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Bayesian Hierarchical Model-Based Information Fusion for Degradation Analysis Considering Non-Competing Relationship

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ABSTRACT Degradation analysis methods are increasingly used in reliability assessment of long-life products. The small sample size problem is worthy of attention in the degradation analysis. Bayesian hierarchical method is employed for fusing multi-source degradation information to handle small sample problems. The competing relationship multiple degradation models have been widely researched and applied. However, there is the non-competing multiple degradation situation exist in mechanical products. In this paper, a Bayesian hierarchical model-based information fusion degradation method with gamma process model is introduced for describing the non-competing relationship multiple degradation process. Therefore, the Bayesian method and the MCMC method are employed for parameters estimation. A numerical example of spool valve of hydraulic system in exoskeleton robot is provided to demonstrate the proposed Bayesian hierarchical model-based information fusion degradation method.

INDEX TERMS Bayesian hierarchical model, uncertainty, exoskeleton robot, reliability, degradation analysis.

I. INTRODUCTION

Nowadays, the complex systems such as industrial, transportation and military are generally required long-life and high-reliability [1]–[4]. Methods such as degradation analysis have been developed to enhance the reliability analysis of these systems [5]–[7]. Due to the advantages in the reliability assessment of long-life, high-reliability products, degradation analysis methods have been widely used in academic research and industrial application [8].

In general, the complex products with long life and high reliability may have multiple performance degradation processes. In the competing relationship multiple degradation models, the products confront a failure when anyone of the performance indicator reaches a predefined threshold [9]–[12]. The multiple degradation processes based on the competing relationship have been widely studied [13]–[15]. Peng *et al.* [16] developed the inverse Gaussian process to characterize the bivariate degradation process considering incomplete observations. Pan and Balakrishnan [17] employed the Gamma process to describe the competing relationship multiple performance characteristics, and Bayesian method is used for parameters estimation. Barker and Newby [18] used a multivariate Wiener process to characterize the degradation process for developing maintenance strategy. Peng *et al.* [19] used Bayesian method to analyze multivariate degradation systems under dynamic conditions. Sari *et al.* [20] developed the generalized linear degradation path models for describing the bivariate degradation model. Pan and Sun [21] proposed a multivariate accelerated degradation test design based on Gamma process for the small sample size problem. Wang *et al.* [22] applied the copula function and stochastic process to construct a residual life prediction model for multivariate degradation products.

However, there is the non-competing multiple degradation situation exist in mechanical products, the competing relationship multiple degradation models may not be suitable for this situation [23]–[26]. To address the problem, a modified multiple degradation processes model is proposed. A classic example, i.e., spool valve of hydraulic system in exoskeleton robot, is investigated in this paper to demonstrate the proposed method. The wear rate of spools and sleeves are two non-competing performance indicators of spool valve [27].

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Earlier spool valve reliability models did not consider the small sample size problem in reliability analysis, but in some cases, small sample size problem is inevitable. A modified model that considers the small sample problem is introduced in this paper. In order to obtain more accurate analysis result under limited degradation observations, it is imperative to take full advantage of the degradation information from Original Equipment Manufacturers (OEMs) and user plants. Degradation information from different sources have the mathematically common, so they can be fused during the reliability analysis [28]–[31].

A hierarchical Bayesian method of gamma process model is introduced into this paper for modeling and analysis degradation process. The hierarchical Bayesian method could fuse degradation information from different sources [32]–[34]. The estimation of model parameters is performed by Bayesian method and Markov chain Monte Carlo (MCMC) method.

The remainder of this paper are organized as follows. Section 2 introduces gamma process model for the non-competing relationship multiple degradation model. In Section 3, a Bayesian hierarchical model-based information fusion method is presented. Section 4 presents the degradation analysis of spool valve of hydraulic system in exoskeleton robot to illustrate the proposed method. Section 5 summarizes the whole work and prospects of the future work.

II. DEGRADATION PROCESS

A. GAMMA PROCESS

Degradation process of produce is usually an evolutionary process. The advantages of stochastic process make it suitable for characterizing the degradation process [35]. The independent increment of gamma process is non-negative, it is consistent with the performance evolvement of the mechanical products. In general, gamma process is used for describing the performance degradation of mechanical products [36]–[38].

Let a stochastic process $\{Y(t), t > 0\}$ obeys gamma process with the following properties

(1) $Y(0) = 0$

(2) Independent increments $\Delta Y(t) = Y(t + \Delta t) - Y(t)$ obey Gamma distribution $\Delta Y(t) \sim Ga(\Delta\eta(t), \lambda)$ with $\Delta\eta(t) = \eta(t + \Delta t) - \eta(t)$.

where $\eta(\cdot)$ is the shape parameter and a right-continuous non-decreasing process in $[0, \infty)$, $\eta(0) = 0$. $\lambda$is scale parameter and $\lambda > 0$.

The probability density function (PDF) of gamma process is shown as

$$f(\Delta Y(t)|\eta(t), \lambda) = \frac{\lambda^{\eta(t)} \Delta Y(t)^{\eta(t)-1} e^{-\lambda \Delta Y(t)}}{\Gamma(\eta(t))} \times I_{(0,\infty)}(\Delta Y(t)) \quad (1)$$

where

$$I_{(0,\infty)}(x) = \begin{cases} 1 & x \in (0, \infty) \\ 0 & x \notin (0, \infty) \end{cases} \quad (2)$$

and $\Gamma(\eta) = \int_0^\infty x^{\eta-1} e^{-x} dx$ is the gamma function.

The pre-specified threshold is defined as C, and the failure time T of the produce is defined as the time at which the degradation indicator first reaches the pre-specified threshold.

$$T = \inf\{t\,|Y(t) \geq C\} \quad (3)$$

The reliability of produce can be expressed as

$$\begin{aligned} R(t|\eta(t), \lambda) &= P(T > t) = P\{X(t) < C\,|\eta(t), \lambda\} \\ &= \int_0^C f(x|\eta(t), \lambda)\, dx \\ &= \int_0^C \frac{\lambda^{\eta(t)} x^{\eta(t)-1} e^{-\lambda x}}{\Gamma(\eta(t))} dx \end{aligned} \quad (4)$$

The PDF of the failure time can be given as

$$\begin{aligned} f(t|C) &= \frac{d}{dt} \frac{\Gamma(\eta t, C/\lambda)}{\Gamma(\eta t)} \\ &= \frac{\eta}{\Gamma(\eta t)} \int_0^{C/\lambda} \left[\ln(x) - \frac{\Gamma'(\eta t)}{\Gamma(\eta t)}\right] x^{\eta t-1} e^{-x} dx \end{aligned} \quad (5)$$

It can be seen that the PDF is very complicated and quite difficult to handle in industrial applications. The Birnbaum-Saunders (B-S) distribution is employed for approximate fitting. After unifying the parameters, the CDF and PDF of B-S distribution can be given as

$$F_{BS} = \Phi\left[\sqrt{\lambda C}\left(\sqrt{\frac{\eta t}{\lambda C}} - \sqrt{\frac{\lambda C}{\eta t}}\right)\right] \quad (6)$$

$$\begin{aligned} f_{BS}(t|\eta, \lambda) &= \frac{\eta\sqrt{\lambda C}}{2\sqrt{2\pi}\lambda C}\left[\left(\frac{\eta t}{\lambda C}\right)^{-\frac{1}{2}} + \left(\frac{\eta t}{\lambda C}\right)^{-\frac{3}{2}}\right] \\ &\quad \cdot \exp\left[-2\lambda C\left(\frac{\eta t}{\lambda C} - 2 + \frac{\lambda C}{\eta t}\right)\right] \end{aligned} \quad (7)$$

B. THE NON-COMPETING RELATIONSHIP MULTIPLE DEGRADATION MODEL

Suppose that the m performance indicators of the produce are mutually-independent, $D_p(t)$ is the degradation data of the Lth degradation processes at the time t. All degradation processes lead to one failure mode, the reliability can be defined as

$$R(t) = \Pr\left\{D_1(t) + D_2(t) + \ldots + D_p(t) < C\right\} \quad (8)$$

If $D_p(t)$ follows a gamma process with shape parameter $\eta_p(t)$ and scale parameter λ with $p = 1, 2, \ldots, L$, when the shape parameter function $\eta_p(t) = \eta_p t$, then the degradation increment ΔD_p follows the gamma distribution $\Delta D_p \sim Ga(\eta_p \Delta t, \lambda)$.

Based on the additivity of the gamma distribution, the non-competing relationship can be obtained as:

$$\sum_{p=1}^{L} \Delta D_p(t) \sim Ga\left(\sum_{p=1}^{L} \eta_p \Delta t, \lambda\right) \quad (9)$$

According to the properties of gamma process, $\left(D_1(t) + D_2(t) + \ldots + D_p(t)\right)$ follows Gamma process with shape parameter function $\eta(t) = \sum_{p=1}^{L} \eta_p t$. The reliability function can be defined as:

$$R(t) = \frac{\int_0^{C\lambda} x^{\sum_{p=1}^{L} \eta_p t - 1} e^{-\lambda} dx}{\Gamma\left(\sum_{p=1}^{L} \eta_p t\right)} \tag{10}$$

III. PARAMETER ESTIMATION

A. DEGRADATION PROCESS ANALYSIS METHOD

In this paper, we analyze the non-competing relationship multiple degradation model of two degradation indicators. The degradation observations of $D_1(t)$ and $D_2(t)$ are monitored for N units. Let $D_1(t_{ij})$ and $D_2(t_{ij})$ denote the jth observation for unit i at time t_{ij} with $j = 1, \ldots, M$ and $i = 1, \ldots, N$. Let $\Delta d_{ij} = D_1(t_{ij}) - D_1(t_{i,j-1})$ and $\Delta d'_{ij} = D_2(t_{ij}) - D_2(t_{i,j-1})$ be the degradation increment. The Δd^p_{ij} follow the gamma distribution $\Delta d^p_{ij} \sim Ga\left(\eta_p \Delta t_{ij}, \lambda\right)$.

The performance degradation data of the non-competing relationship multiple degradation model includes the OEMs data D^O_p and user plants data D^U_p. Denote D^O_p and D^U_p as D_p with $\nu = (\eta_1, \eta_2, \lambda)$, the likelihood function is

$$\begin{aligned} &L(D_1, D_2, \nu|\eta_1, \eta_2, \lambda) \\ &= \prod_{i=1}^{N}\prod_{j=2}^{M} g(\Delta d_{ij}|\eta_1, \lambda) g(\Delta d'_{ij}|\eta_2, \lambda) \\ &= \prod_{i=1}^{N}\prod_{j=2}^{M} \frac{\lambda^{\eta_1 \Delta t_{ij}}}{\Gamma\left(\eta_1 \Delta t_{ij}\right)} \Delta d^{\eta_1 \Delta t_{ij}-1} \exp\left(-\lambda \Delta d_{ij}\right) \\ &\quad \cdot \frac{\lambda^{\eta_2 \Delta t_{ij}}}{\Gamma\left(\eta_2 \Delta t_{ij}\right)} \Delta d'^{\eta_2 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d'_{ij}\right) \end{aligned} \tag{11}$$

where $\Delta d_{ij} = D_1(t_{ij}) - D_1(t_{i,j-1})$, $\Delta d'_{ij} = D_2(t_{ij}) - D_2(t_{i,j-1})$.

Suppose that the prior information is quantified and obtained as prior distribution for degradation model parameters as $\pi(\theta) = \pi(\eta_1, \eta_2, \lambda)$. The posterior distribution could be obtained via the Beysian method to integrate prior information and degradation information as:

$$\begin{aligned} &p(\eta_1, \eta_2, \lambda, \nu\,|D_1, D_2) \propto \pi(\theta) L(D_1, D_2, \nu|\theta) \\ &= \pi(\eta_1, \eta_2, \lambda) L(D_1, D_2, \nu|\eta_1, \eta_2, \lambda) \\ &= \pi(\eta_1, \eta_2, \lambda) \prod_{i=1}^{N}\prod_{j=2}^{M} \frac{\lambda^{\eta_1 \Delta t_{ij}}}{\Gamma\left(\eta_1 \Delta t_{ij}\right)} \Delta d^{\eta_1 \Delta t_{ij}-1}_{ij} \\ &\quad \cdot \exp\left(-\lambda \Delta d_{ij}\right) \frac{\lambda^{\eta_2 \Delta t_{ij}}}{\Gamma\left(\eta_2 \Delta t_{ij}\right)} \Delta d'^{\eta_2 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d'_{ij}\right) \end{aligned} \tag{12}$$

B. BAYESIAN INFORMATION FUSION METHOD

Based on the framework of the Bayesian hierarchical model-based information fusion method, the prior information and multi-source degradation data are integrated to obtain the joint posterior distribution. The realization process is

$$\begin{aligned} &p_I(\eta_1, \eta_2, \lambda, \nu\,\left|D^O_1, D^O_2\right.) \\ &= \pi(\eta_1, \eta_2, \lambda) \prod_{i=1}^{N}\prod_{j=2}^{M} \frac{\lambda^{\eta_1 \Delta t_{ij}}}{\Gamma\left(\eta_1 \Delta t_{ij}\right)} \Delta d^{O\eta_1 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d^O_{ij}\right) \\ &\quad \cdot \frac{\lambda^{\eta_2 \Delta t_{ij}}}{\Gamma\left(\eta_2 \Delta t_{ij}\right)} \Delta d^{O\prime\eta_2 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d^{O\prime}_{ij}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} &p_{II}(\eta_1, \eta_2, \lambda, \nu\,\left|D^U_1, D^U_2, D^O_1, D^O_2\right.) \\ &= p_I(\eta_1, \eta_2, \lambda, \nu\,\left|D^O_1, D^O_2\right.) \\ &\quad \cdot \prod_{i=1}^{N}\prod_{j=2}^{M} \frac{\lambda^{\eta_1 \Delta t_{ij}}}{\Gamma\left(\eta_1 \Delta t_{ij}\right)} \Delta d^{U\eta_1 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d^U_{ij}\right) \\ &\quad \cdot \frac{\lambda^{\eta_2 \Delta t_{ij}}}{\Gamma\left(\eta_2 \Delta t_{ij}\right)} \Delta d^{U\prime\eta_2 \Delta t_{ij}-1}_{ij} \exp\left(-\lambda \Delta d^{U\prime}_{ij}\right) \end{aligned} \tag{14}$$

where D^O_1 and D^O_2 are the OEMs degradation data, $\pi(\eta_1, \eta_2, \lambda)$ is prior distribution quantified and obtained by prior information, $p_I(\eta_1, \eta_2, \lambda, \nu\,\left|D^O_1, D^O_2\right.)$ is the posterior distribution integrated by prior information and the information contained in the OEMs degradation observations. D^U_1 and D^U_2 are the user plants degradation observations, $p_{II}(\eta_1, \eta_2, \lambda, \nu\,\left|D^U_1, D^U_2, D^O_1, D^O_2\right.)$ is the integrated of prior information and multi-source degradation information.

The MCMC method is used to estimate the model parameters. In most applications with Bayesian methods, it is difficult to obtain the posterior distribution. In this paper, we assume that prior distributions of model parameters are non-informative, and the OpenBUGS is used to perform the Gibbs sampling for the parameters estimation.

IV. ILLUSTRATIVE EXAMPLE

The reliability of exoskeleton robot hydraulic systems is constructed by composing subsystems. The reliability of the spool valves directly affects the reliability of the hydraulic system. The main failure mode of the spool valves is the wear degradation of spools and sleeves. When the total wear degradation of spool and sleeve reach predetermined threshold, the spool valves is considered as failure.

The simulated data are generated for demonstrating the applicability of the proposed non-competing relationship degradation model and Bayesian parameter estimation. The wear degradation observations of OEMs are presented in Figure 1, and the wear observations of user plants are shown in Figure 2.

The wear degradation increment of sleeves is $\Delta d_{ij} = D_{SL}(t_{ij}) - D_{SL}(t_{i,j-1})$ which is modeled as $Ga\left(\eta_{SL} \Delta t_{ij}, \lambda\right)$ and the wear degradation increment of spools is $\Delta d'_{ij} = D_{SP}(t_{ij}) - D_{SP}(t_{i,j-1})$ that is modeled as $Ga\left(\eta_{SP} \Delta t_{ij}, \lambda\right)$.

Owing to the limitation on the available prior information, the non-informative prior distribution for parameters of the degradation process model of OEMs, quantized from the

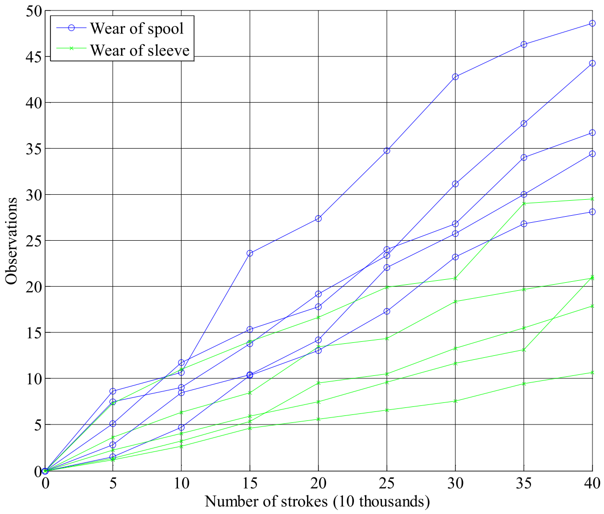

FIGURE 1. **The wear degradation observations of OEMs.**

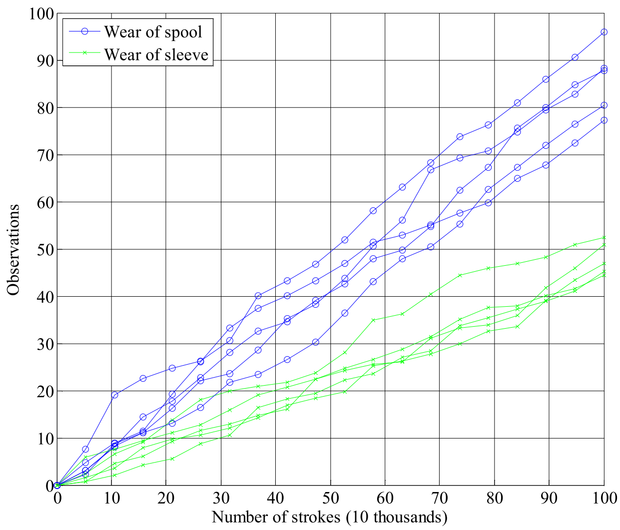

FIGURE 2. **The wear degradation observations of user plants.**

subjective information is shown as

$$\eta_{SL} \sim U(0, 10), \eta_{SP} \sim U(0, 10), \lambda \sim U(0, 10)$$

Uniform distributions for model parameter are used for characterization the non-informative priors with relevant large intervals.

The MCMC method is employed for simulating posterior samples of parameters, The OpengBUGS is adopted to facilitate the implementation of MCMC for the Bayesian hierarchical model-based information fusion degradation analysis of the wear degradation data. The posterior pdf of OEMs degradation model parameters are shown in Figure 3. These posterior distributions transform into prior distribution of model parameters of user plants.

The results for parameter estimation of OEMs degradation are presented in Table 1.

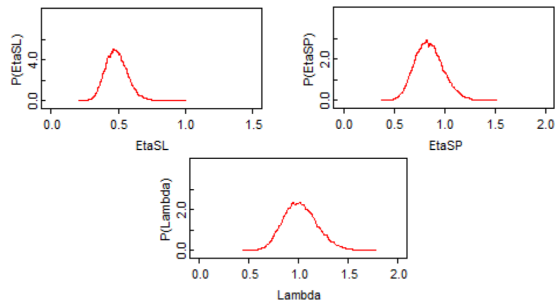

FIGURE 3. **The posterior pdf of model parameters: $\eta_{SL}^{(OEMs)}$, $\eta_{SP}^{(OEMs)}$, $\lambda^{(OEMs)}$.**

TABLE 1. **Statistical summarization of parameters estimation.**

	Mean	Standard deviation	Confidence interval	
			2.5%	97.5%
$\eta_{SL}^{(OEMs)}$	0.4871	0.08084	0.3434	0.6609
$\eta_{SP}^{(OEMs)}$	0.8468	0.1402	0.5968	1.142
$\lambda^{(OEMs)}$	1.017	0.1697	0.7153	1.377

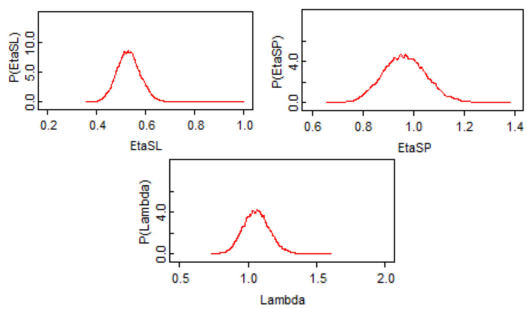

FIGURE 4. **The posterior pdf of model parameters: η_{SL}, η_{SP}, λ.**

The PDFs of posterior distributions of Bayesian hierarchical model-based information fusion degradation analysis model are presented in Figure 4.

Convergence refers to whether the algorithm has reached its target distribution. If the algorithm converges, the generated posterior sample comes from the correct target distribution. Therefore, judging the convergence of the algorithm is an important prerequisite for obtaining the required posterior samples. The trace plots for model parameters are monitored for checking the convergence in this paper. The trace plots of model parameters for the non-competing relationship degradation process are shown in Figure 5.

There is no pattern or irregularity being observed, so we could assume that the Bayesian hierarchical model-based information fusion degradation analysis are converged. The results for parameter estimation given in Table 2.

The pre-specified threshold is $C = 160$, the reliability of the spool valves can be obtained by parameters estimation as presented in Figure 6.

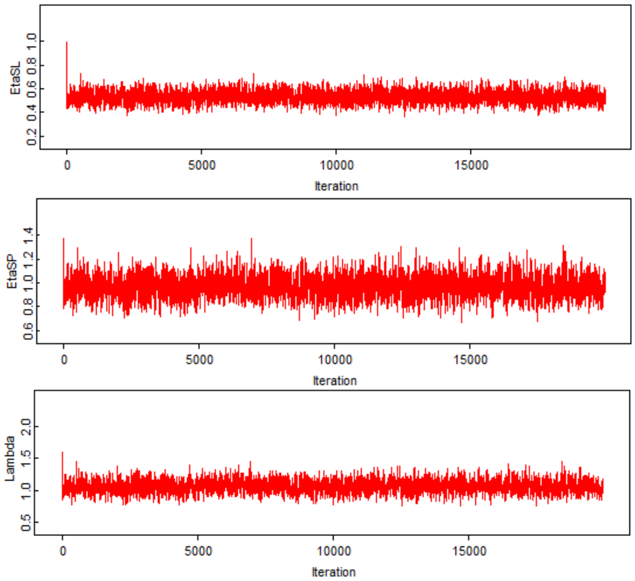

FIGURE 5. The trace plots of degradation model parameters.

TABLE 2. Statistical summarization of parameters estimation.

	Mean	Standard deviation	Confidence interval 5%	Confidence interval 95%
η_{SL}	0.5307	0.0475	0.4407	0.627
η_{SP}	0.9661	0.08663	0.8037	1.143
λ	1.059	0.09568	0.8798	1.253

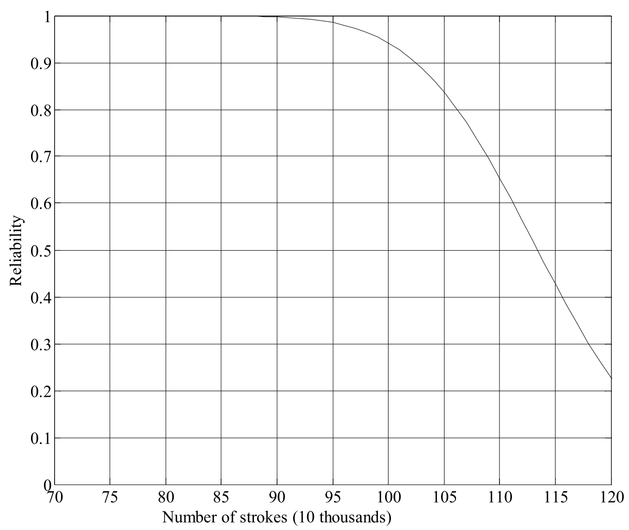

FIGURE 6. Reliability of the spool valves.

V. CONCLUSIONS

In this paper, a Bayesian hierarchical model-based information fusion method considering non-competing relationship is presented. This method is used to characterize the non-competing multiple degradation process under small sample size problems. The characteristics of these degradation process are characterized by gamma process. For fusion the multi-sources degradation information, a Bayesian hierarchical model-based information fusion method is introduced into the framework of degradation analysis. Therefore, the parameters estimation results are obtained via Bayesian method and the MCMC method. The model parameters trace plots are employed for checking the convergence of degradation models. A numerical example of spool valve of hydraulic system in exoskeleton robot is provided to demonstrate the proposed multiple degradation model.

It should be noted that there are some open questions for future work. Due to individual differences are widely present in produces, the random effect could be introduced to the non-competing relationship multiple degradation process. The different working environment of the products in the OEMs and the user plant could be considered into the modeling of the degradation process.

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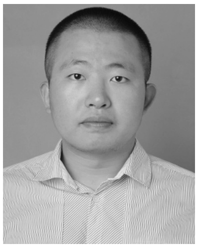

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