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A Bayesian Approach for Degradation Analysis with Individual Differences

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ABSTRACT Reliability assessments of long-life and high-reliability products often face the difficulty of lack of failure time data. Meanwhile, there existing individual differences in produces due to the uncertainties during their whole lifetime, such as manufacturing, assembly, work environment and work load. This paper develops a reliability modeling and assessment method considering individual differences based on performance degradation data. Firstly, the model selection method is employed into the analysis of degradation process. And according to the model selection results, the appropriate degradation model for the degradation process can be chosen. Then, the Bayesian method is applied to establish the parameter estimation method and reliability assessment framework based on the appropriate model. At last, a real engineer example is presented to illustrate the effectiveness of the proposed degradation analysis method with individual differences.

INDEX TERMS Bayesian method, uncertainty, degradation analysis, reliability assessment, individual differences.

I. INTRODUCTION

Heavy-duty machine tools are widely used in the fields of precision manufacturing and assembly, it has crucial impact on the productivity and quality of the products [1]–[3]. The spindle system is an important composing subsystems of heavy-duty machine tools. After decades of development, the useful life and reliability of the spindle system have been greatly improved. This characteristic of spindle system makes the failure time data is difficult to be obtained for an acceptable time and high-confidence reliability assessments, which poses a challenge to traditional reliability assessment methods based on failure data [4], [5]. The failure of spindle system could be attributed to the degradation of the performance characteristics of the components due to the influence of covariates such as service time, working environment and working conditions. Therefore, the degradation analysis could be introduced into reliability assessment for spindle system [6]. In practically, due to the uncertainty of the working environment of the system, the random error in the measurement and the uncertainty of working load, stochastic dynamic is common characteristic of the degradation process. Stochastic process models have great potential for capturing stochastic dynamics in the degradation process, stochastic process models are often used for time-varying degradation processes [7]–[10].

The Wiener process model and the Gamma process model are the two most widely used stochastic process models, while the inverse Gaussian (IG) process is a relatively new stochastic process model for characterizing the degradation process [11]–[13]. Chhikara and Folks [14] introduced Wiener process into produce degradation analysis, and proposed the lifetime distribution of Wiener process obeys IG distribution. Pan *et al.* [15] employed Wiener process model with truncated normal distribution to characterize the degradation process and applied EM algorithms for parameters estimation. Hu *et al.* [16] employed the Wiener process to characterize wind turbine bearing performance degradation process, and proposed the wind turbine bearing residual life prediction method based on the Wiener process model. Abdel-Hameed [17] introduced Gamma process into products reliability analysis to characterize the time-varying degradation processes in 1975. Ye *et al.* [18] studied the semiparametric inference and random effect variables of Gamma process, and obtained estimation of the parameters by EM algorithm. Guida and Penta [19] used the Gamma process model to characterize fatigue-crack-growth and analyzes correlation of model parameters by

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Paris model. The inverse Gaussian process was introduced into degradation analysis by Wang and Xu [20] in 2010. Then, Ye and Chen [21] studies theoretical basis and application of IG process as degradation model and estimates unknown model parameters by maximum likelihood estimation method. Peng *et al.* [22] developed the inverse Gaussian model to characterize product degradation process considers the influence of the time-varying rate.

Although the same type of spindle system of heavy-duty machine tools have commonalities in design, the reliability of the same type of spindle system shows significant individual differences due to uncertainties such as working environment and working load [23]–[25]. To obtain high-confidence reliability assessment results, the individual differences of spindle system should be characterized in degradation analysis methods [26]. For characterizing individual differences, random effects model is generally introduced in the stochastic process model. To this end, some of the parameters in the stochastic process are defined as hyper-parameters, obey some probability distributions [27]. Lawless and Crowder [28] integrated random effects model into Gamma process model. They argued that the individual differences in the produce affect the scale parameters. Wang [29] integrated the random effects model into the basic Wiener process, and EM algorithm is employed to obtain estimation of unknown parameters. For characterizing individual differences, the IG process model with random effect model was also discussed. On the basis of their research, the application of these three stochastic process models integrated random effect model has gradually developed in the degradation analysis [30]–[34].

However, researchers usually used the maximum likelihood estimation method or the EM algorithm for parameter estimation rather than the Bayesian method. Bayesian method has advantages in dealing with small sample size problems and uncertainties [35]–[39]. Moreover, researchers generally directly designated a degradation model to describe the degradation process. They rarely consider using model selection methods to select the most appropriate model. It may lead to misjudgment of the degradation model and errors in the reliability assessment results [40], [41].

Based on the actual needs and characteristics of spindle system, three stochastic process models considering individual differences are used to describe the performance degradation process. The deviance information criterion (DIC) is used as a model selection method for selecting the most suitable one from the three degradation models. A Bayesian framework is developed for the parameter estimation with Markov Chain Monte Carlo (MCMC) method.

The remainder of the article is organized as follows. The degradation process considering the individual differences is presented in Section 2. Section 3 presents the parameter estimation and model selection via Bayesian method. Section 4 demonstrates degradation process considering the individual differences through a real case. Conclusions and possible future directions are discussed in Section 6.

II. DEGRADATION PROCESS

A. WIENER PROCESS WITH RANDOM EFFECTS

Wiener process with drift parameter μ and diffusion parameter σ is employed for degradation process $\{Y(t), t > 0\}$. The independent degradation increment $\Delta Y(t)$ obeys the normal distribution as $N\left(\mu\Delta\tau(t), \sigma^2\Delta\tau(t)\right)$. The PDF of the normal distribution for degradation increment $\Delta Y(t)$ with $Y(0) = 0$ is

$$f(\Delta Y(t)|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi\Delta\tau(t)}}\exp\left(-\frac{(\Delta Y(t) - \mu\Delta\tau(t))^2}{2\sigma^2\Delta\tau(t)}\right) \quad (1)$$

In order to characterize the individual differences of products, Wang [29] introduced the random effects model into the Wiener process through the drift parameter μ and diffusion parameter σ. To indicate the integration of random effects, the Wiener process is modified by

$$\omega = \sigma^{-2} \sim Gamma\left(r^{-1}, \delta\right), \mu|\omega \sim \mathrm{N}(1, \theta/\omega) \quad (2)$$

where ω has the mean δ/r with variance δ/r^2. Accordingly, the marginal density of $Y(t)$can be expressed as

$$f(y) = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(y;\mu,\omega)g_1(\mu;\theta,\omega)\,g_2(\omega;r,\delta)\,\mathrm{d}\omega\mathrm{d}\mu = \frac{\Gamma\left(\delta+\frac{1}{2}\right)}{\sqrt{2\pi r}\Gamma(\delta)\left[\tau^2\theta+\tau\right]^{\frac{1}{2}}}\left[1+\frac{(y-\tau)^2}{2r\left(\tau^2\theta+\tau\right)}\right]^{-\delta-\frac{1}{2}} \quad (3)$$

Let C denote the pre-specified threshold and T denote the failure time, then $T = \inf\{t : Y(t) \geq C\}$. The failure time T follows the IG distribution as $T \sim \mathrm{IG}\left(C/\mu, C^2\omega\right)$. The reliability function can be obtained as

$$\begin{aligned} R(t) &= P(t \leq T) = P(Y(t) < C) \\ &= 1 - P(T \leq t) = 1 - P(Y(t) > C) \\ &= 1 - F_{2\delta}\left[\sqrt{\frac{\delta}{r}}\frac{\tau(t) - C}{\sqrt{\theta\tau(t)^2 + \tau(t)}}\right] \end{aligned} \quad (4)$$

B. GAMMA PROCESS WITH RANDOM EFFECTS

Gamma process with shape parameter η and scale parameter λ is employed for degradation process$\{Y(t), t > 0\}$. The independent degradation increment $\Delta Y(t)$ obeys the Gamma distribution as $Ga(\Delta\eta(t), \lambda)$. The PDF of the Gamma distribution for degradation increment $\Delta Y(t)$ with $Y(0) = 0$ is

$$f(\Delta Y(t)|\eta(t), \lambda) = \frac{\lambda^{\eta(t)}\Delta Y(t)^{\eta(t)-1}e^{-\lambda\Delta Y(t)}}{\Gamma(\eta(t))}I_{(0,\infty)}(\Delta Y(t)) \quad (5)$$

where $\Gamma(\eta) = \int_0^\infty x^{\eta-1}e^{-x}\mathrm{d}x$ is the gamma function.

Since individual differences of products only affect the scale parameter of the Gamma process, shape parameter redefined as $\Delta\eta(t) = \eta\Delta t$ with scale parameter $\lambda = v^{-1}$. Hence, the degradation increments of the model obeys the new Gamma distribution $\Delta Y(t) \sim Ga\left(\eta\Delta t, v^{-1}\right)$ with $v \sim Ga\left(\gamma^{-1}, \delta\right)$ [28]. The marginal density of $Y(t)$can be expressed as

$$f(Y) = \int_0^\infty f\left(Y \mid \eta\Delta t, v^{-1}\right) f\left(v_i^{-1} \middle| \gamma^{-1}, \delta\right) \mathrm{d}v = \frac{B(\eta\Delta t, \delta)^{-1} \gamma^{\delta} Y^{\eta\Delta t - 1}}{(Y+\gamma)^{\eta\Delta t + \delta}} \tag{6}$$

where $B(\eta\Delta t, \delta) = \Gamma(\eta\Delta t)\Gamma(\delta)/\Gamma(\eta\Delta t + \delta)$ is Beta function, $\delta Y(t)/(\gamma\eta\Delta t)$ obeys the F-distribution and $Y(t)/(\gamma + Y(t))$ follows the Beta distribution.

The reliability function of the produce can be expressed as

$$\begin{aligned} R(t) &= P(t < T) = P(Y(t) < C) \\ &= 1 - P(T \le t) = 1 - P(Y(t) \ge C) \\ &= 1 - \frac{B\left(\frac{C}{C+\gamma}; \eta t, \delta\right)}{B(\eta t, \delta)} = F_{2\eta n, 2\delta}\left(\frac{\delta C}{\gamma \eta t}\right) \end{aligned} \tag{7}$$

C. IG PROCESS WITH RANDOM EFFECTS

Employing IG Process for characterizing the performance degradation process, the independent degradation increment $\Delta Y(t)$ obeys the IG distribution as IG $\left(\Delta\Lambda(t), \eta\Delta\Lambda(t)^2\right)$. The PDF of the IG distribution for degradation increment $\Delta Y(t)$ with $Y(0) = 0$ is

$$g(\Delta y(t) \mid \Lambda(t), \eta) = \sqrt{\frac{\eta\Delta\Lambda(t)^2}{2\pi\Delta y(t)^3}} \exp\left(-\frac{\eta(\Delta y(t) - \Delta\Lambda(t))^2}{2\Delta y(t)}\right) \tag{8}$$

In order to describe the individual differences of products in the IG process, it is also necessary introduced the random effects model into the IG process model. Redefining parameters η obeys a Gamma distribution $Ga\left(\gamma^{-1}, \delta\right)$ with mean δ/γ and variance δ/γ^2. Therefore, the marginal density of $Y(t)$can be expressed as

$$f(Y) = \int_0^\infty f(Y \mid \eta) g(\eta) \mathrm{d}\eta = \frac{\Gamma\left(\delta + \frac{1}{2}\right)\gamma^{\delta}}{\Gamma(\delta)\sqrt{2\pi}} \Lambda\Delta t Y^{-\frac{3}{2}} \left[\gamma + \frac{(Y - \Lambda\Delta t)^2}{2Y}\right]^{-\delta - \frac{1}{2}} \tag{9}$$

When this stochastic process model is used to describe the performance evolution of degradation produce, reliability function can be expressed as

$$R(t \mid \Lambda, \gamma, \delta) = \int_{\eta > 0} R(t \mid \Lambda, \eta) g(\eta \mid \gamma, \delta) \mathrm{d}\eta \tag{10}$$

III. PARAMETER ESTIMATION AND MODEL SELECTION

A. BASIC FRAMEWORK OF BAYESIAN PARAMETER ESTIMATION

As illustrated in Figure 1, there is the basic framework of Bayesian parameter estimation.

The stochastic process model is a good tool for the establishment of degradation models. Furthermore, the random effects model is integrated into the stochastic process model for characterizing individual differences in produces. The key step is choosing the appropriate stochastic process model to accurately describe the degradation process. For the construction of likelihood functions, there are three types of model parameters. The fixed parameters θ^F, the parameters θ^R with random effects model and the hyper-parameters θ^H of probability distributions.

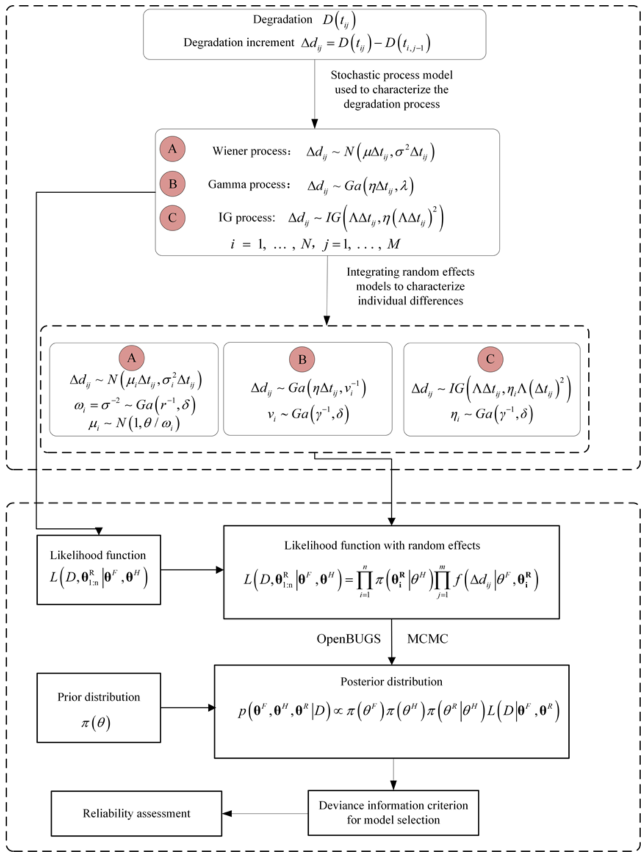

FIGURE 1. Basic framework of Bayesian parameter estimation.

For the acquisition of posterior distribution, the key lies in the quantification and acquisition of prior information. The subjective information and the historical experience information are usually quantified to the informative prior distribution. Under the absence of prior information, the non-informative prior distribution will be used. Then, the prior distribution and likelihood function are integrated via Bayesian method and MCMC method to obtain posterior distribution.

B. BAYESIAN PARAMETER ESTIMATION

Suppose that the degradation observations of N samples with serial number $i, i = 1, \ldots, N$ are observed. All samples were observed in discrete times M with serial number $j, j = 1, \ldots, M$. The degradation $D\left(t_{ij}\right)$ of the ith sample at the jth observation has independent degradation increments $\Delta d_{ij} = D\left(t_{ij}\right) - D\left(t_{i,j-1}\right)$.

1) BAYESIAN PARAMETER ESTIMATION OF WIENER PROCESS WITH RANDOM EFFECTS

Integrating random effects into Wiener process for characterizing individual differences in produces, the degradation increment Δd_{ij} follows the normal distribution $\Delta d_{ij} \sim \mathrm{N}\left(\mu_i \Delta t_{ij}, \sigma_i^2 \Delta t_{ij}\right)$ with $\omega_i = \sigma^{-2} \sim Ga\left(r^{-1}, \delta\right)$ and $\mu_i \sim N\left(1, \theta/\omega_i\right)$. The likelihood function can be obtained as follows

$$\begin{aligned} & L\left(D, \boldsymbol{\mu}, \boldsymbol{\omega} \mid r, \delta, \theta\right) \\ & \quad = \prod_{i=1}^{N} g_1\left(\mu_i \mid \theta, \omega_i\right) g_2\left(\omega_i \mid r, \delta\right) \prod_{j=2}^{M} f\left(\Delta d_{ij} \mid \mu_i, \omega_i\right) \\ & \quad = \prod_{i=1}^{N}\prod_{j=2}^{M}\left[\frac{\Gamma\left(\delta+\frac{1}{2}\right)}{\sqrt{2\pi r}\,\Gamma(\delta)\left[\left(\Delta t_{ij}\right)^2\theta+\Delta t_{ij}\right]^{\frac{1}{2}}}\right] \\ & \quad \left[1+\frac{\left(\Delta d_{ij}-\Delta t_{ij}\right)^2}{2r\left(\left(\Delta t_{ij}\right)^2\theta+\Delta t_{ij}\right)}\right]^{-\delta-\frac{1}{2}} \end{aligned} \tag{11}$$

where $g(\cdot)$ is the PDF of the Gamma distribution, $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_n)$ and $\boldsymbol{\omega} = (\omega_1, \cdots, \omega_n)$ contains random parameters for all samples. The posterior distribution is obtained via the Bayesian method and the MCMC method as follow

$$\begin{aligned} & p\left(r, \delta, \theta, \boldsymbol{\mu}, \boldsymbol{\omega}\right) \propto \pi\left(r, \delta, \theta\right) L\left(D, \boldsymbol{\mu}, \boldsymbol{\omega} \mid r, \delta, \theta\right) \\ & \quad = \pi\left(r, \delta, \theta\right)\prod_{i=1}^{N}\prod_{j=2}^{M}\frac{\Gamma\left(\delta+\frac{1}{2}\right)}{\sqrt{2\pi r}\,\Gamma(\delta)\left[\left(\Delta t_{ij}\right)^2\theta+\Delta t_{ij}\right]^{\frac{1}{2}}} \\ & \quad \left[1+\frac{\left(\Delta d_{ij}-\Delta t_{ij}\right)^2}{2r\left(\left(\Delta t_{ij}\right)^2\theta+\Delta t_{ij}\right)}\right]^{-\delta-\frac{1}{2}} \end{aligned} \tag{12}$$

where $\pi\left(r, \delta, \theta\right)$ is the joint prior distribution of model parameters.

2) BAYESIAN PARAMETER ESTIMATION OF GAMMA PROCESS WITH RANDOM EFFECTS

In order to characterize individual product differences, when the gamma process is used to characterize degradation process, degradation increment Δd_{ij} follows gamma distribution $\Delta d_{ij} \sim Ga\left(\eta\Delta t_{ij}, v_i^{-1}\right)$ with $\Delta t_{ij} = t_{ij} - t_{i,j-1}$. scale parameter v_i obeys another gamma distribution $v_i \sim Ga\left(\gamma^{-1}, \delta\right)$. We can obtain the likelihood function as follows

$$\begin{aligned} & L\left(D, \boldsymbol{v} \mid \eta, \delta, \gamma\right) = \prod_{i=1}^{N} g\left(v_i \mid \delta, \gamma^{-1}\right)\prod_{j=2}^{M} g\left(\Delta d_{ij} \mid \eta\Delta t_{ij}, v_i^{-1}\right) \\ & = \prod_{i=1}^{N}\frac{v_i^{\delta-1}\gamma^{\delta}}{\Gamma(\delta)}\exp\left(-\gamma v_i\right)\prod_{j=2}^{M}\frac{\left(\Delta d_{ij}\right)^{\eta\Delta t_{ij}-1} v_i^{\eta\Delta t_{ij}}}{\Gamma\left(\eta\Delta t_{ij}\right)} \\ & \quad \times \exp\left(-v_i\Delta d_{ij}\right) \end{aligned} \tag{13}$$

where $\boldsymbol{v} = (v_1, \cdots v_n)$ contains the scale parameters of each samples. Let $\pi(\theta) = \pi(\eta, \delta, \gamma)$ is joint prior distribution of parameters, posterior distribution is

$$\begin{aligned} & p\left(\eta, \delta, \gamma, v \mid D\right) \propto \pi(\theta) L\left(D, v \mid \theta\right) \\ & \quad = \pi\left(\eta, \delta, \gamma\right)\prod_{i=1}^{N}\frac{v_i^{\delta-1}\gamma^{\delta}}{\Gamma(\delta)}\exp\left(-\gamma v_i\right) \\ & \quad \prod_{j=2}^{M}\frac{\left(\Delta d_{ij}\right)^{\eta\Delta t_{ij}-1} v_i^{\eta\Delta t_{ij}}}{\Gamma\left(\eta\Delta t_{ij}\right)}\exp\left(-v_i\Delta d_{ij}\right) \end{aligned} \tag{14}$$

3) BAYESIAN PARAMETER ESTIMATION OF IG PROCESS WITH RANDOM EFFECTS

Integrating random effects into the ig process to characterize individual differences, the degradation increment Δd_{ij} follows the ig distribution $\Delta d_{ij} \sim \mathrm{IG}\left(\Lambda\Delta t_{ij}, \eta_i\Lambda\left(\Delta t_{ij}\right)^2\right)$ with $\Delta t_{ij} = t_{ij} - t_{i,j-1}$ and $\eta_i \sim \mathrm{Gamma}\left(\gamma^{-1}, \delta\right)$. Therefore, the likelihood function is

$$\begin{aligned} L\left(D, \boldsymbol{\eta} \mid \Lambda, \gamma, \delta\right) &= \prod_{i=1}^{N} g\left(\eta_i \mid \gamma^{-1}, \delta\right)\prod_{j=2}^{M} f\left(\Delta d_{ij} \mid \Lambda\Delta t_{ij}, \eta_i\right) \\ &= \prod_{i=1}^{N}\prod_{j=2}^{M}\frac{\Gamma\left(\delta+\frac{1}{2}\right)\gamma^{\delta}}{\Gamma(\delta)\sqrt{2\pi}}\Lambda\Delta t\left(\Delta d_{ij}\right)^{-\frac{3}{2}} \\ &\quad \times\left[\gamma+\frac{\left(\Delta d_{ij}-\Lambda\Delta t\right)^2}{2\Delta d_{ij}}\right]^{-\delta-\frac{1}{2}} \end{aligned} \tag{15}$$

where $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_n)$ contains random parameters for all samples, $g(\cdot)$ is the PDF of the Gamma distribution. Quantifying the prior information into joint prior distribution $\pi(\theta) = \pi(\Lambda, \gamma, \delta)$, the posterior distribution could be obtained via the Bayesian method and the MCMC method as follows

$$\begin{aligned} & p\left(\Lambda, \delta, \gamma, \eta \mid D\right) \propto \pi(\theta) L\left(D, \boldsymbol{\eta} \mid \theta\right) \\ & \quad = \pi\left(\Lambda, \gamma, \delta\right)\prod_{i=1}^{N}\prod_{j=2}^{M}\frac{\Gamma\left(\delta+\frac{1}{2}\right)\gamma^{\delta}}{\Gamma(\delta)\sqrt{2\pi}} \\ & \quad \Lambda\Delta t\left(\Delta d_{ij}\right)^{-\frac{3}{2}}\left[\gamma+\frac{\left(\Delta d_{ij}-\Lambda\Delta t\right)^2}{2\Delta d_{ij}}\right]^{-\delta-\frac{1}{2}} \end{aligned} \tag{16}$$

It can be seen from the above equations that the analytical solution of the posterior distribution is difficult to be obtained. The MCMC method is employed for generating samples from the posterior distribution.

C. MODEL SELECTION

Commonly used model selection criteria include the the Akaike information criterion (AIC) [42], the Bayesian information criterion (BIC) [43] and the deviance information criterion (DIC) [44]. The DIC is a standard that can effectively measure the goodness of complex model fitting. According to the advantage in complicated models, DIC is chosen as the model selection method in this paper. It can be defined as follows

$$\mathrm{DIC}(m) = \overline{D(\theta_m, m)} + D\left(\overline{\theta}_m, m\right) = D\left(\overline{\theta}_m, m\right) + 2p_m \tag{17}$$

where $D(\theta_m, m) = -2\log L(y\,|\theta_m, m)$ and $\overline{D(\theta_m, m)}$ is the posterior mean. p_m is the number of valid parameters in model m. $\overline{\theta}_m$ is the posterior mean of the parameters contained in model m.

The DIC can be directly calculated by the MCMC method. Moreover, it can be applied to hierarchical models, hidden variable models, and complex models with inestimable number of estimated parameters. Therefore, DIC is now widely used.

IV. ILLUSTRATIVE EXAMPLE

The spindle system is one of the critical subsystems of the heavy-duty CNC machine tool. The state of spindle system has a great influence on the processing quality of machined parts. The machining accuracy is selected as the performance indicator for the degradation process. For obtaining degradation information of the spindle system, we have monitored machining accuracy of five spindle systems. The degradation observations are presented in Figure 1.

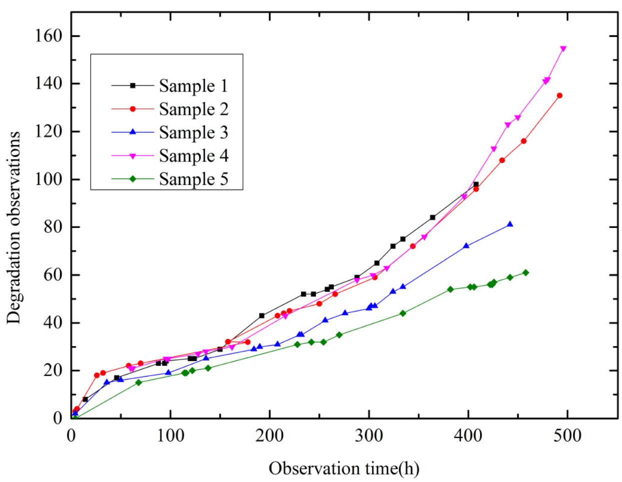

FIGURE 2. Degradation observations of machining accuracy.

The degradation observations of 5 spindle system are monitored, and the serial number is $i, i = 1, \ldots, 5$. The degradation $D\left(t_{ij}\right)$ of the ith sample at the jth observation has independent degradation increments $\Delta d_{ij} = D\left(t_{ij}\right) - D\left(t_{i,j-1}\right)$. The pre-specified threshold of the machining accuracy degradation process is $C = 300$.

After the machining accuracy of the spindle system are obtained, various stochastic process models could be employed to characterize the degradation process. Then MCMC method is used to perform 20000 iterations through OpenBUGS for obtaining the posterior distribution of model parameters. Based on estimation of parameters, the reliability of the spindle system could be assessment. Due to the lack

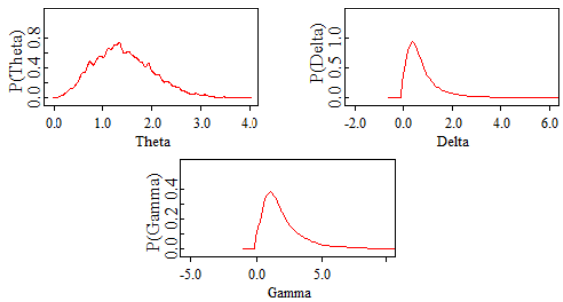

FIGURE 3. The posterior PDFs of θ, δ and r.

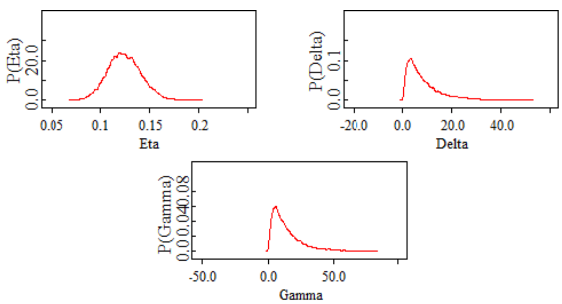

FIGURE 4. The posterior PDFs of η, δ and γ.

of prior information, the model parameters use the uniform distribution to diatribe non-informative prior distribution, which reduces the interference of subjective information for obtaining accurate parameter estimation results.

Integrating random effects into the Wiener process to characterize individual differences in produces, the degradation increment Δd_{ij} follows $\Delta d_{ij} \sim \mathrm{N}\left(\mu_i \Delta t_{ij}, \sigma_i^2 \Delta t_{ij}\right)$ with $\omega_i = \sigma^{-2} \sim Ga\left(r^{-1}, \delta\right)$ and $\mu_i \sim N\left(1, \theta/\omega_i\right)$. The prior distribution is

$$r \sim \text{Uniform}(0, 100), \ \delta \sim \text{Uniform}(0, 100),$$
$$\theta \sim \text{Uniform}(0, 100)$$

The joint posterior distribution are obtained by the MCMC method. The posterior PDFs related to the Wiener process with random effects are shown in Figure 3.

In order to characterize individual product differences, when the Gamma process is employed to characterize degradation process, degradation increment Δd_{ij} follows the Gamma process $\Delta d_{ij} \sim Ga\left(\eta \Delta t_{ij}, v_i^{-1}\right)$ with $\Delta t_{ij} = t_{ij} - t_{i,j-1}$. Scale parameter v_i obeys another Gamma distribution $v_i \sim Ga\left(\gamma^{-1}, \delta\right)$. The joint prior distribution is

$$\eta \sim \text{Uniform}(0, 100), \ \gamma \sim \text{Uniform}(0, 100),$$
$$\delta \sim \text{Uniform}(0, 100)$$

The posterior PDFs related to the Gamma process with random effects are shown in Figure 4.

Integrating random effects into the IG process to characterize individual differences, the degradation increment Δd_{ij} follows the IG distribution $\Delta d_{ij} \sim \mathrm{IG}\left(\Lambda \Delta t_{ij}, \eta_i \Lambda \left(\Delta t_{ij}\right)^2\right)$

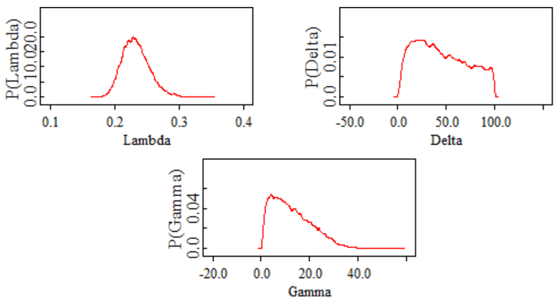

FIGURE 5. The posterior PDFs of Λ, δ and γ.

TABLE 1. The DIC values.

	Wiener process	Gamma process	IG process
DIC	460	417.6	496.9
	Wiener process with random effects	Gamma process with random effects	IG process with random effects
DIC	431.3	404	404.5

TABLE 2. Statistical summarization of parameters estimation.

	Mean	Standard deviation	Confidence interval 2.5%	Confidence interval 97.5%
η	0.1245	0.01682	0.0934	0.1592
δ_v	8.359	7.051	0.8986	27.86
γ_v	14.83	12.55	1.705	50.6

with $\Delta t_{ij} = t_{ij} - t_{i,j-1}$ and $\eta_i \sim \text{Gamma}\left(\gamma^{-1}, \delta\right)$. The joint prior distribution is

$$\Lambda \sim \text{Uniform}\,(0, 100)\,, \ \gamma \sim \text{Uniform}\,(0, 100)\,,$$
$$\delta \sim \text{Uniform}\,(0, 100)$$

The posterior PDFs related to the IG process with random effects are shown in Figure 5.

To obtain accurate reliability assessment results, the DIC is used to select the model that best fits the data set of machining accuracy. The model with the smallest DIC value is considered to be the most appropriate choice. The DIC values for the different degradation models are summarized in Table 1.

It can be seen from Table 1 that Gamma process model integrated random effects model has the smallest DIC, so it can be considered as the most suitable model. At the same time, the DIC of the stochastic process model integrated random effects is smaller than DIC of basic stochastic process. It shows that the stochastic process model integrated random effects is more suitable for characterizing the performance degradation process. It also illustrates the necessity of considering individual differences in degradation analysis.

Therefore, Gamma process model with integrated random effects model is employed to describe the machining accuracy degradation process of the spindle system. Statistical summarization of parameter estimation is given in Table 2.

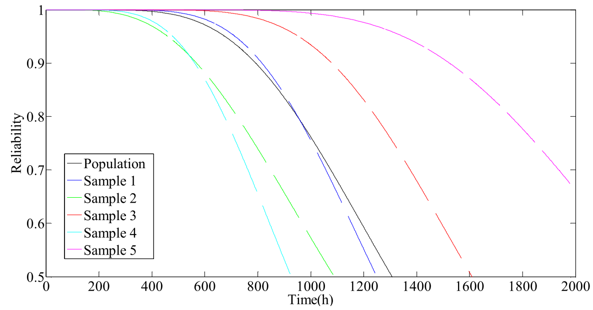

FIGURE 6. Reliability of the spindle system.

Based on the parameter estimation, reliability assessment result of the spindle system can be obtained as shown in Figure 6.

It can be seen from Figure 6 that the reliability of the different spindle system samples vary greatly. Because the reliability assessment of the population considers the uncertainty of the random parameters of all individuals, and the reliability assessment of each sample only considers the uncertainty of its own random parameters.

V. CONCLUSION

In this paper, the reliability modeling and assessment metho based on degradation data is studied under the characteristics of long-life and high-reliability produces and the lack of fault time data. Based on the characteristics of the degradation process, stochastic process models are used to characterize produce degradation process. In order to characterize individual differences, the random effects model is introduced into stochastic process models. The random variables are used to characterize parameters related to individual differences in the stochastic process such that the parameters follow the specific probability distribution. On this basis, the basic framework of parameter estimation and reliability assessment for degradation models based on Bayesian theory is studied.

Some interesting problems deserve further discussion. For instance, the incorporation of different types of prespecified threshold of the degradation processes is interesting to expand the proposed model. In addition, the external shocks on the degradation process is also worth of further investigation.

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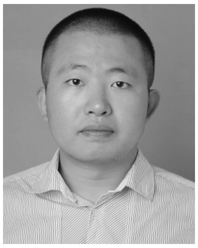

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