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An Extension of Regret Theory Based on Probabilistic Linguistic Cloud Sets Considering Dual Expectations: An Application for the Stock Market

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ABSTRACT The decision of stock selection relies on experts' cognition and investors' behavioral characteristics. This requires the consideration of several conflicting, often fuzzy criteria with uncertainty conditions, which can benefit from multiple attribute decision-making (MADM). Accordingly, an extended regret theory (RT) decision-making method is developed in this study to identify and rank-order superior stocks. First, by extracting the strengths of probabilistic linguistic term sets and cloud models, a novel concept of probabilistic linguistic cloud sets (PLCSs) is proposed to effectively express and handle uncertain preference information. Second, RT is extended to the PLCSs environment. Considering the behavior characteristics of expectation dependence, dual (target and growth) expectations are shown. Third, a distance measure algorithm of PLCSs is defined to calculate the distance between the attribute value and dual expectations, which serves as the basis for the construction of a fuzzy pattern recognition model to determine the optimal membership and attribute weights. Membership is used to modify the RT-based perceived utility, the ranking of alternatives is determined by the modified comprehensive perceived utility. A case study is conducted to demonstrate the proposed method, and its reliability and effectiveness are further verified by comparing it with other methods.

INDEX TERMS Probabilistic linguistic cloud sets, regret theory, dual expectations, stock selection.

I. INTRODUCTION

Stock selection is a key problem in portfolio construction and management [1], [2]. The financial market is a complex and changeable system with many kinds of information. The decision-making for stock selection is affected by various and conflicting attributes and can be treated as a typical multiple attribute decision-making (MADM) problem [3]. It requires a wider perspective including the contribution of nonfinancial parameters that account for behavioral issues such as experts' cognition, investors' behavioral characteristics, assessment of alternatives, and expectations of future [4].

MADM techniques are common methods to construct information and decision evaluation in problems with multiple conflicting goals [3]. These have been extensively investigated in fields such as renewable power sources [5], supply chain management [6], emergency response [7], and electronic commerce [8]. MADM is also an effective tool for financial decision-making. It has attracted increasingly wide attention in the literature and has been applied in many stock exchanges, such as the Taiwan Stock Exchange [9], the Tehran Stock Exchange [10], the Kuala Lumpur Stock Exchange [11], and the Athens Stock Exchange [12]. It is noteworthy that crisp numerical values were often used in early studies to express evaluation information. With the uncertainty of the global investment climate and fierce competition in financial markets, uncertainty has increased in the development of listed companies, leading to greater vagueness and complexity in stock-selection decision-making [1]. Pang *et al.* [13] pointed out that in practical decision-making,

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decision-makers (DMs) may prefer to express their preferences with different degrees and/or distributions of importance, and this information is usually hard to obtain, so they proposed the probabilistic linguistic term sets (PLTSs), which exhibit great flexibility in capturing DMs' hesitant qualitative assessments [14]. Recently, researches have proposed the probabilistic linguistic Choquet integral operator [15] and probabilistic linguistic preference relations [16], these studies have contributed significantly to the development of decision-making with PLTSs.

Several computational methods have been extended to deal with language information, and fuzzy sets theory is frequently used. However, this inevitably leads to the loss and distortion of original information during transformation using a membership function [17]. This can be overcome by the cloud model [18], which allows a stochastic disturbance of the membership degree encircling a determined central value rather than a fixed number [19]. The cloud model utilizes three numerical characteristics to perfectly describe the fuzziness and randomness of qualitative concepts. Based on this, the objective and interchangeable transformation between qualitative concepts and quantitative values become available and clear [20]. Increasingly researchers are paying attention to this model because it can handle uncertainty and avoid information loss during the information gathering process. Wang *et al.* [19] introduced a method to convert linguistic sets of any odd labels to corresponding cloud models. Wang *et al.* [20] converted interval linguistic values to interval integrated clouds, for which they proposed a distance measure. Peng and Wang [21] presented a linguistic intuitionistic cloud model. Li *et al.* [22] combined rough sets and cloud models and applied a rough cloud theory-based method to risk evaluation of failure modes.

To preserve the advantages of the cloud model and distinguish the importance degrees of possible preference information, this study develops an extended cloud model, called probabilistic linguistic cloud set (PLCS). PLCS has the following advantages: (1) It extends the cloud model by adding probabilities without loss of original linguistic evaluation information from DMs. By manifesting the probability distributions of linguistic information, incomplete probability information is acceptable; (2) It improves on probabilistic linguistic term sets by utilizing cloud models instead of linguistic labels or fuzzy numbers. It ideally discloses the uncertain fuzziness of qualitative concepts and effectively handles information distortion that occurs during information fusion. Hence, PLCS is desirable for the expression of DMs' preference information with respect to various attributes, which enables linguistic information to be processed more accurately. How to obtain comprehensive evaluation values for all alternatives through PLCS is a key part of this paper, which can be regarded as the application of decision-making methods.

In addition, it is hard to fairly evaluate alternatives considering only the current realities. For example, focusing only on a stock's current performance and ignoring historical information and future expectations will inevitably affect investment efficiency. Therefore, one should consider the expectations of each alternative under different attributes. This has a moderating effect to ensure fair and accurate solutions to complex problems [23]. This paper considers dual expectations, including target and growth expectations. In decision-making, DMs usually choose a suitable target by combining their experience, the decision environment, and alternative resources. This target expectation is conductive to the rationalization of resource allocations. Growth expectation mainly refers to the future development trend based on historical data, comprehensive development conditions, and the current environment. How to use the moderating effect of dual expectations and make more scientific and reasonable decisions warrant attention, and they are another key part of this paper.

Numerous decision-making methods are mainly based on the assumptions of people being completely rational [24], such as TOPSIS [25] and VIKOR [26]. However, due to the uncertainty and complexity of the decision-making environment, humans' limited cognitive powers, and time pressure, DMs do not completely understand and grasp all relevant information, hence they behave with bounded rationality [27]. The traditional MADM methods based on the expected utility theory where investors are assumed to be fully rational cannot explain the inherent phenomena in the practical application of the stock market. It is necessary to shift the focus of financial studies toward behavioral finance theory. Effective and realistic decision-support models are needed to help investors make good judgments.

Some behavioral decision-making theories such as prospect theory (PT) [28], cumulative prospect theory (CPT) [29], and regret theory (RT) [30], [31] are proposed to explain the behavior and judgment of the individual without the assumption of rationality. However, Nwogugu [32] pointed out that the natural mental processes of human beings can result in decision-making patterns that differ from those predicted by and implicit to PT and CPT. RT is a bounded rationality model that compares outcomes of a given alternative with that of an optional alternative to quantify rejoice and regret, by contrast, it can depict intuitive judgments more consistently and is simpler in parameter setting and calculation [33]. Feelings like rejoice and regret, which are the pillars of RT, are a fact of life. In financial markets, regret is the main driver behind the disposal effect. Moreover, investors will experience regret when their investment yields a lower performance than an obvious alternative investment they could have chosen, so they try to anticipate regret and take it into account in their investment decisions in a consistent manner. Therefore, with the consideration of the influence of regret on decision-making under uncertainty as well as the axiomatic and normative appeal of RT for investment choices [34], it is irrational to ignore RT.

RT has been widely researched and applied in various fields [35]–[38]. To deal with uncertainty and imprecision inherent in the process of decision-making, researchers have introduced RT into uncertain environments. Peng *et al.* [27] developed a Z-number-based RT method to determine the utility, rejoice, and regret values of Z-information. Zhang *et al.* [33] defined a fuzzy regret/rejoice function based on triangular fuzzy numbers and RT to solve fuzzy multi-attribute group decision-making problems with incomplete weight information. Chen *et al.* [39] proposed a MADM model based on fuzzy axiomatic design and RT to model the logistics provider selection problem with uncertain and incomplete information. Nevertheless, there are four shortcomings for RT-based MADM methods: (1) RT-based decision-making research usually focuses on only one kind of decision-making information, but multiple expectation information in actual decisions can moderate perceived utility and ensure fair and accurate decisions. Expectation information from multiple dimensions is necessary but hardly accounted for at present. (2) Attribute evaluation values are usually crisp numbers, Z-numbers, or triangular fuzzy numbers, but relevant work on how to calculate the regret/rejoice of DMs in cloud models environment has not been adequately addressed. (3) RT is rarely applied in investment decisions in stock markets. In fact, an investor who compares stocks pairwise usually feels regret when giving up a better stock. Regret aversion is a typical psychological behavior in investment decision-making that should not be neglected. (4) How to aggregate evaluation information with unknown attribute weights warrants consideration.

The inadequacies described above bring the motivation of this study. The core focus of this paper is to propose an extended RT decision-making method based on PLCSs considering dual expectations to address stock selection problems. The main contributions are as follows. (1) We introduce the concept of PLCS, which is an extension of traditional cloud model to express DMs' preferences by adding complete or incomplete probabilistic distributions to linguistic evaluation clouds. We measure the target and growth expectations and establish corresponding dual expectation matrices. (2) We develop an extended RT in the PLCS environment and quantify the psychological behavior of DMs by the cloud contribution value algorithm from the perspective of regret aversion. (3) We calculate the distances between attribute evaluation information and dual expectations using the proposed distance measure algorithm. On this basis, a fuzzy pattern recognition model is built based on weighted generalized Euclidean distance between the alternative and fuzzy recognition center, and the optimal membership and attribute weights are assigned by solving the model with the cross-iterative algorithm. (4) To balance the perceived utility of each alternative based on the extended RT and the membership between alternative solutions and dual expectations, membership is utilized to modify the perceived utility, thereby obtaining the comprehensive perceived utility of the alternative.

The rest of this paper is structured as follows. Section 2 reviews the basic concepts of the cloud model, proposes the concept of PLCSs, and introduces classical RT. Section 3 presents the details of the proposed method and its implementation. Section 4 demonstrates the applicability and validity of the proposed method through a case study. Section 5 concludes the paper.

II. PROBABILISTIC CLOUD SETS AND REGRET THEORY

A. CLOUD MODEL AND AGGREGATION METHOD

Definition 1 [18]: Let U be the universe of discourse and T be a qualitative concept in U. If $x \in U$ is a random instantiation of T with $x \sim N(Ex, En'^2)$ and $En' \sim N(En, He^2)$, $\mu(x) \in [0, 1]$ is the certainty degree of x belonging to T, as follows,

$$\mu_T(x) = \exp\left(-\frac{(x-Ex)^2}{2En'^2}\right) \tag{1}$$

then the distribution of x in U is defined as a normal cloud (abbreviated as cloud) $C = (Ex, En, He)$. Each $(x, u(x))$ is described as a cloud droplet.

The overall quantitative properties of a concept can be described perfectly in a cloud using three numerical characteristics, consisting of expectation Ex, entropy En, and hyper entropy He. Ex is the mathematical expectation value of cloud drops, which can best represent the qualitative concept. En reflects the randomness and fuzziness of the qualitative concept. He is the degree of uncertainty of En, i.e., the second-order entropy of the entropy [19].

Given two normal cloud models $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$. The operation rules between two clouds covered in this paper are listed as follows:

$$C_1 + C_2 = (Ex_1 + Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2}) \tag{2}$$

$$C_1 - C_2 = (Ex_1 - Ex_2, \sqrt{En_1^2 + En_2^2}, \sqrt{He_1^2 + He_2^2}) \tag{3}$$

$$rC_1 = (rEx_1, \sqrt{r}En_1, \sqrt{r}He_1) \tag{4}$$

Definition 2 [19]: Let $C_i = (Ex_i, En_i, He_i), i \in n$ be n clouds in the same universe U. Then the cloud arithmetic average (CAA) operator is

$$CAA(C_1, C_2, \ldots, C_n) = (\frac{1}{n}\sum_{i=1}^{n} Ex_i, \sqrt{\frac{1}{n}\sum_{i=1}^{n} En_i^2}, \sqrt{\frac{1}{n}\sum_{i=1}^{n} He_i^2}) \tag{5}$$

and the cloud is defined as the arithmetic average cloud.

Definition 3 [19]: Let $C_i = (Ex_i, En_i, He_i)$, $i \in n$ be n clouds in the same universe U. Then the cloud weighted arithmetic average (CWAA) operator is

$$\begin{aligned} CWAA(C_1, C_2, \ldots, C_n) &= \sum_{i=1}^{n} w_i C_i \\ &= (\sum_{i=1}^{n} w_i Ex_i, \sqrt{\sum_{i=1}^{n} w_i En_i^2}, \sqrt{\sum_{i=1}^{n} w_i He_i^2}) \end{aligned} \tag{6}$$

and the cloud is defined as the weighted arithmetic average cloud. $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of C_i, $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. Specifically, if $w_i = \frac{1}{n}(i = 1, 2, \ldots, n)$, then the CWAA operator reduces to the CAA operator.

B. THE CONCEPT OF PROBABILISTIC LINGUISTIC CLOUD SETS

Given the linguistic term set $S = \{S_i | i = -\tau, \ldots, 0, \ldots, \tau, \tau \in N^*\}$ [40] and the effective universe $U = [X_{\min}, X_{\max}]$, then the linguistic information of a $(2\tau + 1)$-label linguistic term set can be converted to adjacent clouds [19]. The collection of these transformed clouds is called a linguistic cloud scale (LCS), denoted by $S(U) = \{C_i | i = -\tau, \ldots, 0, \ldots, \tau, \tau \in N^*\}$, where C_i is the cloud corresponding to the linguistic variable S_i. In addition, as analyzed by Pang et al. [13], DMs may hesitate to express a preference among several possible linguistic variables, moreover, the complete probability distribution on these linguistic variables is usually not so easily obtained. Given these realities, the probabilistic linguistic cloud set (PLCS) is proposed.

Definition 4: Let U be a quantitative universe and $S(U) = \{C_i | i = -\tau, \ldots, 0, \ldots, \tau, \tau \in N^*\}$ be a linguistic cloud scale (LCS). C_i is a cloud representing the qualitative concept in U; $C_0(Ex_0, En_0, He_0)$is described as the center cloud; $(C_{-\tau}(Ex_{-\tau}, En_{-\tau}, He_{-\tau}), \ldots, C_{-1}(Ex_{-1}, En_{-1}, He_{-1}))$ on the left are described as semi-fall clouds reflecting poor qualitative concepts of the clouds; $(C_1(Ex_1, En_1, He_1), \ldots, C_\tau(Ex_\tau, En_\tau, He_\tau))$ on the right as semi-rise clouds reflecting better qualitative concepts. Then a PLCS can be defined as

$$\tilde{C}(p) = \{C^{(k)}(p^{(k)}) | C^{(k)} \in S(U), p^{(k)} \geq 0, \quad k = 1, 2, \ldots, \#C(p), \sum_{k=1}^{\#C(p)} p^{(k)} \leq 1\} \quad (7)$$

where $C^{(k)}(p^{(k)})$ is the cloud model $C^{(k)}$ associated with the probability $p^{(k)}$, and $\#C(p)$ is the number of all different cloud models in $\tilde{C}(p)$.

Note that if $\sum_{k=1}^{\#C(p)} p^{(k)} = 1$, then the complete information of probability distribution of all possible cloud models is obtained; if $\sum_{k=1}^{\#C(p)} p^{(k)} < 1$, then partial ignorance exists because current knowledge cannot provide complete evaluation information, which is not rare in actual MADM problems, in such case, one must consider how to estimate the ignorance of probabilistic information $(1 - \sum_{k=1}^{\#C(p)} p^{(k)})$. Following [13], the normalization of probabilistic information in the PLCS is as follows.

Definition 5: Given a PLCS $\tilde{C}(p) = \{C^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#C(p)\}$ with $\sum_{k=1}^{\#C(p)} p^{(k)} < 1$, the normalization of probabilistic information in $\tilde{C}(p)$ is

$$\hat{p}^{(k)} = p^{(k)} \Big/ \sum_{k=1}^{\#C(p)} p^{(k)}, k \in \#C(p) \quad (8)$$

Then $\tilde{C}(p)$ is transformed to $\hat{C}(p) = \{C^{(k)}(\hat{p}^{(k)}) | k = 1, 2, \ldots, \#C(p)\}$, which satisfies $\sum_{k=1}^{\#C(p)} \hat{p}^{(k)} = 1$.

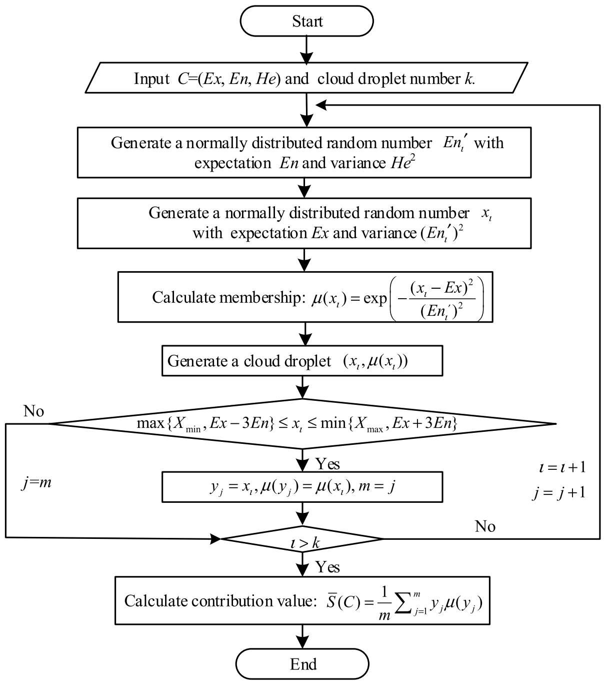

FIGURE 1. Flowchart of the cloud contribution value algorithm.

C. THE SCORE OF PROBABILISTIC LINGUISTIC CLOUD SET

Measurement of the cloud contribution value [19] can be regarded as a mathematical model that transform qualitative concepts into quantitative values by filtering the cloud droplets generated based on the forward cloud generator and calculating the mean of their contribution values. The cloud contribution value algorithm is as follows.

Definition 6: Let $\tilde{C}(p) = \left\{C^{(k)}(p^{(k)}) | k = 1, 2, \ldots, \#C(p)\right\}$ be a PLCS. Then the score $E(\tilde{C}(p))$ of $\tilde{C}(p)$ is

$$E(\tilde{C}(p)) = \bar{S}(\tilde{C}_{syn}), \quad (9)$$

where $\tilde{C}_{syn}$ is the integrated cloud, and $\bar{S}(\tilde{C}_{syn})$ is the contribution value of $\tilde{C}_{syn}$. The three-numerical-tuple characteristics of $\tilde{C}_{syn}$ are:

$$\begin{cases} Ex_{syn} \\ \quad = \hat{p}^{(1)} Ex^{(1)} + \hat{p}^{(2)} Ex^{(2)} + \cdots + p^{(\#C(p))} Ex^{(\#C(p))} \\ En_{syn} \\ \quad = \sqrt{\hat{p}^{(1)} (En^{(1)})^2 + \hat{p}^{(2)} (En^{(2)})^2 + \cdots + p^{(\#C(p))} (En^{(\#C(p))})^2} \\ He_{syn} \\ \quad = \sqrt{\hat{p}^{(1)} (He^{(1)})^2 + \hat{p}^{(2)} (He^{(2)})^2 + \cdots + p^{(\#C(p))} (He^{(\#C(p))})^2} \end{cases} \quad (10)$$

where $\hat{p}^{(k)} = p^{(k)} \Big/ \sum_{k=1}^{\#C(p)} p^{(k)}$, for all $k = 1, 2, \ldots, \#C(p)$. With regard to two PLCSs: $\tilde{C}_1(p)$ and $\tilde{C}_2(p)$, if $E(\tilde{C}_1(p)) \geq E(\tilde{C}_2(p))$, then $\tilde{C}_1(p) \geq \tilde{C}_2(p)$.

D. DISTANCE MEASURE ALGORITHM BETWEEN TWO PROBABILISTIC LINGUISTIC CLOUD SETS

Let $\tilde{C}_1(p) = \left\{C_1^{(k)}(p_1^{(k)})|\ k = 1, 2, \ldots, \#C_1(p)\right\}$ and $\tilde{C}_2(p) = \left\{C_2^{(k)}(p_2^{(k)})|k = 1, 2, \ldots, \#C_2(p)\right\}$ be any two PLCSs. The distance measure algorithm is as follows.

Distance measure algorithm between two PLCSs

Input: two PLCSs $\tilde{C}_1(p)$ and $\tilde{C}_2(p)$and the number of cloud droplets n.
Output: distance $d(\tilde{C}_1(p), \tilde{C}_2(p))$.
Steps:
1. Calculate the clouds $\tilde{C}_{1,syn}$ and $\tilde{C}_{2,syn}$ of $\tilde{C}_1(p)$ and $\tilde{C}_2(p)$ by Eq.(10);
2. Generate n cloud droplets by the forward cloud generator, i.e., $(x_{1\iota}, \mu(x_{1\iota}))$ and $(x_{2\iota}, \mu(x_{2\iota}))$, $\iota = 1, 2, \ldots, n$;
3. Sort two groups of cloud droplets from left to right according to abscissa values $x_{1\iota}$ and $x_{2\iota}$;
4. Filter and retain cloud droplets satisfying $x_{1i} \in [\max\{X_{\min}, Ex_{1,syn} - 3En_{1,syn}\}, \min\{X_{\max}, Ex_{1,syn} + 3En_{1,syn}\}]$ and $x_{2i} \in [\max\{X_{\min}, Ex_{2,syn} - 3En_{2,syn}\}, \min\{X_{\max}, Ex_{2,syn} + 3En_{2,syn}\}]$;
5. Let m_1 and m_2 be the numbers of cloud droplets retained after filtering. if $m_1 \geq m_2$, then randomly select m_2 cloud droplets from m_1 cloud droplets for $\tilde{C}_{1,syn}$, and vice versa;
6. Let $\vartheta = \min\{m_1, m_2\}$. Store respective cloud droplets in sets *Drop1* and *Drop2*, relabel the m_2 cloud droplets as $(x_{j,\tilde{C}_1}, \mu(x_{j,\tilde{C}_1}))$ and $(x_{j,\tilde{C}_2}, \mu(x_{j,\tilde{C}_2}))(j = 1, 2, \cdots, \vartheta)$;
7. Calculate distance $d(\tilde{C}_1(p), \tilde{C}_2(p))$ between $\tilde{C}_1(p)$ and $\tilde{C}_2(p)$: $d(\tilde{C}_1(p), \tilde{C}_2(p)) = d(Drop1, Drop2) = \frac{1}{\vartheta}\sum_{j=1}^{\vartheta}\sqrt{[x_{j,\tilde{C}_1} - x_{j,\tilde{C}_2}]^2 + [u(x_{j,\tilde{C}_1}) - u(x_{j,\tilde{C}_2})]^2}$.

This distance measure algorithm not only makes full use of the numerical characteristics and probability distribution of normal clouds in each PLCS, but also considers the distribution of cloud droplets. Firstly, the integrated clouds are generated by the information in PLCSs. Then, based on the forward cloud generator, cloud droplets are generated, which are further filtered according to the 3σ principle of the normal distribution curve. Later, the retained cloud droplets are arranged and matched in pairs from the perspective of the cloud droplet distribution. Finally, the distance of each pair of cloud droplets in the cloud image is calculated by utilizing the distance formula between two points, so as to measure the distance between two PLCSs. Compared with the distance measure method using the numerical characteristics alone, the way in this paper has higher reliability because it integrates more comprehensive and fine-grained information.

E. REGRET THEORY

According to RT, the DM's perceived utility function consists of a utility function for the current result and a regret/rejoice function. Let x_A and x_B be the consequences of alternatives A and B, respectively. Then the perceived utility for alternative A is defined as

$$U(x_A, x_B) = v(x_A) + R(v(x_A) - v(x_B)) \tag{11}$$

where $v(\cdot)$ is a von Neumann-Morgenstern utility function with $v'(\cdot) > 0$ and $v''(\cdot) < 0$. The utility function $v(x) = x^{\alpha}(0 \leq \alpha \leq 1)$ [29] is usually used to simulate the utility of DMs, where α is the risk aversion coefficient of the DM. A larger α corresponds to a smaller risk aversion value. $R(\cdot)$ is a regret/rejoice function, where $R'(\cdot) > 0$, $R''(\cdot) < 0$, and $R(0) = 0$ [30]. $R'(\cdot) > 0$ represents that $R(\cdot)$ is strictly increasing. Regret aversion, which generates the distinctive predictions of RT, implies that R is concave, as reflected by $R''(\cdot) < 0$. The regret/rejoice function is usually expressed as $R(\Delta v) = 1 - \exp(-\delta \cdot \Delta v)$ [31], where δ denotes the regret aversion coefficient and satisfies $\delta \geq 0$ [41], Δv indicates the difference $(v(x_A) - v(x_B))$ between two utility values for the results (x_A and x_B) of two alternatives (A and B). $R(v(x_A) - v(x_B)) > 0$ means that DM will feel rejoice from choosing A and giving up B; instead, $R(v(x_A) - v(x_B)) < 0$ means the DM will feel regret.

Quiggin [42] improved the applicability and generality of RT by extending it to the selection of the optimal alternative among several. Let $\{A_1, A_2, \ldots, A_m\}$ be a finite set of m alternatives, the result of A_i is x_i, and the DM's perceived utility for alternativeA_i can be defined as

$$U_i = v(x_i) + R(v(x_i) - v(x^*)) \tag{12}$$

where $x^* = \max\{x_i|i = 1, 2, \cdots, n\}$, and $R(v(x_i) - v(x^*))$ represents the regret value, which is always non-positive.

III. EXTENDED REGRET THEORY METHOD FOR PLCSS CONSIDERING DUAL EXPECTATIONS

A. PROBLEM DESCRIPTION AND FRAMEWORK

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a finite set of m alternatives, and $B = \{B_1, B_2, \ldots, B_n\}$ a set of n attributes, whose weight vector is $w = \{w_1, w_2, \ldots, w_n\}^T$, where $w_j \geq 0\ (j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{m} w_j = 1$. The notations $A_i\ (i \in M, M = \{1, 2, \ldots, m\})$ and $B_j\ (j \in N, N = \{1, 2, \ldots, n\})$, respectively, represent the ith alternative and jth attribute.

DMs provide their evaluation information by using $S(U) = \{C_i|i = -\tau, \ldots, 0, \ldots, \tau\}$, which can be expressed by a PLCS: $\tilde{C}_{ij}(p) = \left\{C_{ij}^{(k)}(p_{ij}^{(k)})|\ C_{ij}^{(k)} \in S(U), k \in \#C_{ij}(p)\right\}$. The PLCS denotes the attribute values over the alternative A_i with respect to the attribute B_j, where $C_{ij}^{(k)}$ is the kth cloud of $\tilde{C}_{ij}(p)$; $p_{ij}^{(k)}$ is the probability of $C_{ij}^{(k)}$, $p_{ij}^{(k)} > 0$, $k = 1, 2, \ldots, \#C_{ij}(p)$, $\sum_{k=1}^{\#C_{ij}(p)} p_{ij}^{(k)} \leq 1$, and $\#C_{ij}(p)$ is the number of cloud models in $\tilde{C}_{ij}(p)$. All the PLCSs constitute a decision-making matrix $X = [\tilde{C}_{ij}(p)]_{m\times n}$:

$$X = [\tilde{C}_{ij}(p)]_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_n \end{array} \\ \begin{bmatrix} \tilde{C}_{11}(p) & \tilde{C}_{12}(p) & \cdots & \tilde{C}_{1n}(p) \\ \tilde{C}_{21}(p) & \tilde{C}_{22}(p) & \cdots & \tilde{C}_{2n}(p) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{C}_{m1}(p) & \tilde{C}_{m2}(p) & \cdots & \tilde{C}_{mn}(p) \end{bmatrix} \end{array} \tag{13}$$

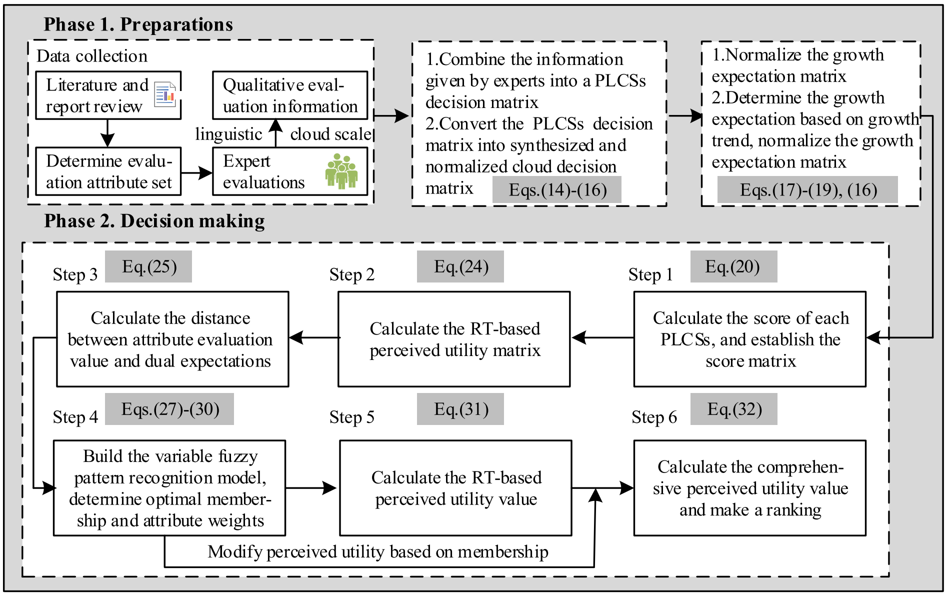

FIGURE 2. Research framework of the proposed method.

where $\tilde{C}_{ij}(p) = \left\{C_{ij}^{(k)}(p_{ij}^{(k)}) | \ k \in \#C_{ij}(p)\right\}$, $i \in m$; $j \in n$.

DMs usually choose a suitable target by considering factors such as their experience, alternative resources, and environment. We call this target expectation, and it can be understood as internal references, which reflects the effort degree of the alternative. Growth expectation mainly refers to the value of the future development trend based on the historical data, comprehensive development conditions, and the environment of the alternative being evaluated. It can be regarded as a time reference that can identify potential risks and predict future development. Let $T = [\tilde{T}_{ij}]_{m\times n}$ and $G = [\tilde{G}_{ij}]_{m\times n}$ be the target expectation matrix and growth expectation matrix, where $\tilde{T}_{ij}$ and $\tilde{G}_{ij}$ are the target and growth expectations, respectively, of alternative A_i with respect to attribute B_j. $\tilde{T}_{ij} \in S(U)$ is given directly, and $\tilde{G}_{ij}$ is obtained indirectly through calculation. Unlike the decision matrix *X, T* and *G* are composed of cloud models rather than PLCSs. The difficulty of this kind of problem lies in how to judge the merits and demerits of alternatives under various attributes according to the decision-making matrix, target expectation matrix, and growth expectation matrix, and how to achieve alternative optimization.

The research framework contains two main phases: the preparation phase and the decision making phase, as shown in Fig.2.

B. GENERATION OF EVALUATING CLOUD AND MEASUREMENT OF DUAL EXPECTATIONS

Without loss of generality, let $\tau = 3$, the linguistic term set can be taken as: $S = \{S_{-3}, S_2, S_1, S_0, S_1, S_2, S_3\}$. Given the effective universe $U = [0, 1]$, these linguistic variables can be converted to seven clouds according to the transformation method in [19]: $C_{-3} = (0, 0.2958, 0.0125)$, $C_{-2} = (0.225, 0.2656, 0.0226)$, $C_{-1} = (0.385, 0.2100, 0.0411)$, $C_0 = (0.5, 0.1922, 0.0470)$, $C_{+1} = (0.615, 0.2100, 0.0411)$, $C_{+2} = (0.775, 0.2656, 0.0226)$, $C_{+3} = (1, 0.2958, 0.0125)$. Then, the corresponding seven-label linguistic cloud scale is $S(U) = \{C_{-3} = \textit{very low}, C_{-2} = \textit{low}, C_{-1} = \textit{slightly low}, C_0 = \textit{fair}, C_{+1} = \textit{slightly high}, C_{+2} = \textit{high}, C_{+3} = \textit{very high}\}$, which is used for DMs to assess the performance of each alternative associated to each attribute.

For the evaluation values $\tilde{C}_{ij}(p) = \left\{C_{ij}^{(k)}(p_{ij}^{(k)}) | \ k = 1, \ldots, \#C_{ij}(p)\right\}$ given by DMs, three steps are necessary before decision-making: probability normalization, synthesis, and attribute value normalization.

Step 1: Normalize $\tilde{C}_{ij}(p)$ to $\tilde{C}_{ij}(\hat{p})$:

$$\tilde{C}_{ij}(\hat{p}) = \left\{C_{ij}^{(k)}(\hat{p}_{ij}^{(k)}) | \ k = 1, 2, \ldots, \#C_{ij}(p)\right\} \tag{14}$$

where $C_{ij}^{(k)} = (Ex_{ij}^{(k)}, En_{ij}^{(k)}, He_{ij}^{(k)})$, $M = \sum_{k=1}^{\#C(p)} p_{ij}^{(k)}$ and $\hat{p}_{ij}^{(k)} = p_{ij}^{(k)} \big/ M$ for all $k = 1, 2, \ldots, \#C_{ij}(p)$.

Step 2: Synthesize $\tilde{C}_{ij}(\hat{p})$ into $\tilde{C}_{ij,syn} = (Ex_{ij,syn}, En_{ij,syn}, He_{ij,syn})$.

$$\begin{cases} Ex_{ij,syn} = \hat{p}_{ij}^{(1)} Ex_{ij}^{(1)} + \hat{p}_{ij}^{(2)} Ex_{ij}^{(2)} + \cdots + p_{ij}^{(\#C(p))} Ex_{ij}^{(\#C(p))} \\ En_{ij,syn} = \sqrt{\hat{p}_{ij}^{(1)} (En_{ij}^{(1)})^2 + \cdots + p_{ij}^{(\#C(p))} (En_{ij}^{(\#C(p))})^2} \\ He_{ij,syn} = \sqrt{\hat{p}_{ij}^{(1)} (He_{ij}^{(1)})^2 + \cdots + p_{ij}^{(\#C(p))} (He_{ij}^{(\#C(p))})^2} \end{cases} \tag{15}$$

Step 3: Normalize $\tilde{C}_{ij,syn}$ to $\tilde{C}_{ij,syn,nor} = (\tilde{E}x_{ij,syn,nor}, \tilde{E}n_{ij,syn,nor} \tilde{H}e_{ij,syn,nor})$.

$$(Ex_{ij,syn,nor}, En_{ij,syn,nor}, He_{ij,syn,nor}) = \begin{cases} \left(\frac{\max_i Ex_{ij,syn} - Ex_{ij,syn}}{\max_i Ex_{ij,syn} - \min_i Ex_{ij,syn}}, \frac{En_{ij,syn}}{X_{\max} - X_{\min}}, \frac{He_{ij,syn}}{X_{\max} - X_{\min}}\right), & \text{for cost } B_j \\ \left(\frac{Ex_{ij,syn} - \min_i Ex_{ij,syn}}{\max_i Ex_{ij,syn} - \min_i Ex_{ij,syn}}, \frac{En_{ij,syn}}{X_{\max} - X_{\min}}, \frac{He_{ij,syn}}{X_{\max} - X_{\min}}\right), & \text{for benefit } B_j \end{cases} \tag{16}$$

It is clear that with the change of expectations, the cloud moves around the horizontal axis, but its sharp does not change. Particularly, for $U = [0, 1]$, $En_{ij,syn,nor} = En_{ij,syn}$ and $He_{ij,syn,nor} = He_{ij,syn}$.

In this paper, the growth level is divided into seven trends according to different development speed of attributes, which are expressed as $trend_\mu (\mu = 1, 2, \ldots, 7)$: extremely significant decline ($\downarrow\downarrow\downarrow$, $trend_1$), significant decline ($\downarrow\downarrow$, $trend_2$), slightly decline ($\downarrow$, $trend_3$), remain stable ($\rightleftarrows$, $trend_4$), slightly growth ($\uparrow$, $trend_5$), significant growth ($\uparrow\uparrow$, $trend_6$), and extremely significant growth ($\uparrow\uparrow\uparrow$, $trend_7$). We can quantify these trends by interval numbers: $[\pi_\mu^L, \ \pi_\mu^R]$.

Let $trend_{ij}$ be the growth trend for alternative A_i with respect to attribute B_j, satisfying $trend_{ij} \in \{trend_\mu | \ \mu = 1, 2, \ldots, 7\}$, with corresponding growth interval $[\pi_{ij}^L, \ \pi_{ij}^R]$ satisfying $[\pi_{ij}^L, \ \pi_{ij}^R] \in \{[\pi_\mu^L, \ \pi_\mu^R] \mid \mu = 1, 2, \ldots, 7\}$. According to the Eqs. (2) and (4), the left and right clouds of growth expectation $(\tilde{G}_{ij}^L, \ \tilde{G}_{ij}^R)$ are calculated by

$$\begin{aligned}\tilde{G}_{ij}^L &= (Ex_{ij,G}^L, \ En_{ij,G}^L, \ He_{ij,G}^L) \\ &= ((1+\pi_{ij}^L) \cdot Ex_{ij,syn}, \ \sqrt{1+\pi_{ij}^L} \cdot En_{ij,syn}, \\ &\quad \sqrt{1+\pi_{ij}^L} \cdot He_{ij,syn}) \end{aligned} \tag{17}$$

$$\begin{aligned}\tilde{G}_{ij}^R &= (Ex_{ij,G}^R, \ En_{ij,G}^R, \ He_{ij,G}^R) \\ &= ((1+\pi_{ij}^R) \cdot Ex_{ij,syn}, \ \sqrt{1+\pi_{ij}^R} \cdot En_{ij,syn}, \\ &\quad \sqrt{1+\pi_{ij}^R} \cdot He_{ij,syn}) \end{aligned} \tag{18}$$

where $Ex_{ij,syn}$, $En_{ij,syn}$, and $He_{ij,syn}$ are obtained from Eq. (15). π_{ij}^L and π_{ij}^R denote the upper and lower limits of the growth trend for alternative $A_i$with respect to attribute B_j.

Then the growth expectation $\tilde{G}_{ij}$ is

$$\begin{aligned}\tilde{G}_{ij} &= (Ex_{ij,G}, \ En_{ij,G}, He_{ij,G}) \\ &= (\frac{Ex_{ij,G}^L + Ex_{ij,G}^R}{2}, \sqrt{\frac{(En_{ij,G}^L)^2 + (En_{ij,G}^R)^2}{2}}, \\ &\quad \sqrt{\frac{(He_{ij,G}^L)^2 + (He_{ij,G}^R)^2}{2}}) \end{aligned} \tag{19}$$

C. CALCULATING THE RT-BASED PERCEIVED UTILITY MATRIX

After converting $\tilde{C}_{ij}(p)$ to $\tilde{C}_{ij,syn,nor}$ by Eqs.(14)-(16), the score of $\tilde{C}_{ij}(p)$ over A_i with respect to B_j can be obtained:

$$E(\tilde{C}_{ij}(p)) = \bar{S}(\tilde{C}_{ij,syn,nor}) \tag{20}$$

where $\bar{S}(\tilde{C}_{ij,syn,nor})$ is the contribution value of $\tilde{C}_{ij,syn,nor}$ calculated according to Fig. 1.

Definition 7: Let $E(\tilde{C}(p))$ be the quantified score of probabilistic linguistic cloud set $\tilde{C}_i(p)$. Suppose that $v(x)$ is the classical utility function, satisfying $v'(x) > 0$ and $v''(x) < 0$. Then the probabilistic linguistic cloud set utility function is defined as $v(E(\cdot))$:$\tilde{C}(p) \to v\left(E(\tilde{C}(p))\right)$.

Definition 7 implies that probabilistic linguistic cloud set utility function $v\left(E(\tilde{C}(p))\right)$ can degenerate to the classical utility function $v(x)$ if the probabilistic linguistic cloud set $\tilde{C}(p)$ is quantified as the crisp number x.

Definition 8: Let A_i be the ith alternative, $\tilde{C}_i(p)$ the evaluation of A_i, and $v(E(\cdot))$ the probabilistic linguistic cloud set utility function as defined in Definition 7. Then the regret/rejoice function based on the probabilistic linguistic cloud set is defined as

$$R_i = R\left[v\left(E(\tilde{C}_i(p))\right) - v\left(E(\tilde{C}^*(p))\right)\right], \quad i \in M, \tag{21}$$

where $\tilde{C}^*(p) = \max\{\tilde{C}_i(p) | i = 1, 2, \ldots, m\}$ is the ideal alternative or ideal point, $R(\cdot)$ is a regret/rejoice function as described in Eq. (11).

Let $\tilde{C}_{ij}(p)$ be the evaluation information in the decision matrix$X = (\tilde{C}_{ij}(p))_{m\times n}$. According to Definition 8,

$$R_{ij} = R\left[v\left(E(\tilde{C}_{ij}(p))\right) - v\left(E(\tilde{C}_j^*(p))\right)\right], \quad i \in M, \ j \in N, \tag{22}$$

where $\tilde{C}_j^*(p) = \max\{\tilde{C}_{ij}(p) | i = 1, 2, \ldots, m\}$ is the ideal point with respect to attribute B_j.

Eq. (22) indicates that the regret/rejoice function based on the PLCS is R_{ij} when choosing alternative A_i instead of the ideal point with respect to attribute B_j.

The perceived utility value over alternative A_i with respect to attribute B_j based on RT is

$$\begin{aligned}U_{ij} &= v\left(E(\tilde{C}_{ij}(p))\right) + R_{ij}\left[v\left(E(\tilde{C}_{ij}(p))\right) - v\left(E(\tilde{C}_j^*(p))\right)\right] \\ & i \in M, \quad j \in N. \end{aligned} \tag{23}$$

This paper uses $v(x) = x^\alpha$ as the utility function and $R(x) = 1 - \exp(-\delta \cdot (x))$ as the regret/rejoice function. Accordingly, Eq. (23) is transformed into

$$\begin{aligned}U_{ij} = -\exp\left\{-\delta \cdot \left[\left(E(\tilde{C}_{ij}(p))\right)^\alpha - \left(E(\tilde{C}_j^*(p))\right)^\alpha\right]\right\} \\ + \left(E(\tilde{C}_{ij}(p))\right)^\alpha + 1, \quad i \in M, \ j \in N. \end{aligned} \tag{24}$$

D. DETERMINING MEMBERSHIP AND ATTRIBUTE WEIGHTS

In this paper, a fuzzy pattern recognition model is used to determine the membership degree between each alternative and dual expectations, as well as the attribute weight vector. Before giving this model, we first need to calculate the distance between the attribute evaluation value and dual expectations. For dual expectations, they do not require probability normalization and synthesis processing, only normalization based on attribute types (Eq. (16)). Using distance measure algorithm, the distances between attribute evaluation value $\tilde{C}_{ij}(p)$ and target and growth expectations ($\tilde{G}_{ij}$ and $\tilde{T}_{ij}$) can be calculated, denoted as $d_{ij}(\tilde{C}_{ij}(p), \tilde{T}_{ij})$ and $d_{ij}(\tilde{C}_{ij}(p), \tilde{G}_{ij})$. Then, the synthetic distance between attribute evaluation value and dual expectations is

$$d_{ij}^{syn} = \varphi \, d(\tilde{C}_{ij}(p), \tilde{T}_{ij}) + (1-\varphi) \, d(\tilde{C}_{ij}(p), \tilde{G}_{ij}) \tag{25}$$

where φ is a parameter that reflects the degree to which the DMs focus on the target expectation, and it can be adjusted according to the DM's preference. A value of $\varphi = 0.5$ means the target and growth expectations are equally important.

A smaller synthetic distance d_{ij}^{syn} indicates a smaller gap between the evaluation value and dual expectations, and a greater d_{ij}^{syn} indicates a larger gap. Therefore, a two-level opposite fuzzy recognition center $Q = [q_{\theta j}]_{2\times n}(\theta = \{1, 2\})$ is established. When $\theta = 1$, then $q_{\theta j} = 0$, which represents the optimal set of attributes; when $\theta = 2$, then $q_{\theta j} = 1$, which represents the worst set of attributes. Let $Y = [y_{\theta i}]_{2\times m}$ be the membership matrix, $y_{\theta i}$ represents the membership of A_i and the fuzzy recognition center, and $y_{1i} = 1 - y_{2i}$ because of the opposition of the recognition center. Then, the weighted generalized Euclidean distance between A_i and fuzzy recognition center can be written as

$$f_i(y, w) = \sum_{\theta=1}^{2}\left\{y_{\theta i}\cdot\sqrt{\sum_{j=1}^{n}[w_j\cdot(d_{ij}^{syn} - q_{\theta j})]^2}\right\}^2 = \sum_{\theta=1}^{2}\left\{y_{\theta i}^2\cdot\sum_{j=1}^{n}[w_j\cdot(d_{ij}^{syn} - q_{\theta j})]^2\right\}, \quad (26)$$

where $w = (w_1, w_2, \cdots, w_n)^T$ is the attribute weight vector.

Obviously, a smaller $f_i(y, w)$ implies a smaller difference between A_i and dual expectations, i.e., better overall recognition of target and growth expectations. Let the set $F(y, w) = [f_1(y, w), f_2(y, w), \ldots, f_m(y, w)]$ be the difference between each alternative and the corresponding dual expectations. Inspired by [43], we built the following model to determine the optimal attribute weight vector $w^* = (w_1^*, w_2^*, \cdots, w_n^*)^T$ and membership matrix $Y^* = [y_{\theta i}^*]_{2\times m}$.

$$\begin{cases} \min\ Z = F(y, w) = \sum_{i=1}^{m} f_i(y, w) \\ \qquad = \sum_{i=1}^{m}\sum_{\theta=1}^{2}\left\{y_{\theta i}^2\cdot\sum_{j=1}^{n}[w_j\cdot(d_{ij}^{syn} - q_{\theta j})]^2\right\} \\ s.t.\ \sum_{\theta=1}^{2} y_{\theta i} = 1, \quad 0 \le y_{\theta i} \le 1 \\ \quad \sum_{j=1}^{n} w_j = 1, \quad 0 \le w_j \le 1 \end{cases} \quad (27)$$

The Lagrange relaxation function is utilized to handle this objective optimization problem.

$$L(y, w, \lambda_y, \lambda_w) = \sum_{i=1}^{m}\sum_{\theta=1}^{2}\left\{y_{\theta i}^2\cdot\sum_{j=1}^{n}[w_j\cdot(d_{ij}^{syn} - q_{\theta j})]^2\right\} - \lambda_y\left(\sum_{\theta=1}^{2} y_{\theta i} - 1\right) - \lambda_w\left(\sum_{j=1}^{n} w_j - 1\right) \quad (28)$$

where λ_y and λ_w are the Lagrange parameters.

Then, let $\partial L/\partial y = \partial L/\partial w = \partial L/\partial \lambda_y = \partial L/\partial \lambda_w = 0$, and we can obtain

$$y_{\theta i} = \left\{\sum_{\kappa=1}^{2}\left(\frac{\sum_{j=1}^{n}[w_j(d_{ij}^{syn} - q_{j\theta})]^2}{\sum_{j=1}^{n}[w_j(d_{ij}^{syn} - q_{j\kappa})]^2}\right)\right\}^{-1} \quad (29)$$

$$w_j = \left\{\sum_{\kappa=1}^{n}\left(\frac{\sum_{i=1}^{m}\sum_{\theta=1}^{2}[y_{\theta i}(d_{ij}^{syn} - q_{j\theta})]^2}{\sum_{i=1}^{m}\sum_{\theta=1}^{2}[y_{\theta i}(d_{i\kappa}^{syn} - q_{\kappa\theta})]^2}\right)\right\}^{-1} \quad (30)$$

To find the optimal membership matrix $Y^* = [y_{\theta i}^*]_{2\times m}$and weight vector $w^* = (w_1^*, w_2^*, \cdots, w_n^*)^T$, we use the cross-iteration algorithm in the variable fuzzy pattern recognition model to solve Eqs. (29) and (30), as shown below.

Cross-iteration algorithm

Steps:

1. Give the iteration accuracy ε of w, generally, let $\varepsilon = 0.0001$;
2. Arbitrarily set the initial weight vector $w^0 = \{w_1^0, w_2^0, \ldots, w_n^0\}$, satisfying $w_j \ge 0$, $\sum_{j=1}^{n} w_j = 1$;
3. Bring w^0 into Eq. (29), and obtain the corresponding initial matrix $y_{\theta i}^0$;
4. Bring matrix $y_{\theta i}^0$ into Eq. (30), and obtain vector $w^1 = \{w_1^1, w_2^1, \ldots, w_n^1\}$;
5. Compare w^1 with w^0. If $\max|w_j^1 - w_j^0| < \varepsilon$ for all $j \in N$, the iteration is ended; otherwise, continue iterating with w^1 as input weight vector;
6. Repeat steps 2-5 until $\max|w_j^{\psi} - w_j^{\psi-1}| < \varepsilon$ is satisfied after ψ iterations.

E. MODIFYING PERCEIVED UTILITY VALUE AND SELECTING OPTIMUM ALTERNATIVE

After determining the optimal attribute weight vector $w^* = (w_1^*, w_2^*, \cdots, w_n^*)^T$, we can obtain the perceived utility value $V(A_i)$ for the alternative $A_i (i \in M)$:

$$V(A_i) = \sum_{j=1}^{n} w_j^* \times U_{ij}, \quad i \in M, \quad (31)$$

where U_{ij} is calculated by Eq. (24). If we only consider the perceived utility value of the alternative, then a larger $V(A_i)$ obviously indicates a better alternative, and all alternatives can be ranked based on perceived utility values.

In the variable fuzzy pattern recognition model, when $\theta = 1$, the membership y_{1i}^* calculated by cross-iteration reflects the closeness of the alternative to dual expectations. Let $Y(A_i) = y_{1i}^*$, if we only consider the membership of the alternative, then a larger $Y(A_i)$ indicates a better alternative. In this paper, the perceived utility and membership are considered simultaneously. If the perceived utility of A_i is greater and the membership is lower, then the perceived utility of A_i

is better. However, if the deviation between the alternative and dual expectations is larger, then A_i is not optimal; and if the perceived utility of alternative A_i is smaller and the membership is higher, then although the alternative is closer to dual expectations, it has poor utility, and A_i is still not optimal. Most studies consider only the perceived utility of the alternatives. In this paper, the perceived utility of the alternative is modified using the membership between the alternative and dual expectations. The modified comprehensive perceived utility for A_i is

$$F(A_i) = \sum_{j=1}^{n} w_j^* \times U_{ij} \times Y(A_i), \quad i \in M. \tag{32}$$

According to Eq. (32), alternatives can be rank-ordered. The optimal alternative is

$$A^* = \{A_i | \max_{1 \le i \le m} F(A_i)\}. \tag{33}$$

IV. AN APPLICATION CASE STUDY

Investment decisions are becoming more complicated with the rapid development of capital markets and the uncertainty of the investment climate. Moreover, one cannot rely solely on technical analysis of relevant parameters with historical data to establish effective models for stock selection. Nonfinancial techniques that account for behavioral concerns, such as investor's expectations, psychological characteristics, and evaluation of alternatives, should also be valued. How to select a high-quality stock from multiple stocks is crucial to investors. In this paper, we examine a real example, using the proposed method to rank 12 listed stocks (A_i, $i = 1, 2, \ldots, 12$) from the Chinese food and beverage industry. A key step in choosing the best-performing investment object is to select appropriate and effective assessment attributes. Accordingly, we first consider the relevant references [1], [11], [44], [45] to provide a list of frequently-used indicators. We then appeal to financial analysts and active investors to narrow the indicators to those that are genuinely effective, as shown in Table 1.

TABLE 1. Attributes for evaluating stocks.

Criterion	Name	Definition
B_1	Return on Equity	Ratio of net income to stockholders' equity, which evaluates how much the company earns on shareholder investment.
B_2	Increase Rate of Main Business Revenue	Ratio of total operating income growth this year to total revenue of the previous year, which reflects the growth and development capacity of a company.
B_3	Current Ratio	Ratio of current assets to current liabilities, which reflects the company's liquidity and explains its ability to meet obligations when they fall due.
B_4	Debt/Equity Ratio	Ratio of total liabilities to total shareholders' equity, which indicates the relative proportion of an entity's equity and debt used to finance its assets. Also known as financial leverage.
B_5	Price/Earnings Ratio	Ratio of market price of each share of common stock to the earnings per share, which is one of the most widely-used tools for determining stock valuation.
B_6	Price/Book Ratio	Ratio of market price of each share of common stock to the book value per share, which reflects the value that market participants attach to a company's equity relative to its book value.

A. IMPLEMENTATION AND COMPUTATION

The implementation process is as follows.

Step 1: Input and process evaluation information.

Four DMs with different levels of experience and knowledge are involved in the decision-making group and provide evaluation information with the seven-label $S(U)$, the numerical characteristics of these cloud models are shown in section 3.2. The evaluation matrices are shown in Table 2. We can find that there are some blanks in the table, reflecting that DMs cannot provide relevant information. The PLCSs decision-making matrix $X = [\tilde{C}_{ij}(p)]_{12\times 6}$ can be obtained by collecting all DMs' evaluation information (see Table 3). Note that B_1, B_2, and B_3 are of benefit type, and the rest are of cost type. According to Eqs. (14)-(16), the synthesized and normalized decision-making matrix $[\tilde{C}_{ij,syn,nor}]_{12\times 6}$ can be calculated (see Table 4).

Step 2: Input and process dual expectation information.

The target expectation of a stock with respect to each attribute is characterized by using the seven-label $S(U)$. For convenient comparison and calculation, the target expectation should be normalized according to Eq. (16), the results are shown in Table 5. Moreover, we set the quantification intervals $[\pi_\mu^L, \pi_\mu^R]$ ($\mu = 1, 2, \ldots, 7$) of growth trend levels ($\downarrow\downarrow\downarrow, \downarrow\downarrow, \downarrow, \rightleftarrows, \uparrow, \uparrow\uparrow, \uparrow\uparrow\uparrow$) as $[-15\%, -10\%]$, $[-10\%, -6\%]$, $[-6\%, -2\%]$, $[-2\%, 2\%]$, $[2\%, 6\%]$, $[6\%, 10\%]$, and $[10\%, 15\%]$. The growth trend of A_i with respect to B_j is identified based on the corresponding geometric average growth rate over the past five years, as shown in Table 6. Then, the growth expectation can be calculated using Eqs. (17)-(19). It is normalized into a comparable one by Eq. (16), as shown in Table 7.

Fig. 3 shows the corresponding normalized cloud figures of attribute evaluation and dual expectations over alternative A_1 with respect to attribute B_1.

Step 3: Calculate the score of $\tilde{C}_{ij}(p)$.

Using the cloud contribution value algorithm and Eq. (20), $E(\tilde{C}_{ij}(p))$ can be calculated, as shown in Fig. 4.

Step 4: Calculate the perceived utility matrix based on RT. The ideal sequence is:

$$\begin{aligned}\tilde{C}^*(p) = (&\{C_{+3}(1.0)\}, \{C_{+3}(1.0)\}, \{C_{+3}(1.0)\},\\ &\{C_{-3}(0.75), C_{-2}(0.25)\}, \{C_{-3}(0.25), C_{-2}(0.25),\\ &C_{+1}(0.25)\}, \{C_{-3}(0.75), C_{-2}(0.25)\}).\end{aligned}$$

We set $\alpha = 0.88$ and $\delta = 0.3$ as in [29] and [33]. Then, the perceived utility matrix $U = (U_{ij})_{12\times 6}$ is obtained according to Eq. (24), as shown in Fig. 5.

TABLE 2. Cloud evaluation matrices provided by four DMs.

	B_1	B_2	B_3	B_4	B_5	B_6
A_1	$C_0, C_{-1}, C_{-1}, C_{-1}$	$C_{-3}, C_{-2}, C_{+2}, C_{+2}$	C_{-1}, C_0, C_0, C_0	$C_{+3}, C_{+2}, -, C_{+3}$	$C_{+1}, C_0, C_{+1}, -$	$C_{-2}, C_{-3}, C_{-3}, C_{-2}$
A_2	$C_{-2}, C_{-3}, C_{-3}, C_{-3}$	C_{-2}, C_0, C_0, C_{+1}	$C_{-2}, C_{-1}, C_{-2}, C_{-1}$	$C_{+2}, C_{+3}, C_{+2}, C_{+1}$	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	C_{-1}, C_0, C_{+1}, C_0
A_3	$C_{+2}, C_{+2}, C_{+1}, C_{+1}$	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	$C_{+3}, C_{+3}, C_{+3}, C_{+2}$	$C_{-2}, C_{-2}, C_{-3}, C_{-3}$	$C_{+2}, C_{+1}, C_{+2}, C_{+1}$	$C_{+3}, -, C_{+1}, C_{+2}$
A_4	C_0, C_0, C_0, C_{+1}	$C_{+1}, C_{-2}, C_{+1}, C_0$	C_0, C_{+1}, C_0, C_0	$C_{-1}, C_{-2}, C_0, C_{-1}$	$C_{-2}, C_{-3}, -, C_{-2}$	$C_{-2}, C_{-2}, C_{-1}, C_{-2}$
A_5	$C_{+1}, C_{+1}, C_0, C_{+2}$	$C_{+2}, -, C_0, C_0$	$C_{+2}, C_{+3}, C_{+2}, -$	$C_{-1}, C_0, C_{-1}, C_{-1}$	$C_{+1}, C_0, C_{-1}, C_{+1}$	$-, C_{+1}, C_{+2}, C_{+2}$
A_6	$C_{-1}, C_{-3}, -, C_{-2}$	$C_{-1}, C_{-1}, C_0, C_{+1}$	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	$C_{-3}, C_{-3}, -, C_{-2}$	$C_{+2}, -, C_{+2}, C_0$	$C_{-3}, C_{-3}, C_{-2}, C_{-3}$
A_7	$C_{+3}, C_{+2}, C_{+1}, C_{+1}$	$C_0, C_{-3}, C_{-3}, C_{-2}$	$-, C_{-3}, C_{-3}, C_{-2}$	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	C_{+2}, C_{+1}, C_0, C_0	$C_{+3}, C_{+2}, C_{+1}, C_{+3}$
A_8	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	$C_{+2}, C_{+3}, C_{+2}, C_{+2}$	$C_{-1}, C_0, C_{-2}, C_{-1}$	$C_0, C_0, C_{-2}, -$	$C_{+2}, C_{+2}, -, C_{+1}$
A_9	$C_{-1}, C_0, C_{-1}, C_{+1}$	$C_{-3}, C_{-3}, C_{-1}, C_{-2}$	C_{+1}, C_0, C_0, C_{+1}	$C_{+3}, C_{+1}, C_{+2}, C_{+2}$	$C_{+3}, C_{+3}, C_{+1}, C_{+2}$	$C_{-1}, C_{-2}, C_{-2}, C_{-1}$
A_{10}	C_{+1}, C_{+1}, C_0, C_0	$C_{+2}, C_{+2}, C_{+2}, C_{+3}$	$C_{+1}, C_{+2}, -, C_{+2}$	$C_{+2}, C_{+2}, C_{+1}, C_{+2}$	$-, C_{+2}, C_{+3}, C_{+1}$	$C_{+1}, -, C_{+1}, C_0$
A_{11}	$C_{+1}, C_{+2}, C_{+3}, C_{+2}$	$-, C_{+1}, C_1, C_0$	$C_{-1}, C_{+1}, C_0, C_{-1}$	$C_{+1}, C_{+2}, C_{+1}, -$	$C_{-1}, C_{-1}, C_{-2}, C_{-2}$	$C_{+2}, C_{+1}, C_0, C_{+2}$
A_{12}	$C_{+3}, C_{+3}, C_{+3}, C_{+3}$	$C_{+1}, C_{+2}, C_{+2}, C_{+3}$	$C_0, C_{+2}, C_{+1}, C_{+1}$	$C_{-3}, C_{-3}, C_{-2}, C_{-3}$	$C_{+3}, C_{+2}, C_{+3}, C_{+3}$	$C_{+3}, C_{+3}, C_{+3}, C_{+2}$

TABLE 3. Probabilistic linguistic cloud decision-making matrix of the group.

	B_1	B_2	B_3	…	B_5
A_1	$\{C_{-1}(0.75),C_0(0.25)\}$	$\{C_{-3}(0.25),C_{-2}(0.25),C_{+2}(0.5)\}$	$\{C_{-1}(0.25),C_0(0.75)\}$	…	$\{C_{-3}(0.5),C_{-2}(0.5)\}$
A_2	$\{C_{-3}(0.75),C_{-2}(0.25)\}$	$\{C_{-2}(0.25),C_0(0.5),C_{+1}(0.25)\}$	$\{C_{-2}(0.5),C_{-1}(0.5)\}$	…	$\{C_{-1}(0.25),C_0(0.5),C_{+1}(0.25)\}$
A_3	$\{C_{+1}(0.5),C_{+2}(0.5)\}$	$\{C_{+3}(1.0)\}$	$\{C_{+2}(0.25),C_{+3}(0.75)\}$	…	$\{C_{+1}(0.25),C_{+2}(0.25),C_{+3}(0.25)\}$
A_4	$\{C_0(0.75),C_{+1}(0.25)\}$	$\{C_{-2}(0.25),C_0(0.25),C_{+1}(0.5)\}$	$\{C_0(0.75),C_{+1}(0.25)\}$	…	$\{C_{-2}(0.5),C_{-1}(0.25)\}$
A_5	$\{C_0(0.25),C_{+1}(0.5),C_{+2}(0.25)\}$	$\{C_0(0.5),C_{+2}(0.25)\}$	$\{C_{+2}(0.5),C_{+3}(0.25)\}$	…	$\{C_{+1}(0.25),C_{+2}(0.5)\}$
A_6	$\{C_{-3}(0.25),C_{-2}(0.25),C_{-1}(0.25)\}$	$\{C_{-1}(0.5),C_0(0.25),C_{+1}(0.25)\}$	$\{C_{+3}(1.0)\}$	…	$\{C_{-3}(0.75),C_{-2}(0.25)\}$
A_7	$\{C_{+1}(0.5),C_{+2}(0.25),C_{+3}(0.25)\}$	$\{C_{-3}(0.5),C_{-2}(0.25),C_0(0.25)\}$	$\{C_{-3}(0.5),C_{-2}(0.25)\}$	…	$\{C_{+1}(0.25),C_{+2}(0.25),C_{+3}(0.5)\}$
A_8	$\{C_{+3}(1.0)\}$	$\{C_{+3}(1.0)\}$	$\{C_{+2}(0.75),C_{+3}(0.25)\}$	…	$\{C_{+1}(0.25),C_{+2}(0.5)\}$
A_9	$\{C_{-1}(0.75),C_0(0.25)\}$	$\{C_{-3}(0.5),C_{-2}(0.25),C_{-1}(0.25)\}$	$\{C_0(0.5),C_{+1}(0.5)\}$	…	$\{C_{-2}(0.5),C_{-1}(0.5)\}$
A_{10}	$\{C_0(0.5),C_{+1}(0.5)\}$	$\{C_{+2}(0.75),C_{+3}(0.25)\}$	$\{C_{+1}(0.25),C_{+2}(0.5)\}$	…	$\{C_0(0.25),C_{+1}(0.5)\}$
A_{11}	$\{C_{+1}(0.25),C_{+2}(0.5),C_{+3}(0.25)\}$	$\{C_0(0.25),C_{+1}(0.5)\}$	$\{C_{-1}(0.5),C_0(0.25),C_{+1}(0.25)\}$	…	$\{C_0(0.25),C_{+1}(0.25),C_{+2}(0.5)\}$
A_{12}	$\{C_{+3}(1.0)\}$	$\{C_{+1}(0.25),C_{+2}(0.5),C_{+3}(0.25)\}$	$\{C_0(0.25),C_{+1}(0.5),C_{+2}(0.25)\}$	…	$\{C_{+2}(0.25),C_{+3}(0.75)\}$

TABLE 4. Synthesized and normalized cloud decision-making matrix.

	B_1	B_2	B_3	B_4	B_5	B_6
A_1	(0.3788,0.2057,0.0427)	(0.3437,0.2735,0.0205)	(0.4288,0.1968,0.0456)	(0.0787,0.2862,0.0165)	(0.5724,0.2057,0.0427)	(0.9366,0.2811,0.0183)
A_2	(0.0000,0.2885,0.0156)	(0.3628,0.2171,0.0407)	(0.2493,0.2394,0.0332)	(0.2212,0.2611,0.0268)	(0.0000,0.2958,0.0125)	(0.5000,0.2013,0.0441)
A_3	(0.6768,0.2394,0.0332)	(1.0000,0.2958,0.0125)	(0.9392,0.2885,0.0156)	(0.9404,0.2212,0.0390)	(0.4220,0.2394,0.0332)	(0.1747,0.2583,0.0279)
A_4	(0.5007,0.1968,0.0456)	(0.3968,0.2212,0.0390)	(0.4910,0.1968,0.0456)	(0.6636,0.0712,0.0091)	(1.0000,0.2583,0.0279)	(0.7504,0.2486,0.0300)
A_5	(0.6040,0.2212,0.0390)	(0.5171,0.2192,0.0406)	(0.8372,0.2759,0.0198)	(0.6212,0.2057,0.0427)	(0.6520,0.2057,0.0427)	(0.2496,0.2486,0.0300)
A_6	(0.1537,0.2583,0.0279)	(0.3761,0.2057,0.0427)	(1.0000,0.2958,0.0125)	(0.9809,0.2862,0.0165)	(0.4368,0.2438,0.0327)	(1.0000,0.2885,0.0156)
A_7	(0.7364,0.2481,0.0318)	(0.0339,0.2657,0.0275)	(0.0000,0.2862,0.0165)	(0.0000,0.2958,0.0125)	(0.5569,0.2171,0.0407)	(0.1085,0.2691,0.0251)
A_8	(1.0000,0.2958,0.0125)	(1.0000,0.2958,0.0125)	(0.8177,0.2735,0.0205)	(0.6636,0.2212,0.0390)	(0.8173,0.2192,0.0406)	(0.2496,0.2486,0.0300)
A_9	(0.3788,0.2057,0.0427)	(0.0000,0.2691,0.0251)	(0.5220,0.2013,0.0441)	(0.2212,0.2611,0.0268)	(0.2110,0.2691,0.0251)	(0.7197,0.2394,0.0332)
A_{10}	(0.5311,0.2013,0.0441)	(0.8009,0.2735,0.0205)	(0.6999,0.2486,0.0300)	(0.2808,0.2528,0.0284)	(0.2923,0.2583,0.0279)	(0.6891,0.2298,0.0361)
A_{11}	(0.7788,0.2611,0.0268)	(0.5009,0.2043,0.0431)	(0.4288,0.2057,0.0427)	(0.3520,0.2298,0.0361)	(0.9615,0.2394,0.0332)	(0.3127,0.2357,0.0351)
A_{12}	(1.0000,0.2958,0.0125)	(0.7537,0.2611,0.0268)	(0.5963,0.2212,0.0390)	(1.0000,0.2885,0.0156)	(0.0778,0.2885,0.0156)	(0.0000,0.2885,0.0156)

Step 5: Calculate the membership and weight vector of attributes.

Using the distance measure algorithm, the distances between attribute evaluation values and the target and growth expectations are calculated, respectively. Taking $\varphi = 0.5$, the synthetic distance d_{ij}^{syn} is calculated (see Table 8) according to Eq. (25) and is used in the variable fuzzy pattern recognition model.

The optimal membership matrix $Y^* = [y_{\theta i}^*]_{2\times 12}$ and attribute weight vector $w^* = (w_1^*, w_2^*, \cdots, w_6^*)^T$ are

TABLE 5. Normalized target expectation matrix.

	B_1	B_2	B_3	B_4	B_5	B_6
A_1	(0.5032,0.2100,0.0411)	(0.6150,0.2100,0.0411)	(0.2065,0.2100,0.0411)	(0.2065,0.2100,0.0411)	(0.5000,0.1922,0.0470)	(0.5032,0.2100,0.0411)
A_2	(0.2065,0.2100,0.0411)	(0.5000,0.1922,0.0470)	(0.3548,0.1922,0.0470)	(0.2065,0.2100,0.0411)	(0.2909,0.2100,0.0411)	(0.7097,0.2656,0.0226)
A_3	(0.7097,0.2656,0.0266)	(1.0000,0.2958,0.0125)	(1.0000,0.2958,0.0125)	(0.7097,0.2656,0.0226)	(0.2909,0.2100,0.0411)	(0.2065,0.2100,0.0411)
A_4	(0.3548,0.1922,0.0470)	(0.5000,0.1922,0.0470)	(0.7097,0.2656,0.0226)	(0.5032,0.2100,0.0411)	(0.5000,0.1922,0.0470)	(0.7097,0.2656,0.0226)
A_5	(0.5032,0.2100,0.0411)	(1.0000,0.2958,0.0125)	(1.0000,0.2958,0.0125)	(1.0000,0.2958,0.0125)	(1.0000,0.2656,0.0226)	(0.5032,0.2100,0.0411)
A_6	(0.0000,0.2656,0.0226)	(0.6150,0.2100,0.0411)	(0.5032,0.2100,0.0411)	(0.7097,0.2656,0.0226)	(1.0000,0.2656,0.0226)	(1.0000,0.2958,0.0125)
A_7	(1.0000,0.2958,0.0125)	(0.0000,0.2958,0.0125)	(0.0000,0.2656,0.0226)	(0.2065,0.2100,0.0411)	(0.5000,0.1922,0.0470)	(0.0000,0.2656,0.0226)
A_8	(1.0000,0.2958,0.0125)	(0.6150,0.2100,0.0411)	(0.7097,0.2656,0.0226)	(1.0000,0.2958,0.0125)	(0.2909,0.2100,0.0411)	(0.0000,0.2656,0.0226)
A_9	(0.3548,0.1922,0.0470)	(0.0000,0.2958,0.0125)	(0.3548,0.1922,0.0470)	(0.3548,0.1922,0.0470)	(0.0000,0.2656,0.0226)	(0.7097,0.2656,0.0226)
A_{10}	(0.7097,0.2656,0.0226)	(0.7750,0.2656,0.0226)	(0.5032,0.2100,0.0411)	(0.3548,0.1922,0.0470)	(0.0000,0.2656,0.0226)	(0.2065,0.2100,0.0411)
A_{11}	(0.7097,0.2656,0.0226)	(0.3850,0.2100,0.0411)	(0.2065,0.2100,0.0411)	(0.0000,0.2656,0.0226)	(0.2909,0.2100,0.0411)	(0.3548,0.1922,0.0470)
A_{12}	(1.0000,0.2958,0.0125)	(0.7750,0.2656,0.0226)	(0.5032,0.2100,0.0411)	(0.7097,0.2656,0.0226)	(0.0000,0.2656,0.0226)	(0.0000,0.2656,0.0226)

TABLE 6. Growth trend of each attribute for each alternative.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
B_1	↑↑	↓	↑↑↑	↑↑	⇄	↓↓	↑↑↑	↓	↓↓↓	↑↑↑	↓	⇄
B_2	↓↓↓	↓↓↓	↑↑	↑↑↑	⇄	↑↑	↑	↑↑	↓↓↓	↓↓↓	↑	⇄
B_3	↑↑↑	↓	↑	↓	↓↓↓	↑↑↑	↑	↓	↑	↑↑	↑	↑
B_4	↓↓↓	↑	↓	↑	↑↑↑	↓↓↓	↑	↑↑	↓	⇄	↓↓	⇄
B_5	↓↓	↑↑↑	↑↑	↑	↓↓↓	↑	↓↓	↑↑↑	↑↑	⇄	↓	↑↑
B_6	↑	⇄	↑↑↑	↑	↓↓↓	↓↓↓	↑↑↑	↑↑	↓	↑↑↑	↓	⇄

TABLE 7. Normalized growth expectation matrix.

	B_1	B_2	B_3	B_4	B_5	B_6
A_1	(0.4153,0.2138,0.0443)	(0.2692,0.2558,0.0192)	(0.4323,0.2087,0.0484)	(0.2338,0.2677,0.0155)	(0.6999,0.1973,0.0409)	(0.9250,0.2867,0.0186)
A_2	(0.0000,0.2827,0.0153)	(0.2843,0.2031,0.0380)	(0.2057,0.2346,0.0325)	(0.2207,0.2663,0.0273)	(0.0000,0.3137,0.0133)	(0.5015,0.2013,0.0441)
A_3	(0.7694,0.2539,0.0352)	(1.0000,0.3074,0.0130)	(0.8630,0.2943,0.0160)	(0.9474,0.2754,0.0179)	(0.4475,0.2488,0.0345)	(0.0732,0.2739,0.0296)
A_4	(0.5466,0.2045,0.0474)	(0.4399,0.2346,0.0414)	(0.4108,0.1928,0.0447)	(0.6621,0.2256,0.0398)	(1.0000,0.2634,0.0284)	(0.7349,0.2536,0.0306)
A_5	(0.6049,0.2212,0.0390)	(0.4831,0.2192,0.0406)	(0.6355,0.2581,0.0186)	(0.5840,0.2182,0.0452)	(0.7916,0.1924,0.0399)	(0.3556,0.2326,0.0281)
A_6	(0.1387,0.2477,0.0267)	(0.3967,0.2138,0.0443)	(1.0000,0.3137,0.0133)	(0.9911,0.2677,0.0155)	(0.4941,0.2487,0.0334)	(1.0000,0.2699,0.0146)
A_7	(0.8363,0.2632,0.0337)	(0.0582,0.2710,0.0281)	(0.0000,0.2919,0.0169)	(0.0000,0.3017,0.0127)	(0.6876,0.2082,0.0390)	(0.0000,0.2854,0.0266)
A_8	(0.9577,0.2898,0.0122)	(1.0000,0.3074,0.0130)	(0.6879,0.2679,0.0201)	(0.6469,0.2299,0.0406)	(0.7943,0.2325,0.0431)	(0.1918,0.2584,0.0312)
A_9	(0.3256,0.1924,0.0399)	(0.0000,0.2517,0.0234)	(0.4797,0.2053,0.0450)	(0.2850,0.2558,0.0262)	(0.2506,0.2796,0.0260)	(0.7306,0.2346,0.0325)
A_{10}	(0.6059,0.2135,0.0468)	(0.6274,0.2558,0.0192)	(0.6707,0.2584,0.0312)	(0.3100,0.2528,0.0284)	(0.4019,0.2583,0.0279)	(0.6411,0.2438,0.0382)
A_{11}	(0.7459,0.2558,0.0262)	(0.4930,0.2083,0.0440)	(0.3941,0.2098,0.0435)	(0.4327,0.2205,0.0346)	(0.9946,0.2346,0.0325)	(0.3471,0.2309,0.0344)
A_{12}	(1.0000,0.2958,0.0125)	(0.6949,0.2611,0.0268)	(0.5479,0.2256,0.0398)	(1.0000,0.2885,0.0156)	(0.1264,0.2999,0.0163)	(0.0107,0.2885,0.0156)

computed by Eqs. (29) - (30). $Y^* = [y^*_{\theta i}]_{2\times 12}$ is shown in Table 9, and $w^* = (0.1628, 0.1879, 0.1601, 0.1767, 0.1287, 0.1838)$.

Step 6: Derive the perceived utility value of each alternative.

After obtaining w^*, the perceived utility value $V(A_i)$ for each alternative is derived according to Eq. (31). The results are listed in the second row of Table 10.

Step 7: Modify the perceived utility value of each alternative based on membership.

The values in the optimal membership matrix Y^* corresponding to $\theta = 1$ clearly represent the membership $Y(A_i)$ (third row, Table 10) between each alternative and dual expectations. The modified comprehensive perceived utility value $F(A_i)$ for each alternative is calculated by Eq. (32), as shown in the last row of Table 10.

Therefore, the ranking of the alternatives is identified as $A_3 \succ A_{12} \succ A_8 \succ A_6 \succ A_4 \succ A_{10} \succ A_{11} \succ A_1 \succ A_9 \succ A_7 \succ A_5 \succ A_2$.

B. COMPARISONS AND DISCUSSIONS

In this section, some comparisons are conducted to demonstrate the characteristics and effectiveness of the proposed method (scenario$_1$).

TABLE 8. Synthetic distance between attribute evaluation value and dual expectations.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
B_1	0.3855	0.2927	0.3631	0.4403	0.3201	0.3001	0.5324	0.1479	0.3390	0.4893	0.3390	0.1347
B_2	0.4149	0.4394	0.1010	0.3858	0.5983	0.4262	0.1922	0.4008	0.1933	0.5311	0.3410	0.2767
B_3	0.4109	0.3177	0.2891	0.5242	0.6185	0.4663	0.1493	0.4680	0.4019	0.3833	0.4150	0.3513
B_4	0.5524	0.1924	0.2641	0.3321	0.5403	0.3277	0.2538	0.4507	0.3114	0.2600	0.5794	0.3054
B_5	0.5343	0.2828	0.3061	0.5353	0.7226	0.6345	0.4971	0.6069	0.3403	0.5357	0.6824	0.2454
B_6	0.4334	0.4191	0.3655	0.2023	0.5064	0.1358	0.3864	0.4463	0.2091	0.5865	0.2740	0.1356

TABLE 9. Optimal membership matrix.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
y^*_{1i}	0.5965	0.7922	0.8643	0.7109	0.4227	0.7459	0.7991	0.6646	0.8533	0.5690	0.6525	0.9008
y^*_{2i}	0.4035	0.2078	0.1357	0.2891	0.5773	0.2541	0.2009	0.3354	0.1467	0.4310	0.3475	0.0992

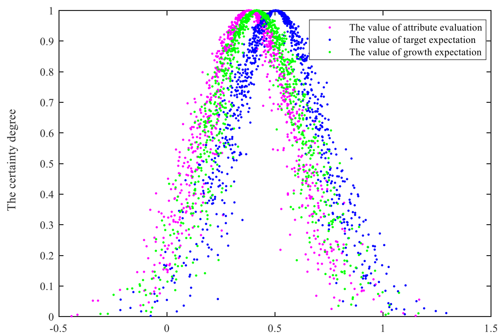

FIGURE 3. Normalized clouds of attribute evaluation and dual expectations over A_1 with respect to B_1.

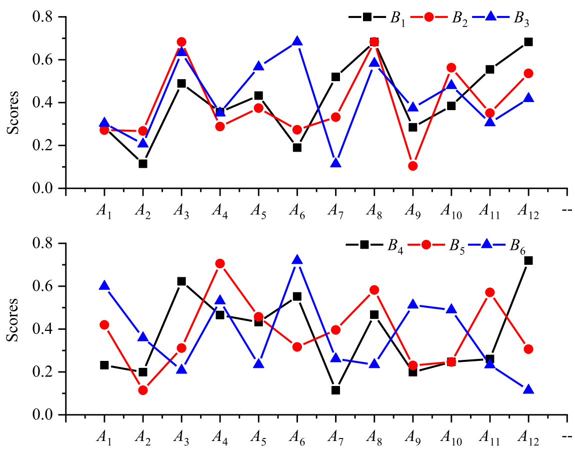

FIGURE 4. Scores of PLCSs over different alternatives with respect to different attributes.

First, for convenient comparison, we calculate the results of the other three scenarios: only consider target expectation (scenario$_2$, $\varphi = 1$), only consider growth expectation (scenario$_3$, $\varphi = 0$), and ignore dual expectations (scenario$_4$). The ranking results for all alternatives under these four scenarios are summarized in Fig. 6.

It can be seen from Fig. 6 that except for the worst-performing alternative (A_2), the alternatives have different

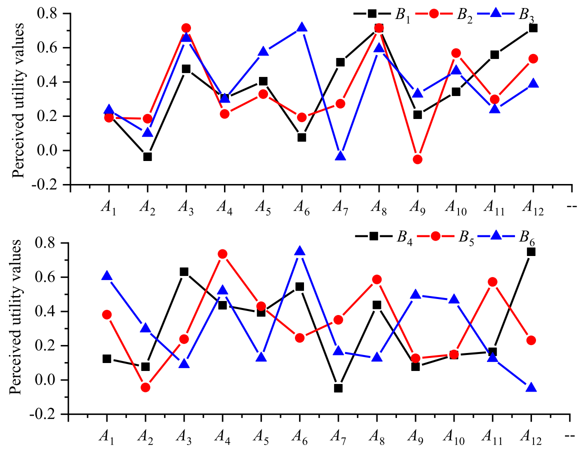

FIGURE 5. Perceived utility values of the alternatives with respect to different attributes.

ranks under these four scenarios. For example, the best alternative obtained by considering dual expectations (scenario$_1$) isA_3, which agrees with the results obtained from scenario$_2$, but it is in the fourth position under scenario$_3$. This difference is due to the memberships obtained under scenario$_1$ and scenario$_2$ that are relatively high (0.8643 and 0.9747, respectively), but the membership is relatively small (0.6754) under scenario$_3$, which results in A_3 losing its first position after modifying perceived utility with membership. In addition, scenario$_4$ represents that there is no membership amendment, and A_8 ranks first and A_3 second, which is different from the result under scenario1 thatA_3 is in the first position and $A_8$the third. This difference is mainly due to perceived utility values and memberships, as shown in Table 10. The perceived utility values of A_3 and A_8 obtained by RT are 0.4755 and 0.5221, respectively, which leads to the performance of A_8 is better than that of A_3 under the scenario$_4$. However, the memberships of A_3 and A_8 obtained by Eqs. (29)-(30) are 0.8643 and 0.6646, respectively, which means A_3 is closer to dual expectations, so when considering two aspects comprehensively, the modified comprehensive perceived utility of A_3 is greater than that ofA_8, and A_3 is superior to A_8.

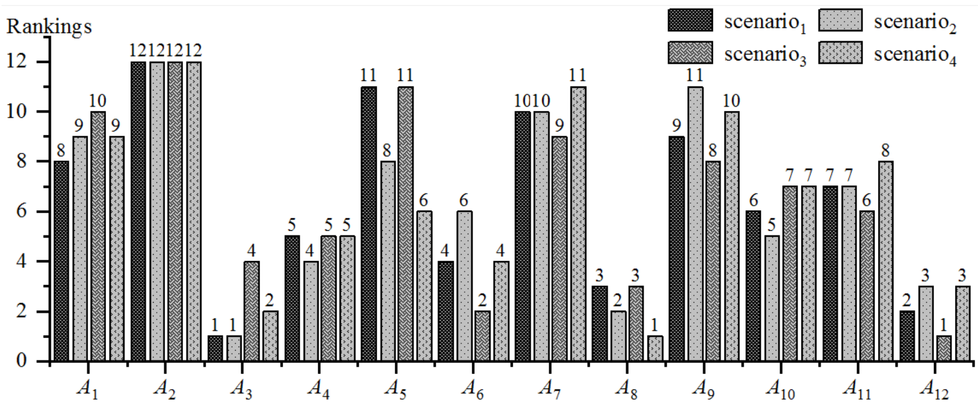

FIGURE 6. Histogram of alternative ranking under different scenarios.

TABLE 10. Perceived utility value, membership, and modified comprehensive perceived utility value.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
$V(A_i)$	0.2893	0.1074	0.4755	0.4047	0.3678	0.4285	0.1959	0.5221	0.1975	0.3676	0.3107	0.4320
$Y(A_i)$	0.5965	0.7922	0.8643	0.7109	0.4227	0.7459	0.7991	0.6646	0.8533	0.5690	0.6525	0.9008
$F(A_i)$	0.1726	0.0851	0.4110	0.2877	0.1555	0.3196	0.1565	0.3470	0.1686	0.2091	0.2027	0.3891

We also compare the proposed RT-based method to the TOPSIS-based and VIKOR-based methods used in MADM problems with PLCSs evaluation. To control the influence of dual expectations on ranking results, the membership $Y(A_i)$ is used to modify the results of the comparison methods; the modified closeness coefficient in the TOPSIS-based method is $CI_{mod}(A_i) = CI(A_i) \cdot Y(A_i)$. The distances $d(A_i, L^+)$ and $d(A_i, L^-)$ are calculated by the distance measure algorithm as in this paper. In the VIKOR-based method, a smaller $Q(A_i)$ indicates a better alternative, which is contrary to decision-making with$Y(A_i)$. So, the modified value of $Q(A_i)$ is $Q_{mod}(A_i) = Q(A_i) \cdot (1 - Y(A_i))$, a smaller value indicates a better alternative.

According to Peng *et al.* [17], a new distance can be adjusted in the cloud model. Let $C_1 = (Ex_1, En_1, He_1)$ and $C_2 = (Ex_2, En_2, He_2)$ be any two cloud models. The new distance based on Peng *et al.* [17] can be set by:

$$d(C_1, C_2) = \left| (1 - \frac{(En_1)^2 + (He_1)^2}{(En_1)^2 + (He_1)^2 + (En_2)^2 + (He_2)^2})Ex_1 - (1 - \frac{(En_2)^2 + (He_2)^2}{(En_1)^2 + (He_1)^2 + (En_2)^2 + (He_2)^2})Ex_2 \right|$$

For convenient comparison with the distance measured in [17], other computations are consistent with the method in this paper. The calculations and ranking results are summarized in Table 11.

From Table 11, it can be seen that the optimal alternative determined by the proposed method is consistent with that from the VIKOR-based method, which illustrates the effectiveness of the proposed method for selecting the optimal alternative. However, there are some differences in ranking among these three methods. We can explain the results as follows. The RT-based method has a prominent feature in considering the regret aversion of DMs. For example, the perceived utility values shown in Fig. 5 indicate that investors will feel more regretful if they select A_2 instead of A_5, resulting in $A_5 \succ A_2$, and the TOPSIS-based method produces the same result, while the VIKOR-based method ranks $A_2 \succ A_5$. In addition, it is necessary to identify the positive and negative ideal points in the TOPSIS-based method without calculating the score of each PLCS. However, only the positive ideal point is necessary for the RT-based method, and the calculation of PLCSs scores is indispensable, which is also a key step in the VIKOR-based method. Therefore, the setting of the ideal reference points is different among these methods, with a non-negligible effect of the results. Our method takes full advantage of the information in PLCSs and reflects the DM's psychological behavior, which is more reasonable in real decision-making.

The best/worst alternative by the method in [17] is consistent with the proposed method, this means that the proposed distance measure algorithm and the distance in [17] can both select the best/worst alternative, largely because the two methods fully utilize the three digital characteristics of cloud models.

The above analysis illustrates that our method can reflect the influence of dual expectations on decision-making results and ignore errors caused by single or no reference expectations. Our method also accounts for the distribution characteristics of cloud droplets and fully uses the information in PLCSs. Another prominent feature of the developed method is that it considers DM's psychological behavior. Thus ranking results obtained by our method is more accurate and convincing.

TABLE 11. Calculation results obtained by different methods.

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
Panel A: TOPSIS-based method												
$d(A_i, L^+)$	0.5619	0.7638	0.3642	0.3867	0.4373	0.3419	0.7800	0.2611	0.6571	0.4522	0.4832	0.3778
$d(A_i, L^-)$	0.4604	0.2403	0.6566	0.6351	0.5710	0.6714	0.2246	0.7558	0.3521	0.5668	0.5426	0.6135
$CI(A_i)$	0.4504	0.2393	0.6432	0.6216	0.5663	0.6626	0.2236	0.7432	0.3489	0.5563	0.5290	0.6189
$CI_{mod}(A_i)$	0.2687	0.1896	0.5560	0.4419	0.2394	0.4942	0.1786	0.4940	0.2977	0.3165	0.3452	0.5575
Ranking	$A_{12} \succ A_3 \succ A_6 \succ A_8 \succ A_4 \succ A_{11} \succ A_{10} \succ A_9 \succ A_1 \succ A_5 \succ A_2 \succ A_7$											
Panel B: VIKOR-based method												
$S(A_i)$	0.5956	0.8220	0.3392	0.4466	0.4900	0.4079	0.7042	0.2760	0.7074	0.4896	0.5619	0.3927
$R(A_i)$	0.1426	0.1628	0.1555	0.1282	0.1474	0.1412	0.1768	0.1474	0.1879	0.1381	0.1478	0.1838
$Q(A_i)$	0.4132	0.7900	0.2865	0.1562	0.3567	0.2293	0.7989	0.1607	0.8951	0.2782	0.4261	0.5728
$Q_{mod}(A_i)$	0.1667	0.1641	0.0389	0.0452	0.2059	0.0583	0.1605	0.0539	0.1313	0.1199	0.1480	0.0568
Ranking	$A_3 \succ A_4 \succ A_8 \succ A_{12} \succ A_6 \succ A_{10} \succ A_9 \succ A_{11} \succ A_7 \succ A_2 \succ A_1 \succ A_5$											
Panel C: Peng's method [17]												
$F(A_i)$	0.1313	0.0928	0.3787	0.3572	0.0955	0.2503	0.1449	0.2724	0.1739	0.1613	0.1908	0.3663
Ranking	$A_3 \succ A_{12} \succ A_4 \succ A_8 \succ A_6 \succ A_{11} \succ A_9 \succ A_{10} \succ A_7 \succ A_1 \succ A_5 \succ A_2$											

V. CONCLUSION

MADM methods are crucial in effective investment decisions, but the existing research has paid little attention to nonfinancial parameters such as DMs' psychological behavior and expectations for the future. From the perspective of regret aversion/rejoice preference, we propose an extended RT decision-making method considering dual expectations to identify and rank-order superior stocks in a complex and uncertain investment environment. The contributions of this study are highlighted in the following four aspects:

(1) Considering the uncertainty and vagueness of the decision-making environment and humans' limited cognition, we define PLCSs as an extension of cloud models, which can ideally disclose the fuzziness of qualitative concepts and effectively handle information loss and distortion that occurs in the information fusion process.

(2) RT is extended to the PLCSs environment to enable calculation of DMs' fuzzy perceived utility value with respect to each alternative. This means that RT is extended to a generalized level, and the effect of some psychological behavior such as regret aversion can be quantified when selecting from multiple alternatives in an uncertain decision environment.

(3) Target and growth expectations are noted to ensure fair, accurate, and suitable results. A proposed distance measure algorithm considers the characteristics of cloud droplet distribution to calculate the distance between attribute evaluation values and target and growth expectations.

(4) A fuzzy pattern recognition model is built for obtaining the optimal membership and optimal attribute weights. A Lagrange relaxation function is constructed to solve the model. RT-based perceived utility is modified based on the optimal membership to obtain the comprehensive perceived utility, which is used for determining the ranking of alternatives.

A case analysis shows that our method provides more feasible and effective outcomes that are consistent with the psychological behavior of human beings. In further research, it would be interesting to consider other behavioral characteristics of DMs, along with nonfinancial parameters, so as to extend the proposed method to a multi-stage decision-making process. We also would like to apply multiple attribute decision-making models to efficient portfolio construction.

REFERENCES

[1] J.-Y. Dong, Y. Chen, and S.-P. Wan, "A cosine similarity based QUALIFLEX approach with hesitant fuzzy linguistic term sets for financial performance evaluation," *Appl. Soft Comput.*, vol. 69, pp. 316–329, Aug. 2018.

[2] M. Ferrara, S. Rasouli, M. Khademi, and M. Salimi, "A robust optimization model for a decision-making problem: An application for stock market," *Oper. Res. Perspect.*, vol. 4, pp. 136–141, Oct. 2017.

[3] A. Hatami-Marbini and F. Kangi, "An extension of fuzzy TOPSIS for a group decision making with an application to Tehran stock exchange," *Appl. Soft Comput.*, vol. 52, pp. 1084–1097, Mar. 2017.

[4] H. Dincer, U. Hacioglu, E. Tatoglu, and D. Delen, "A fuzzy-hybrid analytic model to assess investors' perceptions for industry selection," *Decis. Support Syst.*, vol. 86, pp. 24–34, Jun. 2016.

[5] Y. Wu, C. Xu, and T. Zhang, "Evaluation of renewable power sources using a fuzzy MCDM based on cumulative prospect theory: A case in China," *Energy*, vol. 147, pp. 1227–1239, Mar. 2018.

[6] Ö. Uygun and A. Dede, "Performance evaluation of green supply chain management using integrated fuzzy multi-criteria decision making techniques," *Comput. Ind. Eng.*, vol. 102, pp. 502–511, Dec. 2016.

[7] L. Wang, Z.-X. Zhang, and Y.-M. Wang, "A prospect theory-based interval dynamic reference point method for emergency decision making," *Expert Syst. Appl.*, vol. 42, pp. 9379–9388, Dec. 2015.

[8] P. Liu and F. Teng, "Probabilistic linguistic TODIM method for selecting products through online product reviews," *Inf. Sci.*, vol. 485, pp. 441–455, Jun. 2019.

[9] W.-R. J. Ho, C.-L. Tsai, G.-H. Tzeng, and S.-K. Fang, "Combined DEMATEL technique with a novel MCDM model for exploring portfolio selection based on CAPM," *Expert Syst. Appl.*, vol. 38, pp. 16–25, Jan. 2011.

[10] M. Fasanghari and G. A. Montazer, "Design and implementation of fuzzy expert system for Tehran Stock Exchange portfolio recommendation," *Expert Syst. Appl.*, vol. 37, pp. 6138–6147, Sep. 2010.

[11] A. M. Yaakob, A. Gegov, and S. F. A. Rahman, "Selection of alternatives using fuzzy networks with rule base aggregation," *Fuzzy Sets Syst.*, vol. 341, pp. 123–144, Jun. 2018.

[12] P. Xidonas, G. Mavrotas, and J. Psarras, "Portfolio construction on the Athens Stock Exchange: A multiobjective optimization approach," *Optimization*, vol. 59, no. 8, pp. 1211–1229, 2010.

[13] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Inf. Sci.*, vol. 369, pp. 128–143, Nov. 2016.

[14] Z.-P. Tian, R.-X. Nie, and J.-Q. Wang, "Probabilistic linguistic multi-criteria decision-making based on evidential reasoning and combined ranking methods considering decision-makers' psychological preferences," *J. Oper. Res. Soc.*, vol. 7, pp. 1–18, Jul. 2019.

[15] S.-X. Chen, J.-Q. Wang, and T.-L. Wang, "Cloud-based ERP system selection based on extended probabilistic linguistic MULTIMOORA method and Choquet integral operator," *Comput. Appl. Math.*, vol. 38, p. 88, Jun. 2019.

[16] S.-Z. Luo, H.-Y. Zhang, J.-Q. Wang, and L. Li, "Group decision-making approach for evaluating the sustainability of constructed wetlands with probabilistic linguistic preference relations," *J. Oper. Res. Soc.*, vol. 70, no. 12, pp. 2039–2055, 2019.

[17] H.-G. Peng, H.-Y. Zhang, and J.-Q. Wang, "Cloud decision support model for selecting hotels on TripAdvisor.com with probabilistic linguistic information," *Int. J. Hospitality Manage.*, vol. 68, pp. 124–138, Jan. 2018.

[18] D. Li, C. Liu, and W. Gan, "A new cognitive model: Cloud model," *Int. J. Intell. Syst.*, vol. 24, no. 3, pp. 357–375, Mar. 2009.

[19] J.-Q. Wang, L. Peng, H.-Y. Zhang, and X.-H. Chen, "Method of multi-criteria group decision-making based on cloud aggregation operators with linguistic information," *Inf. Sci.*, vol. 274, pp. 177–191, Aug. 2014.

[20] J.-Q. Wang, J.-J. Peng, H.-Y. Zhang, T. Liu, and X.-H. Chen, "An uncertain linguistic multi-criteria group decision-making method based on a cloud model," *Group Decis. Negotiation*, vol. 24, pp. 171–192, Jan. 2015.

[21] H.-G. Peng and J.-Q. Wang, "Cloud decision model for selecting sustainable energy crop based on linguistic intuitionistic information," *Int. J. Syst. Sci.*, vol. 48, no. 15, pp. 3316–3333, 2017.

[22] J. Li, H. Fang, and W. Song, "Modified failure mode and effects analysis under uncertainty: A rough cloud theory-based approach," *Appl. Soft Comput.*, vol. 78, pp. 195–208, May 2019.

[23] X. Li and X. Chen, "Value determination method based on multiple reference points under a trapezoidal intuitionistic fuzzy environment," *Appl. Soft Comput.*, vol. 63, pp. 39–49, Feb. 2018.

[24] L. Wang, X.-K. Wang, J.-J. Peng, and J. Q. Wang, "The differences in hotel selection among various types of travellers: A comparative analysis with a useful bounded rationality behavioural decision support model," *Tourism Manage.*, vol. 76, Feb. 2020, Art. no. 103961.

[25] D. Walczak and A. Rutkowska, "Project rankings for participatory budget based on the fuzzy TOPSIS method," *Eur. J. Oper. Res.*, vol. 260, pp. 706–714, Jul. 2017.

[26] J. H. Kim and B. S. Ahn, "Extended VIKOR method using incomplete criteria weights," *Expert Syst. Appl.*, vol. 126, pp. 124–132, Jul. 2019.

[27] H.-G. Peng, K.-W. Shen, S.-S. He, H.-Y. Zhang, and J.-Q. Wang, "Investment risk evaluation for new energy resources: An integrated decision support model based on regret theory and ELECTRE III," *Energy Convers. Manage.*, vol. 183, pp. 332–348, Mar. 2019.

[28] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econ., J. Econ. Soc.*, vol. 47, no. 2, pp. 263–291, 1979.

[29] A. Tversky and D. Kahneman, "Advances in prospect theory: Cumulative representation of uncertainty," *J. Risk Uncertainty*, vol. 5, pp. 297–323, Oct. 1992.

[30] G. Loomes and R. Sugden, "Regret theory: An alternative theory of rational choice under uncertainty," *Econ. J.*, vol. 92, no. 368, pp. 805–824, Dec. 1982.

[31] D. E. Bell, "Regret in decision making under uncertainty," *Oper. Res.*, vol. 30, no. 5, pp. 961–981, 1982.

[32] M. Nwogugu, "A further critique of cumulative prospect theory and related approaches," *Appl. Math. Comput.*, vol. 179, pp. 451–465, Aug. 2006.

[33] S. Zhang, J. Zhu, X. Liu, and Y. Chen, "Regret theory-based group decision-making with multidimensional preference and incomplete weight information," *Inf. Fusion*, vol. 31, pp. 1–13, Sep. 2016.

[34] S. Michenaud and B. Solnik, "Applying regret theory to investment choices: Currency hedging decisions," *J. Int. Money Finance*, vol. 27, pp. 677–694, Sep. 2008.

[35] A. Dodonova and Y. Khoroshilov, "Applications of regret theory to asset pricing," Telfer School Manage., Univ. Ottawa, Ottawa, ON, Canada, Working Paper, 2005.

[36] D. Rai, G. K. Jha, P. Chatterjee, and S. Chakraborty, "Material selection in manufacturing environment using compromise ranking and regret theory-based compromise ranking methods: A comparative study," *Universal J. Mater. Sci.*, vol. 1, no. 2, pp. 69–77, 2013.

[37] X. Zhang, H.-M. Liang, F. Fang, and X. Chen, "A method for risky multiple attribute decision making considering regret and rejoicing of the decision maker," *Comput. Ind. Eng.*, vol. 124, pp. 422–434, Oct. 2018.

[38] P. Ji, H.-Y. Zhang, and J.-Q. Wang, "A fuzzy decision support model with sentiment analysis for items comparison in e-commerce: The case study of PConline.com," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 10, pp. 1993–2004, Oct. 2019.

[39] W. Chen, M. Goh, and Y. Zou, "Logistics provider selection for omni-channel environment with fuzzy axiomatic design and extended regret theory," *Appl. Soft Comput.*, vol. 71, pp. 353–363, Oct. 2018.

[40] Z. Xu, "Deviation measures of linguistic preference relations in group decision making," *Omega*, vol. 33, no. 3, pp. 249–254, 2005.

[41] C. E. Laciana and E. U. Weber, "Correcting expected utility for comparisons between alternative outcomes: A unified parameterization of regret and disappointment," *J. Risk Uncertainty*, vol. 36, no. 1, pp. 1–17, Feb. 2008.

[42] J. Quiggin, "Regret theory with general choice sets," *J. Risk Uncertainty*, vol. 8, pp. 153–165, Mar. 1994.

[43] Z. S. Xu, *Uncertain Multi-Attribute Decision Making: Methods and Applications*. Beijing, China: Tsinghua Univ. Press, 2004.

[44] K. Rezaie, V. M. Dalfard, L. Hatami-Shirkouhi, and S. Nazari-Shirkouhi, "Efficiency appraisal and ranking of decision-making units using data envelopment analysis in fuzzy environment: A case study of Tehran stock exchange," *Neural Comput. Appl.*, vol. 23, pp. 1–17, Dec. 2013.

[45] F. Tiryaki and M. Ahlatcioglu, "Fuzzy stock selection using a new fuzzy ranking and weighting algorithm," *Appl. Math. Comput.*, vol. 170, pp. 144–157, Nov. 2005.

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