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Asymptotic Tracking Control for a Class of Pure-Feedback Nonlinear Systems

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ABSTRACT This paper studies the adaptive asymptotic tracking problem for a class of unknown nonlinear systems in pure-feedback form. Different from the traditional literatures which only tackle the bounded tracking problem for pure-feedback systems, this paper investigates the asymptotic tracking problem by developing a novel controller design method. Moreover, the differentiable assumption on nonaffine functions is canceled, and only a mild semi-bounded assumption is required as the controllability condition. By utilizing Lyapunov theorem, it is proved that all the variables of the resulting closed-loop system are semi-globally uniformly ultimately bounded, and the output tracking error can converge to zero asymptotically by choosing design parameters appropriately. Finally, a simulation result is presented to verify the effectiveness of the proposed control scheme.

INDEX TERMS Asymptotic stability, neural networks, nonlinear control systems, pure-feedback systems.

I. INTRODUCTION

In the last several decades, adaptive control techniques have been found to be powerful for controlling the triangle-structural nonlinear systems in terms of either pure-feedback or strict-feedback [1]–[17]. Specifically, pure-feedback systems do not have the explicit control input, which makes the control design very difficult and draws much interest in the control community for a long time [8]–[17]. In [10], to solve the prescribed performance tracking control problem, a low-complexity control scheme is designed for a class of unknown pure-feedback systems. In [11], a predefined-tracking-constrained-based adaptive control scheme is developed for a class of switched stochastic nonlinear systems in the pure-feedback form with dead zone output. By employ the mean value theorem to convert the nonaffine function into an affine form, all these studies referred above have presented a unified and general framework for pure-feedback nonlinear control system design. However, there are still a number of issues should have been further studied, such as, the mean value theorem requires the nonaffine function must be differentiable with respect to the control variables or input. In the hope to overcome these problems, in [12], a pioneering modeling method is presented under the mild assumptions. Instead of utilizing mean value theorem and implicit function theorem, this control method does not require that the nonaffine functions must be differentiable. Subsequently, the controllability conditions are relaxed to semi-bounded and discontinuity in [13] and [14], respectively. In [15], the further research is devoted to a class of more general MIMO pure-feedback nonlinear systems with periodic disturbances.

As for pure-feedback nonlinear systems, it is commonly seen that the system nonlinearities are unknown because of the characteristics of pure-feedback form systems. Moreover, there is usually only a mild assumption on the control directions to be used. The unknown nonlinearities of pure-feedback nonlinear systems suggest that NN or FLS-based control methods are always preferable for them. However, the major drawback of such approximator-based control methods is that, the output tracking error cannot converge to zero asymptotically owing to the presence of approximation error. It is well known that asymptotic tracking has progressed a lot both in theory and practice [18]–[24]. To eliminate the effect of approximation error, a novel neural networks-based adaptive controller is designed for a class of uncertain strict-feedback nonlinear systems [24]. However, the above-mentioned controller design method is limited

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to strict-feedback systems rather than more complex pure-feedback systems. It is worth noting that, for pure-feedback systems, it is very hard to achieve the asymptotic tracking because more sophisticated insight for system structure is needed.

Motivated with the above issues, this study falls in a domain with the asymptotic tracking problem of controlling a class of triangle structure pure-feedback nonlinear systems. The main contributions of this paper are summarized as follows.

1) To the best of the authors' knowledge, it is the first time that the asymptotic tracking problem of pure-feedback systems is achieved;

2) Compared with the existing literature, the assumption on nonaffine functions are much more relaxed, and only the semi-bounded and continuity conditions are required, which makes the control design difficult. To overcome the difficulty, a novel adaptive tracking controller is therefore proposed for pure-feedback systems under this condition;

3) With the help of the Lyapunov stability theorem and Barbalat lemma, all the variables of the resulting closed-loop system are proven to be semi-globally bounded, and the tracking error can asymptotically converge to zero by appropriately choosing the design parameters.

The rest of this paper is organized as follows. Section II gives the problem formulation and preliminaries. In Section III, a modified adaptive neural controller is developed for a class of uncertain pure-feedback nonlinear by using backstepping scheme. The stability analysis of the closed-loop system is given in Section IV. In Section V, simulation study is presented to show the effectiveness of the proposed scheme. Finally, the conclusion is included in Section VI.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider a class of uncertain strict-feedback nonlinear systems of the following form

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i+1}) + \Delta_i(t), & i = 1, 2, \ldots, n-1 \\ \dot{x}_n = f_n(\bar{x}_n, u) + \Delta_n(t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ denotes the state vector of the system; $u \in R$ is system control input; $y \in R$ is system output; $f_i(\cdot)$ are unknown continuous functions; $\Delta_i(t)$ are the unknown external disturbances or uncertainties of the system, $i = 1, \ldots, n$.

The control objective is to design adaptive tracking control such that the system output y asymptotically converges to a desired trajectory y_d and all signals in the closed-loop system are bounded by appropriately choosing design parameters.

To guarantee the controllability, we will invoke the following assumptions, which are standard in backstepping design method.

The main difficulty of this control design problem is that the variables and system input do not appear linearly, which makes the direct feedback linearization difficult or impossible. Define the functions

$$F_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, x_{i+1}) - f_i(\bar{x}_i, 0), \quad i = 1, 2, \ldots, n$$

And denote $x_{n+1} = u$, $\bar{x}_{n+1} = [x_1, x_2, \ldots, x_n, u]^T$ for notation conciseness. Before proceeding to the adaptive fuzzy control design of system (1), let us consider the following assumptions.

Assumption 1: For all x and u, there exist constants l_i, l_i', ϑ_i and ϑ_i' such that

$$\begin{cases} F_i(\bar{x}_i, x_{i+1}) \geq l_i x_{i+1} + \vartheta_i, & x_{i+1} \geq 0 \\ F_i(\bar{x}_i, x_{i+1}) \leq l'_i x_{i+1} + \vartheta'_i, & x_{i+1} < 0 \end{cases} \tag{2}$$

where l_i and l_i' are positive constants, $i = 1, 2, \ldots, n$.

Assumption 2: The desired trajectory y_d is sufficiently smooth function of t, and y_d, $\dot{y}_d$ are bounded, that is, there exists a positive constant B_0 such that $\Pi_0 := \{(y_d, \dot{y}_d, \ddot{y}_d) : (y_d)^2 + (\dot{y}_d)^2 \leq B_0\}$.

Assumption 3: For $1 \leq i \leq n$, there exist an unknown positive constant d_i^* such that $|\Delta_i(t)| \leq d_i^*$.

Lemma 1 [21]: for any $q \in R$ and $\forall \upsilon > 0$, the following inequality holds

$$0 \leq |q| - \frac{q^2}{\sqrt{q^2 + \upsilon^2}} \leq \upsilon \tag{3}$$

A. RBFNN BASICS

The radial basis function neural network (RBFNN) is considered to be used for the controller design in this paper, which is utilized to approximate the continuous function $h(Z)$: $R^n \to R$:

$$h_{nn}(Z) = W^T \psi(Z) \tag{4}$$

where the input vector $Z \in \Omega_Z \subset R^n$, the weights vector $W = [W_1, W_2, \ldots, W_l] \in R^l$, the neural network (NN) node number $l > 1$, and $\psi(Z) = [\psi_1(Z), \ldots, \psi_l(Z)]^T$ with $\psi_i(Z)$ being chosen commonly as a Gaussian function as

$$\psi_i(Z) = \exp\left[\frac{-(Z-\mu_i)^T(Z-\mu_i)}{\eta^2}\right], \quad i = 1, 2, \ldots, l \tag{5}$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ is the center of the receptive field and η is the width of the Gaussian function.

It has been proven that the neural network (4) can approximate any continuous function over a compact set $\Omega_Z \subset R^n$ to any desired accuracy in the form of

$$h(Z) = W^{*T}\psi(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z \subset R^n \tag{6}$$

where W^* is the ideal constant weight vector, and $\varepsilon(Z)$ is the approximation error which is bounded over the compact set, that is, $\|\varepsilon(Z)\| \leq \varepsilon^*$ for $\forall Z \in \Omega_Z$, where $\varepsilon^* > 0$ is an unknown constant. $\varepsilon(Z)$ is denoted as ε to simplify the notation in this paper.

The optimal weight vector W^* is an "artificial" quantity required only for analytical purposes. Typically, W^* is chosen as the value of W that minimizes ε over Ω_Z, that is

$$W^* := \arg \min_{W \in R^l} \left\{ \sup_{Z \in \Omega_Z} |h(Z) - W^T \psi(Z)| \right\} \tag{7}$$

Let $||\cdot||$ denote the 2-norm throughout this paper.

Remark 1: Define the continuous functions $g_{i,1}(\bar{x}_i)$, $g_{i,2}(\bar{x}_i, x_{i+1})$, $g_{i,3}(\bar{x}_i, x_{i+1})$ as follows

$$|F_i(\bar{x}_i, x_{i+1})| \leq g_{i,1}(\bar{x}_i), \quad -a_0 < x_{i+1} < a_0 \tag{8}$$

$$g_{i,2}(\bar{x}_i, x_{i+1}) = \frac{1}{x_{i+1}} \left(F_i(\bar{x}_i, x_{i+1}) - \vartheta_i\right), \quad x_{i+1} \geq a_0 \tag{9}$$

$$g_{i,3}(\bar{x}_i, x_{i+1}) = \frac{1}{x_{i+1}} \left(F_i(\bar{x}_i, x_{i+1}) - \vartheta'\right)_i, \quad x_{i+1} \leq -a_0 \tag{10}$$

where a_0 is any positive constant. From the above definitions, it can seen that they are well-defined continuous functions since x_{i+1} is strictly positive or negative and away from 0 when $g_{i,2}(\bar{x}_i, x_{i+1})$ and $g_{i,3}(\bar{x}_i, x_{i+1})$ are concerned. It follows from (2), (9) and (10) that $g_{i,2}(\bar{x}_i, x_{i+1}) \geq l_i$, $g_{i,3}(\bar{x}_i, x_{i+1}) \geq l'_i$ and

$$F_i(\bar{x}_i, x_{i+1}) = \begin{cases} g_{i,2}(\bar{x}_i, x_{i+1})x_{i+1} + \vartheta_i, & x_{i+1} \geq a_0 \\ l_i x_{i+1} + \mu_{i,1}(t) g_{i,1}(\bar{x}_i) + \mu_{i,2}(t) a_0, & 0 < x_{i+1} < -a_0 \\ l'_i x_{i+1} + \mu_{i,3}(t) g_{i,1}(\bar{x}_i) + \mu_{i,4}(t) a_0, & -a_0 < x_{i+1} < 0 \\ g_{i,3}(\bar{x}_i, x_{i+1}) x_{i+1} + \vartheta'_i, & x_{i+1} \leq -a_0 \end{cases} \tag{11}$$

where $\mu_{i,1}(t) \in [-1, 1]$, $\mu_{i,2}(t) \in [-1, 1]$, $\mu_{i,3}(t) \in [-1, 1]$ and $\mu_{i,4}(t) \in [-1, 1]$ are some unknown bounded functions.

Then, it follows from (11) that $F_i(\bar{x}_i, x_{i+1})$ can be rewritten as the form as follows

$$F_i(\bar{x}_i, x_{i+1}) = G_i(\bar{x}_{i+1}, t)x_{i+1} + \eta_i(\bar{x}_i, t) \tag{12}$$

with

$$G_i(\bar{x}_{i+1}, t) = \begin{cases} g_{i,2}(\bar{x}_i, x_{i+1}), & x_{i+1} \geq a_0 \\ l_i, & 0 < x_{i+1} < -a_0 \\ l'_i, & -a_0 < x_{i+1} < 0 \\ g_{i,3}(\bar{x}_i, x_{i+1}), & x_{i+1} \leq -a_0 \end{cases} \tag{13}$$

$$\eta_i(\bar{x}_i, t) = \begin{cases} \vartheta_i, & x_{i+1} \geq a_0 \\ \mu_{i,1}(t) g_{i,1}(\bar{x}_i) + \mu_{i,2}(t) a_0, & 0 < x_{i+1} < -a_0 \\ \mu_{i,3}(t) g_{i,1}(\bar{x}_i) + \mu_{i,4}(t) a_0, & -a_0 < x_{i+1} < 0 \\ \vartheta'_i, & x_{i+1} \leq -a_0 \end{cases} \tag{14}$$

Noting (13), (14) and the fact that $g_{i,2}(\bar{x}_i, x_{i+1}) \geq l_i$, $g_{i,3}(\bar{x}_i, x_{i+1}) \geq l'_i$, it can be known that

$$\min\{l_i, l'_i\} = g_{i,m} \leq G_i(\bar{x}_{i+1}, t) \leq \max\left\{g_{i,2}(\bar{x}_i, x_{i+1}), g_{i,3}(\bar{x}_i, x_{i+1}), l_i, l'_i\right\} \tag{15}$$

$$|\eta_i(\bar{x}_i, t)| \leq \max\left\{|\vartheta_i|, |\vartheta'_i|, |g_{i,1}(\bar{x}_i)| + a_0, |g_{i,1}(\bar{x}_i)| + a_0\right\} \tag{16}$$

Since $g_{i,2}(\bar{x}_i, x_{i+1})$, $g_{i,3}(\bar{x}_i, x_{i+1})$ and $g_{i,1}(\bar{x}_i)$ are well-defined continuous functions, we use RBFNN to approximate them as follows

$$g_{i,2}(\bar{x}_i, x_{i+1}) = W_{i,2}^{*T} \psi_{i,2}(\bar{x}_i, x_{i+1}) + \varepsilon_{i,2}, \quad \bar{x}_{i+1} \in \Omega_{\bar{x}_{i+1}}, \quad x_{i+1} \geq a_0 \tag{17}$$

$$g_{i,3}(\bar{x}_i, x_{i+1}) = W_{i,3}^{*T} \psi_{i,3}(\bar{x}_i, x_{i+1}) + \varepsilon_{i,3}, \quad \bar{x}_{i+1} \in \Omega_{\bar{x}_{i+1}}, \quad x_{i+1} \leq -a_0 \tag{18}$$

$$g_{i,1}(\bar{x}_i) = W_{i,1}^{*T} \psi_{i,1}(\bar{x}_i) + \varepsilon_{i,1}, \quad \bar{x}_i \in \Omega_{\bar{x}_i} \tag{19}$$

where $\varepsilon_{i,1}$, $\varepsilon_{i,2}$ and $\varepsilon_{i,3}$ are the approximation errors, satisfying $|\varepsilon_{i,1}| \leq \varepsilon_{i,1}^*$, $|\varepsilon_{i,2}| \leq \varepsilon_{i,2}^*$, $|\varepsilon_{i,3}| \leq \varepsilon_{i,3}^*$, with $\varepsilon_{i,1}^*$, $\varepsilon_{i,2}^*$, $\varepsilon_{i,3}^*$ being unknown positive constants.

From [9], it can be known that $\|\psi_1(Z_1)\| \leq s^*$ with s^* being some positive constant. Therefore, we haveP

$$|g_{i,2}(\bar{x}_i, x_{i+1})| \leq \|W_{i,2}^*\| s^* + \varepsilon_{i,1}^*, \quad \bar{x}_{i+1} \in \Omega_{\bar{x}_{i+1}}, x_{i+1} \geq a_0 \tag{20}$$

$$|g_{i,3}(\bar{x}_i, x_{i+1})| \leq \|W_{i,3}^*\| s^* + \varepsilon_{i,3}^*, \quad \bar{x}_{i+1} \in \Omega_{\bar{x}_{i+1}}, x_{i+1} \leq -a_0 \tag{21}$$

$$|g_{i,1}(\bar{x}_i)| \leq \|W_{i,1}^*\| s^* + \varepsilon_{i,1}^*, \quad \bar{x}_i \in \Omega_{\bar{x}_i} \tag{22}$$

Using the above inequalities and noting (15) and (16), we have

$$g_{i,m} \leq G_i(\bar{x}_{i+1}, t) \leq g_{i,M} \tag{23}$$

$$|\eta_i(\bar{x}_i, t)| \leq \eta_i^* \tag{24}$$

with $g_{i,M} = \max\left\{\|W_{i,2}^*\| s^* + \varepsilon_{i,1}^*, \|W_{i,3}^*\| s^* + \varepsilon_{i,3}^*, l_i, l'_i\right\}$, $\eta_i^* = \max\left\{|\vartheta_i|, |\vartheta'_i|, \|W_{i,1}^*\| s^* + \varepsilon_{i,1}^* + a_0, \|W_{i,1}^*\| s^* + \varepsilon_{i,1}^* + a_0\right\}$ being unknown positive constants.

III. ADAPTIVE TRACKING CONTROL

In the framework of backstepping approach, the following change of coordinates is made:

$$\begin{cases} e_1 = x_1 - y_d \\ e_i = x_i - \alpha_{i-1}, \quad i = 2, 3, \ldots, n \end{cases} \tag{25}$$

where e_1 is the tracking error, and α_{i-1} is the virtual control. The recursive design procedure contains n steps. First, at each step of the backstepping design, the intermediate control α_{i-1} is designed to make the corresponding subsystem toward equilibrium position, and at the final step, the stabilization of system (7) can be achieved with the actual control input u being designed.

Step 1: To start, consider the following subsystem of (1) and noting $e_1 = x_1 - y_d$, we have

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{y}_d \\ &= F_1(x_1, x_2) + f_1(x_1, 0) + \Delta_1(t) - \dot{y}_d \\ &= G_1(\bar{x}_2, t)x_2 + \eta_1(\bar{x}_1, t) + f_1(x_1, 0) + \Delta_1(t) - \dot{y}_d \end{aligned} \tag{26}$$

where x_2 is regarded as a virtual control input of this subsystem. Consider the stabilization of subsystem (26) and the follow quadratic Lyapunov function candidate

$$V_{e_1} = \frac{1}{2}e_1^2 \tag{27}$$

The time derivative of V_{e_1} along (26) is

$$\begin{aligned}\dot{V}_{e_1} &= e_1\big(G_1(\bar{x}_2,t)x_2 + \eta_1(\bar{x}_1,t)\\ &\quad + f_1(x_1,0) + \Delta_1(t) - \dot{y}_d\big)\end{aligned} \tag{28}$$

Define a continuous function as

$$h_1(Z_1) = f_1(x_1,0) \tag{29}$$

where $Z_1 = x_1$. Apparently, $h_1(Z_1)$ can be approximated by RBFNN as follows

$$h_1(Z_1) = W_{h,1}^{*T}\psi_{h,1}(Z_1) + \varepsilon_{h,1}, \quad Z_1 \in \Omega_{Z_1} \tag{30}$$

where $\varepsilon_{h,1}$ is the approximation error, satisfying $\left|\varepsilon_{h,1}\right| \le \varepsilon_{h,1}^*$, with $\varepsilon_{h,1}^* > 0$ being unknown positive constant. Then, we can rewritten (28) as

$$\begin{aligned}\dot{V}_{e_1} &= e_1\big(G_1(\bar{x}_2,t)x_2 + \eta_1(\bar{x}_1,t)\\ &\quad + W_{h,1}^{*T}\psi_{h,1}(Z_1) + \varepsilon_{h,1} + \Delta_1(t) - \dot{y}_d\big)\end{aligned} \tag{31}$$

From [9], it can be known that $\left\|\psi_{h,1}(Z_1)\right\| \le s^*$. Noting the boundedness of $\eta_1(\bar{x}_1,t)$, $\varepsilon_{h,1}$, $\Delta_1(t)$, $\dot{y}_d$ and the fact that $W_{h,1}^*$ is a constant vector, we have

$$\left|\eta_1(\bar{x}_1,t) + W_{h,1}^{*T}\psi_{h,1}(Z_1) + \varepsilon_{h,1} + \Delta_1(t) - \dot{y}_d\right| \le M_1 \tag{32}$$

where $M_1 = \eta_1^* + \left\|W_{h,1}^*\right\| s^* + d_1^* + \varepsilon_1^* + B_0$ is an unknown constant.

We construct a virtual control α_1 and the adaptation function $\hat{M}_1$ as follows

$$\alpha_1 = -k_1e_1 - \frac{\lambda_1\hat{M}_1^2e_1}{\sqrt{\hat{M}_1^2e_1^2+\delta^2}} \tag{33}$$

$$\dot{\hat{M}}_1 = \gamma_1\left|e_1\right| \tag{34}$$

where k_1, λ_1, and γ_1 are the positive design parameters; $\hat{M}_1$ is the estimate of M_1; δ is any positive uniform continuous and bounded function, which satisfies

$$\lim_{t\to\infty}\int_0^t \delta(\tau)d\tau \le \delta_1 < +\infty \tag{35}$$

$$\left|\dot{\delta}(t)\right| \le \delta_2 < +\infty \tag{36}$$

where δ_1 and δ_2 are any positive constants.

Define the Lyapunov function candidate

$$V_1 = V_{e_1} + \frac{1}{2\gamma_1}\tilde{M}_1^2 \tag{37}$$

where $\tilde{M}_1 = M_1 - \hat{M}_1$.

In view of (31), (32), and (37), we have

$$\dot{V}_1 \le |e_1|M_1 + G_1(\bar{x}_2,t)e_1(e_2+\alpha_1) - \frac{1}{\gamma_1}\tilde{M}_1\dot{\hat{M}}_1 \tag{38}$$

Choosing $\lambda_1 \ge g_{1,m}^{-1}$ and substituting (33) into (38) yields

$$\begin{aligned}\dot{V}_1 &\le |e_1|M_1 - k_1g_{1,m}e_1^2\\ &\quad - \frac{\hat{M}_1^2e_1^2}{\sqrt{\hat{M}_1^2e_1^2+\delta^2}} + G_1(\bar{x}_2,t)e_1e_2 - \frac{1}{\gamma_1}\tilde{M}_1\dot{\hat{M}}_1\end{aligned} \tag{39}$$

By using Lemma 1 and noting $M_1 = \hat{M}_1 + \tilde{M}_1$, we have

$$\begin{aligned}\dot{V}_1 &\le -k_1g_{1,m}e_1^2 + G_1(\bar{x}_2,t)e_1e_2 + |e_1|\hat{M}_1 + |e_1|\tilde{M}_1\\ &\quad - \frac{\hat{M}_1^2e_1^2}{\sqrt{\hat{M}_1^2e_1^2+\delta^2}} - \frac{1}{\gamma_1}\tilde{M}_1\dot{\hat{M}}_1\\ &\le -k_1g_{1,m}e_1^2 + \delta + G_1(\bar{x}_2,t)e_1e_2\\ &\quad - \frac{1}{\gamma_1}\tilde{M}_1\left(\dot{\hat{M}}_1 - \gamma_1|e_1|\right)\end{aligned} \tag{40}$$

In view of (34), we have

$$\dot{V}_1 \le -k_1g_{1,m}e_1^2 + \delta + g_{1,M}\left|e_1e_2\right| \tag{41}$$

Step i ($2 \le i \le n-1$): A similar procedure is employed recursively for each step $i = 2,\ldots,n-1$. For the sake of brevity, Step i are simplified, with redundant equations and explanations being omitted.

Consider the following subsystem of (1) and noting $e_i = x_i - \alpha_{i-1}$, we have

$$\begin{aligned}\dot{e}_i &= \dot{x}_i - \dot{\alpha}_{i-1}\\ &= G_i(\bar{x}_{i+1},t)x_{i+1} + \eta_i(\bar{x}_i,t)\\ &\quad + f_i(\bar{x}_i,0) + \Delta_i(t) - \dot{\alpha}_{i-1}\end{aligned} \tag{42}$$

where

$$\alpha_{i-1} = \alpha_{i-1}\left(e_{i-1},\hat{M}_{i-1}\right) \tag{43}$$

Therefore, we have that the derivative of α_{i-1} can be expressed as

$$\begin{aligned}\dot{\alpha}_{i-1} &= \frac{\partial\alpha_{i-1}}{\partial e_{i-1}}\dot{e}_{i-1} + \frac{\partial\alpha_{i-1}}{\partial\hat{M}_{i-1}}\dot{\hat{M}}_{i-1}\\ &= \frac{\partial\alpha_{i-1}}{\partial e_{i-1}}\left(f_{i-1}(\bar{x}_{i-1},x_i) + \Delta_{i-1}(t)\right)\\ &\quad + \frac{\partial\alpha_{i-1}}{\partial\hat{M}_{i-1}}\gamma_{i-1}\left|e_{i-1}\right|\end{aligned} \tag{44}$$

Obviously, it follows from (44) that there exists a continuous function $h_i(Z_i)$ such that

$$\left|f_i(\bar{x}_i,0) - \dot{\alpha}_{i-1}\right| \le h_i(Z_i) \tag{45}$$

with $Z_i = \left[\bar{x}_i^T, e_{i-1}, \frac{\partial\alpha_{i-1}}{\partial e_{i-1}}, \frac{\partial\alpha_{i-1}}{\partial\hat{M}_{i-1}}\right]^T$. Apparently, $h_i(Z_i)$ can be approximated by RBFNN as follows

$$h_i(Z_i) = W_{h,i}^{*T}\psi_{h,i}(Z_i) + \varepsilon_{h,i}, \quad Z_i \in \Omega_{Z_i} \tag{46}$$

where $\varepsilon_{h,i}$ is the approximation error, satisfying $\left|\varepsilon_{h,i}\right| \le \varepsilon_{h,i}^*$, with $\varepsilon_{h,i}^* > 0$ being unknown positive constant.

Consider the stabilization of subsystem (42) and the follow quadratic Lyapunov function candidate

$$V_{e_i} = \frac{1}{2}e_i^2 \tag{47}$$

The time derivative of V_{e_i} along (42) is

$$\begin{aligned}\dot{V}_{e_i} \leq\ & e_i\left(G_i(\bar{x}_{i+1}, t)x_{i+1} + \eta_i(\bar{x}_i, t)\right)\\ &+e_i\Delta_i(t) + |e_i|\, h_i(Z_i)\end{aligned} \tag{48}$$

Similar as Step 1, we have

$$\left|\eta_i(\bar{x}_i, t) + W_{h,i}^{*T}\psi_{h,i}(Z_i) + \varepsilon_{h,i} + \Delta_i(t)\right| \leq M_i \tag{49}$$

where $M_i = \eta_i^* + \|W_{h,i}^*\| s^* + d_i^* + \varepsilon_i^*$ is an unknown constant. Using (48) and (49), we obtain

$$\dot{V}_{e_i} \leq e_i G_i(\bar{x}_{i+1}, t)x_{i+1} + |e_i|\, M_i \tag{50}$$

We construct a virtual control α_i and the adaptation function $\hat{M}_i$ as follows

$$\alpha_i = -k_i e_i - \frac{\lambda_i \hat{M}_i^2 e_i}{\sqrt{\hat{M}_i^2 e_i^2 + \delta^2}} \tag{51}$$

$$\dot{\hat{M}}_i = \gamma_i\, |e_i| \tag{52}$$

where k_i, λ_i, and γ_i are the design parameters, and $\hat{M}_i$ is the estimate of M_i.

Define the Lyapunov function candidate

$$V_i = V_{e_i} + \frac{1}{2\gamma_i}\tilde{M}_i^2 \tag{53}$$

where $\tilde{M}_i = M_i - \hat{M}_i$.

In view of (25), (50) and (53), we have

$$\dot{V}_i \leq |e_i|\, M_i + G_i(\bar{x}_{i+1}, t)e_i\left(e_{i+1} + \alpha_i\right) - \frac{1}{\gamma_i}\tilde{M}_i\dot{\hat{M}}_i \tag{54}$$

Choosing $\lambda_i \geq g_{i,m}^{-1}$ and substituting (51) into (54) yields

$$\begin{aligned}\dot{V}_i \leq\ & |e_i|\, M_i - k_i g_{i,m} e_i^2 - \frac{\hat{M}_i^2 e_i^2}{\sqrt{\hat{M}_i^2 e_i^2 + \delta^2}}\\ &+G_i(\bar{x}_{i+1}, t)e_i e_{i+1} - \frac{1}{\gamma_i}\tilde{M}_i\dot{\hat{M}}_i\end{aligned} \tag{55}$$

Similar as Step 1, we have

$$\dot{V}_i \leq -k_i g_{i,m} e_i^2 + \delta + g_{i,M}\, |e_i e_{i+1}| \tag{56}$$

Step n: Consider the following subsystem of (1) and noting $e_n = x_n - \alpha_{n-1}$, we have

$$\begin{aligned}\dot{e}_n &= \dot{x}_n - \dot{\alpha}_{n-1}\\ &= G_n(\bar{x}_{n+1}, t)x_{n+1} + \eta_n(\bar{x}_n, t)\\ &\quad +f_n(\bar{x}_n, 0) + \Delta_n(t) - \dot{\alpha}_{n-1}\end{aligned} \tag{57}$$

Similarly, we have that the derivative of α_{n-1} can be expressed as

$$\begin{aligned}\dot{\alpha}_{n-1} =\ & \frac{\partial\alpha_{n-1}}{\partial e_{n-1}}\left(f_{n-1}(\bar{x}_{n-1}, x_n) + \Delta_{n-1}(t)\right)\\ &+\frac{\partial\alpha_{n-1}}{\partial\hat{M}_{n-1}}\gamma_{n-1}\, |e_{n-1}|\end{aligned} \tag{58}$$

Obviously, it follows from (58) that there exists an continuous function $h_n(Z_n)$ such that

$$|f_n(\bar{x}_n, 0) - \dot{\alpha}_{n-1}| \leq h_n(Z_n) \tag{59}$$

with $Z_n = [\bar{x}_n^T, e_{n-1}, \frac{\partial\alpha_{n-1}}{\partial e_{n-1}}, \frac{\partial\alpha_{n-1}}{\partial\hat{M}_{n-1}}]^T$. Apparently, $h_n(Z_n)$ can be approximated by RBFNN as follows

$$h_n(Z_n) = W_{h,n}^{*T}\psi_{h,n}(Z_n) + \varepsilon_{h,n}, \quad Z_n \in \Omega_{Z_n} \tag{60}$$

where $\varepsilon_{h,n}$ is the approximation error, satisfying $\left|\varepsilon_{h,n}\right| \leq \varepsilon_{h,n}^*$, with $\varepsilon_{h,n}^* > 0$ being unknown positive constant.

Consider the stabilization of subsystem (25) and the follow quadratic Lyapunov function candidate

$$V_{e_n} = \frac{1}{2}e_n^2 \tag{61}$$

Similar as Step 1, we have

$$\left|\eta_n(\bar{x}_n, t) + W_{h,n}^{*T}\psi_{h,n}(Z_n) + \varepsilon_{h,n} + \Delta_n(t)\right| \leq M_n \tag{62}$$

where $M_n = \eta_n^* + \left\|W_{h,n}^*\right\| s^* + d_n^* + \varepsilon_n^*$ is an unknown constant. Using (57), (59) and (62), we obtain

$$\dot{V}_{e_n} \leq e_n G_n(\bar{x}_{n+1}, t)u + |e_n|\, M_n \tag{63}$$

We construct the actual controller u and the adaptation function $\hat{M}_n$ as follows

$$u = -k_n e_n - \frac{\lambda_n \hat{M}_n^2 e_n}{\sqrt{\hat{M}_n^2 e_n^2 + \delta^2}} \tag{64}$$

$$\dot{\hat{M}}_n = \gamma_n\, |e_n| \tag{65}$$

where k_n, λ_n, and γ_n are the positive design parameters, and $\hat{M}_n$ is the estimate of M_n.

Define the Lyapunov function candidate

$$V_n = V_{e_n} + \frac{1}{2\gamma_n}\tilde{M}_n^2 \tag{66}$$

where $\tilde{M}_n = M_n - \hat{M}_n$.

Similar as Step 1, choosing $\lambda_n \geq g_{n,m}^{-1}$, we have

$$\dot{V}_n \leq -k_n g_{n,m} e_n^2 + \delta \tag{67}$$

The design process of adaptive tracking controller has been completed.

IV. STABILITY ANALYSIS

In this section, the main result of this paper is stated as follows.

Theorem 1: Consider the uncertain nonlinear systems (1) and Assumptions 1-3. The virtual controllers are constructed as (33) and (51), with the corresponding adaptation laws given by (34) and (52). The actual controller is given by (64) with the corresponding adaptation laws given by (65). Choose the design parameters to satisfy

$$\begin{cases} k_1 \geq \frac{1}{g_{1,m}}\left(c_0 + \frac{1}{2}\right) \\ k_i \geq \frac{1}{g_{i,m}}\left(g_{i-1,M}^2 + c_0 + \frac{1}{2}\right), & i = 2, \ldots, n \\ \lambda_i \geq g_{i,m}^{-1}, & i = 1, \ldots, n \\ \gamma_i > 0, & i = 1, \ldots, n \end{cases}$$

where c_0 is arbitrary positive constant. Then, all of the signals in the closed-loop system are semi-globally bounded, and the tracking error e_1 can asymptotically converge to zero.

Proof: Choose the Lyapunov function as follows:

$$V = \sum_{i=1}^{n} V_i \tag{68}$$

It follows from (41), (56), (67) that the derivative of V is

$$\dot{V} \leq -\sum_{i=1}^{n} k_i g_{i,m} e_i^2 + \sum_{i=1}^{n-1} g_{i,M} \left| e_i e_{i+1} \right| + n\delta \tag{69}$$

Using the Young's inequality, we have

$$g_{i,M} \left| e_i e_{i+1} \right| \leq \frac{e_i^2}{2} + \frac{g_{i,M}^2 e_{i+1}^2}{2}$$

Then, we have

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^{n} k_i g_{i,m} e_i^2 + \sum_{i=1}^{n-1}\left(\frac{e_i^2}{2} + \frac{g_{i,M}^2 e_{i+1}^2}{2}\right) + n\delta \\ &\leq -c_0 \sum_{i=1}^{n} e_i^2 + n\delta \end{aligned} \tag{70}$$

Integrating (70) over $[0, t]$ yields

$$\begin{aligned} V(t) &\leq V(0) - \int_0^t \left(c_0 \sum_{i=1}^{n} e_i^2(\xi)\right) d\xi + n\int_0^t \delta(\xi) d\xi \\ &\leq V(0) + n\delta_1 \end{aligned} \tag{71}$$

which implies e_i and $\tilde{M}_i$, $i = 1, 2, \ldots, n$ are bounded. In the sequel, we can deduce that x_i, x_n, α_i and u, $i = 1, 2, \ldots, n-1$ are bounded. Therefore, all the signals of closed-loop system are bounded. Moreover, from the first inequality of (71), one has

$$\int_0^t c_0 \sum_{i=1}^{n} e_i^2(\xi) d\xi \leq V(0) - V(t) + n\delta_1 \leq V(0) + n\delta_1 \tag{72}$$

By applying the Barbalat lemma, it is concluded that

$$\lim_{t\to\infty} e_1 = 0 \tag{73}$$

That is, the asymptotic tracking is achieved. This completes the proof of Theorem 1.

Remark 2: In the proof of our results, it can be seen that the inequalities (20), (21) and (22) are used, and these inequalities actually only hold on some compact sets which are represented as $\Omega_{\bar{x}_i}$, $i = 1, \ldots, n$. Define the $\Phi_{\bar{x}_n} = \{\bar{x}_n \,|V(t) \leq V(0) + n\delta_1\}$, it can be seen from (72) that $\Phi_{\bar{x}_n}$ is the compact set which x_i, $i = 1, \ldots, n$ will always converge into. It is also should be noted that we can always choose appropriate design parameters such that $\Phi_{\bar{x}_n} \subset \Omega_{\bar{x}_i}$, $i = 1, \ldots, n$, which means the variables concerned in $\Omega_{\bar{x}_i}$ will stay in $\Omega_{\bar{x}_i}$, and therefore the inequalities (20), (21) and (22) always hold and can be used.

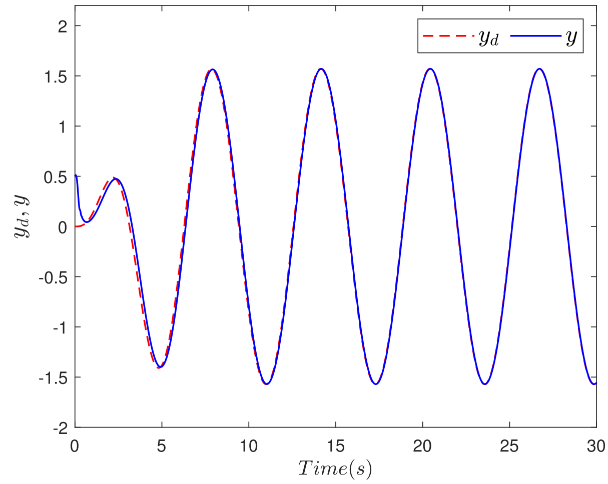

FIGURE 1. Reference signal y_d and system output y.

V. SIMULATION RESULTSION

Consider the dynamics of a one-link manipulator actuated by a brush dc (BDC) motor described as follows[25]:

$$\begin{cases} D\ddot{q} + B\dot{q} + N\sin(q) = I + \Delta_I \\ M\dot{I} = -HI - K_m\dot{q} + V \end{cases} \tag{74}$$

where q, $\dot{q}$ and $\ddot{q}$ are the link angular position, velocity, and acceleration, respectively. I denotes the motor current; Δ_I is the current disturbance; V represents the input control voltage. The parameters values with appropriate units are given in [26] by $D = 1$, $B = 1$, $M = 0.05$, $H = 0.5$, $N = 10$, and $K_m = 10$. Let the torque disturbance to be $\Delta_I = 0.2x_1 \sin(x_2x_3)$ with $x_1 = q$, $x_2 = \dot{q}$ and $x_3 = I$. Define the desired reference signal $y_d = (\pi/2)\sin(t)(1 - e^{-0.1t^2})$.

Therefore, system (74) can be expressed in the following form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (-10\sin(x_1) - x_2) + x_3 + 0.2x_1\sin(x_2x_3) \\ \dot{x}_3 = -10x_2 - 10x_3 + 20\varphi(u) \\ y = x_1 \end{cases} \tag{75}$$

Moreover, $\varphi(u)$ is described as follows

$$\varphi(u) = \begin{cases} u, & u \geq 1.5 \\ 0, & -2.5 < u < 1.5 \\ u, & u \leq -2.5 \end{cases} \tag{76}$$

It can be seen that the non-affine function is non-differentiable with respect to u.

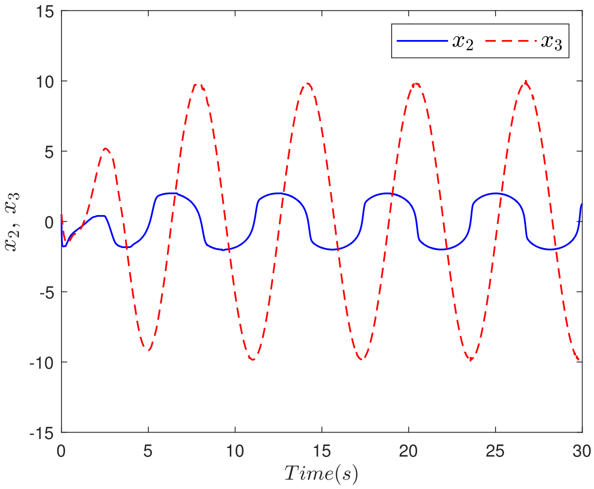

FIGURE 2. System state x_2 and x_3.

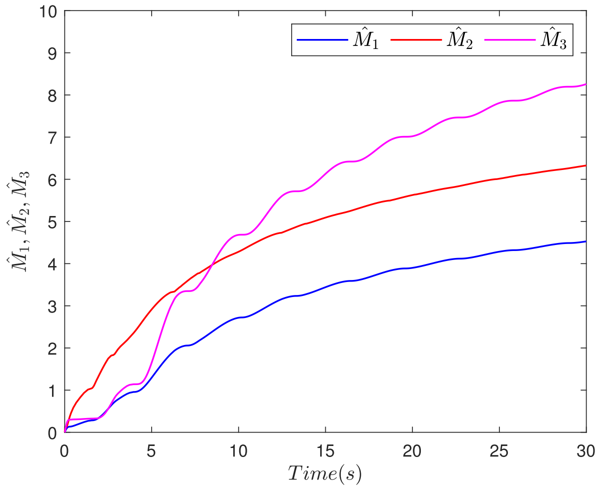

FIGURE 4. Adaptive parameters $\hat{M}_1$, $\hat{M}_2$ and $\hat{M}_3$.

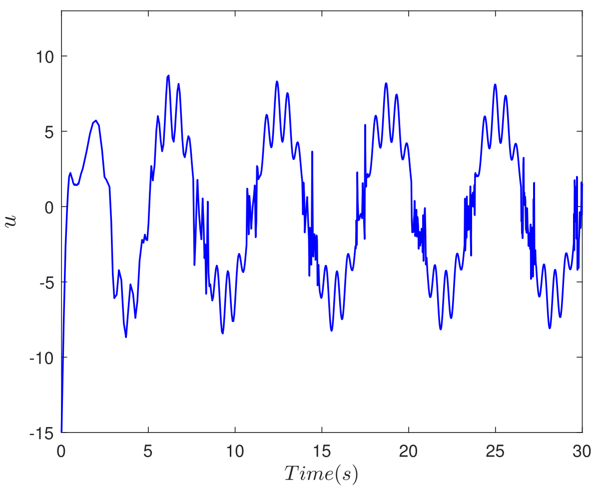

FIGURE 3. Control input u.

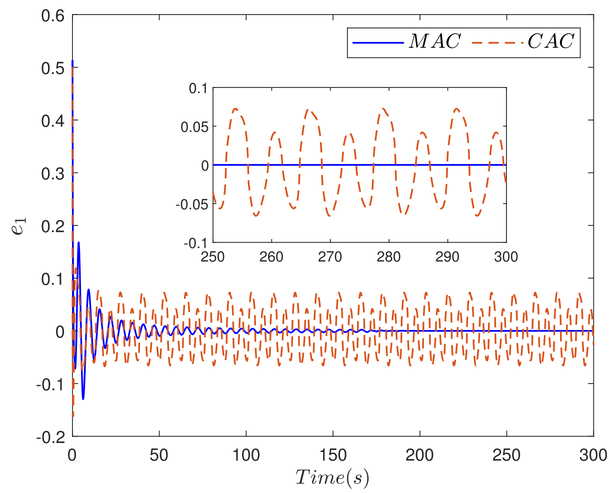

FIGURE 5. Tracking errors e_1.

According to Theorem 1, the adaptive neural controller is chosen as

$$\alpha_1 = -k_1 e_1 - \frac{\lambda_1 \hat{M}_1^2 e_1}{\sqrt{\hat{M}_1^2 e_1^2 + \delta^2}}$$

$$\alpha_2 = -k_2 e_2 - \frac{\lambda_2 \hat{M}_2^2 e_2}{\sqrt{\hat{M}_2^2 e_2^2 + \delta^2}}$$

$$u = -k_3 e_3 - \frac{\lambda_3 \hat{M}_3^2 e_3}{\sqrt{\hat{M}_3^2 e_3^2 + \delta^2}}$$

The adaptive laws are provided by (34) and (52), and the design parameters are selected as $k_1 = k_2 = k_3 = 5$, $\lambda_1 = \lambda_2 = \lambda_3 = 5$, $\delta = 0.001e^{-0.001t}$. The initial conditions are seted as: $[x_1(0), x_2(0), x_3(0)]^T = [0.5, 0.5, 0.5]^T$, $\hat{M}_1(0) = \hat{M}_2(0) = \hat{M}_3(0) = 0$.

The simulation results are shown in Figs. 1-5. It can be readily found that the satisfactory asymptotic tracking performance is obtained from Fig. 1, and the boundedness of x_2, x_3, u, $\hat{M}_1$, $\hat{M}_2$ and $\hat{M}_3$ are shown in Figs. 2-4.

For comparison, the conventional adaptive control (CAC) approach in [12] is performed with the same parameters $k_1 = k_2 = k_3 = 5$, and the corresponding simulation result on the system tracking error is presented in Fig. 5. It is obviously shown in Fig. 5 that, the proposed modified adaptive control (MAC) approach can achieve the better asymptotic tracking compared with CAC, which can only achieve the bounded tracking.

VI. CONCLUSION

In this paper, we concentrate on the asymptotic tracking problem for a class of nth-order SISO pure-feedback nonlinear systems. Different from the exiting results on pure-feedback systems, a novel controller design method is proposed, which can achieve the asymptotic tracking rather than the bounded tracking. Under the conditions of the nonaffine functions being semi-bounded, a modified modeling algorithm is developed to transform the nonaffine function into an affine form. It is proven rigorously via the Lyapunov theorem and Barbalat lemma that the asymptotic tracking performance of a given smooth enough reference signal, as well as the semi-global ultimate uniform boundedness of all the other signals can be guaranteed. Simulation example demonstrates the effectiveness of the proposed method.

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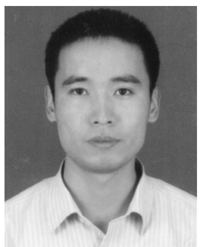

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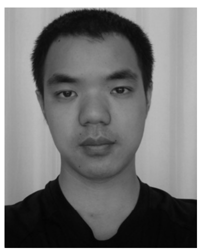

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