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A Generalized Expression for Information Quality of Basic Probability Assignment

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ABSTRACT The information quality is widely used in many applications. However, the existing information quality can only deal with the probability distribution. Compared with probability distribution, the basic probability assignment (BPA) in evidence theory is more efficient to handle uncertainty. As a result, it is necessary to generalize the existing information quality. In this paper, a new expression for information quality is proposed to measure the information quality of BPA. When the BPA degenerates into a probability distribution, the proposed generalized expression for information quality in this paper is degenerated into the information quality proposed by Yager. Numerical examples are used to demonstrate the effectiveness of the generalized expression for information quality. In addition, a weighted average combination rule based on the new expression for information quality is presented. A numerical example in target recognition is illustrated to show its validity in combining conflicting evidence.

INDEX TERMS Information quality, Dempster-Shafer theory, basic probability assignment, Gini entropy, information fusion, target recognition.

I. INTRODUCTION

Dealing with uncertainty is an open issue in real application [1], [2]. Lots of math tools, such as fuzzy sets [3]–[5], belief structure [6], Z-numbers [7], [8] and R-numbers are presented. Probability theory has been heavily studied for hundreds years and is widely used in lots of engineering. As a measure of uncertainty of probability distribution, The information quality proposed by Yager [9] has been applied to decision making [10], [11], pattern classification [12], [13] and maximum fusion [14] etc [15].

There are many different types of uncertainty [16]–[19]. Compared with probability distribution, the basic probability assignment in Dempster-Shafer evidence theory [20], [21] can be seen as a generalization and is more efficient to deal with uncertain information [22].

However, Dempster-Shafer evidence theory [20], [21] as a complete theory for dealing with uncertainty problems has greater flexibility in dealing with uncertainty, while the information quality proposed by Yager [9] does not apply to the measurement of BPA uncertainty. In addition, how to measure the uncertainty of BPA is also an open issue. Therefore, this paper proposes a generalized expression for information quality based on the framework of evidence theory. When the BPA degenerates into a probability distribution, the generalized expression for information quality in this paper is degenerated into the information quality proposed by Yager [9].

The rest of the article is organized as follows. In the second section, the combination rules in the evidence theory, two average combination rules and the information quality proposed by Yager [9] are introduced. In the third section, the generalized expression for information quality proposed in this paper is introduced, and the effectiveness of generalized expression for information quality in measuring BPA uncertainty is illustrated by a large number of examples. In the fourth section, a weighted average combination rule based on information quality will be proposed and a numerical example will be used to illustrate it. The fifth section is a brief conclusion.

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II. PRELIMINARIES

This section will introduce some preliminary knowledge, including evidence theory, combination rules, Gini entropy [23] and the information quality proposed by Yager [9].

A. DEMPSTER-SHAFER EVIDENCE THEORY

Dealing uncertainty and complexity in real world is not inevitable [24]–[27]. Evidence theory is widely used due to its efficiency to model uncertainty [28]–[31]. such as decision making [32], evidential reasoning [33], [34], target recognition [35], risk evaluation [36]. In addition, it provides a bridge to connect different type of uncertain information such as Z-numbers [7], D numbers [37], [38] and belief structure [39]. However, some open issues are not well solved, such as how to determine whether the frame of discernment is incomplete or not is still an open issue [40].

Definition 1: The basic probability assignment is defined as follows, assume A is a subset of X, let A mapping a number m where $m \in [0, 1]$*, and satisfies [20], [21]:*

$$m(\phi) = 0 \quad and \quad \sum_{A \subseteq X} m(A) = 1 \tag{1}$$

The mass m(A) indicates the degree of support for evidence A.

Definition 2: Given two basic probability assignment, the Dempster's combination rule, defined as follows [21]:

$$m(C) = m_1(A) \oplus m_2(B) = \begin{cases} 0 & A \cap B = \emptyset \\ \dfrac{\sum_{A \cap B = C} m_1(A) m_2(B)}{1-k} & A \cap B \neq \emptyset \end{cases} \tag{2}$$

where

$$k = \sum_{A \cap B \neq \emptyset} m_1(A) m_2(B) \tag{3}$$

k is the conflict among the evidences.

B. AVERAGE COMBINATION RULES

Data fusion technology can increase the system performance and improve the efficiency of decision making [41], [42]. When there is a large conflict between the two evidences, the Dempster's combination rule will get counter-intuitive results [43], [44]. To solve this problem, Dubois [45], Yager [46], Smets [47], Murphy [48], Deng et.al [49] and other researchers have proposed new methods [50]. This article would mainly introduce Murphy's average approach [48] and modified average approach proposed by Deng et.al [49].

Murphy proposed a average combination rule [48] that suggests that if n pieces of evidence are available at the same time, the quality can be averaged and the average will be combined $n-1$ times using the Dempster's combination rule.

On the basis of Murphy, A weighted average combination rule proposed by Deng *et al.* [49] based on evidence distance function [51].

The calculation process for the weighted average combination rule proposed by Deng *et al.* [49] is shown in Algorithm 1, as follows:

Algorithm 1 The Weighted Average Combination Rule Proposed by Deng et.al [49]

Given some evidence $m_1, m_2, \ldots, m_n$

step 1	$sim(m_i, m_j) = 1 - d(m_i, m_j)$
step 2	$sup(m_i) = \sum_{j=1 j \neq i}^{n} sim(m_i, m_j)$
step 3	$Crd_i = \frac{sup(m_i)}{\sum_{i=1}^{n} sup(m_i)}$
step 4	$MAE(m) = \sum_{i=1}^{n}(Crd_i \times m_i)$
step 5	combine the evidence n-1 times

C. GINI ENTROPY AND INFORMATION QUALITY

Entropy can be used as a measure of information uncertainty [52]. The greater the uncertainty of information, the larger the value of entropy. There are many measurement methods for entropy, such as Shannon entropy [53], Gini entropy [23] and Deng entropy [54].

Definition 3: Let p_i *be the vector form of the probability distribution, the Gini entropy is defined as follows [23]:*

$$G(p_i) = 1 - \sum_{j=1}^{n} \|p_{ij}\|^2 \tag{4}$$

It can be seen from the Definition 3 that when all $p_j = 1/n$, its value is the largest; when $p_j = 1$, its value is the smallest.

From the definition of the Gini entropy, it can be known that as the value of $\sum_{j=1}^{n} \|p_{ij}\|^2$ increases, the value of the Gini entropy is smaller. The smaller the entropy, the smaller the uncertainty and the larger the information. Therefore, Yager proposed to use $\|p_{ij}\|^2$, named *NegEnt*, as a way to measure information or uncertainty [9].

Definition 4: Given a probability distribution, the information quality is defined as follows [9]:

$$IQ_{p_i} = \|p_i\|^2 = \sum_{j=1}^{n} \|p_{ij}\|^2 \tag{5}$$

where

$$\|p_i\| = \sqrt{(p_i * p_i)} = (\sum_{j=1}^{n} \|p_{ij}\|^2)^{1/2} \tag{6}$$

III. THE PROPOSED METHOD

In this section, the proposed method will be described in detail. The concept of information quality presented by Yager and Petry [9] is efficient to measure the information quality of probability distribution. However, in the much more uncertain situation modelled by BPA, the existing information quality can not work. To solved the question, this paper proposes a generalized expression for information quality to measure the uncertainty of BPA. The generalized expression for information quality will be introduced below.

Definition 5: Given a basic probability allocation, the generalized expression for information quality is defined as follows:

$$IQ_{m_i} = \sum_{A \subseteq X} (\frac{m_i(A)}{2^{|A|} - 1})^2 \tag{7}$$

where m_i is a mass function defined on the frame of discernment X, and $|A|$ is the cardinality of A.

As mentioned above, the generalized expression for information quality is very similar to the information quality proposed by Yager [9], but the belief of each focal element is divided by an item $(2^{|A|}-1)$, which represents the number of potential states in A.

Assume m_1 and m_2 are two probability vectors on the space X and that the relation between these is

$$m_1(a) = m_2(a) - \alpha$$
$$m_1(b) = m_2(b) + \alpha$$

The rest of m_1 and m_2 are the same.

$$\sum\nolimits_{A\subseteq X}(\frac{m_1(A)}{2^{|A|}-1})^2 = (m_2(a)-\alpha)^2 + (m_2(b)+\alpha)^2 + \sum\nolimits_{A\subseteq X, A\neq a,b}(\frac{m_1(A)}{2^{|A|}-1})^2$$

By simplifying:

$$\sum\nolimits_{A\subseteq X}(\frac{m_1(A)}{2^{|A|}-1})^2 - \sum\nolimits_{B\subseteq X}(\frac{m_2(B)}{2^{|A|}-1})^2 = 2\alpha(m_2(b)-m_2(a)) + 2\alpha^2 = 2\alpha((m_2(b)-m_2(a))+\alpha)$$

When $m_2(b) > m_2(a)$, there is $m_1 > m_2$, and the certainty increases. Therefore, the move of the BPA from the focal element with small value to the focal element with large value increases the certainty. When $m_2(b) = m_2(a)$, there is $m_1 > m_2$. Therefore, if two focal element have the same BPA value, shifting some BPA from one focal element to another increases the certainty. When $m_2(b) < m_2(a)$, the situation is complex. If $\mid m_2(a) - m_2(b) \mid > \alpha$, then the certainty decrease. If $\mid m_2(a) - m_2(b) \mid < \alpha$, then the certainty increase.

Example 1: Assuming the mass function $m(a) = 1$, Its information quality can be calculated by Definition 4 and Definition 5 as follows:

$$IQ_p = (1)^2 = 1$$
$$IQ_m = (\frac{1}{2^1-1})^2 = 1$$

Example 2: Given an framework of discernment $X = \{a, b, c, d\}$, where the mass function is $m(a) = m(b) = m(c) = m(d) = 1/4$,

$$IQ_p = (\frac{1}{4})^2 + (\frac{1}{4})^2 + (\frac{1}{4})^2 + (\frac{1}{4})^2 = 0.25$$
$$IQ_m = (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 = 0.25$$

From Example 1 and Example 2, it can be seen that when the BPA degenerates into a probability distribution, the values calculated by the two information qualities are the same. In this case, the generalized expression for information quality is degenerated into the information quality proposed by Yager [9].

Example 3: Given an framework of discernment $X = \{a, b, c, d\}$, where the mass function is

$$m_1 = [(a, 1/4), (b, 1/4), (c, 1/4), (a, d, 1/4)]$$
$$m_2 = [(a, 1/4), (b, 1/4), (c, 1/4), (a, b, d, 1/4)]$$
$$m_3 = [(a, 1/4), (b, 1/4), (c, 1/4), (a, b, c, d, 1/4)]$$
$$IQ_{m_1} = (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^2-1})^2 = 0.1944$$
$$IQ_{m_2} = (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^3-1})^2 = 0.1888$$
$$IQ_{m_3} = (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^1-1})^2 + (\frac{1/4}{2^4-1})^2 = 0.1878$$

By comparison of Example 2 and Example 3, it can been seen that the value of Example 2 is significantly larger than the value of Example 3. This also is correspondent with our intuitive experience. To a certain extent, it can be proved that the generalized expression for information quality has good validity.

Example 4: Given an framework of discernment $X = \{a, b, c, d\}$, where the mass function is $m(a, b, c, d) = 1$,

$$IQ_{m_1} = (\frac{1}{2^4-1})^2 = 0.0044$$

By comparison of Example 3 and Example 4, it can been seen that the value of Example 3 is larger than the value of Example 4. The result is reasonable. Because $m(a, b, c, d) = 1$ represents the information is totally unknown. Therefore, Example 3 has higher information quality value than Example 4.

Example 5: When the identification frame is $X = \{a_1, a_2, \ldots a_N\}$, the value of the information quality is considered in four special cases as follows:

$$m_2(a_1) = m_4(a_2) = \cdots = m_4(a_N) = \frac{1}{N}$$
$$m_3(a_1) = m_2(a_2) = \ldots = m_2(a_N) = m_2(a_1, a_2) = \ldots = m_2(X) = \frac{1}{2^N-1}$$
$$m_4(X) = 1$$

As the value of N changes, the value of the generalized expression for information quality also changes. The law of variation between them will be shown in Figure 1. Furthermore, in order to make the image easier to be understood, the ordinate in Figure 1 will be represented in logarithmic form

From Figure 1, it can be seen that if N is the same, m_3 has the smallest information quality value, that is, it has the greatest uncertainty. As N increases, the value of information quality becomes smaller, which is reasonable.

TABLE 1. Information quality of the above *BPA*.

	m_1	m_2	m_3	m_4	m_5
IQ	0.3800	0.8200	0.3261	0.3261	0.3800

TABLE 2. Results of different combination rules of evidence.

	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
$Dempster$ [21]	$m(A) = 0$ $m(B) = 0.8571$ $m(C) = 0.1429$	$m(A) = 0$ $m(B) = 0.6316$ $m(C) = 0.3684$	$m(A) = 0$ $m(B) = 0.3288$ $m(C) = 0.6712$	$m(A) = 0$ $m(B) = 0.1228$ $m(C) = 0.8772$
$Murphy$ [48]	$m(A) = 0.1543$ $m(B) = 0.7469$ $m(C) = 0.0988$	$m(A) = 0.3500$ $m(B) = 0.5224$ $m(C) = 0.1276$	$m(A) = 0.6027$ $m(B) = 0.2627$ $m(C) = 0.1346$	$m(A) = 0.7958$ $m(B) = 0.0932$ $m(C) = 0.1110$
$Deng\ et.al$ [49]	$m(A) = 0.1543$ $m(B) = 0.7469$ $m(C) = 0.0988$	$m(A) = 0.4861$ $m(B) = 0.3481$ $m(C) = 0.1657$	$m(A) = 0.7773$ $m(B) = 0.0628$ $m(C) = 0.1600$	$m(A) = 0.8909$ $m(B) = 0.0086$ $m(C) = 0.1005$
$proposed\ method$	$m(A) = 0.3330$ $m(B) = 0.5072$ $m(C) = 0.1598$ $m(AC) = 0$	$m(A) = 0.8157$ $m(B) = 0.0883$ $m(C) = 0.0808$ $m(AC) = 0.0152$	$m(A) = 0.9505$ $m(B) = 0.0110$ $m(C) = 0.0291$ $m(AC) = 0.0093$	$m(A) = 0.9859$ $m(B) = 0.0015$ $m(C) = 0.0094$ $m(AC) = 0.0033$

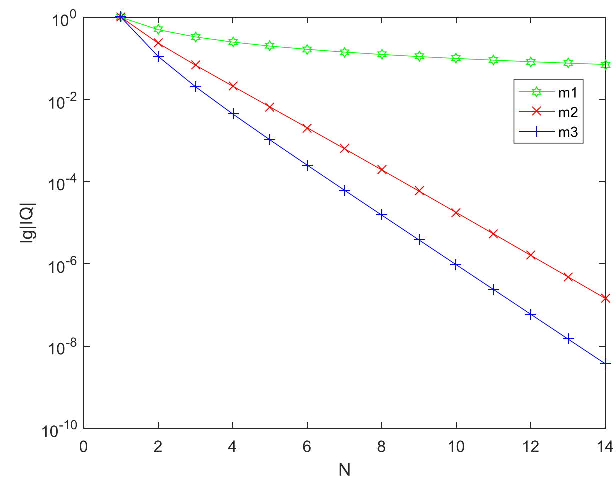

FIGURE 1. Information quality.

IV. APPLICATION

In this section, a weighted average combination rule based on information quality is proposed, an example will be used for the combination. The results will be compared to the results of a combination of three different rules by Dempster, Murphy and Deng *et al.* [21], [48], [49].

For Dempster's combination rule [21], when evidence conflicts, counter-intuitive results will result. This is because of the existence of extreme evidence. Due to the characteristics of information quality, the larger the value of information, the smaller the uncertainty. Therefore, using $\frac{1}{IQ}$ as a weighting factor can be used as a method to reduce the impact of extreme evidence. The calculation process for the proposed weighted average combination rule is shown in Algorithm 2, as follows:

Algorithm 2 The Proposed Weighted Average Combination Rule

Given some evidence $m_1, m_2, \ldots, m_n$

step 1 $IQ_{m_i} = \sum_{A \subseteq X} (\frac{m_i(A)}{2^{|A|}-1})^2$

step 2 $\sum_{i=1}^{n} \frac{1}{IQ_i}$

step 3 $m_w = \sum_{i=1}^{n} \frac{\frac{1}{IQ_i}}{\sum_{i=1}^{n} \frac{1}{IQ_i}} m_i$

step 4 combine the evidence n-1 times which is calculated by Definition 2

An example will be used to illustrate the effectiveness of the method. In the multi-sensor based automatic target recognition system, assuming the real target is A, the system collects five different pieces of evidence from different five sensors, as follows [49]:

$$m_1 = ([A, 0.5], [B, 0.2], [C, 0.3])$$
$$m_2 = ([A, 0], [B, 0.9], [C, 0.1])$$
$$m_3 = ([A, 0.55], [B, 0.1], [A, C, 0.35])$$
$$m_4 = ([A, 0.55], [B, 0.1], [A, C, 0.35])$$
$$m_5 = ([A, 0.6], [B, 0.1], [A, C, 0.3])$$

The generalized expression for information quality of the above five BPAs will be calculated and expressed in Table 1.

The results obtained using the four different combination rules is shown in Table 2.

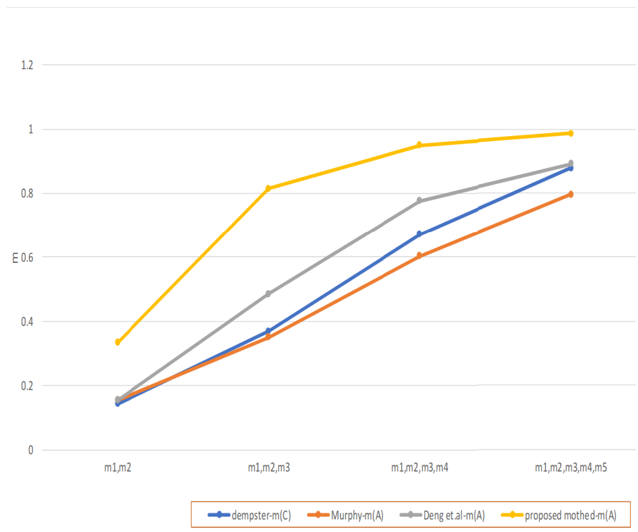

FIGURE 2. Comparison of results of different combination rules.

Table 2 shows that Dempster's combination rules [21] get counter-intuitive results when there is a conflict between the evidences. In the numerical example, m_2 is in high conflict with other evidence. Even if there is more evidence to support Target A, the results of the Dempster's combination rule [21] cannot reflect this fact, which is obviously unreasonable. It can also be seen from the figure that the results obtained using the other three methods are reasonable. By comparing the combined results of m_1 and m_2, it can be seen that the weighted average combination rule based on information quality is effective in reducing the impact of extreme evidence on the results. In addition, Figure 2 plots the evidence of high support in the calculations of the four methods. Through the analysis of Figure 2, the support of Dempster's combination rule for target C is gradually increased, which is counter-intuitive. The other three methods are reasonable, However, when the number of evidence is not adequate, the proposed method is superior to Murphy's average approach [48] and modified average approach proposed by Deng *et al.* [49]. It is shown in figure 2 that the proposed method have much more belief on the target A than the other methods.

V. CONCLUSION

The information quality proposed by Yager [9] can effectively measure the uncertainty of probability distribution. But how to measure the information quality of the basic probability distribution is still an open question. Therefore, this paper proposes a generalized expression for information quality that can be applied to BPA uncertainty measurement. Some examples show the effectiveness of the proposed information quality in BPA uncertainty measurement. In addition, by using $\frac{1}{IQ}$ as the weighting factor, a weighted average combination rule based on generalized expression for information quality is proposed, and its validity is proved by numerical examples in dealing with conflict evidence in target recognition.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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