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Hybrid Harris Hawk Optimization Based on Differential Evolution (HHODE) Algorithm for Optimal Power Flow Problem

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ABSTRACT Harri's Hawk Optimization (HHO) algorithm manifests as a new meta-heuristic algorithm in literature. When we look at studies that have used with this algorithm, we can see that its results in test functions and in the solutions of some test functions in IEEE Congress on Evolutionary Computation (CEC) are much better compared to other heuristic and meta heuristic algorithm results. In this study, an algorithm has been developed which has been hybridized with the mutation operators of Differential Evolution (DE) to further improve the HHO algorithm. This algorithm is named as Hybrid Harris Hawk Optimization based on Differential Evolution (HHODE). Performance of the proposed HHODE algorithm has been first compared with HHO and then compared with the results of other algorithms which have been most commonly used in the literature. In this comparison process, the most commonly used test functions in the literature and some of the other test functions in CEC2005 and CEC2017 as a new application field, have been solved. When the results of the comparison of HHODE with other algorithms are analyzed, it is observed that the balance between the exploratory tendency and exploitative tendency of the algorithm is well consistent. Formula 1 ranking method is used in the order of HHODE according to HHO and other algorithms. When a general evaluation of HHODE was performed, it was found to be an even more powerful algorithm as a result of the combination of strong features of both HHO and DE. The optimal power flow (OPF) problem is one of the most important problems of the modern power system. The HHODE algorithm is proposed to solve the OPF problem, which is considered without valve-point effect and prohibited zones (1) and with prohibited zones (2) in this paper. The effectiveness of the HHODE hybrid algorithm is tested on modified IEEE 30-bus test system. The result of HHODE algorithms are compared with other optimization algorithms in the literature.

INDEX TERMS Harri's hawk optimization, differential evolution, optimization, hybrid algorithms, swarm intelligence, optimal power flow, power system.

I. INTRODUCTION

Optimization is the process of finding the best solution for a problem under certain conditions. Another definition of optimization refers to the process of systematically analyzing or solving a problem by selecting values in a defined range and using them inside a function to minimize or maximize it. With optimization techniques, the decision-making process in the solution of a problem is accelerated and the quality of the decision is increased. In this way, effective, accurate and real-time solutions of problems encountered in real life are achieved. Many algorithms proposed for the solution of optimization problems require mathematical models to construct both the system model and the objective function. Therefore, mathematical models are created according to the structure of the problem. During the creation of these models, limitations related to a cost function and problem which are to be minimized or maximized depending on the decision variables or design parameters are defined. Mathematical (classical) algorithms are algorithms which are designed specifically for the problem or which try to solve the problem by scanning the whole solution space of the problem. However, in real life systems, when situations and problems are analyzed, it is realized that the problems are actually more complicated. Therefore, it is difficult to establish a mathematical equation for solving such complex systems and

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the cost of using it is high. Besides, it is seen that especially in engineering applications, many optimization problems are continuous, discrete, restricted or unrestricted and systems are not linear [1], [2]. Mathematical programming methods such as differential equations and numerical analysis methods used in classical mathematical equation solutions are not successful in solving such problems [3]. Optimization algorithms are used in many areas where there are non-linear problems or mathematical solution equation cannot be created.

The artificial intelligence algorithms which are grouped under the concept of artificial intelligence are used in many optimization problems and engineering applications. When studies conducted with artificial intelligence algorithms are analyzed, it is seen that these algorithms give successful results. As a result of humans' observation of nature, many types of artificial intelligence algorithm have been exposed, with their approaches inspired by animals and nature. When looking at some of the Heuristic and Meta-heuristic algorithms and evolutionary algorithms, it can be seen that many algorithms such as Evolutionary Computation (EC) [4], Tabu Search (TS) [5], [6], Genetic Algorithm (GA) [7]–[9], Simulated Annealing (SA) [10], [11], Particle Swarm Optimization (PSO) [12], [13], Differential Evolution (DE) [14]–[16], Cultural Algorithm (CA) [17], [18], Biogeography Based Optimizer (BBO) [19]–[21], Big-Bang Big-Crunch (BBBC) [22], [23], Central Force Optimization (CFO) [24], [25], Gravitational Search Algorithm (GSA) [26], [27], Socio Evolution and Learning Optimization (SELO) [28], Teaching Learning Based Optimization (TLBO) [29]–[31], Ant Colony Optimization (ACO) [32], [33], Cuckoo Search (CS) [34]–[36] Artificial Bee Colony (ABC) [37], [38], Harris' Hawk Optimization Algorithm (HHO) [39], [40] and Whale Optimization Algorithm (WOA) [41], [42] are put forward and applied to several problems.

However, one of the problems that all heuristic, meta-heuristic, and evolutionary algorithms face is the potential of early convergence or getting stuck in a local minimum point. To overcome this problem, many researchers have developed new hybrid algorithms by combining them with other algorithms that improve the performance and local search method of existing algorithms. Even if only a few of the studies in literature are considered, it is realized that better results are obtained by using hybrid algorithms [43]–[46].

Because Harris' Hawk Optimization (HHO) [39], [40], is used as a new meta-heuristic algorithm and the use of it in a hybrid structure has not been encountered yet. Besides, in this study, HHO and DE are utilized as hybrid (HHODE) and are applied to the benchmark functions which exists in the literature and are mostly compared.

There are many researches in which DE is compared with other algorithms as hybrid algorithms. In addition, these algorithms hybridized with DE have been successfully applied to many real engineering problems [15], [46]–[51]. DE is a meta-heuristic algorithm that uses mutation and crossover schemes for real-valued optimization problems [14]. This algorithm applies both a simple structure and highly effective mutation process. DE uses a mutation process based on the differences of objective vector pairs in a randomly selected goal. The simple mutation process used in the DE algorithm improves the performance of the algorithm and makes it stronger.

In the hybridization of algorithms in literature, most of them lack the equilibrium between the exploration and exploitation phases during the optimization process. During exploration, it is necessary to use the randomly selected operators as much as possible in order for algorithm to do research in the whole area and in various places of the problem's solution space. Thus, after a well-designed discovery process, possession of a rich solution space is ensured in the detection and examination of the best possible solutions in the exploitation phase [52]. In such a structure, of course the exploitation phase is carried out after the exploration phase. Thus, the effectiveness of the exploration phase directly affects the exploitation phase. The optimizer in the application phase focuses on better / high quality possible solutions in the solution space. A well-organized optimizer should be able to strike a reasonable balance between exploration and exploitation tendencies. Otherwise, the possibility of being compressed within the disadvantage of local optimum (LO) and early convergence increases. In this research, Harris' Hawk Optimization (HHO) algorithm is combined with the widely known Differential Evolution (DE) and used in the literature and what we call the algorithm HHODE is formed. In the HHODE algorithm which is a Hybrid algorithm, the balance between exploration and exploitation was attempted to be guaranteed.

Optimal Power Flow (OPF) problem is one of the most important optimization problems of modern power systems. The determination of the control variables is aimed to inequality and equality constraints for optimal operation and planning of the power systems. Many heuristic optimization algorithms have been used to solve complex optimization problems such as economic dispatch, economic and emission dispatch, dynamic economic dispatch and optimal power flow optimization problems. Recently, these heuristic methods have been tested to find the best solution of the OPF problem in the power systems such as GA, GSA, PSO, HS, BBO, DE etc [53], [54], [54]–[56]. In [57], Duman solved OPF problem with and without valve point effect and prohibited zones; forming four different scenarios. The study used symbiotic organisms search (SOS) on power system with IEEE-30 bus. Results of the proposed SOS outperformed various other population-based and evolutionary algorithms from literature. In previous literature there can be seen that implementation of HHO in OPF problems is yet to be found. Kashif Hussain et al. is considered as the first attempt to apply HHO on OPF problem [58].

In this study the HHODE algorithm is provided to find a better solution than other optimizing populated-search algorithms, such as GA, DE, BA, PSO, etc. So HHODE is proposed to solve the OPF problem without valve-point effect and prohibited zones (1) and with prohibited zones (2) in this paper. In the OPF problem it is used to optimize the objective functions related to power generation cost, emission, and power loss on IEEE-30 bus system. The proposed HHODE algorithm applied to test on the IEEE 30-bus standard test system for OPF problem. The results of the proposed method are compared to the other optimization algorithms in the literature.

This paper is organized as follows. In the second part of this study, OPF problem definition, HHO and DE algorithms are presented. In the third chapter, the HHODE algorithm, which is formed as a hybridization of HHO and DE, is presented. In the fourth chapter, HHODE is applied to benchmark problems (CEC2005 and CEC2017), compared with other algorithms and its performance is reviewed and the simulation results of the proposed algorithm in the OPF problem are presented. In the fifth chapter, results and recommendations are evaluated.

II. SCIENTIFIC BACKGROUND

A. OPTIMAL POWER FLOW (OPF) PROBLEM'S DEFINITION

The OPF problem is considered to be an optimization problem that aims to minimize the total fuel cost function under some constraints such as total load, various equality and inequality. An OPF is a minimization problem that is formulated in equation 1.

$$\begin{aligned} \text{Minimize} \quad & f(x,u) \\ \text{Subject to} \quad & g(x,u)=0 \\ & h(x,u)\leq 0 \end{aligned} \tag{1}$$

where $f(x,u)$ is the objective function. x and u are defined as state and control variables, respectively. The given objective function should be achieved by satisfying certain equality and inequality constraints. $g(x,u)=0$ and $h(x,u)\leq 0$ are equality and inequality constraints that are representing.

The state variable x can be defined as in equation 2 where $P_{Gslack}, V_L, Q_G, NPQ, NG, NTL, S$ presents as active power of the generator at slack bus, voltage magnitude of load buses, reactive power of the generators, number of PQ buses, number of generators and number of transmission lines, power of transmission lines, respectively and the control variable u can be defined as in equation 3 where P_G, V_G, T, NT presents as active power output of the generators except at the slack bus, terminal voltage magnitude of the generators, transformer tap ratio and number of tap regulating transformers, respectively.

$$x=\left[P_{Gslack}, V_{L1}..V_{LNPQ}, Q_{G1}..Q_{GNG}, S_{l1}..S_{NL}\right] \tag{2}$$

$$u=\left[P_{G2}..P_{GNG}, V_{G1}..V_{GNG}, T_1..T_{NT}, Q_{C1}..Q_{CNC}\right] \tag{3}$$

Objective Function of the OPF problem is defined as the minimization of the total fuel cost and it can be calculated as in equation 4 where $P_{Gi}$is defined as active power and a_i, b_i, c_i are defined that fuel cost coefficients of the generators.

$$f(x,u)=\left(\sum\nolimits_{i=1}^{N_G} a_iP_{Gi}^2+b_iP_{Gi}+c_i\right)+Penalty(\frac{\$}{h}) \tag{4}$$

Prohibited operating zones (POZs) are occurred in a thermal and hydro-generating unit [57]. The best economy is obtained by avoiding operating in areas and the POZs is formulated as in equation 5 where $P_{Gik}^L = P_{Gi}^{min}$ and $P_{Gik}^U = P_{Gi}^{max}$ and K is described as the number of prohibited zones of generator's unit.

$$P_{Gik}^L \leq P_{Gi} \leq P_{Gik}^U \forall i \in k=1,2,\ldots,K \tag{5}$$

Equality constraints, the load equations are described as equality constraints and the formulations of this are shown in equation 6 and 7 where $N$is the total number of bus, $V_i and V_j$are the voltage magnitude of ith and jth bus, P_{Gi} is active power of ith generators, P_{Di} is demand active power of ith bus, Q_{Di} is demand reactive power of ith bus Q_{Gi} is reactive power of ith generator and θ_{ij} is the voltage angle difference between ith and jth bus.

$$P_{Gi}-P_{Di}-V_i\sum\nolimits_{j=1}^{N} V_j\left[G_{ij}cos\theta_{ij}+B_{ij}sin\theta_{ij}\right]=0 \tag{6}$$

$$Q_{Gi}+Q_{Ci}-Q_{Di}-V_i\sum\nolimits_{j=1}^{N} V_j\left[G_{ij}sin\theta_{ij}-B_{ij}cos\theta_{ij}\right]=0 \tag{7}$$

Inequality constraints, active and reactive power outputs of the generator unit and voltage magnitude are restricted by their lower and upper limits are shown in equations 8,9,10,11 where P_{Gi}^{min} and P_{Gi}^{max} are lower and upper active power values of the ith generating unit and Q_{Gi}^{min} and Q_{Gi}^{max} are lower and upper active power values of the ith generating unit.

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad i=1,\ldots..NG \tag{8}$$

$$Q_{Gi}^{min} \leq Q_{Gi} \leq Q_{Gi}^{max} \quad i=1,\ldots..NG \tag{9}$$

$$V_{Gi}^{min} \leq V_{Gi} \leq V_{Gi}^{max} \quad i=1,\ldots..NG \tag{10}$$

$$V_{Li}^{min} \leq V_{Vi} \leq V_{Vi}^{max} \quad i=1,\ldots..NGQ \tag{11}$$

Transformer tap settings are shown in equation 12 and 13 where T_i^{min} and T_i^{max} are represented as minimum and maximum tap settings, S_{li} and S_{li}^{max} are represented s apparent power flow of branch and maximum apparent power flow limit of each branch.

$$T_i^{min} \leq T_i \leq T_i^{max} \quad i=1,\ldots NT \tag{12}$$

$$S_{li} \leq S_{li}^{max} \quad i=1,\ldots NTL \tag{13}$$

Also the objective function, which is included the penalty terms, is shown in equation 14 where $\lambda_V, \lambda_Q, \lambda_P and \lambda_S$ are penalty factor terms.

$$\begin{aligned} J=&f(x,u)+\lambda_V\sum_{i=1}^{NPQ}(V_{Li}-V_{Li}^{Lim})^2 \\ &+\lambda_P(P_{Gslack}-P_{Gslack}^{lim})^2+\lambda_Q\sum_{i=1}^{NG}(Q_{Gi}-Q_{Gi}^{Lim})^2 \\ &+\lambda_S\sum_{i=1}^{NTL}(S_{li}-S_{li}^{lim})^2 \end{aligned} \tag{14}$$

B. HARRIS' HAWK OPTIMIZATION (HHO)

Strategy of Harris 'Hawk: Harris' Hawk's most important feature in catching its prey is to hunt in groups, collaborating with the Hawks, as opposed to other predators. In this clever strategy, a few Hawks attack from different ways their prey chosen in collaboration for simultaneous delusion. The aim here is to approach the prey in a controlled manner. The attack is desired to be completed in a few seconds. However, sometimes, according to the prey's ability to escape (prey's escape energy and hunting environment), the success of this collaborative attack can be achieved after a few minutes and by a large number of attacks. There is a leader in the collaborative attack. In the event of a fatigued leader when getting away from the prey or during the hunting process of the attack, another Harris' Hawk takes over the leadership. Thus, the process of attack continues until the hunting is successful or the prey completely escapes. This is sometimes used as a tactic. Thus, making the attack from different places confuses and exhausts the prey. The hunting process is completed as the prey which has low energy and has lost its defensive abilities is hunted easily by Harris' Hawk, the leader. HHO is a population-based optimization technique. It can therefore be applied to any optimization problem with appropriate limitations and constraints. The following sections describe the operation logic and process of HHO.

Exploration: In the process of exploration, Harris' Hawk scans and finds his prey (rabbit, etc.) in a hunting environment thanks to his sharp vision. But this is usually not that easy. For this reason, Harris' Hawk waits, observes, tracks and follows the hunting environment for minutes or even hours. In this collaborative strategy, each Harris' Hawk demonstrates the possible solution. The intended solution is the prey itself [40].

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1 \left|X_{rand}(t) - 2r_2 X(t)\right| & q \geq 0.5 \\ (X_{rabbit}(t) - X_m(t)) - r_3(LB + r_4(UB - LB)) & q < 0.5 \end{cases} \tag{15}$$

According to the structure formulated in equation 15 [40] in HHO, each Hawk is placed in a random position and waits to detect the hunt according to two situations. Assuming that these two perching states are (q); in case of q<0.5 the Hawk perches randomly according to the prey's (rabbit) position, and in the case of q≥0.5 it does so according to other Hawks' positions in the hunting area. X(t+1) refers to the position vector of the Hawk in the next iteration, X_rabbit (t) to the position vector of prey (rabbit), and r1, r2, r3, r4 and q are random numbers from 0 to 1 refreshed for each iteration. LB refers to lower limit values, UB to upper limit values, X_rand (t) to the randomly selected Hawk's position in the current population and X_m (t) to the average positions of the Hawks in the current population. Determining random locations was suggested between limits of LB and UB values [40].

$$X_m(t) = \frac{1}{N} \sum\nolimits_{i=1}^{N} X_i(t) \tag{16}$$

As stated in Equation 16, $X_i(t)$ refers to each Hawk's position in each iteration and N represents the total number of Hawks.

Exploration to exploitation: At this stage, according to the energy level of the prey, the HHO algorithm goes from the exploration phase to the exploitation phase. According to the formula given in Equation 17, the energy reduction of the prey is observed [40].

$$E = 2E_0(1 - \frac{t}{T}) \tag{17}$$

In the Equation 17, E refers to the escape energy of the prey, T to the maximum iteration and E_0 to the energy at the starting point. It was accepted as a model here that E_0 changes between -1 and 1 in each iteration. The change from 0 to -1 indicates that the energy of the prey (rabbit) decreases and slows down; the change from 0 to 1 indicates that the prey (rabbit) rests and its energy increases. Of course, normally, as the iterations progress the energy of the prey while escaping will decrease. In case of $|E| \geq 1$ the Hawks will observe to determine the position of the prey (rabbit). This represents HHO's exploration status. In case of $|E| < 1$ this time HHO switch to exploitation, meaning the Hawk is in the attacking phase.

Exploitation: At this stage, Harris' Hawk makes a surprise pounce, a sudden attack on its prey that it has been observing. Of course, in response to this attack, the prey will start to escape to many different directions according to its energy levels. This kind of escape to different directions will continue until the prey completely escapes or the prey is caught. In the proposed HHO [40], at the time of the attack there are four different situations. The prey continuously tries to escape from the threatening situation. The r expression used in the HHO algorithm represents the chance of escape. $r < 0.5$ shows the prey's high probability to escape, $r \geq 0.5$ indicates that the prey is unlikely to escape the last attack. In return to these two possibilities of the prey's situation, Harris' Hawk has two conditions as well: hard besiege or soft besiege. Of course, in the real world, Harris' Hawk tries to reduce his distance to the prey before making the last surprise attacks. The attack will fail if the last attack is carried out when there is no proper distance between the prey and the hunter. In this case, it is very likely that the prey will escape until the hunter(s) get(s) its/their new position. When the hunting process is taken as model, in case of $|E| \geq 0.5$, soft besiege will occur and in case of $|E| < 0.5$, hard besiege will occur.

Soft Besiege: In case of $r \geq 0.5$ and $|E| \geq 0.5$, prey (rabbit) has sufficient escape energy and tries to escape with random maneuvers. But ultimately, it fails to escape. With these soft besieges, Harris' Hawk calmly draws circles, making its prey (rabbit) more tired. So it tries to prepare for the last surprise attack.

$$X(t+1) = \Delta X(t) - E\left|JX_{rabbit}(t) - X(t)\right| \tag{18}$$

$$\Delta X(t) = X_{rabbit}(t) - X(t) \tag{19}$$

In Equation 18 [40] $\Delta X(t)$ represents the difference between the position vector of the prey (rabbit) and its instantaneous position in the t[th] iteration. $J = 2(1 - r_5)$ shows the prey's (rabbit) power of jumping randomly at the time of escape. Here r_5 changes randomly between 0 - 1.

Hard Besiege: In case of $r \geq 0.5$ and $|E| < 0.5$, the prey is now too tired and has very little remaining energy. In this case, Harris' Hawk's turns on the prey is sharper and closer. The next step will result in Hawk attacking the prey. The current position information is now as specified in equation 20 [40].

$$X(t+1) = X_{rabbit}(t) - E\,|\Delta X(t)| \tag{20}$$

Soft besiege with progressive diving: In the case of $|E| \geq 0.5$ but $r < 0.5$, prey (rabbit) has enough energy for a successful escape and the Hawk is in soft besiege situation. In the HHO algorithm, in the mathematical modeling of prey and hunter's movements, levy flight (LF) design was used [40], [58], [59]. In general Harris' Hawk sets its situation in the best position when there is a competitive hunting process, and it organizes the most intense and swift attack on the prey. This logic is also found in the working principle of the HHO algorithm. According to the equation formed in Equation 21, in the case of a soft besiege, the Hawk evaluates the next change in its position. Before making an attack dive, it also evaluates his experiences in other dives and decides whether to make an attack dive or not. In the case where they observe that the prey's energy is high and there is randomness in their escape, they organize irregular, deceptive and strenuous attacks against the prey. Ali Asghar et.al, in their study, assume that according to the mathematical expression given in equation 22 Hawk attacks are LF-based. According to Equation 23, Hawks' new position status in soft besiege changes [40], [59], [60].

$$Y = X_{rabbit}(t) - E\,|JX_{rabbit}(t) - X(t)| \tag{21}$$

$$Z = Y + SxLF(D) \tag{22}$$

$$X(t+1) = \begin{cases} Y if F(Y) < F(X(t)) \\ Z if F(Z) < F(X(t)) \end{cases} \tag{23}$$

Hard besiege with progressive quick dives: In case of $|E| < 0.5$ but $r < 0.5$, prey (rabbit) doesn't have enough energy to escape, and the final besiege process is entered before the final attack which is carried out with the purpose to kill. In terms of prey, this situation is similar to a soft besiege, but this time the Hawks try to quickly reduce the distance of their average positions to the escaping prey. Therefore, in equation 21, the new state of Y is re-formulated as in equation 10. $X_m(t)$ in this new equation is calculated as in equation 16.

$$Y = X_{rabbit}(t) - E\,|JX_{rabbit}(t) - X_m(t)| \tag{24}$$

This attack process, situations of the prey and hunter, what exactly X, Y and Z indicate are shown representatively in Figure 1 [40].

The psudeo code of the classic HHO is given in the algorithm I [40].

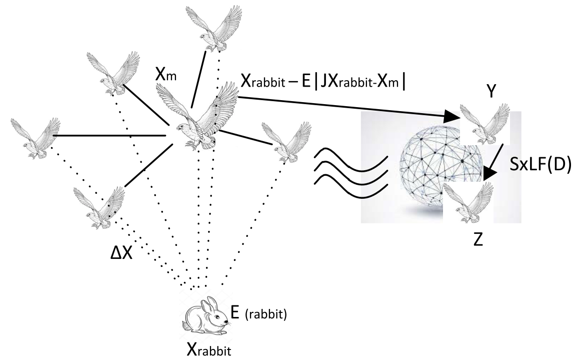

FIGURE 1. Representation of the Harris' Hawk hunting process.

C. DIFFERENTIAL EVOLUTION (DE)

The DE algorithm appears in literature as a simple but powerful population based algorithm. It is used to globally optimize functions which include real valued design parameters. DE uses a mutation process based on the differences of randomly selected objective vector pairs. The simple mutation process used in the DE algorithm improves the performance of the algorithm and makes it stronger. Besides, the DE algorithm can be used quickly, simply, easily and can be easily adapted for the creation of hybrid algorithms. It can be easily adapted to integer, discrete and mixed parameter optimization, can be used for functions related to time/iteration, can produce alternative solutions in a single run and is effective especially in nonlinear constrained optimization problems.

Hybridization of HHO algorithm's exploration stage with the DE algorithm appears to help HHO to produce more efficient and better results. A more detailed explanation of HHODE as a hybrid algorithm is given in section 3. Under this title, explanation of classic DE is given. In the operation process of DE as described in the literature with its classic form, x is the solution that is perturbed such as "random" or "best"; y is the number of difference vectors used to perturb x. Each difference vector reflects the difference between two (randomly) selected but distinct population members. z represents the recombination operator used such as binomial (bin) or exponential (exp) [60]. The five mutation types commonly used in the DE algorithm were given in the following equations of 25, 26, 27, 28 and 29 [61]–[63].

$$DE/rand/1;\ V_i^g = X_{r1}^g + F.X(X_{r2}^g - X_{r3}^g) \tag{25}$$

$$DE/best/1;\ V_i^g = X_{best}^g + F(X_{r1}^g - X_{r2}^g) \tag{26}$$

$$DE/current-to-best/2;\ V_i^g = X_i^g + F(X_{best}^g - X_i^g + X_{r1}^g - X_{r2}^g) \tag{27}$$

$$DE/best/2;\ V_i^g = X_{best}^g + F(X_{r1}^g - X_{r2}^g + X_{r3}^g - X_{r4}^g) \tag{28}$$

$$DE/rand/2;\ V_i^g = X_{r1}^g + F(X_{r2}^g - X_{r3}^g + X_{r4}^g - X_{r5}^g) \tag{29}$$

Mutation generated for each X_i within the $V_i = (V_i^1, V_i^2 \ldots\ldots V_i^N)$ population which appears inside these formulas is a vector. r_1, r_2, r_3, r_4, r_5 values are random

Algorithm 1 Pseudo Code of HHO

Define the population number (N) and nmber of iteration (T) (Input values)
Locations of rabbit and its fitness value (Output values)
Start within random point in population X_i (i=0,1,2....)
while (continue until the conformity value is reached to the desired point)
{

Calculate Hawk's fitness value

Define the position of X_{rabbit}

for (each Hawk (X_i)) (do)

{

//Update the starting enegy (E_0 =2rand()-1) and jumping force (J=2(1-rand())

$E = 2E_0(1 - \frac{t}{T})$ update the E

if ($|E| \geq 1$) **// Exploration Phase**

{

//Update the position according to equation below

$$X(t+1) = \begin{cases} X_{rand}(t) - r_1 |X_{rand}(t) - 2r_2 X(t)| & q \geq 0.5 \\ (X_{rabbit}(t) - X_m(t)) - r_3(LB + r_4(UB - LB)) & q < 0.5 \end{cases}$$

}

if ($|E| < 1$) **// Exploitation Phase**

{

if ($r \geq 0.5$ ve $|E| \geq 0.5$) **//Soft besiege**

{

// Update the position according to equation below

$$X(t+1) = \Delta X(t) - E\,|JX_{rabbit}(t) - X(t)|$$

}

else if ($r \geq 0.5$ ve $|E| < 0.5$)**//Hard besiege**

{

// Update the position according to equation below

$$X(t+1) = X_{rabbit}(t) - E\,|\Delta X(t)|$$

Algorithm 1 *continue:* Pseudo Code of HHO

```
            }
            else if ( r < 0.5 ve |E| ≥ 0.5 )//Soft besiege with dives
            {
                // Update the position according to equation below
```

$$Y = X_{rabbit}(t) - E\left|JX_{rabbit}(t) - X(t)\right|$$
$$Z = Y + SxLF(D)$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases}$$

```
            }
            else if ( r < 0.5 ve |E| < 0.5 )//Hard besiege with dives
            {
                // Update the position according to equation below
```

$$Y = X_{rabbit}(t) - E\left|JX_{rabbit}(t) - X_m(t)\right|$$
$$Z = Y + SxLF(D)$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases}$$

```
            }
        }
        Return X_rabbit
}
```

numbers generated between population size (NP) and 1. F, scale vector and X_{best} are the value X, which indicates the probable solution that has the best conformity value. The DE binomial crossover operator that is used in this study and generally used in the literature is given in equation 30. In this equation, u_i is the produced offspring and CR is the crossing rate, and g is the scalling factor generated independent from X_i. Cauchy distribution is made according to F_i = Cauchy(lop,0.1). The value of F_i is regenerated if $F_I < 0$ or $F_I > 1$. The crossover rate is generated under anormal distribution CR_i = normal(CR,0.1) [61].

$$u_i = \begin{cases} v_i, & if\ (with\ probabilty\ of\ CR) \\ X_i, & otherwise \end{cases} \tag{30}$$

III. PROPOSED ALGORITHM THAT IS HARRIS HAWK OPTIMIZATION BASED ON DIFFERENTIAL EVOLUTION (HHODE) ALGORITHM

The pseudo code of the HHODE algorithm is presented in the following Algorithm 2. In the solution point of this study, the exploration phase is the process of Harris' Hawk scanning and finding its prey (rabbit etc.) in a hunting environment thanks to its sharp vision. But this is usually not that easy. For this reason, Harris' Hawk waits, observes, tracks and follows the hunting environment for minutes or even hours. Due to this situation, the DE mutation operators are applied in the Exploration Phase section corresponding to this process. At this stage in HHO, each Hawk is placed in a random position and waits to detect the prey according to

Algorithm 2 Pseudo Code of HHODE

Define the population number (N) and nmber of iteration (T) (Input values)
Locations of rabbit and its fitness value (Output values)
Start within random point in population X_i (i=0,1,2....)
while (continue until the conformity value is reached to the desired point)

{

Calculate Hawk's fitness value

Define the position of X_{rabbit}

for (each Hawk (X_i)) (do)

{

Update the starting enegy (E_0 =2rand()-1) and jumping force (J=2(1-rand())

$E = 2E_0(1 - \frac{t}{T})$ update the E

if ($|E| \geq 1$) **// Exploration Phase with DE mutation operators**

{

$$X(t+1) = \begin{cases} \begin{pmatrix} X_{r1}^{g} + F.X\left(X_{r2}^{g} - X_{r3}^{g}\right) \\ X_{best}^{g} + F\left(X_{r1}^{g} - X_{r2}^{g}\right) \\ X_{i}^{g} + F\left(X_{best}^{g} - X_{i}^{g} + X_{r1}^{g} - X_{r2}^{g}\right) \\ X_{best}^{g} + F\left(X_{r1}^{g} - X_{r2}^{g} + X_{r3}^{g} - X_{r4}^{g}\right) \\ X_{r1}^{g} + F\left(X_{r2}^{g} - X_{r3}^{g} + X_{r4}^{g} - X_{r5}^{g}\right) \end{pmatrix} q \geq 0.5 \\ (X_{rabbit}(t) - X_m(t)) - r_3(LB + r_4(UB - LB))q < 0.5 \end{cases}$$

}

if ($|E| < 1$) **// Exploitation Phase**

{

if ($r \geq 0.5$ ve $|E| \geq 0.5$) **//Soft besiege**

{

$$X(t+1) = \Delta X(t) - E\left|JX_{rabbit}(t) - X(t)\right|$$

}

else if ($r \geq 0.5$ ve $|E| < 0.5$) **//Hard besiege**

{

Algorithm 2 *continue:* Pseudo Code of HHODE

$$X(t+1) = X_{rabbit}(t) - E\,|\Delta X(t)|$$

}

else if ($r < 0.5$ ve $|E| \geq 0.5$) **//Soft besiege with dives**

{

$$Y = X_{rabbit}(t) - E\,|JX_{rabbit}(t) - X(t)|$$
$$Z = Y + SxLF(D)$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases}$$

}

else if ($r < 0.5$ ve $|E| < 0.5$) **//Hard besiege with dives**

{

$$Y = X_{rabbit}(t) - E\,|JX_{rabbit}(t) - X_m(t)|$$
$$Z = Y + SxLF(D)$$
$$X(t+1) = \begin{cases} Y & if\ F(Y) < F(X(t)) \\ Z & if\ F(Z) < F(X(t)) \end{cases}$$

}

}

Return X_{rabbit}

}

two situations. Assuming that these two perching states are (q); in the case of $q < 0.5$, the Hawk perchs according to the prey (rabbit) the position, and in the case of $q \geq 0.5$, according to other Hawks' position located in the hunting area. In case of $q \geq 0.5$, in calculation of $X(t+1)$ HHODE structure is attained by using five mutation operators of DE.

The proposed algorithm, HHODE, manages an overall population space which is shared by HHO and DE. The differential evolution (DE) optimization has the advantage that, in the course of local search, the diversity in the population.

As specified in Algorithm II in the exploration phase in case of $|E| \geq 1$ and $q \geq 0.5$, in calculation of $X(t+1)$, there are five different mutation operators of DE. Five mutation operators are tested separately in the benchmark problems used in this study. After this stage, in case of $|E| \geq 1$ and $q \geq 0.5$, in calculation of $X(t+1)$, only arbitrarily applying one of these mutation operators of DE will be sufficient to apply. In the 4th section, most commonly used 23 benchmark problems and some of the IEEE CEC2005 and CEC2017 competition functions, the results of the HHODE algorithm are compared to the results of the HHO algorithm. As a result of this comparison, the DE mutation operator in finding the best

TABLE 1. F1-F13 benchmark function results for 30 and 100 dimensions of HHODE.

Prb. /ID	Metric	HHODE 30					HHODE 100				
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2	HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2
F1	AVG	5.848E-143	**1.037E-169**	8.246E-139	2.404E-146	1.077E-126	1.548E-147	**6.453E-177**	7.488E-148	2.560E-140	3.082E-127
	STD	3.203E-142	0.000E+00	4.517E-138	1.037E-145	5.892E-126	8.086E-147	0.000E+00	2.937E-147	1.402E-139	1.247E-126
F2	AVG	1.175E-73	**4.549E-90**	3.530E-75	4.711E-75	1.931E-61	2.213E-75	**3.278E-93**	3.227E-76	5.464E-75	5.262E-64
	STD	6.435E-73	2.491E-89	1.932E-74	1.937E-74	1.058E-60	8.803E-75	1.285E-92	1.755E-75	2.895E-74	2.811E-63
F3	AVG	8.334E-110	**3.134E-138**	9.794E-128	1.347E-107	2.054E-104	1.385E-109	**4.607E-119**	1.142E-103	3.179E-96	9.438E-76
	STD	4.565E-109	1.717E-137	5.364E-127	7.378E-107	1.125E-103	6.780E-109	2.523E-118	6.257E-103	1.741E-95	4.778E-75
F4	AVG	1.868E-74	**8.553E-89**	1.116E-76	4.704E-72	1.340E-65	1.529E-72	**1.822E-86**	5.480E-79	4.382E-72	4.338E-63
	STD	6.690E-74	4.649E-88	5.430E-76	1.899E-71	4.397E-65	6.929E-72	9.977E-86	1.457E-78	1.476E-71	1.630E-62
F5	AVG	1.220E-02	1.384E-02	**8.681E-03**	2.182E-02	3.003E-02	9.836E-03	1.498E-02	**4.607E-03**	1.261E-02	8.818E-03
	STD	8.306E-03	1.246E-02	1.158E-02	6.334E-02	5.206E-02	5.810E-03	2.259E-03	2.080E-03	4.689E-03	3.118E-03
F6	AVG	1.246E-04	6.383E-04	**6.136E-05**	1.399E-04	3.247E-04	3.664E-04	3.251E-04	**1.133E-04**	2.399E-04	1.791E-04
	STD	2.025E-04	5.397E-04	8.841E-05	2.528E-04	7.548E-04	2.059E-04	3.158E-04	2.005E-04	2.472E-04	1.773E-04
F7	AVG	1.949E-04	1.286E-04	**9.031E-05**	1.355E-04	1.521E-04	1.556E-04	1.608E-04	2.303E-04	**1.173E-04**	1.374E-04
	STD	2.345E-04	1.401E-04	9.279E-05	1.395E-04	1.842E-04	1.783E-04	1.749E-04	3.331E-04	8.394E-05	1.345E-04
F8	AVG	-1.244E+04	**-1.217E+04**	-1.253E+04	-1.241E+04	-1.255E+04	-4.185E+04	-4.184E+04	**-4.190E+04**	-4.158E+04	-4.186E+04
	STD	4.220E+02	7.466E+02	1.372E+02	4.897E+02	6.219E+01	2.260E+02	2.505E+02	7.887E+00	1.354E+03	1.563E+02
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
F12	AVG	1.128E-05	2.007E-04	**2.085E-07**	1.322E-05	2.357E-06	8.029E-05	1.672E-04	**1.330E-06**	6.998E-05	4.617E-05
	STD	1.406E-05	1.910E-04	3.320E-07	1.925E-05	4.254E-06	2.153E-04	2.305E-04	2.067E-06	1.952E-04	1.129E-04
F13	AVG	9.261E-05	2.257E-04	**4.587E-06**	1.075E-04	9.153E-05	9.179E-05	1.062E-04	6.431E-06	2.359E-04	**6.276E-06**
	STD	1.755E-04	2.539E-04	4.623E-06	1.963E-04	1.594E-04	2.611E-04	2.663E-04	9.701E-06	3.838E-04	9.417E-06

result was determined. This mutation operator that appeared in literature by being used in HHODE was compared with GA, BBO, DE, PSO, CS, TLBO, BA / BAT, FPA, FA, GWA and MFO algorithms and shown in tables.

The most important factor affecting the performance of each algorithm is the level of computational complexity. By looking at the level of computational complexity of an algorithm, information on its performance before running can be obtained. Of course, it is desirable for a designed algorithm to be simple and have a low level of computational complexity. The computational complexity level of the classical HHO is indicated in the study of Heidari *et al.* [40] as O (Nx (T + TD +1)). Here, O(N) is the the computational complexity of the initialization process of N Hawks. T is the maximum number of iteration and D is the dimension of problem. The computational complexity of the HHODE is the same as in the structure specified in the classical HHO [40]. In this case, the computational complexity of HHODE is O (Nx (T + TD +1)).

IV. EXPERIMENTAL RESULTS

A. BENCHMARKS' FUNCTIONS AND COMPARED ALGORITHMS

In this section, the performance of the proposed HHODE algorithm is first compared with HHO and then compared with the results of other algorithms most commonly used in literature. Benchmark functions are used in this study and most commonly studied in literature [65], [66]. These benchmark functions are divided into three main groups as unimodal (UM), multimodal (MM), and composition (fixed dimension multimodal) (CM). The UM functions (F1-F7) with unique global can best reveal the exploitative (intensification) capacities. The MM functions (F8-F23) can disclose the exploration (diversification) and LO (local optima) avoidance potentials of algorithms. The results of the HHODE proposed in this study is compared with the standard deviation (STD) and average (AVG) results of the HHO, GA, BBO, DE, PSO, CS, TLBO, BA / BAT, FPA, FA, GWA and MFO algorithms [40]. In addition, the problems selected from IEEE CEC 2005 competition [66] (F24-F29). Operating features of GA, PSO, DE and BBO of these algorithms are same as settings stated in Dan Simon's [19] study and features stated in studies of BA [67], FA [68], TLBO [29], GWO [69], FPA [70], CS [34] and MFO [71]. The performance of the proposed HHODE and HHO algorithm is evaluated using a set of problems presented in the CEC2017 competition on real-parameter single objective optimization [72]. In this study, Matlab 17R version Windows 10 operating system, 64-bit processor and 8 GB RAM hardware are used.

The same features are used for the operation of HHODE in order to compare the algorithms state in the previous section. Dimensions of 30, 100, 500 and 1000 is used in F1-F13 test functions. HHODE is operated 30 times and 500 iterations are made for each dimension to get AVG

TABLE 2. F1-F13 benchmark function results for 500 and 1000 dimensions of HHODE.

Prb. /ID	Metric	HHODE 500					HHODE 1000				
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2	HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2
F1	AVG	6.09E-142	**9.891E-165**	1.93E-150	6.31E-141	2.06E-125	1.36E-144	**2.392E-181**	9.82E-146	1.42E-140	1.63E-128
	STD	3.22E-141	1.01E-164	1.05E-149	2.68E-140	1.02E-124	5.87E-144	5.21E-180	5.38E-145	7.00E-140	8.52E-128
F2	AVG	1.67E-72	**1.350E-91**	1.25E-78	8.71E-75	9.81E-62	4.28E-73	**2.067E-89**	4.34E-76	5.55E-73	4.59E-61
	STD	9.02E-72	7.34E-91	4.41E-78	4.70E-74	5.35E-61	2.32E-72	1.13E-88	2.37E-75	2.18E-72	2.32E-60
F3	AVG	6.25E-95	4.12E-125	**5.242E-126**	2.57E-105	8.15E-101	2.23E-91	3.37E-121	**2.632E-121**	7.13E-98	9.63E-95
	STD	3.26E-94	2.73E-124	3.46E-125	1.01E-105	6.23E-101	9.37E-90	5.93E-120	4.98E-120	8.12E-97	1.01E-95
F4	AVG	3.39E-76	**9.188E-90**	5.68E-76	4.64E-71	1.12E-64	5.30E-71	**7.229E-90**	3.51E-76	1.70E-72	2.33E-63
	STD	9.94E-76	5.01E-89	3.09E-75	1.49E-70	5.69E-64	2.90E-70	3.73E-89	1.89E-75	8.90E-72	1.27E-62
F5	AVG	2.61E-01	3.44E-01	**2.124E-01**	8.14E-01	2.31E-01	1.78E-01	8.45E-02	**6.226E-02**	1.57E-01	1.61E-01
	STD	3.61E-01	7.37E-01	2.80E-01	1.63E+00	4.15E-01	2.36E-01	1.59E-01	1.61E-01	2.46E-01	2.22E-01
F6	AVG	2.48E-03	1.15E-03	**4.550E-04**	5.01E-01	6.49E-03	5.85E-03	2.34E-03	**4.653E-04**	5.32E-03	2.12E-03
	STD	2.65E-02	2.45E-03	1.48E-04	1.30E+00	2.66E-02	6.57E-03	5.96E-03	4.13E-04	1.05E-02	2.84E-03
F7	AVG	1.49E-04	9.14E-05	**1.394E-05**	1.23E-04	1.42E-04	1.20E-04	2.37E-04	**9.927E-05**	1.27E-04	2.07E-04
	STD	1.92E-04	8.46E-05	2.24E-05	1.07E-04	1.11E-04	1.33E-04	3.61E-04	1.33E-04	1.07E-04	1.27E-04
F8	AVG	**-2.095E+05**	**-2.095E+05**	**-2.095E+05**	**-2.095E+05**	**-2.094E+05**	**-4.190E+05**	-4.129E+05	**-4.190E+05**	-4.188E+05	-4.189E+05
	STD	9.11E+00	6.29E+00	3.95E+00	4.95E+00	7.80E+01	1.10E+01	9.67E+03	6.51E-01	4.25E+02	1.16E+02
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F12	AVG	2.71E-06	2.65E-06	**7.328E-07**	7.52E-06	7.35E-06	9.72E-06	3.60E-06	**1.135E-06**	3.18E-06	2.95E-06
	STD	9.31E-06	8.69E-06	2.22E-06	1.85E-05	2.35E-06	1.87E-05	3.36E-05	1.29E-06	4.57E-05	3.14E-06
F13	AVG	2.82E-04	3.11E-04	**6.384E-05**	1.31E-04	3.99E-04	6.07E-04	6.55E-04	**2.743E-04**	8.92E-04	4.99E-04
	STD	3.32E-04	3.38E-04	1.31E-04	1.20E-04	2.95E-04	8.23E-04	7.26E-04	2.25E-04	1.97E-03	1.77E-03

error and STD results. In the application of each of the five mutation operators of DE of HHODE, F1-F13 benchmark function results are given for 30 and 100 dimensions in table 1 and for 500 and 1000 dimensions in table 2. HHODE and HHO [40] comparative results of the F1-F13 functions are given in table 3 for 30 and 100 dimensions and in table 4 for 500 and 1000 dimensions.

When the results in Table 1 and 2 are analyzed, it is seen that in the exploration phase of HHODE algorithm.

$$DE/best/1;\ V_i^g = X_{best}^g + F(X_{r1}^g - X_{r2}^g)$$
$$DE/current-to-best/2;\ V_i^g = X_i^g + F(X_{best}^g - X_i^g + X_{r1}^g - X_{r2}^g)$$

mutation operators of DE produce much better results. When literature studies using DE as a hybrid are analyzed, it is seen that the two operators given above gave better results. In this respect, which mutation operator of HHODE is to be chosen corresponds to that stated in the literature. The results of the comparison for same conditions and functions of the results of the HHO algorithm proposed by Heidari *et al.* [40] and HHODE which is the hybrid algorithm that presented in this study, are given in Table 3 and 8 in detail. As shown in these tables, our proposed HHODE algorithm give better results than the HHO [40] algorithm. As a result of the comparison, it is also realized that it is possible to design HHO as a hybrid algorithm.

In the study where the HHO [40] algorithm is compared with other algorithms most commonly used, HHO yielded much better results. It can be immediately understood that HHODE will provide better results when compared with the results of the same algorithms. However, in order to make the evaluations better by presenting the results of the comparison, in this study, we have presented the tables where the results of HHODE are compared with the results of other algorithms. Comparison of F1-F13 benchmark function results with HHO, GA, BBO, DE / BAT, FPA, FA, GWA and MFO algorithms were given in Table 5 for 30 dimensions, in Table 6 for 100 dimensions, in Table 7 500 dimensions and in Table 8 for 1000 dimensions of HHODE.

In the literature, it is seen that new algorithms or hybrid algorithms are applied in the different functions of IEEE CEC20XX competitions series besides the benchmark functions. In this study, by following the same structure, F14-F23 benchmark functions and F24-F29 (C16, C18, C19, C20, C21, C25 and C25) benchmark functions of CEC2005 [67] are used. These comparison results are given in Table 9.

In Table 5, it can be seen that F1-F5, F7, F10-F13 functions for 30 dimensions of HHODE gives better results than

TABLE 3. HHODE and HHO [40] comparative results of F1-F13 functions for 30 and 100 dimensions.

Prb. /ID	Metric	HHODE 30 HHODE/rand/1	HHODE 30 HHODE/best/1	HHODE 30 HHODE/current-to-best/2	HHODE 30 HHODE/best/2	HHODE 30 HHODE/rand/2	HHO 30	HHODE 100 HHODE/rand/1	HHODE 100 HHODE/best/1	HHODE 100 HHODE/current-to-best/2	HHODE 100 HHODE/best/2	HHODE 100 HHODE/rand/2	HHO 100
F1	AVG	5.848E-143	**1.037E-169**	8.246E-139	2.404E-146	1.077E-126	3.95E-97	1.548E-147	**6.453E-177**	7.488E-148	2.560E-140	3.082E-127	1.91E-94
	STD	3.203E-142	0.000E+00	4.517E-138	1.037E-145	5.892E-126	1.72E-96	8.086E-147	0.000E+00	2.937E-147	1.402E-139	1.247E-126	8.66E-94
F2	AVG	1.175E-73	**4.549E-90**	3.530E-75	4.711E-75	1.931E-61	1.56E-51	2.213E-75	**3.278E-93**	3.227E-76	5.464E-75	5.262E-64	9.98E-52
	STD	6.435E-73	2.491E-89	1.932E-74	1.937E-74	1.058E-60	6.98E-51	8.803E-75	1.285E-92	1.755E-75	2.895E-74	2.811E-63	2.66E-51
F3	AVG	8.334E-110	**3.134E-138**	9.794E-128	1.347E-107	2.054E-104	1.92E-63	1.385E-109	**4.607E-119**	1.142E-103	3.179E-96	9.438E-76	1.84E-59
	STD	4.565E-109	1.717E-137	5.364E-127	7.378E-107	1.125E-103	1.05E-62	6.780E-109	2.523E-118	6.257E-103	1.741E-95	4.778E-75	1.01E-58
F4	AVG	1.868E-74	**8.553E-89**	1.116E-76	4.704E-72	1.340E-65	1.02E-47	1.529E-72	**1.822E-86**	5.480E-79	4.382E-72	4.338E-63	8.76E-47
	STD	6.690E-74	4.649E-88	5.430E-76	1.899E-71	4.397E-65	5.01E-47	6.929E-72	9.977E-86	1.457E-78	1.476E-71	1.630E-62	4.79E-46
F5	AVG	1.220E-02	1.384E-02	**8.681E-03**	2.182E-02	3.003E-02	1.32E-02	9.836E-03	1.498E-02	**4.607E-03**	1.261E-02	8.818E-03	2.36E-02
	STD	8.306E-03	1.246E-02	1.158E-02	6.334E-02	5.206E-02	1.87E-02	5.810E-03	2.259E-03	2.080E-03	4.689E-03	3.118E-03	2.99E-02
F6	AVG	1.246E-04	6.383E-04	**6.136E-05**	1.399E-04	3.247E-04	1.15E-04	3.664E-04	3.251E-04	**1.133E-04**	2.399E-04	1.791E-04	5.12E-04
	STD	2.025E-04	5.397E-04	8.841E-05	2.528E-04	7.548E-04	1.56E-04	2.059E-04	3.158E-04	2.005E-04	2.472E-04	1.773E-04	6.77E-04
F7	AVG	1.949E-04	1.286E-04	**9.031E-05**	1.355E-04	1.521E-04	1.40E-04	1.556E-04	1.608E-04	2.303E-04	**1.173E-04**	1.374E-04	1.85E-04
	STD	2.345E-04	1.401E-04	9.279E-05	1.395E-04	1.842E-04	1.07E-04	1.783E-04	1.749E-04	3.331E-04	8.394E-05	1.345E-04	4.06E-04
F8	AVG	-1.244E+04	**-1.217E+04**	-1.253E+04	-1.241E+04	-1.255E+04	−1.25E+04	-4.185E+04	-4.184E+04	**-4.190E+04**	-4.158E+04	-4.186E+04	**−4.19E+04**
	STD	4.220E+02	7.466E+02	1.372E+02	4.897E+02	6.219E+01	1.47E+02	2.260E+02	2.505E+02	7.887E+00	1.354E+03	1.563E+02	2.82E+00
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E-16**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.01E-31	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.01E-31
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00
F12	AVG	1.128E-05	2.007E-04	**2.085E-07**	1.322E-05	2.357E-06	7.35E-06	8.029E-05	1.672E-04	**1.330E-06**	6.998E-05	4.617E-05	4.23E-06
	STD	1.406E-05	1.910E-04	3.320E-07	1.925E-05	4.254E-06	1.19E-05	2.153E-04	2.305E-04	2.067E-06	1.952E-04	1.129E-04	5.25E-06
F13	AVG	9.261E-05	2.257E-04	**4.587E-06**	1.075E-04	9.153E-05	1.57E-04	9.179E-05	1.062E-04	6.431E-06	2.359E-04	**6.276E-06**	9.13E-05
	STD	1.755E-04	2.539E-04	4.623E-06	1.963E-04	1.594E-04	2.15E-04	2.611E-04	2.663E-04	9.701E-06	3.838E-04	9.417E-06	1.26E-04

TABLE 4. HHODE and HHO [40] comparative results of F1-F13 functions for 500 and 1000 dimensions.

Prb. /ID	Metric	HHODE 500 HHODE/rand/1	HHODE 500 HHODE/best/1	HHODE 500 HHODE/current-to-best/2	HHODE 500 HHODE/best/2	HHODE 500 HHODE/rand/2	HHO 500	HHODE 1000 HHODE/rand/1	HHODE 1000 HHODE/best/1	HHODE 1000 HHODE/current-to-best/2	HHODE 1000 HHODE/best/2	HHODE 1000 HHODE/rand/2	HHO 1000
F1	AVG	6.09E-142	**9.891E-165**	1.93E-150	6.31E-141	2.06E-125	1.46E−92	1.36E-144	**2.392E-181**	9.82E-146	1.42E-140	1.63E-128	1.06E−94
	STD	3.22E-141	1.01E-164	1.05E-149	2.68E-140	1.02E-124	8.01E−92	5.87E-144	5.21E-180	5.38E-145	7.00E-140	8.52E-128	4.97E−94
F2	AVG	1.67E-72	**1.350E-91**	1.25E-78	8.71E-75	9.81E-62	7.87E−49	4.28E-73	**2.067E-89**	4.34E-76	5.55E-73	4.59E-61	2.52E−50
	STD	9.02E-72	7.34E-91	4.41E-78	4.70E-74	5.35E-61	3.11E−48	2.32E-72	1.13E-88	2.37E-75	2.18E-72	2.32E-60	5.02E−50
F3	AVG	6.25E-95	4.12E-125	**5.242E-126**	2.57E-105	8.15E-101	6.54E−37	2.23E-91	3.37E-121	**2.632E-121**	7.13E-98	9.63E-95	1.79E−17
	STD	3.26E-94	2.73E-124	3.46E-125	1.01E-105	6.23E-101	3.58E−36	9.37E-90	5.93E-120	4.98E-120	8.12E-97	1.01E-95	9.81E−17
F4	AVG	3.39E-76	**9.188E-90**	5.68E-76	4.64E-71	1.12E-64	1.29E−47	5.30E-71	**7.229E-90**	3.51E-76	1.70E-72	2.33E-63	1.43E−46
	STD	9.94E-76	5.01E-89	3.09E-75	1.49E-70	5.69E-64	4.11E−47	2.90E-70	3.73E-89	1.89E-75	8.90E-72	1.27E-62	7.74E−46
F5	AVG	2.61E-01	3.44E-01	**2.124E-01**	8.14E-01	2.31E-01	3.10E−01	1.78E-01	8.45E-02	**6.226E-02**	1.57E-01	1.61E-01	5.73E−01
	STD	3.61E-01	7.37E-01	2.80E-01	1.63E+00	4.15E-01	3.73E−01	2.36E-01	1.59E-01	1.61E-01	2.46E-01	2.22E-01	1.40
F6	AVG	2.48E-03	1.15E-03	**4.550E-04**	5.01E-01	6.49E-03	2.94E−03	5.85E-03	2.34E-03	**4.653E-04**	5.32E-03	2.12E-03	3.61E−03
	STD	2.65E-02	2.45E-03	1.48E-04	1.30E+00	2.66E-02	3.98E−03	6.57E-03	5.96E-03	4.13E-04	1.05E-02	2.84E-03	5.38E−03
F7	AVG	1.49E-04	9.14E-05	**1.394E-05**	1.23E-04	1.42E-04	2.51E−04	1.20E-04	2.37E-04	**9.927E-05**	1.27E-04	2.07E-04	1.41E−04
	STD	1.92E-04	8.46E-05	2.24E-05	1.07E-04	1.11E-04	2.43E−04	1.33E-04	3.61E-04	1.33E-04	1.07E-04	1.27E-04	1.63E−04
F8	AVG	**-2.095E+05**	**- 2.095E+05**	**- 2.095E+05**	**-2.095E+05**	**-2.094E+05**	**−2.09E+**	**-4.19E+05**	-4.129E+05	**-4.190E+05**	-4.188E+05	-4.189E+05	−
	STD	9.11E+00	6.29E+00	3.95E+00	4.95E+00	7.80E+01	2.84E+	1.10E+01	9.67E+03	6.51E-01	4.25E+02	1.16E+02	1.03E+
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.01E−31	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.01E−31
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+**
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+
F12	AVG	2.71E-06	2.65E-06	**7.328E-07**	7.52E-06	7.35E-06	1.41E−06	9.72E-06	3.60E-06	1.135E-06	3.18E-06	2.95E-06	**1.02E−06**
	STD	9.31E-06	8.69E-06	2.22E-06	1.85E-05	2.35E-06	1.48E−06	1.87E-05	3.36E-05	1.29E-06	4.57E-05	3.14E-06	1.16E−06
F13	AVG	2.82E-04	3.11E-04	**6.384E-05**	1.31E-04	3.99E-04	3.44E−04	6.07E-04	6.55E-04	**2.743E-04**	8.92E-04	4.99E-04	8.41E−04
	STD	3.32E-04	3.38E-04	1.31E-04	1.20E-04	2.95E-04	4.75E−04	8.23E-04	7.26E-04	2.25E-04	1.97E-03	1.77E-03	1.18E−03

other algorithms. The TLBO algorithm for F6 and the CS algorithm for F8 give better results than HHODE. In the remaining F9, HHODE and BBO find same results. When a general evaluation of table 5 results is made, it can be seen that HHODE performs better than other algorithms in all functions except for 2.

TABLE 5. Comparison of F1-F13 benchmark function results for 30 dimensions of HHODE with other algorithms.

Prb./ID		HHODE					HHO	GA	PSO	BBO	FPA	GWO	BAT	FA	CS	MFO	TLBO	DE
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2												
F1	AVG	5.848E-143	**1.037E-169**	8.246E-139	2.404E-146	1.077E-126	3.95E−97	1.03E+03	1.83E+04	7.59E+01	2.01E+03	1.18E−27	6.59E+04	7.11E−03	9.06E−04	1.01E+03	2.17E−89	1.33E−03
	STD	3.203E-142	0.000E+00	4.517E-138	1.037E-145	5.892E-126	1.72E−96	5.79E+02	3.01E+03	2.75E+01	5.60E+02	1.47E−27	7.51E+03	3.21E−03	4.55E−04	3.05E+03	3.14E−89	5.92E−04
F2	AVG	1.175E-73	**4.549E-90**	3.530E-75	4.711E-75	1.931E-61	1.56E−51	2.47E+01	3.58E+02	1.36E−03	3.22E+01	9.71E−17	2.71E+08	4.34E−01	1.49E−01	3.19E+01	2.77E−45	6.83E−03
	STD	6.435E-73	2.491E-89	1.932E-74	1.937E-74	1.058E-60	6.98E−51	5.68E+00	1.35E+03	7.45E−03	5.55E+00	5.60E−17	1.30E+09	1.84E−01	2.79E−02	2.06E+01	3.11E−45	2.06E−03
F3	AVG	8.334E-110	**3.134E-138**	9.794E-128	1.347E-107	2.054E-104	1.92E−63	2.65E+04	4.05E+04	1.21E+04	1.41E+03	5.12E−05	1.38E+05	1.66E+03	2.10E−01	2.43E+04	3.91E−18	3.97E+04
	STD	4.565E-109	1.717E-137	5.364E-127	7.378E-107	1.125E-103	1.05E−62	3.44E+03	8.21E+03	2.69E+03	5.59E+02	2.03E−04	4.72E+04	6.72E+02	5.69E−02	1.41E+04	8.04E−18	5.37E+03
F4	AVG	1.868E-74	**8.553E-89**	1.116E-76	4.704E-72	1.340E-65	1.02E−47	5.17E+01	4.39E+01	3.02E+01	2.38E+01	1.24E−06	8.51E+01	1.11E−01	9.65E−02	7.00E+01	1.68E−36	1.15E+01
	STD	6.690E-74	4.649E-88	5.430E-76	1.899E-71	4.397E-65	5.01E−47	1.05E+01	3.64E+00	4.39E+00	2.77E+00	1.94E−06	2.95E+00	4.75E−02	1.94E−02	7.06E+00	1.47E−36	2.37E+00
F5	AVG	1.220E-02	1.384E-02	**8.681E-03**	2.182E-02	3.003E-02	1.32E−02	1.95E+04	1.96E+07	1.82E+03	3.17E+05	2.70E+01	2.10E+08	7.97E+01	2.76E+01	7.35E+03	2.54E+01	1.06E+02
	STD	8.306E-03	1.246E-02	1.158E-02	6.334E-02	5.206E-02	1.87E−02	1.31E+04	6.25E+06	9.40E+02	1.75E+05	7.78E−01	4.17E+07	7.39E+01	4.51E−01	2.26E+04	4.26E−01	1.01E+02
F6	AVG	1.246E-04	6.383E-04	6.136E-05	1.399E-04	3.247E-04	1.15E−04	9.01E+02	1.87E+04	6.71E+01	1.70E+03	8.44E−01	6.69E+04	6.94E−03	3.13E−03	2.68E+03	**3.29E−05**	1.44E−03
	STD	2.025E-04	5.397E-04	8.841E-05	2.528E-04	7.548E-04	1.56E−04	2.84E+02	2.92E+03	2.20E+01	3.13E+02	3.18E−01	5.87E+03	3.61E−03	1.30E−03	5.84E+03	8.65E−05	5.38E−04
F7	AVG	1.949E-04	1.286E-04	**9.031E-05**	1.355E-04	1.521E-04	1.40E−04	1.91E−01	1.07E+01	2.91E−03	3.41E−01	1.70E−03	4.57E+01	6.62E−02	7.29E−02	4.50E+00	1.16E−03	5.24E−02
	STD	2.345E-04	1.401E-04	9.279E-05	1.395E-04	1.842E-04	1.07E−04	1.50E−01	3.05E+00	1.83E−03	1.10E−01	1.06E−03	7.82E+00	4.23E−02	2.21E−02	9.21E+00	3.63E−04	1.37E−02
F8	AVG	-1.244E+04	-1.217E+04	-1.253E+04	-1.241E+04	-1.255E+04	−1.25E+04	−1.26E+04	−3.86E+03	−1.24E+04	−6.45E+03	−5.97E+03	−2.33E+03	−5.85E+03	**−5.19E+19**	−8.48E+03	−7.76E+03	−6.82E+03
	STD	4.220E+02	7.466E+02	1.372E+02	4.897E+02	6.219E+01	1.47E+02	4.51E+00	2.49E+02	3.50E+01	3.03E+02	7.10E+02	2.96E+02	1.16E+03	1.76E+20	7.98E+02	1.04E+03	3.94E+02
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	9.04E+00	2.87E+02	**0.00E+00**	1.82E+02	2.19E+00	1.92E+02	3.82E+01	1.51E+01	1.59E+02	1.40E+01	1.58E+02
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	4.58E+00	1.95E+01	0.00E+00	1.24E+01	3.69E+00	3.56E+01	1.12E+01	1.25E+00	3.21E+01	5.45E+00	1.17E+01
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**	1.36E+01	1.75E+01	2.13E+00	7.14E+00	1.03E−13	1.92E+01	4.58E−02	3.29E−02	1.74E+01	6.45E−15	1.21E−02
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.01E−31	1.51E+00	3.67E−01	3.53E−01	1.08E+00	1.70E−14	2.43E−01	1.20E−02	7.93E−03	4.95E+00	1.79E−15	3.30E−03
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	1.01E+01	1.70E+02	1.46E+00	1.73E+01	4.76E−03	6.01E+02	4.23E−03	4.29E−05	3.10E+01	0.00E+00	3.52E−02
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	2.43E+00	3.17E+01	1.69E−01	3.63E+00	8.57E−03	5.50E+01	1.29E−03	2.00E−05	5.94E+01	0.00E+00	7.20E−02
F12	AVG	1.128E-05	2.007E-04	**2.085E-07**	1.322E-05	2.357E-06	2.08E−06	4.77E+00	1.51E+07	6.68E−01	3.05E+02	4.83E−02	4.71E+08	3.13E−04	5.57E−05	2.46E+02	7.35E−06	2.25E−03
	STD	1.406E-05	1.910E-04	3.320E-07	1.925E-05	4.254E-06	1.19E−05	1.56E+00	9.88E+06	2.62E−01	1.04E+03	2.12E−02	1.54E+08	1.76E−04	4.96E−05	1.21E+03	7.45E−06	1.70E−03
F13	AVG	9.261E-05	2.257E-04	**4.587E-06**	1.075E-04	9.153E-05	1.57E−04	1.52E+01	5.73E+07	1.82E+00	9.59E+04	5.96E−01	9.40E+08	2.08E−03	8.19E−03	2.73E+07	7.89E−02	9.12E−03
	STD	1.755E-04	2.539E-04	4.623E-06	1.963E-04	1.594E-04	2.15E−04	4.52E+00	2.68E+07	3.41E−01	1.46E+05	2.23E−01	1.67E+08	9.62E−04	6.74E−03	1.04E+08	8.78E−02	1.16E−02

TABLE 6. Comparison of F1-F13 benchmark function results for 100 dimensions of HHODE with other algorithms.

Prb./ID		HHODE					HHO	GA	PSO	BBO	FPA	GWO	BAT	FA	CS	MFO	TLBO	DE
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2												
F1	AVG	1.548E-147	**6.453E-177**	7.488E-148	2.560E-140	3.082E-127	1.91E−94	5.41E+04	1.06E+05	2.85E+03	1.39E+04	1.59E−12	2.72E+05	3.05E−01	3.17E−01	6.20E+04	3.62E−81	8.26E+03
	STD	8.086E-147	0.000E+00	2.937E-147	1.402E-139	1.247E-126	8.66E−94	1.42E+04	8.47E+03	4.49E+02	2.71E+03	1.63E−12	1.42E+04	5.60E−02	5.28E−02	1.25E+04	4.14E−81	1.32E+03
F2	AVG	2.213E-75	**3.278E-93**	3.227E-76	5.464E-75	5.262E-64	9.98E−52	2.53E+02	6.06E+23	1.59E+01	1.01E+02	4.31E−08	6.00E+43	1.45E+01	4.05E+00	2.46E+02	3.27E−41	1.21E+02
	STD	8.803E-75	1.285E-92	1.755E-75	2.895E-74	2.811E-63	2.66E−51	1.41E+01	2.18E+24	3.74E+00	9.36E+00	1.46E−08	1.18E+44	6.73E+00	3.16E−01	4.48E+01	2.75E−41	2.33E+01
F3	AVG	1.385E-109	**4.607E-119**	1.142E-103	3.179E-96	9.438E-76	1.84E−59	2.53E+05	4.22E+05	1.70E+05	1.89E+04	4.09E+02	1.43E+06	4.65E+04	6.88E+00	2.15E+05	4.33E−07	5.01E+05
	STD	6.780E-109	2.523E-118	6.257E-103	1.741E-95	4.778E-75	1.01E−58	5.03E+04	7.08E+04	2.02E+04	5.44E+03	2.77E+02	6.21E+05	6.92E+03	1.02E+00	4.43E+04	8.20E−07	5.87E+04
F4	AVG	1.529E-72	**1.822E-86**	5.480E-79	4.382E-72	4.338E-63	8.76E−47	8.19E+01	6.07E+01	7.08E+01	3.51E+01	8.89E−01	9.41E+01	1.91E+01	2.58E−01	9.31E+01	6.36E−33	9.62E+01
	STD	6.929E-72	9.977E-86	1.457E-78	1.476E-71	1.630E-62	4.79E−46	3.15E+00	3.05E+00	4.73E+00	3.37E+00	9.30E−01	1.49E+00	3.12E+00	2.80E−02	2.13E+00	6.66E−33	1.00E+00
F5	AVG	9.836E-03	1.498E-02	**4.607E-03**	1.261E-02	8.818E-03	2.36E−02	2.37E+07	2.42E+08	4.47E+05	4.64E+06	9.79E+01	1.10E+09	8.46E+02	1.33E+02	1.44E+08	9.67E+01	1.99E+07
	STD	5.810E-03	2.259E-03	2.080E-03	4.689E-03	3.118E-03	2.99E−02	8.43E+06	4.02E+07	2.05E+05	1.98E+06	6.75E−01	9.47E+07	8.13E+02	7.34E+00	7.50E+07	7.77E−01	5.80E+06
F6	AVG	3.664E-04	3.251E-04	**1.133E-04**	2.399E-04	1.791E-04	5.12E−04	5.42E+04	1.07E+05	2.85E+03	1.26E+04	1.03E+01	2.69E+05	2.95E−01	2.65E+00	6.68E+04	3.27E+00	8.07E+03
	STD	2.059E-04	3.158E-04	2.005E-04	2.472E-04	1.773E-04	6.77E−04	1.09E+04	9.70E+03	4.07E+02	2.06E+03	1.05E+00	1.25E+04	5.34E−02	3.94E−01	1.46E+04	6.98E−01	1.64E+03
F7	AVG	1.556E-04	1.608E-04	2.303E-04	**1.173E-04**	1.374E-04	1.85E−04	2.73E+01	3.41E+02	1.25E+00	5.84E+00	7.60E−03	3.01E+02	5.65E−01	1.21E+00	2.56E+02	1.50E−03	1.96E+01
	STD	1.783E-04	1.749E-04	3.331E-04	8.394E-05	1.345E-04	4.06E−04	4.45E+01	8.74E+01	5.18E+00	2.16E+00	2.66E−03	2.66E+01	1.64E−01	2.65E−01	8.91E+01	5.39E−04	5.66E+00
F8	AVG	-4.185E+04	-4.184E+04	-4.190E+04	-4.158E+04	-4.186E+04	−4.19E+04	−4.10E+04	−7.33E+03	−3.85E+04	−1.28E+04	−1.67E+04	−4.07E+03	−1.81E+04	**−2.84E+18**	−2.30E+04	−1.71E+04	−1.19E+04
	STD	2.260E+02	2.505E+02	7.887E+00	1.354E+03	1.563E+02	2.82E+00	1.14E+02	4.75E+02	2.80E+02	4.64E+02	2.62E+03	9.37E+02	3.23E+03	6.91E+18	1.98E+03	3.54E+03	5.80E+02
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	3.39E+02	1.16E+03	9.11E+00	8.47E+02	1.03E+01	7.97E+02	2.36E+02	1.72E+02	8.65E+02	1.02E+01	1.03E+03
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	4.17E+01	5.74E+01	2.73E+00	4.01E+01	9.02E+00	6.33E+01	2.63E+01	9.24E+00	8.01E+01	5.57E+01	4.03E+01
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**	1.82E+01	1.91E+01	5.57E+00	8.21E+00	1.20E−07	1.94E+01	9.81E−01	3.88E−01	1.99E+01	1.66E−02	1.22E+01
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	4.01E−31	4.35E−01	2.04E−01	4.72E−01	1.14E+00	5.07E−08	6.50E−02	2.55E−01	5.23E−02	8.58E−02	9.10E−02	8.31E−01
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	5.14E+02	9.49E+02	2.24E+01	1.19E+02	4.87E−03	2.47E+03	1.19E−01	4.56E−03	5.60E+02	**0.00E+00**	7.42E+01
	STD	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.00E+00	1.05E+02	6.00E+01	4.35E+00	2.00E+01	1.07E−02	1.03E+02	2.34E−02	9.73E−04	1.23E+02	0.00E+00	1.40E+01
F12	AVG	8.029E-05	1.672E-04	**1.330E-06**	6.998E-05	4.617E-05	4.23E−06	4.55E+06	3.54E+08	3.03E+02	1.55E+05	2.87E−01	2.64E+09	4.45E+00	2.47E−02	2.82E+08	3.03E−02	3.90E+07
	STD	2.153E-04	2.305E-04	2.067E-06	1.952E-04	1.129E-04	5.25E−06	8.22E+06	8.75E+07	1.48E+03	1.74E+05	6.41E−02	2.69E+08	1.32E+00	5.98E−03	1.45E+08	1.02E−02	1.88E+07
F13	AVG	9.179E-05	1.062E-04	6.431E-06	2.359E-04	**6.276E-06**	9.13E−05	5.26E+07	8.56E+08	6.82E+04	2.76E+06	6.87E+00	5.01E+09	4.50E+01	5.84E+00	6.68E+08	5.47E+00	7.19E+07
	STD	2.611E-04	2.663E-04	9.701E-06	3.838E-04	9.417E-06	1.26E−04	3.76E+07	2.16E+08	3.64E+04	1.80E+06	3.32E−01	3.93E+08	2.24E+01	1.21E+00	3.05E+08	8.34E−01	2.73E+07

In Table 6, it can be seen that F1-F7, F9, F10, F12 and F13 functions for 100 dimensions of HHODE gives better results than other algorithms. CS algorithm for F8 gives better results than HHODE. For the remaining F11, HHODE and TLBO find same results. When a general evaluation of table 6 results is made, it can be seen that HHODE performs better than other algorithms in all functions except for 1.

In Table 7, it can be seen that n F1-F7, F10, F12 and F13 functions for 500 dimensions of HHODE gives better results than other algorithms. CS algorithm for F8 gives better results than HHODE. For the remaining F9 and F11, HHODE and TLBO find same results. When a general evaluation of Table 7 results is made, it can be seen that HHODE performs better than other algorithms in all functions except for 1.

TABLE 7. Comparison of F1-F13 benchmark function results for 500 dimensions of HHODE with other algorithms.

Prb./ID		HHODE					HHO	GA	PSO	BBO	FPA	GWO	BAT	FA	CS	MFO	TLBO	DE
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2												
F1	AVG	6.09E-142	**9.891E-165**	1.93E-150	6.31E-141	2.06E-125	1.46E−92	6.06E+05	6.42E+05	1.60E+05	8.26E+04	1.42E−03	1.52E+06	6.30E+04	6.80E+00	1.15E+06	2.14E−77	7.43E+05
	STD	3.22E-141	1.01E-164	1.05E-149	2.68E-140	1.02E-124	8.01E−92	7.01E+04	2.96E+04	9.76E+03	1.32E+04	3.99E−04	3.58E+04	8.47E+03	4.93E−01	3.54E+04	1.94E−77	3.67E+04
F2	AVG	1.67E-72	**1.350E-91**	1.25E-78	8.71E-75	9.81E-62	7.87E−49	1.94E+03	6.08E+09	5.95E+02	5.13E+02	1.10E−02	8.34E+09	7.13E+02	4.57E+01	3.00E+08	2.31E−39	3.57E+09
	STD	9.02E-72	7.34E-91	4.41E-78	4.70E-74	5.35E-61	3.11E−48	7.03E+01	1.70E+10	1.70E+01	4.84E+01	1.93E−03	1.70E+10	3.76E+01	2.05E+00	1.58E+09	1.63E−39	1.70E+10
F3	AVG	6.25E-95	4.12E-125	**5.242E-126**	2.57E-105	8.15E-101	6.54E−37	5.79E+06	1.13E+07	2.98E+06	5.34E+05	3.34E+05	3.37E+07	1.19E+06	2.03E+02	4.90E+06	1.06E+00	1.20E+07
	STD	3.26E-94	2.73E-124	3.46E-125	1.01E-105	6.23E-101	3.58E−36	9.08E+05	1.43E+06	3.87E+05	1.34E+05	7.95E+04	1.41E+07	1.88E+05	2.72E+01	1.02E+06	3.70E+00	1.49E+06
F4	AVG	3.39E-76	**9.188E-90**	5.68E-76	4.64E-71	1.12E-64	1.29E−47	9.59E+01	8.18E+01	9.35E+01	4.52E+01	6.51E+01	9.82E+01	5.00E+01	4.06E−01	9.88E+01	4.02E−31	9.92E+01
	STD	9.94E-76	5.01E-89	3.09E-75	1.49E-70	5.69E-64	4.11E−47	1.20E+00	1.49E+00	9.05E−01	4.28E+00	5.72E+00	3.32E−01	1.73E+00	3.03E−02	4.15E−01	2.67E−31	2.33E−01
F5	AVG	2.61E-01	3.44E-01	**2.124E-01**	8.14E-01	2.31E-01	3.10E−01	1.79E+09	1.84E+09	2.07E+08	3.30E+07	4.98E+02	6.94E+09	2.56E+07	1.21E+03	5.01E+09	4.97E+02	4.57E+09
	STD	3.61E-01	7.37E-01	2.80E-01	1.63E+00	4.15E-01	3.73E−01	4.11E+08	1.11E+08	2.08E+07	8.76E+06	5.23E−01	2.23E+08	6.14E+06	7.04E+01	2.50E+08	3.07E−01	1.25E+09
F6	AVG	2.48E-03	1.15E-03	**4.550E-04**	5.01E-01	6.49E-03	2.94E−03	6.27E+05	6.57E+05	1.68E+05	8.01E+04	9.22E+01	1.53E+06	6.30E+04	8.27E+01	1.16E+06	7.82E+01	7.23E+05
	STD	2.65E-02	2.45E-03	1.48E-04	1.30E+00	2.66E-02	3.98E−03	7.43E+04	3.29E+04	8.23E+03	9.32E+03	2.15E+00	3.37E+04	8.91E+03	2.24E+00	3.48E+04	2.50E+00	3.28E+04
F7	AVG	1.49E-04	9.14E-05	**1.394E-05**	1.23E-04	1.42E-04	2.51E−04	9.10E+03	1.43E+04	2.62E+03	2.53E+02	4.67E−02	2.23E+04	3.71E+02	8.05E+01	3.84E+04	1.71E−03	2.39E+04
	STD	1.92E-04	8.46E-05	2.24E-05	1.07E-04	1.11E-04	2.43E−04	2.20E+03	1.51E+03	3.59E+02	6.28E+01	1.12E−02	1.15E+03	6.74E+01	1.37E+01	2.24E+03	4.80E−04	2.72E+03
F8	AVG	-2.095E+05	-2.095E+05	-2.095E+05	-2.095E+05	-2.094E+05	−2.09E+05	−1.31E+05	−1.65E+04	−1.42E+05	−3.00E+04	−5.70E+04	−9.03E+03	−7.27E+04	**−2.10E+17**	−6.29E+04	−5.02E+04	−2.67E+04
	STD	9.11E+00	6.29E+00	3.95E+00	4.95E+00	7.80E+01	2.84E+01	2.31E+04	9.99E+02	1.98E+03	1.14E+03	3.12E+03	2.12E+03	1.15E+04	1.14E+18	5.71E+03	1.00E+04	1.38E+03
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	3.29E+03	6.63E+03	7.86E+02	4.96E+03	7.84E+01	6.18E+03	2.80E+03	2.54E+03	6.96E+03	**0.00E+00**	7.14E+03
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.96E+02	1.07E+02	3.42E+01	7.64E+01	3.13E+01	1.20E+02	1.42E+02	5.21E+01	1.48E+02	0.00E+00	1.05E+02
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**	1.96E+01	1.97E+01	1.44E+01	8.55E+00	1.93E−03	2.04E+01	1.24E+01	1.07E+00	2.03E+01	7.62E−01	2.06E+01
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.01E−31	2.04E−01	1.04E−01	2.22E−01	8.66E−01	3.50E−04	3.25E−02	4.46E−01	6.01E−02	1.48E−01	2.33E+00	2.45E−01
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	5.42E+03	5.94E+03	1.47E+03	6.88E+02	1.55E−02	1.38E+04	5.83E+02	2.66E−02	1.03E+04	**0.00E+00**	6.75E+03
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.32E+02	3.19E+02	8.10E+01	8.17E+01	3.50E−02	3.19E+02	7.33E+01	2.30E−03	4.43E+02	0.00E+00	2.97E+02
F12	AVG	2.71E-06	2.65E-06	**7.328E-07**	7.52E-06	7.35E-06	1.41E−06	2.79E+09	3.51E+09	1.60E+08	4.50E+06	7.42E−01	1.70E+10	8.67E+05	3.87E−01	1.20E+10	4.61E−01	1.60E+10
	STD	9.31E-06	8.69E-06	2.22E-06	1.85E-05	2.35E-06	1.48E−06	1.11E+09	4.16E+08	3.16E+07	3.37E+06	4.38E−02	6.29E+08	6.23E+05	2.47E−02	6.82E+08	2.40E−02	2.34E+09
F13	AVG	2.82E-04	3.11E-04	**6.384E-05**	1.31E-04	3.99E-04	3.44E−04	8.84E+09	6.82E+09	5.13E+08	3.94E+07	5.06E+01	3.17E+10	2.29E+07	6.00E+01	2.23E+10	4.98E+01	2.42E+10
	STD	3.32E-04	3.38E-04	1.31E-04	1.20E-04	2.95E-04	4.75E−04	2.00E+09	8.45E+08	6.59E+07	1.87E+07	1.30E+00	9.68E+08	9.46E+06	1.13E+00	1.13E+09	9.97E−03	6.39E+0

TABLE 8. Comparison of F1-F13 benchmark function results for 1000 dimensions of HHODE with other algorithms.

Prb./ID		HHODE					HHO	GA	PSO	BBO	FPA	GWO	BAT	FA	CS	MFO	TLBO	DE
		HHODE/rand/1	HHODE/best/1	HHODE/current-to-best/2	HHODE/best/2	HHODE/rand/2												
F1	AVG	1.36E-144	**2.392E-181**	9.82E-146	1.42E-140	1.63E-128	1.06E−94	1.36E+06	1.36E+06	6.51E+05	1.70E+05	2.42E−01	3.12E+06	3.20E+05	1.65E+01	2.73E+06	2.73E−76	2.16E+06
	STD	5.87E-144	5.21E-180	5.38E-145	7.00E-140	8.52E-128	4.97E−94	1.79E+05	6.33E+04	2.37E+04	2.99E+04	4.72E−02	4.61E+04	2.11E+04	1.27E+00	4.70E+04	7.67E−76	3.39E+05
F2	AVG	4.28E-73	**2.067E-89**	4.34E-76	5.55E-73	4.59E-61	2.52E−50	4.29E+03	1.79E+10	1.96E+03	8.34E+02	7.11E−01	1.79E+10	1.79E+10	1.02E+02	1.79E+10	1.79E+10	1.79E+10
	STD	2.32E-72	1.13E-88	2.37E-75	2.18E-72	2.32E-60	5.02E−50	8.86E+01	1.79E+10	2.18E+01	8.96E+01	4.96E−01	1.79E+10	1.79E+10	3.49E+00	1.79E+10	1.79E+10	1.79E+10
F3	AVG	2.23E-91	3.37E-121	**2.632E-121**	7.13E-98	9.63E-95	1.79E−17	2.29E+07	3.72E+07	9.92E+06	1.95E+06	1.49E+06	1.35E+08	4.95E+06	8.67E+02	1.94E+07	8.61E−01	5.03E+07
	STD	9.37E-90	5.93E-120	4.98E-120	8.12E-97	1.01E-95	9.81E−17	3.93E+06	1.16E+07	1.48E+06	4.20E+05	2.43E+05	4.76E+07	7.19E+05	1.10E+02	3.69E+06	1.33E+00	4.14E+06
F4	AVG	5.30E-71	**7.229E-90**	3.51E-76	1.70E-72	2.33E-63	1.43E−46	9.79E+01	8.92E+01	9.73E+01	5.03E+01	7.94E+01	9.89E+01	6.06E+01	4.44E−01	9.96E+01	1.01E−30	9.95E+01
	STD	2.90E-70	3.73E-89	1.89E-75	8.90E-72	1.27E-62	7.74E−46	7.16E−01	2.39E+00	7.62E−01	5.37E+00	2.77E+00	2.22E−01	2.69E+00	2.24E−02	1.49E−01	5.25E−31	1.43E−01
F5	AVG	1.78E-01	8.45E-02	**6.226E-02**	1.57E-01	1.61E-01	5.73E−01	4.73E+09	3.72E+09	1.29E+09	7.27E+07	1.06E+03	1.45E+10	2.47E+08	2.68E+03	1.25E+10	9.97E+02	1.49E+10
	STD	2.36E-01	1.59E-01	1.61E-01	2.46E-01	2.22E-01	1.40E+00	9.63E+08	2.76E+08	6.36E+07	1.84E+07	3.07E+01	3.20E+08	3.24E+07	1.27E+02	3.15E+08	2.01E−01	3.06E+08
F6	AVG	5.85E-03	2.34E-03	**4.653E-04**	5.32E-03	2.12E-03	3.61E−03	1.52E+06	1.38E+06	6.31E+05	1.60E+05	2.03E+02	3.11E+06	3.18E+05	2.07E+02	2.73E+06	1.93E+02	2.04E+06
	STD	6.57E-03	5.96E-03	4.13E-04	1.05E-02	2.84E-03	5.38E−03	1.88E+05	6.05E+04	1.82E+04	1.86E+04	2.45E+00	6.29E+04	2.47E+04	4.12E+00	4.56E+04	2.35E+00	2.46E+05
F7	AVG	1.20E-04	2.37E-04	**9.927E-05**	1.27E-04	2.07E-04	1.41E−04	4.45E+04	6.26E+04	3.84E+04	1.09E+03	1.47E−01	1.25E+05	4.44E+03	4.10E+02	1.96E+05	1.83E−03	2.27E+05
	STD	1.33E-04	3.61E-04	1.33E-04	1.07E-04	1.27E-04	1.63E−04	8.40E+03	4.16E+03	2.91E+03	3.49E+02	3.28E−02	3.93E+03	4.00E+02	8.22E+01	6.19E+03	5.79E−04	3.52E+04
F8	AVG	-4.190E+05	-4.129E+05	-4.190E+05	-4.188E+05	-4.189E+05	−4.19E+05	−1.94E+05	−2.30E+04	−2.29E+05	−4.25E+04	−8.64E+04	−1.48E+04	−1.08E+05	**−9.34E+14**	−9.00E+04	−6.44E+04	−3.72E+04
	STD	1.10E+01	9.67E+03	6.51E-01	4.25E+02	1.16E+02	1.03E+02	9.74E+03	1.70E+03	3.76E+03	1.47E+03	1.91E+04	3.14E+03	1.69E+04	2.12E+15	7.20E+03	1.92E+04	1.23E+03
F9	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	8.02E+03	1.35E+04	2.86E+03	1.01E+04	2.06E+02	1.40E+04	7.17E+03	6.05E+03	1.56E+04	**0.00E+00**	1.50E+04
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.01E+02	1.83E+02	9.03E+01	1.57E+02	4.81E+01	1.85E+02	1.88E+02	1.41E+02	1.94E+02	0.00E+00	1.79E+02
F10	AVG	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.882E-16**	**8.88E−16**	1.95E+01	1.98E+01	1.67E+01	8.62E+00	1.88E−02	2.07E+01	1.55E+01	1.18E+00	2.04E+01	5.09E−01	2.07E+01
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.01E−31	2.55E−01	1.24E−01	8.63E−02	9.10E−01	2.74E−03	2.23E−02	2.42E−01	5.90E−02	2.16E−01	1.94E+00	1.06E−01
F11	AVG	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.000E+00**	**0.00E+00**	1.26E+04	1.23E+04	5.75E+03	1.52E+03	6.58E−02	2.83E+04	2.87E+03	3.92E−02	2.47E+04	1.07E−16	1.85E+04
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.63E+03	5.18E+02	1.78E+02	2.66E+02	8.82E−02	4.21E+02	1.78E+02	3.58E−03	4.51E+02	2.03E−17	2.22E+03
F12	AVG	9.72E-06	3.60E-06	1.135E-06	3.18E-06	2.95E-06	**1.02E−06**	1.14E+10	7.73E+09	1.56E+09	8.11E+06	1.15E+00	3.63E+10	6.76E+07	6.53E−01	3.04E+10	6.94E−01	3.72E+10
	STD	1.87E-05	3.36E-05	1.29E-06	4.57E-05	3.14E-06	1.16E−06	1.27E+09	6.72E+08	1.46E+08	3.46E+06	1.82E−01	1.11E+09	1.80E+07	2.45E−02	9.72E+08	1.90E−02	7.67E+08
F13	AVG	6.07E-04	6.55E-04	**2.743E-04**	8.92E-04	4.99E-04	8.41E−04	1.91E+10	1.58E+10	4.17E+09	8.96E+07	1.21E+02	6.61E+10	4.42E+08	1.32E+02	5.62E+10	9.98E+01	6.66E+10
	STD	8.23E-04	7.26E-04	2.25E-04	1.97E-03	1.77E-03	1.18E−03	4.21E+09	1.56E+09	2.54E+08	3.65E+07	1.11E+01	1.40E+09	7.91E+07	1.48E+00	1.76E+09	1.31E−02	2.26E+09

In Table 8, it can be seen that n F1-F7, F10-F13 functions for 1000 dimensions of HHODE gives better results than other algorithms. CS algorithm for F8 gives better results than HHODE. For the remaining F9, HHODE and TLBO find same results. When a general evaluation of Table 8 results is made, it can be seen that HHODE performs better than other algorithms in all functions except for 1.

In Table 9, it can be seen that HHODE gives the same good results for all functions between F14 and F20 with the algorithms that give the best results. HHODE find better results for all functions between the remaining F21 and F29 than

TABLE 9. Comparison of F14-F23 and CEC2005 F24-F29 benchmark function results of HHODE with other algorithms.

Prb./ID		HHODE					HHO	GA	PSO	BBO	FPA	GWO	BAT	FA	CS	MFO	TLBO	DE
		HHODE/ rand/1	HHODE/ best/1	HHODE/ current-to-best/2	HHODE/ best/2	HHODE/ rand/2												
F14	AVG	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E−01**	**9.98E−01**	1.39E+00	**9.98E−01**	9.98E−01	4.17E+00	1.27E+01	3.51E+00	1.27E+01	2.74E+00	**9.98E−01**	1.23E+00
	STD	4.52E-16	3.39E-16	4.52E-16	3.39E-16	4.52E-16	9.23E−01	4.52E−16	4.60E−01	4.52E−16	2.00E−04	3.61E+00	6.96E+00	2.16E+00	1.81E−15	1.82E+00	4.52E−16	9.23E−01
F15	AVG	**3.10E-04**	**3.10E-04**	**3.10E-04**	3.17E-04	3.14E-04	**3.10E−04**	3.33E−02	1.61E−03	1.66E−02	6.88E−04	6.24E−03	3.00E−02	1.01E−03	3.13E−04	2.35E−03	1.03E−03	5.63E−04
	STD	1.49E-06	1.48E-06	6.93E-06	1.92E-05	1.34E-05	1.97E−04	2.70E−02	4.60E−04	8.60E−03	1.55E−04	1.25E−02	3.33E−02	4.01E−04	2.99E−05	4.92E−03	3.66E−03	2.81E−04
F16	AVG	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**−1.03E+00**	−3.78E−01	−1.03E+00	−8.30E−01	**−1.03E+00**	**−1.03E+00**	−6.87E−01	**−1.03E+00**	**−1.03E+00**	**−1.03E+00**	**−1.03E+00**	**−1.03E+00**
	STD	1.65E-09	2.28E-09	2.15E-09	3.87E-10	1.07E-09	6.78E−16	3.42E−01	2.95E−03	3.16E−01	6.78E−16	6.78E−16	8.18E−01	6.78E−16	6.78E−16	6.78E−16	6.78E−16	6.78E−16
F17	AVG	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E−01**	5.24E−01	4.00E−01	5.49E−01	**3.98E−01**	**3.98E−01**	**3.98E−01**	**3.98E−01**	**3.98E−01**	**3.98E−01**	**3.98E−01**	**3.98E−01**
	STD	6.49E-06	7.01E-06	1.56E-05	2.41E-06	2.53E-05	2.54E−06	6.06E−02	1.39E−03	6.05E−02	1.69E−16	1.69E−16	1.58E−03	1.69E−16	1.69E−16	1.69E−16	1.69E−16	1.69E−16
F18	AVG	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	3.10E+00	**3.00E+00**	**3.00E+00**	**3.00E+00**	1.47E+01	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**
	STD	1.43E-06	1.18E-05	1.07E-05	6.33E-06	1.81E-05	0.00E+00	0.00E+00	7.60E−02	0.00E+00	0.00E+00	4.07E−05	2.21E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F19	AVG	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**−3.86E+00**	−3.42E+00	**−3.86E+00**	−3.78E+00	**−3.86E+00**	**−3.86E+00**	−3.84E+00	**−3.86E+00**	**−3.86E+00**	**−3.86E+00**	**−3.86E+00**	**−3.86E+00**
	STD	5.18E-03	6.17E-03	2.48E-03	3.21E-03	4.08E-03	2.44E−03	3.03E−01	1.24E−03	1.26E−01	3.16E−15	3.14E−03	1.41E−01	3.16E−15	3.16E−15	1.44E−03	3.16E−15	3.16E−15
F20	AVG	**-3.322**	**-3.322**	**-3.322**	**-3.322**	**-3.322**	**−3.322**	−1.61351	−3.11088	−2.70774	−3.2951	−3.25866	−3.2546	−3.28105	**−3.322**	−3.23509	−3.24362	−3.27048
	STD	1.36E-04	1.33E-04	7.44E-05	1.48E-04	1.50E-04	0.137406	0.46049	0.029126	0.357832	0.019514	0.064305	0.058943	0.063635	1.77636E−15	0.064223	0.15125	0.058919
F21	AVG	-10.1416	-10.1496	**-10.1501**	-10.1429	-10.1491	−10.1451	−6.66177	−4.14764	−8.31508	−5.21514	−8.64121	−4.2661	−7.67362	−5.0552	−6.8859	−8.64525	−9.64796
	STD	1.39E-02	5.44E-03	4.86E-03	1.45E-02	6.05E-03	0.885673	3.732521	0.919578	2.883867	0.008154	2.563356	2.554009	350.697	1.77636E−15	318.186	176.521	151.572
F22	AVG	-10.4019	-10.4018	**-10.4012**	-10.4018	-10.4012	−10.4015	−5.58399	−6.01045	−9.38408	−5.34373	−10.4014	−5.60638	−9.63827	−5.0877	−8.26492	−10.2251	−9.74807
	STD	7.13E-04	1.64E-03	7.32E-04	2.33E-03	3.14E-03	1.352375	2.605837	1.962628	2.597238	0.053685	0.000678	3.022612	2.293901	8.88178E−16	3.076809	0.007265	1.987703
F23	AVG	-10.5114	-10.4135	**-10.5364**	-10.4550	-10.3926	**−10.5364**	−4.69882	−4.72192	−6.2351	−5.29437	−10.0836	−3.97284	−9.75489	−5.1285	−7.65923	−10.0752	**−10.5364**
	STD	7.24E-02	8.99E-02	8.24E-04	1.46E-01	1.66E-01	0.927655	3.256702	1.742618	378.462	0.356377	1.721889	3.008279	2.345487	1.77636E−15	3.576927	1.696222	8.88E−15
F24	AVG	383.044	386.170	**381.131**	382.131	391.365	396.8256	626.8389	768.1775	493.0129	518.7886	486.5743	1291.474	471.9752	469.0141	412.4627	612.5569	431.0767
	STD	95.837	94.971	70.809	83.840	86.129	79.58214	101.2255	76.09641	102.6058	47.84199	142.9028	150.4189	252.1018	60.62538	68.38819	123.2403	64.1864
F25	AVG	**910**	**910**	**910**	**910**	**910**	**910**	999.4998	1184.819	935.4693	1023.799	985.4172	1463.423	953.8902	910.1008	947.9322	967.088	917.6204
	STD	0	0	0	0	0	0	29.44366	33.02676	9.61349	31.85965	29.95368	68.41612	11.74911	0.036659	27.06628	27.39906	1.052473
F26	AVG	**910**	**910**	**910**	**910**	**910**	**910**	998.9091	1178.34	934.2718	1018.002	973.5362	1480.683	953.5493	910.1252	940.1221	983.774	917.346
	STD	0	0	0	0	0	0	25.27817	35.20755	8.253209	34.87908	22.45008	45.55006	14.086	0.047205	21.68256	45.32275	0.897882
F27	AVG	**910**	**910**	**910**	**910**	**910**	**910**	1002.032	1195.088	939.7644	1010.392	969.8538	1477.919	947.7667	910.1233	945.4266	978.7344	917.3067
	STD	0	0	0	0	0	0	26.66321	23.97978	23.07814	31.51188	19.51721	60.58827	11.18408	0.049732	26.79031	38.22729	0.861945
F28	AVG	860.741	860.379	**860.128**	864.630	864.886	860.8925	1512.467	1711.981	1068.631	1539.357	1337.671	1961.526	1016.389	1340.078	1455.918	1471.879	1553.993
	STD	1.457	0.899	0.213	8.642	8.580	0.651222	94.64553	35.18377	201.9045	42.93441	191.0662	58.46188	270.6854	134.183	36.06884	268.6238	96.35255
F29	AVG	558.083	558.241	**557.526**	560.703	562.312	558.9653	1937.396	2101.145	1897.439	2033.614	1909.091	2221.404	1986.206	1903.852	1882.974	1883.773	1897.031
	STD	2.217	2.278	1.634	8.425	8.488	5.112352	11.25913	29.74533	8.823239	30.2875	6.567542	35.54849	18.88722	185.7944	6.528261	3.493192	4.203909

other algorithms. In this case, HHODE that we are proposed in this study show better results than many algorithms as seen in the tables above.

In Figure 2, the comparison results of HHODE with other algorithms are given as graphs for 30-100-500-1000 dimensions. As it can be understood from the comparison results of HHODE with other algorithms analyzed, it is observed that the balance between the exploratory tendency and exploitative tendency of the algorithm is well consistent.

Besides, in this study, Formula 1 [73] ranking operation which began to be applied in CEC 2010 (competition) was made according to the results of F1-F13 functions of each algorithm. Thus, a global evaluation and ranking operation are carried out between HHODE and HHO, and other algorithms. In brief, it ranks each method for each function result (mean error, variance, etc.), creates a score for each rank (the better rank, the higher score) in each condition, and sums them all. The coefficient evaluations used in the Formula 1 evaluation are as follows; ranking was done from the algorithms which found the best results. $1- > 25$ points, $2- > 18$ points, $3- > 15$ points, $4- > 12$ points, $5- > 10$ points, $6- > 8$ points, $7- > 6$ points, $8- > 4$ points, $9- > 2$ points, $10- > 1$ point and there after 1 point is given. The results of the ranking operation according to the results we find in this study are given in Table 11 according to this scoring system.

When the results in Table 11 are examined, HHODE and HHO rankings are higher than other algorithms. According to all these ranking operations, in the HHODE algorithm, the use of the $DE/current-to-best/2$; $V_i^g = X_i^g + F(X_{best}^g - X_i^g + X_{r1}^g - X_{r2}^g)$ mutation operator of DE is ranked as first in the global evaluation. When a general evaluation of HHODE is performed, the algorithm design presented in this study is found to be an even more powerful algorithm as a result of the combination of strong features of both HHO and DE.

After comparison of HHODE with HHO and other algorithms, the success of the HHODE and HHO is compared in CEC2017 [72] test benchmark functions. HHODE and HHO are operated 30 times, 1000 iterations are made for 10 dimension to get AVG error and STD results are given in Table 10. When the results in Table 12 are examined, according to all these ranking operation, in the HHODE algorithm, the use of the $DE/best/2$; $V_i^g = X_{best}^g + F\left(X_{r1}^g - X_{r2}^g + X_{r3}^g - X_{r4}^g\right)$ mutation operator of DE is ranked as first in the global evaluation.

In the OPF problem, HHODE/current-to-best/2 and HHODE/best/2 mutation operators are applied instead of all

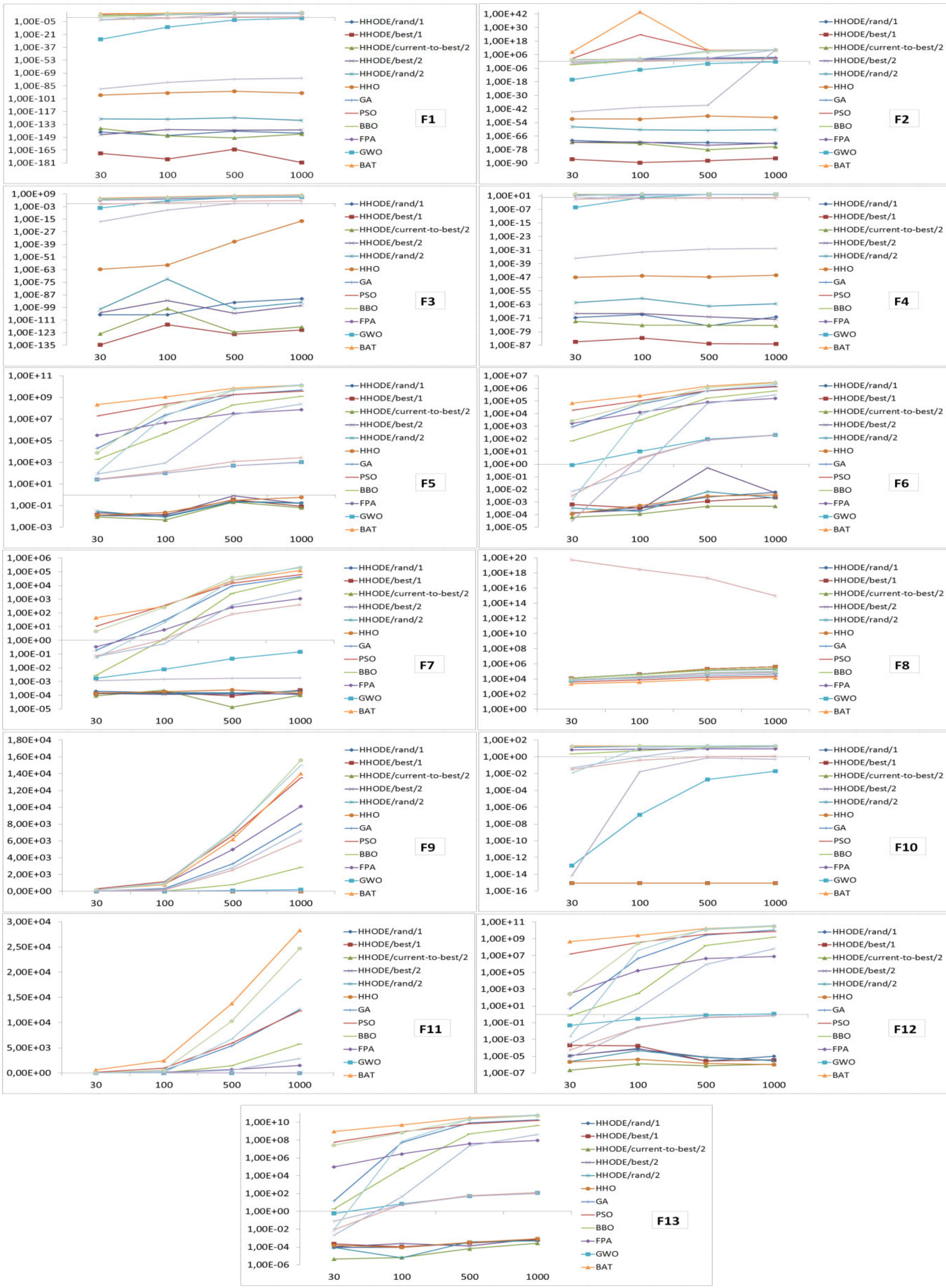

FIGURE 2. Scalability results of the HHODE against to other algorithm in dealing with the F1-F13 functions with 30, 100, 500 and 1000 dimensions.

mutation operators in the HHODE algorithm, the results of which can be seen in table 11 and table 12. Also, classic HHO algorithm is applied to OPF problem to show that HHODE algorithm is better than HHO algorithm. Settings parameters

TABLE 10. HHODE and HHO comparative results of CEC2017 test benchmark functions sets (10 dimension, 30 independent runs, 1000 iteration).

	C1		C2		C3		C4		C5		C6		C7	
	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD
HHO	0,17235	0,37544	0,29282	0,21959	0,19031	0,25433	12,9781	15,1319	8,998726	26,49548	14,8339	16,6018	-478,8629	51,74813
HHODE/rand/1	0,21422	0,30353	0,2527	0,3319	0,17713	0,3466	9,98615	15,4785	6,024549	22,48113	14,123	20,6651	**-456,793**	44,29095
HHODE/best/1	0,22144	0,28116	0,32838	0,24603	0,21844	0,3385	**8,02286**	13,7589	4,965234	22,92379	12,9911	15,1844	-466,1904	46,6471
HHODE/current-to-best/2	0,20057	0,33864	**0,11831**	0,27868	0,26026	0,35074	13,9824	12,6651	8,573295	27,88238	**11,146**	19,7643	-463,6818	53,04259
HHODE/best/2	**0,12418**	0,3315	0,23294	0,29042	**0,1432**	0,37256	15,9992	13,8128	**4,673781**	38,6945	15,4298	19,87	-461,5992	47,23683
HHODE/rand/2	0,12782	0,25584	0,27267	0,31316	0,20869	0,27958	18,9967	11,9094	8,000546	36,71843	29,8926	12,5702	-467,8902	42,04441

	C8		C9		C10		C11		C12		C13		C14	
	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD
HHO	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	31,55821	24,08574	9,9696227	36,614887	19,9163	0,0222776
HHODE/rand/1	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	36,04611	18,21693	12,075731	69,567531	19,9325	0,0257163
HHODE/best/1	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	31,02152	17,82298	19,980744	39,96239	19,9109	0,0230678
HHODE/current-to-best/2	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	**25,04337**	23,63097	6,8746674	46,155976	**19,9072**	0,0310801
HHODE/best/2	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	25,90972	19,86229	5,5835297	453,70773	19,9948	3,36333
HHODE/rand/2	-90,5096	1,45E-14	-0,50959	0,00E+00	-59,754	3,61E-14	-1123,8553	4,63E-13	35,23082	15,70541	**5,5759275**	47,31788	19,9582	0,0910613

	C15		C16		C17		C18		C19		C20		C21	
	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD
HHO	0,21918	0,059934	0,56897	0,21843	0,20325	0,16964	11,0486	12,7165	5,92327	4,35438	0,39837	0,1607	36,86169	33,91445
HHODE/rand/1	0,15964	0,079459	0,71934	0,23251	0,19477	0,16537	16	15,84	6,97176	3,47978	**0,30792**	0,2127	48,33642	31,33777
HHODE/best/1	**0,15199**	0,071222	0,68393	0,31336	0,15417	0,19532	16,2948	11,9711	**2,86825**	5,54476	0,35702	0,19857	**21,99792**	33,67807
HHODE/current-to-best/2	0,20165	0,049468	0,66377	0,26726	**0,10031**	0,18151	17,0948	15,1494	5,48638	3,27008	0,33067	0,19009	32,12926	33,74403
HHODE/best/2	0,19691	0,081786	**0,54626**	0,30276	0,10742	0,19601	14,1208	20,5742	3,57491	3,61389	0,36965	0,22407	46,41265	41,06336
HHODE/rand/2	0,19983	0,071628	0,64412	0,37372	0,15996	0,16708	**11,0121**	14,7562	4,92364	3,54748	0,33501	0,14629	39,06577	33,64937

	C22		C23		C24		C25		C26		C27		C28	
	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD	AVG	STD
HHO	40,41211	5320,13286	19,9947	0,0297817	0,451263	3,18222	2,1886	22,9639	0,58096	0,39396	24,26254	25,98884	13,0317	4,81248
HHODE/rand/1	45,67768	3871,73815	20,0013	0,0920621	0,408373	5,25377	**1,35385**	15,0884	0,25067	0,362	26	26,43592	10,4688	4,39707
HHODE/best/1	33,64654	4344,02904	19,9978	0,0580922	0,403824	3,34579	1,84559	20,6241	**0,13169**	0,40498	**21**	25,53273	10,4303	5,04014
HHODE/current-to-best/2	30,83636	4664,21357	19,9946	0,0591232	0,44621	4,28548	1,7849	18,5591	0,25481	0,24067	29,71339	21,05449	8,8045	5,24472
HHODE/best/2	32,75295	22052,82754	**19,9945**	0,0987444	0,520614	2,9918	1,76761	20,5852	0,1785	0,31501	26,63692	31,03014	**5,24555**	5,06095
HHODE/rand/2	**17,72520**	4199,58527	19,998	0,0734542	**0,280639**	3,62347	1,36669	9,58732	0,27242	0,31871	38	21,13679	7,68802	5,08856

TABLE 11. Formula 1 scores of HHODE, HHO and others algorithms.

Algorithm	Formula 1 score	Rank
HHODE/rand/1	**1097**	**3**
HHODE/best/1	**1252**	**2**
HHODE/current-to-best/2	**1397**	**1**
HHODE/best/2	**1040**	**4**
HHODE/rand/2	**1012**	**6**
HHO	**1019**	**5**
GA	119	17
PSO	184	15
BBO	156	16
FPA	237	11
GWO	319	8
BAT	188	14
FA	209	12
CS	296	9
MFO	190	13
TLBO	555	7
DE	264	10

of the HHO and HHODE algorithms for the OPF problem are;

Number of dimension = 50 and Iteration number = 100

B. OPF RESULTS

The HHODE and HHO algorithms are applied on modified IEEE 30-bus test system for solving the OPF problem and IEEE 30-bus test system data are taken from Refs. [76,77] and the generators data of test system are taken from Ref. [57]

TABLE 12. Formula 1 scores of HHODE and HHO.

Algorithm	Formula 1 score	Rank
HHO	401	6
HHODE/rand/1	414	5
HHODE/best/1	479	2
HHODE/current-to-best/2	469	3
HHODE/best/2	**496**	**1**
HHODE/rand/2	453	4

In order to show the effectiveness of the HHO and HHODE algorithms they are tested on different cases.

Case 1: The OPF problem is solved without valve-point effect and prohibited zones.

Case 2: The OPF problem is solved with prohibited zones.

Case 1: Classic OPF problem without valve-point effect and prohibited zones

The obtained optimum control variables from the HHODE and HHO algorithm for Case 1 are given in Table 13. The result of the HHODE and HHO algorithms and the results of the other methods in the literature are given in Table 14. It is obvious that the result of the proposed HHODE algorithms that use of the HHODE/current-to-best/2 mutation operator

TABLE 13. Optimum control variables for different cases.

Control Variables	Limits Min.	Max.	Case 1 HHO	HHODE/current-to-best/2	HHODE/best/2	Case 2 HHO	HHODE/current-to-best/2	HHO_DE4
P_{G1} (MW)	50	250	176.6269	177.3302	175.0507	181.1390	181.3295	179.4254
P_{G2} (MW)	20	80	49.7567	48.5955	49.0386	44.7467	44.8943	44.8034
P_{G5} (MW)	15	50	21.7887	21.4339	21.9668	21.6236	21.9564	21.9415
P_{G8} (MW)	10	35	18.4951	21.8912	20.0618	18.3758	20.9469	19.6621
P_{G11} (MW)	10	30	13.5667	11.1295	13.4622	13.4782	11.5580	13.5146
P_{G13} (MW)	12	40	12.4072	12.1910	12.9042	13.4278	12.0000	13.2468
V_{G1} (p.u.)	0.95	1.1	1.0767	1.0790	1.0810	1.0799	1.0805	1.0813
V_{G2} (p.u.)	0.95	1.1	1.0570	1.0599	1.0589	1.0537	1.0626	1.0586
V_{G5} (p.u.)	0.95	1.1	1.0206	1.0259	1.0216	1.0268	1.0324	1.0271
V_{G8} (p.u.)	0.95	1.1	1.0270	1.0343	1.0330	1.0382	1.0371	1.0329
V_{G11} (p.u.)	0.95	1.1	1.0750	1.0560	1.0602	1.0461	1.0622	1.0738
V_{G13} (p.u.)	0.95	1.1	1.0797	1.0690	1.0510	1.0704	1.0607	1.0589
$T_{6\text{-}9}$ (p.u.)	0.9	1.1	1.0230	0.9846	1.0146	0.9778	1.0180	1.0004
$T_{6\text{-}10}$ (p.u.)	0.9	1.1	1.0071	1.0255	0.9856	1.0371	0.9544	1.0367
$T_{4\text{-}12}$ (p.u.)	0.9	1.1	1.0087	1.0177	0.9723	1.0131	0.9869	1.0136
$T_{28\text{-}27}$ (p.u.)	0.9	1.1	0.9836	0.9635	0.9834	0.9770	0.9759	0.9793
Fuel Cost ($/hr.)	-	-	801.4228	**800.9959**	801.3732	801.8058	**801.2161**	801.4898
Power losses (MW)	-	-	9.2414	9.1712	9.0844	9.3912	9.2851	9.1939

TABLE 14. Comparison results for Case 1.

Methods	Fuel Cost ($/hr.)	Methods	Fuel Cost ($/hr.)
GA[78]	804.10	EGA[87]	802.06
SA[78]	804.10	FGA[88]	802.00
GA-OPF[79]	803.91	MHBMO[86]	801.985
SGA[80]	803.69	SFLA[89]	801.97
EP-OPF[79]	803.57	PSO[90]	801.89
EP[81]	802.62	Hybrid SFLA-SA[89]	801.79
ACO[82]	802.57	MPSO-SFLA[90]	801.75
IEP[83]	802.46	ABC [91]	801.71
NLP[77]	802.4	BSA [91]	801.63
DE-OPF[84]	802.39	SOS [59]	801.5733
MDE-OPF[84]	802.37	**HHO**	**801.4228**
TS[56]	802.29	**HHODE/current-to-best/2**	**800.9959**
MSFLA[85]	802.287	**HHODE/best/2**	**801.3732**
HBMO[86]	802.211		

TABLE 15. Comparison results for Case 2.

Methods	Fuel Cost ($/hr.)	Methods	Fuel Cost ($/hr.)
GA[89]	809.2314787	BSA[91]	801.85
SA[89]	808.7174786	SOS [59]	801.7649
PSO[89]	806.4331434	**HHO**	**801.8058**
SFLA[89]	806.2155404	**HHODE/current-to-best/2**	**801.2161**
Hybrid SFLA-SA[89]	805.8152356	**HHODE/best/2**	**801.4898**
ABC[91]	804.38		

is better than the HHO and other heuristic algorithms in the literature.

Case 2: Classic OPF problem with prohibited zones

The obtained optimum control variables from the HHODE and HHO algorithm for Case 2 are given in Table 13. The result of the HHODE and HHO algorithms and the results of the other methods in the literature are given in Table 15. It is obvious that the result of the proposed HHODE algorithms that use of the HHODE/current-to-best/2 mutation operator is better than the HHO and other heuristic algorithms in the literature.

V. CONCLUSION AND DISCUSSION

In the hybridization of algorithms in the literature, it is understood that most of the structures lack the equilibrium between the exploration and exploitation stages during the optimization process. The aim of this study is to develop the HHODE algorithm in order to eliminate the lack of equilibrium. In the developed HHODE algorithm, mutation operators of the DE have been used in the exploration phase for the equilibrium phase. By this means, during exploration, a better designed process has been developed for research in the whole area and in various places of the solution space of HHODE. Thus, after a well-designed exploration phase, it is ensured that HHODE has a rich solution space in detecting and examining the best possible solutions in the exploitation phase. In addition, due to the fact that HHODE has a well-organized optimizer structure, it is observed that a reasonable balance could be established between the exploration and exploitation tendencies. If the situation is the opposite, the HHODE algorithm would not be able to find good results by falling into local optimum (LO) or early convergence.

In this study, HHODE has been compared with GA, PSO, DE, BBO, BA, FA, TLBO, GWO, FPA, CS and MFO algorithms which are most commonly used in the literature to observe that HHODE yields successful results. When the results of all tables are examined, it can be seen that HHODE, which is proposed as a new hybrid algorithm, gives better results than other algorithms. Therefore, we believe that it will be useful to use HHODE as an effective hybrid algorithm for optimization problems with researchers.

In this study HHODE algorithm is applied most commonly used 23 benchmark problems and some of the IEEE CEC2005 and CEC2017 competition functions.

The proposed HHODE algorithm (hybrid HHO algorithm with DE) is tested, one of the real engineering problems, on OPF problem within IEEE 30-bus system which is under without valve-point effect and prohibited zones, with prohibited zones test cases. The results of the comparison were that in order to solve the OPF problem, the HHODE algorithm is more effective to find the optimal solution than HHO and other algorithm in the reported that before literature.

In the next studies, success rate can be observed by applying HHODE algorithm in other function tests. In addition, the HHO algorithm can be hybridized with different algorithms and the new hybrid HHO algorithm can be compared with both the classical HHO and the HHODE algorithm that is proposed in this study.

In future works, the observation of the effectiveness of HHODE in other various engineering optimization problems will create new perspectives.

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