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An OW-FCE Model Based on MDE Algorithm for Evaluating Integrated Navigation System

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ABSTRACT The traditional fuzzy comprehensive evaluation (FCE) based on Analytic Hierarchy Process (AHP) has the problem of the applicability of the dimensionless method and the subjectivity of the weighting method, which lead to the inaccurate and unreasonable of evaluation results. An optimization weight (OW) and FCE (OW-FCE) model based on modified differential evolution (MDE) algorithm is proposed. For the navigation system, aiming at the problem that the conversion index of traditional dimensionless method are inaccurate, the linear extremum method and the nonlinear exponential function method are utilized to derive a hybrid dimensionless method of navigation system (NSHD method). When multi-expert weights are adopted to solve the subjectivity of weights, there is uncertainty in each weight, and the weights and OW need to be consistent, which become a difficult problem to solve OW. The optimization model of combination coefficients for OW is established, and the optimal coefficients are solved by MDE. The optimal coefficients and multi-expert weights are used to calculate OW. Taking three sets of Inertial Navigation System/Global Navigation Satellite System (INS/GNSS) integrated navigation systems as an example, the simulation and evaluation have been carried out. The results show that the proposed model is feasible and effective, which can distinguish the system differences and provide technical support for decision makers.

INDEX TERMS Fuzzy comprehensive evaluation, dimensionless method, optimization weight, integrated navigation system, modified differential evolution.

I. INTRODUCTION

As a common decision-making auxiliary mean, evaluation method has become a key technology for selecting the most optimal. Under different application requirements, how to choose the optimal integrated navigation system becomes a core issue for decision makers. Because integrated navigation system can synthesize the advantages of each sub-navigation system, it has attracted wide interests from scholars at home and abroad [1]–[3]. The difference of sensor performance, combination mode, filter and usability of each subsystem lead to different performance of integrated navigation system. The evaluation technology of integrated navigation system can evaluate the system performance, find out the advantages and disadvantages of the system, and provide reference for the system optimization in the future. Therefore, the evaluation

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method plays an important role for selecting the most optimal integrated navigation system.

At this stage, there are a few studies on the evaluation of integrated navigation system, mainly focusing on fusion algorithm of navigation information [4], [5]. In fact, the integrated navigation system is a comprehensive system that contains multi-attribute, multi-type index and has a hierarchical relationship between indexes, which makes the evaluation of integrated navigation system difficult and complex.

The indexes of integrated navigation system have a multilevel relationship, which makes the evaluation complicated. FCE is a method that can quantify qualitative indexes and synthetically evaluate multi-attribute indexes based on fuzzy criteria, which is suitable for multi-level relationship system. However, the evaluation results will be influenced by the traditional FCE based on AHP. In order to solve this problem, a combination weight method [6]–[9], which synthesizes multiple weights, has attracted wide interests from

scholars at home and abroad. By using Shannon entropy theory and AHP method, the subjective and objective weights are combined to establish a variable weight model [6]. A combination method of subjective and objective weights based on game theory is proposed [7]. Game theory holds that Nash equilibrium has the best combination weight. A hybrid multi-attribute decision making method is proposed [8], and a combination weight method based on importance is proposed. An optimization model based on multiple weights and interval weights is proposed to determine the optimal weights of the system [9]. Reference [10] took the randomness of the weight vector itself and the consistency requirement between the weight vectors into account, and used relative entropy theory to express the consistency of the weight vectors. A new weight aggregation method based on particle swarm optimization algorithm is proposed. However, the combination weight methods do not consider the influence of the uncertainty caused by the different weight methods on the combination weight. In order to solve the combination weights, it is necessary to establish the optimization model of combination coefficients. Lagrange operator method applies to a traditional problems [11]. DE [12]–[16] is a common artificial intelligence algorithm for solving the optimization coefficients.

In integrated navigation system, the indexes can be used to calculate the fuzzy relation matrix after they are dimensionless. When some indexes vary, they will have unequal proportional effects, even cross level effects. Linear extremum method is not applicable. At the same time, the nonlinear exponential function method can express the unequal proportion effect, but it cannot describe the problem that some indexes have constant variation rat. Domestic and foreign scholars have carried out in-depth researches on the dimensionless method [17]–[21]. Reference [17] combined with the regression relation of dimensionless results, and proposed a dimensionless evaluation method. Reference [18] used dimensionless eigenvalue of conversion electrodes and improved dimensionless properties. Reference [19] defined a set of dimensionless input parameters with the function of particle density and elastic modulus. Reference [20] proposed an experimental optimization design method for establishing agent model with dimensionless variables, which improved the fidelity of regression model. Reference [21] proposed a nonlinear dimensionless fuzzy processing method to solve the linear dimensionless problem. At present, there is no dimensionless method to meet the requirements of integrated navigation system.

To solve these problems, an OW-FCE model based on MDE is proposed. This work is intended to evaluate integrated navigation system from a systems perspective. This work contain four parts. 1) The existing index system only contain the fusion algorithm index of navigation information. However, the integrated navigation system is a multiattribute, multi-type index system, which has a hierarchical relationship and makes the index system comprehensive. A four-layer index system of integrated navigation system is

designed, which contains the device layer. 2) The existing optimization model of combination weight is complicated. A nonlinear optimization model of combination coefficients for OW is established, which is simple, then the optimal coefficients are solved by MDE. 3) Traditional dimensionless method cannot express the variation of integrated navigation system, which will lead to the inaccurate of indexes. Thus, a NSHD method is deduced to for the integrated navigation system. 4) A case study is demonstrated to verify the effectiveness of the model.

II. INDEX SYSTEM OF INTEGRATED NAVIGATION

From a systems perspective, the indexes of information fusion algorithm cannot fully demonstrate the characteristics of integrated navigation system. Therefore, the extended function indexes are accuracy index, stability index and usability index, which can fully reflect the performance of integrated navigation system. The accuracy indexes include attitude precision index, velocity precision index and position precision index. The stability indexes contain fault tolerance coefficient index and robustness coefficient index. The usability indexes contain manipulation function index, environment adaptability index and communication compatibility index. The most fundamental reason for affecting the accuracy of the navigation system is the device layer error, which extends the index system to the device layer. Taking INS/GNSS integrated navigation system as an example, the indexes of integrated navigation system are summarized as follows.

A. STABILITY INDEX

No matter how excellent the fusion algorithm of integrated navigation system is, when the stability is poor, the contribution of the fusion algorithm to integrated navigation system is very small, even the system cannot work properly. The better the system performance is, the better the stability will be. Generally speaking, fault tolerance coefficient and robustness coefficient are used to express the system stability [04-05].

1) FAULT TOLERANCE COEFFICIENT INDEX

Fault tolerance reflects the ability of the filter to maintain normal operation even when it fails. Fault tolerance is usually reflected in the residual D_k . In normal operation, the residual is zero-mean Gauss white noise with normal distribution, and the covariance matrix is:

$$
E[D_k D_k^T] = H_k P_{k/k-1} H_k^T + R_k.
$$
 (1)

where, $D_{i,k}$ is a component of D_k , which satisfies the following constraint :

$$
VarD_{i,k} \le d\sqrt{\left[H_k P_{k/k-1} H_k^T + R_k\right]}_{ii},\tag{2}
$$

where, *d* is a constant which is determined by the accuracy requirements of data processing. [·]*ii* represents the *i*-th diagonal elements. In fact, formula (2) is satisfied when there are no faults or minor faults in the system. When $D_{i,k}$ is too large or the system failure occurs, formula (2) is no longer satisfied

and the system filter will diverge. Hence, the fault tolerance coefficient can be defined as follows:

$$
D_f = \sum_{i=1}^{n} \min[\frac{d\sqrt{[H_k P_{k/k-1} H_k^T + R_k]}}{Var D_{i,k}}].
$$
 (3)

The larger the D_f is, the stronger the capability of fault tolerant will be. Conversely, the capability of fault tolerant will be poorer.

2) ROBUST COEFFICIENT INDEX

Robustness represents the filter's sensitivity to the object structure of filter and the variation of the parameters. In integrated navigation system, when the system parameters or the external environment varies, the filter can still maintain a certain ability of filtering accuracy. Robustness depends on the state estimation errors, system initial values, system noise and measurement noise. Robustness coefficient can be defined as (4), shown at the bottom of this page, where, \tilde{e}_k = L_k ($X_k - \hat{X}_{k/k}$) and L_k are the linear coefficient matrix and the system state variables. The state variables \hat{X}_0 and X_0 are the initial assumptions and the actual values of the state variables. Respectively, P_0 is a positive definite matrix, reflecting the proximity of \hat{X}_0 and X_0 . The matrix Q_k and R_k represents the matrix of system noise and the matrix of measurement noise, separately. The larger the D_r is, the more sensitive the system is to the parameter variations, and the worse the robustness will be. On the contrary, the robustness will be better.

B. USABILITY INDEX

With the variation of application requirements, the working condition of integrated navigation system also varies. The better the working environment is, the less the impact of the external environment on the system is, the better the adaptability of integrated navigation system will be. In general, the adaptability index is measured by maneuverability index, environment adaptability index and communication compatibility index.

1) Manipulation functions express how easy to operate an integrated navigation system, and the key operational protections that affect system functionality. For the complex integrated navigation systems, the operation should be as simple as possible.

2) The environment adaptability of integrated navigation system is usually measured by the military standard, including waterproof performance, vibration performance, impact performance, temperature performance, humidity performance, electromagnetic interference performance and so

FIGURE 1. The index system of INS/GNSS integrated navigation system.

on. The best the environment adaptability is, the stronger the ability to work in extreme environment will be.

3) Generally, the integrated navigation system does not work alone and need to cooperate with other devices. In the working state, the better the ability of integrated navigation system is, the better the communication compatibility will be.

The index system of INS/GNSS integrated navigation system is established, as shown in figure 1. The index system is divided into four layers, including device layer (D), index layer (C), function layer (B) and target layer (A), which are a three-level structure. The indexes of device layer contain gyro drift (D1), accelerometer bias (D2), initial position error of INS (D3), initial velocity error of INS (D4), position error of GNSS (D5) and velocity error of GNSS (D6). The indexes of device layer and index layer have different dimensions.

III. AN OW-FCE METHOD BASED ON MDE

There are two problems in evaluation of integrated navigation system. 1) Using FCE based on AHP to evaluate integrated navigation system, leads to the subjectivity of evaluation results. 2) Some indexes of integrated navigation system have different dimensions, and the traditional dimensionless method makes the dimensionless values inaccurate. Therefore, the study of OW based on MDE and NSHD method is carried out.

A. OW BASED ON MDE

Aiming at the problem that the evaluation results of FCE are influenced by the subjectivity of weights, the combination

$$
D_r = \{ \sup_{X_0, \{W_k\}, \{V_k\}} \{ \frac{\sum_{k=0}^n \tilde{e}_k^T \tilde{e}_k}{(X_0 - \hat{X}_0)P_0(X_0 - \hat{X}_0)^T} + \sum_{k=0}^n W_k^T Q_k^{-1} W_k + \sum_{k=0}^n V_k^T R_k^{-1} V_k \} \}^{-1},
$$
\n
$$
(4)
$$

method don't take the uncertainty of weights into account, the method of multi-expert weights is used to solve this problem. In order to reduce the potential impact caused by subjective weighting method, multiple methods are introduced to calculate multi-expert weights. Commonly, the subjective weighting methods include AHP method, Delphi method, relative comparison method and serial ratio method [22]. From the perspective of the mathematical statistics, the real weight of each index is a random variable, and the weight values of different weighting methods are only a sample value of the real weight value [10].

1) NONLINEAR OPTIMIZATION MODEL

Due to the differences in the knowledge level, experience of experts, etc., there is the uncertainty in weights. When multiple weights are combined, combination weight also has the uncertainty. Combination coefficients are a form of expression of uncertainty. According to Shannon method [23], entropy is the best measure of uncertainty. Hence, the uncertainty of combination coefficients are expressed by entropy. Assuming that n_0 experts are invited, the weights are calculated according to the different subjective weighting methods. The weight values are $w_1, w_2, \ldots, w_{n_0}$, where $w_i = w_{i1}, w_{i2}, \ldots, w_{in1}, n_1$ denotes the number of index. The combination coefficients of OW are $k_1, k_2, \ldots, k_{n_0}$. OW can be defined as follows:

$$
W = w_1 k_1 + w_2 k_2 + \dots + w_{n_0} k_{n_0} = \sum_{i=1}^{n_0} w_i k_i.
$$
 (5)

Shannon Entropy is used to express the entropy of combination coefficients as follows:

$$
H_1 = -\sum_{i=1}^{n_0} k_i \ln k_i.
$$
 (6)

According to Jaynes's maximum entropy theory [24], the combination coefficients should make Shannon's entropy maximum. An optimization model (P1) is established as follows:

$$
(P1) Max H_1 = -\sum_{i=1}^{n_0} k_i ln k_i,
$$
\n(7)

Subject. to.
$$
\sum_{i=1}^{n_0} k_i = 1,
$$
 (8)

$$
k_i \ge 0, \quad i = 1, 2, \dots, n_0.
$$
 (9)

From the perspective of OW and the single weight, the single weight and OW need to meet the consistency demands. The difference is expressed by the distance between the component of the single vector of weight and the OW vector. The smaller the distance is, the higher the consistency will be. The OW is $W = (W_1, W_2, \dots, W_{n_1})$, and the single weight is $w_i = (w_{i1}, w_{i2}, \dots, w_{in 1})$. *H*₂ can be established as follows:

$$
H_2 = \sum_{j=1}^{n_1} \sum_{i=1}^{n_0} (W_j - w_{ij})^2.
$$
 (10)

Thus, another optimization model (P2) can be established as follows:

$$
(P2) \text{ Min } H_2 = \sum_{j=1}^{n_1} \sum_{i=1}^{n_0} (W_j - w_{ij})^2, \qquad (11)
$$

Subject. to.
$$
\sum_{i=1}^{n_0} k_i = 1,
$$
 (12)

$$
k_i \ge 0
$$
, $i = 1, 2, ..., n_0$, (13)

The optimization models (P1) and (P2) are multi-objective optimization model [25], [26]. The calculation processes of multi-objective optimization model are complicated. Therefore, after converting (P1) to the minimum value, (P1) and (P2) are converted into a single-objective optimization model (P3) by the weighted sum method. The optimization model (P3) can be defined as follows:

$$
(P3) \text{ Min } H = \alpha \sum_{i=1}^{n_0} k_i \ln k_i + \beta \sum_{j=1}^{n_1} \sum_{i=1}^{n_0} (W_j - w_{ij})^2, \quad (14)
$$

Subject. to.
$$
\sum_{i=1}^{n_0} k_i = 1,
$$
 (15)

$$
i, \ge 0, \quad i = 1, 2, \dots, n_0,
$$
 (16)

where, α and β are the model coefficient of model (P1) and model (P2), and they are used to adjust the proportion of optimization models. α and β meets the requirement $\alpha + \beta =$ 1.0. Model (P3) is a nonlinear optimization model.

2) DE

The optimization model (P3) is a nonlinear optimization problem, and the objective function is complicated. Solving the optimal solution is a difficult problem. Lagrange operator method [11] is a common method for solving classical optimization problems, which can solve the traditional problems. So this article solves the nonlinear optimization problem using the popular intelligent algorithm. DE [12]–[15] is a stochastic heuristic search algorithm, which is easy to use, and has strong robustness and global optimization ability. Its unique memory ability enables it to track the current search situation dynamically to adjust the search strategy. It does not need the feature information of the problem. It is suitable for the complicated optimization problems, which are difficult to solve or even impossible to solve by the conventional mathematical optimization methods. The calculation processes of DE are defined as follows:

a: INITIALIZATION

The DE algorithm uses *NP* real-valued parameter vectors whose dimensions are *D*, and uses them as populations for each generation. Each individual of the generation is expressed as $a_{i,G}$ ($i = 1, 2, ..., NP$). Where, *i* denotes the sequence of individuals in the population, *G* denotes the evolutionary algebra, *NP* denotes the population size.

In order to establish the initial search point, the populations must be initialized. Let the variable constraint be

[min a_{ij} , max a_{ij}], then the initial search point is a randomly chosen value within the bounds of the given boundary. The initial search point can be defined as follows:

$$
a_{ji,0} = \text{rand}[0, 1](\max a_{ij} - \min a_{ij})
$$

+
$$
\min a_{ij}(i = 1, 2, ..., NP; j = 1, 2, ..., D), \quad (17)
$$

where, rand [0,1] denotes the generation of uniform random numbers between [0,1].

b: MUTATION OPERATION

For each target vector $a_{i,G}$, the variation vector is:

$$
b_{i,G+1} = a_{r1,G} + F \cdot (a_{r2,G} - a_{r3,G}), \tag{18}
$$

where, the number of random selection r_1 , r_2 and r_3 are different from each other. At the same time, r_1 , r_2 and r_3 , are different from the number of target vector *i*, so $NP \geq 4$ must be satisfied. The mutation operator $F \in [0, 2]$ is a real constant factor to control the size of deviation variables.

c: CROSSOVER OPERATION

In order to increase the diversity of interference vectors, the crossover operation is introduced. The experimental vectors are as follows:

$$
c_{i,G+1} = (c_{1i,G+1}, c_{2i,G+1}, \dots, c_{Di,G+1}),
$$
(19)

$$
c_{ji,G+1} = \begin{cases} b_{ji,G+1}, & \text{if } randd(j) \le CR \text{ or } j = rhe(i) \\ a_{ji,G+1}, & \text{if } randd(j) > CR \text{ and } j \neq mbe(i) \\ (i = 1, 2, \dots, NP; j = 1, 2, \dots, D), \end{cases}
$$
(20)

where, *randd*(*j*) denotes the *j*-th estimate of the random number generator between $[0, 1]$; $\mathit{rnbe}(i) \in (1, 2, \ldots, D)$ denotes a random selection sequence to ensure that $c_{i,G+1}$ is obtained from one parameter of *bi*,*G*+¹ at least. *CR* denotes a crossover operator with a range of values [0, 1].

d: SELECTION OPERATION

To determine if $b_{i,G+1}$ will become a member of the next generation, the test vectors are compared with the target vectors $a_{i,G}$ in the current population according to the greedy criterion. The smaller value of objective function vectors will appear in the next generation population.

e: BOUNDARY CONDITION PROCESSING

In case of boundary constraints, it must be ensured that the values of the new individual are in the feasible domain. A common method is to replace a new individual that does not meet the boundary constraint with a randomly generated vector in the feasible domain.

3) MDE

In DE, because of the role of selection, when the number of evolutionary algebra increases, the differences between individuals will be smaller, which directly affects the diversity of population. The mutation operator F is a constant between 0 and 2. When *F* is too large, the search efficiency of the

algorithm is low, and the accuracy of finding the global optimal solution is low. When F is too small, the diversity of the population is reduced, and it is easy to appear ''premature'' phenomenon. A MDE with an adaptive mutation operator was introduced to solve this problem [16].The adaptive mutation operator is as follows:

$$
\lambda = e^{(1 - \frac{G_m}{1 + G_m - G})} \tag{21}
$$

$$
F = F_0 * 2^{\lambda} \tag{22}
$$

where, F_0 is a mutation operator, G denotes the current number of evolutionary algebra, *G^m* denotes the maximum number of evolutionary algebra. When the algorithm starts running, the adaptive mutation operator has large variation rate, which ensure the diversity of individuals. With the operation of the algorithm, the mutation operator decreases gradually, so as to avoid the destruction of the optimal solution.

B. A NSHD METHOD

The indexes of integrated navigation system have different attributes. The bigger the index value is, the better the index will be, and the index is called the benefit index. The smaller the index value is, the better the index is, and the index is called the cost index. The indexes of integrated navigation system are classified as follows: D1-D6, C1-C3 and C5 are cost indexes, C4 and C6-C8 are benefit indexes. The indexes have different dimensions, so they need to be dimensionless.

1) TRADITIONAL DIMENSIONLESS METHOD

The traditional dimensionless method transforms the index values into dimensionless values between 0 and 1. The common methods are linear extremum method and nonlinear exponential function method. The main idea of the former is that the influence of the index variation value on the quantized results is equal proportion, while the main idea of the latter is that the influence of the index variation value on the quantized results is not equal proportion. The extremum methods of the benefit and cost index are as follows:

$$
y_i = \frac{x_i - \min x_i}{\max x_i - \min x_i},
$$
\n(23)

$$
y_i = \frac{\max x_i - x_i}{\max x_i - \min x_i},\tag{24}
$$

The exponential function methods of the benefit and cost index are as follows:

$$
y_i = h \cdot e^{\left(\frac{x_i - \min x_i}{\max x_i - \min x_i}\right)},\tag{25}
$$

$$
y_i = h \cdot e^{\left(\frac{\max x_i - x_i}{\max x_i - \min x_i}\right)},\tag{26}
$$

where, max x_i and min x_i are the maximum and minimum values of the index x_i . y_i is respectively the dimensionless values of the index x_i , and the range of values is [0,1], h is a coefficient of exponential function.

2) NSHD METHOD

When some indexes of integrated navigation system vary, the variation of index value is not equal proportion to the

variation of the non-quantized value. The variation law of the quantized value is inconsistent with the expected value, the extremum method and the exponential function method are no longer applicable. Taking the gyro drift index of INS as an example, when the gyro drift value is decreased from 0.01°/h to 0.001°/h, the accuracy of INS will increase by an order of magnitude, and the linear extremum method cannot express the technical improvement of the accuracy span level. The transformation result is not accurate. When the gyro drift is decreased from 0.02◦ /h to 0.01◦ /h, the nonlinear exponential function method cannot express the accuracy variation rat.

Aiming at the problem that the traditional dimensionless method is not applicable to the integrated navigation system, a dimensionless new method is proposed by combining the extremum method and the exponential function method. Usually, people are accustomed to converting different dimension indexes into the hundred-rank system, so the conversion results are expressed in the form of percentile. Taking the cost index x_i as an example, the general formula of the dimensionless new method is as follows:

$$
f_1(x_i) = a_1 + b_1 \cdot (\frac{\max x_i - x_i}{\max x_i - \min x_i}) \cdot e^{c_1 \cdot (\frac{\max x_i - x_i}{\max x_i - \min x_i})}, \quad (27)
$$

where, a_1 , b_1 and c_1 are the coefficients, respectively, max x_i and min x_i are the maximum and minimum value of x_i . When $a_1 = c_1 = 0, b_1 = 1$, formula (27) is the extremum function.

In order to maintain the consistency of the hundred-rank system and the evaluation set, the range of values of $f_1(x_i)$ is [10, 100]. In order to solve a_1 , b_1 and c_1 , the following three assumptions are made:

1) When $x_i = \max x_i, f_1(x_i) = 10;$

2) When $x_i = \min x_i, f_1(x_i) = 100;$

3) If $f_1(x_i)$ is the extremum method, when x_i $\frac{\max x_i + \min x_i}{2}$, $f_1(x_i) = 50$. Assuming that $f_1(x_i)$ is the new method, when $x_i = \frac{\max x_i + \min x_i}{2}, f_1(x_i) = 30.$

According to the three assumptions above, we can get *a*¹ $= 10, b_1 = 160/9, c_1 = 4\ln(3/2)$. Substituted the results into the general formula (27), the dimensionless new method of the cost index is formula (28), as shown at the bottom of this page. Similarly, the dimensionless new method of benefit index is formula (29), as shown at the bottom of this page.

According to formula (28) and formula (29), the curves of the dimensionless new method for the cost and the benefit indexes are drawn when x_i are varied from 0 to 10, as shown in figure 2. As can be seen from the graph in figure 2,

FIGURE 2. The curve of the dimensionless new method.

the curves of the dimensionless new method belong to the lower convex curve. For the benefit index, as the index value increases, the slope of the curve becomes larger and larger. The variation value of index has an increasing influence on the result, which is consistent with the variation rule of the benefit index for the integrated navigation system.

Theorem 1 [27]: The necessary and sufficient condition for function $y = f(x)$ to become a dimensionless function is:

1) *y* is related to *x*, and only to *x*;

2) y is independent of the dimension unit of x ;

3) Within the definition domain, the direction of *y* and *x* changes must be coordinated. In another way, for the positive and negative indexes, *y* must be about the monotonic function of \dot{x} .

According to Theorem 1, the dimensionless new method $f_1(x_i)$ is only related to x_i , which satisfies to the first point of Theorem 1. The $f_1(x_i)$ is independent of the dimension x_i , which satisfies to the second point of Theorem 1. According to figure 2, the function curve is convex, monotonously increasing or monotonously decreasing, which accords with the third point of Theorem 1. Therefore, $f_1(x_i)$ is a dimensionless function. $f_1(x_i)$ is defined as a hybrid dimensionless method of navigation system. The new method has the following characteristics:

1) For the cost and benefit indexes, $f(x)$ must be about the monotonic function of *x*;

$$
f_1(x_i) = \begin{cases} 100, & x_i \le \min x_i \\ 10 + 160/9 \cdot \left(\frac{\max x_i - x_i}{\max x_i - \min x_i}\right) e^{4 \ln(3/2) \cdot \left(\frac{\max x_i - x_i}{\max x_i - \min x_i}\right)}, & \min x_i < x_i < \max x_i \\ 10, & x_i \ge \max x_i, \\ 10, & x_i \ge \max x_i \\ 10 + 160/9 \cdot \left(\frac{x_i - \min x_i}{\max x_i - \min x_i}\right) e^{4 \ln(3/2) \cdot \left(\frac{x_i - \min x_i}{\max x_i - \min x_i}\right)}, & \min x_i < x_i < \max x_i \\ 10, & x_i \le \min x_i. \end{cases} \tag{29}
$$

FIGURE 3. Flow chart of multi-level OW-FCE evaluation model.

2) The function value is linked to the hundred-rank system, and the function value conforms to the law of the percentage system;

3) When the index value varies, it has the cross level effects, which is usually applied to the navigation system.

IV. OW-FCE EVALUATION MODEL FOR INTEGRATED NAVIGATION SYSTEM

FCE is a method based on fuzzy mathematics, applying the principle of fuzzy transformation and comprehensively evaluating alternatives based on fuzzy criteria [5], [28]. FCE includes the one-level and multi-level form, and the multilevel FCE is applicable to multi-level relational systems. Therefore, the integrated navigation system is evaluated by the multi-level FCE.

A. ONE-LEVEL FCE

FCE mainly contains three basic elements: index set $U =$ $\{u_1, u_2, \ldots, u_n\}$, alternative set $V = \{v_1, v_2, \ldots, v_m\}$ and fuzzy transformation function. The fuzzy transformation function is defined as follows:

$$
f: U \to F(V), \tag{30}
$$

$$
u_i \mapsto f(u_i) = (r_{i1}, r_{i2}, \dots, r_{im}) \in F(V).
$$
 (31)

The fuzzy relation can be induced by *f* . The fuzzy relation matrix is as follows:

$$
R = \begin{bmatrix} f(u_1) \\ f(u_2) \\ \vdots \\ f(u_n) \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} . \quad (32)
$$

fuzzy relation matrix *R* are combined. The FCE set is defined as follows:

The FCE set can be calculated after the weight *W* and the

$$
T_R: F(U) \to F(V), \tag{33}
$$

$$
W \mapsto T_R(W) = W \circ R,\tag{34}
$$

where, ∘ denotes a fuzzy synthesis operator. There are many kinds of fuzzy synthesis operators. The commonly used fuzzy synthesis operators are summarized as follows:

$$
M(\wedge, \vee) \text{model}: b_j = \vee_{i=1}^n (W_i \wedge r_{ij}), \tag{35}
$$

$$
M(\cdot, \vee) \text{model}: b_j = \vee_{i=1}^n W_i \cdot r_{ij}, \tag{36}
$$

$$
M(\cdot, +)\text{model}: b_j = \sum_{i=1}^{n} W_i \cdot r_{ij}, \tag{37}
$$

where, *W* is the normalized weight, $W = \{W_1, W_2, \ldots, W_n\}$. $∨$ and $∧$ are the minimum value and the maximum value, respectively.

B. MULTI-LEVEL OW-FCE EVALUATION MODEL

The calculation processes of the OW-FCE model can be summarized as shown in figure 3. The flow chart mainly consists of three parts: fuzzy relation matrix, OW and comprehensive evaluation. The processes of the model are as follows:

Step 1 (Establish Index Set and Evaluation Set): According to the characteristics of the index in figure 3, the index system is decomposed, containing the hierarchical structure model of precision (B1), stability (B2) and usability (B3). According to the attribute of the index, the index set *U* and the evaluation set *V* are established.

$$
U = \{u_1, u_2, \dots, u_n\}, \quad V = \{v_1, v_2, \dots, v_m\}, \quad (38)
$$

where, u_i represents the i -th index, and its underlying indexes can be expressed as $u_i = \{u_{i1}, u_{i2}, \ldots u_{in}\}, i = 1, 2, \ldots n$.

Step 2 (Determine Fuzzy Relation Matrix): The simulation tests are carried out according to the index parameters, and the index values are obtained according to the index definition in part 2. The dimensionless values $f_1(x_i)$ are calculated by formula (28) or formula (29). The membership matrix is calculated by the membership function $f_2(vk)$ with normal distribution, and the fuzzy relation matrix *R* is calculated after normalization.

$$
f_2(vk) = e^{-0.005(v_k - f_1(x_i))^2},
$$
\n
$$
R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix}.
$$
\n(39)

Step 3 (Determine OW): A number of industry experts are invited to assign the indexes, and the multi-expert weights are calculated based on AHP method, Delphi method, relative comparison method and serial ratio method. The linear relation of OW shown in formula (5) is established, and the nonlinear optimization model shown in formula (14) - formula (16) is established. The optimal coefficients are solved by MDE. Finally, the OW is calculated.

Step 4 (FCE Set): According to the hierarchical relationship of the index system, OW are divided into different levels. Formula (37) is used to synthesize the fuzzy relation matrix and the OW of first-level, then the first-level FCE set B_i was calculated as followed:

$$
B_i = W_i \circ R_i = [W_{i1}, W_{i2}, \dots, W_{in_1}]
$$

\n
$$
\circ \begin{bmatrix} r_{i11} & r_{i12} & \cdots & r_{i1n} \\ r_{i21} & r_{i22} & \cdots & r_{i2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{im1} & r_{im2} & \cdots & r_{imn} \end{bmatrix}
$$
 (41)

For the integrated navigation system, a multi-level fuzzy comprehensive relationship is established. Taking secondlevel model as an example, after B_i and the OW of firstlevel are synthesized, the second-level FCE set is calculated as follows:

$$
B = W \circ R = W \circ \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix} = W \circ \begin{bmatrix} W_1 \circ R_1 \\ W_2 \circ R_2 \\ \vdots \\ W_m \circ R_m \end{bmatrix}
$$

$$
= [b_1, b_2, \dots, b_n]
$$
(42)

Step 5 (Evaluation Value): The FCE set is a normalized result of the membership degree for each index, which is a relative value of alternative set. Therefore, the evaluation value of integrated navigation system also needs to synthesize the second-level FCE set and the alternative set. The evaluation value can be calculated as follows:

$$
S = B \times V. \tag{43}
$$

The evaluation value S represents the scores of different schemes, which can clearly distinguish the advantages and disadvantages of the system. The calculation processes are analyzed, and the advantages and disadvantages of the system can be found, which can provide reference for further improvement of the system in the future.

V. EVALUATION RESULTS AND ANALYSIS

The integrated navigation system has good accuracy, stability and usability performance. It has been widely used in various types of ships. For the ships, there are a wide variety of integrated navigation systems. According to the application requirements, how to choose the optimal system brings difficulties to the decision makers. Evaluation technology can assist the decision makers to make the optimal decisions. Taking three sets of INS/GNSS integrated navigation system as an example, which is more commonly used in ships, the proposed model is applied to verify the feasibility and effectiveness of the model. In order to verify the effectiveness of the model, a comparative test was carried out. Because the NSHD method cannot be contrasted and verified, so a comparative test of the OW and combination weight [28] was carried out in the device layer (D) of system 1.

A. SIMULATION CONDITIONS AND MULTI-EXPERT **WEIGHTS**

1) THE PARAMETERS OF INTEGRATED NAVIGATION SYSTEM Three sets of integrated navigation systems are system 1, system 2 and system 3. Considering the influence of the device layer index on the performance of integrated navigation system, the best reference system 4 and the worst reference system 5 are established according to the different attributes of the index. Kalman Filter is applied to the three sets of system. Since the usability index cannot be obtained by simulation, it is a qualitative index. The usability index is measured in the range of [1], [10]. The greater the score is, the stronger the ability will be. Three experts in the integrated navigation field of Harbin Engineering University (HEU) were invited to evaluate the index. The system parameters are shown in Table 1.

2) MULTI-EXPERT WEIGHTS

Four experts in the integrated navigation field from HEU are invited to calculate the weights, which include device layer (D), indicator layer (C) and function layer (B), according to four different subjective weighting methods, as shown in Table 2.

B. FUZZY RELATIONAL MATRIX

According to the characteristics of the hundred-rank system, the alternative set is $V = \{10, 20, 30, 40, 50, 60, 70, 80,$ 90, 100}. Taking system 1 as an example, from Table 4 and formula (39), the membership matrix of device layer index, precision index, stability index and usability index are calculated. The membership matrix is normalized into a fuzzy

TABLE 1. The parameters of the device layer and usability index.

TABLE 2. Weight values of four experts.

TABLE 3. Simulation results of integrated navigation system.

TABLE 4. NSHD values.

relation matrix in formula (44) - formula (47), as shown at the bottom of the next page.

C. NSHD METHOD

For the ships, attitude accuracy indexes (C1) contain pitch angle, roll angle and azimuth angle, velocity accuracy indexes (C2) contain eastward speed accuracy and northward speed accuracy, and position accuracy indexes (C3) contain eastward position accuracy and northward position accuracy. The simulation results are shown in Table 3.

For the accuracy index of index layer(C), each indexes have two or three indexes, which will result in two or three dimensionless values. In order to make full use of the information of dimensionless values, the mean value of multiple indexes are taken as dimensionless values. By formula (28) and formula (29), the dimensionless values are shown in table 4.

D. OWs OF INTERGRATED NAVIGATION SYSTEM

The weight information in Table 2 is substituted into formula (14) - formula (16), and a nonlinear optimization model of combination coefficients is established. MDE is used to calculate the combination coefficients of OW. The parameters of MDE are as follows:

Population number is $NP = 50$, variable dimension is $D = 4$, which is the number of experts, maximum evolution algebra is $G_m = 100$, mutation operator is $F_0 = 0.5$, crossover operator is $CR = 0.1$. In order to ensure that the optimization model (P1) and (P2) have the same contribution

TABLE 5. The OW values.

TABLE 6. The FCE set of system 1.

to model (P3), $\alpha = \beta = 0.5$. The coefficients of combination weight W_{11} are 0, 0, 0.5, 0.5. The value of OW and W_{11} are shown in Table 5. The curves of MDE are showed in FIGURE 4.

E. FCE VALUE

For the accuracy index of the index layer, the device layer generates a first-level FCE set and passes to the index layer. At the same time, the simulation obtains the index values. In order to fully integrate the two kinds of information, the weighted sum method is used. Since the simulation data is more reliable than the FCE set, the simulation data is weighted by 0.6 and the FCE set are weighted by 0.4. Because $M(\cdot, +)$ model can synthesize the OW and the fuzzy relation matrix in an all-round way, it has been widely used. Linear summation of fuzzy synthesis operator is adopted. Taking system 1 as an example, the FCE set of all levels is shown in Table 6.

For the device layer of system 1, the evaluation scores of OW and combination weight can be calculated from

FIGURE 4. The curves of MDE.

table 5 and table 6, as showed in table 7. According to Table 7, the score of OW is bigger than the score of combination weight, which indicated that the OW is better than the combination weight.

The evaluation scores of the three sets of integrated navigation systems are shown in Table 8. According to Table 8,

R_{11}	$\begin{bmatrix}\n 0 & 0 & 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703 \\ 0 & 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.7503 \\ 0.5703 & 0.3459 & 0.0772 & 0.0063 & 0.002 & 0 & 0 & 0 & 0 \\ 0.5703 & 0.3459 & 0.0772 & 0.0063 & 0.002 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703 \\ 0 & 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703 \\ 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703\n\end{bmatrix}$ \n	
R_{12}	$\begin{bmatrix}\n 0 & 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703 \\ 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703 \\ 0 & 0 & 0 & 0 & 0.002 & 0.0063 & 0.0772 & 0.3459 & 0.5703\n\end{bmatrix}$,\n	(45)
R_{13}	$\begin{bmatrix}\n 0.6807 & 0.2759 & 0.0411 & 0.0023 & 0 & 0 & 0 & 0 & 0 \\ 0.0512 & 0.2360 & 0.4005 & 0.25 & 0.0574 & 0.0048 & 0.002 & 0 & 0 &$	

TABLE 7. Evaluation scores of the device layer in system 1.

TABLE 8. The evaluation scores.

system 1 has the highest performance score, which indicates that system 1 is the best in three systems. System 2 has the middle performance score, indicating that system 2 is in the middle level of three systems. System 3 has the lowest performance score, indicating that system 3 is the worst in three systems. The results are obviously different, the degree of discrimination is large. It is clear at a glance, which can help decision makers make the optimal decision. The evaluation results are consistent with the simulation results of integrated navigation system, which prove that the proposed model is feasible and effective. The index system contain the stability and usability index, which are more comprehensive.

VI. CONCLUSION

Based on the analysis of the traditional dimensionless methods and the subjectivity of weights, an OW-FCE model based on MDE is proposed. Taking three sets of INS/GNSS integrated navigation systems as an example, the simulations of integrated navigation system are carried out, and the proposed model is used to obtain the evaluation results to verify the feasibility and effectiveness of the model. Finally, the main conclusions of this paper are as follows:

1) This paper establishes a three-level index system, which include the device layer errors. The indexes are more comprehensive and reasonable, which can assist the decision makers to accurately confirm the good and bad indexes.

2) The combination of the extremum method and the exponential function method is used to derive the NSHD method, which is more reasonable and more applicable.

3) A nonlinear optimization model of combination coefficients of OW is established. The optimal coefficients are solved by MDE, the weighted sum method of the optimal coefficients and multi-expert weights are used to obtain the OW. The OW matrix is collective intelligence, objective and reasonable.

4) The evaluation results are expressed in the hundred-rank system, which are straightforward and consistent with the actual situation and simulation results. An OW-FCE model based on MDE has broad application prospects in evaluating integrated navigation system.

In the future, some aspects of the evaluation model for integrated navigation system need to be improved. For example, the experimental data of integrated navigation systems will be used to calculate the weights or a more optimized OW will be proposed. In order to achieve goal 1, how to process and make use of a large amount of measured data becomes a difficult problem. To goal 2, the intelligent algorithm with better performance will be introduced.

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