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Covering-Based Rough Fuzzy, Intuitionistic Fuzzy and Neutrosophic Nano Topology and Applications

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ABSTRACT In recent years, ''mathematical orientations on real-life problems'', which continue to increase, began to make a significant impact. Information systems for many decision-making problems consist of uncertain, incomplete, indeterminate and indiscernible structures and components. Classical set theory and interpretation methods fail to represent, express and solve the problems of these types or cause to make wrong decisions. For this reason, in this study, we provide definitions and methods to present information and problem representations in more detail and precision. This paper introduces three new topologies called covering-based rough fuzzy, covering-based rough intuitionistic fuzzy and covering-based rough neutrosophic nano topology. Some fundamental definitions such as open set, closed set, interior, closure and basis are given. Neutrosophic definitions and properties are mainly investigated. We give some real life applications of covering-based rough neutrosophic nano topology in the final part of the paper and an explanatory example of decision making application by defining core point.

INDEX TERMS Approximation space, core point, covering-based topology, fuzzy nano topology, fuzzy sets, intuitionistic nano topology, intuitionistic sets, neutrosophic nano topology, neutrosophic sets, rough decision making, location selection problem.

I. INTRODUCTION

The world of science has been working on and producing uncertain, incomplete, indeterminate, and indiscernible structures since crisp structures are understood to be unable to solve most real-life problems. In the last decade, uncertain representations, which have found enormous applications in engineering, medicine, computers, space research and even social sciences, make them feel that they will take more space in the future. In this sense, we would like to contribute to these recent developments with a study that includes both topology, generalized fuzzy and decision making. In this section, we will introduce the main uncertain information systems from the past to the present and explain why they need to work immediately after we talk about their applications.

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Rough set theory is a way of representing and reasoning imprecision and uncertain information in data [1]. It deals with the approximation of sets constructed from descriptive data elements. This is helpful while aiming to explore decision rules, important features, and minimization of conditional attributes. While an information set represents the probabilistic uncertainty due to vagueness, rough set theory is widely used to represent imprecision due to incomplete knowledge.

Traditional rough approaches are based on equivalence relations, but this condition is not fulfilled in some cases. So, the approximations have been broaden to the similarity relation based rough sets [2], [3], the tolerance relation based rough sets [4], the dominance relation based rough sets [5], the arbitrary binary relation based rough sets [6], [7], [9]. An attractive and inherent research aim in rough set theory is to study rough set theory by means of topology. In fact, Polkowski [10] made a sign that topological aspects of rough set theory were recognized early within the scope of topology of partitions. In 1988, Skowron [11] and Wiweger [12] discussed this issue separately for the traditional rough set theory. Topological spaces obtained by rough sets bottomed on information were systems putted up and characterized by Polkowski [13]. Kortelainen [14] paid regard to relationships among topological spaces, modified sets, and rough sets based on a pre-order. Skowron et al. [4], [15] generalized the classical approximation spaces to tolerance approximation spaces, and discussed the problems of attribute reduction in these spaces [16]. Lashin et al. [17] introduced the topology generated by a sub-base, and defined a topological rough membership function by the sub-base of the topology. In addition, connections between fuzzy rough set theory and fuzzy topology were also investigated [18]–[20]. General topology is accepted as an introduction to the understanding of topology, and the basis of general topology is the topological space, which is generally regarded as a representation of the universal space, and in particular the geometric shape in the concepts of mathematical analysis. The general topology has become the appropriate framework for each collection associated with relationships. Topology is also a strong mathematical instrument to examine into information systems, rough sets and so on [11], [17].

Recently, some of the works featured in the scope of this article and stand out are as follows. Zhan and his collaborations have studied on multi-criteria decision making problems of covering-based general multi-granulation intuitionistic fuzzy rough sets, covering based variable precision (I,T)-fuzzy rough sets, covering-based intuitionistic fuzzy rough sets, novel fuzzy rough set models [22]–[28]. Zhang, Yang and their collaborations have studied decision making problems of generalized interval neutrosophic rough sets, covering-based generalized IF rough sets, a hybrid model of single valued neutrosophic sets and rough sets, hesitant fuzzy linguistic rough set, and Merger and acquisition target selection based on interval neutrosophic multi-granulation rough sets [29]–[33].

This paper introduces some new topologies having properties of nano and covering. Furthermore, we give a decision making example of bus stations location on neutrosophic environment by using core points of the defined topologies.

Our main motivations in this study are that:

(i) We have shown that not only the classical approach spaces, but also the different mathematical structures of these spaces provide opportunities for decision-making.

(ii) The applications of approximation spaces and rough sets on decision making are generally interpreted in the field of medicine. In this work, we give an example of decision making on location selection. On the one hand, with the proposed new definitions, we presented wider versions of the approach space topologies and gave an example that can be applied to real life problems.

The proposed method has two important advantages.

(i) Firstly, since the method can be applied to neutrosophic data, more detailed evaluations can be made on the information.

(ii) Another important advantage is the complexity of the algorithm and the process step is not too much by disabling the process can only have ideas with the core point.

As the definitions in the paper are new, it is very difficult to compare with other publications and studies. In particular, the core definition in our study differs from the core definition in the classical sense for rough sets and approximation spaces. To compare the method with some existing methods, Lellis Thivagar and his collaborations [34]–[36], which provides intensive studies especially in the fields of Nano Topology and its applications, has given the definition of Nano topology within the framework of neutrosopy, but with this structure, they pointed out that one could find applications in areas such as Geographical Information Systems (GIS) field including remote sensing, object reconstruction from airborne laser scanner, real time tracking, routing applications and modeling cognitive agents [37]. They did not give a specific application example. Moreover, the definition of core used in many methods is different from the definition of core used in our study. On the other hand, while a decision-making process is generally in the form of algorithms consisting of five or six steps, our method can only make a definition with the core definition.

II. PRELIMINARIES

Fuzzy set notion was putted forward by Zadeh in 1965 [21]. Fuzzy sets and fuzzy logic have been performed in numerous real life applications to manage vagueness until today. On a universe K, a fuzzy set A defined, uses a value $\mu_A(u) \in [0, 1]$ to give the membership grade of A. Intuitionistic fuzzy set concept was introduced by Atanassov [38] in 1986. The concept is a generalization of fuzzy sets and provably equivalent to interval valued fuzzy sets. The concept takes both truth-membership $T_A(x)$ and falsity-membership $F_A(x)$, with $T_A(x), F_A(x) \in [0, 1]$ and $0 \leq T_A(x) + F_A(x) \leq 1$ into consideration. In [39], Smarandache introduced neutrosophy in 1995. Indeterminacy is quantified explicitly and truth-membership, indeterminacy membership and falsity-membership are independent In neutrosophic set. A neutrosophic set A defined on universe K. $x = x(T, I, F) \in A$ with T, I and F being the real standard or non-standard subsets of $]^-0, 1^+[$. T is the degree of truth-membership function in the set A, I is the indeterminacy-membership function in the set A and F is the falsity-membership function in the set A. In [1], Pawlak introduced the rough set theory in 1982. Rough set theory addresses vagueness and uncertainty in data analysis and information systems. It gives some ways to obtain the deciding factors from data.

Let K be a non-empty set and R be an equivalence relation on K, and (K, R) be an approximation space, and let $X \subseteq K$,

1) in A, the lower approximation of X is the set

$$\underline{X} = \{x \in K : [x]_R \subset X\}.$$

2) In A, the upper approximation of X is the set

$$\overline{X} = \{x \in K : [x]_R \wedge X \neq \phi\}.$$

3) In A, the boundary region of X is the set

$$BN(X) = \overline{X} - \underline{X}.$$

If (K, R) is an approximation space with $X, Y \subseteq K$, then

1) X and Y are roughly bottom-equal in A, written $(X \eqsim Y)$, $\iff \underline{X} = \underline{Y}$.
2) X and Y are roughly top-equal in A, written $(X \simeq Y)$, $\iff \overline{X} = \overline{Y}$.
3) X and Y are roughly equal in A, written $X \approx Y$, $\iff \underline{X} = \underline{Y}$ and $\overline{X} = \overline{Y}$.

$\approx$ is an equivalence relation on the power set of K. The family of all equivalence classes of the rough relation $\approx$ is denoted by

$$R_{\approx} = \{[X] : X \subseteq K\},$$

where $[X]$ is a rough set, its elements are subsets of K having the same lower approximation and the same upper approximation. For each rough set $X \subseteq K$, we write $X = (\underline{X}, \overline{X})$. Note that $\phi = (\phi, \overline{\phi})$ and $K = (\underline{K}, \overline{K})$. So ϕ and K are rough sets. If $X, Y \in R_{\approx}$, then $Y \subseteq_{\approx} X \iff \underline{Y} \subseteq \underline{X}$ and $\overline{Y} \subseteq \overline{X}$, and Y will be called a rough subset of X. The family of all rough subsets of X in (K, R) is called rough power set of X.

If (K, R) is an approximation space and X, Y are rough subsets of K, then the rough union, rough intersection and rough complement are defined as follows:

1) $X \vee Y = (\underline{X} \vee \underline{Y}, \overline{X} \vee \overline{Y})$.
2) $X \wedge Y = (\underline{X} \wedge \underline{Y}, \overline{X} \wedge \overline{Y})$.
3) $X^c = (K \setminus \overline{X}, K \setminus \underline{X}) = K - X$.

In [40], Bryniarski defined the notion of covering-based rough sets, which is an extension of the classical Pawlak's rough set. If C is a family of non-empty subsets of a non-empty set K such that $\cup C = K$, then C is called a **covering** of K. Bryniarski defined the lower and upper approximations and the boundary region in a similar way as Pawlak.

By a covering approximation space (K, C), we mean that K is a non-empty set and C is a covering of K satisfying the following approximation condition: $\forall A, B \subset C$ such that $A \subset B$, $\exists X \subset K$ with $A = \underline{X}$, $B = \overline{X}$. If $X \subset K$, then the ordered pair $(\underline{X}, \overline{X})$ is the covering-based rough set of X. The definition of the covering rough subsets in any covering approximation space (K, C) is similar to definition of rough subsets in any approximation space (K, R), [41].

Definition 1 [41]: Let (K, C) be a covering approximation space and τ be a subfamily of the family of all covering rough subsets of $X = (\underline{X}, \overline{X}, BN(X))$ having the following properties:

1) $X, \emptyset \in \tau$.
2) Infinite union of the elements of τ is in τ.
3) Finite intersection of elements of τ is in τ.

Then τ is called a covering-based nano topology on X.

III. COVERING-BASED ROUGH FUZZY NANO TOPOLOGY

Definition 2 [21]: Let A be a non-empty set. A fuzzy set X is of the form $X = \{< a : \mu_X(a) >, a \in A\}$, where $0 \leq \mu_X(a) \leq 1$ is the degree of membership of each $a \in A$ to the set X.

Definition 3 [37]: Let R be an equivalence relation on a non-empty set X. Let F be a fuzzy set in X with the membership function μ_F. Then the fuzzy nano lower, fuzzy nano upper approximation of F and fuzzy nano boundary of F in the approximation (X, R) denoted by $\underline{\mathscr{F}}(F)$, $\overline{\mathscr{F}}(F)$ and $B_{\mathscr{F}}(F)$ are respectively defined as follows:

1) $\underline{\mathscr{F}}(F) = \{< x, \mu_{\underline{R}(A)}(x) > /y \in [x]_R, x \in X\}$
2) $\overline{\mathscr{F}}(F) = \{< x, \mu_{\overline{R}(A)}(x) > /y \in [x]_R, x \in X\}$
3) $B_{\mathscr{F}}(F) = \overline{\mathscr{F}}(F) - \underline{\mathscr{F}}(F)$

where $\mu_{\underline{R}(A)}(x) = \wedge_{y \in [x]_R}(y)$ and $\mu_{\overline{R}(A)}(x) = \vee_{y \in [x]_R}(y)$.

Definition 4 [37]: Let R be an equivalence relation on a non-empty set X and F be a fuzzy set in X. Suppose that the collection $\tau_{\mathscr{F}}(F) = \{0_F, 1_F, \underline{\mathscr{F}}(F), \overline{\mathscr{F}}(F), BF(F)\}$ forms a topology. Then it is said to be a fuzzy nano topology. We call $(X, \tau_{\mathscr{F}}(F))$ the fuzzy nano topological space. The elements of $\tau_{\mathscr{F}}(F)$ are called fuzzy nano open sets.

Definition 5: Let (K, C) be a covering approximation space where K is a non-empty set and let X be a subset of K. Let A be a fuzzy set in K with the membership function μ_A. Then the covering-based rough fuzzy nano lower, covering-based rough fuzzy nano upper approximation of A and covering rough based fuzzy nano boundary of A in the approximation (K, C) denoted by $\underline{\mathscr{F}}_{C_X}(A)$, $\overline{\mathscr{F}}_{C_X}(A)$ and $\mathscr{F}_{BN(X)}(A)$ are respectively defined as follows:

1) $\underline{\mathscr{F}}_{C_X}(A) = \{< k, \mu_{C_{\underline{X}(A)}}(k) > /y \in [k]_{C_{\underline{X}}}, k \in K\}$
2) $\overline{\mathscr{F}}_{C_X}(A) = \{< k, \mu_{C_{\overline{X}(A)}}(k) > /y \in [k]_{C_{\overline{X}}}, k \in K\}$
3) $\mathscr{F}_{BN(X)}(A) = \{< k, \mu_{BN(X)(A)}(k) > /y \in [k]_{BN(X)}, k \in K\}$

where $\mu_{C_{\underline{X}(A)}}(k) = \wedge_{y \in [k]_{C_{\underline{X}}}}(y)$, $\mu_{C_{\overline{X}(A)}}(k) = \vee_{y \in [k]_{C_{\overline{X}}}}(y)$ and $\mu_{BN(X)(A)}(k) = \vee_{y \in [k]_{BN(X)}}(y)$. Addition to this, $C_{\underline{X}}$ is the lower approximation of X with respect to C, $C_{\overline{X}}$ is the upper approximation of X with respect to C and $C_{BN(X)} = C_{\overline{X}} \setminus C_{\underline{X}}$.

If $\tau_{\mathscr{F}}(C, X, A) = \{0_F, 1_F, \underline{\mathscr{F}}_{C_X}(A), \overline{\mathscr{F}}_{C_X}(A), \mathscr{F}_{BN(X)}(A)\}$ forms topology, $\tau_{\mathscr{F}}(C, X, A)$ is called **covering-based rough fuzzy nano topology**. The elements of $\tau_{\mathscr{F}}(C, X, A)$ are called covering-based rough fuzzy nano open sets.

Example 6: $\tau_{\mathscr{F}}(C, X, A)$ defines a topology for given a universe $K = \{P_1, P_2, P_3\}$, a covering set $C = \{\{P_1, P_2\}, \{P_2, P_3\}\}$, a subset $X = \{P_1, P_3\}$, and a fuzzy set $A = \{< P_1, 0 >, < P_2, 1 >, < P_3, 0.3 >\}$. Then,

$$\begin{aligned}
C_{\underline{X}} &= \emptyset \\
C_{\overline{X}} &= \{\{P_1, P_2\}, \{P_2, P_3\}\} \\
C_{BN(X)} &= \overline{\mathscr{F}}_{C_X}(A) \\
\underline{\mathscr{F}}_{C_X}(A) &= \emptyset = 0_F, \\
\overline{\mathscr{F}}_{C_X}(A) &= \{< P_1, 1 >, < P_2, 1 >, < P_3, 1 >\}, \\
\mathscr{F}_{BN(X)}(A) &= \overline{\mathscr{F}}_{C_X}(A)
\end{aligned}$$

Finally, $\tau_{\mathscr{F}}(C, X, A)$ defines a topology.

If $B = \{< x, \mu_B(x) >: x \in K\}$ is a fuzzy set, its complement is $B^c = \{< x, 1 - \mu_B(x) >: x \in K\}$. $[\tau_{\mathscr{F}}(C, X, A)]^c$ is a set containing B^c **?**for every $B \in \tau_{\mathscr{F}}(C, X, A)$.

IV. COVERING-BASED ROUGH INTUITIONISTIC FUZZY TOPOLOGY

Definition 7 [38]: An intuitionstic set X in a non-empty set A is of the form $X = \{< a : \mu_X(a), \nu_X(a) >, a \in A\}$, where $\mu_X(a)$ and $\nu_X(a)$ represent the degree of membership function and the degree of non-membership respectively of each $a \in A$ to the set X and $0 \leq \mu_X(a) + \nu_X(a) \leq 1$ for all $a \in A$.

Definition 8 [37]: Let R be an equivalence relation on a non-empty set X. Let F be an intuitionistic set in X with the membership function μ_F and the non-membership function ν_F. The intuitionistic nano lower, intuitionistic nano upper approximation and intuitionistic nano boundary of F in the approximation (X, R) denoted by $\underline{I}(F)$, $\overline{I}(F)$ and $B_I(F)$, respectively, are defined as follows:

1) $\underline{I}(F) = \{< x, \mu_{\underline{R}(A)}(x), \nu_{\underline{R}(A)}(x) > /y \in [x]_R, x \in X\}$
2) $\overline{I}(F) = \{< x, \mu_{\overline{R}(A)}(x), \nu_{\overline{R}(A)}(x) > /y \in [x]_R, x \in X\}$
3) $B_I(F) = \overline{I}(F) - \underline{I}(F)$.

Definition 9: Let (K, C) be a covering approximation space in a non-empty set K. Assume that X is a subset of K. Let A be an intuitionistic fuzzy set in K with the membership function μ_A and the non-membership function ν. Then the covering based rough intuitionistic fuzzy nano lower, covering based rough intuitionistic fuzzy nano upper approximation of A and covering based rough intuitionistic fuzzy nano boundary of A in the approximation (K, C) denoted by $\underline{\mathscr{IF}}_{C_X}(A)$, $\overline{\mathscr{IF}}_{C_X}(A)$ and $\mathscr{IF}_{BN(X)}(A)$, respectively, are defined as follows:

1) $\underline{\mathscr{IF}}_{C_X}(A) = \{< k, (\mu_{C_{\underline{X}(A)}}(k), \nu_{C_{\underline{X}(A)}}(k)) > /y \in [k]_{C_{\underline{X}}}, k \in K\}$
2) $\overline{\mathscr{IF}}_{C_X}(A) = \{< k, (\mu_{C_{\overline{X}(A)}}(k), \nu_{C_{\overline{X}(A)}}(k)) > /y \in [k]_{C_{\overline{X}}}, k \in K\}$
3) $\mathscr{IF}_{BN(X)}(A) = \{< k, (\mu_{BN(X)(A)}(k), \nu_{BN(X)(A)}(k)) > /y \in [k]_{BN(X)}, k \in K\}$

where $\mu_{C_{\underline{X}(A)}}(k) = \wedge_{y \in [k]_{C_{\underline{X}}}}(y)$, $\mu_{C_{\overline{X}(A)}}(k) = \vee_{y \in [k]_{C_{\overline{X}}}}(y)$ and $\mu_{BN(X)(A)}(k) = \vee_{y \in [k]_{BN(X)}}(y)$.

$\nu_{C_{\underline{X}(A)}}(k) = \vee_{y \in [k]_{C_{\underline{X}}}}(y)$, $\nu_{C_{\overline{X}(A)}}(k) = \wedge_{y \in [k]_{C_{\overline{X}}}}(y)$ and $\nu_{BN(X)(A)}(k) = \wedge_{y \in [k]_{BN(X)}}(y)$.

Addition to this, $C_{\underline{X}}$ is the lower approximation of X with respect to C, $C_{\overline{X}}$ is the upper approximation of X with respect to C and $C_{BN(X)} = C_{\overline{X}} \setminus C_{\underline{X}}$.

If $\tau_{\mathscr{IF}}(C, X, A) = \{0_F, 1_F, \underline{\mathscr{IF}}_{C_X}(A), \overline{\mathscr{IF}}_{C_X}(A), \mathscr{IF}_{BN(X)}(A)\}$ forms topology, $\tau_{\mathscr{IF}}(C, X, A)$ is called **covering based rough intuitionistic fuzzy nano topology**. The elements of $\tau_{\mathscr{IF}}(C, X, A)$ are called covering based rough intuitionistic fuzzy nano open sets.

V. COVERING BASED ROUGH NEUTROSOPHIC NANO TOPOLOGY

Definition 10 [43]: Let A be a non-empty set. A *neutrosophic set* N is defined as:

$$N = \{(a, T(a), I(a), F(a)) : a \in A\}$$

where $T_N : A \longmapsto]^-0, 1^+[$ is a truth-membership function, $I_N : A \longmapsto]^-0, 1^+[$ is an indeterminacy-membership function and $F_N : A \longmapsto]^-0, 1^+[$ is a falsity-membership function.

Definition 11 [43]: Let A be a non-empty set. A *single valued neutrosophic set* N is defined as:

$$N = \{(a, T(a), I(a), F(a)) : a \in A\}$$

where $T_N : A \longmapsto [0, 1]$, is a truth-membership function, $I_N : A \longmapsto [0, 1]$ is an indeterminacy-membership function and $F_N : A \longmapsto [0, 1]$ is a falsity-membership function with $0 \leq T_N(a) + I_N(a) + F_N(a) \leq 3$. We denote a single valued neutrosophic number by $x = (T, I, F)$.

Definition 12 [42]: Let X be a non-empty set. Let $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$ and $B = \{< x : T_B(x), I_B(x), F_B(x) >, x \in X\}$ be neutrosophic sets. Then the following statements hold:

1) $0_N = \{< x, 0, 0, 1 >: x \in X\}$ and $1_N = \{< x, 1, 0, 0 >: x \in X\}$.
2) $A \subseteq B$ iff $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_B(x) \leq F_A(x)$ for all $x \in X$.
3) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
4) $A^C = \{< x, F_A(x), 1 - I_A(x), T_A(x) >: x \in X\}$.
5) $A \cap B = \{x, T_A(x) \wedge T_B(x), I_A(x) \wedge I_B(x), F_A(x) \vee F_B(x)$ for all $x \in X\}$.
6) $A \cup B = \{x, T_A(x) \vee T_B(x), I_A(x) \vee I_B(x), F_A(x) \wedge F_B(x)$ for all $x \in X\}$.

Definition 13 [44]: Let R be an equivalence relation on a non-empty set X. Let A be a neutrosophic set in X such that μ_A is the membership function, υ_A is the indeterminacy function and ω_A is the non-membership function. Then in the approximation (X, R), the lower and the upper approximations of A denoted by $\underline{N}(A)$ and $\overline{N}(A)$, respectively, are defined as follows:

$$\underline{N}(A) = \{< x, \mu_{\underline{N}(A)}, \upsilon_{\underline{N}(A)}, \omega_{\underline{N}(A)} >: y \in [x]_R, x \in X\}$$
$$\overline{N}(A) = \{< x, \mu_{\overline{N}(A)}, \upsilon_{\overline{N}(A)}, \omega_{\overline{N}(A)} >: y \in [x]_R, x \in X\}$$

$$\mu_{\underline{N}(A)}(x) = \bigwedge_{y \in [x]_R} \mu_A(y), \quad \upsilon_{\underline{N}(A)}(x) = \bigwedge_{y \in [x]_R} \upsilon_A(y),$$
$$\omega_{\underline{N}(A)}(x) = \bigvee_{y \in [x]_R} \omega_A(y)$$

and

$$\mu_{\overline{N}(A)}(x) = \bigvee_{y \in [x]_R} \mu_A(y), \quad \upsilon_{\overline{N}(A)}(x) = \bigvee_{y \in [x]_R} \mu_A(y),$$
$$\omega_{\overline{N}(A)}(x) = \bigwedge_{y \in [x]_R} \mu_A(y),$$

Thus, $0 \leq \mu_{\underline{N}(A)} + \upsilon_{\underline{N}(A)} + \omega_{\underline{N}(A)} \leq 3$ and $0 \leq \mu_{\overline{N}(A)} + \upsilon_{\overline{N}(A)} + \omega_{\overline{N}(A)} \leq 3$ where $\bigvee$ means *max* operator and $\bigwedge$ means *min* operator. $\mu_A(y)$, $\upsilon_A(y)$ and $\omega_A(y)$ are the membership, indeterminacy and non-membership of y with respect to A. It is fairly easy to show that $\overline{N}(A)$ and $\underline{N}(A)$ are two neutrosophic sets in X.

Definition 14 [37]: Let R be an equivalence relation on a non-empty set X. Let A be a neutrosophic set in X and if the collection $\tau_{N(A)} = \{0_N, 1_N, \overline{N}, \underline{N}\}$ forms a topology then it is said to be a rough neutrosophic topology. We call $(X, \tau_{N(A)})$ rough neutrosophic topological space. The elements of $(X, \tau_{N(A)})$ are called rough neutrosophic topological open sets.

Example 15: Let $X = \{P_1, P_2, P_3, P_4\}$, $X/R = \{\{P_1, P_2\}, \{P_3, P_4\}\}$, and

$$A = \{< P_1, (0.8, 0.7, 0.4) >, < P_2, (0.2, 0.3, 0.4) >, < P_3, (0.1, 0.6, 0.2) >, < P_4, (0, 0.4, 0.9) >\}.$$

Then

$$\begin{aligned}
\underline{N}(A) &= \{< P_1, (0.2, 0.3, 0.4) >, < P_2, (0.2, 0.3, 0.4) >, < P_3, (0, 0.4, 0.9) >, < P_4, (0, 0.4, 0.9) >\}\\
\overline{N}(A) &= \{< P_1, (0.8, 0.7, 0.4) >, < P_2, (0.8, 0.7, 0.4) >, < P_3, (0.1, 0.6, 0.2) >, < P_4, (0.1, 0.6, 0.2) >\}\\
\underline{N}(A) \cap \overline{N}(A) &= \{< P_1, (0.2, 0.3, 0.4) >, < P_2, (0.2, 0.3, 0.6) >, < P_3, (0, 0.4, 0.9) >, < P_4, (0, 0.4, 0.9) >\} = \underline{N}(A)\\
\underline{N}(A) \cup \overline{N}(A) &= \{< P_1, (0.8, 0.7, 0.4) >, < P_2, (0.8, 0.7, 0.4) >, < P_3, (0.1, 0.6, 0.2) >, < P_4, (0.1, 0.6, 0.2) >\} = \overline{N}(A)\\
0_N \cap \overline{N}(A) &= 0_N, \quad 0_N \cap \underline{N}(A) = 0_N,\\
0_N \cup \overline{N}(A) &= \overline{N}(A), \quad 0_N \cup \underline{N}(A) = \underline{N}(A)\\
1_N \cap \overline{N}(A) &= \overline{N}(A), \quad 1_N \cap \underline{N}(A) = \underline{N}(A),\\
1_N \cup \overline{N}(A) &= 1_N, \quad 1_N \cup \underline{N}(A) = 1_N\\
0_N \cap 1_N &= 0_N, \quad 0_N \cup 1_N = 1_N
\end{aligned}$$

Therefore, $(X, \tau_{N(A)}) = \{0_N, 1_N, \overline{N}(A), \underline{N}(A)\}$ forms topology.

Definition 16: [44] Let R be an equivalence relation on a non-empty set X. Let A be a neutrosophic set in X such that μ_A is the membership function, υ_A is the indeterminacy function and ω_A is the non-membership function. Then in the approximation (X, R), the lower, the upper and the boundary approximations of A denoted by $\underline{N}(A)$, $\overline{N}(A)$ and $BN(A)$, respectively, are defined as follows:

$$\begin{aligned}
\underline{N}(A) &= \{< x, \mu_{\underline{N}(A)}, \upsilon_{\underline{N}(A)}, \omega_{\underline{N}(A)} > :\ y \in [x]_R, x \in X\}\\
\overline{N}(A) &= \{< x, \mu_{\overline{N}(A)}, \upsilon_{\overline{N}(A)}, \omega_{\overline{N}(A)} > :\ y \in [x]_R, x \in X\}\\
BN(A) &= \overline{N}(A) - \underline{N}(A)
\end{aligned}$$

where;

$$\mu_{\underline{N}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y), \quad \upsilon_{\underline{N}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y),$$
$$\omega_{\underline{N}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y)$$

and

$$\mu_{\overline{N}(A)}(x) = \bigvee_{y\in[x]_R} \mu_A(y), \quad \upsilon_{\overline{N}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y),$$
$$\omega_{\overline{N}(A)}(x) = \bigwedge_{y\in[x]_R} \mu_A(y),$$

Thus, $0 \leq \mu_{\underline{N}}(A) + \upsilon_{\underline{N}(A)} + \omega_{\underline{N}(A)} \leq 3$ and $0 \leq \mu_{\overline{N}(A)} + \upsilon_{\overline{N}(A)} + \omega_{\overline{N}(A)} \leq 3$ where $\bigvee$ means *max* operator and $\bigwedge$ means *min* operator. $\mu_A(y)$, $\upsilon_A(y)$ and $\omega_A(y)$ are the membership, indeterminacy and non-membership of y with respect to A. It is not difficult to see that $\overline{N}(A)$, $\underline{N}(A)$ and $BN(A)$ are three neutrosophic sets in X.

Definition 17: Let K be a non-empty set, (K, C) be a covering approximation space and X be a subset of K. Let A be a neutrosophic set in K such that μ_A is the membership function, υ_A is the indeterminacy function and ω_A is the non-membership function. Then the covering based rough neutrosophic nano lower, covering based rough neutrosophic nano upper approximation of A and covering based rough neutrosophic nano boundary of A in the approximation (K, C) denoted by $\underline{\mathcal{N}}_{C_X}(A)$, $\overline{\mathcal{N}}_{C_X}(A)$ and $\mathcal{N}_{BN(X)}(A)$, respectively, are defined as follows:

$$\begin{aligned}
\underline{\mathcal{N}}_{C_X}(A) &= \{< k, (\mu_{C_{\underline{X}(A)}})(k), \nu_{C_{\underline{X}(A)}})(k), \omega_{C_{\underline{X}(A)}})(k)) > /y \in [k]_{C_{\underline{X}}}, k \in K\}\\
\overline{\mathcal{N}}_{C_X}(A) &= \{< k, (\mu_{C_{\overline{X}(A)}}(k), \nu_{C_{\overline{X}(A)}}(k), \omega_{C_{\overline{X}(A)}}(k)) > /y \in [k]_{C_{\overline{X}}}, k \in K\}\\
\mathcal{N}_{BN(X)}(A) &= \{< k, (\mu_{BN(X)(A)}(k), \nu_{BN(X)(A)}(k), \omega_{BN(X)(A)}(k)) > /y \in [k]_{BN(X)}, k \in K\}
\end{aligned}$$

where

$$\mu_{C_{\underline{X}(A)}}(k) = \wedge_{y\in[k]_{C_{\underline{X}(A)}}} \mu(y), \quad \nu_{C_{\underline{X}(A)}}(k) = \wedge_{y\in[k]_{C_{\underline{X}(A)}}} \nu(y),$$
$$\omega_{C_{\underline{X}(A)}}(k) = \vee_{y\in[k]_{C_{\underline{X}}}} \omega(y), \quad \mu_{C_{\overline{X}(A)}}(k) = \vee_{y\in[k]_{C_{\overline{X}(A)}}} \mu(y),$$
$$\nu_{C_{\overline{X}(A)}}(k) = \vee_{y\in[k]_{C_{\overline{X}(A)}}} \nu(y), \quad \omega_{C_{\overline{X}(A)}}(k) = \wedge_{y\in[k]_{C_{\overline{X}}}} \omega(y),$$
$$\mu_{BN(X)(A)}(k) = \vee_{y\in[k]_{BN(X)}} \mu(y),$$
$$\nu_{BN(X)(A)}(k) = \vee_{y\in[k]_{BN(X)}} \nu(y),$$

and $\omega_{BN(X)(A)}(k) = \wedge_{y\in[k]_{BN(X)}} \omega(y)$. Addition to this, $C_{\underline{X}}$ is the lower approximation of X with respect to C, $C_{\overline{X}}$ is the upper approximation of X with respect to C and $C_{BN(X)} = C_{\overline{X}} \setminus C_{\underline{X}}$.

If $\tau_{\mathcal{N}}(C, X, A) = \{0_N, 1_N, \underline{\mathcal{N}}_{C_X}(A), \overline{\mathcal{N}}_{C_X}(A), \mathcal{N}_{BN(X)}(A)\}$ forms topology, $\tau_{\mathcal{N}}(C, X, A)$ is called **covering based rough neutrosophic nano topology**.

Example 18: $\tau_{\mathcal{N}}(C, X, A)$ defines a topology for a given universe $K = \{P_1, P_2, P_3\}$, a covering set $C = \{\{P_1, P_2\}, \{P_2, P_3\}\}$, a subset $X = \{P_1, P_3\}$, and a neutrosophic set $A = \{< P_1, (0, 0.2, 1) >, < P_2, (1, 0, 0.6) >, < P_3, (0.3, 0.4, 0, 5) >\}$. Then, $C_{\underline{X}} = \emptyset$, $C_{\overline{X}} = \{\{P_1, P_2\}, \{P_2, P_3\}\}$, $C_{BN(X)} = C_{\overline{X}}$, $\underline{\mathcal{N}}_{C_X}(A) = \emptyset = 0_N$,

$\overline{\mathcal{N}}_{C_X}(A) = \{< P_1, (1, 0.2, 0.6) >, < P_2, (1, 0.2, 0.6) >, < P_2, (1, 0.4, 0.6) >, < P_3, (1, 0.4, 0.6) >\}$, and $\mathcal{N}_{BN(X)}(A) = \overline{\mathcal{N}}_{C_X}(A)$.

Finally, it is not difficult to check that $\tau_{\mathcal{N}}(C, X, A)$ defines a topology. The complement of $\tau_{\mathcal{N}}(C, X, A)$ is $[\tau_{\mathcal{N}}(C, X, A)]^c = \{0_N, 1_N, \{< P_1, (0.6, 0.8, 1) >, < P_2, (0.6, 0.8, 1) >, < P_2, (0.6, 0.6, 1) >, < P_3, (0.6, 0.6, 1) >\}\}$.

Definition 19: Let $\tau_{\mathcal{N}}(C, X, A)$ be covering based rough neutrosophic nano topology. The elements of $\tau_{\mathcal{N}}(C, X, A)$ are called covering based rough neutrosophic nano open sets.

Definition 20: Let $\tau_{\mathcal{N}}(C, X, A)$ be covering based rough neutrosophic topology. The elements of $[\tau_{\mathcal{N}}(C, X, A)]^c$ are called covering based rough neutrosophic nano closed sets.

Definition 21: If $\tau_{\mathcal{N}}(C, X, A)$ is a covering based rough neutrosophic nano topological space on an universe K and B be any neutrosophic subset of K. Then the covering based rough neutrosophic nano interior of B is defined as the union of all covering based rough neutrosophic nano open subsets of B and it is denoted by $\mathcal{N}_{int}(B)$. $\mathcal{N}_{int}(B)$ is the largest covering based rough neutrosophic nano open subset of B.

Example 22: Consider the covering based rough neutrosophic nano topology $\tau_{\mathcal{N}}(C, X, A)$ in Example 4.9. Let $B = \{< P_2, (0.6, 0.8, 1) >, < P_2, (0.6, 0.6, 1) >, < P_3, (0.6, 0.6, 1) >\}$. Then $\mathcal{N}_{int}(B) = \{< P_1, (0.6, 0.8, 1) >, < P_2, (0.6, 0.8, 1) >, < P_2, (0.6, 0.6, 1) >, < P_3, (0.6, 0.6, 1) >\}\}$. If $B = \{< P_2, (0.6, 0.8, 0) >, < P_2, (0.3, 0.6, 1) >, < P_3, (0.6, 0.5, 1) >\}$, then $\mathcal{N}_{int}(B) = \emptyset$.

Definition 23: If $\tau_{\mathcal{N}}(C, X, A)$ is a covering based rough neutrosophic nano topological space on an universe K and B be any neutrosophic subset of K. The covering based rough neutrosophic nano closure of B is defined as the intersection of all covering based rough neutrosophic nano closed sets containing B and it is denoted by $\mathcal{N}_{cl}(B)$. $\mathcal{N}_{cl}(B)$ is the smallest covering based rough neutrosophic nano closed set containing B.

Definition 24: Let $\tau_{\mathcal{N}}(C, X, A)$ be a covering based rough neutrosophic nano topological space on an universe K and $\mathscr{B}$ be a collection of subsets of $\tau_{\mathcal{N}}(C, X, A)$. If the collection of all unions members of of $\mathscr{B}$ is a covering based rough neutrosophic nano topological space on K, then it is called a base for $\tau_{\mathcal{N}}(C, X, A)$.

Definition 25: [45]: Let (K, C) be a covering based approximation space. For any $x \in K$, the neighbourhood of x is defined by $Neighbor(x) = \{M \in C \mid x \in M\}$.

By Definition 4.16, we give the following definition of the neighbourhood in our topology.

Definition 26: Let $\tau_{\mathcal{N}}(C, X, A)$ be a covering based rough neutrosophic nano topological space on a universe K, we define the neighbourhood of x as follows:

$$Neighbor(x) = \bigcap\{K \in \tau_{\mathcal{N}}(C, X, A) \mid x \in K\}.$$

Definition 27: Let $\tau_{\mathcal{N}}(C, X, A)$ be a covering based rough neutrosophic nano topological space on a universe K. If x is in each covering based rough neutrosophic nano set, then x is called a *core point*.

VI. APPLICATIONS

Thivagar and his associates showed that rough set, approximation space and especially nano topology has many real life applications in medical diagnosis, digital image segmentation, pattern recognition, nutrition modelling and recruitment process [34]–[36]. Generally, the symptoms of the patients or the opinions of the experts about certain attributes are evaluated on a table. In most of these studies, the core and the basis of the topologies of the approximation spaces formed by the lower and upper approximations, and boundaries are the determinants of the decision making with the data obtained from this table on a discourse. However, in some problems with more complex input and output, fuzzy, intuitionistic fuzzy or triple (neutrosophic) input or output are the features that are revealed. Especially in the field of pattern recognition, high-order spectra are used for the analysis and processing of triple inputs and outputs. When analysing these patterns, these triple inputs or outputs should be processed as data directly on their own property rather than being used as an attribute. In addition to pattern recognition, the use of higher order spectrum finds wide application in many areas such as diabetes diagnosis, heart rate, biomedical signals, radar HRRP target recognition [46]–[50]. More generally, *rough neutrosophic nano topology* can be used for analyzing and decision-making process on discourses with ternary data. Moreover, on the discourse of the data, *covering based rough neutrosophic nano topology* can be used to classify the discourse's parts in order to maintain the integrity of the discourse. When using *covering based rough neutrosophic nano topology*, the topology base will help to make a decision, just like other analyses via rough set and topology. Here, we give a simple application for a possible use of the topology.

Example 28: A city hall wants to make layout plan for bus stations with the intention of landscaping. Five bus stations that are indicated by $K = \{s_1, s_2, s_3, s_4, s_5\}$ are desired to be placed in the city. For some reasons, (information sharing, connection, distances to touristy areas, or convenience for fast transport of disabled individuals etc.) some buses moving between these stations are grouped to cover all stations of the city in terms of each station's situations. Let this covering set C be $\{\{s_1, s_2, s_3\}, \{s_2, s_4\}, \{s_3, s_5\}\}$. Moreover, they also offer a subset of non-surrendered options under certain options, and this subset is $X = \{s_2, s_3, s_5\}$. The city hall council presents an offer for the city hall's bus station placement and asks an expert A to prepare an evaluation report. The expert presents the report in the form of neutrosophic values because the expert evaluates many parameters. The report of the expert for the stations in accordance with the offer of the city hall is as follows: $A = \{< s_1, (0.2, 0.6, 0.3) >, < s_2, (0, 0.1, 0) >, < s_3, (0, 0, 1) >, < s_4, (1, 0, 0) >, < s_5, (0, 0.2, 1) >\}$.

As the first stage for checking whether the assessment of the expert is valid or not, we regard it as a *covering based rough neutrosophic nano topology* form in accordance with the offer and the report. If this structure forms the topology, we say that there is compatibility between the offer and the report. Now, let's examine it.

$$\begin{aligned}
C_{\overline{X}} &= \{\{s_2, s_4\}\{s_3, s_5\}, \{s_1, s_2, s_3\}\} \\
C_{\underline{X}} &= \{\{s_3, s_5\}\} \\
C_{BN(X)} &= \{\{s_1, s_2, s_3\}, \{s_2, s_4\}\} \\
\overline{\mathcal{N}}_{C_X}(A) &= \{< s_1, (0,0,1) >, < s_2, (0,0,1) >, \\
&\quad < s_3, (0,0,1) >, < s_4, (0,0,1) >, \\
&\quad < s_5, (0,0,1) >\} \\
\underline{\mathcal{N}}_{C_X}(A) &= \{< s_3, (0,0,1) >, < s_5, (0,0,1) >\} \\
\mathcal{N}_{BN(X)}(A) &= \{< s_1, (0,0,1) >, < s_2, (0,0,1) >, \\
&\quad < s_3, (0,0,1) >, < s_4, (0,0,1) >\}
\end{aligned}$$

Then, it forms a *covering based rough neutrosophic nano topology*. On the other hand, there are two core points $< s_3, (0,0,1) >$ and $< s_5, (0,0,1) >$ in the topology. These points indicate that $< s_3, (0,0,1) >$ and $< s_5, (0,0,1) >$ definitely wrong decisions for the planned placement of s_3 and s_5.

VII. CONCLUSION

Some new topologies and their definitions on covering property have been given in this paper. We gave new topology definitions that combine the sets such fuzzy, intuitionistic fuzzy, neutrosophic and rough with nano topology which are useful in many decision making problems. We showed that they are suitable to apply to many real life problems after the given definitions, and in the last part of the paper we gave the decision making application for a bus station placement problem. we hope that the work in this paper constitutes a new basis for new studies and applications.

In the future, we will define new points addition to core point definition and study decision making problems by using the points. Another future plan is to extend this study with interval valued and bipolar neutrosophic sets and their topologies and implement them on computer.

AUTHOR CONTRIBUTIONS

Conceptualization, M.A. and S.T.; Methodology, M.A., S.T., C.O and F.S; Validation, C.O., S.T., M.A. and F.S; Investigation, M.A. and S.T.; Resources, C.O., S.T., M.A. and F.S.; Writing-Original Draft Preparation, M.A., S.T. and C.O.; Writing-Review and Editing, C.O., S.T., M.A. and F.S.; Supervision, C.O. and F.S.

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