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A PAPR Reduction Method With EVM Constraints for OFDM Systems

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ABSTRACT In orthogonal frequency division multiplexing (OFDM) systems, high peak to average power ratio (PAPR) is one of its core issues. The OFDM signal with high PAPR passes high-power amplifier (HPA) will result in severely nonlinear distortions, which not only decreases HPA power efficiency, but also increases the whole system's bit error rate (BER). In order to attain the OFDM signal with lower PAPR and better BER, we consider PAPR optimization with error vector magnitude (EVM) constrains in this paper. Different from the second order conic programming (SOCP) scheme with high complexity proposed in the literature, we present an innovative approach based on linearized alterative direction method of multipliers (LADMM) to deal with the PAPR optimization. The proposed LADMM algorithm is a simple and efficient method with FFT/IFFT complexity in every iteration. We also prove LADMM algorithm's convergence and then perform simulations. Simulation results demonstrate that LADMM algorithm not only acquires larger PAPR reduction, but also obtains better BER.

INDEX TERMS Error vector magnitude (EVM), linearized alterative direction method of multipliers (LADMM), orthogonal frequency division multiplexing (OFDM), peak to average power ratio (PAPR).

I. INTRODUCTION

As a multi-carrier transmission technique, orthogonal frequency division multiplexing (OFDM) is extensively adopted to next-generation wireless communication systems, because it provides a lot of benefits, such as higher bandwidth efficiency, modulation and demodulation's implementation by discrete Fourier transform (DFT) and inverse discrete Fourier transform (IDFT), immunity to multipath environments [1]. However, the high peak to average power ratio (PAPR) of the transmitted signal is one of the primary flaws of the OFDM systems. The high PAPR makes OFDM to be very sensitive to nonlinear effects touched off by transmitter power amplifier. When the high PAPR OFDM signal is passed through a power amplifier followed a limited linearity, it will give rise to a significant loss of power efficiency, severe in-band distortion and out-of-band noise.

Plenty of research works carried out for the issue, and many methods and techniques have been put forward to decrease the PAPR of the OFDM signal. These methods can basically be divided into three categories, i.e. coding, signal scrambling, and signal distortion [2]. The work of [3] discusses in detail the PAPR reduction techniques.

Coding technologies [4], such as complement block code (CBC) [5], low density parity-check (LDPC) code [6], etc, provide better PAPR reduction, but the scheme's complexity is too high to be applied in practice.

Signal scrambling scheme is based on improving signal's statistical properties to reduce the OFDM signal's PAPR. The representative schemes are selected mapping (SLM) [7], [8] and partial transmit sequence (PTS) [9], [10]. The PTS technology is discussed in detail in the literature [11]. The two methods can achieve better PAPR reduction, but getting an optimal solution needs to solve a combination optimization. Hence the high computational complexity restricts their practical applications. Another flaw of the two methods is to send side information to receiver, which reduces data throughput.

Signal distortion technology has attracted much attention because it can be directly embedded into OFDM system and does not need to change the mechanism of transceiver. Clipping OFDM signals under a given PAPR is the easiest scheme [12], which degrades the whole system's

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bit-error-rate (BER) and leads to severe in-band distortion and great out-of-band radiation. In order to lower out-of-band radiation, clipping and filtering (CF) methods is presented [13]. While lowering out-of-band radiation, the CF method also results in peak regrowth. So iterative CF (ICF) scheme is proposed to reduce peak regrowth [14]. The common disadvantage of all CF or ICF schemes is that they are unable to diminish in-band distortion, thus increasing the whole system's BER.

Tone reservation (TR) approach presented in [15] is an undistorted technique. However, achieving optimized peak-cancelling signals in the TR technique require to tackle a quadratically constrained quadratic program, that is a very high computational burden. To avoid this flaw, TR algorithm of integrating ICF technology was proposed in [16]. The method first clips time domain OFDM signal under a predefined clipping level, and then filters the clipped signal making clipping noise only on protected subcarriers. However, the performance of this scheme is difficult to meet the actual requirements. In order to get a satisfied PAPR reduction, these schemes require very large iterations. To accelerate the clipping-filtering technique's convergence, the adaptive-scaling (AS) TR method with a predefined threshold was also presented in [16], [17]. But the AS algorithm has two major flaws, i.e. 1) pre-selecting the optimal clipping threshold is very hard work, because many factors, including subcarrier number, position and size of reserved tones are closely relative to the optimal target clipping threshold. 2) different clipping threshold may lead to different PAPR decrease performance. To conquer two major flaws of the AS algorithm, an adaptive amplitude clipping (AAC) TR method [18] and adaptive iterative clipping and filtering (AICF) method [19] were put forward to reduce PAPR. Regardless of the initial clipping level, and different clipping levels almost gain same PAPR reduction.

Recently, signal distortion schemes based on convex optimization theory [20] have been widely used in PAPR reduction of OFDM signals. An error vector magnitude (EVM) optimization using second order conic programming (SOCP) [21], [22] and semi-definite programming (SDP) [23] was tackled. The SOCP scheme were also used to minimized the PAPR of OFDM signals with EVM constraints [24]. But SOCP and SDP methods have high computational complexity, so the two schemes can not apply in practical systems. A simplified ICF (SICF) approach was proposed to deal with the EVM optimization [25]. Fast iterative shrinkage-thresholding algorithm (FISTA) [26] and alterative direction method of multipliers (ADMM) [27] were presented to tackle the optimal OFDM signals with IFFT/FFT complexity. In this paper, we reconsider to minimized the PAPR of OFDM signals with EVM constraints. We focus on developing a low complexity algorithm to tackle the optimization. The contributions of the paper are as follows:

(1), Although the similar question had been solved by SOCP method in [24], high complexity of the SOCP method restricts its application. Here, we tackle the issue and provide a simple and efficient solution with a lower complexity.

(2), we develop an innovative approach based on linearized alterative direction method of multipliers (LADMM) [28]–[30] with low complexity to proceed the optimization. Firstly, we apply the proximal operator to tackle subproblems, and then supply the closed-form solution of each subproblem of the LADMM algorithm which is different from the ADMM in [27]. Furthermore, we provide a theoretical proof to ensure the convergence of the presented LADMM algorithm and analyze its complexity on every iteration.

Simulation results indicate that proposed LADMM method not only acquires larger PAPR reduction, but also obtains better bit error ratio (BER) performance compared with those existing technologies.

This paper is organized as follows. In Section II, the OFDM system model is introduced and the corresponding optimization is described. The LADMM is described in Section III. In Section IV, The LADMM's convergence is proposed. The performances of LADMM are evaluated by computer simulations in Section V. Finally, conclusions are drawn in Section VI.

Notations: in the following, $\|\cdot\|_2$ and $\|\cdot\|_\infty$ stand for a vector's Euclidean and ℓ_∞ norm, respectively. $E[\cdot]$ represents a random variable's expectation. $<\bullet>$ denotes the inner product of two real vectors. $(\cdot)^T$ and $(\cdot)^H$ represent the transpose, the conjugate transpose of a vector or a matrix, respectively.

II. SYSTEM MODEL

Let us take into account an OFDM system with N subcarriers. By carried out IDFT to the data signals $\boldsymbol{X} = [X_0, \ldots, X_{N-1}]^T$, we achieve the discrete-time baseband OFDM signal $\boldsymbol{x} = [x_0, \ldots, x_{JN-1}]^T$ with oversampling factor J, that is

$$\boldsymbol{x} = \boldsymbol{F}\boldsymbol{X}, \tag{1}$$

where $\boldsymbol{F}$ is a $JN \times N$ IDFT matrix, whose (m, j)-th entry is $F_{m,j} = \frac{1}{\sqrt{JN}} e^{\frac{j2\pi mj}{JN}}$.

The PAPR of an OFDM signal $\boldsymbol{x}$ is defined as the ratio of maximal power to average one,

$$\text{PAPR}(\boldsymbol{x}) = \frac{\|\boldsymbol{x}\|_\infty^2}{E[\|\boldsymbol{x}\|_2^2]}. \tag{2}$$

where $\|\cdot\|_2$ and $\|\cdot\|_\infty$ stand for a vector's Euclidean and ℓ_∞ norm, respectively. $E[\cdot]$ represents a random variable's expectation.

Error vector magnitude (EVM) is adopted to measure the in-band distortion of an OFDM signal, which is denoted as

$$\text{EVM} = \frac{\|\boldsymbol{F}^H\boldsymbol{x} - \boldsymbol{X}_0\|_2}{\|\boldsymbol{X}_0\|_2} \tag{3}$$

where X_0 represents the ideal data signal. In order to minimize in-band distortion and obtain a satisfactory PAPR, An EVM optimization with PAPR constraint was addressed by SOCP method in [21], [22]. To decrease the SOCP computational complexity, a simplified ICF (SICF) scheme was also presented in [25]. However, most modern digital communication standards have a predefined EVM threshold. So we consider the following optimization.

$$\min_{x\in\mathbb{C}^{JN}} \|x\|_\infty^2 \quad \text{s.t. EVM} \le \epsilon \tag{4}$$

Using (3), the EVM constraint on x can be represented as follows.

$$\min_{x\in\mathbb{C}^{JN}} \|x\|_\infty^2 \quad \text{s.t. } \|F^H x - X_0\|_2 \le \epsilon\|X_0\|_2 \tag{5}$$

where ϵ is an EVM threshold. If the transmitted OFDM signal meets the EVM constraint, then the received signal has an acceptable BER after decoding [24].

We have the following comments on the optimization (5).

(1), Because directly minimizing PAPR will result in a complicated non-convex optimization, we replace the PAPR with $\|x\|_\infty^2$, rather than $\|x\|_\infty$ in [26] as the objective function in (5). So we can get a convex optimization and achieve a globally optimal solution. Another reason for this substitution is that we can achieve an exact solution of the proximal operator of $\|x\|_\infty^2$. Solving the proximal operator of $\|x\|_\infty$ in [26] is only an approximate solution. (The definition of the proximal operator will be given in Section III).

(2), Low complexity algorithms, such as FISTA [26] and ADMM [27] can not deal with (5) directly. We develop the LADMM algorithm to tackle the optimization (5) in the next section. We provide the closed-form solution of each subproblem in the LADMM algorithm, then prove the proposed LADMM algorithm is convergent and analyze its complexity on every iteration is about $\mathcal{O}(JN\log(JN))$.

III. LINEARIZED ACCELERATE PROXIMAL GRADIENT METHOD

A. ALTERATIVE DIRECTION METHOD OF MULTIPLIERS

Linearized alterative direction method of multipliers (LADMM) comes from alterative direction method of multipliers (ADMM) which was discussed in [31], [32]. The core of ADMM algorithm is that it can decompose complex problems into simple sub-problems, and each sub-problem is easy to solve or has a closed-form solution. In order to utilize ADMM, we introduce an auxiliary variable y, so the (5) can be reformulated as

$$\min_{x\in\mathbb{C}^{JN}, y\in\mathbb{C}^N} \|x\|_\infty^2 \quad \text{s.t. } \|y\|_2 \le \epsilon\|X_0\|_2,\; F^H x - X_0 = y \tag{6}$$

The augmented Lagrangian function of (6) is:

$$L_\mu(x, y, u) = \|x\|_\infty^2 - \Re(u^H(F^H x - X_0 - y)) + (\mu/2)\|F^H x - X_0 - y\|_2^2 \tag{7}$$

where $u \in \mathbb{C}^N$ is the Lagrangian multiplier and $\mu > 0$ is a penalty parameter.

The resulting ADMM is the the following

$$x^{i+1} = \arg\min_x L_\mu(x, y^i, u^i), \tag{8a}$$
$$y^{i+1} = \arg\min_{y\in\mathcal{M}} L_\mu(x^{i+1}, y, u^i), \tag{8b}$$
$$u^{i+1} = u^i - \mu(F^H x^{i+1} - X_0 - y^{i+1}). \tag{8c}$$

where $\mathcal{M} = \{y : \|y\|_2 \le \epsilon\|X_0\|_2\}$.

B. SOLVING SUBPROBLEM (8A)

Based on (7), subproblem (8a) is equivalent to solve

$$x^{i+1} = \arg\min_x \|x\|_\infty^2 + (\mu/2)\|F^H x - X_0 - y^i - \frac{u^i}{\mu}\|_2^2 \tag{9}$$

Although the subproblem (9) can be solved by the fist-order algorithm such as FISTA or ADMM, this will lead to a double loop algorithm for tackling the optimization (5), i.e. the outer layer is the ADMM algorithm and the inner layer is the ADMM or the FISTA algorithm. It is evident that the double loop algorithm will lead to a high computational complexity. Here, our goal is to find an algorithm with low complexity and each subproblem has a a closed-form solution. Because $\|x\|_\infty^2$ is non-smooth and FF^H is not an identity matrix, so the closed-form solution of (9) is not available for the original ADMM. Generally speaking, it is not necessary to exactly solve the original subproblem (9) under the condition of ensuring the convergence of iterations (8). So we try to simplify the subproblem as much as possible to achieve a closed-form solution of the subproblem (9). Motivated by the objective, we propose to approximate the subproblem (9) by linearizing the quadratic term in (9), which is also widely used to deal with the constrained linear least-squares problem [28], nuclear norm minimization [29] and Dantzig selector [30]. More specifically, we have

$$\begin{aligned}\frac{1}{2}\|F^H x - X_0 - y^i - \frac{u^i}{\mu}\|_2^2 &\approx \frac{1}{2}\|F^H x^i - X_0 - y^i - \frac{u^i}{\mu}\|_2^2 \\ &\quad + \Re(g_i^H(x - x^i)) \\ &\quad + \frac{1}{2\lambda}\|x - x^i\|_2^2\end{aligned} \tag{10}$$

where $\lambda > 0$ is a proximal parameter (satisfying the condition $0 < \lambda < \rho(FF^H)$, and ρ expresses the spectral radius of a matrix, that will be explained in Lemma 4.3 of Section IV) and

$$g_i = F(F^H x^i - X_0 - y^i - \frac{u^i}{\mu}) \tag{11}$$

is the gradient of $\frac{1}{2}\|\boldsymbol{F}^H\boldsymbol{x} - \boldsymbol{X}_0 - \boldsymbol{y}^i - \frac{\boldsymbol{u}^i}{\mu}\|_2^2$ at $\boldsymbol{x} = \boldsymbol{x}^i$. Plugging (10) into (9) and ignoring the first item $\frac{1}{2}\|\boldsymbol{F}^H\boldsymbol{x}^i - \boldsymbol{X}_0 - \boldsymbol{y}^i - \frac{\boldsymbol{u}^i}{\mu}\|_2^2$ on the right of (10), we acquire the following approximation to (9)

$$\begin{aligned} \boldsymbol{x}^{i+1} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_\infty^2 + \frac{\mu}{2\lambda}\|\boldsymbol{x} - (\boldsymbol{x}^i - \lambda \boldsymbol{g}_i)\|_2^2 \\ &= prox_{\|\boldsymbol{x}\|_\infty^2}(\boldsymbol{x}^i - \lambda \boldsymbol{g}_i, \frac{\lambda}{\mu}) \end{aligned} \tag{12}$$

where the proximal operator $prox_f : \mathbf{R}^m \to \mathbf{R}^m$ of a function f with parameter τ is denoted as [32]

$$prox_f(\boldsymbol{v}, \tau) = \arg\min_{\boldsymbol{x}}(f(\boldsymbol{x}) + 1/(2\tau)\|\boldsymbol{x} - \boldsymbol{v}\|_2^2). \tag{13}$$

Let $f(\boldsymbol{x}) = \|\boldsymbol{x}\|_\infty^2$, $\boldsymbol{v} = \boldsymbol{x}^i - \lambda\boldsymbol{g}_i$. By modifying the scheme in [33], we can achieve an exact solution of the proximal operator of $f(\boldsymbol{x})$.

Specially, let $\boldsymbol{c}$ be a rearrangement of $|\boldsymbol{v}|$ in descending order, i.e. $c_{\pi(1)} \geq c_{\pi(2)} \geq c_{\pi(3)} \geq \cdots \geq c_{\pi(i)} \geq \cdots \geq c_{\pi(JN)}$. Let $s_i = \sum_{k=1}^{i} c_{\pi(k)}$ for $i = 1, 2, \ldots, JN$, and $j = \max\{i : c_{\pi(i)} \geq \frac{s_i}{i+2\tau}\}$. Set $\beta = \frac{s_j}{j+2\tau}$, we obtain the proximal operator of $f(\boldsymbol{x})$ by the following formula

$$prox_f(\boldsymbol{v}, \tau) = \begin{cases} \beta \dfrac{\boldsymbol{v}_{\pi(i)}}{|\boldsymbol{v}_{\pi(i)}|}, & \text{if } i \leq j, \\ \boldsymbol{v}_{\pi(i)}, & \text{otherwise.} \end{cases} \tag{14}$$

C. SOLVING SUBPROBLEM (8B)

Based on (7), subproblem (8*b*) is equivalent to solve

$$\begin{aligned} \boldsymbol{y}^{i+1} &= \arg\min_{\boldsymbol{y}\in\mathcal{M}} L_\mu(\boldsymbol{x}^{i+1}, \boldsymbol{y}, \boldsymbol{u}^i) \\ &= \arg\min_{\boldsymbol{y}\in\mathcal{M}} (\mu/2)\|\boldsymbol{F}^H\boldsymbol{x}^{i+1} - \boldsymbol{X}_0 - \boldsymbol{y} - \frac{\boldsymbol{u}^i}{\mu}\|_2^2 \end{aligned} \tag{15}$$

Let $\omega^i = \boldsymbol{F}^H\boldsymbol{x}^{i+1} - \boldsymbol{X}_0 - \frac{\boldsymbol{u}^i}{\mu}$, we can achieve the solution of (15) as follows.

$$\boldsymbol{y}^{i+1} = \begin{cases} \omega^i, & \|\omega^i\|_2 \leq \epsilon\|\boldsymbol{X}_0\|_2, \\ \dfrac{\epsilon\|\boldsymbol{X}_0\|_2}{\|\omega^i\|_2}\omega^i, & \text{otherwise.} \end{cases} \tag{16}$$

Through the above derivation and analysis, we can find that the main difference between LADMM and ADMM is the solution of subproblem (9). By linearizing the quadratic term of (9) and using the proximal operator, we obtain the closed-form solution of (9), which is not available for the ADMM method [27]. So we call the algorithm linearized ADMM (LADMM) in order to distinguish it from the original ADMM. The presented LADMM algorithm is summed up in Algorithm 1.

D. COMPLEXITY ANALYSIS OF LADMM

We use the order of complexity of an algorithm to evaluate the complexity of this algorithm, which is also used in [25], [27], [31]–[33]. In Algorithm 1, the computational burden of each iteration is very low. In Step 2.1, we first need to compute $\boldsymbol{g}_i$ by (11). Because the matrix $\boldsymbol{F}$ and $\boldsymbol{F}^H$ are Fourier-type operators, so evaluating $\boldsymbol{g}_i$ does not need to carry out matrix-vector products, it can be efficiently computed by FFT and IFFT with complexity $\mathcal{O}(JN\log(JN))$. For the proximal operation in step 2.2, we acquire to calculate $|\boldsymbol{v}|$ and perform sort operations, which have an $\mathcal{O}(JN)$ and $\mathcal{O}(JN\log(JN))$ computational complexity, respectively. Evaluating j and β acquire $\mathcal{O}(JN)$ complexity, respectively. Similarly, calculating the the proximal operation (14) also needs $\mathcal{O}(JN)$ complexity. So the computational complexity of evaluating $\boldsymbol{x}^{i+1}$ is about $\mathcal{O}(JN\log(JN))$ complex multiplications. In Step 2.3, we first calculate ω^i which need to perform an FFT operation. so it needs $\mathcal{O}(JN\log(JN))$ complex multiplications. And then computing $\boldsymbol{y}^{i+1}$ which has $\mathcal{O}(JN)$ complex multiplications. Finally, in step 2.4, because $\boldsymbol{F}^H\boldsymbol{x}^{i+1}$ has been achieved in Step 2.3, updating the Lagrangian multiplier $\boldsymbol{u}^{i+1}$ only takes JN complex multiplications. A few complex vector additions or subtractions is negligible. By the above analysis, the whole complexity of Algorithm 1 is about $\mathcal{O}(JN\log(JN))$. Compared with the SOCP with complexity $\mathcal{O}(N^3)$, the proposed algorithm has lower computational cost and is a simple and efficient method with the computational complexity $\mathcal{O}(JN\log(JN))$.

Algorithm 1 LADMM Algorithm

1: Input $\boldsymbol{X}_0$, parameters $\mu > 0, \epsilon > 0, 0 < \lambda < \rho(\boldsymbol{F}\boldsymbol{F}^H)$, and initial solutions $\boldsymbol{x}^0, \boldsymbol{y}^0, \boldsymbol{u}^0$, $i = 0$.
2: While stopping condition is not satisfied
Step 2.1 Compute $\boldsymbol{g}_i$ by (11),
Step 2.2 Compute $\boldsymbol{x}^{i+1}$ by (12) and (14),
Step 2.3 Compute $\boldsymbol{y}^{i+1}$ by (16),
Step 2.4 Update $\boldsymbol{u}^{i+1}$ by (8c),
Step 2.5 Update $i \leftarrow i + 1$.
3: **Output** $\boldsymbol{x}$.

E. CONNECTIONS TO FISTA AND ADMM

Although FISTA and ADMM can not tackle (5) directly, the two methods solve the unconstrained alternative of (5):

$$\min_{\boldsymbol{x}\in\mathbb{C}^{JN}} \|\boldsymbol{x}\|_\infty + \frac{1}{2\alpha}\|\boldsymbol{F}^H\boldsymbol{x} - \boldsymbol{X}_0\|_2^2 \tag{17}$$

where $\alpha > 0$ is a regularized parameter.

Let $r(\boldsymbol{x}) = \frac{1}{2\alpha}\|\boldsymbol{F}^H\boldsymbol{x} - \boldsymbol{X}_0\|_2^2$ and $q(z) = \|z\|_\infty$. The iterations of the FISTA consist of the following steps:

$$\begin{aligned} \boldsymbol{\theta} &= \boldsymbol{y}_k - \frac{1}{\alpha L}\boldsymbol{F}(\boldsymbol{F}^H\boldsymbol{y}_k - \boldsymbol{X}_0) \\ \boldsymbol{x}_k &= P_L(\boldsymbol{y}_k), \\ t_{k+1} &= \frac{1 + \sqrt{1 + 4t_k^2}}{2} \\ \boldsymbol{y}_{k+1} &= \boldsymbol{x}_k + \frac{t_k - 1}{t_{k+1}}(\boldsymbol{x}_k - \boldsymbol{x}_{k-1}) \end{aligned} \tag{18}$$

where L is the smallest Lipschitz constant for the gradient $\nabla r(\boldsymbol{x})$ of the function $r(\boldsymbol{x})$ and the proximal map

$$P_L(\boldsymbol{y}) = \arg\min_z \left\{ \|z\|_\infty + \frac{L}{2}\|z - (\boldsymbol{y} - \frac{1}{\alpha L}\boldsymbol{F}(\boldsymbol{F}^H y - \boldsymbol{X}_0)\|_2^2 \right\}$$

For ADMM, the augmented Lagrangian associated with the problem (17) is

$$L_\rho(\boldsymbol{x}, z, \boldsymbol{c}) = r(\boldsymbol{x}) + q(z) - \Re(\boldsymbol{c}^H(\boldsymbol{x}-z)) + (\rho/2)\|\boldsymbol{x}-z\|_2^2 \quad (19)$$

where $\boldsymbol{c} \in \mathbb{C}^{JN}$ is the Lagrangian multiplier and $\rho > 0$ is a penalty parameter. The ADMM can be expressed as

$$\begin{aligned} \boldsymbol{x}^{i+1} &= \arg\min_{\boldsymbol{x}} L_\rho(\boldsymbol{x}, z^i, \boldsymbol{c}^i), \\ z^{i+1} &= \arg\min_{z} L_\rho(\boldsymbol{x}^{i+1}, z, \boldsymbol{c}^i), \\ \boldsymbol{c}^{i+1} &= \boldsymbol{c}^i - \rho(\boldsymbol{x}^{i+1} - z^{i+1}). \end{aligned} \quad (20)$$

The corresponding iterations of the ADMM include the following steps:

$$\begin{aligned} \boldsymbol{x}^{i+1} &= (I - \frac{1}{1+\alpha\rho}\boldsymbol{F}\boldsymbol{F}^H)(\alpha^{-1}\boldsymbol{F}X_0 + \rho z^i + \boldsymbol{c}^i), \\ z^{i+1} &= prox_q(\boldsymbol{x}^{i+1} - \frac{\boldsymbol{c}^i}{\rho}, \frac{1}{\rho}), \\ \boldsymbol{c}^{i+1} &= \boldsymbol{c}^i - \rho(\boldsymbol{x}^{i+1} - z^{i+1}). \end{aligned} \quad (21)$$

Based on (18), the main computational cost of the FISTA algorithm consists of two parts. Firstly, evaluating $\boldsymbol{\theta}$ requires a pair of FFT/IFFT with roughly computational complexity $\mathcal{O}(JN\log(JN))$. The computational complexity of calculating the proximal map $P_L(\boldsymbol{y})$ is about $\mathcal{O}(JN\log(JN))$. So the overall complexity of the FISTA algorithm is about $\mathcal{O}(JN\log(JN))$. According to (21), the computational complexity of the ADMM algorithm is dominant by $\boldsymbol{x}^{i+1}$, which needs two IFFT and an FFT with complexity $\mathcal{O}(JN\log(JN))$. While calculating z^{i+1} also requires $\mathcal{O}(JN\log(JN))$ complex multiplications. Hence, the proposed LADMM-based PAPR reduction algorithm have the same computational complexity as the FISTA and ADMM i.e. $\mathcal{O}(JN\log(JN))$. But the LADMM algorithm has two advantages. Firstly, the algorithm 1 provides a closed solution for the proximal operator of the function $\|\boldsymbol{x}\|_\infty^2$, but the FISTA [26] gives only a truncated solution for the proximal operator of the function $\|\boldsymbol{x}\|_\infty$. Moreover, the FISTA algorithm in [26] also requires to calculate the smallest Lipschitz constant L of the gradient $\nabla r(\boldsymbol{x})$, which has a very large computational cost. Secondly, the LADMM algorithm can deal with more complex constrained optimization problems which is not easy to solve for the ADMM and FISTA algorithms.

IV. ANALYSIS OF GLOBAL CONVERGENCE

We provide the global convergence of the LADMM method in the subsection by referring to the methods of [28]–[30]. Because standard theory of convex optimization operate on real numbers and the complex numbers are used in (6), we expand every vector and matrix to a real-imaginary form which will be denoted in boldface. As an example, the vector $\boldsymbol{x} \in \mathbb{C}^{JN}$ has the following expanded form $x \in \Re^{2JN}$, where

$$x = [\Re x_1, \Im x_1, \Re x_2, \Im x_2, \cdots, \Re x_{JN}, \Im x_{JN}]^T \quad (22)$$

$\mathcal{M} = \{\boldsymbol{y} : \|\boldsymbol{y}\|_2 \leq \epsilon\|\boldsymbol{X}_0\|_2\}$ has the following expanded form $\mathbb{M} = \{y : \|y\|_2 \leq \epsilon\|X_0\|_2\}$. The matrix $\boldsymbol{F}$ has the following expanded form $F \in \Re^{2JN\times 2N}$, where every element $F_{m,j}$ of $\boldsymbol{F}$ is replaced by the following 2×2 block

$$\begin{bmatrix} \Re F_{m,j}, & -\Im F_{m,j} \\ \Im F_{m,j}, & \Re F_{m,j} \end{bmatrix}. \quad (23)$$

Using theory of convex optimization, solving (6) is equivalent to finding$(x^*, y^*, u^*) \in \Psi := \mathbb{R}^{2JN} \times \mathbb{M} \times \mathbb{R}^{2N}$ and $h(x^*) \in \partial(\|x^*\|_\infty^2)$ such that the following condition are satisfied:

$$\begin{cases} h(x^*) - Fu^* = 0, \\ < y' - y^*, u^* > \geq 0, \quad \forall y' \in \mathbb{M}, \\ F^T x^* - X_0 - y^* = 0. \end{cases} \quad (24)$$

where ∂ represents the subdifferential operator of a non-smooth convex function.

Let Ψ^* be the set of elements satisfying conditions (24) in Ψ, $\boldsymbol{\xi}^* = (x^*, y^*, u^*) \in \Psi^*$ and $h(x) \in \partial(\|x\|_\infty^2)$. Then solving (24) corresponds to finding $\boldsymbol{\xi}^* \in \Psi^*$ and $h(x^*) \in \partial(\|x^*\|_\infty^2)$, such that

$$< \boldsymbol{\xi} - \boldsymbol{\xi}^*, A(\boldsymbol{\xi}^*) > \geq 0, \quad \forall \boldsymbol{\xi} \in \Psi \quad (25)$$

where $\boldsymbol{\xi} = (x, y, u)$ and

$$A(\boldsymbol{\xi}) = \begin{pmatrix} h(x) - Fu \\ u \\ F^T x - X_0 - y \end{pmatrix}. \quad (26)$$

In order to set up the LADMM's global convergence, we need to prove the following lemma.

Lemma 1: Let the matrices $H = \begin{bmatrix} -\mu F \\ \mu I \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} \frac{\mu}{\lambda}I - \mu FF^T & 0 & 0 \\ 0 & \mu I & 0 \\ 0 & 0 & \frac{1}{\mu}I \end{bmatrix}$ then the sequence $\boldsymbol{\xi}^i$ generated by the Algorithm 1 satisfies the following inequality:

$$< \boldsymbol{\xi} - \boldsymbol{\xi}^{i+1}, A(\boldsymbol{\xi}^{i+1}) - B(\boldsymbol{\xi}^i - \boldsymbol{\xi}^{i+1}) + H(y^i - y^{i+1}) > \geq 0, \quad \forall \boldsymbol{\xi} \in \Psi. \quad (27)$$

Proof: Based on the iteration (12), (16) and (8c), we obtain $\boldsymbol{\xi}^{i+1} \in \Psi$, $h(x^{i+1}) \in \partial(\|x^{i+1}\|_\infty^2)$, and

$$\begin{cases} (x - x^{i+1})^T[h(x^{i+1}) + \mu g_i + \frac{\mu}{\lambda}(x^{i+1} - x^i)] \geq 0, \\ (y - y^{i+1})^T u^{i+1} \geq 0, \\ (u - u^{i+1})^T[(F^T x^{i+1} - y^{i+1} - X_0) - \frac{1}{\mu}(u^i - u^{i+1})] \geq 0. \end{cases} \quad (28)$$

Using (8c) and the expanded form of (11), we can obtain the first inequality of (28) as follows.

$$\begin{aligned} h(x^{i+1}) &+ \mu g_i + \frac{\mu}{\lambda}(x^{i+1} - x^i) \\ &= h(x^{i+1}) - Fu^{i+1} - \mu F(y^i - y^{i+1}) \\ &\quad + \mu(FF^T - \frac{1}{\lambda}I)(x^i - x^{i+1}) \end{aligned} \quad (29)$$

By the definitions of $A(\boldsymbol{\xi})$ in (26) and the matrices B, H, combining (28) and (29) will result in (27) immediately. ∎

Based on Lemma 4.1, we have the following lemma.

Lemma 2: let ξ^i be the sequence generated by the Algorithm 1 and $\xi^* \in \Psi^*$, the matrix B is defined by Lemma 4.1, then we have

$$< \xi^i - \xi^*, B(\xi^i - \xi^{i+1}) > \geq < \xi^i - \xi^{i+1}, B(\xi^i - \xi^{i+1}) > - < u^i - u^{i+1}, y^i - y^{i+1} > \quad (30)$$

Proof: Let $\xi = \xi^*$ in (27), we obtain

$$< \xi^* - \xi^{i+1}, A(\xi^{i+1}) - B(\xi^i - \xi^{i+1}) + H(y^i - y^{i+1}) > \geq 0. \quad (31)$$

By the equality $F^T x - X_0 = y$, (31) is equivalent to

$$< \xi^{i+1} - \xi^*, B(\xi^i - \xi^{i+1}) > \geq < A(\xi^{i+1}), \xi^{i+1} - \xi^* > - \mu < F^T x^{i+1} - y^{i+1} - X_0, y^i - y^{i+1} > . \quad (32)$$

Noting (8c), we have $F^T x^{i+1} - y^{i+1} - X_0 = \frac{1}{\mu}(u^i - u^{i+1})$. Plugging it into (32), we achieve

$$< \xi^{i+1} - \xi^*, B(\xi^i - \xi^{i+1}) > \geq < A(\xi^{i+1}), \xi^{i+1} - \xi^* > - < u^i - u^{i+1}, y^i - y^{i+1} > . \quad (33)$$

On the other hand, we can achieve

$$\begin{aligned} &< A(\xi^{i+1}) - A(\xi^*), \xi^{i+1} - \xi^* > \\ &= < \begin{pmatrix} h(x^{i+1}) - Fu^{i+1} - h(x^*) + Fu^* \\ u^{i+1} - u^* \\ F^T x^{i+1} - y^{i+1} - F^T x^* + y^* \end{pmatrix} \begin{pmatrix} x^{i+1} - x^* \\ y^{i+1} - y^* \\ u^{i+1} - u^* \end{pmatrix} > \\ &= (x^{i+1} - x^i)^T \left(h(x^{i+1}) - h(x^*) \right) \geq 0 \end{aligned} \quad (34)$$

where the final inequality utilizes the monotonicity of the subdifferential operator of the convex function $\|x\|_\infty^2$. Since ξ^* is a solution of (24), we have by means of (25) and (34)

$$< A(\xi^{i+1}), \xi^{i+1} - \xi^* > \geq < A(\xi^*), \xi^{i+1} - \xi^* > \geq 0 \quad (35)$$

Plugging (35) into (33), we can achieve

$$< \xi^{i+1} - \xi^*, B(\xi^i - \xi^{i+1}) > \geq - < u^i - u^{i+1}, y^i - y^{i+1} > . \quad (36)$$

Using the identity $\xi^{i+1} - \xi^* = (\xi^{i+1} - \xi^i) + (\xi^i - \xi^*)$, we obtain (30) immediately. ■

For $\forall \xi, \zeta \in \Psi$, we define an inner product by the following formula

$$< \xi, \zeta >_B = \xi^T B \zeta \quad (37)$$

where the matrix B is denoted in Lemma 4.1 and the induced norm by $\|\xi\|_B^2 = < \xi, \xi >_B$. According to the definition of induced norm, we must guarantee that matrix **B** is positive definite, so we restrict $0 < \lambda < 1/\rho(FF^T)$, where $\rho(FF^T)$ represents the spectral radius of FF^T. Then we have the following lemma.

Lemma 3: let ξ^i be the sequence generated by the Algorithm 1, if $0 < \lambda < 1/\rho(FF^T)$, then we have

(1) $\lim_{i \to \infty} \|\xi^i - \xi^{i+1}\|_B = 0$.

(2) The sequence $\{\xi^i\}$ is bounded.

(3) For $\forall \xi^* \in \Psi^*$, the sequence $\|\xi^i - \xi^*\|_B$ is monotonically non-increasing and thus converges.

Proof: By (37), (30) can be rewritten as

$$< \xi^i - \xi^*, \xi^i - \xi^{i+1} >_B \geq \|\xi^i - \xi^{i+1}\|_B^2 - < u^i - u^{i+1}, y^i - y^{i+1} > \quad (38)$$

By the identity $\xi^{i+1} = \xi^i - (\xi^i - \xi^{i+1})$ and (30), (38), we have

$$\begin{aligned} &\|\xi^i - \xi^*\|_B^2 - \|\xi^{i+1} - \xi^*\|_B^2 \\ &= 2 < \xi^i - \xi^*, \xi^i - \xi^{i+1} >_B - \|\xi^i - \xi^{i+1}\|_B^2 \\ &\geq \|\xi^i - \xi^{i+1}\|_B^2 - 2 < u^i - u^{i+1}, y^i - y^{i+1} > \end{aligned} \quad (39)$$

Recalling the second inequality of (28), $\forall y \in \mathbb{M}$, we have

$$\begin{cases} (y - y^{i+1})^T u^{i+1} \geq 0, \\ (y - y^i)^T u^i \geq 0. \end{cases} \quad (40)$$

Substituting y in the above first and the second inequality with $y^i$and y^{i+1}, respectively, we obtain

$$\begin{cases} (y^i - y^{i+1})^T u^{i+1} \geq 0, \\ (y^{i+1} - y^i)^T u^i \geq 0. \end{cases} \quad (41)$$

Adding up the above two inequalities, we have

$$< u^i - u^{i+1}, y^i - y^{i+1} > \leq 0 \quad (42)$$

Combining (39), we get

$$\|\xi^i - \xi^*\|_B^2 - \|\xi^{i+1} - \xi^*\|_B^2 \geq \|\xi^i - \xi^{i+1}\|_B^2. \quad (43)$$

By the above inequality (43), the statements of the lemma follow immediately. ■

Now, we prove the global convergence of Algorithm 1.

Theorem 1: For any $\mu > 0$ and an arbitrary initial point $\xi^0 = (x^0, y^0, u^0)$, the sequence $\xi^i = (x^i, y^i, u^i)$ generated by Algorithm 1 with $0 < \lambda < 1/\rho(FF^T)$ converges to $\xi^* = (x^*, y^*, u^*)$, where (x^*, y^*) is a solution of (6).

Proof: It can be drawn from (1) of Lemma 4.3 that

$$\begin{cases} \lim_{i \to \infty} \|x^i - x^{i+1}\| = 0, \\ \lim_{i \to \infty} \|y^i - y^{i+1}\| = 0, \\ \lim_{i \to \infty} \|u^i - u^{i+1}\| = 0. \end{cases} \quad (44)$$

Based on the (2) of Lemma 4.3, $\xi^i = (x^i, y^i, u^i)$ has at least a limit point that is denoted by $\xi^\infty = (x^\infty, y^\infty, u^\infty)$. Let ξ^{i_k} be a subsequence converging to ξ^∞, i.e.

$$\begin{cases} \lim_{k \to \infty} \|x^{i_k} - x^{i_k+1}\| = 0, \\ \lim_{k \to \infty} \|y^{i_k} - y^{i_k+1}\| = 0, \\ \lim_{k \to \infty} \|u^{i_k} - u^{i_k+1}\| = 0. \end{cases} \quad (45)$$

Next, we demonstrate that ξ^∞ satisfies the optimality condition (25). In fact, based on (45) and (27), we can draw that

$$\lim_{k \to \infty} < \xi - \xi^{i_k+1}, A(\xi^{i_k+1}) > \geq 0, \forall \xi \in \Psi \quad (46)$$

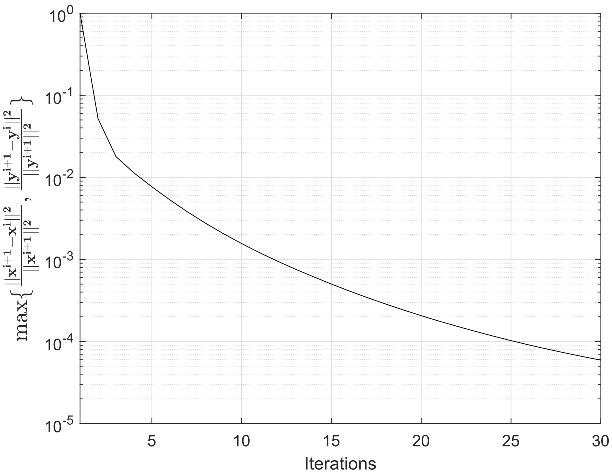

FIGURE 1. The convergence performance of the LADMM algorithm.

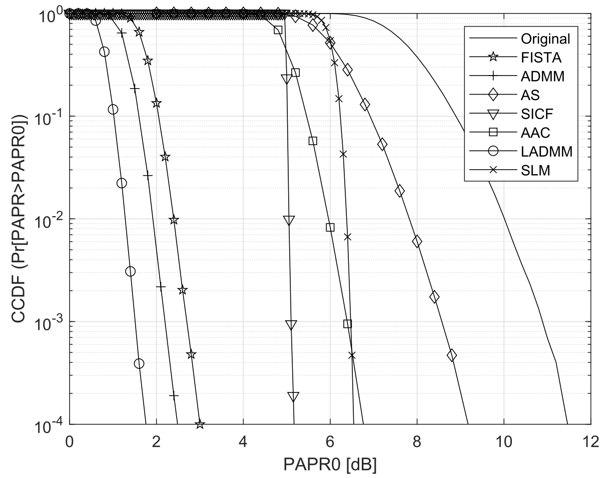

FIGURE 2. PAPR comparison of different schemes for 16-QAM modulation.

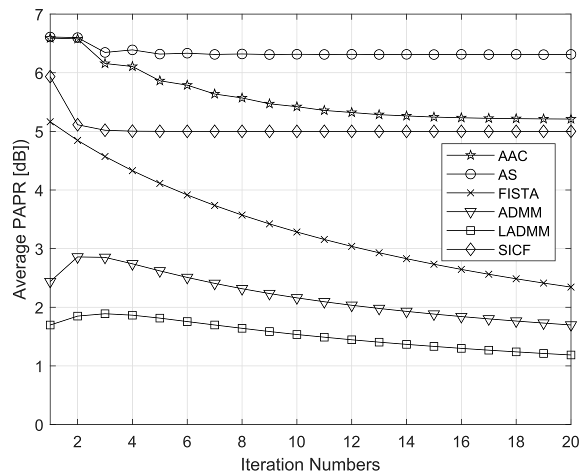

FIGURE 3. Average PAPR reduction versus iterations.

So any limit point of $\boldsymbol{\xi}^{i_k}$ is a solution of (25), i.e. $\boldsymbol{\xi}^{\infty}$ satisfies the optimality condition (25).

On the other hand, combing (3) of lemma4.3 and (43), it can be derived that

$$\|\boldsymbol{\xi}^i - \boldsymbol{\xi}^{\infty}\|_B^2 \geq \|\boldsymbol{\xi}^{i+1} - \boldsymbol{\xi}^{\infty}\|_B^2, \forall k \geq 0 \tag{47}$$

The fact implies that $\boldsymbol{\xi}^{\infty}$ is the only limit point of the sequence $\boldsymbol{\xi}^i$, and so $\boldsymbol{\xi}^i$ converges to $\boldsymbol{\xi}^{\infty}$. Hence (x^{∞}, y^{∞}) is a solution of (6) . ■

V. SIMULATION RESULTS

The computer simulations of the presented LADMM algorithm are carried out in the section. 10^4 OFDM symbols with 16-QAM modulation are randomly produced so as to achieve CCDF of PAPR, Oversampled factor is $J = 4$. We compare the LADMM algorithm with the AS, the FISTA, the AAC, the SICF, the SLM and the ADMM algorithms,. In the LADMM algorithm, $\mu = 0.01$, $\lambda = 0.99$, $\epsilon = 0.0316$. The stopping criterion of the LADMM algorithm is

$$\max\{\frac{\|\boldsymbol{x}^{i+1} - \boldsymbol{x}^i\|^2}{\|\boldsymbol{x}^{i+1}\|^2}, \frac{\|\boldsymbol{y}^{i+1} - \boldsymbol{y}^i\|^2}{\|\boldsymbol{y}^{i+1}\|^2}\} \leq tol$$

where tol is a tolerance.

The convergence performance of the LADMM algorithm is shown in Fig. 1. We select $\max\{\frac{\|\boldsymbol{x}^{i+1}-\boldsymbol{x}^i\|^2}{\|\boldsymbol{x}^{i+1}\|^2}, \frac{\|\boldsymbol{y}^{i+1}-\boldsymbol{y}^i\|^2}{\|\boldsymbol{y}^{i+1}\|^2}\}$ as the relative residual error of the LADMM algorithm and set $tol = 10^{-4}$. From the curve in Fig. 1, we can find that the relative residual error decreases rapidly. About 25 iterations, the tol is satisfied. This fact is consistent with our convergence theorem 4. However, this may be a big computational burden for a practical system. We will demonstrate the PAPR and BER of the LADMM algorithm can achieve better performances at about 10 iterations in Fig. 5 and Fig. 6.

The PAPR decrease performance of different methods, i.e. the LADMM, the FISTA, the AS, the AAC, the SICF, the SLM and the ADMM algorithms are compared in Fig. 2. The computational complexity is the same for all schemes. The PAPR of unclipping OFDM signal is 11.5 dB. The PAPR reduction performance of the AS, the AAC, the SLM, the SICF, the FISTA and the ADMM algorithms are about 2.3 dB, 4.7 dB, 5.0 dB, 6.5 dB, 8.5 dB and 9.0 dB, better than that of the original OFDM signal, respectively. Compared with the PAPR of the AS, the AAC, the SLM, the SICF, the FISTA and the ADMM algorithms, the LADMM method obtains an approximate 7.4 dB, 5.0 dB, 4.7 dB, 3.2 dB, 1.2 dB, and 0.7 dB PAPR decrease gain, respectively.

The average PAPR reduction performance of different algorithms with the same iteration numbers are compared in Fig. 3. We can find that when the iteration number is 3 the AS and the SICF algorithms converge to about 6.3 dB and 5 dB, respectively. Continuing to increase iteration, the two algorithms do not have any performance improvement. After 14 iterations, PAPR of the AAC converges to about 5.2 dB. Among all the curves, the FISTA algorithm has the fastest descent rate. For the ADMM and the LADMM algorithms, the two curves show similar trend.

But we should note the fact that PAPR decrease performance of the LADMM algorithm is the best for all the same iteration numbers.

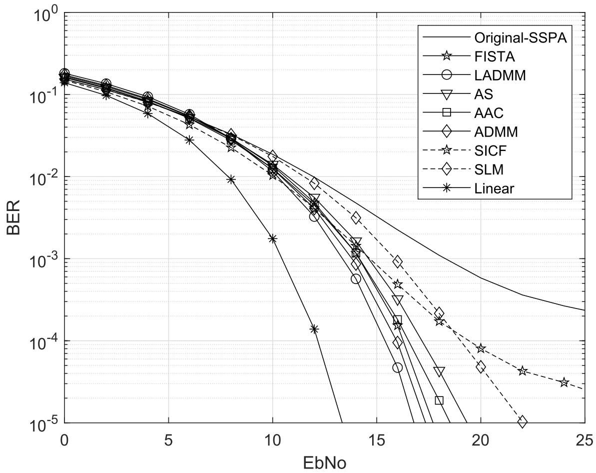

FIGURE 4. **BER performance comparison of different methods for 16-QAM modulation.**

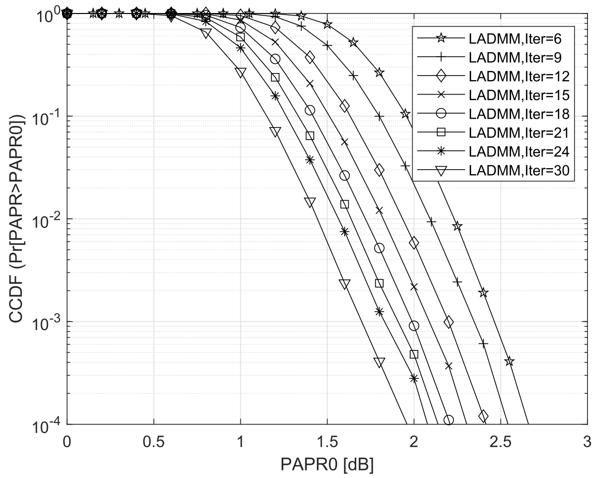

FIGURE 5. **PAPR reduction versus iterations for 16-QAM modulation.**

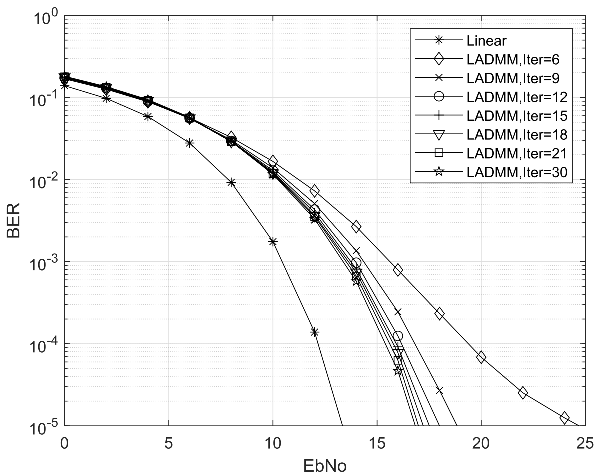

FIGURE 6. **BER versus iterations for 16-QAM modulation.**

To compare BER performance of the whole system, we make the optimized signal to pass through a solid-state power amplifier (SSPA). Usually, the operation point of the HPA is far away from the saturation region, and is reduced by certain input power back-off (IBO). The IBO is denoted as the saturation power of the HPA and the average power of the input signal. We set the IBO = 4 dB in the simulations.

We compare BER performance of different approaches over an additive white Gaussian noise (AWGN) channel in Fig. 4. Here, the word "Linear" represents the BER of the original OFDM signal without PAPR reduction and any distortion. The word "Original-SSPA" expresses the BER obtained by the original OFDM signal of no PAPR reduction passing through the SSPA. We can see that the BERs of the SICF and the SLM algorithms are inferior to other methods except the "Original-SSPA". This may be due to the fact that two methods have a cut-off CCDF (refer to Fig. 2). The OFDM signal of no PAPR reduction and distortion has a BER level at approximate 13.5×10^{-5} dB. However, when the SSPA with IBO = 4 dB is employed, the BER loss for the LADMM, the ADMM, the FISTA, the AAC, the AS and the SLM algorithms are approximately 3.2 dB, 3.8 dB, 4.1 dB, 5.0 dB, 5.8 dB and 8.5 dB, respectively, compared with the ideal BER curve. The BER of the LADMM approach is about 0.6 dB, 0.9 dB, 1.8 dB, 2.6 dB and 5.3 dB, better than ones of the ADMM, the FISTA, the AAC, the AS and the SLM algorithms at 10^{-5} BER level, respectively.

In Fig. 5, we simulate the PAPR decrease performance of different iterations on the LADMM algorithm. This can be found that as iterations increase, the PAPR reduction performances become better. For example, when iteration numbers are 6, 9, 12 and 15, corresponding PAPR are approximate 2.63 dB, 2.51 dB, 2.38 dB and 2.25 dB at CCDF = 10^{-4}, respectively. The PAPR reduction gaps are about 0.12 dB. The gap between the PAPR obtained at iteration number 30 (the LADMM algorithm converges) and that achieved at iteration number 6 is only 0.83 dB.

In Fig. 6, we evaluate the BER performances of the LADMM algorithm on different iterations over an AWGN channel. The BER of the OFDM signal without PAPR reduction and distortion is also ploted in Fig. 6 as a reference. It is showed that as iterations increase, the BER performances become better. For example, when iteration numbers are 6, 9, 12 and 15, corresponding BER are approximate 24.8 dB, 18.6 dB, 18 dB and 17.5 dB at the BER of 10^{-5}, respectively. Combining with Fig. 5, we can find that with the increasement of iterations, the better the PAPRs are, the better the BERs become. But more iterations do not lead to more PAPR and BER gains. In order to decrease computational cost for practical system, we can make a trade-off between PAPR reduction, BER and iterations.

In Fig. 7 and Fig. 8, we also evaluate the PAPR and BER performances in 64-QAM modulation for different schemes. We can find that the PAPR performance in 64-QAM modulation is similar to that in 16-QAM modulation. But for 64-QAM modulation, BER performances of the AS, the AAC, the SICF and the SLM methods drop rapidly, which may be due to the high PAPRs of these methods

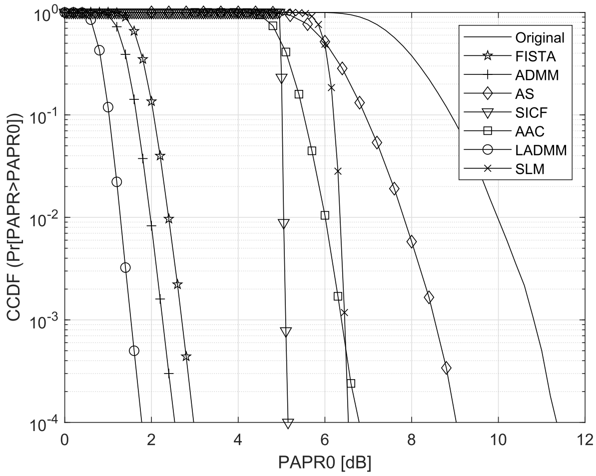

FIGURE 7. PAPR comparison of different schemes for 64-QAM modulation.

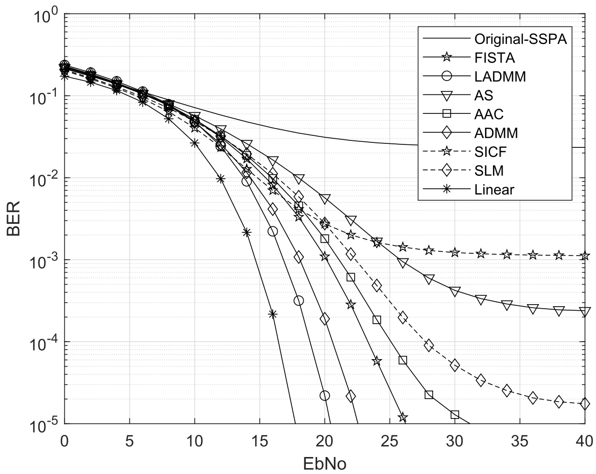

FIGURE 8. BER performance comparison of different methods for 64-QAM modulation.

and their sensitivity to high-order constellations. However, the proposed LADMM scheme still achieves the best BER performance, which shows that this method is robust to PAPR and BER performance of higher-order constellation.

VI. CONCLUSION

The PAPR reduction with an EVM constraint for OFDM systems is took into account in the paper. Unlike the SOCP method with a huge computational burden, We present an innovative approach based on linearized alterative direction method of multipliers (LADMM) to tackle the PAPR optimization. Based on the ADMM method, we first get the LADMM algorithm by linearizing the quadratic term of subproblem. Because every subproblem of the LADMM algorithm has a closed-form solution, this gives rise to its simplicity and high efficiency. Then we analyse the LADMM algorithm's complexity. The analysis results show that the computational complexity is about $\mathcal{O}(JN\log(JN))$ in every iteration. We also give and prove the LADMM algorithm's convergence theorem. At last, we compare the LADMM algorithm with the existing technologies on PAPR and BER performances. Simulation results demonstrate that LADMM algorithm not only acquires larger PAPR reduction, but also obtains better BER.

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