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A Reliability Model of Micro-Engines Subject to Natural Degradation and Dependent Zoned Shocks

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ABSTRACT Most Micro-Electro-Mechanical Systems (MEMS) experience natural degradation and random shocks simultaneously, and their failures are mainly the results of competing soft and hard failure processes. For some MEMS devices like micro-engines, considering that they have resistance against small shock loads, shocks can be categorized into three shock zones according to their magnitudes: safety zone, damage zone, and fatal zone. The fatal shocks can cause hard failure immediately, and the damage shocks can (i) increase the degradation level and (ii) reduce the hard failure threshold. In this paper, the effect (ii) is described by a dependence between the classifications of damage shocks and fatal shocks: after surviving a damage shock, the probability of a shock in fatal zone increases and in damage zone decreases. Due to the dependence, the Poisson process widely used in previous studies is unsuitable. A dependent zoned shock model is developed, where a Hindrance model, which is a special type of Markov point process, is first introduced to describe the damage shocks, and a Cox process is used to model the fatal shocks. Then, a general reliability model of micro-engines subject to degradation and dependent zoned shocks is developed. When the micro-engine degrades linearly, an analytical reliability model is derived. Finally, a numerical example is conducted to illustrate the effectiveness of the developed model.

INDEX TERMS Reliability modeling, dependent zoned shocks, degradation process, Hindrance model, Cox process, MEMS reliability.

I. INTRODUCTION

MEMS has been effectively applied to many intelligent mechatronic systems, such as vehicles [1], [2], and sensors [3], [4], and its reliability is widely studied [5], [6]. Many researchers at Sandia National Laboratories have investigated the failure modes of MEMS by performing reliability tests [5]. The MEMS device used in the reliability testing is the electrostatically driven micro-actuator (i.e. micro-engine) [7]. The micro-engine consists of multiple orthogonal linear comb drive actuators which are connected to a rotating gear mechanically as shown in FIGURE. 1. By applying voltages, the linear displacement of the comb drives is transformed into circular motion of the gear via a pin joint. The gear rotates about a hub, which is anchored to the substrate [8].

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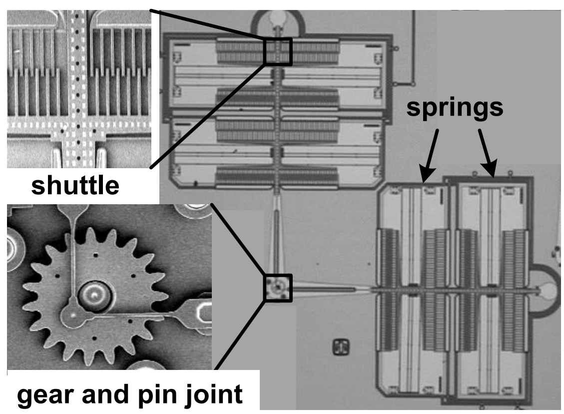

FIGURE 1. **The scanning electron microscopy image of the micro-engine [8].**

Based on the experiments on a reliability testing infrastructure [9], the dominant failure mechanism is visible wear on rubbing surfaces, which often leads to either broken pin joints or seized the micro-engine. The wear degrades the performance of MEMS [10], and the micro-engine will lose its intended function (i.e. soft failure) when the wear volume is beyond the threshold value. The micro-engine is not only subject to wear but also experiences random shocks. Tanner *et al.* [11] conduct a reliability analysis of a micro-engine in shock environments. Random shocks will cause wear debris, which will accelerate the wear on rubbing surfaces and contribute to soft failure. Moreover, the misalignment of the springs may occur and a large enough shock could result in a spring fracture (i.e. hard failure). Therefore, they will be subject to the soft and hard failures, which are competing, and any of them can lead to system failure [12], [13]. In addition, these two failures are dependent since they are both dependent on random shocks [14], and they can be referred to as dependent competing failure processes (DCFPs) [15], [16].

To develop a reliability model of the micro-engine with the DCFPS, Peng *et al.* [17], Che *et al.* [18], and Song *et al.* [15] assume that failures of a micro-engine generally occur in two modes: soft failures caused jointly by continuous smooth wear and additional abrupt wear debris due to shocks, and hard failures (i.e. spring fractures) caused by a sudden size from the same shock process. A cumulative shock model is put forward to calculate the total degradation and a probabilistic model is developed to calculate the reliability in their literature.

In the models above, every shock has a detrimental effect on the degradation process. However, the micro-engine is typically designed to resist against shock loads with small magnitude due to system structures and material strength, and consequently the shock loads under a certain magnitude cannot influence the degradation process at all [19], [20]. To consider this problem, Jiang *et al.* [19], Fan *et al.* [21], An and Sun [20] and Wei *et al.* [22] propose zoned shock models, where shocks are divided into safety zone, damage zone, and fatal zone based on their magnitudes. They perform reliability analysis of DCFPs through considering that only shock loads larger than a certain level can cause additional damage on degradation. In their models, they describe the shocks as a homogenous Poisson process (HPP) with the intensity λ. The shocks in the safety zone, damage zone, and fatal zone also follow a HPP with intensity λP_1, λP_2, and λP_3 respectively, where P_1, P_2, and P_3 are the probabilities that the ith shock belongs to the safety, damage, and fatal zones, respectively, and they satisfy that $P_1 + P_2 + P_3 = 1$.

Although Refs [19]–[22] propose the zoned shock models to evaluate system reliability with DCFPs, they assume that the system strength (i.e. hard failure threshold) is a constant value. In fact, a damage shock will reduce the system strength, and then shock loads will exceed the system strength more easily [23], [24]. Based on the results of the reliability test in [11], for a micro-engine in shock environments, a shock in fatal zone can cause the spring fracture (i.e. hard failure), and a shock in damage zone can cause the misalignment of the springs which will reduce their strength and make the springs more vulnerable to fatal shocks. Rafiee *et al.* [24] and Hao *et al.* [25] consider that the nonfatal shocks will weaken the strength of the micro-engine through reducing its hard failure threshold.

In this paper, the shock process is considered to contain fatal shocks that can cause hard failure immediately [26], damage shocks that can influence the micro-engine in two ways: (i) damaging the micro-engine by increasing the wear debris, and (ii) weakening the system strength by reducing the hard failure threshold, and safety shocks that have no effect on the micro-engine. After the micro-engine survives a damage shock, the decreased hard failure threshold makes the micro-engine more vulnerable to hard failure. To develop the reliability model of the micro-engine, this condition is described as a novel dependence between the classifications of damage shocks and fatal shocks: when the hard failure threshold decreases, the probability of a shock in fatal zone (i.e. P_3) increases and in damage zone (i.e. P_2) decreases, and accordingly the occurrence rate of fatal shocks (i.e. λP_3) increases and the occurrence rate of damage shocks (i.e. λP_2) decreases. Therefore, the arrived damage shocks will facilitate the occurrence of fatal shocks while hinder the occurrence of damage shocks, and the damage shocks and fatal shocks are dependent. To the best of our knowledge, the dependence is first studied to model DCFPs. For a micro-engine in MEMS, its reliability may be overestimated if ignoring the dependence.

In this paper, a dependent zoned shock model is developed to build the reliability model of the micro-engine subject to DCFPs with decreasing hard failure threshold. In the mentioned models above, the shock process is assumed to be a HPP or nonhomogeneous Poisson process (NHPP), which is a completely random process. However, due to the dependence, the HPP or NHPP will not be a suitable choice to describe the damage shocks and fatal shocks. In the dependent zoned shock model, a Hindrance model is first introduced to describe the damage shocks. The Hindrance model, a special type of Markov point process (MPP), has a property that the arrival points can hinder the points arriving in the future [27]. In addition, a Cox process is used to describe the fatal shocks whose intensity is random and is dependent on the number of arrived damage shocks. Such process describes a Poisson process with a random intensity that might represent a random environment or a field that influences the Poisson locations of the points. The Cox process is first developed by Cox [28], and recently, it is used to model the random shocks by many studies [29]–[32]. Based on the dependent zoned shock model, a general reliability model of system subject to degradation and dependent zoned shocks is developed. When the micro-engine degrades linearly, an analytical reliability model is derived. Finally, a numerical example is conducted to illustrate the effectiveness of the analytical reliability model.

The remainder of this article is organized as follows. Section 2 describes the failure processes of the micro-engine and the dependent zoned shocks. In Section 3, we propose the reliability model of the micro-engine with degradation process and dependent zoned shocks. Section 4 presents a numerical example and conducts the sensitivity analysis of the important parameters in the reliability model. Finally, Section 5 concludes the work.

II. DESCRIPTION OF MICRO-ENGINE FAILURE PROCESSES

In this paper, we develop a reliability model of the micro-engine in MEMS developed at Sandia National Laboratories. The micro-engine is subject to degradation and random shocks simultaneously. Based on the reliability test of the micro-engine at different levels of shocks [11], the micro-engine exhibits no damage and functions smoothly after the test of a low level of shock loads; the shocks with the magnitude larger than a certain threshold can cause additional wear debris on rubbing surfaces, in addition, they can lead to the spring misalignment; and a huge shock will lead to a spring fracture. As shown in FIGURE. 2, similar to studies [19]–[21], the random shocks can be divided into safety, damage, and fatal zones according to their magnitudes. The shock belongs to safety, damage, and fatal zones when its magnitude W is less than the safety zone threshold D_S, between D_S and the fatal zone threshold (i.e. hard failure threshold) D_F, and larger than D_F, respectively.

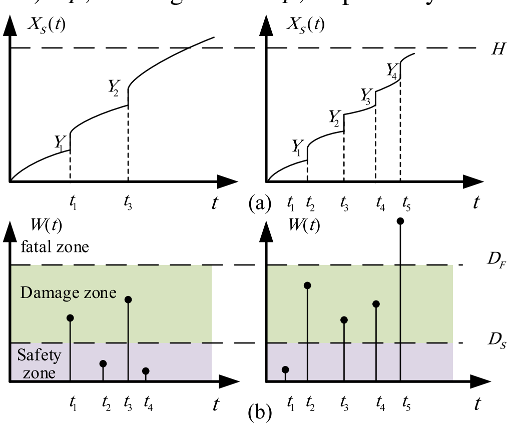

FIGURE 2. Two dependent competing failure processes: (a) soft failure process due to degradation process, and (b) hard failure process due to fatal shocks.

The micro-engine's failures are mainly the results of competing soft and hard failure processes. Soft failure will occur when the total wear volume $X_S(t)$, which consists of the continual wear and the additional wear debris Y caused by damage shocks, is beyond the threshold value H. Hard failure will occur immediately when a fatal shocks arrives. The soft and hard failures are dependent since they are both related to random shocks [14]. Then, the micro-engine experiences DCFPs.

In many studies [19]–[21], random shocks arrive according to a Poisson process with the intensity $\lambda(t)$, and the arrival shocks can be classified into one of the three zones based on their magnitude. According to the decomposition of Poisson process [33], the arrival of shocks in the safety zone, damage zone, and fatal zones also follows a Poisson process with rate $\lambda(t) P_1$, $\lambda(t) P_2$, and $\lambda(t) P_3$ respectively. Based on D_S and D_F, P_1, P_2, and P_3 can be calculated through the distribution of W_i, which is an i.i.d. random variable.

In fact, the micro-engine experiences dependent damage shocks and fatal shocks. The damage shocks can change the spring's structure and reduce its strength (i.e. the hard failure threshold D_F), and then make the springs easier to be fractured by a shock. In this paper, the condition is considered as the dependence between the classifications of damage shocks and fatal shocks. After the micro-engine survives a damage shocks, D_F will decrease, while the safety zone threshold D_S will not change. Consequently, P_2 will decrease to $P_{2,i}$ and P_3 will increase to $P_{3,i}$ when a system withstands i damage shocks. As shown in FIGURE. 3, when the micro-engine withstands a damage shock at t_1 and t_3, D_F decreases at t_1 and t_3, and then P_2 decreases to $P_{2,1}$ and $P_{2,2}$ respectively, and P_3 increases to $P_{3,1}$ and $P_{3,2}$ respectively. The arrival damage shocks can facilitate the occurrence of fatal shocks while can hinder the occurrence of damage shocks. Therefore, the dependence can be considered in a statistical way that the intensity of fatal shocks and the intensity of damage shocks are dependent on the number of arrival damage shocks.

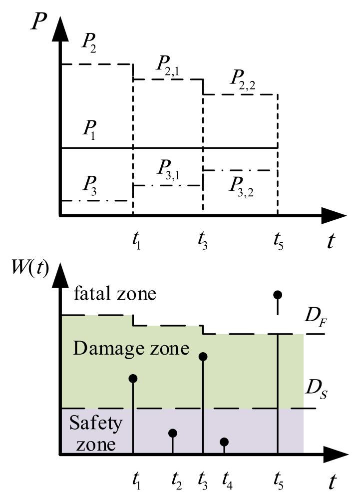

FIGURE 3. The dependence between damage shocks and fatal shocks.

To describe the dependent zoned shocks, the following assumptions are utilized to develop the reliability model.

1. Random shocks arrive with the intensity function $\lambda(t)$. The probability that a shock belongs to the safety zone is constant, and the intensity of safety shocks is $\lambda_1(t) = \lambda(t) P_1$, which is not influenced by damage shocks.
2. The dependence that the arrived damage shocks hinder the occurrence of themselves is considered as $P_{2,i} = (1 - \gamma i) P_2$, and then $\lambda_{2,i}(t) = \lambda(t) P_{2,i} = (1 - \gamma i) \lambda_2(t)$, where $\lambda_2(t) = \lambda(t) P_2$ and it is the initial intensity of damage shocks, $\lambda_{2,i}(t)$ is the intensity of damage shocks after system survived i damage shocks, and γ is a Hindrance factor. Since $P_{2,i}$ cannot be negative, $P_{2,i}$ and $\lambda_{2,i}(t)$ is equal to 0 if $i = n_\gamma$

where $n_\gamma = \gamma^{-1}$. n_γ is assumed to be an integer, and it is the maximum number of arrival damage shocks.

3. The dependence that the arrived damage shocks facilitate the occurrence of fatal shocks is considered as $P_{3,i} = P_3 + \gamma i P_2$, and then $\lambda_{3,i}(t) = \lambda(t) P_{3,i} = \lambda_3(t) + \gamma i \lambda_2(t)$, where $\lambda_3(t) = \lambda(t) P_3$ and it is the initial intensity of fatal shocks, and $\lambda_{3,i}(t)$ is the intensity of fatal shocks after system survived i damage shocks.

Yang *et al.* [34], Wang and Pham [35], and Cha *et al.* [36] assume that the hard failure rate will increase after surviving a shocks. Cha *et al.* [36] assume that the sudden failure rate λ_t is linearly dependent on the number of arrived shocks $N(t)$, $\lambda_t = \lambda_0 + \eta N(t)$, where λ_0 is the initial failure rate and η is a constant parameter. Similarly, we assume that the probability of a shock resulting in hard failure is linearly dependent on the number of arrived damage shocks i, $P_{3,i} = P_3 + \eta i$. Then we can obtain $P_{2,i} = P_2 - \eta i$ since $P_1 + P_{2,i} + P_{3,i} = 1$ and P_1 is constant. In this paper, η is denoted as $\eta = \gamma P_2$, and then we can obtain $\lambda_{3,i}(t) = \lambda(t) P_{3,i} = \lambda_3(t) + \gamma i \lambda_2(t)$, and $\lambda_{2,i}(t) = (1 - \gamma i) \lambda_2(t)$.

III. RELIABILITY MODELING FOR THE MICRO-ENGINE SUBJECT TO DEGRADATION AND DEPENDENT ZONED SHOCKS

A. MODELING DEPENDENT ZONED SHOCKS

As discussed above, many studies [19]–[21] assume that the arrival of shocks in the safety zone, damage zone, and fatal zone follows a HPP or NHPP with rate $\lambda(t) P_1$, $\lambda(t) P_2$, and $\lambda(t) P_3$ respectively. However, HPP or NHPP, which is a completely random process [27], may be not suitable for the dependent damage shocks and fatal shocks. Each point in HPP or NHPP is stochastically independent of all the other points, while damage shocks and fatal shocks are dependent and their occurrences are dependent on the number of arrived damage shocks. To describe the dependence, a dependent zoned shock model is developed. The damage shocks hinder the occurrence of themselves, which can be described by a Hindrance model. In the Hindrance model, the arrival points can hinder the points arriving in the future [27]. The intensity of fatal shocks is dependent on the number of arrived damage shocks. A Doubly Stochastic Poisson Process (DSPP), also called conditional Poisson Process and Cox process, can be used to describe the fatal shocks. Such process can describe a Poisson process with a random intensity which is influenced by a random environment or a field.

1) MODELING DAMAGE SHOCKS BY A HINDRANCE MODEL

The damage shock process $\{N_2(t), t \geq 0\}$ is described as a Hindrance model with the intensity function $\lambda_{2,i}(t)$. Let $N_2(t)$ denote the number of arrival damage shocks before time t, and the transition probability from the state $N_2(s) = i$ to the state $N_2(t) = j$ is represented by $P_{ij}(s,t)$. For a damage shock process, we are only interested in $P_i(t)$, which is the transition probability from the state $N_2(0) = 0$ to the state $N_2(t) = i$, i.e., $P_i(t)$ is the probability that i damage shocks have arrived by time t. Since the arrival damage shocks can hinder the damage shock process, the intensity function of the Hindrance model can be assumed as

$$\lambda_{2,i}(t) = (1 - \gamma i) \lambda_2(t), \quad \text{for } 0 \leq i \leq n_\gamma \tag{1}$$

Then, $P(N_2(t) = i)$ can be derived as where $a^{(k)} = a(a-1)\ldots(a-k+1)$, and $\Lambda_2(t) = \int_0^t \lambda_2(u) du$.

Readers can see [37] for a more detailed proof of Eq. (2), as shown at the bottom of this page. Based on Eq. (2), when γ^{-1} is an integer, we can see that $P_i(t)$ is a binomial distribution, $B\left(\gamma^{-1}, 1 - \exp(-\gamma \Lambda_2(t))\right)$, which promotes the reliability modeling and the numerical solution.

2) MODELING FATAL SHOCKS BY A COX PROCESS

The intensity of fatal shocks is dependent on the number of the arrived damage shocks. For the Cox process $\{N_3(t), t \geq 0\}$, its intensity is affected by an external process i.e. the damage shock process, which is a random process and follows a MPP. The detailed properties of the Cox process can be found in [38].

In this paper, the condition intensity function of the Cox process is assumed as

$$\lambda_{3,i}(t) = \lambda_3(t | N_2(t) = i) = \lambda_3(t) + \gamma i \lambda_2(t), \quad \text{for } 0 \leq i \leq n_\gamma. \tag{3}$$

Let t_i denote the arrival time of the ith damage shock, and then, based on the properties of Cox process, for $0 \leq i \leq n_\gamma$, we can obtain the following conditional probability

$$\begin{aligned} &P(N_3(t_0, t_1) = k_1, \ldots, N_3(t_i, t) = k_{i+1} | N_2(t) = i, t_0, \ldots, t_i, t) \\ &= \prod_{j=1}^{i} P\left(N_3(t_{j-1}, t_j) = k_j | t_{j-1}, t_i\right) P(N_3(t_i, t) = k_{i+1} | t_i, t), \end{aligned} \tag{4}$$

where $N_3(t_{i-1}, t_i)$ is the number of fatal shocks between t_{i-1} and t_i, and t_0 is equal to 0. In the time interval $[t_{i-1}, t_i)$, the condition intensity function is $\lambda_{3,i-1}(t)$, and the conditional probability $P(N_3(t_{i-1}, t_i) = k_i | t_{i-1}, t_i)$ can be derived as

$$\begin{aligned} &P(N_3(t_{i-1}, t_i) = k_i | t_{i-1}, t_i) \\ &\quad = \exp\left(-\Lambda_{3,i-1}(t_{i-1}, t_i)\right) \frac{\left(\Lambda_{3,i-1}(t_{i-1}, t_i)\right)^{k_i}}{k_i!}, \end{aligned} \tag{5}$$

$$\begin{aligned} P(N_2(t) = i) &= P_i(t) \\ &= \frac{\left(-\gamma^{-1} + (i-1)\right)^{(i)} (\exp(-\gamma \Lambda_2(t)) - 1)^i \exp(-(1-\gamma i)\Lambda_2(t))}{i!} \\ &= C_{\gamma^{-1}}^{i} (1 - \exp(-\gamma \Lambda_2(t)))^i (\exp(-\gamma \Lambda_2(t)))^{\gamma^{-1} - i}, \end{aligned} \tag{2}$$

where $\Lambda_{3,i-1}(t_{i-1}, t_i)$ is given by

$$\Lambda_{3,i-1}(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} \lambda_{3,i-1}(u)du. \quad (6)$$

Therefore, Eq. (4) can be expressed as

$$\begin{aligned} &P(N_3(t_0, t_1) = k_1, \ldots, N_3(t_i, t) = k_{i+1} | N_2(t) = i, t_0, \ldots, t_i, t) \\ &= \exp\left(-\sum_{j=1}^{i} \Lambda_{3,j-1}(t_{j-1}, t_j) - \Lambda_{3,i}(t_i, t)\right) \\ &\prod_{j=1}^{i} \frac{(\Lambda_{3,j-1}(t_{j-1}, t_j))^{k_j} (\Lambda_{3,i}(t_i, t))^{k_{i+1}}}{k_j! k_{i+1}!}. \end{aligned} \quad (7)$$

B. MODELING FOR HARD FAILURES AND SOFT FAILURES

1) MODELING FOR HARD FAILURES DUE TO FATAL SHOCKS

In this paper, the hard failure occurs according to the extreme shock model, i.e., the hard failure will occur immediately if a fatal shock arrives. Therefore, the probability that no hard failure occurs at time t is the probability that no fatal shocks arrives by time t, and it can be expressed as

$$\begin{aligned} P_{NH}(t) &= P(N_3(t) = 0) \\ &= \sum_{i=0}^{n_\gamma} P(N_3(t) = 0 | N_2(t) = i) P(N_2(t) = i), \end{aligned} \quad (8)$$

where $P(N_3(t) = 0 | N_2(t) = i)$ is the conditional probability that no hard failure occurs by time t given that i damage shocks have arrived. For $0 \le i \le n_\gamma$, it can be derived as

$$\begin{aligned} &P(N_3(t) = 0 | N_2(t) = i) \\ &= P\begin{pmatrix} N_3(t_0, t_1) = 0, \ldots, N_3(t_{i-1}, t_i) = 0, \\ N_3(t_i, t) = 0 | N_2(t) = i, t_0, \ldots, t_i, t < t_{i+1} \end{pmatrix} \\ &\quad \times P(t_0, \ldots, t_i, t < t_{i+1} | N_2(t) = i) \\ &= P\begin{pmatrix} N_3(t_0, t_1) = 0, \ldots, N_3(t_{i-1}, t_i) = 0, N_3(t_i, t) = 0 \\ |N_2(t) = i, t_0, \ldots, t_i, t < t_{i+1} \end{pmatrix} \\ &\quad \times \frac{P(t < t_{i+1} | t_i) \times P(t_i | t_{i-1}) \times \ldots \times P(t_2 | t_1) P(t_1)}{P(N_2(t) = i)}, \end{aligned} \quad (9)$$

where $P(t < t_{i+1} | t_i)$ is the conditional probability that there is no damage shock arriving in the time interval $(t_i, t]$ given t_i. $P(t_i | t_{i-1})$ is the conditional probability that the ith damage shock arrives in an infinitesimal interval dt_i at time t_i given t_{i-1}, and it can be derived as

$$P(t_i | t_{i-1}) = P_{i-1i-1}(t_{i-1}, t_i) \lambda_{2,i-1}(t_i) dt_i. \quad (10)$$

Then, Eq. (9) can be derived as (11), shown at the bottom of this page.

Based on Eq. (8), the probability that no hard failure occurs can be derived as

$$\begin{aligned} &P_{NH}(t) \\ &= P(N_3(t) = 0) \\ &= P(N_3(t) = 0 | N_2(t) = 0) P(N_2(t) = 0) \\ &\quad + \sum_{i=1}^{n_\gamma} P(N_3(t) = 0 | N_2(t) = i) P(N_2(t) = i) \\ &= \exp(-\Lambda(t)) \\ &\quad + \sum_{i=1}^{n_\gamma} \int_0^t \cdots \int_{t_{i-2}}^t \int_{t_{i-1}}^t \\ &\quad \times \exp\begin{pmatrix} -\sum_{j=1}^{i} (\Lambda_{3,j-1}(t_{j-1}, t_j) + \Lambda_{2,j-1}(t_{j-1}, t_j)) \\ -\Lambda_{3,i}(t_i, t) - \Lambda_{2,i}(t_i, t) \end{pmatrix} \\ &\quad \times \prod_{j=1}^{i} \lambda_{2,j-1}(t_j) dt_i dt_{i-1} \ldots dt_1 \\ &= \exp(-\Lambda(t)) \begin{pmatrix} 1 + \sum_{i=1}^{n_\gamma} \int_0^t \cdots \int_{t_{i-2}}^t \int_{t_{i-1}}^t \prod_{j=1}^{i} (1 - (j-1)\gamma) \\ \lambda_2(t_j) dt_i dt_{i-1} \ldots dt_1 \end{pmatrix}, \end{aligned} \quad (12)$$

where $\Lambda(t)$ is given by $\Lambda(t) = \int_0^t \lambda_2(u) + \lambda_3(u)du$.

$$\begin{aligned} &P(N_3(t) = 0 | N_2(t) = i) \\ &= \int_0^t \cdots \int_{t_{i-2}}^t \int_{t_{i-1}}^t P\begin{pmatrix} N_3(t_0, t_1) = 0, \ldots, N_3(t_{i-1}, t_i) = 0, \\ N_3(t_i, t) = 0 | t_0, \ldots, t_i, t < t_{i+1} \end{pmatrix} \\ &\quad \times \frac{\prod_{j=1}^{i} P_{j-1j-1}(t_{j-1}, t_j) \lambda_{2,j-1}(t_j) P_{ii}(t_i, t)}{C_{\gamma^{-1}}^{i} (1 - \exp(-\gamma \Lambda_2(t)))^i (\exp(-\gamma \Lambda_2(t)))^{\gamma^{-1} - i}} dt_i dt_{i-1} \ldots dt_1 \\ &= \frac{\int_0^t \cdots \int_{t_{i-2}}^t \int_{t_{i-1}}^t \exp\begin{pmatrix} -\sum_{j=1}^{i} (\Lambda_{3,j-1}(t_{j-1}, t_j) + \Lambda_{2,j-1}(t_{j-1}, t_j)) \\ -\Lambda_{3,i}(t_i, t) - \Lambda_{2,i}(t_i, t) \end{pmatrix} \times \prod_{j=1}^{i} \lambda_{2,j-1}(t_j) dt_i dt_{i-1} \ldots dt_1}{C_{\gamma^{-1}}^{i} (1 - \exp(-\gamma \Lambda_2(t)))^i (\exp(-\gamma \Lambda_2(t)))^{\gamma^{-1} - i}}. \end{aligned} \quad (11)$$

2) MODELING FOR SOFT FAILURES DUE TO DEGRADATION AND DAMAGE SHOCKS

The soft failure will occur when the total degradation, $X_S(t)$, exceeds a threshold value H. $X_S(t)$ is the sum of the continual degradation $X(t)$ and the abrupt degradation shifts Y_i due to the shocks. When the ith damage shock arrives, Y_i can accumulate instantaneously, such as the wear debris caused by the damage shock load. The sudden damage increments Y_i for $i = 1, 2, \ldots, n_\gamma$, can be measured by the damage sizes of the damage shock loads, and they are assumed to be random variables. The cumulative degradation increments caused by random shocks by time t, denoted as $S(t)$, are expressed as

$$S(t) = \begin{cases} \sum_{i=1}^{N_2(t)} Y_i, & \text{if } N_2(t) > 0 \\ 0 & \text{if } N_2(t) = 0. \end{cases} \quad (13)$$

According to the cumulative shock model, the total degradation can be obtained as

$$X_S(t) = X(t) + S(t) = X(t) + \sum_{i=1}^{N_2(t)} Y_i. \quad (14)$$

Then, the probability that no soft failure occurs at time t (i.e., the total degradation is less than the threshold value H), $P_{NS}(t)$, can be expressed as

$$P_{NS}(t) = P(X_S(t) < H) = \sum_{i=0}^{n_\gamma} P\left(X(t) + \sum_{j=0}^{i} Y_j | N_2(t) = i\right) P(N_2(t) = i). \quad (15)$$

Let $G(x, t)$ denote the cumulative distribution function (CDF) of $X(t)$ at time t, $f_Y(y)$ denote the probability density function (PDF) of Y_i, and $f_Y^{<m>}(y)$ denote the PDF of the sum of m i.i.d. Y_i variables. The CDF of $X_S(t)$ can be derived by utilizing a convolution integral [1] and $P(X_S(t) < H | N_2(t) = i)$ can be derived as

$$P(X_S(t) < H | N_2(t) = i) = \int_0^H G(H-u) f_Y^{<i>}(u)\, du. \quad (16)$$

Based on Eq. (15), the probability that no soft failure occurs can be derived as

$$P_{NS}(t) = \sum_{i=0}^{n_\gamma} \int_0^H G(H-u) f_Y^{<i>}(u)\, du \times C_{\gamma^{-1}}^{i} (1 - \exp(-\gamma \Lambda_2(t)))^i (\exp(-\gamma \Lambda_2(t)))^{\gamma^{-1} - i}. \quad (17)$$

C. SYSTEM RELIABILITY ANALYSIS

In this section, we discuss the development of the general reliability model of the micro-engine subject to degradation process and dependent zoned shocks. The micro-engine experiences dependent competing soft and hard failures, and either one will cause the system to fail. Therefore, the system reliability is the probability that no fatal shock occurs and the total degradation is less than the threshold level H at time t. The general system reliability can be expressed as

$$R(t) = P(N_3(t) = 0, X_S(t) < H). \quad (18)$$

The reliability model considers two types of dependence: (a) the dependence between damage shocks and the fatal shocks and (b) the dependence between the damage shocks and the degradation process. Therefore, the events $N_3(t) = 0$, $X_S(t) < H$ are dependent, since they are both dependent on the damage shocks. However, the two events are conditional-independent given the number of arrival damage shocks $N_2(t)$ [21]. Thus, Eq. (18) can be expressed as the total probability formula of $N_2(t)$, and we obtain Eq. (19).

$$\begin{aligned} R(t) &= \sum_{i=0}^{n_\gamma} P(N_3(t) = 0, X_S(t) < H | N_2(t) = i) P(N_2(t) = i) \\ &= \sum_{i=0}^{n_\gamma} P(N_3(t) = 0 | N_2(t) = i) \\ &\quad \times P(X_S(t) < H | N_2(t) = i) P(N_2(t) = i) \end{aligned} \quad (19)$$

The expressions for $P(N_3(t) = 0 | N_2(t) = i)$, $P(X_S(t) < H | N_2(t) = i)$ and $P(N_2(t) = i)$ can be determined by using Eq. (11), (16), and (2), respectively.

There is a special case, i.e. $N_2(t) = 0$. $P(N_3(t) = 0 | N_2(t) = 0)$ can be expressed as

$$P(N_3(t) = 0 | N_2(t) = 0) = \exp(-\Lambda(t)), \quad (20)$$

and $P(X_S(t) < H | N_2(t) = 0)$ can be expressed as

$$P(X_S(t) < H | N_2(t) = 0) = G(H, t). \quad (21)$$

Therefore, the system reliability model (Eq. 19) can be derived as

$$\begin{aligned} R(t) &= \exp(-\Lambda(t)) G(H, t) \\ &\quad + \sum_{i=1}^{n_\gamma} \exp(-\Lambda(t)) \int_0^t \ldots \int_{t_{i-2}}^t \int_{t_{i-1}}^t \prod_{j=1}^{i} (1 - (j-1)\eta) \\ &\quad \times \lambda_2(t_j)\, dt_i dt_{i-1} \ldots dt_1 \int_0^H G(H-u) f_Y^{<i>}(u)\, du. \end{aligned} \quad (22)$$

Eq. (22) is a general reliability model for the micro-engine experiencing degradation process and dependent zoned shocks. Based on the results of [8], the micro-engine wears linearly. In this paper, a linear path model is used to model the continuous degradation process, and then an analytical reliability model can be derived based on Eq. (22).

$$X(t) = \varphi + \beta t, \quad (23)$$

where the initial value of φ is a constant and the wear rate β is a random variable following a normal distribution, $\beta \sim N(\mu_\beta, \sigma_\beta^2)$. In addition, The random shocks follow a HPP with the intensity λ. In this model, Y_i is assumed to be an i.i.d. normal random variable, $Y_i \sim N(\mu_Y, \sigma_Y^2)$. The assumptions above or similar ones have been made by many researchers (e.g., [15], [17], [39], [40]). According to the decomposition of Poisson process [33], the initial intensity of damage shocks is a constant $\lambda_2 = \lambda p_2$, and the initial intensity of damage shocks is also a constant $\lambda_3 = \lambda p_3$.

Based on the assumptions above, the expressions for $P(N_3(t) = 0|N_2(t) = i)$, $P(X_S(t) < H|N_2(t) = i)$, and $P(N_2(t) = i)$ can be derived as analytical expressions. The intensity function of damage shocks after i arrival damage shocks is $\lambda_{2,i} = (1-\gamma i)\lambda_2$, and the intensity function of fatal shocks after i arrival damage shocks is $\lambda_{3,i} = \lambda_3 + \gamma i \lambda_2$. Then $P(N_3(t) = 0|N_2(t) = i)$ can be derived as (24), shown at the bottom of this page.

Based on the linear path degradation model and the assumptions that β is a normal distribution, $\beta \sim N(\mu_\beta, \sigma_\beta^2)$ and Y_i is also a normal distribution, $Y_i \sim N(\mu_Y, \sigma_Y^2)$. $P(X_S(t) < H|N_2(t) = i)$ can be expressed as:

$$P(X_S(t) < H|N_2(t) = i) = P\left(X(t) + \sum_{i=1}^{i} Y_i < H\right) = \Phi\left(\frac{H - (\mu + \mu_\beta t + i\mu_{Y_i})}{\sqrt{\sigma_\beta^2 t^2 + i\sigma_i^2}}\right), \quad (25)$$

where $\Phi(\bullet)$ is the CDF of a standard normal random variable.

Due to $\lambda_{2,i} = (1-\gamma i)\lambda_2$, $P(N_2(t) = i)$ can be derived as

$$P(N_2(t) = i) = C_{\gamma^{-1}}^{i}(1 - \exp(-\lambda_2 t))^i (\exp(-\eta\lambda_2 t))^{\gamma^{-1}-i}. \quad (26)$$

Finally, $P_{NH}(t)$, $P_{NS}(t)$, and $R(t)$ can be calculated according to Eqs. (12), (17), and (22), and then we obtain Eqs. (27)-(29) respectively.

$$P_{NH}(t) = \exp(-\lambda_3 t - \lambda_2 t) + \sum_{i=1}^{n_\gamma}\prod_{j=1}^{i}(1-(j-1)\gamma)\frac{\exp(-\lambda_3 t - \lambda_2 t)(\lambda_2 t)^i}{i!}, \quad (27)$$

$$P_{NS}(t) = \Phi\left(\frac{H - (\mu + \mu_\beta t)}{\sigma_\beta t}\right)\exp(-\lambda_2 t) + \sum_{i=1}^{n_\gamma}\Phi\left(\frac{H - (\mu + \mu_\beta t + i\mu_{Y_i})}{\sqrt{\sigma_\beta^2 t^2 + i\sigma_i^2}}\right) \times C_{\gamma^{-1}}^{i}(1 - \exp(-\lambda_2 t))^i(\exp(-\eta\lambda_2 t))^{\gamma^{-1}-i}, \quad (28)$$

$$\begin{aligned} R(t) &= \sum_{i=0}^{n_\gamma} P(N_3(t) = 0, X_S(t) < H|N_2(t) = i) \times P(N_2(t) = i) \\ &= \exp(-\lambda_3 t)\,\Phi\left(\frac{H - (\mu + \mu_\beta t)}{\sigma_\beta t}\right)\exp(-\lambda_2 t) \\ &\quad + \sum_{i=1}^{n_\gamma}\prod_{j=1}^{i}(1-(j-1)\gamma)\frac{\exp(-\lambda_3 t - \lambda_2 t)(\lambda_2 t)^i}{i!} \\ &\quad \times \Phi\left(\frac{H - (\mu + \mu_\beta t + i\mu_{Y_i})}{\sqrt{\sigma_\beta^2 t^2 + i\sigma_i^2}}\right), \end{aligned} \quad (29)$$

IV. RESULT AND DISCUSSION

A micro-engine in the MEMS developed at Sandia National Laboratories is used to illustrate the proposed reliability model. The micro-engine is subject to natural degradation (i.e. wear) and three-zone shocks: safety shocks, damage shocks, and fatal shocks. In addition, the arrived damage shocks hinder the occurrence of themselves while facilitate the occurrence of fatal shocks. The micro-engine experiences two DCFPs: soft failure due to continual wear process and wear debris resulting from damage shocks, and hard failure due to spring fracture resulting from fatal shocks.

Based on the data in [8], [17], along with some reasonable assumptions, the values of corresponding parameters for the proposed model are shown in TABLE 1.

A. RESULTS AND ANALYSIS

For the proposed model considering degradation process and dependent zoned shocks, the reliability function $R(t)$ in Eq. (29) is plotted in FIGURE. 4. In addition, we can also obtain the micro-engine's reliability without the dependence between damage shocks and fatal shocks by setting $\gamma = 0$, i.e., the hard failure threshold will not be reduced by damage shocks. In general, the reliability of the micro-engine decreases as the dependence is considered. As expected,

$$\begin{aligned} &P(N_3(t) = 0|N_2(t) = i) \\ &= \frac{\exp(-\lambda_3 t - \lambda_2 t)\prod_{j=1}^{i}(1-(j-1)\eta)\int_0^t \cdots \int_{t_{i-2}}^t \int_{t_{i-1}}^t \lambda_2^i dt_i dt_{i-1} \ldots dt_1}{C_{\gamma^{-1}}^{i}(1 - \exp(-\gamma\lambda_2 t))^i(\exp(-\gamma\lambda_2 t))^{\gamma^{-1}-i}} \\ &= \frac{\prod_{j=1}^{i}(1-(j-1)\gamma)\exp(-\lambda_3 t - \lambda_2 t)(\lambda_2 t)^i}{i!C_{\gamma^{-1}}^{i}(1 - \exp(-\gamma\lambda_2 t))^i(\exp(-\gamma\lambda_2 t))^{\gamma^{-1}-i}}. \end{aligned} \quad (24)$$

TABLE 1. Corresponding parameters for the reliability analysis of the micro-engines.

Parameters	Value	Sources
H	$0.00125\ \mu m^3$	[8, 17]
φ	$0\ \mu m^3$	[8, 17]
β	$\beta \sim N(8.4803\times10^{-9}, 6.0016\times10^{-10})\mu m^3$	[8, 17]
Y	$Y \sim N(1.0\times10^{-4}, 2.0\times10^{-10})\mu m^3$	[17]
λ	2.5×10^{-5}	[17]
p_2	0.85	Assumption
p_3	0.05	Assumption
γ	0.05	Assumption

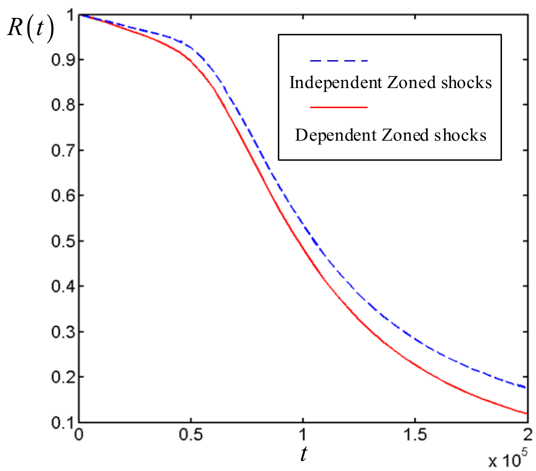

FIGURE 4. The system reliability with and without the dependence.

the reliability with the dependence is lower than the reliability without the dependence. This can be explained by the fact that when considering the dependence, the damage shocks will reduce the spring's strength, and the spring fracture (i.e. the hard failure) will occur more easily.

Based on Eq. (27) and (28), the probability of no hard failures $P_{NH}(t)$ and the probability of no soft failures $P_{NS}(t)$ are plotted in FIGURE. 5. It can be easily observed that $P_{NH}(t)$ is almost equal to $R(t)$ before time $t = 5 \times 10^5$, and $P_{NS}(t)$ and $R(t)$ decrease sharply after $t = 5 \times 10^5$. The main mathematical and physical reasons for the phenomenon are:

1. At the beginning, since the wear volume is far from the threshold H, soft failure seldom occurs and hard failure is the dominant failure mode. Therefore, $P_{NH}(t)$ is almost equal to $R(t)$ and they decrease slowly.
2. As time goes on, the wear volume will increase and approach the threshold H. Hence, soft failure is more likely to occur after $t = 5 \times 10^5$, and $P_{NS}(t)$ and $R(t)$ will both decrease sharply.

B. SENSITIVITY ANALYSIS

The micro-engine experiences two DCFPs with the dependence between damage shocks and fatal shocks. Therefore, the Hindrance factor γ is an important parameter. Sensitivity

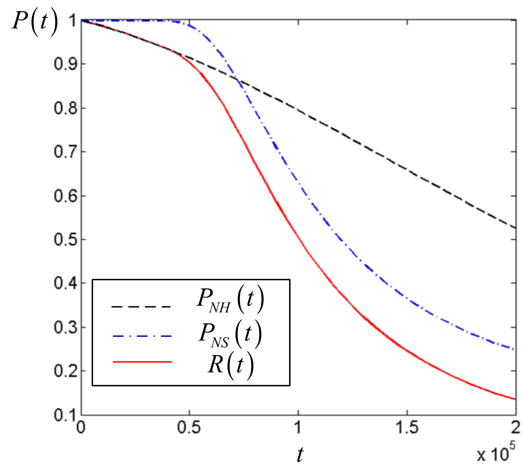

FIGURE 5. The probability of no hard failures $P_{NH}(t)$, the probability of no soft failures $P_{NS}(t)$, and reliability function $R(t)$.

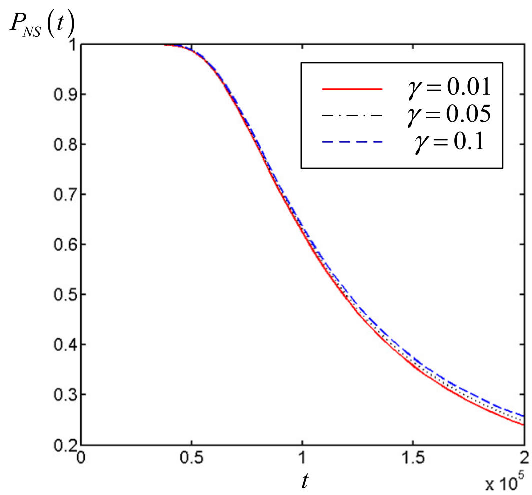

FIGURE 6. Sensitivity analysis of $P_{NS}(t)$ on γ.

analysis is conducted to measure the impacts of γ on the soft failure, hard failure, and reliability functions. In addition, the parameters H and λ influence the micro-engine's reliability significantly. The sensitivity analysis of the system reliability on H and λ is also performed.

1) SENSITIVITY ANALYSIS OF THE HINDRANCE FACTOR

The parameter γ represents the hindrance and facilitation effects of arrived damage shocks on the intensity of the damage shocks and the fatal shocks respectively. To study its effect on the soft failure, hard failure, and system failure, we calculate $P_{NS}(t)$, $P_{NH}(t)$, and $R(t)$ under three different levels of dependence (i.e. $\gamma = 0.01, 0.05, 0.1$) and the results are plotted in FIGUREs. 6-8.

As you can see in FIGURE. 6, $P_{NS}(t)$ under three different levels of dependence are almost the same before $t = 5 \times 10^5$, and $P_{NS}(t)$ increases as γ increases after $t = 5 \times 10^5$. The main mathematical and physical reasons for the phenomenon are:

1. At the beginning, the number of arrival damage shocks is relatively small, even 0. There is little hindrance effect on the damage shocks. In addition, the wear

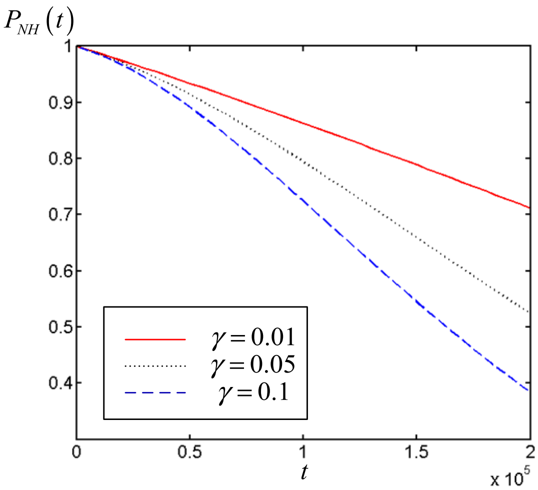

FIGURE 7. Sensitivity analysis of $P_{NH}(t)$ on γ.

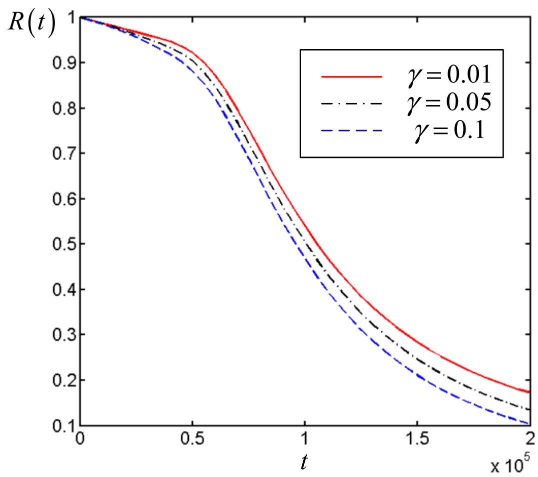

FIGURE 8. Sensitivity analysis of $R(t)$ on γ.

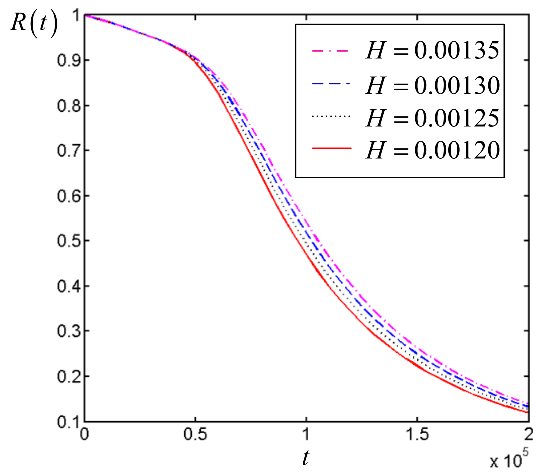

FIGURE 9. Sensitivity analysis of $R(t)$ on H.

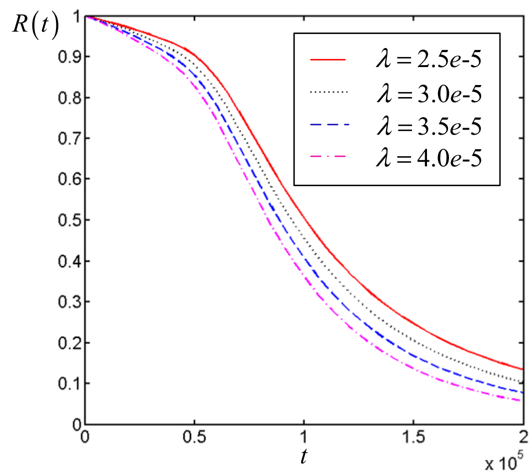

FIGURE 10. Sensitivity analysis of $R(t)$ on λ.

volume is far from the threshold H. Hence, the effects of three different levels of dependence on soft failure are almost the same.

2. As time goes on, the number of arrival damage shocks increases. When γ is relatively large, the intensity of damage shocks will be hindered significantly by the arrived damage shocks. Hence, fewer damage shocks will arrive and the wear debris caused by damage shocks will decrease, which lead to a higher $P_{NS}(t)$.

As can be observed in FIGURE. 7, the parameter γ influences hard failure function more than soft failure, and $P_{NH}(t)$ decreases as γ increases. When γ is relatively large, the intensity of fatal shocks will be facilitated more significantly by the arrived damage shocks. Thus, more fatal shocks will arrive, which leads to more hard failures.

As shown in FIGURE. 8, the system reliability is sensitive to the parameter γ, and $R(t)$ decreases as γ increases. It indicates that the system reliability can be improved through decreasing γ.

2) SENSITIVITY ANALYSIS OF OTHER IMPORTANT PARAMETERS

The wear degradation failure threshold value H and the arrival rate of random shocks λ are also important parameters that affect the micro-engine's reliability. The sensitivity analysis of the system reliability on these two parameters is shown in FIGUREs. 9 and 10 respectively.

FIGURE. 9 shows that the threshold value H influence system reliability significantly. As H increases from 0.00120 μm^3 to 0.00135 μm^3, $R(t)$ shifts to right. It can be inferred that the reliability performance is better for a high value of H.

In FIGURE. 10, it can be seen that $R(t)$ is sensitive to the parameter λ. When λ increases from 2.5×10^{-5} to 4×10^{-5}, the reliability curves shift to the left. It indicates that the reliability decreases when micro-engines operates in an environment with higher occurrence intensity of random shocks.

V. CONCLUSION

This paper proposes a reliability model for a micro-engine subject to natural degradation and dependent zoned shocks.

In this paper, the dependence between damage shocks and fatal shocks is first studied to consider the fact that a damage shock reduces the hard failure threshold and then shock loads will exceed the system strength more easily. A dependent zoned shock model is developed, where a Hindrance model is first introduced to describe damage shocks which are hindered by themselves arrived previously, and a Cox process is utilized to model fatal shocks whose intensity is dependent on the number of arrived damage shocks. Then, an analytical reliability function of the micro-engine is derived, and it may evaluate the reliability more precisely. The results and sensitivity analysis show that the dependence between damage shocks and fatal shocks lead to more hard failures and then results in a lower system reliability. For future investigation, we will extend the dependent zoned shock model with a more complex dependence where not only the damage shocks but also the degradation can reduce the hard failure threshold. In addition, the maintenance can be included in this model to enhance system reliability.

APPENDIX

ABBREVIATION

DCFPs	Dependent competing failure processes
MPP	Markov point process
HPP	Homogenous Poisson process
NHPP	Nonhomogeneous Poisson process
DSPP	Doubly Stochastic Poisson Process

NOTATION

H	Soft failure threshold
D_F	Hard failure threshold
D_S	Safety zone threshold
$N_1(t)$	Number of safety shocks that have arrived by time t
$N_2(t)$	Number of damage shocks that have arrived by time t
$N_3(t)$	Number of fatal shocks that have arrived by time t
$\lambda(t)$	Intensity of random shocks
$\lambda_1(t)$	Initial intensity of safety shocks
$\lambda_2(t)$	Initial intensity of damage shocks
$\lambda_3(t)$	Initial intensity of fatal shocks
$\lambda_{2,i}(t)$	The intensity of damage shocks after i damage shocks arrive
$\lambda_{3,i}(t)$	The intensity of fatal shocks after i damage shocks arrive
$P_{2,i}$	The probability that the shock belongs to the damage zone after i damage shocks arrive
$P_{3,i}$	The probability that the shock belongs to the fatal zone after i damage shocks arrive
W_i	Magnitude of the ith shock load
Y_i	Damage caused by the ith shock
$S(t)$	Cumulative shock damage size at t
$X(t)$	Continuous degradation at t
$X_S(t)$	Total degradation at t
$G(x,t)$	The cumulative distribution function of $X(t)$
$f_Y^{<m>}(y)$	PDF of the sum of m independent and identically distributed (i.i.d.) Y_i variables
n_γ	The maximum number of arrival damage shocks.
γ	Inhibition factor
$P_{NH}(t)$	The probability that no hard failure by time t
$P_{NS}(t)$	The probability that no soft failure by time t

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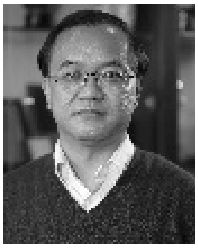

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