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Iterative FEBI-PO Hybrid Method for Analyzing Scattering Problems of Complex Structures Having Junctions With Large-Scale Platform

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ABSTRACT An iterative hybrid of finite element boundary integral (FEBI) method and physical optics (PO) is presented for scattering problems of complex structures having junctions with electrically large-scale platform. The treatment of basis functions on the junctions between complex structures and platform is discussed in detail. Moreover, the multilevel fast multipole algorithm and the adaptive cross approximation method are employed to accelerate the solution of FEBI and the computation of interaction between FEBI and PO regions, respectively. Compared with FEBI method, the hybrid method can efficiently reduce the number of unknowns and the computational complexity while preserving acceptable accuracy. In addition, the alternative implementation of FEBI and PO can also maintain good independence of the two methods. Some numerical results are discussed to demonstrate the efficiency and accuracy of the approach.

INDEX TERMS Electromagnetic scattering, finite element boundary integral, hybrid method, physical optics.

I. INTRODUCTION

The numerical simulation of electromagnetic scattering problems is really important for various engineering designs and applications. The properties of scatterers, such as electrically large scale, complicated geometries, complex anisotropic and heterogeneous materials, bring many challenges to the accuracy and efficiency of numerical methods. Among the general methods, the finite element method (FEM) [1] is recognized as the most versatile one, owning to its capability in handling electromagnetic problems with complicated geometries and complex materials. However, for solving unbounded problems with FEM, an artificial boundary must be introduced to truncate the infinite region, and an absorbing boundary condition needs to be imposed on the artificial boundary for getting a unique solution, which usually results in a large computational domain and truncation error. To overcome this difficulty, the finite element boundary integral (FEBI) [2]–[4] becomes a research hotspot and developed rapidly, which is the hybrid of FEM and boundary integral equation method. The boundary integral equation, which is solved with the method of moments (MoM) [5], can incorporate the Sommerfeld radiation condition exactly with the use of an appropriate Green's function. So the discretized domain can be kept to the minimum. However, this method is difficult to implement for objects with complex structures or inhomogeneous mediums. Besides, it generates a full impedance matrix. FEBI can be considered as the mutual complement of finite element method and the integral equation method. In the FEBI, an artificial boundary is first introduced to enclose the object, which can be just the surface of the object. FEM is used to deal with the interior region of the artificial boundary containing complex structures or inhomogeneous mediums, while the boundary integral equation method is used to describe the fields in the free space of exterior region. The fields in these two regions are coupled at the boundary via field continuity conditions. The applications of these full-wave methods mentioned above are often limited by their immense requirement of computational resource and computational complexity.

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To solve the scattering and radiation problems of complex structures with electrical large-scale PEC platform, these full-wave methods are extended by hybridizing with some high frequency techniques [6]–[9], such as physical optics (PO), uniform theory of diffraction (UTD), and so on. Among them, the hybridization of MoM and PO is a classical method [10]–[12], developed for onboard antenna problems, whose implement has two different ways. One is the conventional way, in which the contribution of PO region is coupled into the MoM impedance matrix. However, if the PO region is electrically large, the computation of PO contribution is still time consuming. The other one is the iterative way, whose representative is the efficient iterative MoM-PO (EI-MoM-PO) method proposed in [13]. In the method, the PO contribution is iteratively considered as an additional voltage source of the MoM region, and the iterative process is terminated when the relative error of the MoM current is smaller than a given threshold. In addition, the multilevel fast multipole algorithm (MLFMA) is adapted to further increase the efficiency and accuracy of the IE-MoM-PO method [14]. However, the MoM limits the capability of MoM-PO method in solving the complex structures. One approach is to substitute MoM with other Full-wave methods. A hybrid FEBI-MoM-PO formulation is discussed to mainly analyze radiation problems in [15]. In the paper, the contributions of PO region and FEBI region are coupled into a whole impedance matrix. Moreover, a robust domain decomposition preconditioner is applied to solve the impedance matrix system. Different from the direct coupling, an iterative hybrid of FEBI and PO is presented in [16] to study shielding effectiveness of a cavity above large platform. Then, [17] adopts the iterative hybrid of FEBI and PO for analyzing scattering problem of an inhomogeneous cavity with large platform.

It should be mentioned that the iterative hybrid FEBI-PO method considered in [16], [17] only deals with the problem when the cavity and the platform are unconnected. In this paper, we focus on discussing the iterative hybrid FEBI-PO method in analyzing scattering problems of complex structures having junctions with large-scale PEC platform, which can both preserve the efficient simulation of complex structures and handle the electrically large-scale PEC platform simultaneously. To further enhance the efficiency of the hybrid method, the multilevel fast multipole algorithm (MLFMA) [18] is employed to improve the solution of FEBI method, and the adaptive cross approximation (ACA) [19] is applied to accelerating the calculation of interaction between FEBI and PO regions, which remarkably reduces its memory usage and CPU time cost. In addition, since the FEBI and the PO are implemented alternatively, the advantages of the two methods can be kept effectively. The impedance matrix of FEBI only needs to be calculated once in the whole iteration process, while there is no need to solve any equation for getting the electrical current in PO regions. From the numerical examples, it is illuminated that, compared with FEBI method, the iterative method converges very fast, and can efficiently reduce the number of unknowns and the computational complexity while preserving acceptable accuracy for solving the scattering problems of complex structures having junctions with large platform.

The remainder of the paper is organized as follows. In Section 2, the procedure of the hybrid FEBI-PO method is discussed. Some numerical examples are shown in Section 3 to demonstrate the performance of the iterative FEBI-PO method. Finally, we make a conclusion in Section 4.

II. THE ITERATIVE HYBRID FEBI-PO METHOD

In this section, we first present the procedure of the iterative hybrid FEBI-PO method in details. Then, since the complex structure has junctions with large-scale PEC platform, we put stress on the discussion of basis functions on junctions. In addition, the ACA is adopted to enhance the computational efficiency for coupled matrix of FEBI and PO regions.

A. PROCEDURE OF THE ITERATIVE FEBI-PO METHOD

Consider the problem of electromagnetic wave scattering by an inhomogeneous and arbitrarily-shaped complex structure on a large-scale PEC platform. Since there are junctions between the structure and the platform, the complex structure and its surrounding area on the platform are selected as the FEBI region, and the left major part of the platform is the PO region.

The details of iterative hybrid FEBI-PO method have been described in [16], whose procedure can be summarized as follows.

Step 1: Divide the FEBI and PO regions.

Step 2: Solve the scattering problem of FEBI region with the original incident wave, and obtain the currents of FEBI region.

The matrix equation for FEBI region can be written as

$$\begin{bmatrix} K_{II} & K_{IS} & 0 \\ K_{SI} & K_{SS} & B \\ 0 & P & Q \end{bmatrix} \begin{bmatrix} E_I \\ E_S \\ \bar{H}_S \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix} \tag{1}$$

where $\left[E_I^T \; E_S^T \; \bar{H}_S^T \right]$ is the solution vector of FEBI region, corresponding to the electric and magnetic fields. The first two equations are resulted from FEM, while the last equation is deduced by boundary integral equation. The vector on the right hand side of (1), is the discrete coefficients of excitation. For convenience, we introduce an abbreviation for (1),

$$[Z]\,[X] = [V]\,. \tag{2}$$

Step 3: Calculate the n**th current in PO region induced by the latest FEBI-region currents and the original incident wave.**

The nth current in the PO region can be expanded with Rao-Wilton-Glisson (RWG) vector basis functions [20] and the aim of this step is to determine the coefficients of RWG basis functions. Assume that the expansion formula of the nth current in the PO region is

$$J^{PO,n} = \sum_{i=1}^{M_P} c_i^{PO,n} g_i^{PO} \tag{3}$$

where $J^{PO,n}$ is the PO-region current at nth iteration, g_i^{PO} is ith RWG basis function in PO region, M_P is the number of basis functions, and $c_i^{PO,n}$ is the coefficient of ith RWG basis function.

The coefficients $c_i^{PO,n}$ in (3) can be obtained with the means given in [21], namely,

$$c_k^{PO,n} = \tau_k^{inc} - \sum_{i=1}^{M_1} a_i^{FEBI,n-1} \tau_{i,k}^J - \sum_{i=1}^{M_2} b_i^{FEBI,n-1} \tau_{i,k}^M \tag{4}$$

and the symbols

$$\tau_k^{inc} = \left(\hat{t}_k^+ + \hat{t}_k^-\right) \cdot \delta_{inc} \hat{n} \times \bar{H}^{inc}(r_k) \tag{5}$$

$$\tau_{i,k}^J = \left(\hat{t}_k^+ + \hat{t}_k^-\right) \cdot \delta_{i,k}^J \hat{n} \times \tilde{K}\left(g_i^{FEBI}\right)(r_k) \tag{6}$$

$$\tau_{i,k}^M = \left(\hat{t}_k^+ + \hat{t}_k^-\right) \cdot \delta_{i,k}^M \hat{n} \times L\left(g_i^{FEBI}\right)(r_k) \tag{7}$$

The three terms on the right hand side of (4) denote the impressed magnetic field, magnetic fields generated by the surface electric and magnetic currents of FEBI region, respectively. The vector r_k is the middle point of the kth edge in the PO region, $\hat{n}$ is the outward unit vector normal to PO region at r_k, $\hat{t}_k^{\pm}$ are the unit vector perpendicular to the kth common edge in triangles $T_k^{\pm}$, respectively. g_i^{FEBI} is the ith RWG basis function in FEBI region. M_1 and M_2 are the numbers of RWG basis functions defined for expanding electric current and magnetic current in FEBI region, respectively. $a_i^{FEBI,n-1}$ and $b_i^{FEBI,n-1}$ are the $(n-1)$th solved coefficients of ith RWG basis function for electric current and magnetic current in FEBI region, respectively. δ denotes the shadowing effect, which can be determined with the algorithm in [22]. The operator L and $\tilde{K}$ are

$$L(X) = -ik \int_{S'} \left[X(r') + \frac{1}{k^2} \nabla\nabla' \cdot X(r')\right] G(r, r') dS' \tag{8}$$

$$\tilde{K}(X) = \int_{S'} X(r') \times \nabla G(r, r') dS' \tag{9}$$

where $G(r, r') = \frac{e^{ik|r-r'|}}{4\pi|r-r'|}$ is the Green's function.

Step 4: Calculate the disturbed excitationcaused by the PO-region current for FEBI region.

The vector of disturbed excitation is defined in [16],

$$\Delta_k^n = -\left\langle \hat{n} \times g_k^{FEBI}, L\left(J^{PO,n}\right)\right\rangle - \left\langle g_k^{FEBI}, L\left(J^{PO,n}\right)\right\rangle + \left\langle \hat{n} \times g_k^{FEBI}, \tilde{K}\left(J^{PO,n}\right)\right\rangle \tag{10}$$

where Δ_k^n is the kth element of the induced excitation at the nth iteration step, $\hat{n}$ denotes the exterior unit normal vector on the surface of FEBI region.

Step 5: Solve the scattering problem of FEBI region with the modified excitation, which is obtained by adding the disturbed excitation, and get the*n*thcurrents of FEBI region.

The matrix equation for FEBI region with modified excitation is

$$[Z][X] = [V] + \left[\Delta_k^n\right]. \tag{11}$$

Here the impedance matrix $[Z]$ and the vector $[V]$ are invariable in the whole iteration process. Only the vector $\left[\Delta_k^n\right]$ changes in each iteration step.

Step 6: Check whether the iteration should continue.

The relative error is defined in [16], which is

$$\varepsilon_n = \frac{\left\|\left[X_n^{FEBI}\right] - \left[X_{n-1}^{FEBI}\right]\right\|}{\left\|\left[X_{n-1}^{FEBI}\right]\right\|}, \quad n = 1, 2, \cdots, \tag{12}$$

where the symbol $\|\cdot\|$ denotes the norm, the vector $\left[X_n^{FEBI}\right]$ is the nth iterative solution of FEBI region. If the error is less than the threshold, exit the iteration. Otherwise, $n := n + 1$ and go to Step 3.

With regard to the solution of the FEBI matrix system, we decouple the FEM part from the boundary integral part and acquire two submatrix systems with less unknowns. With the submatrix system of FEM part, the unknowns related to FEM can be expressed in terms of that associated to boundary integral part. Then, a reduced matrix system only related to boundary integral part is obtained, whose solution is accelerated with MLFMA. Thereafter, we can gain the unknowns of FEM part with the solved unknowns of boundary integral part and the FEM matrix system.

B. BASIS FUNCTIONS ON JUNCTIONS

In Step 2 and Step 5 of the iterative hybrid FEBI-PO method, FEBI method is employed to solve the scattering problems. Therefore, we use the tetrahedral mesh to discretize the FEBI region and take vector basis functions for edge elements [1] to discretize electric and magnetic fields. With the triangle discretization on the surface inherited from the tetrahedral mesh of the FEBI region, the equivalent surface currents can be expanded in terms of the RWG vector basis functions. It should be mentioned that there is the relationship between the two kinds of basis functions, i.e.

$$\hat{n} \times N_e = g_e, \tag{13}$$

where N_e is the vector basis function for edge e to discretize electric (magnetic) fields in the FEBI region, and g_e is the RWG basis function for edge e to discretize electric (magnetic) currents on the surface of the FEBI region, whose expression is

$$g_e(r) = \begin{cases} \dfrac{L_e}{2A_e^+}\left(r - r_e^+\right), & r \in T_e^+, \\ -\dfrac{L_e}{2A_e^-}\left(r - r_e^-\right), & r \in T_e^-, \\ 0, & \text{otherwise}, \end{cases} \tag{14}$$

where L_e is the length of edge e, $A_e^{\pm}$ is the area of triangle $T_e^{\pm}$, and $r_e^{\pm}$ is the free vertex of triangle $T_e^{\pm}$.

According to [23], an edge on the triangular mesh is called a junction, if more than two domains or surfaces meet at this

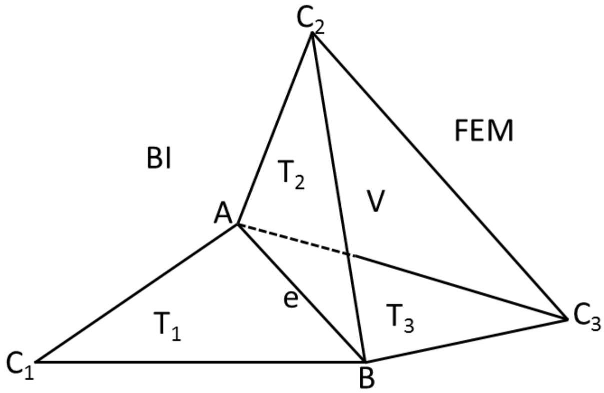

FIGURE 1. An example of a junction of three triangles.

edge. Following the definition, some kinds of junctions are often encountered, when the complex structure is on the PEC platform. Fig. 1 illustrates an example of a junction between the complex structure and the PEC platform, which is denoted by e. The edge e also can be regarded as a junction of triangles T_1, T_2 and T_3.The tetrahedron V belongs to the discretization mesh of the complex structure. Triangles T_2 and T_3 of V are on the boundary surface of the complex structure, while triangles T_1 and T_3 are on the surface of PEC platform. Since the magnetic surface current M vanishes on metallic surfaces, it is not necessary to define the basis function for magnetic surface current associated to e. Therefore, we merely discuss the basis functions for electric surface current J and electromagnetic fields (E and H) in the tetrahedron assigned to junctions.

In addition, (13) indicates that the tangential component of the vector basis function N_e on the surface assigned to edge e for discretizing the magnetic field in FEBI region is just the oriented RWG basis function g_e associated to edge e for discretizing the electric current on the surface. So the discussion of basis functions assigned to junctions should be based on the boundary conditions of electromagnetic fields and the continuity property of the surface currents, which are influenced by the features of platform and the complex structure. The geometry and material complexity of the structure allows us to assume that it has inhomogeneous dielectric domains and its surface is not closed metallic. We mainly consider that the PEC platform is open or closed and the triangle T_2 is metallic or dielectric, which results in the four cases given in Fig. 2.

In Fig. 2(a), the PEC platform is open and the triangle T_2 is metallic. Since the tangential component of magnetic field is not continuous across metallic surfaces, the electric surface current J has independent values on the opposite sides of metallic surfaces. Hence, three RWG basis functions can be defined to describe the electric surface current associated to junction e, whose $T_e^{\pm}$ are listed in Table 1.

FEM is used in FEBI region and a matrix equation is obtained, whose number of unknowns is larger than that of equations. Thus, boundary integral equations defined on the surface for the exterior of FEBI region are needed to make the

TABLE 1. Parameters for three RWG basis functions assigned to *e*.

g_i	T_i^+	T_i^-
g_1	T_1	T_3
g_2	T_3	T_2
g_3	T_2	T_1

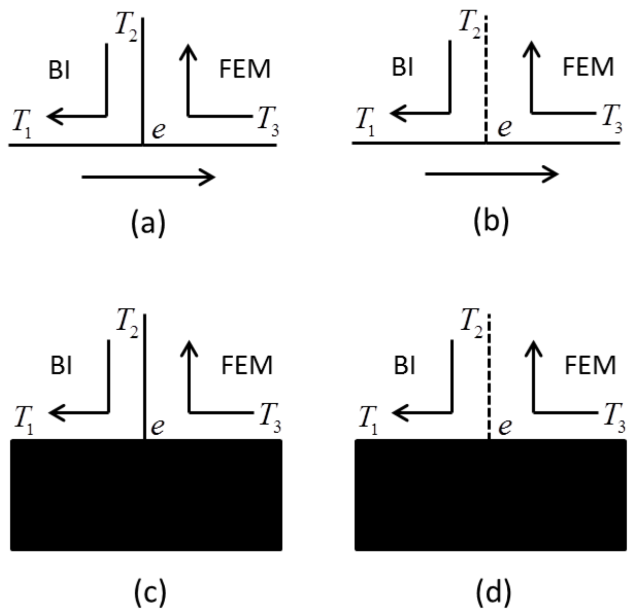

FIGURE 2. The sectional drawings of junctions and basis functions for electric surface current density assigned to junctions in four cases. solid lines denote triangle on open metallic surface, black blocks denote triangle on the surface of closed metallic domain, and dashed lines denote triangle on dielectric surface. The intersections of solid or dashed lines indicate the junction *e*. The arrows denote the oriented basis functions. (a).The case with open platform and metallic triangle T_2. (b). The case with open platform and dielectric triangle T_2. (c). The case with closed platform and metallic triangle T_2. (d). The case with closed platform and dielectric triangle T_2.

matrix system uniquely solvable. Furthermore, with respect to the discretization of the boundary integral equations, there are three RWG basis functions assigned to edge e. Since they are linear dependent, we can adopt arbitrary two RWG basis functions for discretizing the boundary integral equations. It means that we should remove one RWG basis function to guarantee that the rank of impedance matrix is full. In fact, once the vector basis function N_e assigned to edge e is employed in FEM for discretizing the magnetic field, it is equivalent to that the related RWG basis function g_2 is used in the impedance matrix. Therefore, we preserve g_2 and remove any one of g_1 and g_3.

Fig. 2(b) displays the case with the open PEC platform and dielectric triangle T_2. The discretization in the interior of FEBI region with FEM is similar to the foregoing case. As for the discretization of boundary integral equations for the exterior of FEBI region, on the opposite sides of metallic surfaces, the electric surface current J still has independent values, so the RWG basis functions assigned to junction e on the opposite sides of metallic surface have independent unknown coefficients. However, since the tangential component of magnetic field is continuous across the dielectric surface and the orientation of RWG basis functions depends on the exterior normal vector of the surface, the unknown coefficients of the oriented RWG basis functions assigned to edge e on the opposite sides of dielectric surface have the same value, i.e., the unknown coefficients of g_2 and g_3 are the same. Actually, the coefficient of g_2 is that of the vector basis function N_e assigned to edge e defined in the FEBI region. Thus, by adding the corresponding columns of N_e and g_3 in the matrix equations together, these unknowns can be combined to a single one.

The cases with the closed platform are shown in Fig. 2(c) and 2(d), respectively. The closed PEC platform implies that there is no need to define the RWG basis function g_1 and only the other two RWG basis functions, g_2 and g_3 are left. In FEBI region, the vector basis function N_e assigned to edge e is used by FEM, which corresponds to g_2, while the RWG basis function g_3 is used in the discretization of boundary integral equations for the exterior of FEBI region.

C. ACA FOR ACCELERATING THE INTERACTION BETWEEN FEBI AND PO REGIONS

In Step 4 of the iterative hybrid FEBI-PO method, we need to calculate the disturbed excitation caused by the PO-region current for FEBI region. We assume that the PO-region current at nth iteration has the following expansion,

$$J^{PO,n} = \sum_{i=1}^{M_P} c_i^{PO,n} g_i^{PO} \tag{15}$$

where M_P is the number of unknowns in PO region, $c_i^{PO,n}$ is the expansion coefficients and g_i^{PO} is ith RWG basis function in PO region.

With (15), the disturbed excitation can be rewritten as

$$\Delta_k^n = \sum_{i=1}^{M_P} c_i^{PO,n} \left(-\left\langle \hat{n} \times g_k, L\left(g_i^{PO}\right)\right\rangle - \left\langle g_k, L\left(g_i^{PO}\right)\right\rangle + \left\langle \hat{n} \times g_k, \tilde{K}\left(g_i^{PO}\right)\right\rangle \right) \tag{16}$$

The integral terms in (16) reflect the interaction between FEBI and PO regions, which all correspond to full matrices. The electrically large scale of PEC platform leads to a great number of unknowns in PO region. Even though the size of FEBI region is electrically moderate, the computational complexity and memory requirement of the matrices of the three integral terms are still considerable. Therefore, ACA algorithm [19] is adopted here to accelerate the computation of the integral terms in (16) and reduce the relevant memory requirement.

III. NUMERICAL EXAMPLE

Several numerical examples are displayed in this section to demonstrate the efficiency and accuracy of the iterative FEBI-PO hybrid method. The numerical results of the MLFMA, FEBI or the MoM-PO hybrid method are taken as benchmarks. Regarding discretization, the mesh sizes are about 0.05λ, 0.1λ and 0.3λ for FEM-region, BI-region and PO-region, respectively, where λ is the wavelength of incident plane wave. Both the threshold in step 6 of the procedure of the iterative hybrid FEBI-PO method described in section II and the terminating tolerance ε in ACA algorithm are chosen as $1.0e-3$.

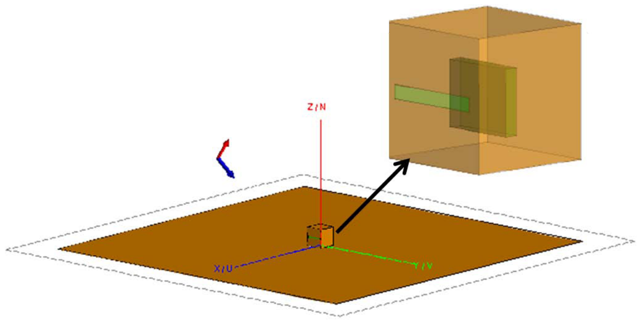

FIGURE 3. The schematic of an inhomogeneous cavity on a large-scale platform.

A. AN INHOMOGENEOUS CAVITY ON A LARGE-SCALE PLATFORM

We first study an inhomogeneous cavity on a large platform, as shown in Fig. 3. The cavity is a cubic and its length is 1m. A small aperture is on the cavity, whose width and length are 0.1m and 0.8m, respectively. Moreover, there is a dielectric slab within the cavity. The slab size is 0.1m × 0.6m × 0.5m and its relative permittivity is 1.5. The size of the platform under the cavity is 20m ×20m. The propagation and polarization directions of the incident plane wave are illustrated in Fig. 3, i.e., the polarization direction $(\theta, \varphi) = (45°, 180°)$ and the propagation direction $(\theta, \varphi) = (135°, 180°)$, where θ and φ denote the coordinate elevation angle and azimuth angle, respectively.

TABLE 2. Discretizations for FEBI and FEBI-PO.

Method	Tetrahedron number	Triangle number on cavity surface	Triangle number on the platform	Total Triangle number
FEBI	74857	6040	110098	116138
FEBI-PO	74857	6040	12348	18388

In the example, to apply iterative FEBI-PO method, the inhomogeneous cavity and its surrounding area on the platform are regarded as FEBI region, and the left part of the platform is considered as PO region. Assume that the frequency of incident wave is 0.3GHz. Table 2 provides the discretization for FEBI and iterative FEBI-PO. It is displayed that, by employing PO method, the hybrid method of FEBI-PO can reduce the number of unknowns remarkably, compared

TABLE 3. The number of unknowns and the CPU itme for FEBI and FEBI-PO.

Method	Unknowns for FEM	Unknowns for BI	Unknowns for PO	CPU time (second)
FEBI	74857	173982	--	60231.55
FEBI-PO	74857	9251	18380	6862.94

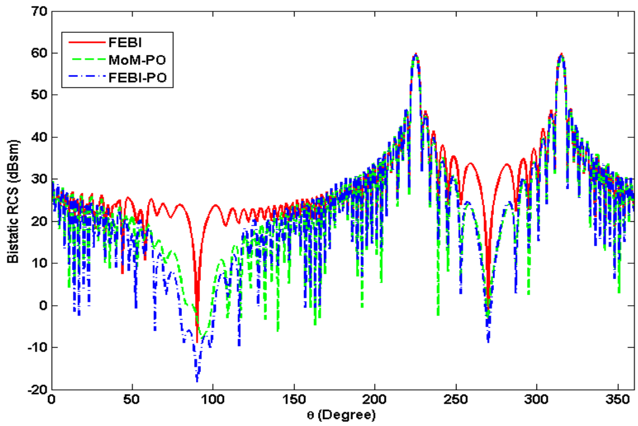

FIGURE 4. The bistatic RCS at 0.3GHz for plane wave scattering from the inhomogeneous cavity on a large platform provided by FEBI, MoM-PO and FEBI-PO.

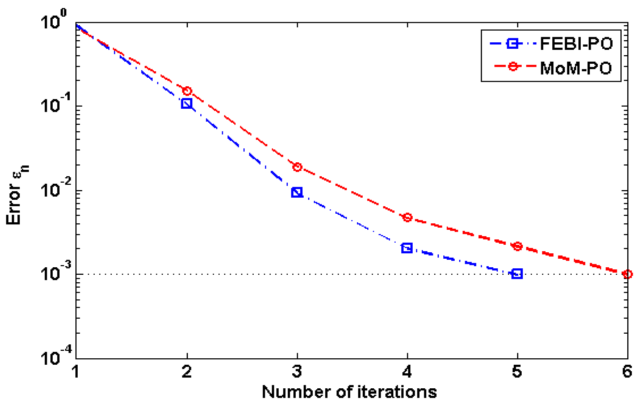

FIGURE 5. The iteration history of the two hybrid methods for the cavity on a large platform when the threshold is chosen as 0.001.

with FEBI. Fig. 4 displays the numerical results of bistatic radar cross section (RCS) for the model, provided by FEBI, iterative MoM-PO and iterative FEBI-PO, respectively. It can be seen that the result of iterative FEBI-PO agrees well with that of iterative MoM-PO, and results of these two hybrid methods are quite consistent with that of FEBI except for the values in the neighborhoods of 90° and 270°, which is caused by the inaccuracy of PO near these two angles. The iteration history of the MoM-PO and FEBI-PO hybrid methods are given in Fig. 5. Both iterative hybrid methods converge to $1.0e-3$ quickly for this example. The number of unknowns and the CPU time cost for FEBI and iterative FEBI-PO are shown in Table 3. The CPU time used by iterative FEBI-PO

TABLE 4. The unknowns in different regions and reduced dimension of ACA with different frequency.

Frequency (GHz)	Unknowns of BI region	Unknowns of PO region	Reduced dimension of ACA
0.1	1092	2088	83
0.15	2274	4641	97
0.2	4167	8170	120
0.25	6615	12920	140
0.3	9251	18380	168

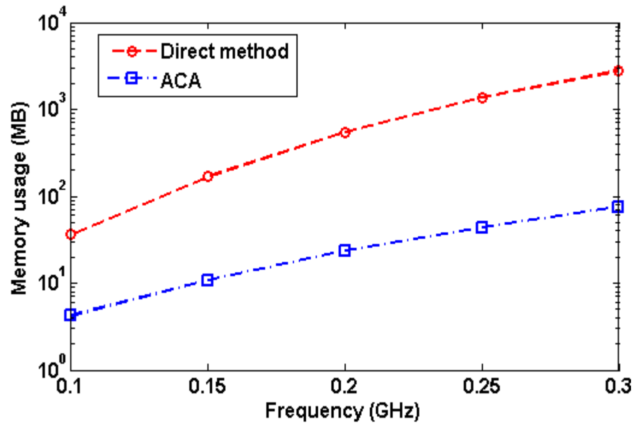

FIGURE 6. The memory usages of direct method and ACA for calculating the coupling matrix with the frequency varying from 0.1GHz to 0.3GHz.

is about 11.4 percent of that used by FEBI, which indicates the significant efficiency of the iterative FEBI-PO.

Then, for exhibiting the superiority of ACA in accelerating the computation and reducing the memory requirement, we analyze the numerical results of the example with the frequency varying from 0.1GHz to 0.3GHz. Table 4 displays the unknowns of BI region and PO region, which correspond to the row and column dimensions of the coupling matrix. Besides, the reduced dimension of ACA is also given in Table 4. Fig. 6 and Fig. 7 demonstrate the memory usages and CPU time costs of direct method and ACA for calculating the coupling matrix at different frequency, respectively. The memory usage is reduced over 90% and CPU time is saved over 80% for the example. It indicates that ACA can dramatically enhance the efficiency of iterative FEBI-PO by reducing the CPU time cost and memory usage.

B. A SHIP MODEL

The second example is the scattering problem of a ship model in Fig. 8, where the cabin is on the platform. The dimension of the cabin is 7.2m × 4.85m × 4.1m and there is a curvilinear aperture on the cabin surface, whose width and length are 0.1 and 0.8, respectively. A dielectric seat is within the cabin, whose relative permittivity is 1.5. Moreover, the dimension of ship body is about 130m ×23.3m ×20m. The incident plane wave impinges on the ship, whose polarization

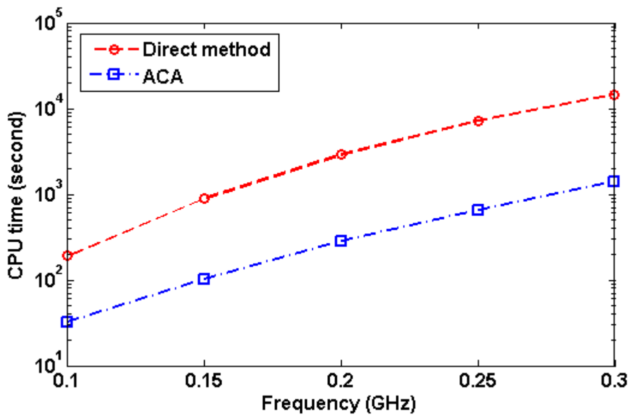

FIGURE 7. The CPU time costs of direct method and ACA for calculating the coupling matrix with the frequency varying from 0.1GHz to 0.3GHz.

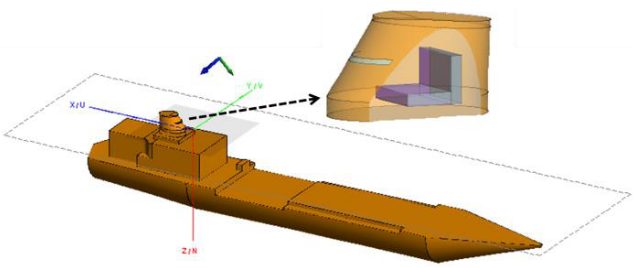

FIGURE 8. The schematic of a ship model consisting of a cabin and a large-scale ship body.

TABLE 5. Discretizations for FEBI and FEBI-PO.

Method	Tetrahedron number	Triangle number on the cabin surface	Triangle number on the ship body	Total Triangle number
FEBI	241496	13255	196620	209875
FEBI-PO	240892	13255	22152	35407

direction $(\theta, \varphi) = (45°, 0°)$ and propagation direction $(\theta, \varphi) = (45°, 180°)$.

When the frequency of incident wave is 0.1GHz, the FEBI, iterative MoM-PO and iterative FEBI-PO are adopted to give the numerical results of bistatic RCS for the ship model (see Fig. 9). The discretizations for FEBI and iterative FEBI-PO are presented in Table 5. The number of triangles for discretizing the ship body is reduced evidently by using iterative FEBI-PO, compared with FEBI. Furthermore, the number of unknowns and the CPU time cost for FEBI and iterative FEBI-PO are given in Table 6. It reveals that iterative FEBI-PO is considerably efficient in reducing the number of unknowns and CPU time.

Fig. 9 illustrates that the iterative FEBI-PO is capable of providing a solution with broadly equivalent accuracy, compared with iterative MoM-PO. Moreover, we also show the iteration history of iterative FEBI-PO and MoM-PO for the ship model in Fig. 10. The iterative FEBI-PO just needs 7 iterations to reach the convergence, while iterative MoM-PO takes 12 iterations to arrive at the threshold for this example.

TABLE 6. The number of unknowns and the CPU itme for FEBI and FEBI-PO.

Method	Unknowns for FEM	Unknowns for BI	Unknowns for PO	CPU time (second)
FEBI	277966	315094	--	146520.6
FEBI-PO	277966	20164	33228	36242.04

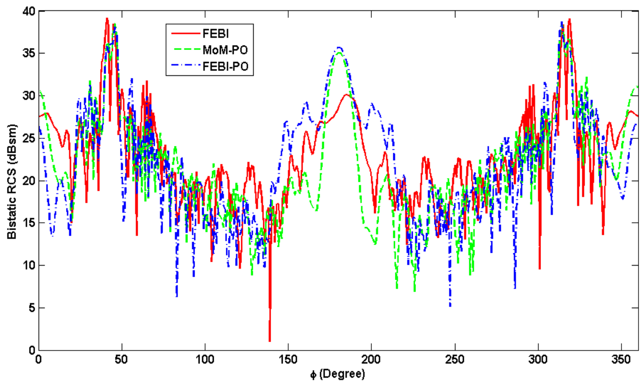

FIGURE 9. The bistatic RCS at 0.1GHz for plane wave scattering from the ship model provided by FEBI, MoM-PO and FEBI-PO.

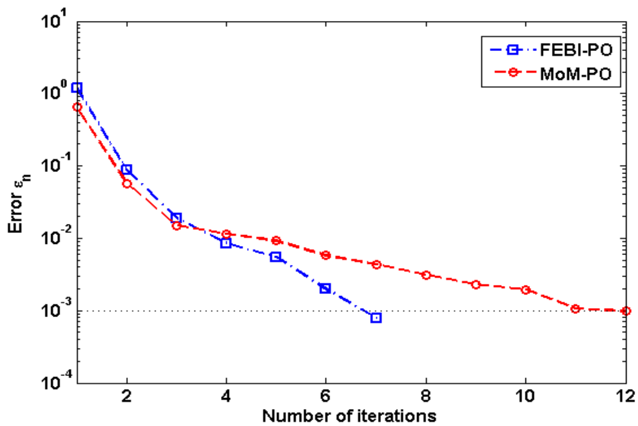

FIGURE 10. The iteration history of the two hybrid methods for the ship model when the threshold is chosen as 0.001.

IV. CONCLUSION

To both preserve the efficient simulation for complex structures and handle the electrically large-scale PEC platform, we developed an iterative hybrid of FEBI and PO method. The whole regions are divided into FEBI region and PO region, according to the structure and material properties. In the iterative process, the FEBI region and PO region are alternatively solved. The interaction of the two regions is realized by updating the disturbed excitation in one region with the latest computed current of the other region. The MLFMA is used for the speedup of the solution in FEBI region, while the ACA is also employed to accelerate the computations of interaction between FEBI and PO regions. Numerical experiments illuminate that the developed method converges fast and can efficiently reduce the number of unknowns and the computational complexity while providing acceptable accuracy, compared with FEBI method. Therefore, the paper provides an efficient means for analyzing scattering problems of complex structures having junctions with electrically large-scale platform.

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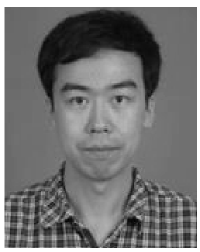

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