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Recursive Least Squares Parameter Estimation Algorithms for a Class of Nonlinear Stochastic Systems With Colored Noise Based on the Auxiliary Model and Data Filtering

LONGJIN WANG[1] AND YAN HE[1,2]
[1]College of Electrical and Mechanical Engineering, Qingdao University of Science and Technology, Qingdao 266061, China
[2]High Performance Carbon Material Preparation and Application Engineering Laboratory of Shandong Province, Qingdao 266061, China

Corresponding author: Yan He (heyan_sd@163.com)

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ABSTRACT This paper considers the parameter identification for a class of nonlinear stochastic systems with colored noise. We filter the input-output data by using an estimated noise transfer function and obtain two identification models, one containing the parameters of the noise model, and the other containing the parameters of the system model. A data filtering based recursive generalized extended least squares algorithm is proposed by using the data filtering technique, and a recursive generalized extended least squares algorithm is derived for comparison. Finally, an example is given to support the proposed algorithms. Compared with the recursive generalized extended least squares algorithm, the data filtering based recursive generalized extended least squares algorithm can not only reduce the computational burden, but also enhance the parameter estimation accuracy.

INDEX TERMS Parameter estimation, bilinear system, data filtering, least squares, recursive identification.

I. INTRODUCTION

Mathematical models are the basis of controller design [1]–[4] and system analysis [5]–[7]. Many parameter estimation methods have been proposed for different systems [8]–[10] such as linear systems [11]–[13] and nonlinear systems [14]–[16]. Nonlinear systems have received much attention in the area of signal modeling and system identification for the past decade [17]. Ma et al studied the hierarchical identification algorithm for multivariate Hammerstein systems by using the modified Kalman filter [18] and filtering-based multistage recursive identification algorithm for an input nonlinear output-error autoregressive system by using the key term separation technique [19]. The auxiliary model identification idea can handle the identification problems in the presence of the unmeasurable variables in the information vectors [20]. In this aspect, Guo et al. proposed a recursive least squares algorithm for pseudo-linear ARMA systems using the auxiliary model [21]; Li et al. derived an auxiliary model based least squares iterative algorithm for parameter estimation of bilinear systems using interval-varying measurements [22].

The least squares algorithms contain the least squares based iterative (LSI) algorithms [23]–[28] and the recursive least squares (RLS) algorithms [29], [30], which are suitable for the off-line and on-line parameter estimation. Xu et al. proposed an iterative parameter estimation algorithm for signal models based on measured data [31]. Wang *et al.* [32] derived recursive least squares and gradient algorithms for Hammerstein-Wiener systems. Information filtering has wide applications in many areas, e.g., parameter identification [33] and signal processing [34]. Some filtering based identification algorithms have been proposed during the past decade. Ding et al. derived an iterative parameter identification algorithm for pseudo-linear systems with ARMA noise using the data filtering technique [35]. Pan *et al.* [36] derived a filtering based multi-innovation extended stochastic gradient algorithm for multi-variable control systems.

The bilinear system is a special class of nonlinear stochastic systems which widely exist in biological engineering [37], communication engineering [38] and nuclear engineering [39], and the model structure includes the products

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of the inputs and the states. Many identification algorithms have been proposed for the bilinear systems [40], [41]. Zhang et al. proposed several state-space recursive identification algorithms for the bilinear systems including a state filtering-based least squares algorithm with the hierarchical identification principle [42], a hierarchical approach for joint parameter and state estimation algorithm [43] and a combined state and parameter estimation algorithm [44], which can directly provide the state-space model, but the computational complexity increases as the dimensions of the parameter vectors increase. Some identification methods can be applied to many fields such as transportation and control community [45]–[47].

The iterative identification algorithms are suitable for the off-line parameter estimation. The state-space identification algorithms which are suitable for on-line parameter estimation can directly provide the state-space models, but the computational complexity increase as the dimensions of the parameter vectors increase. Different from the iterative algorithms and the recursive state-space identification algorithms, this paper derives an recursive identification algorithm using the data filtering technique to reduce the computational burden and enhance the parameter estimation accuracy. The main contributions of this paper are as follows.

- Using the data filtering technique, an F-RGELS algorithm is presented to enhance the parameter estimation accuracy and to reduce the computational burden, which is suitable for on-line parameter estimation.
- The computational efficiency comparison is discussed between the RGELS algorithm and the F-RGELS algorithm to illustrate the high efficiency of the F-RGELS algorithm.

The rest of this paper is organized as follows. Section II describes the identification model of a bilinear system with colored noise. Section III proposes a RGELS algorithm by using the auxiliary model. Section IV derives an F-RGELS algorithm by using the data filtering technique and discusses the computational efficiency of the proposed algorithms. Section V provides an example to verify the effectiveness of the proposed algorithms. Finally, we offer some concluding remarks in Section VI.

II. SYSTEM DESCRIPTION AND IDENTIFICATION MODEL

Let us define some notations first. "$A =: X$" or "$X := A$" stands for "A is defined as X"; $\boldsymbol{I}_n$ denotes an identity matrix of size $n \times n$; $\mathbf{1}_n$ denotes an $n \times 1$ vector whose elements are all unity; z denotes a unit forward shift operator with $z\boldsymbol{x}(t) = \boldsymbol{x}(t+1)$ and $z^{-1}\boldsymbol{x}(t) = \boldsymbol{x}(t-1)$. The bilinear system can be expressed as

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t)u(t) + \boldsymbol{l}u(t), \tag{1}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t), \tag{2}$$

where $\boldsymbol{x}(t) := [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the system input, $y(t) \in \mathbb{R}$ is the system output, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times n}$, $\boldsymbol{l} \in \mathbb{R}^n$ and $\boldsymbol{c} \in \mathbb{R}^{1\times n}$ are the system parameter matrices and vectors:

$$\boldsymbol{A} := \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$\boldsymbol{l} := \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{n-1} \\ l_n \end{bmatrix} \in \mathbb{R}^{n\times 1}, \quad \boldsymbol{B} := \begin{bmatrix} 0 \\ \boldsymbol{b} \end{bmatrix} \in \mathbb{R}^{n\times n},$$

$$\boldsymbol{c} := [1, 0, 0, \cdots, 0] \in \mathbb{R}^{1\times n},$$

$$\boldsymbol{b} := [-b_n, -b_{n-1}, -b_{n-2}, \cdots, -b_1] \in \mathbb{R}^{1\times n}.$$

Based on the work in [33] to eliminate the state variables in (1) and (2), the input-output expression of the bilinear system can be equivalently written as

$$[A(z)+u(t-n)B(z)]y(t)=C(z)u(t)+D(z)u(t-n)u(t), \tag{3}$$

where

$$A(z) := 1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_nz^{-n}, \quad a_i \in \mathbb{R},$$
$$B(z) := b_1z^{-1} + b_2z^{-2} + \cdots + b_nz^{-n}, \quad b_i \in \mathbb{R},$$
$$C(z) := c_1z^{-1} + c_2z^{-2} + \cdots + c_nz^{-n}, \quad c_i \in \mathbb{R},$$
$$D(z) := d_2z^{-2} + d_3z^{-3} + \cdots + d_nz^{-n}, \quad d_i \in \mathbb{R}.$$

The parameters a_i, b_i, l_i and the coefficients c_i, d_i have the following relations:

$$[c_n, \cdots, c_2, c_1] := [l_n + a_{n-1}l_1 + a_{n-2}l_2 + \cdots + a_1l_{n-1}, \cdots, l_2 + a_1l_1, l_1] \in \mathbb{R}^{1\times n},$$
$$[d_n, \cdots, d_3, d_2] := [b_{n-1}l_1 + b_{n-2}l_2 + \cdots + b_1l_{n-1}, \cdots, b_1l_1] \in \mathbb{R}^{1\times(n-1)}.$$

As the practical processes are usually disturbed by stochastic noises, we introduce a noise term $\omega(t) \in \mathbb{R}$ to (3), and then we obtain a bilinear system with colored noise

$$[A(z)+u(t-n)B(z)]y(t)=C(z)u(t)+D(z)u(t-n)u(t)+\omega(t), \tag{4}$$

where $\omega(t)$ is a stochastic noise sequence with zero mean, which may be a moving average (MA) process, an autoregressive (AR) process or an autoregressive moving average process (ARMA). Without loss of generality, we consider the stochastic noise as an ARMA noise.

Consider an ARMA noise,

$$\omega(t) = \frac{F(z)}{E(z)}v(t), \tag{5}$$

where $v(t) \in \mathbb{R}$ is a white noise sequence with zero mean, $E(z)$ and $F(z)$ are polynomials in z^{-1}, and

$$E(z) := 1 + e_1z^{-1} + e_2z^{-2} + \cdots + e_{n_e}z^{-n_e}, \quad e_i \in \mathbb{R},$$
$$F(z) := 1 + f_1z^{-1} + f_2z^{-2} + \cdots + f_{n_f}z^{-n_f}, \quad f_i \in \mathbb{R}.$$

Assume that the orders n, n_e and n_f are all known, and $y(t)=0, u(t)=0$ and $v(t)=0$ as $t\leq 0$. Define the parameter vector $\boldsymbol{\theta}$ and the information vector $\boldsymbol{\varphi}(t)$ as

$$\boldsymbol{\theta} := \begin{bmatrix} \boldsymbol{\theta}_s \\ \boldsymbol{\theta}_n \end{bmatrix} \in \mathbb{R}^{n_0}, \quad \boldsymbol{\varphi}(t) := \begin{bmatrix} \boldsymbol{\varphi}_s(t) \\ \boldsymbol{\varphi}_n(t) \end{bmatrix} \in \mathbb{R}^{n_0},$$
$$n_0 := 4n + n_e + n_f - 1,$$

where

$$\begin{aligned}
\boldsymbol{\theta}_s &:= [a_1, a_2, \cdots, a_n, b_1, b_2, \cdots, b_n, c_1, c_2, \cdots, c_n, d_2, \\
&\quad d_3, \cdots, d_n]^{\mathrm{T}} \in \mathbb{R}^{n_1}, \quad n_1 := 4n-1, \\
\boldsymbol{\theta}_n &:= [e_1, e_2, \cdots, e_{n_e}, f_1, f_2, \cdots, f_{n_f}]^{\mathrm{T}} \in \mathbb{R}^{n_2}, \\
&\quad n_2 := n_e + n_f, \\
\boldsymbol{\varphi}_s(t) &:= [-y(t-1), -y(t-2), \cdots, -y(t-n), \\
&\quad -u(t-n)y(t-1), -u(t-n)y(t-2), \cdots, \\
&\quad -u(t-n)y(t-n), u(t-1), u(t-2), \cdots, \\
&\quad u(t-n), u(t-n)u(t-2), u(t-n)u(t-3), \cdots, \\
&\quad u(t-n)u(t-n)]^{\mathrm{T}} \in \mathbb{R}^{n_1}, \\
\boldsymbol{\varphi}_n(t) &:= [-\omega(t-1), -\omega(t-2), \cdots, -\omega(t-n_e), \\
&\quad v(t-1), v(t-2), \cdots, v(t-n_f)]^{\mathrm{T}} \in \mathbb{R}^{n_2}.
\end{aligned}$$

According to the above definitions, Equation (4) can be written as

$$\begin{aligned}
y(t) &= \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\boldsymbol{\theta}_s + \boldsymbol{\varphi}_n^{\mathrm{T}}(t)\boldsymbol{\theta}_n + v(t) &(6)\\
&= \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\boldsymbol{\theta}_s + \omega(t) &(7)\\
&= \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} + v(t). &(8)
\end{aligned}$$

Equation (8) is the identification model of the bilinear system in (4). The objective of this paper is to develop new recursive algorithms for estimating the parameter vectors $\boldsymbol{\theta}_n$ and $\boldsymbol{\theta}_s$ in (6) using the measured input-output data $\{u(i), y(i) : i = 1, 2, \cdots, t\}$.

In what follows, a RGELS algorithm is derived for the bilinear system with colored noise. Furthermore, a F-RGELS algorithm is presented to reduce the computational burden and enhance the parameter estimation accuracy. A simulation example is provided to evaluate the estimation accuracy and the computational efficiency of the proposed algorithms.

III. THE RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

In this section, a RGELS algorithm is proposed based on the input-output representation of the bilinear system with colored noise by using the auxiliary model.

Use the input-output data to define the stacked vector $\boldsymbol{Y}_t$ and the stacked matrix $\boldsymbol{\Phi}_t$ as

$$\boldsymbol{Y}_t := \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(t) \end{bmatrix} \in \mathbb{R}^{t}, \quad \boldsymbol{\Phi}_t := \begin{bmatrix} \boldsymbol{\varphi}^{\mathrm{T}}(1) \\ \boldsymbol{\varphi}^{\mathrm{T}}(2) \\ \vdots \\ \boldsymbol{\varphi}^{\mathrm{T}}(t) \end{bmatrix} \in \mathbb{R}^{t\times n_0}.$$

According to (8), define a quadratic criterion function:

$$J_1(\boldsymbol{\theta}) := \| \boldsymbol{Y}_t - \boldsymbol{\Phi}_t\boldsymbol{\theta} \|^2 . \tag{9}$$

Minimizing $J_1(\boldsymbol{\theta})$ and letting its partial derivative with respect to $\boldsymbol{\theta}$ be zero, we can obtain the recursive relations of computing $\hat{\boldsymbol{\theta}}(t)$:

$$\begin{aligned}
\hat{\boldsymbol{\theta}}(t) &= \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)[y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1)], &(10)\\
\boldsymbol{L}(t) &= \boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)[1 + \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\boldsymbol{\varphi}(t)]^{-1}, &(11)\\
\boldsymbol{P}(t) &= [\boldsymbol{I}_{n_0} - \boldsymbol{L}(t)\boldsymbol{\varphi}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1). &(12)
\end{aligned}$$

However, the information vector $\boldsymbol{\varphi}(t)$ in (10)–(12) contains the unmeasurable terms $\omega(t-i)$ $(i = 1, 2, \cdots, n_e)$ and $v(t-i)$ $(i = 1, 2, \cdots, n_f)$, and then Equation (10) cannot give the estimate $\hat{\boldsymbol{\theta}}(t)$. The solution is to replace the unknown items $\omega(t-i)$ and $v(t-i)$ in $\boldsymbol{\varphi}(t)$ with their corresponding estimates $\hat{\omega}(t-i)$ and $\hat{v}(t-i)$.

From (7), we have $\omega(t) = y(t) - \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\boldsymbol{\theta}_s$. Replacing $\boldsymbol{\theta}_s$ with its estimate $\hat{\boldsymbol{\theta}}_s(t)$, the estimate of $\omega(t)$ can be computed by

$$\hat{\omega}(t) = y(t) - \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t).$$

From (8), we have $v(t) = y(t) - \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta}$. Replacing $\boldsymbol{\varphi}(t)$ and $\boldsymbol{\theta}$ with $\hat{\varphi}(t)$ and $\hat{\boldsymbol{\theta}}(t)$, respectively, the estimate of $v(t)$ can be computed by

$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t).$$

Replacing $\boldsymbol{\varphi}(t)$ in (10)–(12) with its estimate $\hat{\boldsymbol{\varphi}}(t)$, we can derive the following recursive least squares relations:

$$\begin{aligned}
\hat{\boldsymbol{\theta}}(t) &= \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)[y(t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1)], &(13)\\
\boldsymbol{L}(t) &= \boldsymbol{P}(t-1)\hat{\boldsymbol{\varphi}}(t)[1 + \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\varphi}}(t)]^{-1}, &(14)\\
\boldsymbol{P}(t) &= [\boldsymbol{I}_{n_0} - \boldsymbol{L}(t)\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1). &(15)
\end{aligned}$$

Combining (10)–(15), we can summarize the recursive generalized extended least squares (RGELS) algorithm for the bilinear system as

$$\begin{aligned}
\hat{\boldsymbol{\theta}}(t) &= \hat{\boldsymbol{\theta}}(t-1) + \boldsymbol{L}(t)[y(t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t-1)], &(16)\\
\boldsymbol{L}(t) &= \boldsymbol{P}(t-1)\hat{\boldsymbol{\varphi}}(t)[1 + \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\varphi}}(t)]^{-1}, &(17)\\
\boldsymbol{P}(t) &= [\boldsymbol{I}_{n_0} - \boldsymbol{L}(t)\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)]\boldsymbol{P}(t-1), &(18)\\
\hat{\boldsymbol{\varphi}}(t) &= \begin{bmatrix} \boldsymbol{\varphi}_s(t) \\ \hat{\boldsymbol{\varphi}}_n(t) \end{bmatrix}, &(19)\\
\boldsymbol{\varphi}_s(t) &= [-y(t-1), -y(t-2), \cdots, -y(t-n), \\
&\quad -u(t-n)y(t-1), -u(t-n)y(t-2), \cdots, \\
&\quad -u(t-n)y(t-n), u(t-1), u(t-2), \cdots, \\
&\quad u(t-n), u(t-n)u(t-2), \\
&\quad u(t-n)u(t-3), \cdots, \\
&\quad u(t-n)u(t-n)]^{\mathrm{T}}, &(20)\\
\hat{\boldsymbol{\varphi}}_n(t) &= [-\hat{\omega}(t-1), -\hat{\omega}(t-2), \cdots, -\hat{\omega}(t-n_e), \\
&\quad \hat{v}(t-1), \hat{v}(t-2), \cdots, \hat{v}(t-n_f)]^{\mathrm{T}}, &(21)\\
\hat{\omega}(t) &= y(t) - \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t), &(22)\\
\hat{v}(t) &= y(t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t). &(23)
\end{aligned}$$

The multiplications and additions of the RGELS algorithm is given in Table 1. The computation procedures of the RGELS algorithm in (16)–(23) are listed in the following.

TABLE 1. The flop amounts of the algorithms.

Algorithms	Multiplications	Additions	Flops
$\hat{\boldsymbol{\theta}}(t)$	$\hat{\boldsymbol{\theta}}(t)=\hat{\boldsymbol{\theta}}(t-1)+\boldsymbol{L}(t)e(t)$	n_0	n_0
	$e(t):=y(t)-\hat{\boldsymbol{\varphi}}^{\rm T}(t)\hat{\boldsymbol{\theta}}(t-1)$	n_0	n_0
$\boldsymbol{L}(t)$	$\boldsymbol{L}(t)=\boldsymbol{\xi}(t)[1+\hat{\boldsymbol{\varphi}}^{\rm T}(t)\xi(t)]^{-1}$	$2n_0$	n_0
	$\boldsymbol{\xi}(t):=\boldsymbol{P}(t-1)\hat{\boldsymbol{\varphi}}(t)$	n_0^2	$n_0^2-n_0$
$\boldsymbol{P}(t)$	$\boldsymbol{P}(t)=\boldsymbol{P}(t-1)-\boldsymbol{L}(t)\boldsymbol{\xi}^{\rm T}(t)$	n_0^2	n_0^2
$\hat{\omega}(t)$	$\hat{\omega}(t)=y(t)-\boldsymbol{\varphi}_s^{\rm T}(t)\hat{\boldsymbol{\theta}}_s(t)$	n_1	n_1
$\hat{v}(t)$	$\hat{v}(t)=y(t)-\hat{\boldsymbol{\varphi}}^{\rm T}(t)\hat{\boldsymbol{\theta}}(t)$	n_0	n_0
Sum	$2n_0^2+5n_0+n_1$	$2n_0^2+3n_0+n_1$	
Total flops	$N_1:=4n_0^2+8n_0+2n_1$		

1) Initialize: let $t=n_0$, $\hat{\boldsymbol{\theta}}(0)=\mathbf{1}_{n_0}/p_0$, $\boldsymbol{P}(0)=p_0\boldsymbol{I}_{n_0}$, $\hat{\omega}(t-i)=0$ $(i=1,2,\cdots,n_e)$ and $\hat{v}(t-i)=0$ $(i=1,2,\cdots,n_f)$, and p_0 is taken to be a large number, e.g., $p_0=10^6$.
2) Collect the input-output data $u(t)$ and $y(t)$. Form $\boldsymbol{\varphi}_s(t)$, $\hat{\boldsymbol{\varphi}}_n(t)$ and $\hat{\boldsymbol{\varphi}}(t)$ by (20), (21) and (19), respectively.
3) Compute the gain vector $\boldsymbol{L}(t)$ by (17) and the covariance matrix $\boldsymbol{P}(t)$ by (18).
4) Update the parameter estimate $\hat{\boldsymbol{\theta}}(t)$ by (16).
5) Compute $\hat{\omega}(t)$ by (22) and $\hat{v}(t)$ by (23).
6) Increase t by 1 and go to Step 2.

The methods proposed in this paper can be extended to study the parameter estimation problems of time-varying systems, nonlinear systems and multi-variable systems [48]–[51], and can be applied to other literatures [52]–[55].

IV. THE FILTERING BASED RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

Using the rational fraction $\frac{E(z)}{F(z)}$ to filter the input-output data of the bilinear system can change the structure of the noise model and enhance the parameter estimation accuracy. As $\frac{E(z)}{F(z)}$ is unknown, its estimate $\frac{\hat{E}(z)}{\hat{F}(z)}$ is generally used to filter the input-output data. The identification method based on the filtered data is called the F-RGELS algorithm.

For the bilinear system in (4), define the filtered input $u_f(t)$ and the filtered output $y_f(t)$ as

$$u_f(t):=\frac{E(z)}{F(z)}u(t),\quad y_f(t):=\frac{E(z)}{F(z)}y(t).$$

Multiplying the both sides of (4) by $\frac{E(z)}{F(z)}$, we have

$$[A(z)+u(t-n)B(z)]\,y_f(t)=[C(z)+u(t-n)D(z)]\,u_f(t)+v(t). \tag{24}$$

Define the filtered information vector $\boldsymbol{\varphi}_f(t)$ as

$$\begin{aligned}\boldsymbol{\varphi}_f(t):=[&-y_f(t-1),-y_f(t-2),\cdots,-y_f(t-n),\\&-u(t-n)y_f(t-1),-u(t-n)y_f(t-2),\cdots,\\&-u(t-n)y_f(t-n),u_f(t-1),u_f(t-2),\cdots,\\&u_f(t-n),u(t-n)u_f(t-2),u(t-n)u_f(t-3),\cdots,\\&u(t-n)u_f(t-n)]^{\rm T}\in\mathbb{R}^{n_1}.\end{aligned}$$

Thus, the filtered model in (4) can be written as

$$y_f(t)=\boldsymbol{\varphi}_f^{\rm T}(t)\boldsymbol{\theta}_s+v(t). \tag{25}$$

Define the stacked vector $\boldsymbol{Y}_{ft}$ and the stacked matrix $\boldsymbol{\Phi}_{ft}$ as

$$\boldsymbol{Y}_{ft}:=\begin{bmatrix}y_f(1)\\y_f(2)\\\vdots\\y_f(t)\end{bmatrix}\in\mathbb{R}^t,\quad \boldsymbol{\Phi}_{ft}:=\begin{bmatrix}\boldsymbol{\varphi}_f^{\rm T}(1)\\\boldsymbol{\varphi}_f^{\rm T}(2)\\\vdots\\\boldsymbol{\varphi}_f^{\rm T}(t)\end{bmatrix}\in\mathbb{R}^{t\times n_1}.$$

Define the criterion function:

$$J_2(\boldsymbol{\theta}_s):=\|\boldsymbol{Y}_{ft}-\boldsymbol{\Phi}_{ft}\boldsymbol{\theta}_s\|^2.$$

Minimizing $J_2(\boldsymbol{\theta}_s)$ and letting its partial derivative with respect to $\boldsymbol{\theta}_s$ be zero, we can obtain the following recursive least squares relations:

$$\hat{\boldsymbol{\theta}}_s(t)=\hat{\boldsymbol{\theta}}_s(t-1)+\boldsymbol{L}_s(t)[y_f(t)-\boldsymbol{\varphi}_f^{\rm T}(t)\hat{\boldsymbol{\theta}}_s(t-1)], \tag{26}$$

$$\boldsymbol{L}_s(t)=\boldsymbol{P}_s(t-1)\boldsymbol{\varphi}_f(t)[1+\boldsymbol{\varphi}_f^{\rm T}(t)\boldsymbol{P}_s(t-1)\boldsymbol{\varphi}_f(t)]^{-1}, \tag{27}$$

$$\boldsymbol{P}_s(t)=[\boldsymbol{I}_{n_1}-\boldsymbol{L}_s(t)\boldsymbol{\varphi}_f^{\rm T}(t)]\boldsymbol{P}_s(t-1). \tag{28}$$

Note that the polynomials $E(z)$ and $F(z)$ are unknown, so are the filtered input-output data $u_f(t)$ and $y_f(t)$, and the filtered information vector $\boldsymbol{\varphi}_f(t)$. Thus, the estimate of $\boldsymbol{\theta}_s(t)$ is impossible to compute directly. Here, we replace the unknown variables with their estimates to implement the recursive computation.

Use the estimates of noise states to define the stacked vector $\boldsymbol{W}_t$ and the stacked matrix $\boldsymbol{\Phi}_{nt}$ as

$$\boldsymbol{W}_t:=\begin{bmatrix}\hat{w}(1)\\\hat{w}(2)\\\vdots\\\hat{w}(t)\end{bmatrix}\in\mathbb{R}^t,\quad \boldsymbol{\Phi}_{nt}:=\begin{bmatrix}\hat{\boldsymbol{\varphi}}_n^{\rm T}(1)\\\hat{\boldsymbol{\varphi}}_n^{\rm T}(2)\\\vdots\\\hat{\boldsymbol{\varphi}}_n^{\rm T}(t)\end{bmatrix}\in\mathbb{R}^{t\times n_2}.$$

According to the noise model in (5), define the criterion function:

$$J_3(\boldsymbol{\theta}_n):=\|\boldsymbol{W}_t-\boldsymbol{\Phi}_{nt}\boldsymbol{\theta}_n\|^2.$$

Minimizing $J_3(\boldsymbol{\theta}_n)$ and letting its partial derivative with respect to $\boldsymbol{\theta}_n$ be zero, we can obtain the following recursive least squares relations:

$$\hat{\boldsymbol{\theta}}_n(t)=\hat{\boldsymbol{\theta}}_n(t-1)+\boldsymbol{L}_n(t)[w(t)-\boldsymbol{\varphi}_n^{\rm T}(t)\hat{\boldsymbol{\theta}}_n(t-1)], \tag{29}$$

$$\boldsymbol{L}_n(t)=\boldsymbol{P}_n(t-1)\boldsymbol{\varphi}_n(t)[1+\boldsymbol{\varphi}_n^{\rm T}(t)\boldsymbol{P}_n(t-1)\boldsymbol{\varphi}_n(t)]^{-1}, \tag{30}$$

$$\boldsymbol{P}_n(t)=[\boldsymbol{I}_{n_2}-\boldsymbol{L}_n(t)\boldsymbol{\varphi}_n^{\rm T}(t)]\boldsymbol{P}_n(t-1). \tag{31}$$

Replacing the parameter vector $\boldsymbol{\theta}_s$ and the information vector $\boldsymbol{\varphi}(t)$ with their estimates $\hat{\boldsymbol{\theta}}_s(t)$ and $\hat{\boldsymbol{\varphi}}(t)$, respectively. The estimates $\hat{\omega}(t)$ and $\hat{v}(t)$ can be computed by

$$\hat{\omega}(t) = y(t) - \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t),$$
$$\hat{v}(t) = y(t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t).$$

Use $\hat{\omega}(t)$ and $\hat{v}(t)$ to construct the estimate of $\boldsymbol{\varphi}_n(t)$:

$$\hat{\boldsymbol{\varphi}}_n(t) = [-\hat{\omega}(t-1), -\hat{\omega}(t-2), \cdots, -\hat{\omega}(t-n_e), \hat{v}(t-1), \hat{v}(t-2), \cdots, \hat{v}(t-n_f)]^{\mathrm{T}}.$$

Using the parameter estimate $\hat{\boldsymbol{\theta}}(t)$ to construct the estimates of $E(z)$ and $F(z)$:

$$\hat{E}(t,z) = 1 + \hat{e}_1(t)z^{-1} + \hat{e}_2(t)z^{-2} + \cdots + \hat{e}_{n_e}(t)z^{-n_e},$$
$$\hat{F}(t,z) = 1 + \hat{f}_1(t)z^{-1} + \hat{f}_2(t)z^{-2} + \cdots + \hat{f}_{n_f}(t)z^{-n_f}.$$

Filtering $u(t)$ and $y(t)$ with $\hat{E}(t,z)$ and $\hat{F}(t,z)$, we can obtain the estimates $\hat{u}_f(t)$ and $\hat{y}_f(t)$:

$$\hat{u}_f(t) = u(t) + \sum_{i=1}^{n_e} e_i u(t-i) - \sum_{j=1}^{n_f} e_j u(t-j),$$
$$\hat{y}_f(t) = y(t) + \sum_{i=1}^{n_e} e_i y(t-i) - \sum_{j=1}^{n_f} e_j y(t-j).$$

Using $\hat{u}_f(t)$ and $\hat{y}_f(t)$ to construct the estimate of $\boldsymbol{\varphi}_f(t)$:

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_f(t) = [&-\hat{y}_f(t-1), -\hat{y}_f(t-2), \cdots, -\hat{y}_f(t-n),\\ &-u(t-n)\hat{y}_f(t-1), -u(t-n)\hat{y}_f(t-2), \cdots,\\ &-u(t-n)\hat{y}_f(t-n), \hat{u}_f(t-1), \hat{u}_f(t-2), \cdots,\\ &\hat{u}_f(t-n), u(t-n)\hat{u}_f(t-2), u(t-n)\hat{u}_f(t-3), \cdots,\\ &u(t-n)\hat{u}_f(t-n)]^{\mathrm{T}}.\end{aligned}$$

Replacing $\boldsymbol{\varphi}_f(t)$, $\omega(t)$ and $\boldsymbol{\varphi}_n(t)$ in (26)–(31) with their estimates $\hat{\boldsymbol{\varphi}}_f(t)$, $\hat{\omega}(t)$ and $\hat{\boldsymbol{\varphi}}_n(t)$, respectively, the filtering based recursive generalized extended least squares (F-RGELS) algorithm can be summarized as

$$\hat{\boldsymbol{\theta}}_s(t) = \hat{\boldsymbol{\theta}}_s(t-1) + \boldsymbol{L}_s(t)[\hat{y}_f(t) - \hat{\boldsymbol{\varphi}}_f^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t-1)], \tag{32}$$
$$\boldsymbol{L}_s(t) = \boldsymbol{P}_s(t-1)\hat{\boldsymbol{\varphi}}_f(t)[1 + \hat{\boldsymbol{\varphi}}_f^{\mathrm{T}}(t)\boldsymbol{P}_s(t-1)\hat{\boldsymbol{\varphi}}_f(t)]^{-1}, \tag{33}$$
$$\boldsymbol{P}_s(t) = [\boldsymbol{I}_{n_1} - \boldsymbol{L}_s(t)\hat{\boldsymbol{\varphi}}_f^{\mathrm{T}}(t)]\boldsymbol{P}_s(t-1), \tag{34}$$
$$\hat{u}_f(t) = u(t) + \sum_{i=1}^{n_e} e_i u(t-i) - \sum_{j=1}^{n_f} e_j u(t-j), \tag{35}$$
$$\hat{y}_f(t) = y(t) + \sum_{i=1}^{n_e} e_i y(t-i) - \sum_{j=1}^{n_f} e_j y(t-j), \tag{36}$$
$$\begin{aligned}\hat{\boldsymbol{\varphi}}_f(t) = [&-\hat{y}_f(t-1), -\hat{y}_f(t-2), \cdots, -\hat{y}_f(t-n),\\ &-u(t-n)\hat{y}_f(t-1), -u(t-n)\hat{y}_f(t-2), \cdots,\\ &-u(t-n)\hat{y}_f(t-n), \hat{u}_f(t-1), \hat{u}_f(t-2), \cdots,\\ &\hat{u}_f(t-n), u(t-n)\hat{u}_f(t-2),\\ &u(t-n)\hat{u}_f(t-3), \cdots, (t-n)\hat{u}_f(t-n)]^{\mathrm{T}},\end{aligned} \tag{37}$$
$$\hat{\boldsymbol{\theta}}_n(t) = \hat{\boldsymbol{\theta}}_n(t-1) + \boldsymbol{L}_n(t)[\hat{\omega}(t) - \hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_n(t-1)], \tag{38}$$
$$\boldsymbol{L}_n(t) = \boldsymbol{P}_n(t-1)\hat{\boldsymbol{\varphi}}_n(t)[1 + \hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}(t)\boldsymbol{P}_n(t-1)\hat{\boldsymbol{\varphi}}_n(t)]^{-1}, \tag{39}$$
$$\boldsymbol{P}_n(t) = [\boldsymbol{I}_{n_2} - \boldsymbol{L}_n(t)\hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}(t)]\boldsymbol{P}_n(t-1), \tag{40}$$
$$\begin{aligned}\hat{\boldsymbol{\varphi}}_n(t) = [&-\hat{\omega}(t-1), -\hat{\omega}(t-2), \cdots, -\hat{\omega}(t-n_e),\\ &\hat{v}(t-1), \hat{v}(t-2), \cdots, \hat{v}(t-n_f)]^{\mathrm{T}},\end{aligned} \tag{41}$$
$$\hat{\omega}(t) = y(t) - \boldsymbol{\varphi}_s^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t), \tag{42}$$
$$\hat{v}(t) = (t) - \hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t), \tag{43}$$
$$\begin{aligned}\boldsymbol{\varphi}_s(t) = [&-y(t-1), -y(t-2), \cdots, -y(t-n),\\ &-u(t-n)y(t-1), -u(t-n)y(t-2), \cdots,\\ &-u(t-n)y(t-n), u(t-1), u(t-2), \cdots,\\ &u(t-n), u(t-n)u(t-2), u(t-n)u(t-3), \cdots,\\ &u(t-n)u(t-n)]^{\mathrm{T}},\end{aligned} \tag{44}$$
$$\hat{\boldsymbol{\varphi}}(t) = \begin{bmatrix}\boldsymbol{\varphi}_s(t)\\ \hat{\boldsymbol{\varphi}}_n(t)\end{bmatrix}, \tag{45}$$
$$\begin{aligned}\hat{\boldsymbol{\theta}}_s(t) = [&\hat{a}_1(t), \hat{a}_2(t), \cdots, \hat{a}_n(t), \hat{b}_1(t), \hat{b}_2(t), \cdots, \hat{b}_n(t),\\ &\hat{c}_1(t), \hat{c}_2(t), \cdots, \hat{c}_n(t),\\ &\hat{d}_2(t), \hat{d}_3(t), \cdots, \hat{d}_n(t)]^{\mathrm{T}},\end{aligned} \tag{46}$$
$$\begin{aligned}\hat{\boldsymbol{\theta}}_n(t) = [&\hat{e}_1(t), \hat{e}_2(t), \cdots, \hat{e}_{n_e}(t),\\ &\hat{f}_1(t), \hat{f}_2(t), \cdots, \hat{f}_{n_f}(t)]^{\mathrm{T}}.\end{aligned} \tag{47}$$

The computation procedures of the F-RGELS algorithm in (32)–(47) are listed in the following.

1) Initialize: let $t = n_0$, and set the initial values $\hat{\boldsymbol{\theta}}_s(0) = \mathbf{1}_{n_1}/p_0$, $\boldsymbol{P}_s(0) = p_0\boldsymbol{I}_{n_1}$, $\hat{\boldsymbol{\theta}}_n(0) = \mathbf{1}_{n_2}/p_0$, $\boldsymbol{P}_n(0) = p_0\boldsymbol{I}_{n_2}$, $\hat{\omega}(t-i) = 0$ $(i = 1, 2, \cdots, n_e)$ and $\hat{v}(t-i) = 0$ $(i = 1, 2, \cdots, n_f)$, and p_0 is taken to be a large number, e.g., $p_0 = 10^6$.
2) Collect the input-output data $u(t)$ and $y(t)$. Form $\boldsymbol{\varphi}_s(t)$, $\hat{\boldsymbol{\varphi}}_n(t)$ and $\hat{\boldsymbol{\varphi}}(t)$ by (44), (41) and (45), respectively.
3) Compute $\hat{\omega}(t)$ and $\hat{v}(t)$ by (42) and (43). Compute the gain vector $\boldsymbol{L}_n(t)$ and the covariance matrix $\boldsymbol{P}_n(t)$ by (39) and (40).
4) Update the parameter estimate $\hat{\boldsymbol{\theta}}_n(t)$ by (38).
5) Compute $\hat{u}_f(t)$ and $\hat{y}_f(t)$ by (35) and (36), respectively. Construct $\hat{\boldsymbol{\varphi}}_f(t)$ by (37).
6) Compute the gain vector $\boldsymbol{L}_s(t)$ and the covariance matrix $\boldsymbol{P}_s(t)$ by (33) and (34).
7) Update the parameter estimate $\hat{\boldsymbol{\theta}}_s(t)$ by (32).
8) Increase t by 1 and go to Step 2.

The computational efficiency of the F-RGELS algorithm are shown in Table 2. The proposed methods proposed in this paper can combine other methods [56]–[59] to study the parameter estimation problems of different systems with colored noises [60]–[69] such as signal modeling and communication networked systems [70]–[72].

The computational efficiency is usually counted by the flop (the floating point operation). Here, an addition, a multiplication, a subtraction, a division all is a flop. In general, a division is considered as a multiplication and a subtraction is considered as an addition. Thus, the computational amount of an identification algorithm can be expressed by adds and multiplications. The total flop numbers of the RGELS algorithm

TABLE 2. The computational efficiency of the F-RGELS algorithm.

Algorithms	Multiplications	Additions	Flops
$\hat{\boldsymbol{\theta}}_s(t)$	$\hat{\boldsymbol{\theta}}_s(t)=\hat{\boldsymbol{\theta}}_s(t-1)+\boldsymbol{L}_s(t)e_s(t)$	n_1	n_1
	$e_s(t):=\hat{y}_f(t)-\hat{\boldsymbol{\varphi}}_f^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t-1)$	n_1	n_1
$\boldsymbol{L}_s(t)$	$\boldsymbol{L}_s(t)=\boldsymbol{\xi}_s(t)[1+\hat{\boldsymbol{\varphi}}_f^{\mathrm{T}}(t)\xi_s(t)]^{-1}$	$2n_1$	n_1
	$\boldsymbol{\xi}_s(t):=\boldsymbol{P}_s(t-1)\hat{\boldsymbol{\varphi}}_f(t)$	n_1^2	$(n_1-1)n_1$
$\boldsymbol{P}_s(t)$	$\boldsymbol{P}_s(t)=\boldsymbol{P}_s(t-1)-\boldsymbol{L}_s(t)\boldsymbol{\xi}_s^{\mathrm{T}}(t)$	n_1^2	n_1^2
$\hat{\boldsymbol{\theta}}_n(t)$	$\hat{\boldsymbol{\theta}}_n(t)=\hat{\boldsymbol{\theta}}_n(t-1)+\boldsymbol{L}_n(t)e_n(t)$	n_2	n_2
	$e_n(t):=\hat{\omega}(t)-\hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_n(t-1)$	n_2	n_2
$\boldsymbol{L}_n(t)$	$\boldsymbol{L}_n(t)=\boldsymbol{\xi}_n(t)[1+\hat{\boldsymbol{\varphi}}_n^{\mathrm{T}}(t)\boldsymbol{\xi}_n(t)]^{-1}$	$2n_2$	n_2
	$\boldsymbol{\xi}_n(t):=\boldsymbol{P}_n(t-1)\hat{\boldsymbol{\varphi}}_n(t)$	n_2^2	$(n_2-1)n_2$
$\boldsymbol{P}_n(t)$	$\boldsymbol{P}_n(t)=\boldsymbol{P}_n(t-1)-\boldsymbol{L}_n(t)\boldsymbol{\xi}_n^{\mathrm{T}}(t)$	n_2^2	n_2^2
$\hat{\omega}(t)$	$\hat{\omega}(t)=y(t)-\boldsymbol{\varphi}_s^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}_s(t)$	n_1	n_1
$\hat{v}(t)$	$\hat{v}(t)=y(t)-\hat{\boldsymbol{\varphi}}^{\mathrm{T}}(t)\hat{\boldsymbol{\theta}}(t)$	n_0	n_0
$\hat{u}_f(t)$	$u(t)+\sum_{i=1}^{n_e}e_iu(t-i)-\sum_{j=1}^{n_f}e_ju(t-j)$	n_2	n_2+2
$\hat{y}_f(t)$	$y(t)+\sum_{i=1}^{n_e}e_iy(t-i)-\sum_{j=1}^{n_f}e_jy(t-j)$	n_2	n_2+2
Sum		$2n_1^2+2n_2^2+6n_0+n_2$	$2n_1^2+2n_2^2+4n_0+n_2+4$
Total flops		$N_2:=4n_1^2+4n_2^2+10n_0+2n_2+4$	

TABLE 3. The flop amounts of the algorithms.

Algorithms	Multiplications	Additions	Flops
RGELS	$2n_0^2+5n_0+n_1$	$2n_0^2+3n_0+n_1$	$N_1:=4n_0^2+8n_0+2n_1$
F-RGELS	$2n_1^2+2n_2^2+6n_0+n_2$	$2n_1^2+2n_2^2+4n_0+n_2+4$	$N_2:=4n_1^2+4n_2^2+10n_0+2n_2+4$

TABLE 4. The RGELS estimates and their errors with $\sigma^2=1.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.69861	0.63656	0.21738	-0.12578	1.06248	-2.14550	0.15646	-0.01853	0.09872	11.61724
200	0.70340	0.61462	0.21670	-0.12550	0.99589	-2.20674	0.11666	-0.04747	0.11450	8.66280
500	0.72821	0.62380	0.24169	-0.11644	0.89421	-2.24204	0.16196	-0.12812	0.08387	6.26723
1000	0.72086	0.63088	0.24568	-0.10624	0.88213	-2.29525	0.16826	-0.11681	0.12439	5.36887
2000	0.74124	0.64491	0.23388	-0.10253	0.86686	-2.28715	0.20110	-0.17115	0.11330	5.23157
3000	0.74765	0.64672	0.23074	-0.10159	0.86369	-2.25086	0.20070	-0.15256	0.14625	5.18746
True values	0.71000	0.63000	0.20000	-0.18000	0.90000	-2.30000	0.16000	-0.20000	0.20000	

TABLE 5. The F-RGELS estimates and their errors with $\sigma^2=1.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.70709	0.65213	0.20941	-0.17526	1.12618	-2.17761	0.13380	-0.14019	0.22459	9.98774
200	0.69818	0.61559	0.20540	-0.17299	1.01717	-2.21831	0.11409	-0.06203	0.30441	8.58182
500	0.72091	0.62626	0.21934	-0.17570	0.91240	-2.22202	0.14072	-0.16962	0.18653	3.38729
1000	0.70806	0.63090	0.21799	-0.17671	0.87759	-2.28962	0.13827	-0.14836	0.16904	2.65133
2000	0.71336	0.63236	0.20439	-0.18139	0.86195	-2.31911	0.16909	-0.17777	0.17787	2.01614
3000	0.71589	0.63113	0.20143	-0.18197	0.86052	-2.29662	0.17253	-0.16721	0.19944	1.99068
True values	0.71000	0.63000	0.20000	-0.18000	0.90000	-2.30000	0.16000	-0.20000	0.20000	

and the F-RGELS algorithm are $N_1=4n_0^2+8n_0+2n_1$ and $N_2=4n_1^2+4n_2^2+10n_0+2n_2+4$, respectively.

The flops of the RGELS algorithm and the F-RGELS algorithm are listed in Table 3, where $n_0=n_1+n_2$, and the flop difference between the RGELS algorithm and the F-RGELS algorithm is

$$\begin{aligned}N_1-N_2&=8n_1n_2-4n_2-4\\&=4n_2(n_1-1)+4(n_1n_2-1)>0.\end{aligned}$$

$N_1>N_2$ means that the F-RGELS algorithm is more flop-efficient than the RGELS algorithm.

V. EXAMPLE

Consider the following bilinear system:

$$\begin{aligned}&[A(z)+u(t-n)B(z)]y(t)\\&\quad=[C(z)+u(t-n)D(z)]u(t)+\omega(t),\ E(z)\omega(t)\\&\quad=F(z)v(t),\\&A(z)=1+a_1z^{-1}+a_2z^{-2}=1+0.71z^{-1}+0.63z^{-2},\\&B(z)=b_1z^{-1}+b_2z^{-2}=0.2z^{-1}-0.18z^{-2},\\&C(z)=c_1z^{-1}+c_2z^{-2}=0.9z^{-1}-2.3z^{-2},\\&D(z)=d_2z^{-2}=0.16z^{-2},\\&E(z)=1+e_1z^{-1}=1-0.2z^{-1},\\&F(z)=1+f_1z^{-1}=1+0.2z^{-1}.\end{aligned}$$

In simulation, the input signal $\{u(t)\}$ adopts a persistent excitation sequence with unit variance and zero mean. $\{v(t)\}$ is a white noise sequence with zero mean and variance $\sigma^2=1.0^2$ and $\sigma^2=3.0^2$, respectively. Applying the RGELS algorithm in (16)–(23) and the F-RGELS algorithm in (32)–(47) to compute the parameter estimate $\hat{\boldsymbol{\theta}}(t)$ of the bilinear system. The parameter estimates and their errors are shown in Tables 4–7, where $\delta(t):=\|\hat{\boldsymbol{\theta}}(t)-\boldsymbol{\theta}\|/\|\boldsymbol{\theta}\|$ is the

TABLE 6. The RGELS estimates and their errors with $\sigma^2 = 3.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.71604	0.68181	0.20670	-0.10635	1.48935	-1.88673	0.09044	-0.09627	0.09829	27.73571
200	0.70035	0.63299	0.20445	-0.09530	1.21389	-2.06466	-0.00479	-0.11777	0.11495	16.78927
500	0.74458	0.63866	0.25230	-0.09506	0.89288	-2.17166	0.10992	-0.18730	0.07382	8.03366
1000	0.73166	0.64156	0.25403	-0.08998	0.84658	-2.28352	0.10839	-0.15357	0.12008	6.01300
2000	0.74340	0.65012	0.24285	-0.09083	0.79530	-2.33432	0.21855	-0.18636	0.13326	6.62561
3000	0.75001	0.64691	0.23645	-0.09430	0.78739	-2.25970	0.22653	-0.16586	0.16650	6.63231
True values	0.71000	0.63000	0.20000	-0.18000	0.90000	-2.30000	0.16000	-0.20000	0.20000	

TABLE 7. The F-RGELS estimates and their errors with $\sigma^2 = 3.0^2$.

t	a_1	a_2	b_1	b_2	c_1	c_2	d_2	e_1	f_1	δ (%)
100	0.74461	0.70948	0.19345	-0.17574	1.58854	-1.86824	0.03254	-0.18907	0.15376	30.92522
200	0.71542	0.64500	0.19315	-0.15947	1.24352	-1.99486	-0.02801	-0.10950	0.23988	18.91934
500	0.74893	0.64461	0.23147	-0.16973	0.92232	-2.04985	0.08132	-0.20041	0.17242	10.07362
1000	0.72185	0.64312	0.23115	-0.17027	0.83249	-2.25470	0.07716	-0.17218	0.15690	4.93475
2000	0.72679	0.64251	0.21649	-0.17517	0.78680	-2.35786	0.18661	-0.18805	0.17911	5.03265
3000	0.72943	0.63501	0.20910	-0.18215	0.78163	-2.29288	0.19870	-0.16252	0.21578	4.96583
True values	0.71000	0.63000	0.20000	-0.18000	0.90000	-2.30000	0.16000	-0.20000	0.20000	

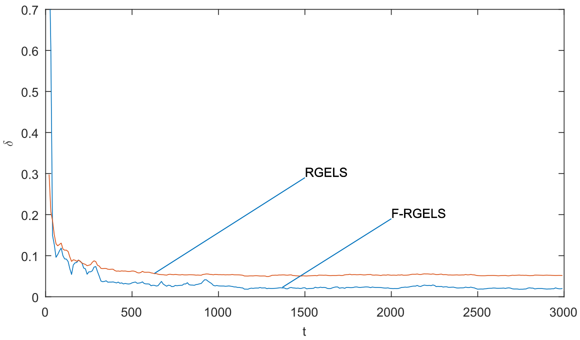

FIGURE 1. The estimation errors δ versus t with $\sigma^2 = 1.0^2$.

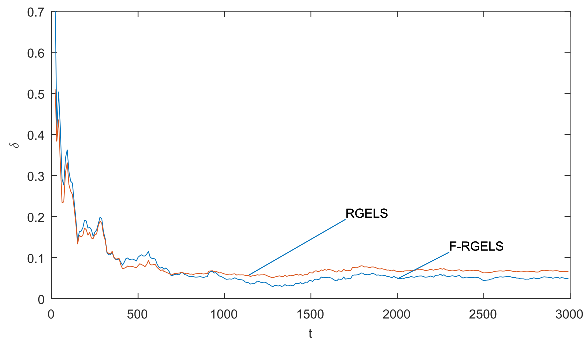

FIGURE 2. The estimation errors δ versus t with $\sigma^2 = 3.0^2$.

parameter estimation error. The estimation errors δ versus time are shown in Figures 1 and 2.

In order to validate the parameter estimation accuracy, we use the RGELS estimates and the F-RGELS estimates to construct the estimated model, respectively, that is

$$\hat{y}(t) = y(t) - [\hat{A}(z) + u(t-n)\hat{B}(z)]\frac{\hat{E}(z)}{\hat{F}(z)}y(t) + [\hat{C}(z) + u(t-n)\hat{D}(z)]\frac{\hat{E}(z)}{\hat{F}(z)}u(t). \quad (48)$$

Define $\hat{y}_f(t) := \frac{\hat{E}(z)}{\hat{F}(z)}y(t)$ and $\hat{u}_f(t) := \frac{\hat{E}(z)}{\hat{F}(z)}u(t)$, and then Equation (48) can be expressed as

$$\hat{y}(t) = y(t) - [\hat{A}(z) + u(t-n)\hat{B}(z)]\hat{y}_f(t) + [\hat{C}(z) + u(t-n)\hat{D}(z)]\hat{u}_f(t).$$

Using the RGELS estimates in Table 6 at $t = 3000$ to construct the RGELS estimated model

$$\begin{aligned}
\hat{y}_1(t) &= y(t) - [\hat{A}_1(z) + u(t-n)\hat{B}_1(z)]\hat{y}_{f1}(t) \\
&\quad + [\hat{C}_1(z) + u(t-n)\hat{D}_1(z)]\hat{u}_{f1}(t), \\
\hat{y}_{f1}(t) &= \frac{\hat{E}_1(z)}{\hat{F}_1(z)}y(t), \quad \hat{u}_{f1}(t) = \frac{\hat{E}_1(z)}{\hat{F}_1(z)}u(t), \\
\hat{A}_1(z) &= 1 + 0.75001z^{-1} + 0.64691z^{-2}, \\
\hat{B}_1(z) &= 0.23645z^{-1} - 0.09430z^{-2}, \\
\hat{C}_1(z) &= 0.78739z^{-1} - 2.25970z^{-2}, \\
\hat{D}_1(z) &= 0.22653z^{-2}, \\
\hat{E}_1(z) &= 1 - 0.16586z^{-1}, \\
\hat{F}_1(z) &= 1 + 0.16650z^{-1}.
\end{aligned}$$

Using the F-RGELS estimates in Table 7 at $t = 3000$ to construct the F-RGELS estimated model

$$\begin{aligned}
\hat{y}_2(t) &= y(t) - [\hat{A}_2(z) + u(t-n)\hat{B}_2(z)]\hat{y}_{f2}(t) \\
&\quad + [\hat{C}_2(z) + u(t-n)\hat{D}_2(z)]\hat{u}_{f2}(t), \\
\hat{y}_{f2}(t) &= \frac{\hat{E}_2(z)}{\hat{F}_2(z)}y(t), \quad \hat{u}_{f2}(t) = \frac{\hat{E}_2(z)}{\hat{F}_2(z)}u(t), \\
\hat{A}_2(z) &= 1 + 0.72943z^{-1} + 0.63501z^{-2}, \\
\hat{B}_2(z) &= 0.20910z^{-1} - 0.18215z^{-2}, \\
\hat{C}_2(z) &= 0.78163z^{-1} - 2.29288z^{-2}, \\
\hat{D}_2(z) &= 0.19870z^{-2}, \\
\hat{E}_2(z) &= 1 - 0.16252z^{-1}, \\
\hat{F}_2(z) &= 1 + 0.21578z^{-1}.
\end{aligned}$$

In order to validate these estimated models, we use the rest 100 data from $t = 3001$ to $t = 3100$ to compute the predicted output $\hat{y}_i(t)$. The actual output $y(t)$, the predicted output $\hat{y}_i(t)$ and their error $\hat{y}_i(t) - y(t)$ are shown in Figures 3–4 for the RGELS algorithm and the F-RGELS algorithm. Figures 3–4 show that the predicted output are very close to the actual output of the bilinear system. This demonstrates that the identification models capture the characteristics of the bilinear system.

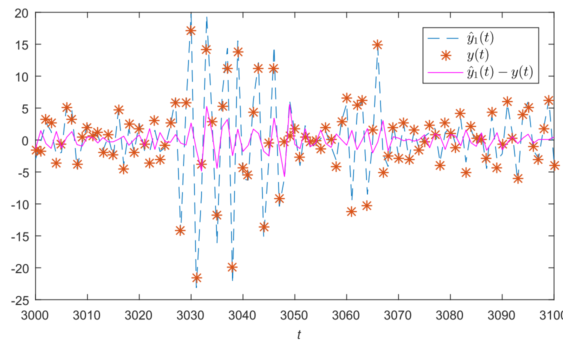

FIGURE 3. The predicted output $\hat{y}_1(t)$, the actual output $y(t)$ and their error $\hat{y}_1(t) - y(t)$ versus t based on the RGELS estimates.

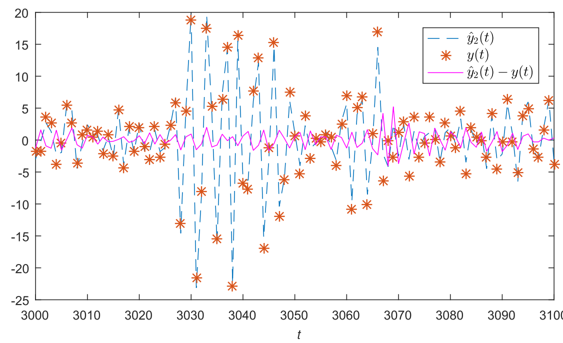

FIGURE 4. The predicted output $\hat{y}_2(t)$, the actual output $y(t)$ and their error $\hat{y}_2(t) - y(t)$ versus t based on the F-RGELS estimates.

From the computational loads in Tables 1–3, and the simulation results in Tables 4–7 and Figures 1–4, we can draw the follow conclusions.

- The parameter estimation errors given by the RGELS algorithm and the F-RGELS algorithm become smaller with t increasing – see Tables 4–7.
- As the noise variance decreases, the parameter estimation errors given by the RGELS algorithm and the F-RGELS algorithm become small – see Figures 1 and 2 and Tables 4–7.
- Compared with the RGELS algorithm, the F-RGELS algorithm can not only reduce the computational amount, but also enhance the estimation accuracy effectively – see Tables 1–7 and Figures 1 and 2.
- The outputs of the estimated models approach those of the actual system – see Figures 3 and 4.

VI. CONCLUSION

A filtering based recursive generalized extended least squares (F-RGELS) algorithm is presented to reduce the computational burden and enhance the parameter estimation accuracy by using the data filtering technique, and a recursive generalized extended least squares algorithm is derived for comparison. The proposed algorithms have the following features. Compared with the RGLES algorithm, the F-RGELS algorithm can not only reduce the computational burden, but also enhance the parameter estimation accuracy. Using the data filtering technique, the bilinear system is divided into two subsystems and the information vector dimension decrease significantly. Then, the F-RGELS algorithm has lower computational burden than the RGELS algorithm. The proposed recursive least squares estimation algorithms for a class of nonlinear stochastic systems with colored noise using the input-output data filtering can combine other estimation algorithms [73]–[76] and mathematical tools [77]–[80] to explore new parameter identification methods of linear, bilinear and nonlinear stochastic systems with colored noise and can be applied to other fields [81]–[84] such as information processing and communication systems [85]–[88].

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LONGJIN WANG received the Ph.D. degree in control science and engineering from Harbin Engineering University, in 2009. From 2009 to 2013, he was an Engineer with China Shipbuilding Heavy Industry Corporation. Since 2013, he has been an Associate Professor of control science and engineering with the Qingdao University of Science and Technology. His current research interests include system identification and motion control of marine crafts.

YAN HE received the Ph.D. degree in power and engineering thermophysics from the Huazhong University of Science and Technology, in 2005. She has been a Professor with the Qingdao University of Science and Technology, where she is also the Academic Leader in the first-tier discipline of power engineering and engineering thermophysics. Her current research interests include thermal transformation, energy development and utilization, and industrial process identification.