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# Consensus-Based Multi-Person Decision Making Using Consistency Fuzzy Preference Graphs

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**ABSTRACT** This paper presents consensus based multi-person decision making (MPDM) using consistency graphs (additive consistent and order consistent) in a fuzzy environment. At the first level, consistency analysis is put forward after defining consistent fuzzy preference graph (CFPG) with the help of additive transitivity. This analysis further leads us to determine priority weights vector of the decision-makers (DMRs) after evaluation consistency weights. At the second level, the consensus analysis helps us to determine whether the selection process should be initiated or not. If the consensus degree amongst DMRs does not reach a minimum acceptable level, then the enhancement mechanism plays a central role to improve the consensus level by giving suitable suggestions to DMRs. In the end, the weighted sum operator (WSO) is used to get aggregated consistency fuzzy preference graph ($A_g$CFPG) and the order consistency property provides us sufficient information to rank the alternatives.

**INDEX TERMS** Fuzzy graph (FG), fuzzy preference graph (FPG), additive consistent fuzzy preference graph (ACFPG), order consistent fuzzy preference graph (OCFPG), fuzzy compatibility graph (FCG).

## I. INTRODUCTION AND PRELIMINARY RESULTS

Decision making (DM) is a rational process being used to choose the best option(s) from a set of different options, it pledges when someone has to do something but does not know what. Everyone experiences DM situations in his/her daily life: commonly, to shopping, to select what to eat, and to decide whom or what to vote for in a referendum or election. DM can be classified in numerous diverse groups under certain individualities as the source(s) for the statistics and the preference representation layouts that are used to handle the decision problem. In our structure, the selection of the best alternative(s) from a prearranged set $X = \{x_1, x_2, \ldots, x_n\}$, $n \geq 2$ of possible alternatives is the goal.

DM is not only the situation for an individual, where he/she provides a pairwise assessment of alternatives, but some problems have to be described by a group of DMRs who work together to conclude the best option(s). The procedure to solve DM problem(s) with multiple experts is called group decision making (GDM) or also known as MPDM. Preference relation is the most common representation format used in GDM because it is a valuable tool in modeling decision processes, when we have to combine DMRs' preferences into group preferences [1].

In fuzzy framework, a DMR allocates numerical value from [0, 1] to each pair of alternatives which shows the preference degree of one alternative over the other. A very natural question arises while assigning the values: which conditions have to be satisfied in order to obtain consistent results in final ranking? Because inconsistency leads decision making procedure to unreliable conclusions that is why it is important to study conditions under which consistency is associated. On the other hand, perfect consistency is hard to achieve in reality, particularly when ranking a set with large number of alternatives. Consistency is directly associated to transitivity property, and various such properties in literature and consistency may be shown accordingly.

Several procedures on consistency measure and enhancement of preference values have been offered in a successive way [2]–[7], [9], [13]. While to handle MPDM problems

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in authentic manner, the consensus measure plays crucial role. Numerous consensus models have been proposed in literature: Herrera-Viedma *et al.* [8], in 2002, presented a consensus based scheme to handle GDM situations in different preference formats, utility values, and multiplicative preference relations. In 2007, Herrera-Viedma *et al.* [10] proposed a procedure to investigate the consistency level and consensus measure for incomplete fuzzy preference relations and a feedback mechanism was introduces to improve the consistency and consensus degrees, simultaneously. While in 2013, Xia *et al.* [11] proposed the multiplicative consistency based consensus of reciprocal preference matrices and examined an algorithm to enhance consensus level for given preferences. Wu and Chiclana [12], in 2014, proposed a visual information feedback mechanism for GDM problems with triangular fuzzy complementary preference relations to identify experts, alternatives and corresponding preference values that contribute less to consensus. To provide a general framework for existing methods, in 2015, Xia and Chen [13] defined a consensus index of individual pairwise comparison matrices and developed two consensus improving methods by introducing a general aggregation operator based on Abelian linearly ordered group. In 2016, Zhang *et al.* [14] developed a consensus building method based on multiplicative consistency for GDM with IRPRs. Zhang and Pedrycz [15], in 2018, presented goal programming models in order to enhance consistency and consensus measures for intuitionistic fuzzy and multiplicative preference relations, respectively. In 2019, Atiq-ur-Rehman *et al.* [16] proposed a consensus-based hybrid technique for multi-person decision making.

A graph is a way to represent a specific affiliation between the objects and provides an idea to observe the level of the association between any two objects of a universe of discourse. If proper weights of relationship between the objects are given, then the problem can be solved by using a weighted graph. But in a natural sense, most of the situations carry relationships in fuzzy environment. For instance, if $L$ shows certain locations in a city and the construction of a network of roads between elements of $L$ is the aim, then the costs of construction of the links are fuzzy. But by using the topography and local factors, the costs can be compared to some extent and fuzzy relations can be formed. Thus, fuzzy graph models are more helpful and realistic in natural situations.

In 1973, the first definition of fuzzy graph was proposed by Kaufmann [17], based on Zadeh's fuzzy relations [18]. But in 1975, the foundations of fuzzy graph theory were laid by Rosenfeld [19] after introducing fuzzy analogs of a number of basic graph-theoretic notions carrying with, subgraphs, paths, connectedness, groups, bridges, cut-vertices, forests, and trees. In 1994, Mordeson and Peng [20] investigated and proposed some operations on fuzzy graphs. In 2009, Gani and Radha [21] measured the degrees of the vertices graphs and the resultant fuzzy graphs obtained under the operations defined in [20]. In 2012, Akram and Davvaz [22] investigated the Intuitionistic fuzzy graphs and defined the strong Intuitionistic graphs. In 2017, Ashraf *et al.* [23] Naz *et al.* [24] proposed the notions of single valued neutrosophic graphs and their use in multi criteria decision-making. More recently, Akram et al. introduced many new concepts related to m-polar fuzzy graph, fuzzy soft graph, rough fuzzy graph, neutrosophic graph and their extensions [25], [26].

*Definition 1 Directed Graph [27]*: A directed graph $G_d$ is a pair $G_d = (V, E)$, where $V$ is the set of vertices and $E$ is the set of arcs. Each element $(a, b)$ of $E$ is the ordered pair, which denotes the arc from the vertex $a$ to $b$, while the pair $(b, a)$ means the opposite direction arc.

In numerous applications, each edge in a graph carries a connected numerical value, called a weight which is usually nonnegative in nature. Both, directed and undirected graphs, may be weighted.

*Definition 2 Fuzzy Graph*: A fuzzy graph $G_f = (V_f, E_f, w_{V_f}, w_{E_f})$ is a weighted graph together with a pair of functions $w_{V_f} : V_f \longrightarrow [0, 1]$ and $w_{E_f} : E_f \longrightarrow [0, 1]$, known as vertex-wieght function and edge-weight function respectively, where $V_f$ is called the fuzzy set of vertices and $E_f$ is the fuzzy set of edges.

If a fuzzy graph $G_f$ carries a function $w_{E_f}$ such that $w_{E_f} : E_f \longrightarrow 0$ i.e., for all edges $e \in E_f$, $w_{E_f}(e) = 0$, then $G_f$ is a vertex-weighted fuzzy graph. On the other hand, if $w_{V_f} : V_f \longrightarrow 0$ i.e., for all vertices $v \in V_f$, $w_{V_f}(v) = 0$, then $G_f$ is an edge-weighted fuzzy graph.

*Definition 3 Fuzzy Preference Graph*: A directed graph $G_{fp} = (A, E_f, w_{E_f})$ together with a function $w_{E_f} : E_f \longrightarrow [0, 1]$, is called fuzzy preference graph (FPG), where $A$ is a set of alternatives (nodes) and $E_f$ is a collection of edges $e_{ij} (i, j \in N)$ of alternative $a_i$ to alternative $a_j$, and edge-weight function $w_{E_f}$ assigns the weight $w_{ij} \in [0, 1]$ to the edge $e_{ij}$ and denotes the degree of preference of alternative $a_i$ to the alternative $a_j$, such that:

$$w_{ij} + w_{ji} = 1.$$

*Note:-* $w_{ij} = 0.5$ indicates that alternatives $a_i$ and $a_j$ are equally preferred. If $w_{ij} > 0.5$, then alternative $a_i$ is superior to alternative $a_j$ while $w_{ij} < 0.5$ shows that $a_i$ is not preferable over $a_j$. If $w_{ij} = 1$, then the alternative $a_i$ has a definite preference over the alternative $a_j$.

*Definition 4 Consistent Fuzzy Preference Graph*: A fuzzy preference graph $\widetilde{G}_{fp} = (A, \widetilde{E}_f, w_{\widetilde{E}_f})$ is said to be consistent fuzzy preference graph (CFPG), if there exist a transitive function $Tr$ such that:

$$w_{ik} = Tr(w_{ij}, w_{jk}),$$

for all intermediate alternatives $A_j$ and $w_{ik}, w_{ij}, w_{jk} \in [0, 1]$ with $i \neq j \neq k$.

*Definition 5 Additive Consistent Fuzzy Preference Graph*: A FPG is said to be additive consistent if for all intermediate alternatives

$$w_{ik} = w_{ij} + w_{jk} - 0.5 \tag{1}$$

holds (i.e., $T(w_{ij}, w_{jk}) = w_{ij} + w_{jk} - 0.5$), for instance, as shown in the following Figure 1.

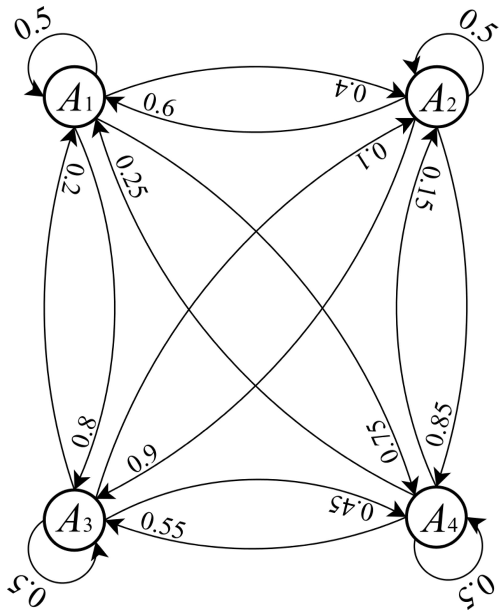
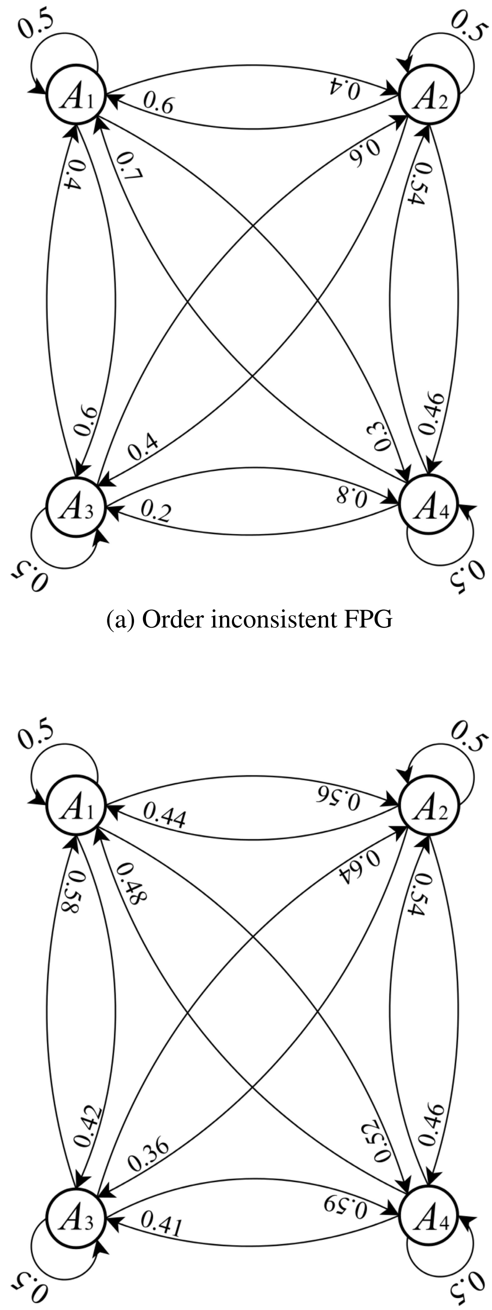



**FIGURE 1.** **Additive consistent fuzzy preference graph (ACFPG).**




(a) Order inconsistent FPG

(b) Order consistent FPG

**FIGURE 2.** **Order inconsistent and order consistent FPGs, respectively.**

*Definition 6 Order Consistent FPG*: An FPG $\overline{G}_{fp} = (A, \overline{E}_f, w_{\overline{E}_f})$ together with a function $w_{\overline{E}_f} : \overline{E}_f \longrightarrow [0, 1]$ which assigns weights to edges in such a way that $w_{ik} \leq w_{il}$ for all $1 \leq i \leq n$ and $k, l \in \{1, 2, \ldots, n\}$ $((n > 1) \in N)$, is called order consistent FPG (OCFPG).

For example, FPG in Figure 2(a) is not order consistent because if we observe, $w_{12} = 0.6$ and $w_{14} = 0.3$ indicate that alternative $A_4$ is preferable to alternative $A_2$ while $w_{32} = 0.6$ and $w_{34} = 0.8$ result in that alternative $A_2$ has preference to alternative $A_4$, and hence, order consistency is voilated. But, the Figure 2(b) is an OCFPG which provides that $w_{i3} \leq w_{i1}$, $w_{i1} \leq w_{i4}$ and $w_{i4} \leq w_{i2}$ for all $1 \leq i \leq 4$.

- $w_{i3} \leq w_{i1}$ indicates that alternative $A_3$ is prefered to alternative $A_1$;
- $w_{i1} \leq w_{i4}$ indicates that alternative $A_1$ is prefered to alternative $A_4$;
- $w_{i4} \leq w_{i2}$ indicates that alternative $A_4$ is prefered to alternative $A_2$.

Therefore, $A_3 \succ A_1 \succ A_4 \succ A_2$ is the preference order

*Definition 7 Consistency FPG*: A FPG that conform the both, additive consistency and order consistency at the same time, is known consistency FPG.

*Proposition 8*: If a FPG carries a set $A$ of $n$ vertices (alternatives) i.e., $A = \{A_1, A_2, \ldots, A_n\}$ with $w_{ik}$, used to denote the preference weight of alternative $A_i$ to alternative $A_k$ such that $w_{ik} + w_{ki} = 1$, $w_{ii} = 0.5$ and $w_{ik} = w_{ij} + w_{jk} - 0.5$, then a consistency FPG $\widetilde{\widetilde{G}}_{fp}$ can be formed using all intermediate alternatives $A_j$ based on given FPG by repeated application of

$$\widetilde{\widetilde{w}}_{ik} = \frac{1}{2(n-2)} \sum_{\substack{j=1 \\ j \neq i \neq k}}^{n} (w_{ij} - w_{ji} + w_{jk} - w_{kj}) + 0.5 \tag{2}$$

with $\widetilde{\widetilde{w}}_{ik} + \widetilde{\widetilde{w}}_{ki} = 1$.

*Proof:* Based on Eq. (1), following $(n-2)$ equations can be established against preference weigt $w_{ik}$ for all $j \neq i \neq k$:

$$\begin{aligned} w_{ik} &= w_{i1} + w_{1k} - 0.5, \\ w_{ik} &= w_{i2} + w_{2k} - 0.5, \\ &\;\;. \\ &\;\;. \\ w_{ik} &= w_{in} + w_{nk} - 0.5. \end{aligned}$$

The average of all the above equations results in:

$$\begin{aligned}
\overline{w}_{ik} &= \frac{(w_{ik} + w_{ik} + \ldots + w_{ik}(n-2)\text{ times})}{n-2} \\
&= \frac{1}{n-2}\{(w_{i1} + w_{1k} - 0.5) + (w_{i2} + w_{2k} - 0.5)\} \\
&\quad + \ldots + \frac{1}{n-2}\{(w_{in} + w_{nk} - 0.5)\} \\
&= \frac{1}{n-2}\left\{\left(\frac{w_{i1} + w_{i1}}{2} + \frac{w_{1k} + w_{1k}}{2}\right)\right\} \\
&\quad + \frac{1}{n-2}\left\{\left(\frac{w_{i2} + w_{i2}}{2} + \frac{w_{2k} + w_{2k}}{2}\right)\right\} \\
&\quad + \ldots + \frac{1}{n-2}\left\{\left(\frac{w_{in} + w_{in}}{2} + \frac{w_{nk} + w_{nk}}{2}\right)\right\} \\
&\quad - 0.5 \\
&= \frac{1}{2(n-2)}\{(w_{i1} - w_{1i} + w_{1k} - w_{k1} + 2)\} \\
&\quad + \frac{1}{2(n-2)}\{(w_{i2} - w_{2i} + w_{2k} - w_{k2} + 2)\} \\
&\quad + \ldots + \frac{1}{2(n-2)}\{(w_{in} - w_{ni} + w_{nk} - w_{kn} + 2)\} \\
&\quad - 0.5 \\
&= \frac{1}{2(n-2)}\{(w_{i1} - w_{1i} + w_{1k} - w_{k1})\} \\
&\quad + \frac{1}{2(n-2)}\{(w_{i2} - w_{2i} + w_{2k} - w_{k2})\} \\
&\quad + \ldots + \frac{1}{2(n-2)}\{(w_{in} - w_{ni} + w_{nk} - w_{kn})\} \\
&\quad + \frac{1}{2(n-2)}\{2\,(n-2) - (n-2)\} \\
\overline{w}_{ik} &= \frac{1}{2(n-2)}\{(w_{i1} - w_{1i} + w_{1k} - w_{k1})\} \\
&\quad + \frac{1}{2(n-2)}\{(w_{i2} - w_{2i} + w_{2k} - w_{k2})\} \\
&\quad + \ldots + \frac{1}{2(n-2)}\{(w_{in} - w_{ni} + w_{nk} - w_{kn})\} \\
&\quad + 0.5 \\
&= \frac{1}{2(n-2)}\{(w_{i1} - w_{1i} + w_{1k} - w_{k1})\} \\
&\quad + \frac{1}{2(n-2)}\{(w_{i2} - w_{2i} + w_{2k} - w_{k2})\} \\
&\quad \times v \ldots + \{w_{in} - w_{ni} + w_{nk} - w_{kn})\} + 0.5 \\
&= \frac{1}{2(n-2)}\sum_{\substack{j=1 \\ j\neq i\neq k}}^{n}(w_{ij} - w_{ji} + w_{jk} - w_{kj}) + 0.5.
\end{aligned}$$

Hence, we can establish

$$\widetilde{\widetilde{w}}_{ik} = \frac{1}{2(n-2)}\sum_{\substack{j=1 \\ j\neq i\neq k}}^{n}(w_{ij} - w_{ji} + w_{jk} - w_{kj}) + 0.5.$$

*Definition 8 Fuzzy Compatibility Graph*: A fuzzy compatibility graph (FCG) is an ordered pair $G_{fc} = (A, \xi)$ together with a function $\xi : A \times A \longrightarrow [0, 1]$, where $A$ is a set of nodes (elements or alternatives), such that

(i). $a \in A$ implies that $\xi(a, a) = 1$; (reflexivity)

(ii). for all $(a, b) \in A \times A$, $\xi(a, b) = \xi(b, a)$. (symmetry)

## II. GROUP DECISION MAKING USING FUZZY PREFERENCE GRAPHS

This section presents an hybrid consistency and consensus based GDM using FPGs under the transitive consistency, and final decision is established based on order consistency property. Assume that there are set $A = \{A_1, A_2, \ldots, A_n\}$ of $n$ alternatives (vertices) and set $D = \{D_1, D_2, \ldots, D_m\}$ of $m$ DMRs. Suppose that $G^q_{fp} = (A, E^q_f, w_{E^q_f})$ be a FPG provided by the decision maker $D_q$, where $w_{E^q_f} : E^q_f \longrightarrow [0, 1]$ which assigns the weight $w^q_{ik} \in [0, 1]$ to the edge $e^q_{ik}$ to represent the preference degree of alternative $A_i$ to alternative $A_k$. The proposed GDM procedure is elaborated with several stages as follows:

### A. CONSISTENCY ANALYSIS

Undeniably, consistency is the substantial issue to admit when information is given by the expert, the deficiency of consistency in DM with the data leads to an inconsistent conclusion. The consistency fuzzy preference graph $\widetilde{\widetilde{G}}^q_{fp}$ parallel to FPG $G^q_{fp}$, $q = 1, 2, \ldots, m$, can be constructed with the repeated application of Eq.(2), and then we can measure the consistency degree of $G^q_{fp}$ based on its edge-weights' similarity with the corresponding weights of $\widetilde{\widetilde{G}}^q_{fp}$ by computing the distances between them. Following three stages play role to evaluate the consistency degree of a FPG:

1) *Construction of fuzzy compatibility graph*: At this level, we construct FCG $G^q_{fc}$, which represents the consistency degree of pairs of vertices (alternatives) in $G^q_{fp}$, by using

$$\xi^q_{ik} = 1 - d(w^q_{ik}, \widetilde{\widetilde{w}}^q_{ik}), \tag{3}$$

where $d(w^q_{ik}, \widetilde{\widetilde{w}}^q_{ik})$ shows the distance (error) measured by $\left|w^q_{ik} - \widetilde{\widetilde{w}}^q_{ik}\right|$. Apparently, the higher the value of $\xi^q_{ik}$, the more consistent $w^q_{ik}$ is with respect to the rest of the preference weights take in alternatives $A_i$ and $A_k$.

2) *Consistency degree of a particular alternative*: The consistency degree related to a particular alternative $A_i$, $1 \leq i \leq n$, of $G^q_{fp}$ is measured by taking the average of compatibility weights of alternative $A_i$ to rest of the alternatives as:

$$cd(A_i) = \frac{1}{n-1}\sum_{\substack{k=1 \\ k\neq i}}^{n}\xi^q_{ik}, \tag{4}$$

where $cd(A_i) \in [0, 1]$. When $cd(A_i) = 1$, then all the preference weights related to alternative $A_i$ are fully consistent, if not, the lower $cd(A_i)$ the more inconsistent these preference weights are.

3) *Consistency degree of $G^q_{fp}$*: Finally, the average of all consistency degrees against alternatives $A_i$, $1 \leq i \leq n$,

results in the consistency degree of $G_{fp}^{q}$ as:

$$cd(G_{fp}^{q}) = \frac{1}{n}\sum_{i=1}^{n} cd(A_i), \tag{5}$$

where $cd(G_{fp}^{q}) \in [0, 1]$. If $cd(G_{fp}^{q}) = 1$, the FPG $G_{fp}^{q}$ is fully consistent, else, the lower $cd(G_{fp}^{q})$ the more inconsistent FPG is.

### B. ASSIGNING PRIORITY WEIGHTS TO DMRs

Once, the consistency degrees of $G_{fp}^{q}$, $1 \leq q \leq m$, are measured, it is rational to allocate the higher weights to the DMRs carrying FPGs with larger consistency degrees correspondingly. Following relation can be used to assign the weights to the DMRs based on their provided FPGs, and known as consistency weights

$$C_w(D_q) = \frac{cd(G_{fp}^{q})}{\sum\limits_{q=1}^{m} cd(G_{fp}^{q})}, \tag{6}$$

while $\sum\limits_{q=1}^{m} C_w(D_q) = 1$. Moreover, if DMRs carry predefined priority weights $\beta = \{\beta_1, \beta_2, \ldots, \beta_m\}$, then the final priority weights to DMRs will be assigned by emerging $\beta_q$, $1 \leq q \leq m$, and respective consistency weights $C_w(D_q)$, $1 \leq q \leq m$, under the relation

$$w(D_q) = \frac{\beta_q \times C_w(D_q)}{\sum\limits_{q=1}^{m} \left(\beta_q \times C_w(D_q)\right)}, \tag{7}$$

where $\sum\limits_{q=1}^{m} w(D_q) = 1$. If DMRs came without having experts predefined priority weights $\beta_q$, then the consistency weights will be taken as the priority weights of DMRs.

### C. CONSENSUS ANALYSIS

As it is revealed in Section 1 that the consensus plays an important role while a number of DMRs intract to reach a decision, hence, measure the consensus among the DMRs is necessary. In this context, the fuzzy compatibility graphs $G_{fc}^{qr}$ for every pair $(D_q, D_r)$, $(q = 1, 2, \ldots, m-1;\ r = q+1, \ldots, m)$, are to be constructed using

$$\xi_{ik}^{qr} = 1 - \left|w_{ik}^{q} - w_{ik}^{r}\right|. \tag{8}$$

Then the collective FCG $G_{fc}$ is obtained after aggregating all $G_{fc}^{qr}$ by applying following formula

$$\xi_{ik} = \frac{2}{m(m-1)}\sum_{q=1}^{m-1}\sum_{r=q+1}^{m} \xi_{ik}^{qr}, \tag{9}$$

where every compatibility weight $\xi_{ik}$, $1 \leq i, k \leq n$, represents the consensus measure among DMRs for pair of alternatives $A_i$ and $A_k$. The consensus degree amongst DMRs for a particular alternative $A_i$ is estimated by taking the average of its compatibility weights to rest of alternatives as:

$$CD(A_i) = \frac{1}{n-1}\sum_{\substack{k=1\\k\neq i}}^{n} \xi_{ik}. \tag{10}$$

Finally, the consensus level amongst DMRs on the given infromation can be measured by the average of consensus degrees of all alternatives $A_i$, $1 \leq i \leq n$, as

$$CD(G_{fc}) = \frac{1}{n}\sum_{i=1}^{n} CD(A_i). \tag{11}$$

Once the consensus degree amongst DMRs is estimated, it entails to compare with pre-settled consensus level $\mu$ (say). If $CD(G_{fc}) \geq \mu$, then an acceptable level of consensus is reached and the selection process initiates, otherwise, a enhancement mechanism originates to reach at an acceptable level.

### D. ENHANCEMENT MECHANISM

The main objective of enhancement mechanism is to provide comprehensive information to DMRs to improve their preference weights and reach an acceptable consensus level. We have to identify the pairs of alternatives which have to enhance their preference weights, in this context, following formula helps us:

$$R_{imp}^{q} = \{(A_i, A_k) \mid \xi_{ik} < CD(G_{fc})\}, \tag{12}$$

for $i \neq k \in \{1, 2, \ldots, n\}$. The system indorses that the respective DMR has to enhance the preference weights if they are smaller than the mean values of evaluations of the rest of DMRs, or decrease them if they are larger than the mean values.

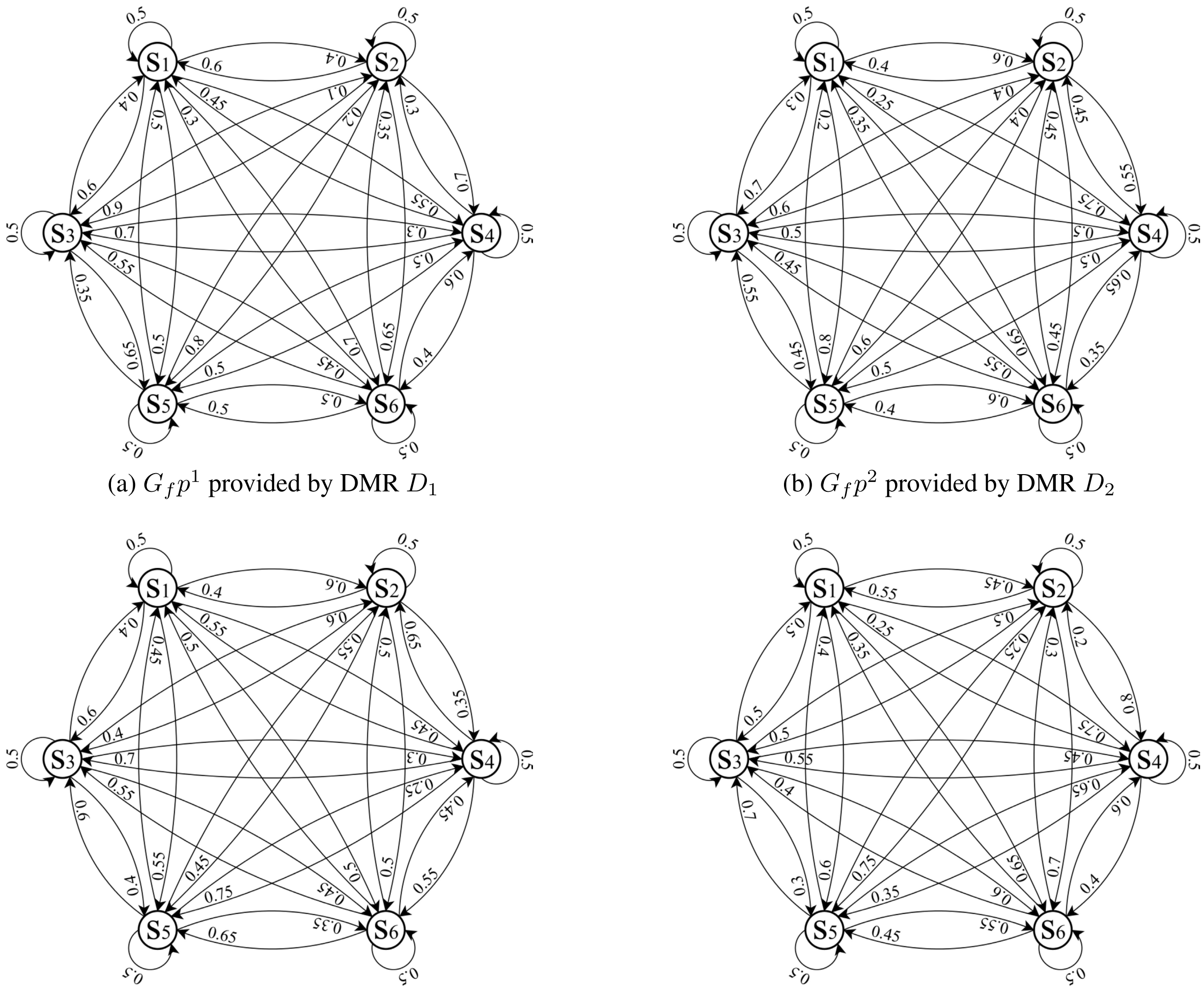
### E. SELECTION PHASE

After having an acceptable consensus level amongst all DMRs, the selection procedure is to be initiated to rank all the alternatives to choose the best one. But, it may often that the information provided by each DM is weighted differently. Therefore, when the priority weights for DMRs are estimated, their informations require to be aggregated into global one. In this regard, we obtain a final consistency FPG $\widetilde{G}_{fp}^{c}$ carrying preference weights obtained from the weighted sum of the corresponding preference weights from $\widetilde{G}_{fp}^{q}$, $1 \leq q \leq m$ as:

$$\widetilde{w}_{ik}^{c} = \sum_{q=1}^{m} w(D_q) \times \widetilde{w}_{ik}^{q}, \tag{13}$$

where $1 \leq i, k \leq n$. Hence, definition of order consistency will result in ranking of alternatives.

To validate the proposed method, consider a situation in which four DMRs, $D = \{D_1, D_2, D_3, D_4\}$, interact to rank the six franchises, $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$, of a famous private school system in a city based on "quality education" and "service structure". Quality plays a central role




(a) $G_fp^1$ provided by DMR $D_1$ (b) $G_fp^2$ provided by DMR $D_2$

(c) $G_fp^3$ provided by DMR $D_3$ (d) $G_fp^4$ provided by DMR $D_4$

**FIGURE 3.** FPGs provided by the decision makers $D_1$, $D_2$, $D_3$ and $D_4$, respectively.

for customers before availing any type of service and in measuring the performance of the institute. Education sector is the service sector which is considered to be the back bone of national and economic development. The DMRs provide their average information in form fuzzy preference graphs $G^q_{fp}$ shown in the Figure 3, where vertices denote the franchises of the school system and weighted directed edges pairwise preference values, as:

To reach the acceptable result, following steps have to be performed:

**(i) Consistency analysis:** Consistency analysis derive us to measure and allocate the weights to the experts in order to have quality information. For this purpose, Eq. (2) help us to form consistency FPGs (ACFPGs and OCFPGs) $\widetilde{\overline{G}}^q_{fp}$, $1 \leq q \leq 4$, against given $G^q_{fp}$ respectively, and are shown in the following Figure 4:

Now, we measure the consistency degree of FPGs by forming their FCGs using Eq. (3), such as, the FCG $G^1_{fc}$ for $G^1_{fp}$ against DMR $D_1$ is shown in Figure 5:

and the application of Eq. (4) and Eq. (5) results in

$$cd(G^1_{fc}) = 0.9313.$$

Similarly, consistency degrees for $G^q_{fc}$, $q = 2, 3, 4$, provided by the DMRs $D_2$, $D_3$ and $D_4$ can be evaluated using Eq. (3) to Eq. (5), and are given as:

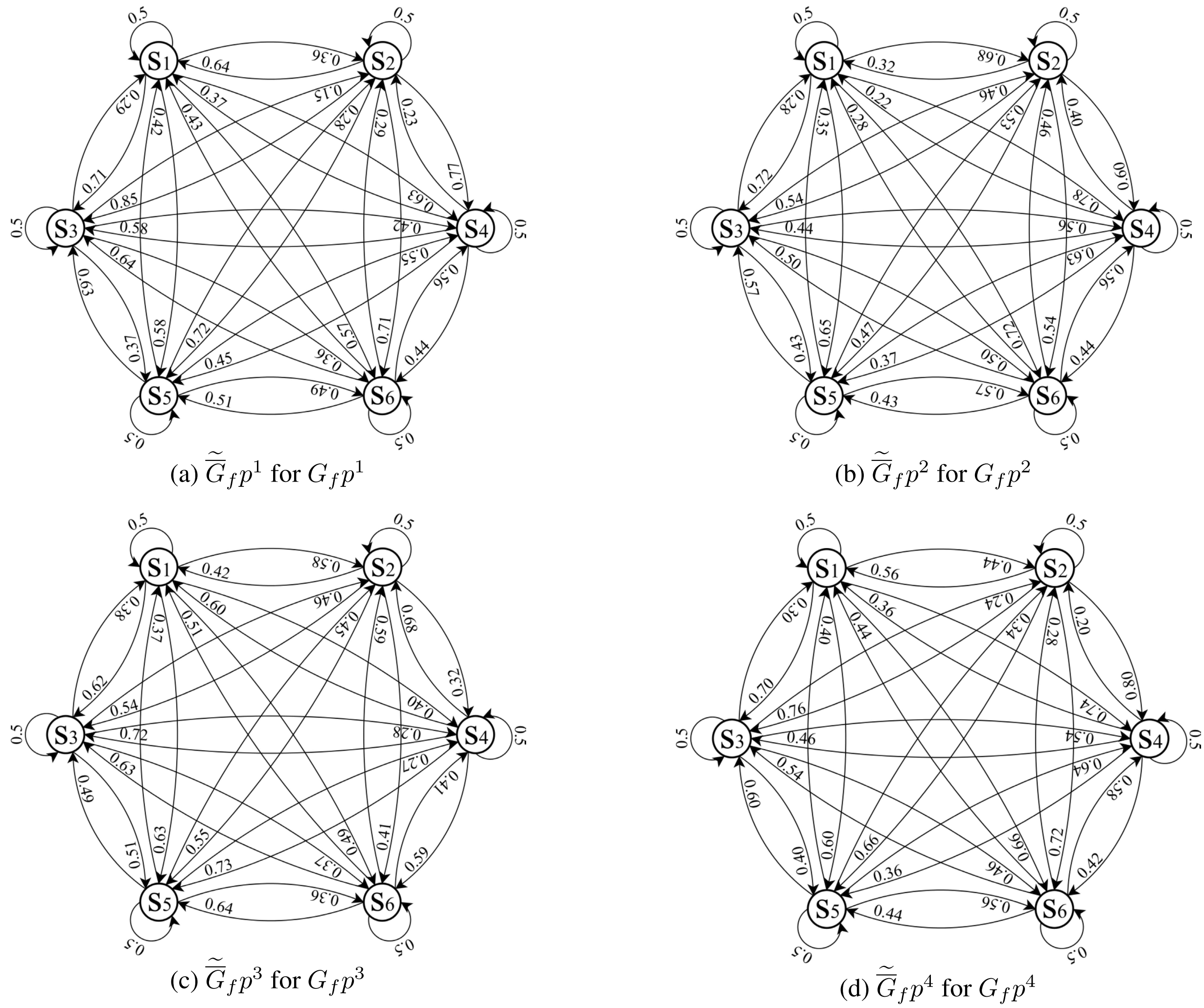
$$cd(G^2_{fc}) = 0.9393;\ cd(G^3_{fc}) = 0.9453;\ cd(G^4_{fc}) = 0.9353.$$

**(ii) Priority weights to DMRs:** Here, the consistency weights will be used as final weights of DMRs because there is no any predefined weights vector. Therefore, Eq. (6) implies that

$$C_w(D_1) = 0.2483,\ C_w(D_2) = 0.2504,$$
$$C_w(D_3) = 0.2520 \text{ and } C_w(D_4) = 0.2493.$$

**(iii) Consensus analysis:** The aggregated FCG $G_{fc}$ is constructed using Eqs. (8-9) and is given in Figure 6:

and, furthermore, Eqs. (10-11) result in global consensus $CD(G_{fc}) = 0.85$ amongst DMRs. If it is greater or equal to the




(a) $\widetilde{\overline{G}}_f p^1$ for $G_f p^1$

(b) $\widetilde{\overline{G}}_f p^2$ for $G_f p^2$

(c) $\widetilde{\overline{G}}_f p^3$ for $G_f p^3$

(d) $\widetilde{\overline{G}}_f p^4$ for $G_f p^4$

**FIGURE 4.** Consistency FPGs against decision makers $D_1$, $D_2$, $D_3$ and $D_4$, respectively.

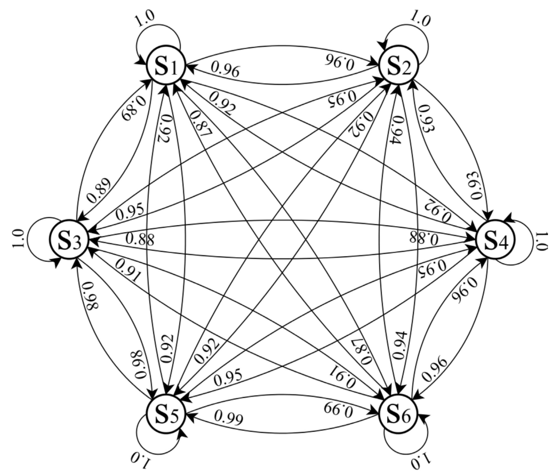



**FIGURE 5.** FCG $G_f c^1$ for FPG $G_f p^1$.

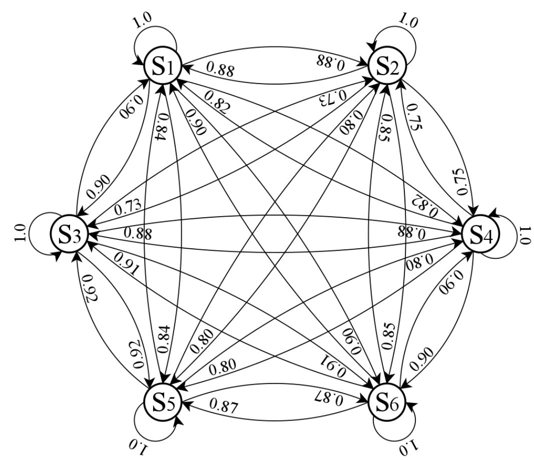



**FIGURE 6.** Aggregated $G_f c$.

threshold consensus degree $\eta$, then decision problem enters into the selection phase otherwise some DMRs are suggested to enhance their information under expression (12).

**(iv) Selection phase:** For $CD(G_{fc}) = 0.85$, an acceptable consensus level, the collective consistency FPG $\widetilde{\overline{G}}^c_{fp}$ is constructed using Eq. (2) and Eq. (13) given as in Figure 7:

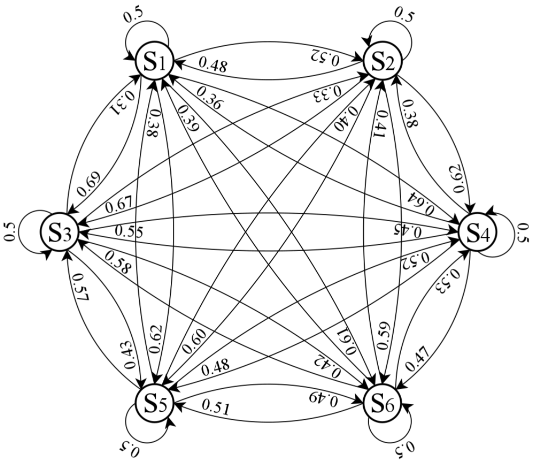



**FIGURE 7.** Consistency FPG $\tilde{\tilde{G}}_{fp^c}$.

As Figure 7 is an OCFPG which provides the information that $\tilde{\tilde{w}}_{i1} \leq \tilde{\tilde{w}}_{i2}$, $\tilde{\tilde{w}}_{i2} \leq \tilde{\tilde{w}}_{i6}$, $\tilde{\tilde{w}}_{i6} \leq \tilde{\tilde{w}}_{i5}$, $\tilde{\tilde{w}}_{i5} \leq \tilde{\tilde{w}}_{i4}$ and $\tilde{\tilde{w}}_{i4} \leq \tilde{\tilde{w}}_{i3}$ for all $1 \leq i \leq 6$ and are interpreted as:

- $\tilde{\tilde{w}}_{i1} \leq \tilde{\tilde{w}}_{i2}$ indicates that franchise $S_1$ is prefered over franchise $S_2$;
- $\tilde{\tilde{w}}_{i2} \leq \tilde{\tilde{w}}_{i6}$ indicates that franchise $S_2$ is prefered over franchise $S_6$;
- $\tilde{\tilde{w}}_{i6} \leq \tilde{\tilde{w}}_{i5}$ indicates that franchise $S_6$ is prefered over franchise $S_5$;
- $\tilde{\tilde{w}}_{i5} \leq \tilde{\tilde{w}}_{i4}$ indicates that franchise $S_5$ is prefered over franchise $S_4$;
- $\tilde{\tilde{w}}_{i4} \leq \tilde{\tilde{w}}_{i3}$ indicates that franchise $S_4$ is prefered over franchise $S_3$.

Hence, the final preference ranking of all the franchises is $S_1 \succ S_2 \succ S_6 \succ S_5 \succ S_4 \succ S_3$ which leads us to $S_1$ as the best franchise.

## III. CONCLUSION

In this paper, some graphical notions of multi-person decision making using consensus based information in fuzzy environment have been proposed. Additive transitivity is used to construct the consistent FPGs, and then consistency analysis is made to measure the consistency level of the iformation provided by DMRs respectively by. This analysis also leaded us to evaluate the consistency weights, and then the final priority weights of the DMRs which are taking part in decision problem. Additionally, an enhancement mechanism is used to provide us much knowledge to accelerate the execution of a higher consensus level. After getting a satisfactory consensus degree amongst DMRs, the entire process entered into the selection phase to rank all the alternatives to choose the best one. Order consistency property played a central role in ranking the alternatives. At the end, a graphical example is used to gain a greater insight into the multi-person decision problems while data is being taken from fuzzy environment.

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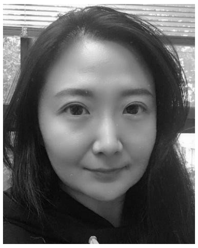

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