Efficient Detection Scheme for Physical-Layer Network Coding in Multiway Relay Channels

HAO LI [1], **(Student Member, IEEE), XIAO-WEN CHANG** [2],
YUNLONG CAI [3], **(Senior Member, IEEE),**
AND BENOIT CHAMPAGNE [4], **(Senior Member, IEEE)**

[1]Department of Electrical Computer Engineering, McGill University, Montreal, QC H3A 2K6, Canada
[2]School of Computer Science, McGill University, Montreal, QC H3A 2K6, Canada
[3]College of Information Science and Electrical Engineering, Zhejiang University, Zhejiang 310027, China
[4]Department of Electrical Computer Engineering, McGill University, Montreal, QC H3A 2K6, Canada

Corresponding author: Hao Li (hao.li3@mail.mcgill.ca)

ABSTRACT In this paper, we propose a novel scheme for physical-layer network coding (PNC) in multiway relay channels (MWRC) from the perspective of sequential multiuser detection. We consider an uplink MWRC scenario where N users, each equipped with a single antenna, simultaneously transmit their signal to a relay equipped with K antennas ($K < N$). Extraction of the network codes from the superimposed user signals at the relay node is formulated as an under-determined linear system. To solve this problem with low decoding complexity, the proposed method combines successive interference cancellation (SIC) with Babai estimation for regularized integer least squares (ILS). Specifically, SIC decoding is first employed to detect a selected subset of stronger user signals and remove their interfering effects. Babai estimation is then applied to extract the remaining user signals, which is formulated as an ILS problem with reduced dimension. We develop a power allocation scheme to enhance the performance of both SIC and ILS steps, and discuss an optimal user pairing strategy based on the average decoding error probability. Numerical results demonstrate the performance improvement of the proposed method in extracting network codes from multiple superimposed user signals.

INDEX TERMS Babai estimation, multiway relay channel (MWRC), physical-layer network coding (PNC), successive interference cancellation (SIC).

I. INTRODUCTION

Physical-layer network coding (PNC) [1], [2], which exploits the broadcast nature of wireless channels to improve the network throughput, has drawn considerable attention in recent years [3]–[5]. In a classic half-duplex two way relay channel (TWRC) scenario, two end users attempt to exchange information with the help of a relay. In contrast to the conventional network coding (NC) scheme [6], [7], which requires 2 time slots for up-link and 1 time slot for downlink transmissions [8], PNC in TWRC only consumes 2 time slots in total. By exploiting the additive nature of electromagnetic (EM) waves at the physical layer, PNC allows users to send signals simultaneously to the relay using only 1 time slot. After extracting and decoding the superimposed user signals,

the relay encodes this information into an NC signal and broadcasts it in a subsequent time slot. Upon reception of the broadcast NC signal, each user decodes the desired signal from the other user by employing its self-information. Compared with the conventional NC scheme, PNC leads to a 33% throughput improvement. Hence, it provides an appealing solution to meet the exacting demands of various applications envisaged for 5th generation (5G) wireless networks and beyond, such as streaming 4K video, machine-to-machine communications, on-line cloud sharing, etc. [9]–[12]. To take full advantage of PNC for these applications, several studies have been carried out with focus on specific TWRC issues, such as: the design of symbol mapping [13], [14], the effect of time or phase synchronization [15]–[19] , and channel estimation [20]–[22].

As a natural extension to TWRC, the use of PNC in multiway relay channels (MWRC) [23], where multiple users share

The associate editor coordinating the review of this manuscript and approving it for publication was Lin Bai.

information through a single relay, has been less studied. The superposition of multiple, say $N > 2$, user signals at the relay increases the difficulty of extracting network codes due to the mutual interference. The use of relay equipped with multiple antennas, say K, provides a simple solution to the MWRC problem since the spatial diversity can be exploited to diminish interference. Most of the literature on PNC in MWRC [24]–[26] focuses on implementation scenarios where the number of relay antennas is greater than the number of users, i.e., $K \geq N$. To some degree, this assumption defeats the inherent idea behind PNC, i.e., that the boost in throughput should result from natural coding in the wireless medium rather than from the additional cost of space-time processing. From this perspective, it would seem worthwhile to consider the case $K < N$ for PNC in MWRC, where the throughput gain is not solely obtained by the spatial diversity.

To the best of our knowledge, only a limited number of studies have addressed the problem of PNC in MWRC in the case $K < N$. In [27], the multiway relay network is decomposed into smaller building blocks, or atoms, over which existing TWRC techniques can be applied. In [28], a similar concept is considered where an opportunistic transmission protocol selects pairs of users for sequential transmission. Nonetheless, these approaches require at least $N - 1$ time slots for up-link transmission in an N-way relay channel. In [29], constellation design for simultaneous transmission of user signals in MWRC is formulated as a constrained optimization, where the aim is to maximize the minimal distance among the set of network coded symbols. However, this scheme is designed for AWGN channels where the multiple user signals barely suffer from channel distortions and their constellations are correctly superimposed at the relay. Aside from the intricate design, the scheme's complexity also increases rapidly when the number of users or the modulation order becomes large. In general, we find that existing approaches to the multiway PNC problems tend to follow concepts advanced for TWRC PNC, and rely on directly obtaining network codes from the superimposed signals at the relay. However, unlike the TWRC scenario, this task becomes extremely challenging when the number of colliding signals increases, requiring: the use of special scheduling via the decomposition of the network into smaller subnets, mitigation of the multiuser interference for each code extraction, or complicated signaling designs allowing the relay to unambiguously resolve codewords from a large superimposed constellation. Consequently, these approaches often turn out to either have limited efficiency or suffer from high complexity.

In this paper, we propose a novel scheme for PNC in MWRC, aiming to address these challenges from a different perspective, i.e. sequential multi-user detection (MUD). The benefits of doing so are twofold: 1) we still treat the MWRC as a natural encoder within the wireless medium, which is consistent with the inherent idea of PNC; 2) the use of MUD offers a powerful framework for the extraction of the network codes with relatively low complexity.

To be specific, we consider an uplink MWRC scenario where N users, each equipped with single antenna, simultaneously transmit signals to a relay equipped with K antennas, with emphasis on the case $K < N$. In contrast to existing approaches which seek to directly obtain the network codes from the superimposed user signals at the relay, we formulate this problem as an under-determined linear system in terms of the user symbols (from which the network codes can be easily obtained). To solve this problem with low decoding complexity, the proposed method combines successive interference cancellation (SIC) with Babai estimation [30] for regularized integer least squares (ILS). Specifically, SIC decoding is first employed to estimate a selected subset of stronger user signals and remove their interfering effects. Babai estimation is then applied to provide a solution to an ILS problem with reduced dimension, allowing the extraction of the remaining weaker user signals. We develop a power allocation scheme to enhance the performance of both the SIC and ILS detection steps, and discuss the optimal user pairing strategy based on the average decoding error probability. Through simulations, it is shown that the proposed method can lead to notable performance improvement in the extraction of network codes from superimposed user signals in MWRC.

The rest of the paper is organized as follows. The underlying MWRC system model is introduced in Section II. The proposed scheme for PNC in MWRC is developed in Section III by building on the SIC and regularized ILS problem formulations. The power allocation schemes for the SIC and ILS steps, as well as the proposed user pairing strategy are developed in Section IV. Supporting simulation results are presented in Section V, followed by conclusion in Section VI.

Notations: We use bold lower-case and upper-case letters for vectors and matrices, respectively. $\mathbf{A} = [a_{ij}]_{M \times N}$ denotes an $M \times N$ matrix with a_{ij} as the $(i, j)^{th}$ entry, while $\mathbf{I}$ and $\mathbf{0}$ are identity and zero matrices of appropriate dimensions. $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose and Hermitian transpose of a matrix $\mathbf{A}$. $\text{diag}(d_1, \ldots, d_n)$ returns a square diagonal matrix with diagonal entries $d_1, \ldots, d_n$. $||\cdot||$ and $|\cdot|$ refer to the Euclidean norm of vectors and modulus of scalars, respectively.

II. SYSTEM MODEL

As illustrated in Fig. 1, we consider a half-duplex multiway relay network where N users share information with each other through a common relay R. User terminals are equipped with single antenna while the relay is equipped with $K < N$ antennas.[1] We assume that there is no direct link among users, i.e., information exchange between two users needs to go through the relay. We consider radio transmission over narrow-band, i.e., frequency flat, slow fading channels. As a common assumption adopted in most existing works on PNC in TWRC [18], [31], perfect channel estimation and time synchronization are available for any node in the network.

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[1] The use of $K \geq 2$ makes it possible to exploit spatial diversity. While K can take on any integer value, the main focus in this work is on the case $K < N$.
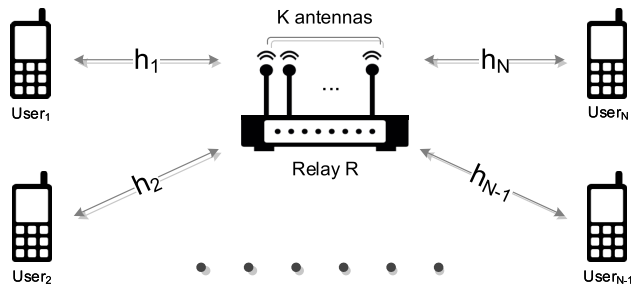
FIGURE 1. Illustration of the MWRC system model.

Data transmission proceeds in two stages, namely: multiple access (MA) or up-link and broadcast (BC) or downlink.

A. MA STAGE

1) SIMULTANEOUS TRANSMISSION
In the MA stage, all users simultaneously transmit signals to the relay so that only a single time slot is consumed. The signal transmitted by the ith user is given by $\sqrt{P_i}s_i$, where s_i is a discrete random modulation symbol with zero mean and unit variance, while P_i is the allocated power to this user. To simplify the exposition, binary phase shift keying (BPSK) modulation is assumed, i.e., $s_i \in \{-1, +1\}$, although generalization to other symbol constellations are possible. The superimposed signals received at the relay are represented by:

$$\mathbf{y} = \mathbf{HAs} + \mathbf{n}_R, \tag{1}$$

where $\mathbf{y} = [y_1, \ldots, y_K]^T \in \mathbb{C}^{K \times 1}$ is the vector of received signals at the relay antennas, $\mathbf{s} = [s_1, \ldots, s_N]^T \in \{-1, +1\}^{N \times 1}$ is the vector of user symbols, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_N] \in \mathbb{C}^{K \times N}$ is the channel matrix with column $\mathbf{h}_i \in \mathbb{C}^{K \times 1}$ representing the channel vector between the ith user and the relay, $\mathbf{A} = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_N})$, and $\mathbf{n}_R \in \mathbb{C}^{K \times 1}$ is an additive noise vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}$.

2) RELAY DETECTION AND NETWORK CODING
After the transmission, the relay generates $N - 1$ valid codeword by selecting $N - 1$ pairs of users and assigning to each pair a network code based on the received signal vector $\mathbf{y}$. Different strategies are possible for the selection of the $N - 1$ user pairs [32], represented by (i, j) where $i, j \in \{1, 2, \ldots, N\}, i < j$. In the sequel, we denote by $\mathcal{C}$ the selected set of $N - 1$ signal pairs. For instance, a straightforward approach consists in forming pairs by sequentially selecting neighboring user signals, i.e.:

$$\mathcal{C}_{seq} = \{(i, i + 1) \mid i = 1, 2, \ldots, N - 1\}. \tag{2}$$

Alternative strategies that lead to improved performance are considered in Section IV-D.

For each pair $(i, j) \in \mathcal{C}$, the next step consists in estimating the selected user signals s_i and s_j. To elucidate this step, let

us expand (1) as follows:

$$\mathbf{y} = \underbrace{\left(\mathbf{h}_i \sqrt{P_i} s_i + \mathbf{h}_j \sqrt{P_j} s_j \right)}_{\text{desired signals}} + \underbrace{\left(\sum_{\substack{m=1 \\ m \notin \{i,j\}}}^{N} \mathbf{h}_m \sqrt{P_m} s_m + \mathbf{n}_R \right)}_{\text{interference+noise}}, \tag{3}$$

where the first term contains the desired signal pair while the second term represents multi-user interference and noise. Based on (3), in the literature on PNC, estimation of the desired signal pair is typically formulated as a constrained least-squares problem, i.e.:

$$\{\hat{s}_i, \hat{s}_j\} = \arg \min_{s_i, s_j \in \{-1, +1\}} ||\mathbf{y} - \mathbf{h}_i \sqrt{P_i} s_i - \mathbf{h}_j \sqrt{P_j} s_j||^2. \tag{4}$$

Clearly, the multi-user interference may severely degrade the quality of these estimates when N increases.

At last, a valid network code is generated for each pair of estimated signals by applying a so-called mixing function, represented by $s_{ij} = \phi(\hat{s}_i, \hat{s}_j)$, where s_{ij} denotes the resulting network code. A common choice of mixing function $\phi(\cdot)$, which is used in this work for simplicity, is the bit-wise modulo-2 sum on the logical values of the estimated signals. Equivalently, this corresponds to multiplication in the finite field $\{-1, +1\}$, i.e.:

$$s_{ij} = \phi(\hat{s}_i, \hat{s}_j) = \hat{s}_i \hat{s}_j. \tag{5}$$

Consequently, by proceeding in this manner for every selected pair $(i, j) \in \mathcal{C}$, a finite sequence of $N - 1$ codewords is generated, i.e., $\{s_{ij}\}_{(i,j) \in \mathcal{C}}$.

B. BC STAGE
During this stage, the relay broadcasts the network codes in $\{s_{ij}\}_{(i,j) \in \mathcal{C}}$ to the users. Since any uplink performance gain or loss will be accordingly passed on to the BC stage, the choice of the downlink transmission approach does not directly affect the performance evaluation of the detection schemes at the relay. For simplicity, we therefore consider a conventional scheme for the BC stage, namely, sequential broadcast using one time slot per code so that $N - 1$ time slots are needed for a complete broadcast of the code sequence. Specifically, if s_{ij} is broadcast in a given time slot, the signal received at user m is given by:

$$r_m = g_m \sqrt{P_r} s_{ij} + n_m, \tag{6}$$

where P_r is the total transmit power of the relay, $g_m \in \mathbb{C}$ is the downlink channel gain, and $n_m \in \mathbb{C}$ is the downlink noise. The channel gain g_m includes the combined effect of beamforming or pre-coding at the relay followed by parallel transmission from the K relay antennas to user m. Under the slow fading assumption, g_m remains constant during the BC stage.

Based on the observation of r_m in the corresponding time slot, user m obtains an estimate of s_{ij} (which we assume error-free for simplify). Once the broadcast phase is completed after $N - 1$ time slots, user m stores the detected code sequence $\{s_{ij}\}_{(i,j) \in \mathcal{C}}$ and then uses its self-information s_m to obtain the complete signal vector $\mathbf{s}$. For instance,

if $\mathcal{C} = \mathcal{C}_{seq}$, user m obtains $\mathbf{s}$ by iteratively recovering each user signal from the detected sequence $\{s_{ij}\}_{(i,j)\in\mathcal{C}_{seq}}$, which is described as:

$$s_{k+1} = \phi(s_{k,k+1}, s_k), \quad \text{for } k = m, m+1, \ldots N-1$$
$$s_{l-1} = \phi(s_{l-1,l}, s_l), \quad \text{for } l = m, m-1, \ldots 2 \qquad (7)$$

where the initial input s_m is the self-information of user m. An example of such process is presented in Fig. 2. For the recovery to be effective, the selection strategy (i.e., set $\mathcal{C}$) must guarantee that all user signals are retrievable at an arbitrary user terminal. If $\mathcal{C}$ is not well designed, e.g., replacing (3, 4) by (1, 3) in $\mathcal{C}_{seq}$, the code sequence is invalid since the signal vector $\mathbf{s}$ can not be fully recovered in the BC stage.
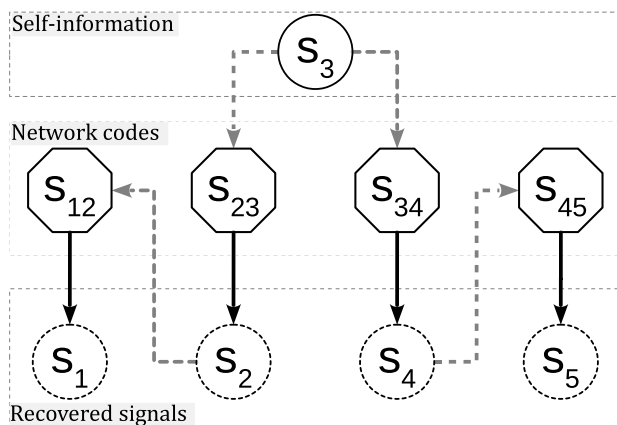
FIGURE 2. Example of the iterative recovery process for $\mathcal{C}_{seq}$ at user 3. User 3 first uses its self-information to decode s_2 and s_4 and then detects s_1 and s_5.

III. THE PROPOSED METHOD

According to (3)-(4), the presence of multi-user interference in the superimposed signals at the relay decreases the reliability of the estimated network codes $\{s_{ij}\}_{(i,j)\in\mathcal{C}}$, especially when N is large. In this section, we discuss methods to overcome this issue with the relay detection by first investigating the SIC process, Babai estimation for ILS, and the regularized ILS. We then develop the proposed method to resolve the user symbols from the superimposed signals in the MA stage.

A. SUCCESSIVE INTERFERENCE CANCELLATION

SIC can be used at the relay to mitigate the interfering effect, by discriminating signals on the basis of their relative power levels. Specifically, the user signal with the largest received power is first detected and its interfering effect removed from the observation; the detection then proceeds sequentially to the next strongest signal. SIC will be used here as a preliminary step to remove part of the multi-user interference in (4), thereby facilitating the subsequent estimation of weaker signals by a more sophisticated ILS technique.

Under the assumption of known channel state information and power allocation scheme,[2] let us arrange the user signals

[2]Power allocation is discussed in detail in Section IV.

in ascending order of their received power at the relay. That is, define $\varrho_i = ||\mathbf{h}_i||^2 P_i$ and assume that $\varrho_i < \varrho_j, \forall i < j$. Also let γ_i denote the signal-to-noise-plus-interference ratio (SINR) under the assumption of perfect signal cancellation at each iteration of the SIC process, i.e.:

$$\gamma_i = \frac{\varrho_i}{\sum_{j<i} \varrho_j + \sigma^2}. \qquad (8)$$

Using this notation, the SIC detection can be described by Algorithm 1, where γ_o is a threshold for detection. At the ith iteration, when $\gamma_i > \gamma_0$ the algorithm detects the dominant user signal s_i as the solution of LS problem (9) and then cancels its effect from the current observation vector in (10). Ideally, this process repeats for each iteration until the weakest signal s_1 is detected. Hence, with a proper power allocation scheme, an estimated signal vector consisting of all user symbols can be detected, as given by $\hat{\mathbf{s}} = [\hat{s}_1, \ldots, \hat{s}_N]^T$.

Algorithm 1 SIC

1: $i = N$, $\mathbf{y}^{(i)} \leftarrow \mathbf{y}$
2: **while** $i \geq 1$ **do**
3:     **if** $\gamma_i > \gamma_0$ **then**
4:         Detecting s_i from $\mathbf{y}^{(i)}$:

$$\hat{s}_i \leftarrow \arg\min_{\xi \in \{-1, +1\}} ||\mathbf{y}^{(i)} - \mathbf{h}_i\sqrt{P_i}\xi||^2. \qquad (9)$$

5:         Removing influence of s_i:

$$\mathbf{y}^{(i-1)} \leftarrow \mathbf{y}^{(i)} - \mathbf{h}_i\sqrt{P_i}\hat{s}_i. \qquad (10)$$

6:     **else** Break **while**
7:     **end if**
8:     $i = i - 1$.
9: **end while**

Note that the SIC process does not depend on the number of relay antennas K. Algorithm 1 is thus always executable as long as the condition on γ_0 is satisfied. In theory, this process can recover the user signals at the relay for systems with arbitrary dimensions including underdetermined ones, i.e., $N > K$. In practice, however, the estimation accuracy of $\hat{s}_i$ highly depends on the relative strength of the term $\mathbf{h}_i\sqrt{P_i}s_i$ in $\mathbf{y}^{(i)}$ at each iteration. Specifically, the accuracy of $\hat{\mathbf{s}}$ depends on the power difference among the received user signals at the relay. With a given power limit P_T on the user terminals (i.e., $P_i \leq P_T$) and fixed channel gains, when the number N of the colliding user signals increases, the conditions for successful application of SIC cannot be maintained. Either the power difference between user signals is too small or the weaker signals are dominated by noise and interference. To overcome this critical dilemma, we propose to use a complementary ILS-based solution for the detection of weaker signals.

B. BABAI ESTIMATION FOR OVERDETERMINED ILS

MUD in linear systems can also be achieved by solving an ILS problem for the unknown user signals [33]–[36].

Specifically, upon receiving the superimposed signal vector $\mathbf{y}$ in (1), the relay may attempt to solve the following problem:

$$\min_{\mathbf{s}\in\mathcal{B}} ||\mathbf{y} - \mathbf{HAs}||^2, \tag{11}$$

where $\mathcal{B}$ is the constraint set of the transmitted signal vector $\mathbf{s}$. Here, $\mathcal{B}$ is determined by the constellation of user signal symbols, i.e., $\mathcal{B} = \{-1, +1\}^N$. A solution to (11) can be obtained by the reduction and search processes. When the system is overdetermined,[3] i.e., $K \geq N$, matrix $\mathbf{H}$ can be reduced by an QR decomposition with column pivoting, written as:

$$\mathbf{H\Pi} = [\mathbf{Q}_1, \mathbf{Q}_2]\begin{bmatrix} \mathbf{R} \\ \mathbf{0} \end{bmatrix} = \mathbf{Q}_1\mathbf{R} \tag{12}$$

where Π is a permutation matrix of order N decided by algorithms such as LLL-P, V-BLAST, or SQRD (see [34] and the references therein), $\mathbf{Q} = [\mathbf{Q}_1, \mathbf{Q}_2] \in \mathbb{C}^{K\times K}$ is unitary, and $\mathbf{R} \in \mathbb{C}^{N\times N}$ is upper triangular. The reduced ILS problem is expressed as:

$$\min_{\bar{\mathbf{s}}\in\mathcal{B}} ||\bar{\mathbf{y}} - \bar{\mathbf{R}}\bar{\mathbf{s}}||^2 \tag{13}$$

where $\bar{\mathbf{y}} = \mathbf{Q}_1^H\mathbf{y}$, $\bar{\mathbf{R}} = \mathbf{R\Pi^H A\Pi}$, and $\bar{\mathbf{s}} = \Pi^H\mathbf{s}$. For later convenience, we define the permuted matrix $\bar{\mathbf{A}} = \Pi^H\mathbf{A\Pi} = \mathrm{diag}(\sqrt{\bar{P}_1}, \ldots, \sqrt{\bar{P}_N})$, where $\bar{P}_i$ denotes the power allocated to $\bar{s}_i$. Note that the optimization problem (11) is NP-hard. To efficiently find a reasonable estimate of $\bar{\mathbf{s}}$, a suboptimal solution called the Babai point $\bar{\mathbf{s}}^B$ is of interest instead [34], [37]. The determination of $\bar{\mathbf{s}}^B$ involves a sequence of element-wise (binary searches) where at each step, starting from the bottom row of $\bar{\mathbf{y}} - \bar{\mathbf{R}}\bar{\mathbf{s}}$ and moving up, previously detected signals are canceled followed by nulling of the residual. This process can be described as follows:

$$\bar{s}_i = \left\lfloor \left(\bar{y}_i - \sum_{j=i+1}^{N} \bar{r}_{ij}\bar{s}_j\right)/\bar{r}_{ii} \right\rceil, \tag{14}$$

for $i = N, \ldots, 1$, where $\lfloor \cdot \rceil$ denotes the nearest integer in $\{-1, +1\}$. The Babai point for estimating $\mathbf{s}$ is thus $\mathbf{s}^B = \Pi\bar{\mathbf{s}}^B$.

C. FORMULATION OF REGULARIZED ILS
According to (14), Babai estimation finds a solution to the problem (11) for the over-determined system, i.e., when $K \geq N$; however, it fails for the under-determined system, i.e., when $K < N$. In such a case, (12) generates a $K \times N$ upper-trapezoidal matrix $\mathbf{R}$ that has $N - K + 1$ nonzero entries in the Kth row. The additional non-zero terms create difficulties in the successive estimation of $\bar{s}_i$ for $i = N, \ldots, K$ in (14). Specifically, following the QR decomposition of (1), we obtain:

$$\bar{y}_i = \bar{r}_{ii}\bar{s}_i + \sum_{j=i+1}^{N} \bar{r}_{ij}\bar{s}_j + \bar{n}_i, \tag{15}$$

where $\bar{n}_i$ is the ith entry of $\bar{\mathbf{n}} = \mathbf{Q}_1^H\mathbf{n}_R$. The interfering term $\sum_{j=i+1}^{N} \bar{r}_{Kj}\bar{s}_j$ in (15) causes error propagation in the

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[3]An overdetermined system is often defined as one for which $K > N$; however, to simplify the presentation, we include the case $K = N$ as part of this condition.

successive estimation process, undermining the reliability of the estimates $\bar{s}_i$.

To overcome this problem, we can regularize the cost function in (11) and formulate the regularized problem as an over-determined system with extended dimensions, to which (14) can be applied. Specifically, the ILS problem (11) is equivalent to:

$$\min_{\mathbf{s}\in\mathcal{B}} ||\mathbf{y} - \mathbf{HAs}||^2 + \lambda||\mathbf{As}||^2, \tag{16}$$

where $\lambda > 0$ is a regularization parameter. Since $||\mathbf{As}||^2$ is constant under the constraint $\mathbf{s} \in \mathcal{B}$, if $\mathbf{s}^*$ is the optimal solution to (11), it is also the solution to (16). Once λ is determined, problem (16) can be formulated as:

$$\min_{\mathbf{s}\in\mathcal{B}} ||\check{\mathbf{y}} - \check{\mathbf{H}}\mathbf{As}||^2, \tag{17}$$

where $\check{\mathbf{y}} = [\mathbf{y}^T, \mathbf{0}_{N\times 1}^T]^T$ and $\check{\mathbf{H}} = [\mathbf{H}^T, \sqrt{\lambda}\mathbf{I}_{N\times N}]^T$. Since the dimension of $\check{\mathbf{H}}$ is extended to $(K+N)\times N$, Babai estimation can be applied directly to solve (17). We thus can obtain a reduced system similar to (13), but with corresponding variables given by $\bar{\mathbf{y}} = \check{\mathbf{Q}}_1^H\check{\mathbf{y}}$, $\bar{\mathbf{R}} = \check{\mathbf{R}}\check{\Pi}^H\mathbf{A}\check{\Pi}$, $\bar{\mathbf{s}} = \check{\Pi}^H\mathbf{s}$, where matrices $\check{\mathbf{Q}} = [\check{\mathbf{Q}}_1, \check{\mathbf{Q}}_2]$, $\check{\mathbf{R}}$ and $\check{\Pi}$ now refer to the QR decomposition of $\check{\mathbf{H}}$ as in (12).

The regularization parameter λ, whose choice is usually related to the noise variance σ^2, affects the efficiency of general search algorithms such as sphere decoding [38], [39]. In the context of Babai estimation, the regularized system corresponding to (16) is written as:

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \sqrt{\lambda}\mathbf{I} \end{bmatrix}\mathbf{As} + \begin{bmatrix} \mathbf{n}_R \\ -\sqrt{\lambda}\mathbf{As} \end{bmatrix}. \tag{18}$$

Since the noise and the signal vectors $\mathbf{n}_R$ and $\mathbf{s}$ are independent, $\check{\mathbf{n}}_R = [\mathbf{n}_R^T, (-\sqrt{\lambda}\mathbf{As})^T]^T$ has the covariance matrix $\check{\Sigma} = \mathrm{diag}(\sigma^2, \ldots, \sigma^2, \lambda P_1, \ldots, \lambda P_N)$. After the QR reduction, the variance of the ith entry of $\bar{\mathbf{n}} = \check{\mathbf{Q}}_1^H\check{\mathbf{n}}_R$ is a weighted arithmetic mean of the diagonal entries of $\check{\Sigma}$, given by:

$$\check{\mathbf{q}}_i^H\check{\Sigma}\check{\mathbf{q}}_i = \sum_{j=1}^{K}\check{q}_{ij}^2\sigma^2 + \sum_{l=1}^{N-K}\check{q}_{i,K+l}^2\lambda P_l, \tag{19}$$

where $\check{\mathbf{q}}_i$ is the ith column vector of $\check{\mathbf{Q}}_1$. On the one hand, a small value of λ is desirable to reduce the noise variance and its harmful effects on the estimation accuracy. On the other hand, too small a value of α can make the matrix $\check{\mathbf{H}}$ ill-conditioned, which may create numerical instability in the estimation process. In practice, we find that λ should be chosen so that the noise variances for $\bar{\mathbf{n}}$ and $\mathbf{n}_R$ are on the same level. In light of the power constraint $P_i \leq P_T$ and considering the variance expression in (19), we adopt $\lambda = \sigma^2/P_T$ as the regularization parameter for the Babai estimation in this paper.

D. THE PROPOSED ALGORITHM
There still remain some issues regarding SIC and Babai estimation. As pointed out earlier, under power constraint on

the user terminals, i.e. $P_i \leq P_T$, the performance of SIC will degrade as N increases. Besides, the ILS regularization in (16) is achieved via the introduction of a non-informative term $\lambda||\mathbf{As}||^2$, which is equivalent to the injection of extraneous noise in an augmented system as seen in (18)-(19). When $N \gg K$, this noise becomes a dominant factor in the solution of (17), which ultimately limits the ability of Babai estimation to resolve the user signals.

To overcome these problems, we propose a new algorithm that combines SIC and Babai estimation to efficiently solve problem (11) in the under-determined case $K < N$. The algorithm derivation relies on two main ideas. First, we note from (15) that after the QR decomposition, there only remain $N - K + 1$ superimposed users signals in $\bar{y}_K$, instead of the N original signals. From the perspective of SIC, this operation is beneficial since it allows for larger power differences among the remaining signals. This suggests that we only apply SIC to extract $\{\bar{s}_i | i = K, \ldots, N\}$ from $\bar{\mathbf{y}}$, thereby taking advantage of the reduced number of colliding signals. Second, if a portion of the user signals, say $N - L > 0$, can be removed from the detection before the application of Babai estimation, this will reduce the regularized system dimension, i.e., lessen the gap between N and K from $N - K$ to $L - K$. Consequently, this will limit the harmful effects of the non-informative term $\lambda||\mathbf{As}||^2$ on the solution of the regularized ILS problem, as discussed above.

To be specific, the proposed algorithm, which is presented as Algorithm 2, starts with a QR reduction of the original system (11). Under the assumption $K < N$, this yields:

$$\bar{\mathbf{y}} = \mathbf{Q}_1^H \mathbf{y}, \quad \bar{\mathbf{A}} = \Pi^H \mathbf{A} \Pi, \quad \bar{\mathbf{R}} = \mathbf{R}\bar{\mathbf{A}}, \quad (20)$$

where $\mathbf{Q} = [\mathbf{Q}_1] \in \mathbb{C}^{K \times K}$ is unitary and Π is a permutation matrix of order N so that $|\bar{r}_{K,i}| \leq |\bar{r}_{K,j}|, \forall i < j \in \{K, \ldots, N\}$. The original system is now reduced to (13) where $\bar{\mathbf{s}} = \Pi^H \mathbf{s}$.

The SIC detection is then executed, aiming to resolve the last $N - K + 1$ signals $\{\bar{s}_i | i = K, \ldots, N\}$ of $\bar{\mathbf{s}}$ from $\bar{y}_K$. Let γ_i denote the SINR at iteration i, and let L denote the first value of i, as this index decreases from N to K, at which the condition on the required SINR for SIC deteciton is no longer met, i.e. $\gamma_L < \gamma_0$ for some threshold γ_0. At this point, the SIC process is interrupted and we set $\bar{\mathbf{s}}_L = [\mathbf{0}_{1 \times L}, \bar{s}_{L+1}, \ldots, \bar{s}_N]^T$. The algorithm then cancels the effect of the detected signals in $\bar{\mathbf{s}}_L$ from $\bar{\mathbf{y}}$ and generates related quantities as follows:

$$\begin{aligned} \mathbf{y}_T &= \bar{\mathbf{y}} - \bar{\mathbf{R}}\bar{\mathbf{s}}_L, \quad \mathbf{s}_T = \mathbf{T}\bar{\mathbf{s}} \\ \mathbf{H}_T &= \mathbf{R}\mathbf{T}^T, \qquad \mathbf{A}_T = \mathbf{T}\bar{\mathbf{A}}\mathbf{T}^T \end{aligned} \quad (21)$$

where $\mathbf{T} = [\mathbf{I}_L, \mathbf{0}_{L \times (N-L)}]$ is a truncation matrix.

The above steps lead us to the following truncated ILS problem with reduced dimension:

$$\min_{\mathbf{s}_T \in \{-1,+1\}^L} ||\mathbf{y}_T - \mathbf{H}_T\mathbf{A}_T\mathbf{s}_T||^2, \quad (22)$$

where the aim is to determine the remaining user signals in $\mathbf{s}_T$. If $L = K$, the problem is overdetermined (see footnote 3) and Babai estimation as described in (14) can be applied directly

to (22). If $L > K$, however, the truncated problem is underdetermined and ILS regularization of (22) is needed prior to Babai estimation.

Proceeding as in Section III-C, we define:

$$\check{\mathbf{y}}_T = [\mathbf{y}_T^T, \mathbf{0}_{L \times 1}^T]^T, \quad \check{\mathbf{H}}_T = [\mathbf{H}_T^T, \sqrt{\lambda}\mathbf{I}_L]^T. \quad (23)$$

We next perform the QR decomposition of $\check{\mathbf{H}}_T$, which gives the unitary matrix $\check{\mathbf{Q}}_T = [\check{\mathbf{Q}}_{T1}, \check{\mathbf{Q}}_{T2}] \in \mathbb{C}^{(K+L)\times(K+L)}$, the permutation matrix $\check{\Pi}_T$ of order L based on the CH algorithm in [33], and the upper triangular matrix $\check{\mathbf{R}}_T = \check{\mathbf{Q}}_{T1}^H \check{\mathbf{H}}_T \check{\Pi}_T$. With the help of this decomposition, the regularized system is reduced to

$$\min_{\bar{\mathbf{s}}_T \in \{-1,+1\}^L} ||\bar{\mathbf{y}}_T - \bar{\mathbf{R}}_T\bar{\mathbf{s}}_T||^2, \quad (24)$$

where $\bar{\mathbf{s}}_T = \check{\Pi}_T^H \mathbf{s}_T$ and

$$\bar{\mathbf{y}}_T = \check{\mathbf{Q}}_{T1}^H \check{\mathbf{y}}_T, \quad \bar{\mathbf{R}}_T = \check{\mathbf{R}}_T\bar{\mathbf{A}}_T, \quad \bar{\mathbf{A}}_T = \check{\Pi}_T \mathbf{A}_T \check{\Pi}_T^H \quad (25)$$

At this point, Babai estimation can be applied to solve (24).

IV. POWER ALLOCATION AND USER PAIRING

In this section, a power allocation scheme for the proposed method that combines Babai estimation and SIC is developed. An optimal selection strategy for user pairs is also presented based on the decoding error probability.

A. POWER ALLOCATION FOR BABAI ESTIMATION

The comprehensive performance analysis of Babai estimation for overdetermined ordinary and box-constrained ILS problems can be found in [33], [34], [40]. Key results from these studies indicate that in the absence of a power allocation mechanism and when the noise level is small, the success probability, i.e., $\Pr(\mathbf{s}^B = \mathbf{s})$ where $\mathbf{s}^B$ is the Babai point, is upper bounded by a function of the determinant of the upper triangular factor from the QR decomposition (see (11)-(12)). Furthermore, the upper bound is reached if the diagonal entries of this matrix factor are identical.

In the context of Algorithm 2, Babai estimation is applied to (24) where the upper triangular matrix $\bar{\mathbf{R}}_T$ (following the QR decomposition of $\check{\mathbf{H}}_T$) is given by:

$$\bar{\mathbf{R}}_T = \check{\mathbf{R}}_T\bar{\mathbf{A}}_T = \begin{bmatrix} \sqrt{\bar{P}_1'}\check{r}_{11} & \cdots & \sqrt{\bar{P}_L'}\check{r}_{1,L} \\ & \ddots & \vdots \\ & & \sqrt{\bar{P}_L'}\check{r}_{L,L} \end{bmatrix}, \quad (26)$$

where $\bar{\mathbf{A}}_T = \text{diag}(\sqrt{\bar{P}_1'}, \ldots, \sqrt{\bar{P}_L'})$ is the corresponding power allocation matrix of $\bar{\mathbf{s}}_T$ after the permutation $\check{\Pi}_T$. According to the stated property of the success probability, it is desirable that the diagonal entries of $\bar{\mathbf{R}}_T$ be equal, i.e.:

$$\bar{P}_i'|\check{r}_{ii}|^2 = \eta, \quad \forall i \in \{1, \ldots, L\}, \quad (27)$$

where η is a constant. However, when considering the power constraint on the user terminals, i.e. $\bar{P}_i' \leq P_T$, power allocation based on (27) is not always practical. Indeed, when

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Algorithm 2 The proposed algorithm

1: **procedure** QR-factorization:
2:     Input: $\mathbf{H}$, $\mathbf{A}$, $\mathbf{y}$.
3:     QR decomposition: $[\mathbf{Q}, \mathbf{R}, \Pi] = \text{qr}(\mathbf{H})$
4:     Return: $\bar{\mathbf{y}}$, $\bar{\mathbf{A}}$, $\bar{\mathbf{R}}$.   ▷ Refer to (20)
5: **end procedure**
6: **procedure** Partial detection using SIC
7:     Initialize: $i = N$, $\bar{y}^{(i)} \leftarrow \bar{y}_K$.
8:     **while** $i > K$ **do**
9:         **if** $\gamma_i \geq \gamma_0$ **then**
10:             Detect $\bar{s}_i$ from $y^{(i)}$:

$$\bar{s}_i \leftarrow \arg \min_{\xi \in \{-1, +1\}} |\bar{y}^{(i)} - \bar{r}_{K,i}\xi|^2.$$

11:             Cancel the effect of $\bar{s}_i$:

$$\bar{y}^{(i-1)} \leftarrow \bar{y}^{(i)} - \bar{r}_{K,i}\bar{s}_i.$$

12:         **else** Break **while**
13:         **end if**
14:         $i = i - 1$.
15:     **end while**
16:     Return $L = i$, $\bar{\mathbf{s}}_L = [\mathbf{0}, \bar{s}_{L+1}, \bar{s}_{L+2}, \dots, \bar{s}_N]^T$.
17:     Removing the effect of $\bar{\mathbf{s}}_L$.
18:     Obtain: $\mathbf{y}_T$, $\mathbf{H}_T$, $\mathbf{A}_T$.   ▷ Refer to (21)
19: **end procedure**
20: **procedure** regularized ILS
21:     Regularization of the truncated system.
22:     Obtain: $\check{\mathbf{y}}_T$, $\check{\mathbf{H}}_T$ ▷ Refer to (23)
23:     QR reduction of the regularized ILS system.
24:     Obtain: $\bar{\mathbf{y}}_T$, $\bar{\mathbf{R}}_T$, $\bar{\mathbf{A}}_T$ ▷ Refer to (25)
25:     Apply Babai estimation to solve:

$$\min_{\bar{\mathbf{s}}_T \in \{-1, +1\}^{L-1}} ||\bar{\mathbf{y}}_T - \bar{\mathbf{R}}_T \bar{\mathbf{s}}_T||^2$$

                                 ▷ Refer to (14)

26:     Return: $\bar{\mathbf{s}}_T$
27: **end procedure**

---

matrix $\check{\mathbf{R}}_T$ is ill-conditioned, i.e. $\max\{|\check{r}_{ii}|\}/\min\{|\check{r}_{ii}|\} \gg 1$, the fulfillment of (27) for larger $|\check{r}_{ii}|$ requires the transmitted power $\bar{P}'_i$ needs to be very small, causing the additive noise to become overwhelming during the detection.

Instead of equalizing the diagonal entries of $\bar{\mathbf{R}}_T$, we could try to maximize the signal-to-noise ratio (SNR) of each received user signal. Assuming perfect cancellation at each iteration of the Babai estimation, the SNR of the ith user signal in $\bar{\mathbf{s}}_T$ is given by:

$$\rho_i = \bar{P}'_i |\check{r}_{ii}|^2 / \sigma_i^2, \quad i = 1, \dots, L, \tag{28}$$

where σ_i^2 is the noise variance after the regularization and the QR reduction, which is given by $\sigma_i^2 = \check{\mathbf{q}}_{T1i}^H \check{\Sigma}_T \check{\mathbf{q}}_{T1i}$, where $\check{\Sigma}_T = \text{diag}(\sigma^2, \dots, \sigma^2, \lambda\bar{P}_1, \dots, \lambda\bar{P}_L)$ and $\check{\mathbf{q}}_{T1i}$ is the ith column vector of $\check{\mathbf{Q}}_{T1}$ in (25). While the SNR in (28) is maximized by allocating maximum power to each user, i.e. $\bar{P}'_i = P_T$, this approach cannot be employed here. Indeed,

we note from (15) that some of the user signals contained in $\mathbf{s}_T$, specifically $\bar{s}_i$ for $i = K, \dots, L$, will interfere during the SIC process of Algorithm 2, which is aimed at removing the last $N - L$ signals in $\bar{\mathbf{s}}_L$ from $\bar{\mathbf{y}}$ prior to the Babai estimation. Hence, the elaboration of an adequate power allocation also requires consideration of the SIC process, which is discussed in next.

B. POWER ALLOCATION FOR SIC

Let's consider the power allocation for the user signals detected during the SIC process, i.e., $\{\bar{s}_i | i = L + 1, \dots, N\}$. Referring to (8), (20) and the SIC procedure in Algorithm 2, the SINR of $\bar{s}_i$ is given by:

$$\gamma_i = \frac{\bar{P}_i |r_{K,i}|^2}{\sum_{j=K}^{i-1} \bar{P}_j |r_{K,j}|^2 + \sigma^2}, \tag{29}$$

where the noise variance σ^2 is preserved by the unitary transformation $\mathbf{Q}_1$. Assuming perfect interference removal, a reliable estimate of $\bar{s}_i$ is achieved if it is sufficiently stronger than the remaining interference plus noise. This requires γ_i to exceed a given threshold, say $\gamma_i \geq \gamma_0$, $i = L + 1, \dots, N$. On the one hand, the greater the value of γ_i, the more reliable is the estimate of $\bar{s}_i$ is. On the other hand, each $\bar{s}_i$ (except for $\bar{s}_N$) is seen as interference to its predecessors in the SIC process, suggesting that γ_i for $i = L + 1, \dots, N - 1$ be also restrained. Considering this trade-off, a suitable approach for the selection of the powers $\bar{P}_i$ under the constraint $\bar{P}_i \leq P_T$, is to maximize the minimum value of γ_i [41]. The problem can be formulated as:

$$\max_{\{\bar{P}_i\}_{i=L+1}^N} \quad \min \{\gamma_i | i = L + 1, \dots, N\} \tag{30a}$$

$$\text{s.t.} \quad \bar{P}_i \leq P_T, \quad i = L + 1, \dots, N, \tag{30b}$$

where in this discussion, the values of $\bar{P}_K, \dots, \bar{P}_L$ are assumed to be fixed. We find that the direct solution of such a maximin problem is quite challenging. Alternatively, we therefore replace problem (30) by the following more tractable form, where the goal is to maximize a lower bound on the SINRs γ_i:

$$\max_{\{\bar{P}_i\}_{i=L+1}^N, \gamma} \quad \gamma \tag{31a}$$

$$\text{s.t.} \quad \frac{\bar{P}_i |r_{K,i}|^2}{\sum_{j=K}^{i-1} \bar{P}_j |r_{K,j}|^2 + \sigma^2} \geq \gamma, \tag{31b}$$

$$\bar{P}_i \leq P_T, \quad i = L + 1, \dots, N. \tag{31c}$$

To solve problem (31), we first express constraint (31b) into the following form:

$$\bar{P}_i |r_{K,i}|^2 \geq \gamma \left(\sum_{j=K}^{i-2} \bar{P}_j |r_{K,j}|^2 + \sigma^2 + \bar{P}_{i-1} |r_{K,i-1}|^2 \right) \tag{32}$$

where the last term is constrained as:

$$\bar{P}_{i-1} |r_{K,i-1}|^2 \geq \gamma \left(\sum_{j=K}^{i-2} \bar{P}_j |r_{K,j}|^2 + \sigma^2 \right). \tag{33}$$

Substituting (33) into (32) and proceeding iteratively for $i = N, \ldots, L+2$, we obtain a relaxed set of conditions:

$$\bar{P}_i |r_{K,i}|^2 \geq \gamma(\gamma+1)^{i-L-1}\varsigma^2, \quad i = L+1, \ldots, N, \quad (34)$$

where

$$\varsigma^2 = \sum_{j=K}^{L} \bar{P}_j |r_{K,j}|^2 + \sigma^2 \quad (35)$$

is the variance of the interfering terms from the perspective of $\bar{s}_{L+1}$. For the largest value of γ satisfying each inequality in (34), we have:

$$\bar{P}_i |r_{K,i}|^2 = \gamma(\gamma+1)^{i-L-1}\varsigma^2, \quad i = L+1, \ldots, N. \quad (36)$$

Since $\bar{P}_i |r_{K,i}|^2 / \bar{P}_{i-1} |r_{K,i-1}|^2 = \gamma+1 > 1$, we infer that $\bar{P}_i |r_{K,i}|^2$ is decreasing as index i runs from N to $L+1$. Note that signal $\bar{s}_N$ is the first one to be canceled and hence does not interfere with the detection of the remaining signals. For this reason, the maximum power can be assigned to this signal, i.e., $\bar{P}_N = P_T$, such that:

$$P_T |r_{K,N}|^2 = \gamma(\gamma+1)^{N-L-1}\varsigma^2, \quad (37)$$

from which γ can be obtained.

C. POWER ALLOCATION FOR PROPOSED ALGORITHM
Referring to (25)-(24), let us represent permutation matrix $\check{\Pi}_T$ of order L by a bijection π of $\{1, \ldots, L\}$ onto itself, mapping index i of $\mathbf{s}_T$ to index $j = \pi(i)$ of $\bar{\mathbf{s}}_T$, so that $\bar{s}_i = \bar{s}_{Tj}$ and accordingly, $\bar{P}_i = \bar{P}'_j$. Since the signals $\bar{s}_i$ for $i = K, \ldots, L$ are detected as part of the Babai estimation process, it is desirable for their SNRs to be as large as possible. This suggests that the value of ς^2 in (35), which involves $\bar{P}_i$ for $i = K, \ldots, L$, can not be too small. However, considering (37), we see that under a power constraint, this requirement on ς^2 conflicts with that of maximizing γ.

To overcome this difficulty, we modify the single objective problem (31) into a multi-objective optimization problem over γ for the SIC process and ρ_i, $i = 1, \ldots, L$, for the Babai estimation. The modified problem is formulated as:

$$\max_{\{\bar{P}_i\}_{i=1}^L, \gamma} \quad [\gamma, \rho_1, \ldots, \rho_L], \quad (38a)$$

$$\text{s.t.} \quad \bar{P}_T |r_{K,N}|^2 = \gamma(\gamma+1)^{N-L-1}\varsigma^2 \quad (38b)$$

$$\bar{P}_i \leq P_T, \quad \text{for } i = 1, \ldots, L, \quad (38c)$$

where ς^2 is given in (35) and $\bar{P}_i = \rho_j \sigma_j^2 / |\check{r}_{jj}|^2$ with $j = \pi(i)$.

There exist several approaches [42] to solve the multi-objective problem (38). Here, we adopt the weighted-sum method, which aggregates the multiple objectives into a single objective via a linear combination with positive weight. Since all user signals are of equal importance in the network coding process, it is fair to consider that variables γ and ρ_i are also equally important. In addition, γ has an influence on the detection performance of $N - L$ user signals. Hence, we

assign the weight $N - L$ to γ and a unit weight to each ρ_i for $i = 1, \ldots, L$. The problem (38) is then converted into:

$$\max_{\{\bar{P}_i\}_{i=1}^L, \gamma} \quad (N-L)\gamma + \sum_{j=1}^{L} \rho_j \quad (39a)$$

$$\text{s.t.} \quad \bar{P}_T |r_{K,N}|^2 = \gamma(\gamma+1)^{N-L-1}\varsigma^2, \quad (39b)$$

$$\bar{P}_i \leq P_T, \quad \text{for } i = 1, \ldots, L. \quad (39c)$$

Since problem (39) involves a single objective with non-linear constraint, a standard nonlinear programming solver can be applied to obtain the solution, denoted as $\{\bar{P}_i^*\}_{i=1}^L$ and γ^*. The power allocation in terms of $\bar{P}_i$ is thus given as:

$$\bar{P}_i = \begin{cases} \bar{P}_i^*, & i = 1, \ldots, L \\ \gamma^*(\gamma^*+1)^{i-L-1}\varsigma^{*2}/|r_{K,i}|^2, & i = L+1, \ldots, N, \end{cases} \quad (40)$$

where $\varsigma^{*2} = \sum_{i=K}^{L} \bar{P}_i^* |r_{K,i}|^2 + \sigma^2$, and $\gamma_0 = \gamma^*$ is set as the threshold for stopping the SIC process. Finally, the desired powers P_i of the transmitted signals in (1) can be obtained by applying an inverse permutation of Π in (20) to $\bar{P}_i$ for $i = 1, \ldots, N$.

D. USER PAIRING STRATEGY
As explained in Section II, the detected user signals at the relay need to be encoded into network codes. The pairing strategy, represented by the set $\mathcal{C}$ of pairs (i, j), will influence the decoding performance of user terminals in the BC stage. For this reason, it is worth considering an optimal selection strategy for the user pairs at the relay.
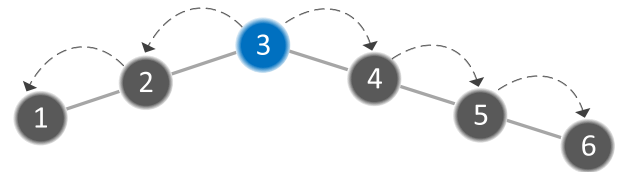

FIGURE 3. Illustration of the tree structure of the sequential pairing strategy $\mathcal{C}_{seq}$ in a 6-way relay network. The graph demonstrates a process where user 3 decodes all other user signals by proceeding through the branches in two directions.

From the perspective of graph theory, a valid strategy $\mathcal{C}$ can be represented as a tree, where the N users and the $N-1$ user pairs form the vertices and edges respectively, and each pair of vertices in the tree is connected by a unique path. Taking a 6-way relay network as an example, the graph of a sequential pairing strategy $\mathcal{C}_{seq}$ (see (2)) takes the form of the tree depicted in Fig. 3. The iterative decoding process in a specific user terminal, as given by (7) for $\mathcal{C}_{seq}$, is equivalent to a walk in tree, starting from the vertex of that user, and progressing towards the other users. In general, when a user terminal i decodes the signal of a user j, the corresponding path along the tree $\mathcal{C}$ is given by:

$$\mathcal{L}_{(i,j)} = \{(i, a), (a, b), \ldots, (d, j)\} \subset \mathcal{C}, \quad (41)$$

where $a, b, \ldots, d$ represent the intermediate vertices.

Let us denote by ψ_{ij} the decoding error probability of signal s_j in user terminal i. Essentially, ψ_{ij} depends on the network codes $s_{\alpha\beta}$, where $(\alpha, \beta) \in \mathcal{L}_{(i,j)}$. The reliability of $s_{\alpha\beta}$ is related to both up-link and down-link transmissions, which can be characterized by error probabilities $\psi_{(\alpha,\beta)}^{ul}$ and $\psi_{(\alpha,\beta)}^{dl,i}$ respectively. For binary signaling, a correct code $s_{\alpha\beta}$ is generated in the MA stage when two correct or two wrong decisions are made on s_α and s_β. Hence, $\psi_{(\alpha,\beta)}^{ul}$ can be expressed as:

$$\psi_{(\alpha,\beta)}^{ul} = \psi_{e\alpha}(1 - \psi_{e\beta}) + \psi_{e\beta}(1 - \psi_{e\alpha}), \quad (42)$$

where $\psi_{e\alpha}$ and $\psi_{e\beta}$ represent the error probabilities for the estimation of signals s_α and s_β at the relay. The probability $\psi_{(\alpha,\beta)}^{dl,i}$ is determined by the quality of the link from the relay to user i during the BC stage, as represented by (6). Under the slow fading assumption for the downlink channels, the error probabilities $\psi_{(\alpha,\beta)}^{dl,i}$ can be regarded as identical for different edges $(\alpha, \beta) \in \mathcal{L}_{(i,j)}$.

We note that an odd number of incorrect codes along $\mathcal{L}_{(i,j)}$ leads to a wrong decision on s_j, while an even number leads to a right decision. Hence, obtaining the exact value of ψ_{ij} requires a careful consideration of the error propagation effects which entails high computational costs. For simplicity, we approximate these effects by adopting a high SNR assumption as proposed in [43], i.e., ignoring even numbers of propagated errors whose small probabilities have negligible influence on ψ_{ij}. Numerical results later demonstrate that our approximation approach has a comparable effect to that observed in [43]. Assuming independence of the uplink and the downlink transmissions, ψ_{ij} is hence given by:

$$\psi_{ij} = 1 - \prod_{(\alpha,\beta)\in\mathcal{L}_{(i,j)}} (1 - \psi_{(\alpha,\beta)}^{ul})(1 - \psi_{(\alpha,\beta)}^{dl,i}). \quad (43)$$

For a given pairing strategy C, the average decoding error probability can be expressed as:

$$\psi_e^{avg} = \frac{1}{N^2 - N} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \psi_{ij}, \quad (44)$$

where ψ_{ij} is given by (43). Finally, an optimal strategy C^* can be found by an extensive search over all valid pairing strategies C so that the minimum ψ_e^{avg} is achieved:

$$C^* = \arg\min_{C} \psi_e^{avg}. \quad (45)$$

In (45), the search space consists of all the spanning trees of a simple undirected connected graph on N vertices [44], the size of which is determined by Cayley's Formula [45]. In this work, the various error probabilities entering the calculation of the objective (44) are obtained through a simulation approach.

V. SIMULATION EXPERIMENTS AND DISCUSSIONS
This section presents numerical evaluation results of the proposed scheme for PNC in MWRC. To this end, we use

Monte Carlo experiments based on the following system configuration. BPSK signaling is adopted at both the relay and the user terminals, whose maximum transmitting power is normalized to $P_T = 1$. We assume the various radio links to be Rayleigh fading, i.e., the entries of the channel matrix $\mathbf{H}$ in (1) are modeled as independent complex circular Gaussian random variables with zero mean and unit variance. The noise variance at each receiving antenna is adjusted to obtain the desired SNR level. To simplify the discussion and minimize the effects of indirect factors in the performance comparison among the different estimation schemes, we consider uncoded systems as in, e.g., [46].
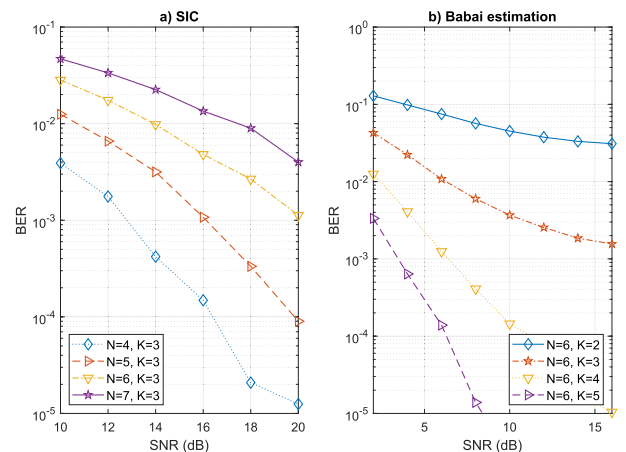

FIGURE 4. BER performance of conventional SIC and Babai estimation for different choices of N and K.

A. CONVENTIONAL SIC AND BABAI ESTIMATION
We begin by investigating the bit-error-rate (BER) performance of the conventional SIC and the Babai estimation in solving underdetermined systems. Fig. 4(a) shows the BER performance of SIC for $K = 3$ relay antennas and different numbers of user N. We can observe that as N increases from 4 to 7 in this case the BER increases rapidly at any given SNR level. This effect, which is caused by the reduction of the power difference between the user signals as N increases, illustrates the fundamental limitation of SIC discussed in Section III-D. Fig. 4(b) shows the BER performance of Babai estimation for $N = 6$ users and different numbers of antennas K. It is seen that as K increases from 2 to 5, the BER also increases rapidly at any given SNR level. This result illustrates the harmful effects of the non-informative term in (16) when the system becomes severely rank deficient.

Next, we investigate the effect of the regularization parameter λ in (16) on the performance for Babai estimation. Fig. 5 illustrates the BER versus $\lambda = \ell\sigma^2$ with $\ell \in \{0.05, 0.1, 0.2, 0.5, 1, 1.5, 5, 10, 20\}$ for different SNR levels under configuration $N = 6, K = 4$. It can be seen that the best performance is obtained when $\lambda \approx \sigma^2/P_T = \sigma^2$, which supports our proposed choice for this parameter in Section III-C.
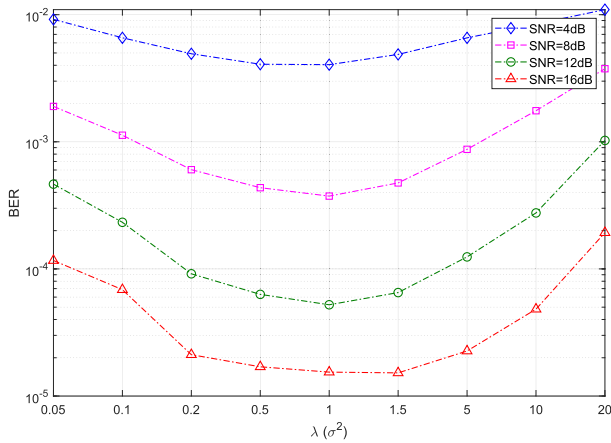
FIGURE 5. Impact of λ on the BER performance for Babai estimation.

FIGURE 7. Throughput performance of the proposed algorithm and the Atom I building block approach [27].
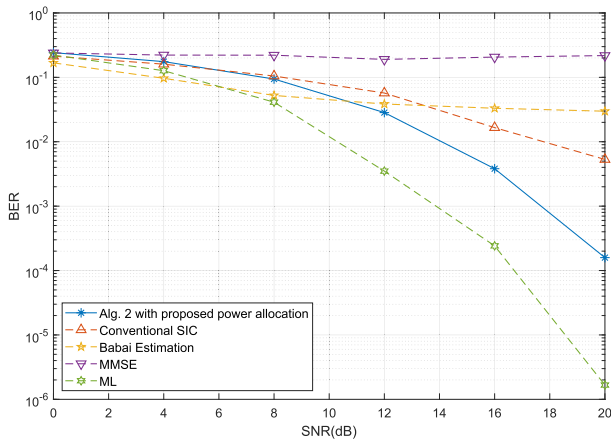

FIGURE 6. BER performance of the proposed algorithm, conventional SIC, Babai estimation, MMSE estimation and ML estimation for $N = 6$ and $K = 2$.

In Fig. 7, we investigate the throughput performance of the proposed algorithm, which is defined as the rate of correct signal detection at the relay. For the sake of comparison, we also consider the Atom I building block structure in [27], which decomposes the network into $N - 1$ subnets to allow simultaneous transmission from only 2 users at a time (TWRC). This is in contrast to the proposed algorithm which allows simultaneous transmission from all N users (MWRC) during the MA stage. Specifically, Fig. 7 shows the throughput versus SNR for $K = 4$ and $N \in \{6, 8\}$, as obtained by transmitting 50×10^3 symbols from each user within a common time interval for both schemes. The results clearly illustrate the advantages of the proposed MWRC detection algorithm, which leads to a nearly 4dB SNR gain over the Atom I TWRC scheme. Indeed, under the constraint of a fixed transmission time, Atom I must reduce the amount of time allocated to each subnet, which tends to increase the error rate for the individual user signals.

Next, we compare the decoding complexity of Algorithm 2 to a constellation design approach, i.e., the minimal constellation distance maximization in [29]. The decoding complexity here refers to the number of constellation points that need to be searched at the relay for signal detection, assuming BPSK signaling. In our algorithm, due to its sequential nature, this number is simply $2N$; for the algorithm in [29], the complexity is obtained by simulations.[4] Fig. 8 illustrates the decoding complexities versus the number of users N for the two approaches. We note that our approach has a linear growth in complexity with N, which is a prominent advantage over the constellation design method. The latter needs to generate a large size constellation to avoid the codeword ambiguity as N increases, e.g., it is 64 when $N = 6$ in this experiment. Due to this nature, its decoding complexity grows exponentially with N.

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[4]We note that the method in [29] assumes an additive white Gaussian noise (AWGN) channel and does not perform well over Rayleigh channels. For this reason, AWGN is used in order to evaluate its decoding complexity. We also note that the case of BPSK corresponds to the choice $M = 2$ in [29].

B. EVALUATIONS OF PROPOSED METHOD

Fig. 6 compares the BER performance of the proposed Algorithm 2 to the conventional SIC and Babai estimation in the case $N = 6$ and $K = 2$. Results of minimum mean square error (MMSE) and ML estimations are also presented as benchmarks for comparison. A specific power allocation scheme is used for each one of these methods as exposed in Section IV, in order to optimize performance. From the result, we note that the Babai estimation slightly outperforms the other two methods in the low SNR range from 0dB to 12dB, but its advantage quickly disappears afterward. Beyond this point, our proposed Algorithm 2 with power allocation (40) exhibits the best performance, with a rapid falloff in BER as the SNR increases. Although the performance of SIC is superior to Babai estimation at high SNR, the gains are limited compared to the proposed algorithm. From the result, we also note that conventional MMSE estimation is not effective in solving under-determined problem while the ML estimation achieves the best performance at a cost of exhaustive search for the optimal solution.
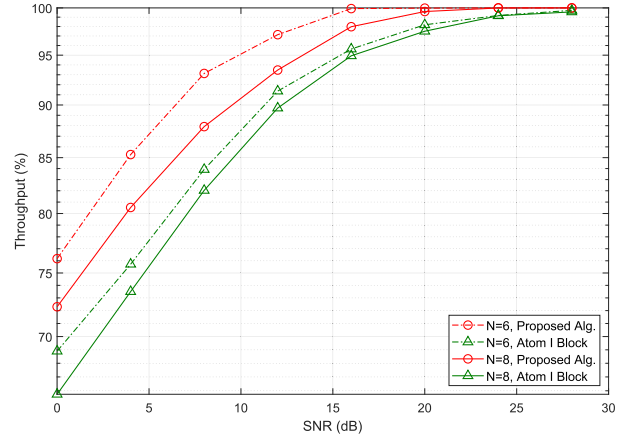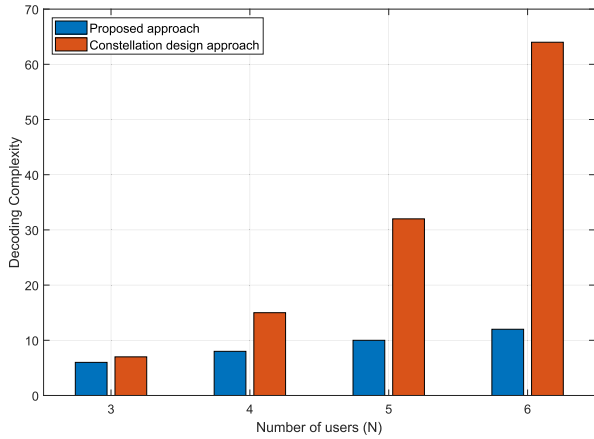
FIGURE 8. Decoding complexity for the proposed algorithm and the constellation design approach [29].

FIGURE 9. CPU time for the proposed algorithm and the searching algorithm.

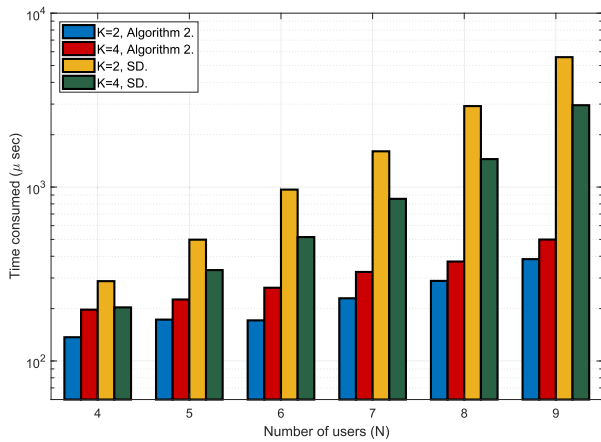**FIGURE 10.** BER performance and decoding complexity of the proposed algorithm with QPSK modulation.

FIGURE 11. Comparison between the approximation approach used in ψ_{ij} in (43) and the one adopted in [43].

The effectiveness of Algorithm 2 in reducing the computational complexity of the estimation process is further demonstrated by comparing it to the sphere decoding (SD) approach [47], which is a widely used iterative search algorithm for solving ILS problems. Fig. 9 shows the consumed CPU time per user symbol versus the number of users N for both algorithms under $K \in \{2, 4\}$. We note that the CPU time of Algorithm 2 is significantly less than that consumed by SD. Indeed, due to the system's rank deficiency, the latter algorithm needs to search back and forth repeatedly to detect a given user signal, which is often costly. Meanwhile, the proposed algorithm can better cope with the rank deficiency by combining two different sequential process, namely SIC and Babai estimation, which simplifies the search process and allows for a significant reduction in complexity.

In order to demonstrate the applicability of the proposed algorithm with higher modulation schemes, we also conduct an experiment using the QPSK modulation. Fig. 10(a) shows the BER performance of BPSK and QPSK modulations when $N = 6$ and $K = 2$. From the result, a 3 to 4 *dB* performance loss for the QPSK modulation is observed which is reasonable
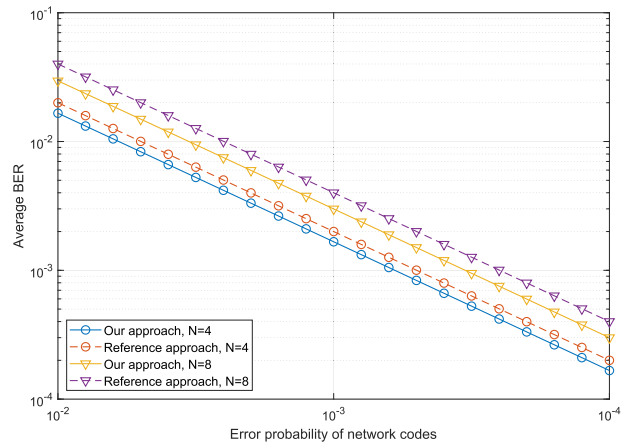
due to the existence of the multi-user interference. Fig. 10(b) shows the decoding complexity versus the number of users for the two modulation schemes. Although higher than that of the BPSK modulation, the complexity of the QPSK modulation remains linear with respect to N which is still efficient.

C. USER PAIRING STRATEGY

In this part, we investigate the effect of the selected pairing strategy $\mathcal{C}$ on the performance of network coding in MWRC. To begin, we validate the approximation used in the derivation of the error probability (43) by comparing the results with the approach from [43], which also relies on the SNR assumption. Fig. 11 shows the average downlink BER versus the error probability of sequentially paired network codes at the relay, when evaluated according to the two approximation approaches for $N \in \{4, 8\}$. From this figure, we find that the BER curves for the two approaches exhibit a similar linear trend and remain reasonably close to each other. This shows that our approach has a similar effect to that of [43] when approximating the error propagation.
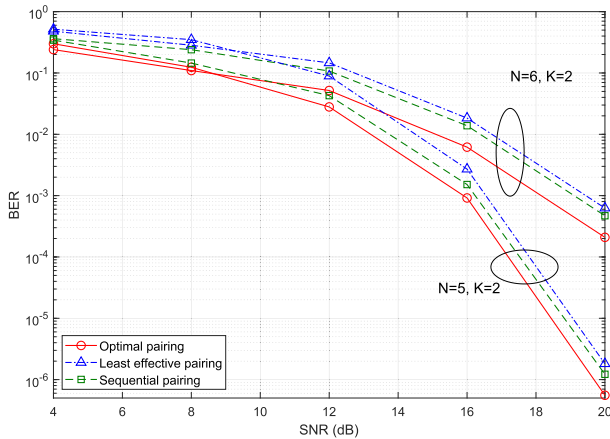
FIGURE 12. BER performance for different user pairing strategies.

Next, we consider three different strategies: the sequential strategy $\mathcal{C}_{seq}$ defined in (2) and commonly adopted in the literature [29]; the proposed optimal strategy $\mathcal{C}^*$ defined as the optimal spanning tree minimizing the average decoding error probability in (45); and a so-called least effective strategy $\mathcal{C}_w$, defined as the spanning tree that yields the worst performance in (45). Fig. 12 shows the BER versus SNR obtained with these three strategies for two different system configurations, corresponding to $K = 2$ and $N \in \{5, 6\}$. It is observed that the proposed optimal strategy $\mathcal{C}^*$ leads to a notable reduction in the BER when compared to the other two strategies, especially in the case $N = 6$, where the conventional strategy $\mathcal{C}_{seq}$ provides only a marginal improvement over the least efficient strategy $\mathcal{C}_w$.
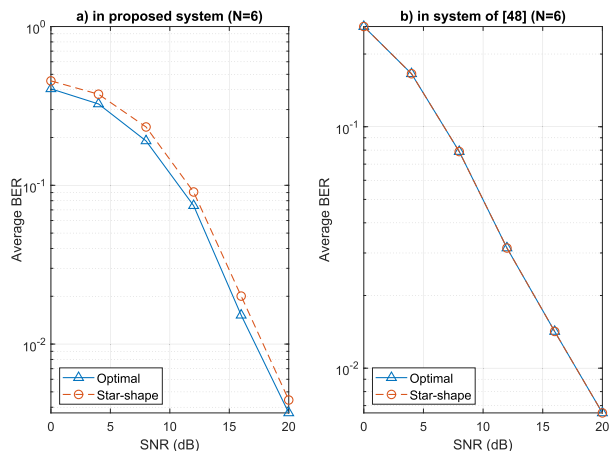

FIGURE 13. Comparisons between the optimal strategy and the star-shape strategy of [48]. The corresponding BER performances of the two strategies are evaluated in both the proposed system and the non-simultaneous decode-and-forward system of [48].

Finally, we compare our optimal pairing strategy with the star-shape pairing strategy in [48], which pairs the signal from the user having the maximum channel gain with each one of the remaining signals. We incorporate each strategy in both the proposed system and the non-simultaneous system of [48], and then evaluate the corresponding average BER versus SNR. Fig. 13(a) and Fig. 13(b) respectively show the

performance comparison between the two pairing strategies in each system. Due to the existence of multiuser interference in our system, the use of the signal with the maximum channel gain in the star-shape strategy does not yield the optimal performance, as shown in Fig. 13(a). In contrast, in a system with non-simultaneous transmission where multiuser interference does not exist, the performance of both strategies is identical, as seen in Fig. 13(b).

VI. CONCLUSION

In this paper, we considered an uplink MWRC scenario for PNC, where N users, each with single antenna, simultaneously transmit their signals to a relay equipped with K antennas ($K < N$). A novel detection scheme was proposed at the relay to iteratively resolve user signals and remove their interfering effects from the signal set. The proposed scheme combines a conventional SIC and a regularized ILS solution using Babai estimation to solve an underdetermined ILS problem. We developed a power allocation scheme for the proposed method, and also investigated an optimal user pairing strategy to reduce the decoding error rate at the user terminals. Simulation results revealed the achievable performance improvement of the proposed method in the detection of network codes for PNC in MWRC. While this paper focused on the signal processing aspects of PNC in MWRC, the further consideration of related information theoretic aspects (e.g., achievable rate region), remains an interesting avenue for future work.

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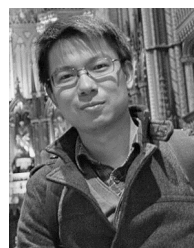
HAO LI (S'15) received the B.S. degree from Shanghai Jiaotong University, Shanghai, China, in 2011, and the M.S. degree from the University of Western Ontario, London, Ontario, Canada, in 2013. He is currently pursuing the Ph.D. degree in electrical and computer engineering with McGill University, Montreal, Quebec, Canada. His research interests are in the area of wireless network, with a focus on the system design for the relay communications and network coding in the physical layer.
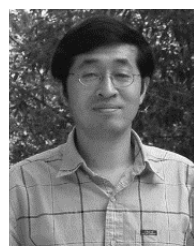
XIAO-WEN CHANG received the B.S. and M.S. degrees in computational mathematics from Nanjing University, Nanjing, China, in 1986 and 1989, respectively, and the Ph.D. degree (Dean's Honor List) in computer science from McGill University, Montreal, Quebec, Canada, in 1997. He is currently an Associate Professor with the School of Computer Science, McGill University. His research interests are in the area of scientific computing, with the emphasis on numerical linear algebra and its applications. He has published more than 40 articles in refereed journals.

YUNLONG CAI (S'07–M'10–SM'16) received the B.S. degree in computer science from Beijing Jiaotong University, Beijing, China, in 2004, the M.Sc. degree in electronic engineering from the University of Surrey, Guildford, U.K., in 2006, and the Ph.D. degree in electronic engineering from the University of York, York, U.K., in 2010. From 2010 to 2011, he was a Postdoctoral Fellow with the Electronics and Communications Laboratory, Conservatoire National des Arts et Métiers, Paris, France. From 2016 to 2017, he was a Visiting Scholar with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, USA. Since 2011, he has been with the College of Information science and Electronic Engineering, Zhejiang University, Hangzhou, China, where he is currently an Associate Professor. His research interests include transceiver design for multiple-antenna systems, sensor array processing, adaptive filtering, full-duplex communications, cooperative and relay communications, and wireless information and energy transfer.

BENOIT CHAMPAGNE (S'87–M'89–SM'03) received the B.Ing. degree in engineering physics from Cole Polytechnique de Montréal, Montreal, QC, Canada, in 1983, the M.Sc. degree in physics from the Université de Montréal, in 1985, and the Ph.D. degree in electrical engineering from the University of Toronto, Toronto, ON, Canada, in 1990.

From 1990 to 1999, he was an Assistant and then an Associate Professor with INRS-Telecommunications, Université du Quebec, Montreal. Since 1999, he has been with McGill University, Montreal, where he is currently a Full Professor with the Department of Electrical and Computer Engineering. From 2004 to 2007, he also served as an Associate Chair of Graduate Studies with the Department of Electrical and Computer Engineering, McGill University. He is the author or coauthor of more than 200 refereed publications. His research has been funded by the Natural Sciences and Engineering Research Council of Canada, the ''Fonds de Recherche sur la Nature et les Technologies'' from the Government of Quebec, and some major industrial sponsors, including Nortel Networks, Bell Canada, InterDigital, and Microsemi. His research interests include the study of advanced algorithms for the processing of information-bearing signals by digital means and several topics in statistical signal processing, such as detection and estimation, sensor array processing, adaptive filtering, and applications thereof to broadband communications and audio processing.

Dr. Champagne has also served on the Technical Committees of several international conferences in the fields of communications and signal processing. In particular, he has served as the Co-Chair of the Wide Area Cellular Communications Track for the IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications (Toronto, September 2011), as the Co-Chair of the Antenna and Propagation Track for the IEEE Vehicular Technology Conference-Fall, (Los Angeles, CA, USA, September 2004), and as the Registration Chair for the IEEE International Conference on Acoustics, Speech and Signal Processing (Montreal, May 2004). He has served as an Associate Editor for the IEEE SIGNAL PROCESSING LETTERS, the IEEE TRANSCATIONS ON SIGNAL PROCESSING, and the *EURASIP Journal on Applied Signal Processing*.

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