IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received November 11, 2019, accepted November 21, 2019, date of publication November 28, 2019, date of current version December 23, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2956486

Stabilization for Rectangular Descriptor Fractional Order Systems

HUANXIA LIU, CHONG LIN, (Senior Member, IEEE), BING CHEN, AND WENTAO GENG
The Institute of Complexity Science, Qingdao University, Qingdao 266071, China

Corresponding author: Chong Lin (linchong_2004@hotmail.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61673227 and Grant 61873137, and in part by the Taishan Scholar Project under 201911 and the Natural Science Foundation of Shandong Province of China under Grant ZR2016FM06.

ABSTRACT This paper focuses on the stabilization problem for rectangular descriptor fractional order systems (FOSs) with $0 < \alpha \leq 1$. Firstly, a fractional order dynamic compensator is constructed to make the closed-loop systems square descriptor FOSs. Secondly, two types of input signals of the compensator are considered, which are state input signal case and output input signal case. Thirdly, a necessary and sufficient condition is proposed for the state input signal case, while a new method of controller design is given for the output input signal case in which the output matrix of the system need not to be full row rank, which reduces the conservativeness of existing methods. Finally, an efficient iterative algorithm for solving the resultant matrix inequalities is proposed, and a numerical example is offered to verify the advantage and feasibility of the results.

INDEX TERMS Rectangular descriptor FOS, stabilization, fractional order dynamic compensator, iterative algorithm.

I. INTRODUCTION

Fractional calculus, originated in the 17th century, is a classical mathematical concept and can be regarded as a generalization of ordinary calculus [1]. Since Podlubny et al. introduced fractional calculus theory into control theory in the 1990s, the research on fractional order control systems has been greatly developed. Due to fractional order model has been widely used in control systems, signal processing, biomedical systems and other fields, more and more researchers have been engaged in this field [2]. The basic stability and stabilization problems for fractional order control systems have been studied in [3]–[5].

Descriptor systems originated in 1960's are more complicated than normal systems. Considering the ability to describe impulsive behavior and non-dynamic constraints is more accurately, descriptor systems have been attracted a lot of attentions. With the efforts of scholars, many research results about descriptor systems have been achieved. The admissibility of descriptor FOSs with fractional order $0 < \alpha \leq 1$ and $1 < \alpha < 2$ has been studied in [6]–[10]. Necessary and sufficient conditions of observer based stabilization for descriptor FOSs were studied in [11]. About stabilization problem of descriptor FOSs as well as uncertain case, singular value decomposition method is used in [12] and in [14], which output matrix of the systems need to be full row rank. By normalizing the systems before controller designs method is studied in [13]–[15]. Efficient iterative algorithms are built in [16].

However, all of those achievements considered the square descriptor FOSs. Rectangular descriptor systems, where the number of equations and state variables may not be equal, have broader descriptions and more complex behaviors than square descriptor systems [17]. So far, the results for rectangular descriptor systems are extremely abundant, like generalized regularity and regularizability, impulse controllability and observability, estimation and observer design [18]–[23]. What's more, a new feedback structure to stabilize rectangular descriptor systems by dynamic output feedback (dynamic compensator) plus state feedback is proposed in [26]. But all of those work are reported in case of integer order. Until 2019, Zhao et al. for the first time studied the stabilization of rectangular descriptor FOSs by designing fractional order dynamic compensator [13]. Unfortunately, the controller design requires additional constraints, that increases the conservatism.

In view of the above achievements, we focus on the stabilization problem for rectangular descriptor FOSs with $0 < \alpha \leq 1$. The main contributions are highlighted as follows:

The associate editor coordinating the review of this manuscript and approving it for publication was Jianquan Lu.

1. We design a fractional order dynamic compensator to make the closed-loop system a square descriptor FOS, and study two types of input signals of the compensator, which are state input signal case and output input signal case.

2. We propose a necessary and sufficient condition to solve the state input signal case for the first time.

3. We give a new method of controller design for the output input signal case, and provide an LMI based algorithm to solve the corresponding conditions.

The paper is organized as follows: In Section II, we introduce some definitions, useful lemmas and the problem formulation. Section III, presents the main results. Section IV gives numerical example to illustrate the effectiveness of our results. Section V is the conclusion.

Notation: In this paper, $X < 0$ (respectively, $X > 0$) denotes a symmetric negative (respectively, positive) definite matrix. M^{-1} and M^T denote the inverse and the transpose of matrix M, respectively. I denotes identity matrix with compatible dimension. Symbol " $*$ " denotes a symmetric term in a matrix. R^n denotes the n dimensional space and $R^{m\times n}$ denotes the $m \times n$ dimensional matrix space. Define $sym(M) = M + M^T$.

II. PROBLEM FORMULATION

In this section, we introduce some definitions, useful lemmas and the problem formulation.

Definition 2.1 [24]: The Caputo calculus operator is defined as:

$$D^{\alpha}f(t) = \frac{d^{\alpha}f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{\alpha+1-n}},$$

where $D^{\alpha}f(t)$ denotes the α order ($\alpha > 0$) derivative of function $f(t)$, n is a positive integer satisfying $n-1 < \alpha \le n$. $\Gamma(\cdot)$ is the Gamma function which is defined as:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}dt.$$

Lemma 2.1 [7]: Descriptor FOS $ED^{\alpha}x(t) = Ax(t)$ ($E \in R^{n\times n}$, $rank(E) = r_0 \le n$) with $0 < \alpha \le 1$ is admissible, if and only if there exist matrices $X_1,\ X_2 \in R^{r_0\times r_0}$, $X_3 \in R^{(n-r_0)\times r_0}$ and $X_4 \in R^{(n-r_0)\times(n-r_0)}$ such that

$$\begin{bmatrix} X_1 & X_2 \\ -X_2 & X_1 \end{bmatrix} > 0,$$

$$sym\{MAN(aX - bY)\} < 0, \tag{1}$$

where $a = \sin(\frac{\pi}{2}\alpha)$, $b = \cos(\frac{\pi}{2}\alpha)$, $X = \begin{bmatrix} X_1 & 0 \\ X_3 & X_4 \end{bmatrix}$, $Y = \begin{bmatrix} X_2 & 0 \\ 0 & 0 \end{bmatrix}$ and $M,\ N \in R^{n\times n}$ are non-singular matrices satisfying

$$MEN = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad MAN = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}. \tag{2}$$

Remark 2.1: Note that the eigenvalues of matrix MAN and $(MAN)^T$ are identical. Consequently, the following conclusion can be derived easily:

The descriptor FOS which described in Lemma 2.1 is admissible, if and only if there exists a matrix $P_0 \in P_{\alpha}^{n\times n}$ such that

$$sym\{P_0^T MAN\} < 0, \tag{3}$$

where $P_{\alpha}^{n\times n} = \{aX - bY\}$, a, b, X, Y, M and N are defined in lemma 2.1.

Lemma 2.2 [25]: Let $\Psi,\ W,\ R$ be given matrices with appropriate dimensions. Then

$$\begin{cases} \Psi < 0 \\ \Psi + sym\left\{WR^T\right\} < 0 \end{cases} \tag{4}$$

hold, if and only if there exists an appropriately dimensional matrix G such that

$$\begin{bmatrix} \Psi & W + RG^T \\ * & -sym\{G\} \end{bmatrix} < 0. \tag{5}$$

Consider the following rectangular descriptor FOS:

$$\begin{cases} ED^{\alpha}x(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}, \quad 0 < \alpha \le 1, \tag{6}$$

where $x(t) \in R^n$, $u(t) \in R^{n_u}$ and $y(t) \in R^{n_y}$ denote, respectively, the state vector, input and output vector of the system. $E \in R^{m\times n}$ is the system descriptor matrix satisfying $rank(E) = r \le \min\{m, n\}$. $A \in R^{m\times n}$, $B \in R^{m\times n_u}$ and $C \in R^{n_y\times n}$ are known real constant matrices.

In order to realize our purpose, we design the following fractional order compensator:

$$\begin{cases} E_kD^{\alpha}x_k(t) = A_kx_k(t) + B_kv(t) \\ u(t) = C_kx_k(t) + D_kv(t) \end{cases}, \quad 0 < \alpha \le 1, \tag{7}$$

where $x_k(t) \in R^{n_k}$ is state vector of the compensator, $A_k,\ B_k,\ C_k$ and D_k are the compensator gain matrices to be determined, $E_k \in R^{m_k\times n_k}$ is known and $rank(E_k) = r_k \le \min\{m_k, n_k\}$, $v(t) \in R^q$ is the input signal in compensator.

Combing (2) with (1), the closed-loop descriptor FOS is obtained as:

$$\begin{bmatrix} E & 0 \\ 0 & E_k \end{bmatrix} D^{\alpha}\begin{pmatrix} x(t) \\ x_k(t) \end{pmatrix} = \begin{bmatrix} A & BC_k \\ 0 & A_k \end{bmatrix}\begin{pmatrix} x(t) \\ x_k(t) \end{pmatrix} + \begin{bmatrix} BD_k \\ B_k \end{bmatrix} v(t). \tag{8}$$

To ensure that system (8) is a square descriptor FOS, which means matrix $\begin{bmatrix} E & 0 \\ 0 & E_k \end{bmatrix}$ is square, the dimension of matrices E and E_k must meet

$$\hat{n} = n + n_k = m + m_k. \tag{9}$$

With the help of the compensator (7), rectangular descriptor FOS (6) is converted into a square descriptor FOS (8). Next, we will consider the stabilization of square descriptor FOS (8) according to the input signal in compensator.

III. MAIN RESULTS

In this section, two types of input signals in compensator (7) are proposed.

Case 1: Input signal $v(t) = x(t)$.

In this case, it means that the system state $x(t)$ is available for use in the compensator [26]. Then descriptor FOS (8) is equivalent to:

$$\hat{E}D^{\alpha}X(t) = \left(\hat{A} + \hat{B}K\right)X(t), \tag{10}$$

where

$$\hat{E} = \begin{bmatrix} E & 0 \\ 0 & E_k \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \hat{B} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix},$$
$$K = \begin{bmatrix} D_k & C_k \\ B_k & A_k \end{bmatrix}, \quad X(t) = \begin{pmatrix} x(t) \\ x_k(t) \end{pmatrix}. \tag{11}$$

Theorem 3.1: The closed-loop descriptor FOS (10) is admissible, if and only if there exist matrices $Z_0 \in P_{\alpha}^{\hat{n}\times\hat{n}}$ and Q with appropriate dimension such that

$$sym\{M\hat{A}NZ_0 + M\hat{B}Q\} < 0, \tag{12}$$

where matrices $\hat{A}$, $\hat{B}$ are defined in (11), M and N satisfy

$$M\hat{E}N = \begin{bmatrix} I_{r+r_k} & 0 \\ 0 & 0 \end{bmatrix}, \quad M\hat{A}N = \begin{bmatrix} \hat{A}_1 & \hat{A}_2 \\ \hat{A}_3 & \hat{A}_4 \end{bmatrix}. \tag{13}$$

In this case, the compensator parameter matrices are given by

$$K = QZ_0^{-1}N^{-1}. \tag{14}$$

Proof: The proof is similar to that of Theorem 3.2 in [7], and thus it is omitted here.

Remark 3.1: Since rectangular descriptor FOS (6) does not allow to consider the state feedback control directly, we convert rectangular descriptor FOS (6) into square descriptor FOS (8) under compensator (7) firstly, and then adopt state vector of rectangular descriptor FOS (6) as the input signal of compensator (7), so as to effectively solve the state feedback control problem of rectangular descriptor FOS.

Case 2: Input signal $v(t) = y(t)$.

In this case, the system output vector $y(t)$ is used in the compensator. Then descriptor FOS (8) is equivalent to:

$$\hat{E}D^{\alpha}X(t) = \left(\hat{A} + \hat{B}K\hat{C}\right)X(t), \tag{15}$$

where $\hat{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$, and $\hat{E}, \hat{A}, \hat{B}, K, X(t)$ are defined in (11).

Theorem 3.2: Suppose the system (10) is admissible. If there exist matrices G, H with appropriate dimensions and $P \in P_{\alpha}^{\hat{n}\times\hat{n}}$, such that

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & -sym\{G\} \end{bmatrix} < 0, \tag{16}$$

then the closed-loop descriptor FOS (15) is admissible.

where

$$\Omega_1 = sym\{P^T M\hat{A}N + P^T M\hat{B}K_1 N\},$$
$$\Omega_2 = P^T M\hat{B} + N^T\hat{C}^T H^T - N^T K_1^T G^T,$$

and $K_1 = QZ_0^{-1}N^{-1}$ is an intermediate matrix which is derived from (14), matrices Q, Z_0 satisfy (12), and M, N satisfy (13).

In this case, the compensator parameter matrices are given by

$$K = G^{-1}H. \tag{17}$$

Proof: Suppose (16) and (17) hold. Substituting K_1 into (16), by Lemma 2.2 we have

$$\begin{aligned} &\Omega_1 + sym\left\{P^T M\hat{B}(N^T\hat{C}^T H^T G^{-T} - N^T {K_1}^T)^T\right\}, \\ &= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K_1 N\right\} \\ &\quad + sym\left\{P^T M\hat{B}(G^{-1}H\hat{C}N - K_1 N)\right\}, \\ &= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K_1 N - P^T M\hat{B}K_1 N \right. \\ &\quad \left. + P^T M\hat{B}G^{-1}H\hat{C}N\right\}, \\ &= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K\hat{C}N\right\}, \\ &= sym\left\{P^T M(\hat{A} + \hat{B}K\hat{C})N\right\}, \\ &< 0. \end{aligned} \tag{18}$$

By Remark 2.1 and (18), it follows that the closed-loop descriptor FOS (15) is admissible. This completes the proof of the theorem.

It is worth noting that in Theorem 3.2, we need to get the intermediate matrix K_1 from Theorem 3.1 firstly, then substitute it into (16). However, Theorem 3.1 can only give one fixed K_1 from many solutions, which may lead to (16) being infeasible. Next we will propose a method to solve this problem.

Theorem 3.3: Suppose there exist matrices $K(i)$, ΔK with appropriate dimensions and $Z \in P_{\alpha}^{\hat{n}\times\hat{n}}$, such that

$$\begin{aligned} \Lambda(i) \triangleq sym &\left\{M\hat{A}NZ + M\hat{B}K(i)NZ \right. \\ &\left. + M\hat{B}\Delta K\right\} < 0. \end{aligned} \tag{19}$$

If there exist matrices G, H with appropriate dimensions and $P \in P_{\alpha}^{\hat{n}\times\hat{n}}$, such that

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & -sym\{G\} \end{bmatrix} < 0, \tag{20}$$

then the closed-loop descriptor FOS (15) is admissible.

where

$$\Omega_1 = sym\{P^T M\hat{A}N + P^T M\hat{B}K_1 N\},$$
$$\Omega_2 = P^T M\hat{B} + N^T\hat{C}^T H^T - N^T K_1^T G^T,$$
$$K_1 = K(i) + \Delta K Z^{-1}N^{-1},$$

matrices M and N satisfy (13).

In this case, the compensator parameter matrices are given by

$$K = G^{-1}H. \tag{21}$$

Proof: Suppose (19)-(21) hold, we can get matrices $K(i)$, ΔK and Z from (19). Defining $K_1 = K(i) + \Delta K Z^{-1} N^{-1}$ and substituting K_1 into (20), by Lemma 2.2 we have

$$
\begin{aligned}
&\Omega_1 + sym\left\{P^T M\hat{B}(N^T \hat{C}^T H^T G^{-T} - N^T {K_1}^T)^T\right\},\\
&= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K_1 N\right\}\\
&\quad + sym\left\{P^T M\hat{B}(G^{-1}H\hat{C}N - K_1 N)\right\},\\
&= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K_1 N - P^T M\hat{B}K_1 N\right.\\
&\quad \left. + P^T M\hat{B}G^{-1}H\hat{C}N\right\},\\
&= sym\left\{P^T M\hat{A}N + P^T M\hat{B}K\hat{C}N\right\},\\
&= sym\left\{P^T M(\hat{A} + \hat{B}K\hat{C})N\right\},\\
&< 0. \qquad (22)
\end{aligned}
$$

By Remark 2.1 and (22), it follows that the closed-loop descriptor FOS (15) is admissible. This completes the proof of the theorem.

Next, we will give an algorithm to solve the (19) and (20) in Theorem 3.3.

Algorithm 3.1:

Step 1: Set $i = 0$. Let $K(i) = 0$.

Step 2: Solve the following optimization problem with respect to matrices Z, ΔK and λ.

$$
\begin{aligned}
&min \lambda,\\
&s.t.\ \Lambda(i) < \lambda, \qquad (23)
\end{aligned}
$$

where $\Lambda(i)$ is defined as in (19).

Step 3: Let $K(i+1) = K(i) + \Delta K Z^{-1} N^{-1}$. If $\lambda < 0$, stop, and $K(i+1)$ is the gain matrix K_1. Else, set $i = i + 1$ and go to Step2.

Step 4: Substituting K_1 into (20). If inequality (20) is feasible, then we can get the gain matrix K. Else, set $i = i + 1$ and go to Step2.

Remark 3.2: When solving output feedback control problems, the controller design methods in [12], [13] requires $B^T P = \hat{P} B^T$ or $PC = C\hat{P}$ for some matrix $\hat{P}$. Obviously, such requirements are conservative. In contrast, the method proposed in this paper does not require these constraints.

Remark 3.3: The methods in [13]–[15] normalize the descriptor matrix firstly, then design controllers for the normal FOSs. Our method in Theorem 3.2 needs not normalize the descriptor matrix. Moreover, Theorem 3.3 together with Algorithm 3.1 effectively reduces the conservativeness of Theorem 3.2.

IV. SIMULATION EXAMPLE

In this section, we use a numerical example to verify the present methods.

Example 4.1. Consider rectangular descriptor FOS (6) with parameters:

$$
E = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & -2 \end{bmatrix},
$$

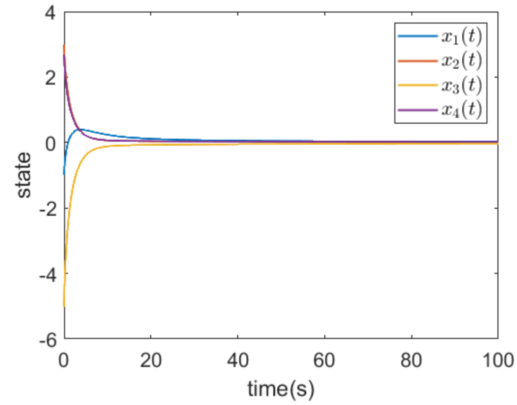


FIGURE 1. State curves for system in Example 4.1.

$$
B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}.
$$

Let $\alpha = 0.75$, $E_k = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. This example is borrowed from [13]. Since the output matrix C has not full row rank, the method in [13] is not feasible. By using the function *reff* in MATLAB, it is easy to get nonsingular matrices M and N satisfying (13):

$$
M = \begin{bmatrix} 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix},
$$

$$
N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.
$$

We first consider Case 1. Set the initial values $X_0 = \begin{bmatrix} -1 & 3 & 0 & 0 & 0.5 \end{bmatrix}^T$. By using LMI Toolbox in MATLAB to solve Theorem 3.1, the feasible solutions can be found as

$$
Z_0 = \begin{bmatrix} 16.34 & -13.52 & -0.05 & 0 & 0 \\ -13.52 & 16.28 & 0.01 & 0 & 0 \\ -0.05 & 0.01 & 11.68 & 0 & 0 \\ 0.62 & -24.61 & 0.20 & -1.06 & -1.51 \\ 3.94 & 12.87 & 0 & -1.51 & 0.84 \end{bmatrix},
$$

$$
Q = \begin{bmatrix} -2.31 & -7.53 & 0.01 & 1.52 & -0.49 \\ -0.20 & 0.61 & -6.08 & 0.39 & -6.39 \\ 0.19 & -0.13 & -6.39 & -0.39 & 0.31 \end{bmatrix}.
$$

Then the compensator parameter matrices can be obtained as

$$
K = \begin{bmatrix} 0.02 & 0.04 & -0.12 & -0.90 & 0.04 \\ 0.07 & 1.56 & 1.06 & 0.04 & 0.22 \\ -0.15 & -0.73 & -0.53 & -0.34 & -0.47 \end{bmatrix}.
$$

With the matrix K, we can see from Fig.1 and Fig.2, state curves for system and compensator tend to zero, respectively.

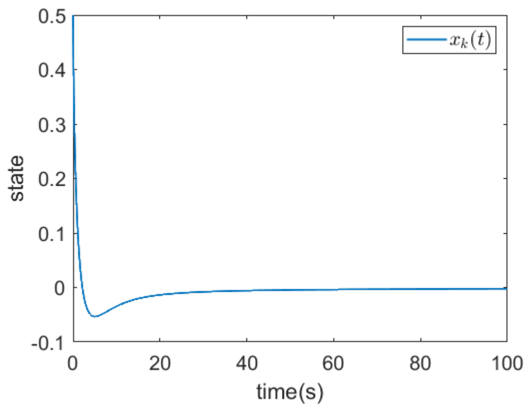


FIGURE 2. State curve for compensator in Example 4.1.

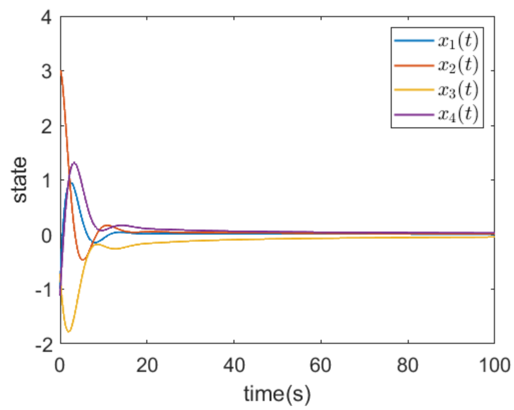


FIGURE 3. State curves for system in Example 4.1.

We next consider Case 2. Set the initial values $X_0 = \begin{bmatrix} -1 & 3 & 0 & 0 & 0.5 \end{bmatrix}^T$. By using LMI Toolbox in MATLAB to solve Theorem 3.2, the feasible solutions can be found as

$$P = \begin{bmatrix} 0.15 & 0 & -0.79 & 0 & 0 \\ 0 & 0.07 & 0.57 & 0 & 0 \\ -0.79 & 0.59 & 21.28 & 0 & 0 \\ 0.14 & -0.14 & -1.33 & -0.21 & -0.27 \\ -2.13 & -2.37 & 1.28 & -0.27 & 0.35 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.01 & 0.01 & -0.46 \\ -0.01 & -0.01 & -0.61 \\ -0.03 & -0.03 & -10.85 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.09 & 0.06 & 1.04 \\ 0.06 & 0.19 & 1.41 \\ 1.04 & 1.41 & 28.03 \end{bmatrix}.$$

Then the compensator parameter matrices can be obtained as

$$K = \begin{bmatrix} -1.81 & -1.81 & -13.65 \\ -0.35 & -0.36 & -2.85 \\ -1.02 & -1.02 & -7.35 \end{bmatrix}.$$

With the matrix K, we can see from Fig.3 and Fig.4, state curves for system and compensator tend to zero, respectively.

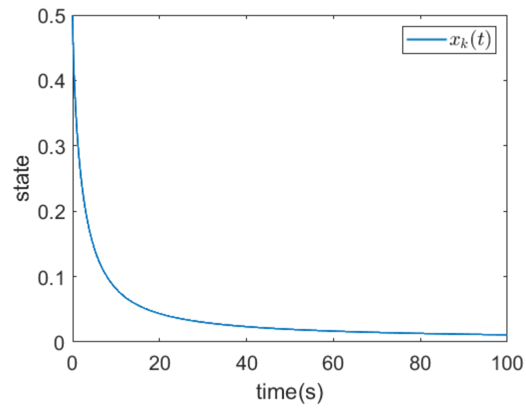


FIGURE 4. State curve for compensator in Example 4.1.

V. CONCLUSION

In this paper, the stabilization problem for rectangular descriptor FOSs with $0 < \alpha \leq 1$ is studied. A fractional order dynamic compensator is constructed to make the closed-loop system square descriptor FOSs. Two types of input signals of the compensator are considered, i.e., case 1 is that the input signal of compensator is the state vector of the rectangular descriptor FOS, and case 2 is that the input signal of compensator is the output vector of the rectangular descriptor FOS. A necessary and sufficient condition is proposed to solve the stabilization problem for Case 1. Consequently, a new method of controller design for Case 2 is given. A numerical example is provided to illustrate the effectiveness and advantage of the proposed design schemes. The stabilization problem for rectangular descriptor FOSs with $1 < \alpha < 2$ is the future work.

REFERENCES

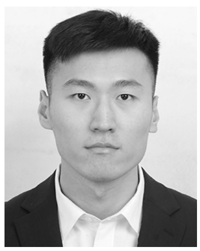
[1] K. B. Oldham and J. Spanier, "The fractional calculus," *Math. Gazette*, vol. 56, no. 247, pp. 396–400, 1974.
[2] I. Podlubny, *Fractional Differential Equations*. New York, NY, USA: Academic, 1999.
[3] Y. Li, Y. Q. Chen, and I. Podlubny, "Mittag-Leffler stability of fractional order nonlinear dynamic systems," *Automatica*, vol. 45, pp. 1965–1969, 2009.
[4] M. S. Tavazoei and M. Haeri, "A note on the stability of fractional order systems," *Math. Comput. Simul.*, vol. 79, no. 5, pp. 1566–1576, Jan. 2009.
[5] J. G. Lu and Y. Q. Chen, "Robust stability and stabilization of fractional-order interval systems with the fractional order α: The $0 < \alpha < 1$ case," *IEEE Trans. Autom. Control*, vol. 55, no. 1, pp. 152–158, Jan. 2010.
[6] Y. Yu, Z. Jiao, and C.-Y. Sun, "Sufficient and necessary condition of admissibility for fractional-order singular system," *Acta Autom. Sinica*, vol. 39, no. 12, pp. 2160–2164, Dec. 2013.
[7] X. Zhang and Y. Chen, "Admissibility and robust stabilization of continuous linear singular fractional order systems with the fractional order α: The $0 < \alpha < 1$ case," *ISA Trans.*, vol. 82, pp. 42–50, Nov. 2018.
[8] S. Marir, M. Chadli, and D. Bouagada, "New admissibility conditions for singular linear continuous-time fractional-order systems," *J. Franklin Inst.*, vol. 354, no. 2, pp. 752–766, Jan. 2017.
[9] Y. Wei, B. Du, Y. Chen, S. Cheng, and Y. Wang, "Necessary and sufficient admissibility condition of singular fractional order systems," in *Proc. 35th Chin. Control Conf.*, Chengdu, China, Jul. 2016, pp. 85–88.
[10] X. Zhang, Y. Lv, and C. Lin, "Admissibility and robust stabilization of fractional-order derivative uncertain linear continuous singular systems," in *Proc. Chin. Control Conf.*, 2018, pp. 10218–10223.

[11] C. Lin, B. Chen, P. Shi, and J.-P. Yu, "Necessary and sufficient conditions of observer-based stabilization for a class of fractional-order descriptor systems," *Syst. Control Lett.*, vol. 112, pp. 31–35, Feb. 2018.
[12] Y. Ji and J. Qiu, "Stabilization of fractional-order singular uncertain systems," *ISA Trans.*, vol. 56, pp. 53–64, May 2015.
[13] Z. L. Zhao and X. F. Zhang, "Output feedback stabilization of uncertain rectangular descriptor fractional order systems with $0 < \alpha < 1$," *IEEE Access*, vol. 7, pp. 108948–108956, 2019.
[14] Y. Wei, P. W. Tse, Z. Yao, and Y. Wang, "The output feedback control synthesis for a class of singular fractional order systems," *ISA Trans.*, vol. 69, pp. 1–9, Jul. 2017.
[15] I. N'Doye, M. Darouach, M. Zasadzinski, and N.-E. Radhy, "Robust stabilization of uncertain descriptor fractional-order systems," *Automatica*, vol. 49, pp. 1907–1913, Jun. 2013.
[16] Y. Guo, C. Lin, B. Chen, and Q.-G. Wang, "Stabilization for singular fractional-order systems via static output feedback," *IEEE Access*, vol. 6, pp. 71678–71684, 2018.
[17] L. R. Fletcher, "Regularizability of descriptor systems," *Int. J. Syst. Sci.*, vol. 17, pp. 843–847, Dec. 1986.
[18] G. Duan and Y. Chen, "Generalized regularity and regularizability of rectangular descriptor systems," *J. Control Theory Appl.*, vol. 5, pp. 159–163, May 2007.
[19] J. Y. Ishihara and M. H. Terra, "Impulse controllability and observability of rectangular descriptor systems," *IEEE Trans. Autom. Control*, vol. 46, no. 6, pp. 991–994, Jun. 2001.
[20] G. Zhang, "Regularizability, controllability and observability of rectangular descriptor systems by dynamic compensation," in *Proc. Amer. Control Conf.*, Jun. 2006, pp. 4393–4398.
[21] M. K. Gupta, N. K. Tomar, and S. Bhaumik, "Full- and reduced-order observer design for rectangular descriptor systems with unknown inputs," *J. Franklin Inst.*, vol. 352, pp. 1250–1264, Mar. 2015.
[22] C. Lin, J. Chen, B. Chen, L. Guo, and Z. Zhang, "Fuzzy normalization and stabilization for a class of nonlinear rectangular descriptor systems," *Neurocomputing*, vol. 219, pp. 263–268, Jan. 2017.
[23] C. Lin, J. Chen, B. Chen, L. Guo, and Z. Zhang, "Stabilization for a class of rectangular descriptor systems via time delayed dynamic compensator," *J. Franklin Inst.*, vol. 356, pp. 1944–1954, Mar. 2019.
[24] M. Caputo, "Linear models of dissipation whose Q is almost frequency independent," *Ann. Geophys.*, vol. 19, no. 4, pp. 383–393, 1966.
[25] S. M. Saadni, M. Chaabane, O. Bachelier, and D. Mehdi, "Robust stability and stabilization of a class of singular systems with multiple time-varying delays," *IFAC Proc. Volumes*, vol. 38, no. 1, pp. 161–166, 2005.
[26] G. Zhang, Z. Zuo, W. Liu, and Q. Zhang, "Stabilization of rectangular descriptor systems," in *Proc. Control Conf.*, Jul. 2008, pp. 777–781.

HUANXIA LIU received the B.Sc. degree in mathematics and information science from Yantai University, China, in 2016. She is currently pursuing the master's degree with Qingdao University, China. Her research interest is in the general area of systems and control theory, especially in fractional order systems.

CHONG LIN received the B.Sc. and M.Sc. degrees in applied mathematics from Northeastern University, Shenyang, China, in 1989 and 1992, respectively, and the Ph.D. degree in electrical and electronic Engineering from Nanyang Technological University, Singapore, in 1999. From 1999 to 2006, he held a research positions at The University of Hong Kong and the National University of Singapore. He had short visit at Brunel University, in 2007, and the University of Johannesburg, in 2019. Since 2006, he has been a Professor with the Institute of Complexity Science, Qingdao University, China. He has published over 100 international journal articles and two books. His research interests include systems analysis and control, robust control, and fuzzy control.

BING CHEN received the B.A. degree in mathematics from Liaoning University, Shenyang, China, in 1982, the M.A. degree in mathematics from the Harbin Institute of Technology, Harbin, China, in 1991, and the Ph.D. degree in electrical engineering from Northeastern University, Shenyang, in 1998. He is currently a Professor with the Institute of Complexity Science, Qingdao University, Qingdao, China. He has published over 100 international journal articles. His research interests include nonlinear control systems, robust control, and adaptive fuzzy control.



WENTAO GENG received the B.Eng. degree in automation from Qingdao University, China, in 2015, where he is currently pursuing the master's degree. His research interest is in the general area of systems and control theory, especially in fractional order systems.