , where  $\sigma_j$  is a bounded positive constant.

*Assumption 2:* It is assumed that all the subsystem states are available by various sensors.

*Remark 1:* Actually, the Assumption 1 is quite general in the literature to indicate that the faults are constant or time-varying at a limited changing rate (see [20]). It typically says that fault diagnosis observer is designed for the situations where the system is not “exploding”, which is not really a restriction.

*Remark 2:* In practice, the tension  $t_j$  and angular velocity  $\omega_j$  can be measured via the tensiometers and tachometers, respectively. The real-time radius  $R_j$  can be obtained by the radius measurement device. Then, the web velocity  $v_j$  can be obtained by  $v_j = R_j \omega_j$ . That is, the Assumption 2 is reasonable.

### III. DESIGN OF ROBUST DISTRIBUTED FAULT DIAGNOSIS OBSERVERS

#### A. DESIGN OF DISTURBANCE ATTENUATION BASED DISTRIBUTED FAULT DIAGNOSIS OBSERVER

In this subsection, the idea of disturbance observer is utilized to design the distributed fault estimation observer. Meanwhile, the  $H_\infty$  technique is introduced to deal with the multiple disturbances. Then, the fault estimation observer for the  $j$ th subsystem can be designed as follows.

$$\begin{cases} \dot{\hat{x}}_j(t) = K_{1j}F_j(\xi_j(t) - K_{1j}x_j(t)) + K_{1j}(A_jx_j + B_ju_j + \Phi_j(x_j) + \delta_j(x_j, x_k, x_l, x_m)), \\ \hat{f}_j(t) = \xi_j(t) - K_{1j}x_j(t), \end{cases} \quad (11)$$

where,  $\xi_j(t)$  is the auxiliary variable,  $\hat{f}_j(t)$  is the estimation of actuator fault  $f_j(t)$ ,  $K_{1j}$  is the  $j$ th fault estimation observer gain matrix to be determined later. Then, based on the estimated actuator fault  $\hat{f}_j(t)$ , the observer for subsystem (8) can be presented by

$$\begin{cases} \dot{\hat{x}}_j = A_j\hat{x}_j + B_ju_j + \Phi_j(\hat{x}_j) + \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m) + F_j\hat{f}_j(t) + M_{1j}(y_j - \hat{y}_j), \\ \hat{y}_j = C_j\hat{x}_j, \end{cases} \quad (12)$$

where,  $M_{1j}$  is the observer gain matrix to be designed later. Obviously, if the state estimation  $\hat{x}_j$  is convergent to the actual state  $x_j$ , the proposed fault estimation algorithm is correct.

Define  $e_{x_j}(t) = x_j(t) - \hat{x}_j(t)$ ,  $e_{f_j}(t) = f_j(t) - \hat{f}_j(t)$  and  $z_{1j}(t) = C_j e_{x_j}(t) + C_{f_j} e_{f_j}(t)$ . Then, from the system (8), (11) and (12), the estimation error dynamic subsystem can be obtained as follows.

$$\begin{cases} \dot{e}_{x_j}(t) = (A_j - M_{1j}C_j)e_{x_j}(t) + F_j e_{f_j}(t) + D_j d_j(t) + H_j v_j(t) + (\Phi_j(x_j) - \Phi_j(\hat{x}_j)) + (\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)), \\ \dot{e}_{f_j}(t) = K_{1j}D_j d_j(t) + K_{1j}F_j e_{f_j}(t) + K_{1j}H_j v_j(t) + \dot{f}_j(t), \\ z_{1j}(t) = C_j e_{x_j}(t) + C_{f_j} e_{f_j}(t), \end{cases} \quad (13)$$

Then, sufficient condition of asymptotic stability with the ability to attenuate the multiple disturbances for system (13) will be given in Theorem 1. Meanwhile, the observer gain matrices  $K_{1j}$  and  $M_{1j}$  can be obtained by solving LMI.

*Theorem 1:* Considering the web-winding subsystem (8) with the disturbances in (2), for given parameters  $\alpha_{ji} > 0$  ( $j = 1, 2, \dots, N; i = 1, 2, 3$ ), the Lipschitz constant  $\gamma_{max}^j > 0$  and  $\kappa_j > 0$ , if there exist matrices  $Q_{j1} = Q_{j1}^T > 0$ ,  $Q_{j2} = Q_{j2}^T > 0$ ,  $Q_{j1}M_{1j}$  and  $Q_{j2}K_{1j}$  such that the following inequality holds, there exist observers in the form of (11) and (12) such that the augmented estimation error subsystem (13) is asymptotically stable with the  $H_\infty$  performance level  $\alpha_{ji}$ .

$$\begin{bmatrix} \Psi_{j11} & \Psi_{j12} & Q_{j1}D_j & Q_{j1}H_j & 0 & Q_{j1} \\ \star & \Psi_{j22} & Q_{j2}K_{1j}D_j & Q_{j2}K_{1j}H_j & Q_{j2} & 0 \\ \star & \star & -\alpha_{j1}^2 I & 0 & 0 & 0 \\ \star & \star & \star & -\alpha_{j2}^2 I & 0 & 0 \\ \star & \star & \star & \star & -\alpha_{j3}^2 I & 0 \\ \star & \star & \star & \star & \star & \Psi_{j66} \end{bmatrix} < 0 \quad (14)$$

where,  $\Psi_{j11} = Q_{j1}A_j + A_j^T Q_{j1} - Q_{j1M_{1j}}C_j - C_j^T Q_{j1M_{1j}} + C_j^T C_j + (\sum_{i=1}^N \gamma_{max}^i + \kappa_j)I$ ,  $\Psi_{j12} = Q_{j1}F_j + C_j^T C_{fj}$ ,  $\Psi_{j22} = Q_{j2K_{1j}}F_j + F_j^T Q_{j2K_{1j}}^T + C_{fj}^T C_{fj}$ ,  $\Psi_{j66} = -(\kappa_j + N\gamma_{max}^j)^{-1}I$ ,  $I$  denotes the identify matrix with appropriate dimensions, and the symmetric terms in a symmetric matrix are denoted by  $\star$ . Moreover, the observer gain matrices can be obtained by  $M_{1j} = Q_{j1}^{-1}Q_{j1M_{1j}}$  and  $K_{1j} = Q_{j2}^{-1}Q_{j2K_{1j}}$ .

*Proof:* Choose the following Lyapunov function

$$V_j(t) = e_{x_j}^T(t)Q_{j1}e_{x_j}(t) + e_{f_j}^T(t)Q_{j2}e_{f_j}(t), \quad (15)$$

where,  $Q_{j1}$  and  $Q_{j2}$  are the real symmetric positive definite matrices. For any  $e_{x_j}(t) \neq 0$  and  $e_{f_j}(t) \neq 0$ ,  $V_j(t) > 0$ . Then, by differentiating (15) with respect to time along the error trajectory (13) we can get

$$\begin{aligned} \dot{V}_j(t) = & e_{x_j}^T(t)\text{sym}(Q_{j1}A_j - Q_{j1M_{1j}}C_j)e_{x_j}(t) \\ & + 2e_{x_j}^T(t)Q_{j1}F_j e_{f_j} + 2e_{x_j}^T(t)Q_{j1}D_j d_j(t) \\ & + 2e_{x_j}^T(t)Q_{j1}H_j v_j(t) + 2e_{f_j}^T Q_{j2K_{1j}} H_j v_j(t) \\ & + 2e_{x_j}^T(t)Q_{j1}(\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)) \\ & + 2e_{x_j}^T(t)Q_{j1}(\Phi_j(x_j) - \Phi_j(\hat{x}_j)) + 2e_{f_j}^T Q_{j2K_{1j}} F_j \\ & + 2e_{f_j}^T Q_{j2K_{1j}} D_j d_j(t) + 2e_{f_j}^T Q_{j2} \dot{f}_j, \end{aligned} \quad (16)$$

where,  $Q_{j1M_{1j}} = Q_{j1}M_{1j}$ ,  $Q_{j2K_{1j}} = Q_{j2K_{1j}}$  and  $\text{sym}(\cdot)$  denotes that  $\text{sym}(\Theta) = \Theta + \Theta^T$ . From (9) we can obviously get that

$$\begin{aligned} & 2e_{x_j}^T(t)Q_{j1}(\Phi_j(x_j) - \Phi_j(\hat{x}_j)) \\ & \leq 2 \| e_{x_j}^T(t)Q_{j1} \| * \|(\Phi_j(x_j) - \Phi_j(\hat{x}_j))\| \\ & \leq 2\kappa_j \| e_{x_j}^T(t)Q_{j1} \| * \|x_j - \hat{x}_j\| \\ & \leq e_{x_j}^T(t)(\kappa_j Q_{j1} Q_{j1} + \kappa_j I)e_{x_j}(t). \end{aligned}$$

That is,

$$2e_{x_j}^T(t)Q_{j1}(\Phi_j(x_j) - \Phi_j(\hat{x}_j)) \leq e_{x_j}^T(t)(\kappa_j Q_{j1} Q_{j1} + \kappa_j I)e_{x_j}(t). \quad (17)$$

It follows from (10) that the following inequality can be obtained

$$\begin{aligned} & 2e_{x_j}^T(t)P_{j1}(\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)) \\ & \leq 2 \| e_{x_j}^T(t)P_{j1} \| * (\gamma_j \|e_{x_j}(t)\| \\ & \quad + \gamma_k \|e_{x_k}(t)\| + \gamma_l \|e_{x_l}(t)\| + \gamma_m \|e_{x_m}(t)\|) \\ & \leq 2 \| e_{x_j}^T(t)P_{j1} \| * \sum_{i=1}^N \gamma_{max}^i \|e_{x_i}(t)\| \\ & \leq 2\gamma_{max}^j \sum_{i=1}^N \| e_{x_j}^T(t)P_{j1} \| * \|e_{x_i}(t)\| \\ & \leq \gamma_{max}^j \sum_{i=1}^N (e_{x_j}^T(t)P_{j1}P_{j1}e_{x_j}(t) + e_{x_i}^T(t)e_{x_i}(t)) \\ & \leq N\gamma_{max}^j e_{x_j}^T(t)P_{j1}P_{j1}e_{x_j}(t) + \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t), \end{aligned}$$

i.e.,

$$\begin{aligned} & 2e_{x_j}^T(t)P_{j1}(\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)) \\ & \leq N\gamma_{max}^j e_{x_j}^T(t)P_{j1}P_{j1}e_{x_j}(t) + \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t), \end{aligned} \quad (18)$$

where,  $\gamma_{max}^j = \max\{\gamma_j, \gamma_k, \gamma_l, \gamma_m\}$ . Then, by (17) and (18), (16) can be further evaluated as

$$\begin{aligned} \dot{V}_j(t) \leq & e_{x_j}^T(t)(\text{sym}(Q_{j1}A_j - Q_{j1M_{1j}}C_j) + \kappa_j I \\ & + (\kappa_j + N\gamma_{max}^j)Q_{j1}Q_{j1})e_{x_j}(t) + 2e_{x_j}^T(t)Q_{j1}F_j e_{f_j} \\ & + 2e_{x_j}^T(t)Q_{j1}D_j d_j(t) + 2e_{x_j}^T(t)Q_{j1}H_j v_j(t) \\ & + 2e_{f_j}^T Q_{j2K_{1j}} H_j v_j(t) + 2e_{f_j}^T Q_{j2K_{1j}} F_j \\ & + 2e_{f_j}^T Q_{j2K_{1j}} D_j d_j(t) + 2e_{f_j}^T Q_{j2} \dot{f}_j \\ & + \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t). \end{aligned} \quad (19)$$

It is provided that the system (13) has a generalized  $H_\infty$  performance index, i.e.,

$$\|z_{1j}(t)\|^2 < \alpha_{j1}^2 \|d_j(t)\|^2 + \alpha_{j2}^2 \|v_j(t)\|^2 + \alpha_{j3}^2 \|\dot{f}_j(t)\|^2$$

Obviously, the generalized  $H_\infty$  performance index is equivalent to

$$\begin{aligned} J_{j\infty}(t) = & \int_0^\infty (\|z_{1j}(t)\|^2 - \alpha_{j1}^2 \|d_j(t)\|^2 \\ & - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\dot{f}_j(t)\|^2) dt < 0. \end{aligned}$$

Due to that  $V_j(t) > 0$ , then under the zero initial conditions we can get that

$$\begin{aligned} J_{j\infty}(t) \leq & \int_0^\infty (\dot{V}_j(t) + \|z_{1j}(t)\|^2 - \alpha_{j1}^2 \|d_j(t)\|^2 \\ & - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\dot{f}_j(t)\|^2) dt. \end{aligned}$$

Then, for the overall system we can get that

$$\begin{aligned} \sum_{j=1}^N J_{j\infty}(t) \leq & \int_0^\infty \sum_{j=1}^N (\dot{V}_j(t) + \|z_{1j}(t)\|^2 - \alpha_{j1}^2 \|d_j(t)\|^2 \\ & - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\dot{f}_j(t)\|^2) dt. \end{aligned} \quad (20)$$

Due to the fact that

$$\sum_{j=1}^N \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t) = \sum_{j=1}^N \sum_{i=1}^N \gamma_{max}^i e_{x_j}^T(t)e_{x_j}(t),$$

then (20) can be further evaluated as

$$\sum_{j=1}^N J_{j\infty}(t) \leq \int_0^\infty \left\{ \sum_{j=1}^N \Phi_j^T \Pi_j \Phi_j \right\} dt.$$

where,

$$\Phi_j = [e_{x_j}^T(t) \ e_{f_j}^T(t) \ d_j^T(t) \ v_j^T(t) \ \dot{f}_j^T(t)]^T,$$

$$\Pi_j = \begin{bmatrix} \Pi_{j11} & \Pi_{j12} & Q_{j1}D_j & Q_{j1}H_j & 0 \\ \star & \Pi_{j22} & Q_{j2K_{1j}}D_j & Q_{j2K_{1j}}H_j & Q_{j2} \\ \star & \star & -\alpha_{j1}^2 I & 0 & 0 \\ \star & \star & \star & -\alpha_{j2}^2 I & 0 \\ \star & \star & \star & \star & -\alpha_{j3}^2 I \end{bmatrix},$$

$$\Pi_{j11} = Q_{j1}A_j + A_j^T Q_{j1} - Q_{j1M_{1j}}C_j - C_j^T Q_{j1M_{1j}}^T + C_j^T C_j$$

$$+ \left( \sum_{i=1}^N \gamma_{max}^i + \kappa_j \right) I + (\kappa_j + N \gamma_{max}^j) Q_{j1} Q_{j1},$$

$$\Pi_{j12} = Q_{j1}F_j + C_j^T C_{fj},$$

$$\Pi_{j22} = Q_{j2K_{1j}}F_j + F_j^T Q_{j2K_{1j}}^T + C_{fj}^T C_{fj}.$$

Obviously, the sufficient condition for  $\sum_{j=1}^N J_{j\infty}(t) < 0$  is  $\Pi_j < 0$ . By using Schur complement, the inequality (14) can be obtained. This ends the proof.

From the above description, it is obvious that the faults can be estimated by the designed distributed fault estimation observer. Meanwhile, the multiple disturbances are attenuated by  $H_\infty$ . It belongs to one of the disturbance attenuation techniques. In order to further improve the robustness of fault diagnosis system to the disturbances or uncertainties, the disturbance compensation based distributed fault estimation method, which incorporates the fault estimation and disturbances compensation together, will be studied in the following subsection.

### B. DESIGN OF DISTURBANCE COMPENSATION BASED DISTRIBUTED FAULT DIAGNOSIS OBSERVER

For the subsystem (8), the following composite observer will be constructed to estimate the system faults and the uncertain disturbance simultaneously, with which the disturbance  $d_j(t)$  and the uncertain effects  $v_j(t)$  can be rejected and attenuated, respectively. Then, the disturbance compensation based distributed fault estimation observer can be formulated as

$$\begin{cases} \dot{r}_j(t) = (W_j + \Gamma_{2j}D_jG_j)(r_j(t) - \Gamma_{2j}x_j(t)) + \Gamma_{2j}(A_jx_j + B_ju_j + \Phi_j(x_j) + \delta_j(x_j, x_k, x_l, x_m) + F_j\hat{f}_j(t)), \\ \dot{\xi}_j(t) = K_{2j}F_j(\xi_j(t) - K_{2j}x_j(t)) + K_{2j}(A_jx_j + B_ju_j + \Phi_j(x_j) + \delta_j(x_j, x_k, x_l, x_m) + D_j\hat{d}_j(t)), \\ \hat{f}_j(t) = \xi_j(t) - K_{2j}x_j(t), \\ \hat{w}_j(t) = r_j(t) - \Gamma_{2j}x_j(t), \\ \hat{d}_j(t) = G_j\hat{w}_j(t), \end{cases} \quad (21)$$

where,  $\hat{w}_j(t)$  and  $\hat{d}_j(t)$  are the estimation of  $w_j(t)$  and  $d_j(t)$ , respectively;  $r_j(t)$  and  $\xi_j(t)$  are the auxiliary variables;  $\Gamma_{2j}$  and  $K_{2j}$  are the  $j$ th observer gain matrices to be determined later. Then, based on the estimated actuator fault  $\hat{f}_j(t)$  and external disturbances  $\hat{d}_j(t)$ , the observer for subsystem (8) can be presented as

$$\begin{cases} \dot{\hat{x}}_j = A_j\hat{x}_j + B_ju_j + \Phi_j(\hat{x}_j) + \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m) + D_j\hat{d}_j(t) + F_j\hat{f}_j(t) + M_{2j}(y_j - \hat{y}_j), \\ \hat{y}_j = C_j\hat{x}_j, \end{cases} \quad (22)$$

where,  $M_{2j}$  is the  $j$ th observer gain matrix to be designed later.

By defining the estimation error vector  $e_{\varpi_j}(t) = \varpi_j(t) - \hat{\varpi}_j(t)$ , the estimation error system can be obtained from the system (8), (21) and (22) to show as

$$\begin{cases} \dot{e}_{x_j}(t) = (A_j - M_{2j}C_j)e_{x_j}(t) + F_j e_{f_j}(t) + D_j G e_{\varpi_j} + H_j v_j(t) + (\Phi_j(x_j) - \Phi_j(\hat{x}_j)) + (\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)), \\ \dot{e}_{\varpi_j}(t) = (W_j + \Gamma_{2j}D_jG_j)e_{\varpi_j}(t) + \Gamma_{2j}F_j e_{f_j}(t) + R_j \xi_j(t) + \Gamma_{2j}H_j v_j(t), \\ \dot{e}_{f_j}(t) = K_{2j}D_jG_j e_{\varpi_j}(t) + K_{2j}F_j e_{f_j}(t) + K_{2j}H_j v_j(t) + \dot{f}_j(t). \end{cases} \quad (23)$$

Define  $\bar{e}_j(t) = [e_{\varpi_j}^T(t) \ e_{f_j}^T(t)]^T$  and the auxiliary reference output  $z_{2j}(t)$ . Then, system (23) can be equivalently expressed as

$$\begin{cases} \dot{e}_{x_j}(t) = (A_j - M_{2j}C_j)e_{x_j}(t) + D_j G e_{\varpi_j} + F_j e_{f_j}(t) + H_j v_j(t) + (\Phi_j(x_j) - \Phi_j(\hat{x}_j)) + (\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)), \\ \dot{\bar{e}}_j(t) = \bar{A}_j \bar{e}_j(t) + \bar{R}_j \xi_j(t) + \bar{H}_j v_j(t) + \bar{I}_j \dot{f}_j(t) \\ z_{2j}(t) = C_j e_{x_j}(t) + \bar{C}_j \bar{e}_j(t) = C_j e_{x_j}(t) + C_{\varpi_j} e_{\varpi_j}(t) + C_{f_j} e_{f_j}(t), \end{cases} \quad (24)$$

where,

$$\bar{A} = \begin{bmatrix} W_j + \Gamma_{2j}D_jG_j & \Gamma_{2j}F_j & K_{2j}D_jG_j & K_{2j}F_j \\ R_j & & & \\ 0 & & & \\ \Gamma_{2j} & & & \\ K_{2j} & & & \end{bmatrix},$$

$$\bar{R}_j = \begin{bmatrix} R_j \\ 0 \end{bmatrix}, \quad \bar{I}_j = \begin{bmatrix} 0 \\ I_j \end{bmatrix},$$

$$\bar{H}_j = \begin{bmatrix} \Gamma_{2j} \\ K_{2j} \end{bmatrix} H_j, \quad \bar{C}_j = [C_{\varpi_j} \ C_{f_j}].$$

Let  $D_{GF_j} = [D_jG_j \ F_j]$ ,  $\bar{W}_j = \begin{bmatrix} W_j & 0 \\ 0 & 0 \end{bmatrix}$ ,  $\Gamma_{2K_{2j}} = \begin{bmatrix} \Gamma_{2j} \\ K_{2j} \end{bmatrix}$ . Then, sufficient condition of asymptotic stability with the ability to attenuate the norm bounded uncertain disturbances for system (23) or (24) will be given in Theorem 2. Meanwhile, the observer gain matrices  $\Gamma_{2j}$ ,  $K_{2j}$  and  $M_{2j}$  can be obtained by solving the LMI.

**Theorem 2:** For the web-winding subsystem (8) with the disturbances in (2), the given the parameters  $\alpha_{ji} > 0$  ( $j = 1, 2, \dots, N; i = 1, 2, 3$ ), the Lipschitz constant  $\gamma_{max}^j > 0$  and  $\kappa_j > 0$ , if there exist matrices  $P_{j1} = P_{j1}^T > 0$ ,  $P_{j2} = P_{j2}^T > 0$ ,  $P_{j1M_{2j}}$  and  $P_{j2\Gamma_{2j}K_{2j}}$  such that

$$\begin{bmatrix} \Omega_{j11} & \Omega_{j12} & 0 & P_{j1}H_j & 0 & P_{j1} \\ \star & \Omega_{j22} & P_{j2}\bar{I}_j & P_{j2\Gamma_{2j}K_{2j}}H_j & P_{j2}\bar{R}_j & 0 \\ \star & \star & -\alpha_{j1}^2 I & 0 & 0 & 0 \\ \star & \star & \star & -\alpha_{j2}^2 I & 0 & 0 \\ \star & \star & \star & \star & -\alpha_{j3}^2 I & 0 \\ \star & \star & \star & \star & \star & \Omega_{j66} \end{bmatrix} < 0, \quad (25)$$

where,  $\Omega_{j11} = P_{j1}A_j + A_j^T P_{j1} - P_{j1M_{2j}}C_j - C_j^T P_{j1M_{2j}}^T + C_j^T C_j + \left( \sum_{i=1}^N \gamma_{max}^i + \kappa_j \right) I$ ,  $\Omega_{j12} = P_{j1}D_{GF_j} + C_j^T \bar{C}_j$ ,  $\Omega_{j22} = P_{j2}\bar{W}_j + \bar{W}_j^T P_{j2} + P_{j2\Gamma_{2j}K_{2j}}D_{GF_j} + D_{GF_j}^T P_{j2\Gamma_{2j}K_{2j}}^T + \bar{C}_j^T \bar{C}_j$ ,

$\Omega_{j66} = -(\kappa_j + N\gamma_{max}^j)^{-1}I$ , notations  $I$  and  $\star$  denote the same meanings as Theorem 1. Then, there exist observers in the form of (21) and (22) such that the augmented estimation error subsystem (23) or (24) is asymptotically stable with the  $H_\infty$  performance level  $\alpha_{ji}$ . Moreover, the observer gain matrices can be obtained by  $M_{2j} = P_{j1}^{-1}P_{j1}M_{2j}$  and  $\Gamma_{2jK_{2j}} = P_{j2}^{-1}P_{j2}\Gamma_{2jK_{2j}}$ .

*Proof:* Choose the following Lyapunov functions

$$V_{j1}(t) = e_{x_j}^T(t)P_{j1}e_{x_j}(t), \quad (26)$$

$$V_{j2}(t) = \bar{e}_j^T(t)P_{j2}\bar{e}_j(t), \quad (27)$$

where,  $P_{j11}$  and  $P_{j12}$  are the real symmetric positive definite matrices. For any  $e_{x_j}(t) \neq 0$  and  $\bar{e}_j(t) \neq 0$ ,  $V_{j1}(t) > 0$  and  $V_{j2}(t) > 0$ . Then, by differentiating (26) and (27) with respect to time along the error trajectory (24) we can get

$$\begin{aligned} \dot{V}_{j1}(t) &= e_{x_j}^T(t)\text{sym}(P_{j1}A_j - P_{j1}M_{2j})e_{x_j}(t) \\ &+ 2e_{x_j}^T(t)P_{j1}D_{GF_j}\bar{e}(t) + 2e_{x_j}^T(t)P_{j1}H_jv_j(t) \\ &+ 2e_{x_j}^T(t)P_{j1}(\Phi_j(x_j) - \Phi_j(\hat{x}_j)) \\ &+ 2e_{x_j}^T(t)P_{j1}(\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)), \end{aligned} \quad (28)$$

and

$$\begin{aligned} \dot{V}_{j2}(t) &= \bar{e}_j^T(t)\text{sym}(P_{j2}\bar{W}_j + P_{j2}\Gamma_{2jK_{2j}}D_{GF_j})\bar{e}_j(t) \\ &+ 2\bar{e}_j^T(t)P_{j2}\bar{R}_j\bar{\xi}_j(t) + 2\bar{e}_j^T(t)P_{j2}\bar{I}_j\bar{f}_j(t) \\ &+ 2\bar{e}_j^T(t)P_{j2}\Gamma_{2jK_{2j}}H_jv_j(t), \end{aligned} \quad (29)$$

where,  $P_{j1}M_{2j} = P_{j1}M_{2j}$  and  $P_{j2}\Gamma_{2jK_{2j}} = P_{j2}\Gamma_{2jK_{2j}}$ .

From (9) we can obviously get that

$$2e_{x_j}^T(t)P_{j1}(\Phi_j(x_j) - \Phi_j(\hat{x}_j)) \leq e_{x_j}^T(t)(\kappa_j P_{j1}P_{j1} + \kappa_j I)e_{x_j}(t). \quad (30)$$

Similarly to (18), we can get

$$\begin{aligned} &2e_{x_j}^T(t)P_{j1}(\delta_j(x_j, x_k, x_l, x_m) - \delta_j(\hat{x}_j, \hat{x}_k, \hat{x}_l, \hat{x}_m)) \\ &\leq N\gamma_{max}^j e_{x_j}^T(t)P_{j1}P_{j1}e_{x_j}(t) + \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t), \end{aligned} \quad (31)$$

where,  $\gamma_{max}^j = \max\{\gamma_j, \gamma_k, \gamma_l, \gamma_m\}$ . Then, it follows from (30) and (31) that (28) can be further evaluated as

$$\begin{aligned} \dot{V}_{j1}(t) &\leq e_{x_j}^T(t)\text{sym}(P_{j1}A_j - P_{j1}M_{2j}) + \kappa_j I \\ &+ (N\gamma_{max}^j + \kappa_j)P_{j1}P_{j1}e_{x_j}(t) \\ &+ 2e_{x_j}^T(t)P_{j1}D_{GF_j}\bar{e}_j(t) + 2e_{x_j}^T(t)P_{j1}H_jv_j(t) \\ &+ \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t). \end{aligned} \quad (32)$$

Subsequently, a new Lyapunov function candidate is chosen as  $V_j(t) = V_{j1}(t) + V_{j2}(t)$ . It is provided that the system (24) has a generalized  $H_\infty$  performance index, i.e.,

$$\|z_{2j}(t)\|^2 < \alpha_{j1}^2 \|d_j(t)\|^2 + \alpha_{j2}^2 \|v_j(t)\|^2 + \alpha_{j3}^2 \|\dot{f}_j(t)\|^2$$

Then, a generalized  $H_\infty$  performance index is equivalent to

$$\begin{aligned} J_{j\infty}(t) &= \int_0^\infty (\|z_{2j}(t)\|^2 - \alpha_{j1}^2 \|\dot{f}_j(t)\|^2 \\ &\quad - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\xi_j(t)\|^2) dt < 0. \end{aligned}$$

Due to that  $V_j(t) > 0$ , then under the zero initial conditions we can get that

$$\begin{aligned} J_{j\infty}(t) &\leq \int_0^\infty (\dot{V}_j(t) + \|z_{2j}(t)\|^2 - \alpha_{j1}^2 \|\dot{f}_j(t)\|^2 \\ &\quad - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\xi_j(t)\|^2) dt. \end{aligned} \quad (33)$$

According to (29), (32) and (33), we can get that

$$\begin{aligned} &J_{j\infty}(t) \\ &\leq \int_0^\infty \{e_{x_j}^T(t)(\text{sym}(P_{j1}A_j - P_{j1}M_{2j}) \\ &\quad + (N\gamma_{max}^j + \kappa_j I)P_{j1}P_{j1})e_{x_j}(t) + 2e_{x_j}^T(t)P_{j1}H_jv_j(t) \\ &\quad + 2e_{x_j}^T(t)P_{j1}D_{GF_j}\bar{e}_j(t) + \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t) \\ &\quad + \bar{e}_j^T(t)\text{sym}(P_{j2}\bar{W}_j + P_{j2}\Gamma_{2jK_{2j}}D_{GF_j})\bar{e}_j(t) \\ &\quad + 2\bar{e}_j^T(t)P_{j2}\bar{R}_j\bar{\xi}_j(t) + 2\bar{e}_j^T(t)P_{j2}\bar{I}_j\bar{f}_j(t) \\ &\quad + 2\bar{e}_j^T(t)P_{j2}\Gamma_{2jK_{2j}}H_jv_j(t) + e_{x_j}^T(t)C_j^T C_j e_{x_j}(t) \\ &\quad + e_{x_j}^T(t)C_j^T \bar{C}_j \bar{e}_j(t) + \bar{e}_j^T(t)\bar{C}_j^T C_j e_{x_j}(t) + \bar{e}_j^T(t)\bar{C}_j^T \bar{C}_j \bar{e}_j(t) \\ &\quad - \alpha_{j1}^2 \dot{f}_j^T(t)\dot{f}_j(t) - \alpha_{j2}^2 v_j^T(t)v_j(t) - \alpha_{j3}^2 \xi_j^T(t)\xi_j(t)\} dt. \end{aligned}$$

Subsequently, the generalized  $H_\infty$  performance index for the overall web-winding system is equivalent to

$$\begin{aligned} \sum_{j=1}^N J_{j\infty}(t) &\leq \int_0^\infty \sum_{j=1}^N (\dot{V}_j(t) + \|z_{2j}(t)\|^2 - \alpha_{j1}^2 \|\dot{f}_j(t)\|^2 \\ &\quad - \alpha_{j2}^2 \|v_j(t)\|^2 - \alpha_{j3}^2 \|\xi_j(t)\|^2) dt. \end{aligned} \quad (34)$$

Additionally,

$$\sum_{j=1}^N \gamma_{max}^j \sum_{i=1}^N e_{x_i}^T(t)e_{x_i}(t) = \sum_{j=1}^N \sum_{i=1}^N \gamma_{max}^i e_{x_j}^T(t)e_{x_j}(t). \quad (35)$$

Then, by (33)-(35) and Schur complement we can get

$$\sum_{j=1}^N J_{j\infty}(t) \leq \int_0^\infty \{ \sum_{j=1}^N \Lambda_j^T \Upsilon_j \Lambda_j \} dt,$$

where,  $\Lambda_j = [e_{x_j}^T(t) \quad \bar{e}_j^T(t) \quad \dot{f}_j^T(t) \quad v_j^T(t) \quad \xi_j^T(t)]^T$ ,  $\Upsilon_j$  is the matrix on the left side of the inequality (25). Obviously, the sufficient condition for  $\sum_{j=1}^N J_{j\infty}(t) < 0$  is  $\Upsilon_j < 0$ . This ends the proof.

#### IV. SIMULATION STUDY

As a typical representation of the multi-motor systems, the three-motor web-winding system can express all important dynamic characteristics of web-winding systems containing unwinding, rewinding and speed traction. Then,



the simulation for three-motor web-winding is conducted in this section to illustrate the effectiveness of the designed distributed fault diagnosis methods. The simulations are carried out on Simulink. It is assumed that the brown paper is used as the web material, and the corresponding parameters are given as follows:  $n_1 = n_2 = n_3 = 1$ ,  $E = 1.5 \times 10^7 N/m^2$ ,  $h = 1.2 \times 10^{-4} m$ ,  $L_1 = L_2 = 1.2 m$ ,  $t_0 = 2 N$ ,  $0.0018 m \leq R_1 \leq 0.1 m$ ,  $R_2 = 0.02 m$ ,  $0.0018 m \leq R_3 \leq 0.1 m$ ,  $\rho = 800 kg/m^3$ ,  $b = 0.1 m$ ,  $b_{f1} = 0.005 Nm/rad/s$ ,  $b_{f2} = 0.0065 Nm/rad/s$ ,  $b_{f3} = 0.0046 Nm/rad/s$ ,  $J_{c1} + n_1^2 J_{m1} = 6.76 \times 10^{-6} kg \cdot m^2$ ,  $J_2 = 8.67 \times 10^{-4} kg \cdot m^2$ ,  $J_{c3} + n_3^2 J_{m3} = 6.76 \times 10^{-6} kg \cdot m^2$ . Meanwhile, the external disturbance vector  $d_j(t)$  caused by rotating motor is assumed to be periodic and is a harmonic disturbance described by (2) with  $\varpi_j(0) = \hat{\varpi}_j(0) = [0 \ 0]^T$ ,  $W_j = W$ ,  $R_j = R$  and

$$W = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad G_2 = [1 \ 0], \quad G_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

In the simulation, we select  $\xi_j(t)$  and  $v_j(t)$  as the random signal with upper 2-norm bound 1 and chose the weighting matrices  $C_{\varpi_j}$  and  $C_{f_j}$  as

$$C_{\varpi_1} = C_{\varpi_3} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_{\varpi_2} = [1 \ 0],$$

$$C_{f_1} = [0 \ 1]^T, \quad C_{f_2} = 1, \quad C_{f_3} = [0 \ 1]^T.$$

**A. SOLUTIONS OF TWO METHODS**

For  $\alpha_{j1} = \alpha_{j2} = \alpha_{j3} = 1.5$ ,  $\kappa_j = \gamma_{max}^j = 1 \times 10^4$ , the gain matrices  $K_{1j}$  and  $M_{1j}$  of disturbance attenuation based distributed fault estimation observer in Subsection III-A are achieved by solving LMI (14) in Theorem 1 as follows

$$K_{11} = [-0.0000 \quad -105.2851],$$

$$M_{11} = 1.0e + 04 * \begin{bmatrix} 2.4420 & -0.0083 \\ -0.0083 & 2.4407 \end{bmatrix},$$

$$K_{12} = -105.2851, \quad M_{12} = 2.4407e + 04,$$

$$K_{13} = [0.0000 \quad -105.2851],$$

$$M_{13} = 1.0e + 04 * \begin{bmatrix} 2.4420 & 0.0084 \\ 0.0084 & 2.4407 \end{bmatrix}.$$

Similarly, by solving the LMI (25) in Theorem 2, the gain matrices  $\Gamma_{2j}$ ,  $K_{2j}$  and  $M_{2j}$  of distributed fault estimation observer in Subsection III-B can be obtained as

$$\Gamma_{21} = \begin{bmatrix} -2.8611 & 0.9427 \\ -0.1145 & -2.1464 \end{bmatrix}, \quad K_{21} = \begin{bmatrix} 1.0618 \\ -2.5396 \end{bmatrix}^T,$$

$$M_{21} = \begin{bmatrix} 593.8089 & -74.9309 \\ -74.1676 & 589.0015 \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} -2.9907 \\ -1.8982 \end{bmatrix}, \quad K_{22} = -2.7989, \quad M_{22} = 582.7551,$$

$$\Gamma_{23} = \begin{bmatrix} -2.8611 & 0.9427 \\ -0.1145 & -2.1464 \end{bmatrix}, \quad K_{23} = \begin{bmatrix} 1.0618 \\ -2.5396 \end{bmatrix}^T,$$

$$M_{23} = \begin{bmatrix} 593.5714 & 74.2354 \\ 73.4646 & 589.2391 \end{bmatrix}.$$

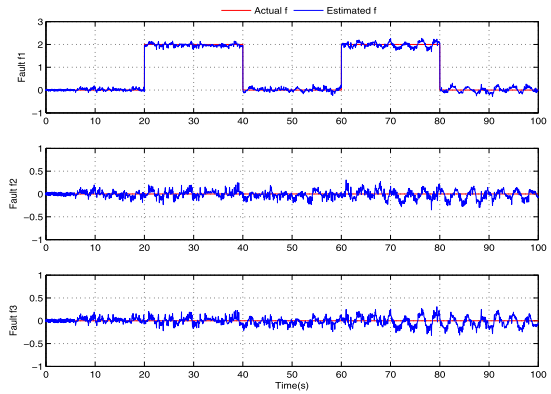


FIGURE 3. Disturbance attenuation based method under Case 1.1.

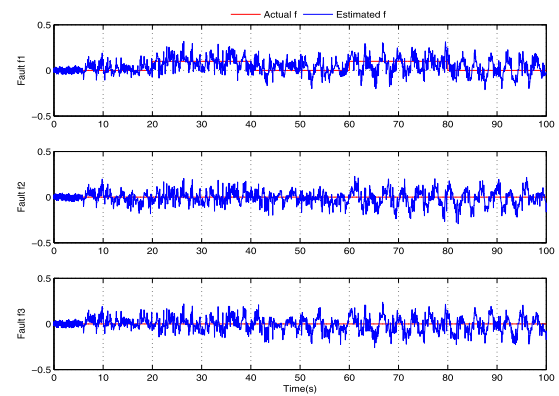


FIGURE 4. Disturbance attenuation based method under Case 1.2.

**B. COMPARISON SIMULATION RESULTS AND ANALYSIS**

In this subsection, the effectiveness of the proposed disturbance attenuation based distributed fault estimation method in Subsection III-A and the disturbance compensation based distributed fault estimation method in Subsection III-B are verified. In practice, the uneven wear or fracture of the bearing and linkages will lead to the web-winding system actuator intermittent faults. Due to that the web-winding system is driven timely by motors, then it is reasonable to assume that the system faults caused by the driven motor are periodic. Additionally, the step function is chosen to simulate the web-winding system actuator lock-in-place. In order to comprehensively test the proposed distributed fault diagnosis methods, the following three cases will be considered in simulation. Simulation results are shown in Figures. 3-14.

Case 1: An actuator abrupt fault occurring at the first subsystems as follows.

$$Case\ 1.1 : \begin{cases} f_1 = 2, & 20s < t \leq 40s, 60s < t \leq 80s \\ f_2 = f_3 = 0, & 0s \leq t \leq 100s \end{cases}$$

$$Case\ 1.2 : \begin{cases} f_1 = 0.1, & 20s < t \leq 40s, 60s < t \leq 80s \\ f_2 = f_3 = 0, & 0s \leq t \leq 100s \end{cases}$$

Figures 3-6 show the fault estimation results under Case 1. Obviously, amplitude of the fault in Case 1.2 is smaller

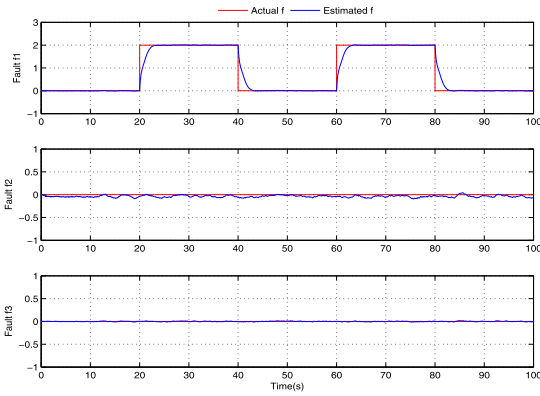


FIGURE 5. Disturbance compensation based method under Case 1.1.

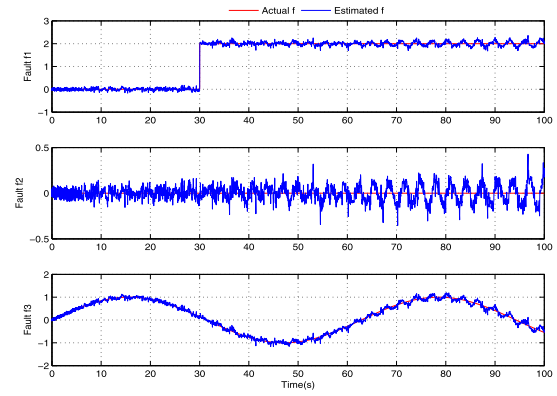


FIGURE 7. Disturbance attenuation based method under Case 2.1.

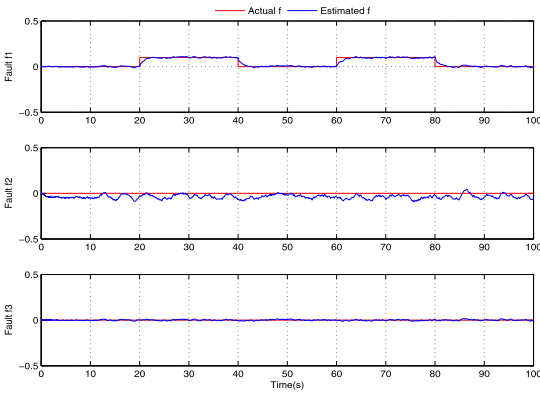


FIGURE 6. Disturbance compensation based method under Case 1.2.

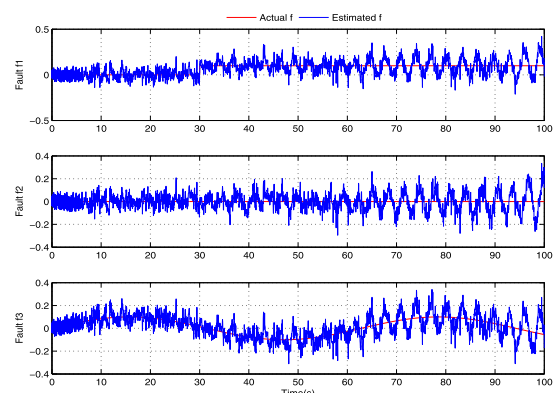


FIGURE 8. Disturbance attenuation based method under Case 2.2.

than it in Case 1.1, which is used to imitate the incipient intermittent fault. Figure 3 and Figure 5 show the distributed fault estimation results under Case 1.1. It is obvious that the proposed two distributed fault estimation methods can track the system actual faults, and the estimation time of the disturbance attenuation based method in Subsection III-A is shorter than the disturbance compensation based method in Subsection III-B. But the accuracy of disturbance compensation based method is higher than the disturbance attenuation based method. Figure 4 and Figure 6 are the fault estimation results under Case 1.2. Obviously, it is difficult to distinguish the incipient fault from the multiple disturbances or uncertainties by disturbance attenuation based method in Subsection III-A. Nevertheless, the incipient fault can be estimated with a high accuracy via the disturbance compensation based method in Subsection III-B.

Case 2: Abrupt fault and time-varying sinusoidal fault existing simultaneously in the first and third subsystems.

$$\begin{aligned}
 \text{Case 2.1 : } & \begin{cases} f_1 = 2, & 30s < t \leq 100s \\ f_2 = 0, & 0s \leq t \leq 100s \\ f_3 = \sin(0.1t), & 0s \leq t \leq 100s \end{cases} \\
 \text{Case 2.2 : } & \begin{cases} f_1 = 0.1, & 30s < t \leq 100s \\ f_2 = 0, & 0s \leq t \leq 100s \\ f_3 = 0.1\sin(0.1t), & 0s \leq t \leq 100s \end{cases}
 \end{aligned}$$

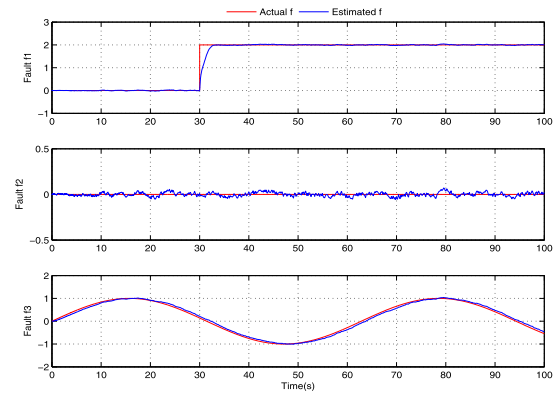


FIGURE 9. Disturbance compensation based method under Case 2.1.

Similarly to the Case 1, aiming at the faults with a large amplitude, the proposed two methods can track the system actual faults from Figure 7 and Figure 9. However, the estimation accuracy of the method in Subsection III-B is higher than the method in Subsection III-A. From Figure 8 and Figure 10, it further verify that the disturbance compensation based fault estimation method in Subsection III-B is more suitable to estimate the system incipient faults than the method in Subsection III-A.

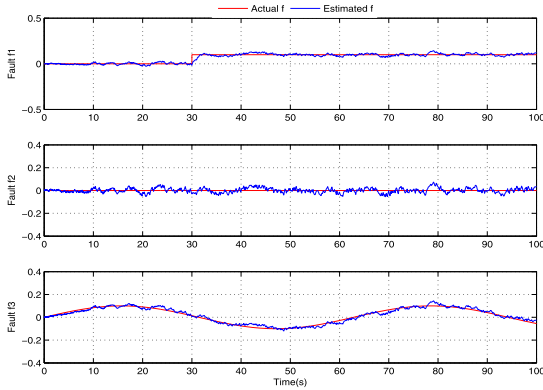


FIGURE 10. Disturbance compensation based method under Case 2.2.

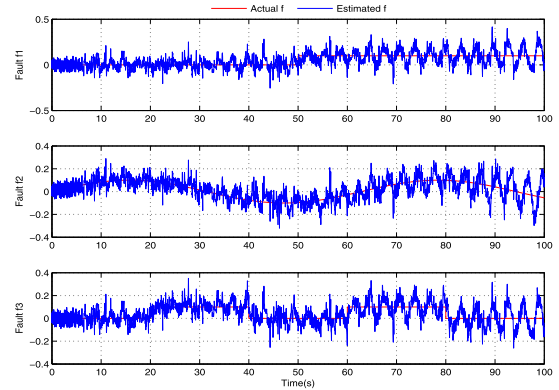


FIGURE 12. Disturbance attenuation based method under Case 3.2.

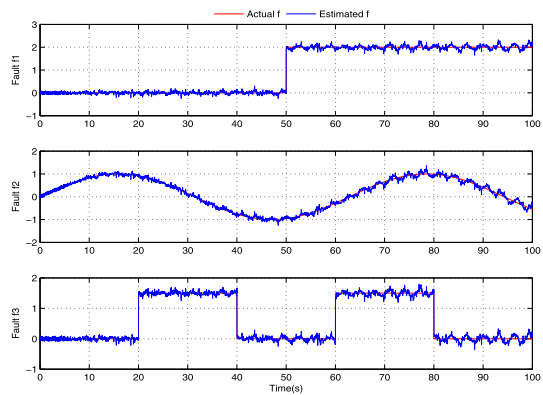


FIGURE 11. Disturbance attenuation based method under Case 3.1.

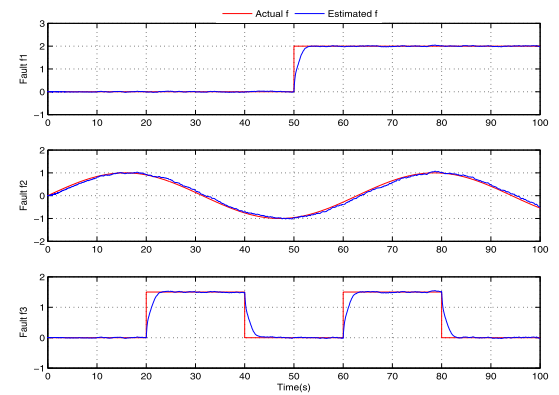


FIGURE 13. Disturbance compensation based method under Case 3.1.

Case 3: Three subsystems suffer from the abrupt fault, time-varying sinusoidal fault and intermittent fault, respectively.

$$Case\ 3.1: \begin{cases} f_1 = 2, & 50s < t \leq 100s \\ f_2 = \sin(0.1t), & 0s \leq t \leq 100s \\ f_3 = 1.5, & 20s < t \leq 40s, 60s < t \leq 80s \end{cases}$$

$$Case\ 3.2: \begin{cases} f_1 = 0.1, & 50s < t \leq 100s \\ f_2 = 0.1\sin(0.1t), & 0s \leq t \leq 100s \\ f_3 = 0.1, & 20s < t \leq 40s, 60s < t \leq 80s \end{cases}$$

The Case 1 and Case 2 are the fault estimation results under the situations that the system suffers from one or two subsystem faults. In order to further verify the effectiveness of the design two methods, Case 3 assumes that the three subsystems suffers from various faults at different time, and Figures 11-14 show the simulation results. It is similar to Case 1 and Case 2, the disturbance compensation based method in Subsection III-B can track the system actual faults, and it is suitable to deal with the incipient or minor faults. The disturbance attenuation based distributed fault estimation method in Subsection III-A can be used to estimate the system fault with a big amplitude.

Overall, from the simulation results in Figures 3-14 we can get that if the system suffers from the faults among the

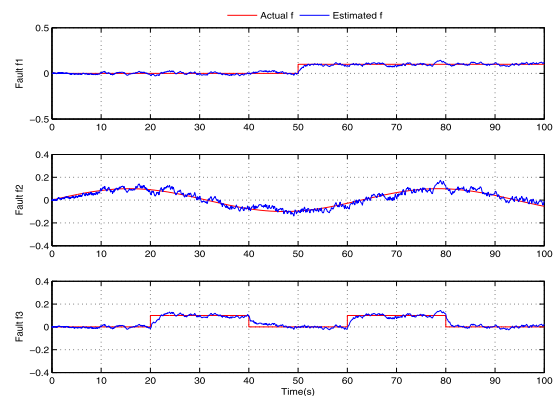


FIGURE 14. Disturbance compensation based method under Case 3.2.

Cases 1-3 with big amplitudes, the proposed two methods all can track the system actual faults. However, if the system suffers from incipient faults, the method in Subsection III-A will have a worse performance, or even cannot estimate the system faults. The method in Subsection III-B has a better robustness to the disturbances or uncertainties and can be used for the incipient or minor faults with a high accuracy. Despite that the fault estimation time of the method in Subsection III-B is longer than the former one, it is less than 4s and can meet the actual requirement. It is also clear that the proposed

disturbance compensation based distributed fault diagnosis method not only can obtain the objective of fault detection, but also can realize the fault isolation and fault estimation. It can be used in the field of CBM and fault tolerant control of the web-winding systems.

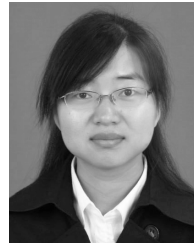
## V. CONCLUSION

In this paper, by regarding the large-scale web-winding system as a synthetic system with several dynamic subsystems, the problem of distributed fault diagnosis for web-winding system is studied based on the methods of disturbance compensation or disturbance attenuation. Then, aiming at each subsystem, the disturbance compensation based fault diagnosis method and disturbance attenuation based distributed fault diagnosis method are designed by considering the multiple disturbances and various system actuator faults. Meanwhile, sufficient conditions of asymptotic stability of the estimation error systems are derived based on the Lyapunov theory, and the observer gain matrices are obtained by solving the LMIs. Finally, simulations results show that the proposed distributed fault diagnosis methods can realize the aims of fault detection, fault isolation and fault estimation, and it can provide the decision-making for CBM and fault tolerant control. However, in this study the  $H_\infty$  performance suppression level  $\alpha_{ji}$  are selected without optimization for the two methods. Therefore, we will focus on the optimal problem in the future work. Beyond that, we will further be concentrated on the experimental tests.

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