IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 31, 2019, accepted November 9, 2019, date of publication November 13, 2019, date of current version November 22, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2953244

Dissipativity Analysis of Complex-Valued Stochastic Neural Networks With Time-Varying Delays

MEIJUAN LIU, XIANGRONG WANG, ZIYE ZHANG, AND ZHEN WANG
College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China
Corresponding authors: Meijuan Liu (liumeijuanqd@126.com) and Xiangrong Wang (xrwang2011@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61503222, in part by the Science and Technology Support Plan for Youth Innovation of Colleges and Universities of Shandong Province of China under Grant 2019KJI005, and in part by the Shandong University of Science and Technology (SDUST) Public Visiting Scholar Foundation.

ABSTRACT This paper considers the dissipativity analysis problem for complex-valued stochastic neural networks (CVSNNs) with time-varying delays. By constructing the Lyapunov functions, using Jensen inequality and stochastic analysis techniques, several sufficient conditions for the exponential dissipativity and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-dissipativity in the mean square are obtained in terms of linear matrix inequalities (LMIs). Compared with the existing ones, in our work, some results which are more applicable for CVSNNs with time-varying delays case are derived. Finally, two numerical examples are provided to illustrate the effectiveness and improvement of our theoretical results.

INDEX TERMS Complex-valued stochastic neural network, time-varying delays, dissipativity, Itô formula.

I. INTRODUCTION

For more than one decade, stability analysis of neural networks(NNs) has drawn more and more attentions due to their wide applications in numerous areas such as signal processing, image processing, pattern recognition, combinatorial optimization and moving object speed detection, see [1], [2], [12]–[14], [38], [41], etc. On the other hand, in the electronic circuit implementation of the NNs, time-delays caused by the finite switching speed of amplifiers and communication time, and it may be an important source of oscillation, divergence, instability or other performance of NNs. It is natural that neural networks with time-delay received considerable attentions, and many results have been reported in the literatures [3]–[8], [48], [54], etc. In recent years, stability of complex-valued neural networks(CVNNs) have found widespread applications in various fields such as, but not limited to, optoelectronics, filtering, speech synthesis, remote sensing, artificial neural information processing and others. Compared with real-valued neural networks, the states, connection weights and activation functions of complex-valued neural networks are defined in the complex domain, which can provide a simple and natural way to maintain the physical characteristics of the original problems [25], [33], [35], [36], [39], [40], [52], etc.

However, during the research of stability, scholars found that it is not all the trajectories of dynamical neural networks all can approach a single stable equilibrium point, and also some dynamical neural networks have no equilibrium point. In the course of dealing with these problems, the concept of dissipativity plays a key role. The idea of dissipativity theory was first introduced in [28] by Willems, and it was generalized in [20]. Dissipativity theory is very useful tool for control technique like robotics, active vibration damping, electromechanical systems, circuit theory and inverse control [11], [16], [18], [26], etc. For example, in [10], by using the framework of Filippov solution, differential inclusion theory, an appropriate Lyapunov-Krasovskii functional and LMI technique, the problem of dissipativity analysis for memristor-based complex-valued neural networks with time-varying delays is investigated extensively and several new sufficient conditions for global dissipativity, global exponential dissipativity and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity are derived. Some novel sufficient conditions that guarantee the dissipativity of complex-valued BAM neural networks are obtained in [15] by using the inequality techniques, Halanay inequality, and upper right Dini derivative concepts.

The associate editor coordinating the review of this manuscript and approving it for publication was Yanzheng Zhu.

It is well known that the external stochastic interference is another important source of oscillation, divergence, instability or other performance of NNs. It has an destructive impact on the state of the neural network system. Hence it is necessary to take into account the effect of stochastic on the neural network [42]–[46], [50], [53] and solve this kind of problems. For a long time, stochastic systems have been widely concerned by researchers from various background. For instance, the dissipativity analysis of stochastic neural network system have been the very hot research topic. However, it should be pointed out that, in large amount of the existing literatures, the dissipativity problem of neural networks has been investigated mainly for deterministic real-valued/complex-valued neural network with or without time-delay [9]–[11], [32]. In [51], dissipativity for discrete-time switched systems is investigated, one new concept of decomposable dissipativity is proposed and the energy changes of subsystems and energy dissipation in the whole switched system are explained. In [11], some new sufficient conditions on global dissipativity and global exponential dissipativity of memristor-based complex-valued neural networks have been derived. And in [9], other novel sufficient conditions that guarantee the dissipativity of complex-valued bidirectional associative memory neural networks are obtained by using the inequality techniques, Halanay inequality, and upper right Dini derivative concepts. So it is natural that the dissipativity problem of stochastic neural network has been investigated.

And up to now, many significant results have been proposed regarding the dissipativity of real-valued stochastic neural networks(RVSNNs) [17], [22], [27], [47], [49], etc. In [24], authors investigated the global dissipativity of real-valued stochastic system. By constructing several proper Lyapunov functionals combining with Jensen inequality, Itô formula and some analytic techniques, several sufficient conditions for the global dissipativity in means of such stochastic neural networks are derived in LMIs forms. In [19], the problems of global dissipativity and global exponential dissipativity are investigated for discrete-time stochastic neural networks with time-varying delays and general activation functions. By using similar method, several new delay-dependent criteria for checking the global dissipativity and global exponential dissipativity of the addressed neural networks are established. While in [21], dissipativity analysis is also discussed for discrete-time stochastic neural networks with time-varying discrete and finite-distributed delays. The discreted Jensen inequality and lower bounds lemma are adopted to deal with the involved finite sum quadratic terms, and a sufficient condition is derived to ensure the considered neural networks to be strictly $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-dissipative, which is delay-dependent in the sense that it depends on not only the discrete delay but also the finite-distributed delay.

In this paper, we study the problem of dissipativity analysis of CVSNNs with time-varying delays. By use of stochastic integral inequalities, some appropriate Lyapunov function and linear matrix inequality technique, some sufficient conditions are derived to guarantee global exponential dissipativity and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-dissipativity of CVSNNs with time-varying delays. Finally, two numerical examples are given to demonstrate the effectiveness of the present results.

This paper is organized as follows: In section 2, an CVSNNs with time-varying delays is proposed, and some notations, preliminary definitions and lemmas are presented. In section 3, sufficient conditions for globally exponential dissipative and $(\mathcal{Q}, \mathcal{R}, \mathcal{S})$-dissipative of CVSNNs with time-varying delays are presented. In section 4, two numerical examples are established to illustrate the effective of our proposed theoretical results. Finally, conclusions are presented in Section 5.

Notation: Throughout this paper, let A be the complex-valued matrix, the superscript $*$ and T denote the matrix complex conjugate transposition and matrix transposition, respectively. Let z be a complex number, z^* denote the conjugate transpose of z, $|\cdot|$ is the Euclidean norm in C^n, and $\|z\|^2 = z^*z$, $|z| = [|z_1|, |z_2|, \ldots, |z_n|]^{\mathrm{T}}$. The notion $A > 0$ (respectively, $A \geq 0$) means that A is a complex symmetric and positive (respectively,semi-definite) matrix. I_n is identity matrix. As matrix A, $\lambda_{\min}(A)$ represents the smallest eigenvalue of A.

Denote by $\mathcal{L}^2_{\mathscr{F}_0}([-\tau, 0]; C^n)$ the family of all $\mathscr{F}_0$ measurable $C([-\tau, 0]; C^n)$ random variables $\varphi = \{\varphi(s) : -\tau \leq s \leq 0\}$ such that $\sup_{-\tau\leq s\leq 0} E\|\varphi(s)\|^2 < +\infty$ where $E[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure $\mathcal{P}$. $\mathcal{L}[0, +\infty)$ represents the space of square-integrable vector functions over $[0, +\infty)$.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the following complex-valued stochastic neural networks with time-varying delay:

$$dz(t) = [-Dz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + u(t)]dt + \sigma(t, z(t), z(t-\tau(t)))dw(t), \quad (1)$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^{\mathrm{T}} \in C^n$ is the state vector, $f(z(t)) = [f_1(z_1(t)), f_2(z_2(t)), \ldots, f_n(z_n(t))]^{\mathrm{T}} \in C^n$ and $f(z(t-\tau(t))) = [f_1(z_1(t-\tau(t))), f_2(z_2(t-\tau(t))), \ldots, f_n(z_n(t-\tau(t)))]^{\mathrm{T}} \in C^n$ are the vector-valued activation functions without and with the varying-time delay. $D = \mathrm{diag}(d_1, d_2, \ldots, d_n) \in R^{n\times n}$ with $d_j > 0 (j = 1, 2, \ldots, n)$ is the self-feedback connection weight matrix. $A = (a_{jk}) \in C^{n\times n}$ and $B = (b_{jk}) \in C^{n\times n}$ are the connection weight matrices without and with varying-time delay, respectively. $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^{\mathrm{T}} \in C^n$ is the external input vector. $\sigma(z(t), z(t-\tau(t))) = (\sigma_1(z_1(t), z_1(t-\tau(t))), \sigma_2(z_2(t), z_2(t-\tau(t))), \ldots, \sigma_n(z_n(t), z_n(t-\tau(t))))^{\mathrm{T}} \in C^{n\times n}$ is the diffusion coefficient function and $w(t) = (w_1(t), w_2(t), \ldots, w_n(t))^{\mathrm{T}}$ is an n-dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t>0}, \mathcal{P})$ with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. The initial condition of system (1) is given by

$$z(s) = \varphi(s), \quad -\tau \leq s \leq 0, \ \varphi(s) \in \mathcal{L}^2_{\mathscr{F}_0}([-\tau, 0], C^n). \quad (2)$$

Assumption 1: $\tau(t)$ is time-varying delay which satisfies

$$0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq d < 1 \tag{3}$$

with τ and d are constants.

Assumption 2: The neuron activation functions $f_j(\cdot)$ satisfy $f_j(0) = 0$ $(j = 1, 2, \ldots, n)$ and the following Lipschitz conditions

$$|f_j(z)| \leq l_j|z|, \quad \forall z \in C,\ j = 1, 2, \ldots, n. \tag{4}$$

From (4), it is clear to get

$$f^*(z)f(z) \leq z^* L^{\mathrm{T}} L z, \tag{5}$$

where $L = \mathrm{diag}(l_1, l_2, \ldots, l_n)$ and l_j is a constant.

Assumption 3: Moreover, we assume that

$$\begin{aligned} &\mathrm{trace}\ \sigma^*(z, z^\tau)\sigma(z, z^\tau) \\ &\quad \leq z^* Q_1 z + (z^\tau)^* Q_2 z^\tau, \quad \forall z, z^\tau \in C^n, \end{aligned} \tag{6}$$

where Q_1, Q_2 are positive Hermitian matrices.

Definition 1: The complex-valued stochastic neural networks (1) are said to be globally exponential dissipative system if there exists a compact set $S^* \supset S$ in C^n such that $\forall z_0 \in \mathcal{L}^2_{\mathscr{F}_0}([-\tau, 0], C^n)\backslash S^*$, there exist $\varepsilon > 0$ and a constant $M(z_0)$ such that

$$\inf_{\tilde{z} \in S^*} E[\|z(t, t_0, z_0) - \tilde{z}\|] \leq e^{-\varepsilon(t-t_0)} E[M(z_0)].$$

Moreover, the set S^* is called globally exponential attractive set.

Now we introduce another definition on dissipativity. Let the energy supply function of neural network (1) be defined by

$$\begin{aligned} E\{G(u, z, T)\} = E\{\langle z, \mathcal{Q}z\rangle\} + 2E\{\langle z, \mathcal{S}u\rangle\} \\ + E\{\langle u, \mathcal{R}u\rangle\}, \quad T \geq 0, \end{aligned} \tag{7}$$

where $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ are Hermitian matrices, $\mathcal{Q} \leq 0$ and

$$\langle z, v\rangle_{\mathrm{T}} = \int_0^T z^{\mathrm{T}} v dt, \quad T \geq 0.$$

Definition 2: The complex-valued stochastic neural networks (1) are said to be strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative if, for any $T \geq 0$ and some scalar $\gamma > 0$, under zero initial state, the following inequality

$$E\{G(u, z, T)\} \geq \gamma E\{\langle u, u\rangle_{\mathrm{T}}\}$$

holds for any nonzero input $u \in \mathcal{L}^2[0, \infty)$.

Lemma 1 (Jensen Inequality): For any constant Hermitian matrix $\mathcal{R} \in C^{n\times n}$ with $\mathcal{R} > 0$ and vector function $u(s)$: $[a, b] \rightarrow C^n$ with scalars $a < b$, such that the following inequalities hold

$$\begin{aligned} &(\int_a^b u(s)ds)^* \mathcal{R}(\int_a^b u(s)ds) \\ &\quad \leq (b-a)\int_a^b u^*(s)\mathcal{R}u(s)ds, \\ &(\int_a^b \int_{t+\lambda}^t u(s)dsd\lambda)^* \mathcal{R}(\int_a^b \int_{t+\lambda}^t u(s)dsd\lambda) \\ &\quad \leq \frac{1}{2}(b^2 - a^2)\int_a^b \int_{t+\lambda}^t u^*(s)\mathcal{R}u(s)dsd\lambda. \end{aligned}$$

Lemma 2 [18]: Let $(\Omega, \mathscr{F}, \mathcal{P})$ be a probability space, X an integrable real-valued random variable and $\Psi : R \rightarrow R$ a measurable convex function, with $E(\|X\|) < \infty$. Then $\Psi(E(X)) \leq E(\Psi(X))$.

Lemma 3 [34]: Given a Hermitian matrix Θ, then $\Theta < 0$ is equivalent to

$$\begin{bmatrix} \Theta^R & -\Theta^I \\ \Theta^I & \Theta^R \end{bmatrix} < 0,$$

where $\Theta^R = \mathrm{Re}(\Theta)$ and $\Theta^I = \mathrm{Im}(\Theta)$.

Lemma 4 (Schur Complement): The LMI

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} < 0$$

with $\Sigma_{11} = \Sigma_{11}^*$, $\Sigma_{22} = \Sigma_{22}^*$, is equivalent of the following conditions:

$$\begin{aligned} &(1)\ \Sigma_{22} < 0,\ \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^* < 0, \\ &(2)\ \Sigma_{11} < 0,\ \Sigma_{22} - \Sigma_{12}^*\Sigma_{11}^{-1}\Sigma_{12} < 0. \end{aligned}$$

Remark 2: We suppose that the activation functions and the diffusion coefficient functions are all not explicitly expressed by separating real-imaginary parts throughout this paper. Moreover, obviously, the case with separable functions on real and imaginary parts is the special case of our inseparable case and it can be dealt with by the methods in real number domain, as stated in [19].

III. MAIN RESULTS

In this section, we will derive several sufficient conditions which ensure the CVSNNs (1) to be dissipative. Let

$$\begin{aligned} z(t) &= \mathrm{Re}(z) + i\mathrm{Im}(z), \quad u(t) = u^R(t) + iu^I(t), \\ A &= A^R + iA^I, \quad B = B^R + iB^I, \end{aligned} \tag{8}$$

and for convenience, denote

$$z_t = z(t),\ z_{t-\tau(t)} = z(t - \tau(t)),\ z_{t-\tau} = z(t - \tau).$$

Theorem 1: Suppose that Assumptions 1, 2 and 3 hold. If there exist Hermitian matrices $P_j > 0 (j = 1, \ldots, 5)$, $N > 0$, diagonal matrices $G > 0$, $H > 0$, $J > 0$, a positive real constant ρ and any appropriate dimension matrices $M_l (l = 1, 2, 3)$, such that the following LMIs hold:

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & P_1 A & P_1 B & \Phi_{17} \\ * & \Phi_{22} & 0 & 0 & \Phi_{27} \\ * & * & \Phi_{44} & 0 & 0 \\ * & * & * & \Phi_{55} & 0 \\ * & * & * & * & \Phi_{77} \end{bmatrix} < 0, \tag{9}$$

$$P_1 \leq \rho I, \tag{10}$$

where

$$\begin{aligned} \Phi_{11} &= \varepsilon P_1 - (DP_1 + P_1 D) + \rho Q_1 + 2N \\ &\quad + e^{\varepsilon\tau}(P_2 + P_4) + 2M_1 + G\hat{L}, \\ \Phi_{22} &= \rho Q_2 - (1 - d)P_2 - 2M_2 + H\hat{L}, \\ \Phi_{44} &= e^{\varepsilon\tau}(P_3 + P_5) + G, \end{aligned}$$

$$\Phi_{55} = (d-1)P_3 - H,\ \Phi_{77} = -2M_3,$$
$$\Phi_{12} = M_2 - M_1^*,\ \Phi_{17} = M_3 - M_1^*,$$
$$\Phi_{27} = -M_3 - M_2^*,$$

then the CVSNNs (1) are said to be globally exponentially dissipative and $S = \{z : \|Ez\| \leq \frac{1}{\lambda_{\min}(N)}\|u_t P_1\|, z \in C^n\}$ is said to be the attractive set.

Proof: Consider the following Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{11}$$

where

$$V_1(t) = e^{\varepsilon t} z_t^* P_1 z_t,$$
$$V_2(t) = \int_{t-\tau(t)}^{t} e^{\varepsilon(s+\tau)} z_s^* P_2 z_s ds + \int_{t-\tau(t)}^{t} e^{\varepsilon(s+\tau)} f^*(z_s) P_3 f(z_s) ds,$$
$$V_3(t) = \int_{t-\tau}^{t} e^{\varepsilon(s+\tau)} z_s^* P_4 z_s ds + \int_{t-\tau}^{t} e^{\varepsilon(s+\tau)} f^*(z_s) P_5 f(z_s) ds.$$

Taking Itô formula to $V(t)$, then we can get

$$dV(t) = LV(t)dt + e^{\varepsilon t}(z_t^* P_1 \sigma(z_t, z_{t-\tau(t)}) + \sigma^*(z_t, z_{t-\tau(t)}) P_1 z_t) dW_t, \tag{12}$$

where

$$\begin{aligned}LV(t) = {} & \varepsilon e^{\varepsilon t} z_t^* P_1 z_t \\ & + e^{\varepsilon t}\text{trace}[\sigma^*(z_t, z_{t-\tau(t)}) P_1 \sigma(z_t, z_{t-\tau(t)})] \\ & + e^{\varepsilon t}[-Dz_t + Af(z_t) + Bf(z_{t-\tau(t)}) + u_t]^* P_1 z_t \\ & + e^{\varepsilon t} z_t^* P_1[-Dz_t + Af(z_t) + Bf(z_{t-\tau(t)}) + u_t] \\ & + e^{\varepsilon(t+\tau)} z_t^* P_2 z_t + e^{\varepsilon(t+\tau)} f^*(z_t) P_3 f(z_t) \\ & - (1 - \dot{\tau(t)}) e^{\varepsilon[t+\tau-\tau(t)]} z_{t-\tau(t)}^* P_2 z_{t-\tau(t)} \\ & - (1 - \dot{\tau(t)}) e^{\varepsilon[t+\tau-\tau(t)]} f^*(z_{t-\tau(t)}) P_3 f(z_{t-\tau(t)}) \\ & + e^{\varepsilon(t+\tau)} z_t^* P_4 z_t - e^{\varepsilon t} z_{t-\tau}^* P_4 z_{t-\tau} + e^{\varepsilon(t+\tau)} \\ & \times f^*(z_t) P_5 f(z_t) - e^{\varepsilon t} f^*(z_{t-\tau}) P_5 f(z_{t-\tau}).\end{aligned}$$

Based on (3), (6) and (10)

$$\begin{aligned}LV(t) \leq {} & e^{\varepsilon t}\{z_t^*[\varepsilon P_1 + e^{\varepsilon\tau}(P_2 + P_4) - (D^* P_1 + P_1 D) \\ & + \rho Q_1] z_t + z_{t-\tau(t)}^*[\rho Q_2 - (1-d)P_2] z_{t-\tau(t)} \\ & + z_{t-\tau}^*(-P_4) z_{t-\tau} + f^*(z_t)[e^{\varepsilon\tau}(P_3 + P_5)] f(z_t) \\ & + f^*(z_{t-\tau(t)})(d-1) P_3 f(z_{t-\tau(t)}) + f^*(z_{t-\tau}(-P_5) \\ & \times f(z_{t-\tau}) + [(f^*(z_t) A^* P_1 z_t + (z_t^* P_1 A f(z_t)) \\ & + (f^*(z_{t-\tau(t)}) B^* P_1 z_t + (z_t^* P_1 B f(z_{t-\tau(t)})) \\ & + (u_t^* P_1 z_t + z_t^* P_1 u_t)]\}.\end{aligned} \tag{13}$$

According to (4), for any $j = 1, 2, \ldots, n$, we have

$$|f_j(z_j(t))| \leq l_j |z_j(t)|.$$

Let $G = \text{diag}(g_1, g_2, \ldots, g_n) > 0$, we can write

$$g_j f_j^*(z_j(t)) f_j(z_j(t)) - g_j l_j^2 z_j^*(t) z_j(t) \leq 0,$$

for all $j = 1, 2, \ldots, n$. Also we define $\hat{L} = L^\top L$, thus

$$f^*(z_t) G f(z_t) - z_t^* G \hat{L} z_t \leq 0. \tag{14}$$

Similarly, for diagonal matrices $H > 0$ and $J > 0$, we can obtain

$$\begin{aligned}f^*(z_{t-\tau(t)}) H f(z_{t-\tau(t)}) - z_{t-\tau(t)}^* H \hat{L} z_{t-\tau(t)} \leq 0, \\ f^*(z_{t-\tau}) J f(z_{t-\tau}) - z_{t-\tau}^* J \hat{L} z_{t-\tau} \leq 0.\end{aligned} \tag{15}$$

Consider the Newton-Leibniz formulation

$$z_t - z_{t-\tau(t)} = \int_{t-\tau(t)}^{t} dz_s,$$

then we have

$$2e^{\varepsilon t}[z_t - z_{t-\tau(t)} - \int_{t-\tau(t)}^{t} dz_s]^*[M_1 z_t + M_2 z_{t-\tau(t)} + M_3 \int_{t-\tau(t)}^{t} dz_s)] = 0. \tag{16}$$

From the inequalities (14) – (16),

$$\begin{aligned}& -f^*(z_t) G f(z_t) + z_t^* G \hat{L} z_t - f^*(z_{t-\tau(t)}) H f(z_{t-\tau(t)}) \\ & + z_{t-\tau(t)}^* H \hat{L} z_{t-\tau(t)} - f^*(z_{t-\tau}) J f(z_{t-\tau}) \\ & + z_{t-\tau}^* J \hat{L} z_{t-\tau} + 2e^{\varepsilon t}[z_t - z_{t-\tau(t)} - \int_{t-\tau(t)}^{t} dz_s]^*[M_1 z_t \\ & + M_2 z_{t-\tau(t)} + M_3 \int_{t-\tau(t)}^{t} dz_s)] \geq 0.\end{aligned} \tag{17}$$

Then, substituting (17) into (13), we can get

$$\begin{aligned}& LV(t) \\ & \leq e^{\varepsilon t}\{z_t^*[\varepsilon P_1 + e^{\varepsilon\tau}(P_2 + P_4) - (D^* P_1 + P_1 D) \\ & \quad + \rho Q_1 + G\hat{L} + 2M_1 + 2N] z_t + z_{t-\tau(t)}^*[(d-1)P_2 + \rho Q_2 \\ & \quad - 2M_2 + H\hat{L}] z_{t-\tau(t)} + z_{t-\tau}^*(-P_4 + J\hat{L}) z_{t-\tau} + f^*(z_t) \\ & \quad \times [e^{\varepsilon\tau}(P_3 + P_5) - G] f(z_t) + f^*(z_{t-\tau(t)})[(d-1)P_3 - H] \\ & \quad \times f(z_{t-\tau(t)}) + f^*(z_{t-\tau})[-P_5 - G] f(z_{t-\tau}) + (\int_{t-\tau(t)}^{t} dz_s)^* \\ & \quad \times [-M_3](\int_{t-\tau(t)}^{t} dz_s) + [f^*(z_t) A^* P_1 z_t + z_t^* P_1 A f(z_t) \\ & \quad + f^*(z_{t-\tau(t)} B^* P_1 z_t + z_t^* P_1 B f(z_{t-\tau(t)}) + 2z_t^* M_2 z_{t-\tau(t)} \\ & \quad + 2z_t^* M_3 \int_{t-\tau(t)}^{t} dz_s - 2z_{t-\tau(t)}^* M_1 z_t \\ & \quad - 2(\int_{t-\tau(t)}^{t} dz_s)^* M_1 z_t - 2z_{t-\tau(t)}^* M_3 \int_{t-\tau(t)}^{t} dz_s \\ & \quad - 2(\int_{t-\tau(t)}^{t} dz_s)^* M_2 z_{t-\tau(t)}\} + e^{\varepsilon t}[u_t^* P_1 z_t \\ & \quad + z_t^* P_1 u_t - 2z_t^* N z_t].\end{aligned} \tag{18}$$

Denote that

$$\eta^* = \Big\{z_t^*, z_{t-\tau(t)}^*, z_{t-\tau}^*, f^*(z_t), f^*(z_{t-\tau(t)}), f^*(z_{t-\tau}), (\int_{t-\tau(t)}^{t} dz_s)^*\Big\}.$$

Thus, taking the expectation to (12), and from (18), we have

$$\begin{aligned} EdV(t) &= E(LV(t)dt) \\ &\leq Ee^{\varepsilon t}\eta^*\tilde{\Phi}\eta + Ee^{\varepsilon t}[-2z_t^*Nz_t + u_t^*P_1z_t + z_t^*P_1u_t] \\ &\leq Ee^{\varepsilon t}[-2z_t^*Nz_t + u_t^*P_1z_t + z_t^*P_1u_t] \\ &\leq e^{\varepsilon t}[-2\lambda_{\min}(N)\|Ez_t\|^2 + 2\|Ez_t\|\|u_t^*P_1\|] \\ &\leq 0, \end{aligned} \tag{19}$$

where $Ez_t \in C^n \setminus S$ and

$$\tilde{\Phi} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & P_1A & P_1B & 0 & \Phi_{17} \\ * & \Phi_{22} & 0 & 0 & 0 & 0 & \Phi_{27} \\ * & * & \Phi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & 0 \\ * & * & * & * & \Phi_{55} & 0 & 0 \\ * & * & * & * & * & \Phi_{66} & 0 \\ * & * & * & * & * & * & \Phi_{77} \end{bmatrix}$$

$\Phi_{33} = -P_4 - J\hat{L}, \quad \Phi_{66} = -P_5 - J.$

From (9), obviously, $\tilde{\Phi} < 0$, then, integrating two sides of the inequality (19) from 0 to an arbitrary $t > 0$, we have

$$\begin{aligned} EV(z(t)) &= EV(z(0)) + \int_0^t ELV(z(s))ds \\ &\leq EV(z(0)). \end{aligned} \tag{20}$$

On the other hand, from the definition of $V(z(t))$, it is easy to know that

$$V(z(t)) \geq e^{\varepsilon t}z^*(t)P_1z(t).$$

Then we can get

$$EV(z(0)) \geq e^{\varepsilon t}Ez_t^*P_1z_t.$$

By using Lemma 2,

$$EV(z(0)) \geq e^{\varepsilon t}\lambda_{\min}(P_1)\|Ez_t\|^2,$$

also

$$\|Ez_t\| \leq [e^{\varepsilon t}\lambda_{\min}(P_1)]^{\frac{-1}{2}}[EV(z(0))]^{\frac{1}{2}},$$

therefore, by Definition 1, the CVSNNs (1) are global exponential dissipative in mean square and S is an attractive set. This completes the proof.

Remark 3: The complex-valued LMI cannot be tested straightly by LMI tool box. In the following corollary, we provide the equivalent LMIs in real number domain which can be solved with the help of standard available numerical.

Corollary 1. Suppose that Assumptions 1, 2 and 3 hold. If there exist positive Hermitian matrices $P_{j1} + iP_{j2}(j = 1, \ldots, 5)$, $N^{\mathrm{R}} + iN^{\mathrm{I}}$, diagonal matrices $G > 0, H > 0, J > 0$, a positive real constant ρ, and any appropriate dimension matrices $M_{l1} + iM_{l2}(l = 1, 2, 3)$, such that the following LMIs hold:

$$\begin{bmatrix} \Phi^R & -\Phi^I \\ \Phi^I & \Phi^R \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} P_{11} - \rho I & -P_{12} \\ P_{12} & P_{11} - \rho I \end{bmatrix} < 0, \tag{22}$$

where

$$\Phi^R = \begin{bmatrix} \Phi_{11}^R & \Phi_{12}^R & \Phi_{14}^R & \Phi_{15}^R & \Phi_{17}^R \\ * & \Phi_{22}^R & 0 & 0 & \Phi_{27}^R \\ * & * & \Phi_{44}^R & 0 & 0 \\ * & * & * & \Phi_{55}^R & 0 \\ * & * & * & * & 0 \\ * & * & * & * & \Phi_{77}^R \end{bmatrix},$$

$$\Phi^I = \begin{bmatrix} \Phi_{11}^I & \Phi_{12}^I & \Phi_{14}^I & \Phi_{15}^I & \Phi_{17}^I \\ \Phi_{21}^I & \Phi_{22}^I & 0 & 0 & \Phi_{27}^I \\ \Phi_{41}^I & 0 & \Phi_{44}^I & 0 & 0 \\ \Phi_{51}^I & 0 & 0 & \Phi_{55}^I & 0 \\ \Phi_{71}^I & \Phi_{72}^I & 0 & 0 & \Phi_{77}^I \end{bmatrix},$$

with

$$\begin{aligned}
\Phi_{11}^R &= \varepsilon P_{11} - (D^*P_{11} + P_{11}D) + e^{\varepsilon\tau}(P_{21} + P_{41}) \\
&\quad + \rho Q_{11} + 2N^{\mathrm{R}} + G\hat{L} + 2M_{11}, \\
\Phi_{12}^R &= M_{21} - M_{11}^\top,\ \Phi_{14}^R = P_{11}A^R - P_{12}A^I, \\
\Phi_{15}^R &= P_{11}B^R - P_{12}B^I,\ \Phi_{17}^R = M_{31} - M_{11}^\top, \\
\Phi_{27}^R &= -M_{31} - M_{21}^\top,\ \Phi_{55}^R = (d-1)P_{31} - H, \\
\Phi_{22}^R &= (d-1)P_{21} + \rho Q_{21} - 2M_{21} + H\hat{L}, \\
\Phi_{44}^R &= e^{\varepsilon\tau}(P_{31} + P_{51}) - G,\ \Phi_{77}^R = -2M_{31}, \\
\Phi_{11}^I &= \varepsilon P_{12} - (D^*P_{12} + P_{12}D) + e^{\varepsilon\tau}(P_{22} + P_{42}) \\
&\quad + \rho Q_{12} + 2N^{\mathrm{I}} + 2M_{12}, \\
\Phi_{12}^I &= M_{22} + M_{12}^\top,\ \Phi_{14}^I = P_{11}A^I + P_{12}A^R, \\
\Phi_{15}^I &= P_{11}B^I + P_{12}B^R,\ \Phi_{17}^I = M_{32} + M_{12}^\top, \\
\Phi_{21}^I &= -M_{22}^\top - M_{12},\ \Phi_{27}^I = -M_{32} + M_{22}^\top, \\
\Phi_{22}^I &= (d-1)P_{22} + \rho Q_{22} - 2M_{22}, \\
\Phi_{41}^I &= -(A^R)^\top(P_{12})^\top - (A^I)^\top(P_{11})^\top, \\
\Phi_{51}^I &= (B^R)^\top P_{12} - (B^I)^\top P_{11}, \\
\Phi_{44}^I &= e^{\varepsilon\tau}(P_{32} + P_{52}) - G, \\
\Phi_{55}^I &= (d-1)P_{32},\ \Phi_{71}^I = -M_{32}^\top - M_{12}, \\
\Phi_{72}^I &= M_{32}^\top - M_{22},\ \Phi_{77}^I = -2M_{32},
\end{aligned}$$

then the CVSNNs (1) are said to be a globally exponentially dissipative system and the set $S_1 = \{\mathrm{Re}(z) : E\|\mathrm{Re}(z)\| = \frac{1}{\lambda_{\min}(N^{\mathrm{R}})}\|u^RP_{11} - u^IP_{12}\|\}$, $S_2 = \{\mathrm{Im}(z) : E\|\mathrm{Im}(z)\| = \frac{1}{\lambda_{\min}(N^{\mathrm{I}})}\|u^RP_{12} + u^IP_{11}\|\}$ are the positive invariant and globally attractive sets.

Proof: Using Lemma 3, if $\Phi < 0$ in Theorem 1 is equivalent to $\begin{bmatrix} \Phi^R & -\Phi^I \\ \Phi^I & \Phi^R \end{bmatrix} < 0$ in (21). Moreover, from (19) and (21), we can write

$$\begin{aligned} EdV(t) &= ELV(t)dt \\ &\leq Ee^{\varepsilon t}\eta^*\Phi\eta + 2Ee^{\varepsilon t}[-z_t^*(N^{\mathrm{R}} + iN^{\mathrm{I}})z_t \\ &\quad + z_t^*(P_{11} + iP_{12})(u_t^R + iu_t^I)] \end{aligned}$$

$$\begin{aligned}&\le 2e^{\varepsilon t}\|Ez_t\|[-\lambda_{\min}(N^{\mathrm{R}})\|Ez_t\|+(P_{11}u_t^R-P_{12}u_t^I)]\\&\quad+i2e^{\varepsilon t}\|Ez_t\|[-\lambda_{\min}(N^{\mathrm{I}})\|Ez_t\|+(P_{11}u_t^I+P_{12}u_t^R)]\\&\le 2e^{\varepsilon t}\|Ez_t\|\{[-\lambda_{\min}(N^{\mathrm{R}})\|E\mathrm{Re}(z_t)\|+(P_{11}u_t^R\\&\quad-P_{12}u_t^I)]+i[-\lambda_{\min}(N^{\mathrm{I}})\|E\mathrm{Im}(z_t)\|\\&\quad+(P_{11}u_t^I+P_{12}u_t^R)]\}\le 0.\end{aligned}\tag{23}$$

Then, we can say that the CVSNNs (1) are said to be globally exponentially dissipative and $S_1=\{\mathrm{Re}(z):E\|\mathrm{Re}(z)\|=\frac{1}{\lambda_{\min}(N^{\mathrm{R}})}\|u^RP_{11}-u^IP_{12}\|\}$, $S_2=\{\mathrm{Im}(z):E\|\mathrm{Im}(z)\|=\frac{1}{\lambda_{\min}(N^{\mathrm{I}})}\|u^RP_{12}+u^IP_{11}\|\}$ are positive invariant and globally attractive sets. This completes the proof.

In the following Theorem 2, we will give a new result for stochastic neural networks system (1) from another point of view.

Theorem 2. Suppose that Assumptions 1, 2 and 3 hold. If there exist positive scalars $\alpha,\rho_1,\rho_2,\rho_3$, Hermitian matrices $\bar P_j>0 (j=1,\ldots,7)$, and diagonal matrices $G>0, H>0$, such that the following LMIs hold:

$$\Psi<0,\ \Lambda<0,\ \Delta<0,\tag{24}$$
$$\bar P_1\le\rho_1I,\ \bar P_6\le\rho_2I,\ \bar P_7\le\rho_3I,\tag{25}$$

where

$$\Psi=\begin{bmatrix}\Psi_{11}&\bar P_1A&\bar P_1B&\Psi_{17}\\ *&\Psi_{44}&0&0\\ *&0&\Psi_{55}&0\\ *&0&0&\Psi_{77}\end{bmatrix},$$
$$\Lambda=\rho_1\bar Q_2+\rho_2\tau^2\bar Q_2+\rho_3\frac{1}{4}\tau^2\bar Q_2-(1-d)\bar P_4+H\hat L,$$
$$\Delta=\tau^2\bar P_7-\bar P_6$$

with

$$\begin{aligned}\Psi_{11}&=-(D\bar P_1+\bar P_1D)+\rho_1\bar Q_1+\rho_2\tau^2\bar Q_1\\&\quad+\rho_3\frac{1}{4}\tau^2\bar Q_1+\bar P_4+G\hat L-\mathcal{Q},\\ \Psi_{14}&=\bar P_1A,\ \Psi_{15}=\bar P_1B,\ \Psi_{17}=\bar P_1-\mathcal{S},\\ \Psi_{44}&=\bar P_3+\bar P_5-G,\ \Psi_{55}=(d-1)\bar P_5-H,\\ \Psi_{77}&=\alpha I-\mathcal{R},\end{aligned}$$

then the CVSNNs (1) are strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative.

Proof: Consider the following Lyapunov function as

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t),\tag{26}$$

where

$$\begin{aligned}V_1(t)&=z^*(t)\bar P_1z(t),\\ V_2(t)&=\int_{t-\tau(t)}^t z^*(s)\bar P_2z(s)ds\\&\quad+\int_{t-\tau(t)}^t f^*(z(s))\bar P_3f(z(s))ds,\\ V_3(t)&=\int_{t-\tau}^t z^*(s)\bar P_4z(s)ds\\&\quad+\int_{t-\tau}^t f^*(z(s))\bar P_5f(z(s))ds,\\ V_4&=\tau\int_{-\tau}^0\int_{t+\theta}^t dz^*(s)\bar P_6dz(s)d\theta\\ V_5&=\frac{1}{2}\tau^2\int_{-\tau}^0\int_\theta^0\int_{t+\lambda}^t dz^*(s)\bar P_6dz(s)d\lambda d\theta.\end{aligned}$$

Using Itô formula to (26), one can get that $dV(t)=LV(t)dt+e^{\varepsilon t}(z_t^*\bar P_1\sigma(z_t,z_t^\tau)+\sigma^*(z_t,z_t^\tau)\bar P_1z_t)dW_t$, where

$$\begin{aligned}&LV(t)\\&=\mathrm{trace}[\sigma^*(z_t,z_{t-\tau(t)})\bar P_1\sigma(z_t,z_{t-\tau(t)})]\\&\quad+[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]^*\bar P_1z_t\\&\quad+z_t^*\bar P_1[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]+z_t^*\bar P_2z_t\\&\quad+f^*(z_t)\bar P_3f(z_t)-(1-\dot\tau(t))z_{t-\tau(t)}^*\bar P_2z_{t-\tau(t)}\\&\quad-(1-\dot\tau(t))f^*(z_{t-\tau(t)})\bar P_3f(z_{t-\tau(t)})+z_t^*\bar P_4z_t\\&\quad-z_{t-\tau}^*\bar P_4z_{t-\tau}+f^*(z_t)\bar P_5f(z_t)-f^*(z_{t-\tau})\\&\quad\times\bar P_5f(z_{t-\tau})+\tau^2\mathrm{trace}[\sigma^*(z_t,z_{t-\tau(t)})\bar P_6\sigma(z_t,z_{t-\tau(t)})]\\&\quad-\tau(\int_{t-\tau}^t\mathrm{trace}[\sigma^*(z_s,z_{s-\tau(s)})\bar P_6\sigma(z_s,z_{s-\tau(s)})ds)]\\&\quad+\frac{1}{4}\tau^4\mathrm{trace}[\sigma^*(z_t,z_{t-\tau(t)})\bar P_7\sigma(z_t,z_{t-\tau(t)})]\\&\quad-\frac{1}{2}\tau^2\int_{-\tau}^0\int_\theta^0 dz^*(t+\lambda)\bar P_7dz(t+\lambda)d\lambda d\theta.\end{aligned}\tag{27}$$

According to (3) and (25),

$$\begin{aligned}&LV(t)\\&\le z_t^*\rho_1\bar Q_1z_t+z_{t-\tau(t)}^*\rho_1\bar Q_2z_{t-\tau(t)}\\&\quad+[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]^*\bar P_1z_t\\&\quad+z_t^*\bar P_1[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]\\&\quad+z_t^*\bar P_2z_t+f^*(z_t)\bar P_3f(z_t)-z_{t-\tau(t)}^*\bar P_4z_{t-\tau(t)}+z_t^*\bar P_4z_t\\&\quad-f^*(z_{t-\tau(t)})\bar P_5f(z_{t-\tau(t)})-(1-d)z_{t-\tau(t)}^*\bar P_2z_{t-\tau(t)}\\&\quad+f^*(z_t)\bar P_5f(z_t)-(1-d)f^*(z_{t-\tau(t)})\bar P_3f(z_{t-\tau(t)})\\&\quad-\tau(\int_{t-\tau}^t\mathrm{trace}[\sigma^*(z_s,z_{s-\tau(s)})\bar P_6\sigma(z_s,z_{s-\tau(s)}))]ds\\&\quad+\tau^2z_t^*\rho_2\bar Q_1z_t+\tau^2z_{t-\tau(t)}^*\rho_2\bar Q_2z_{t-\tau(t)}+\frac{1}{4}\tau^2[z_t^*\rho_3\bar Q_1z_t\\&\quad+\tau^2z_{t-\tau(t)}^*\rho_3\bar Q_2z_{t-\tau(t)}]+\tau^2[\int_{t-\tau}^t\sigma(z_s,z_{s-\tau(s)})ds]^*\\&\quad\times\bar P_7[\int_{t-\tau}^t\sigma(z_s,z_{s-\tau(s)})ds].\end{aligned}\tag{28}$$

Substituting (6) into (28), and using Lemma 1, one has

$$\begin{aligned}&LV(t)+\alpha u_t^*u_t-[z_t^*\mathcal{Q}z_t+2z_t^*\mathcal{S}u_t+u_t^*\mathcal{Q}u_t]\\&\le z_t^*\rho_1\bar Q_1z_t+z_{t-\tau(t)}^*\rho_1\bar Q_2z_{t-\tau(t)}\\&\quad+[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]^*\bar P_1z_t\\&\quad+z_t^*\bar P_1[-Dz_t+Af(z_t)+Bf(z_{t-\tau(t)})+u_t]+z_t^*\bar P_2z_t\\&\quad+f^*(z_t)\bar P_3f(z_t)-z_{t-\tau(t)}^*\bar P_4z_{t-\tau(t)}-f^*(z_{t-\tau(t)})\bar P_5\\&\quad\times f(z_{t-\tau(t)})+z_t^*\bar P_4z_t-(1-d)z_{t-\tau(t)}^*\bar P_2z_{t-\tau(t)}\\&\quad+f^*(z_t)\bar P_5f(z_t)-(1-d)f^*(z_{t-\tau(t)})\bar P_3f(z_{t-\tau(t)})\\&\quad+\tau^2z_t^*\rho_2\bar Q_1z_t+\tau^2z_{t-\tau(t)}^*\rho_2\bar Q_2z_{t-\tau(t)}\end{aligned}$$

$$-(\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)})ds)^*\bar{P}_6(\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)}))ds)$$
$$+\frac{1}{4}\tau^2[z_t^*\rho_3\bar{Q}_1z_t+\tau^2z_{t-\tau(t)}^*\rho_3\bar{Q}_2z_{t-\tau(t)}]$$
$$+\tau^2[\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)})ds]^*\bar{P}_7[\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)})ds]$$
$$+\alpha u_t^*u_t-[z_t^*\mathcal{Q}z_t+2z_t^*\mathcal{S}u_t+u_t^*\mathcal{Q}u_t]. \quad (29)$$

On the basis of (14) and (15), we can get

$$LV(t)+\alpha u_t^*u_t-[z_t^*\mathcal{Q}z_t+2z_t^*\mathcal{S}u_t+u_t^*\mathcal{Q}u_t]$$
$$\le z_t^*(-D\bar{P}_1-\bar{P}_1D+\rho_1\bar{Q}_1+\bar{P}_2+\bar{P}_4+\rho_2\tau^2\bar{Q}_1$$
$$+\rho_3\frac{1}{4}\tau^2\bar{Q}_1+G\hat{L})z_t+z_{t-\tau(t)}^*(\rho_1\bar{Q}_2+\rho_2\tau^2\bar{Q}_2$$
$$+\rho_3\frac{1}{4}\tau^2\bar{Q}_2)z_{t-\tau(t)}^*+z_{t-\tau}^*(-\bar{P}_4)z_{t-\tau}+f^*(z_t)$$
$$\times(\bar{P}_3+\bar{P}_5-G)f(z_t)+f^*(z_{t-\tau(t)})[(d-1)\bar{P}_5-H]$$
$$\times f(z_{t-\tau(t)})+f^*(z_{t-\tau})(-\bar{P}_5)f(z_{t-\tau})+\alpha u_t^*u_t$$
$$+[\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)})ds]^*(\tau^2\bar{P}_7-\bar{P}_6)$$
$$\times\int_{t-\tau}^{t}\sigma(z_s,z_{s-\tau(s)})ds+[f^*(z_t)A^*\bar{P}_1z_t+z_t^*\bar{P}_1Af(z_t)$$
$$+f^*(z_t)B^*\bar{P}_1z_t+z_t^*\bar{P}_1Bf(z_t)+u_t^*\bar{P}_1z_t+z_t^*\bar{P}_1u_t]$$
$$-[z_t^*\mathcal{Q}z_t+u_t^*\mathcal{R}u_t+2z_t^*\mathcal{S}u_t]. \quad (30)$$

Let $\eta^*=\Big\{z_t^*,z_{t-\tau(t)}^*,z_{t-\tau}^*,f^*(z_t),f^*(z_{t-\tau(t)}), f^*(z_{t-\tau}),u_t^*,[\int_{t-\tau}^{t}\sigma(z_s,z_{t-\tau(s)})]^*\Big\}$,

then, (30) can be expressed as

$$LV(t)+\alpha u_t^*u_t-[z_t^*\mathcal{Q}z_t+2z_t^*\mathcal{S}u_t+u_t^*\mathcal{Q}u_t] \le \eta^*\hat{\Psi}\eta, \quad (31)$$

where

$$\hat{\Psi}=\begin{bmatrix}\Psi_{11}&0&0&\bar{P}_1A&\bar{P}_1B&0&\Psi_{17}&0\\0&\Lambda&0&0&0&0&0&0\\0&0&\Psi_{33}&0&0&0&0&0\\ *&0&0&\Psi_{44}&0&0&0&0\\ *&0&0&0&\Psi_{55}&0&0&0\\0&0&0&0&0&\Psi_{66}&0&0\\ *&0&0&0&0&0&\Psi_{77}&0\\0&0&0&0&0&0&0&\Delta\end{bmatrix},$$

$\Psi_{33}=-\bar{P}_4,\ \Psi_{66}=-\bar{P}_3.$

Because of the equivalence of $\hat{\Psi}<0$ and (24), from the inequality (31), we have

$$LV(t)+\alpha u_t^*u_t\le z_t^*\mathcal{Q}z_t+2z_t^*\mathcal{S}u_t+u_t^*\mathcal{Q}u_t. \quad (32)$$

Both integrating (32) from 0 to T, and then taking the expectation, under the initial conditions, we can obtain

$$E\{G(u,z,T)\}=E\{\langle z,\mathcal{Q}z\rangle\}+2E\{\langle z,\mathcal{S}u\rangle\} +E\{\langle u,\mathcal{R}u\rangle\},\quad T\ge 0. \quad (33)$$

Therefore, based on Definition 2, the CVSNNs (1) are strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative system. The proof is completed.

Remark 4: In practice, the influence of stochastic interference on the stability of the system has been paid more and more attention. In real-valued models, some related results have arisen, such as the global dissipativity, exponential dissipativity and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity about real-valued stochastic discrete-time neural networks [19], [21], [23], [31]. But, in complex domain, there has been few information on this issue. Further, the result for the dissipativity of complex-valued neural networks with stochastic interference has been not reported. In Theorems 1 and 2, we study exponential dissipativity and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity and derive the corresponding sufficient conditions.

Remark 5: Sufficient conditions that guarantee $(\mathcal{Q},\mathcal{S},\mathcal{R})$ dissipativity of the CVSNNs (1) are derived in terms of complex-valued LMIs. Similar to Theorem 1, based on Lemma 3, the sufficient conditions for strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity of the CVSNNs (1) can be verified by the following Corollary 2 in terms of real-valued LMIs. These LMIs can be solved with the help of standard available numerical.

Corollary 2: Suppose that Assumptions 1, 2, 3 hold, and $\mathcal{Q}=\mathcal{Q}_1+i\mathcal{Q}_2$, $\mathcal{S}=\mathcal{S}_1+i\mathcal{S}_2$, $\mathcal{R}=\mathcal{R}_1+i\mathcal{R}_2$. If there exist positive scalars $\alpha,\rho_1,\rho_2,\rho_3$, Hermitian matrices $\bar{P}_{j1}+i\bar{P}_{j2}>0(j=1,\ldots,7)$, and diagonal matrices $G>0$, $H>0$, such that the following LMIs hold:

$$\begin{bmatrix}o^R&-o^I\\o^I&o^R\end{bmatrix}<0, \quad (34)$$
$$\begin{bmatrix}\bar{P}_{11}-\rho_1I&-\bar{P}_{12}\\\bar{P}_{12}&\bar{P}_{11}-\rho I\end{bmatrix}<0, \quad (35)$$
$$\begin{bmatrix}\bar{P}_{61}-\rho_2I&-\bar{P}_{62}\\\bar{P}_{62}&\bar{P}_{61}-\rho I\end{bmatrix}<0, \quad (36)$$
$$\begin{bmatrix}\bar{P}_{71}-\rho_3I&-\bar{P}_{72}\\\bar{P}_{72}&\bar{P}_{71}-\rho I\end{bmatrix}<0, \quad (37)$$

for $o=\Psi,\Lambda,\Delta$, where

$$\Psi^R=\begin{bmatrix}\Psi_{11}^R&\Psi_{14}^R&\Psi_{15}^R&\Psi_{17}^R\\ *&\Psi_{44}^R&0&0\\ *&0&\Psi_{55}^R&0\\ *&0&0&\Psi_{77}^R\end{bmatrix},$$
$$\Psi^I=\begin{bmatrix}\Psi_{11}^I&\Psi_{14}^I&\Psi_{15}^I&\Psi_{17}^I\\\Psi_{41}^I&\Psi_{44}^I&0&0\\\Psi_{51}^I&0&\Psi_{55}^I&0\\\Psi_{71}^I&0&0&\Psi_{77}^I\end{bmatrix},$$

with

$$\Psi_{11}^R=-(D^*\bar{P}_{11}+\bar{P}_{11}D)+(\bar{P}_{21}+\bar{P}_{41})+\rho_1\bar{Q}_{11}+\rho_2\tau^2\bar{Q}_{11}+\frac{1}{4}\rho_3\tau^2\bar{Q}_{11}+G\hat{L}-\mathcal{Q}_1,$$
$$\Psi_{44}^R=\bar{P}_{31}+\bar{P}_{51}-G,\ \Psi_{55}^R=(d-1)\bar{P}_{51}-H,$$
$$\Psi_{77}^R=\alpha I-\mathcal{R}_1,\ \Psi_{14}^R=\bar{P}_{11}A^R-\bar{P}_{12}A^I,$$
$$\Psi_{15}^R=\bar{P}_{11}B^R-\bar{P}_{12}B^I,\ \Psi_{17}^R=\bar{P}_{11}-\mathcal{S}_1,$$
$$\Psi_{11}^I=-(D^*\bar{P}_{12}+\bar{P}_{12}D)+(\bar{P}_{22}+\bar{P}_{42})+\rho_1\bar{Q}_{12}+\rho_2\tau^2\bar{Q}_{12}+\frac{1}{4}\rho_3\tau^2\bar{Q}_{12}-\mathcal{Q}_2,$$

$$\Psi_{44}^I = \bar{P}_{32} + \bar{P}_{52},\ \Psi_{55}^I = (d-1)\bar{P}_{52},\ \Psi_{77}^I = -\mathcal{R}_2,$$
$$\Psi_{14}^I = \bar{P}_{11}A^I + \bar{P}_{12}A^R,\ \Psi_{15}^I = \bar{P}_{11}B^I + \bar{P}_{12}B^R,$$
$$\Psi_{17}^I = \bar{P}_{12} - \mathcal{S}_2,\ \Psi_{41}^I = -A^{RT}\bar{P}_{12}^T - A^{IT}\bar{P}_{11}^T,$$
$$\Psi_{71}^I = -\bar{P}_{12}^T + \mathcal{S}_2^T,\ \Psi_{51}^I = -B^{RT}\bar{P}_{12}^T - B^{IT}\bar{P}_{11}^T,$$

and

$$\Lambda^R = \rho_1\bar{Q}_{21} + \rho_2\tau^2\bar{Q}_{21} + \frac{1}{4}\rho_3\tau^2\bar{Q}_{21} + (d-1)\bar{P}_{41} + H\hat{L},$$
$$\Lambda^I = \rho_1\bar{Q}_{22} + \rho_2\tau^2\bar{Q}_{22} + \frac{1}{4}\rho_3\tau^2\bar{Q}_{22} + (d-1)\bar{P}_{42},$$
$$\Delta^R = \tau^2\bar{P}_{71} - \bar{P}_{61},\ \Delta^I = \tau^2\bar{P}_{72} - \bar{P}_{62},$$

then the CVSNNs (1) are strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative.

Remark 6: Indeed, in the existing literatures, the stochastic interference on stochastic neural network always is always treated as Brownian motion for propose of obtaining the appropriate conclusion. We would like to point out that it is possible to extend our main results to more neural networks with more general stochastic process, such as Markov jump process and Levy jump process, and the corresponding work will be done in the near future.

IV. NUMERICAL EXAMPLES

In this section, we will give two examples to show the effectiveness of our results.

Example 1: Consider a two-dimensional CVSNNs as follows:

$$dz(t) = [-Dz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + u(t)]dt + \sigma(t, z(t), z(t-\tau(t)))dw(t), \quad (38)$$

with the following parameter values

$$D = \begin{bmatrix} 5.0 & 0 \\ 0 & 5.0 \end{bmatrix},\quad L = \begin{bmatrix} \frac{1}{2.45} & 0 \\ 0 & \frac{1}{2.45} \end{bmatrix},$$
$$A = \begin{bmatrix} -0.3-0.2i & 0.8+0.5i \\ -0.3-0.15i & 0.7+0.8i \end{bmatrix},$$
$$B = \begin{bmatrix} 0.9+0.5i & -0.3+0.9i \\ 0.8-0.3i & -0.3+0.9i \end{bmatrix}.$$

For this system, the activation functions are the form as $f(z(\cdot)) = \tanh(z(\cdot))$, the diffusion coefficient function $\sigma(z(t), z(t-\tau(t))) = 0.01 * [0.2z(t) + 0.2z(t-\tau(t))]$ and $\tau(t) = 0.1 + 0.1 * \sin t$, $\tau = 0.2, d = 0.1, \varepsilon = 0.0262$. According to Corollary 1, it can be verified that this system is dissipative. We take $u = [4+2i, 4+2i]^*$ and the initial conditions $z_1 = 0.5 - 0.25i, z_2 = 0.3 - 0.2i$.

Then using standard available numerical packages to solve the LMIs (21) and (22), we can obtain the following feasible solution such as

$$P1 = \begin{bmatrix} 11.3876 & -10.3522-3.1006i \\ -10.3522+3.1006i & 13.2917i \end{bmatrix},$$
$$P2 = \begin{bmatrix} 22.2466 & -12.3579+1.8329i \\ -12.3579-1.8329i & 26.8132 \end{bmatrix},$$

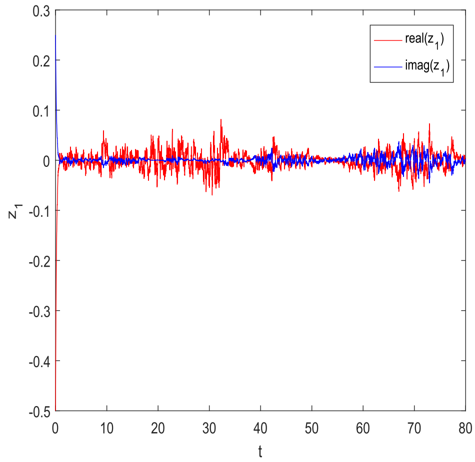

FIGURE 1. The trajectories of real and imaginary parts of variables z_1 for the stochastic neural network system without external energy inputs in example 1.

$$P3 = \begin{bmatrix} 0.5690 & 0.0878-0.005i \\ 0.0878+0.005ii & 0.1885 \end{bmatrix},$$
$$P4 = \begin{bmatrix} 20.0304 & -13.1818+1.1003i \\ -13.1818-1.1003i & 23.611 \end{bmatrix},$$
$$P5 = \begin{bmatrix} 0.5966 & 0.0816+0.003i \\ 0.0816-0.003i & 0.1923 \end{bmatrix},$$
$$N = \begin{bmatrix} 58.1933 & -60.3428+5.338i \\ -60.3428-5.338i & 75.0341 \end{bmatrix},$$
$$M1 = \begin{bmatrix} 126+0.1138i & 469.18+7.2188i \\ -4716.1-7.2253i & 15.5-0.0721i \end{bmatrix},$$
$$M2 = \begin{bmatrix} -20.8-0.0735i & 4724.4-1.8932i \\ -4683.6-1.7335i & -24.8+0.2743i \end{bmatrix},$$
$$M3 = \begin{bmatrix} 16.1-0.3929i & 7328.8-2.6072 \\ -7357.2+2.4465i & 19.2-0.3441i \end{bmatrix},$$
$$G = \begin{bmatrix} 1.6752 & 0 \\ 0 & 1.6752 \end{bmatrix},\quad \rho = 55.6489$$
$$H = \begin{bmatrix} 1.7274 & 0 \\ 0 & 1.7274 \end{bmatrix},\quad J = \begin{bmatrix} 0.7477 & 0 \\ 0 & 0.7477 \end{bmatrix}.$$

Therefore, by Corollary 1, we know that the CVSNNs (38) with the above given parameters is globally exponentially dissipative. And it is easy to compute that the positive invariant and global attractive set are $S = \{z : E\|z_t\| \leq 0.3368\}$. The Figs.3 and 4 depict the state response of the CVSNNs (38).

Example 2: Consider a two-dimensional CVSNNs as follows:

$$dz(t) = [-Dz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + u(t)]dt + \sigma(t, z(t), z(t-\tau(t)))dw(t), \quad (39)$$

with the following parameter values

$$A = \begin{bmatrix} 2.5-2.5i & 2.5-3.8i \\ -4-2.7i & 3.8+2.5i \end{bmatrix},$$

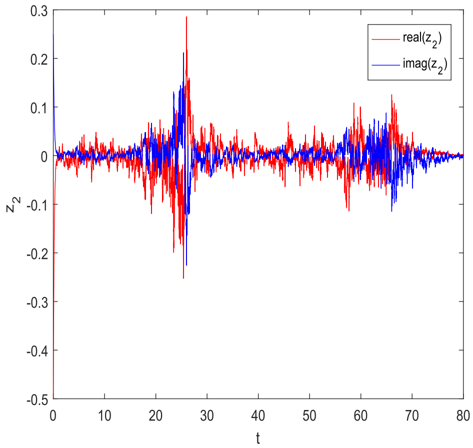

FIGURE 2. **The trajectories of real and imaginary parts of variables z_2 for the stochastic neural network system without external energy inputs in example 1.**

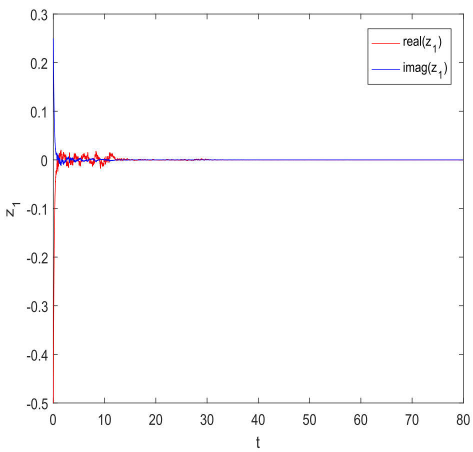

FIGURE 3. **The trajectories of real and imaginary parts of variables z_1 for the stochastic neural network system under external energy inputs with random initial conditions in example 1.**

$$B = \begin{bmatrix} -3.8+4.5i & -1.5+2.8i \\ -0.4+3.8i & -2.8-1.1i \end{bmatrix},$$
$$D = \begin{bmatrix} 5.5 & 0 \\ 0 & 5.5 \end{bmatrix}, \quad u = \begin{bmatrix} -4+4i \\ -4+2i \end{bmatrix},$$

and $f(z(\cdot))$, $\tau(t)$, $\sigma(z(t), z(t-\tau(t)))$ are chosen as the same in Example 1. Moreover, choosing

$$\mathcal{Q} = \begin{bmatrix} -3.8+4.5i & -1.5+2.8i \\ -0.4+3.8i & -2.8-1.1i \end{bmatrix},$$
$$\mathcal{S} = \begin{bmatrix} 0.3+0.5i & -0.6-0.2i \\ 0.4-0.6i & 0.5+0.3i \end{bmatrix},$$
$$\mathcal{R} = \begin{bmatrix} 4.5 & -0.5-i \\ -0.5+i & 2.5 \end{bmatrix}, u = \begin{bmatrix} 1-i \\ 1-i \end{bmatrix},$$
$$Q1 = \begin{bmatrix} 4.2 & 2.3 \\ 2.3 & 2.0 \end{bmatrix}, Q1 = \begin{bmatrix} 4.7 & 2.6 \\ 2.6 & 2.2 \end{bmatrix},$$

Then using standard available numerical packages to solve the LMIs (34)-(37), we can obtain the following feasible

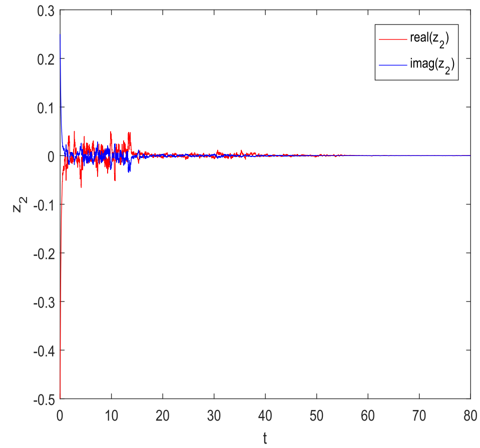

FIGURE 4. **The trajectories of real and imaginary parts of variables z_2 for the stochastic neural network system under external energy inputs with random initial conditions in example 1.**

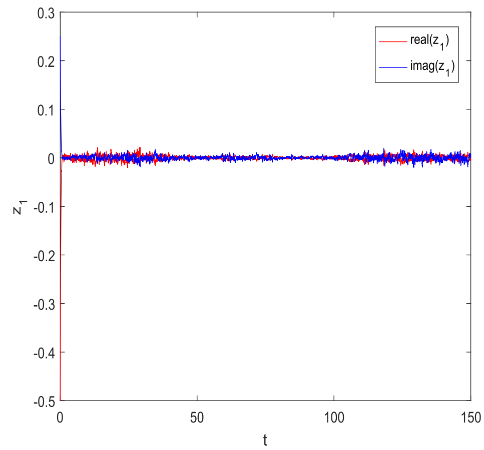

FIGURE 5. **The trajectories of real and imaginary parts of variables z_1 for the stochastic neural network system without external energy inputs in example 2.**

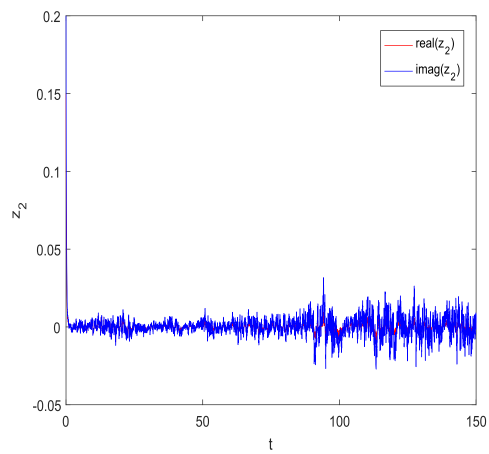

FIGURE 6. **The trajectories of real and imaginary parts of variables z_2 for the stochastic neural network system without external energy inputs in example 2.**

solution such as

$$\bar{P}_1 = 10^{-4} * \begin{bmatrix} 29 & -5-1.288i \\ -5+1.288i & 23 \end{bmatrix},$$

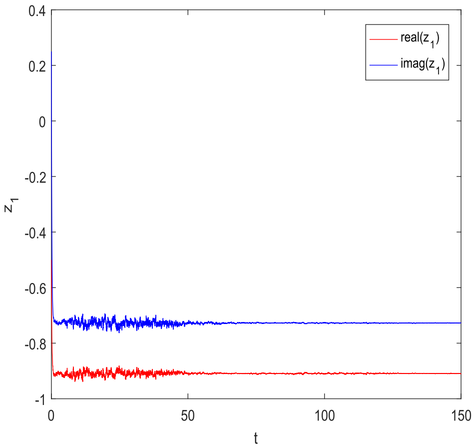

FIGURE 7. The trajectories of real and imaginary parts of variables z_1 for the stochastic neural network system under external energy inputs with random initial conditions in example 2.

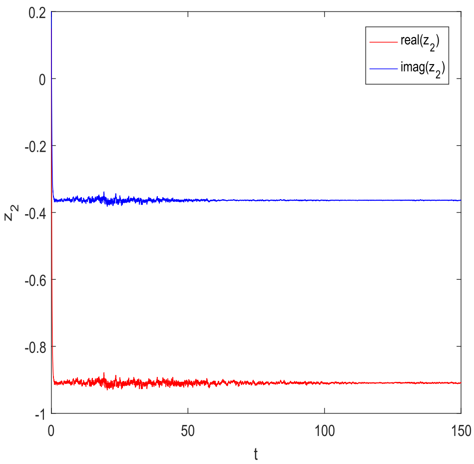

FIGURE 8. The trajectories of real and imaginary parts of variables z_2 for the stochastic neural network system under external energy inputs with random initial conditions in example 2.

$$\bar{P}_2 = \begin{bmatrix} 0.2727 & 0.1617 + 0.0299i \\ 0.1617 - 0.0299i & 0.1241 \end{bmatrix},$$

$$\bar{P}_3 = \begin{bmatrix} 0.0722 & 0.107 \\ 0.107 & 0.0664 \end{bmatrix},$$

$$\bar{P}_4 = \begin{bmatrix} 0.4959 & 0.2810 + 0.0724i \\ 0.2810 - 0.0724i & 0.2352 \end{bmatrix},$$

$$\bar{P}_5 = \begin{bmatrix} 0.0566 & 0.007 - 10^{-5} * 6.07i \\ 0.007 + 10^{-5} * 6.07i & 0.0603 \end{bmatrix},$$

$$\bar{P}_6 = \begin{bmatrix} 0.0437 & 0 \\ 0 & 0.0437 \end{bmatrix}, \bar{P}_7 = \begin{bmatrix} 0.0163 & 0 \\ 0 & 0.0163 \end{bmatrix},$$

$$G = \begin{bmatrix} 0.1959 & 0 \\ 0 & 0.1959 \end{bmatrix}, H = \begin{bmatrix} 0.0368 & 0 \\ 0 & 0.0368 \end{bmatrix},$$

and $\rho_1 = 0.0047$, $\rho_2 = 0.0607$, $\rho_3 = 0.05554$, $\alpha = 0.0084$. Therefore, the stochastic neural network system (39) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative, and the Figs.7 and 8 depict the state response of the CVSNNs (39) with the initial conditions $z_1 = -0.5 + 0.15i$, $z_2 = 0.3 - 0.2i$.

Remark 7. The different choices of the diffusion coefficient function ratio can make the stability of dissipative system very different. Any change in external stochastic interference can make the dissipative system very unstable. Therefore, we choose 0.01 as the diffusion coefficient function ratio in contrast to the big ones.

V. CONCLUSION

In this paper, the problem of exponential dissipativity and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity for complex-value stochastic neural networks with time-varying delays have been investigated. By using the inequality techniques and stochastic analysis techniques, some sufficient conditions for exponential dissipativity and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity of CVSNNs have been derived in terms of LMIs. Moreover, the global attractive sets are obtained which are positive invariant sets. Complex-valued LMIs can be checked numerically by using the effective LMI toolbox in MATLAB. Two numerical examples were provided to illustrate the effectiveness of the proposed results.

REFERENCES

[1] M. M. Gupta, L. Jin, and N. Homma, *Static and Dynamic Neural Networks: From Fundamentals to Advanced Theory*. New York, NY, USA: Wiley, 2003.
[2] Y. Fu and T. Chai, "Nonlinear adaptive decoupling control based on neural networks and multiple models," *Int. J. Innov. Comput. Inf. Control*, vol. 8, no. 3, pp. 1867–1878, Mar. 2012.
[3] Y. He, Q.-G. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with time-varying delay," *Automatica*, vol. 43, no. 2, pp. 371–376, Feb. 2007.
[4] P. G. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, Jan. 2011.
[5] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: Application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.
[6] Z. Zhang, C. Lin, and B. Chen, "New stability criteria for linear time-delay systems using complete LKF method," *Int. J. Syst. Sci.*, vol. 46, no. 2, pp. 377–384, Jan. 2015.
[7] J. Chen, C. Lin, B. Chen, and Q.-G. Wang, "Fuzzy-model-based admissibility analysis and output feedback control for nonlinear discrete-time systems with time-varying delay," *Inf. Sci.*, vols. 412–413, pp. 116–131, Oct. 2017.
[8] J. Chen, C. Lin, B. Chen, and Q.-G. Wang, "Improved stability criterion and output feedback control for discrete time-delay systems," *Appl. Math. Model.*, vol. 52, pp. 82–93, Dec. 2017.
[9] C. Rajivganthi, F. A. Rihan, and S. Lakshmanan, "Dissipativity analysis of complex-valued BAM neural networks with time delay," *Neural Comput. Appl.*, vol. 31, no. 1, pp. 127–137, Jan. 2019.
[10] X. Li, R. Rakkiyappan, and G. Velmurugan, "Dissipativity analysis of memristor-based complex-valued neural networks with time-varying delays," *Inf. Sci.*, vol. 294, pp. 645–665, Feb. 2015.
[11] R. Rakkiyappan, G. Velmurugan, X. Li, and D. O'Regan, "Global dissipativity of memristor-based complex-valued neural networks with time-varying delays," *Neural Comput. Appl.*, vol. 27, pp. 629–649, Apr. 2016.
[12] Y. Zou, R. V. Donner, N. Marwan, J. F. Donges, and J. Kurths, "Complex network approaches to nonlinear time series analysis," *Phys. Rep.*, vol. 787, pp. 1–97, Jan. 2019.
[13] Y. Zhu and W. X. Zheng, "Multiple Lyapunov functions analysis approach for discrete-time switched piecewise-affine systems under dwell-time constraints," *IEEE Trans. Autom. Control*, to be published, doi: 10.1109/TAC.2019.2938302.
[14] J. Chen, C. Lin, B. Chen, and Q.-G. Wang, "Mixed H_∞ and passive control for singular systems with time delay via static output feedback," *Appl. Math. Comput.*, vol. 293, pp. 244–253, Jan. 2017.

[15] S. Ramasamy and G. Nagamani, "Dissipativity and passivity analysis for discrete-time complex-valued neural networks with leakage delay and probabilistic time-varying delays," *Int. J. Adapt. Control Signal Process.*, vol. 31, no. 6, pp. 876–902, Jun. 2017.
[16] G. Velmurugan, R. Rakkiyappan, V. Vembarasan, J. Cao, and A. Alsaedi, "Dissipativity and stability analysis of fractional-order complex-valued neural networks with time delay," *Neural Netw.*, vol. 86, pp. 42–53, Feb. 2017.
[17] M. D. S. Aliyu, "Dissipativity and stability of nonlinear jump systems," in *Proc. ACC*, San Diego, CA, USA, vol. 2, Jun. 1999, pp. 795–799.
[18] G. Wang, J. Cao, and L. Wang, "Global dissipativity of stochastic neural networks with time delay," *J. Franklin Inst.*, vol. 346, pp. 794–807, Oct. 2009.
[19] Q. Song, "Stochastic dissipativity analysis on discrete-time neural networks with time-varying delays," *Neurocomputing*, vol. 74, no. 5, pp. 838–845, Feb. 2011.
[20] D. J. Hill and P. J. Moylan, "Dissipative dynamical systems: Basic input-output and state properties," *J. Franklin Inst.*, vol. 309, pp. 327–357, May 1980.
[21] Z.-G. Wu, P. Shi, H. Su, and J. Chu, "Dissipativity analysis for discrete-time stochastic neural networks with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 3, pp. 345–355, Mar. 2013.
[22] T. Radhika and G. Nagamani, "Dissipativity analysis of stochastic memristor-based recurrent neural networks with discrete and distributed time-varying delays," *Netw., Comput. Neural Syst.*, vol. 27, no. 4, pp. 237–267, Jul. 2016.
[23] T. Rajpurohit and W. M. Haddad, "Dissipativity theory for nonlinear stochastic dynamical systems," *IEEE Trans. Autom. Control*, vol. 62, no. 4, pp. 1684–1699, Apr. 2017.
[24] Z.-G. Wu, J. H. Park, H. Su, and J. Chu, "Dissipativity analysis of stochastic neural networks with time delays," *Nonlinear Dyn.*, vol. 70, no. 1, pp. 825–839, Oct. 2012.
[25] Z. Zhang, X. Liu, C. Lin, and B. Chen, "Finite-time synchronization for complex-valued recurrent neural networks with time delays," *Complexity*, vol. 2018, Dec. 2018, Art. no. 8456737.
[26] Z.-G. Wu, J. Lam, H. Su, and J. Chu, "Stability and dissipative analysis of static neural networks with time delay," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 2, pp. 199–210, Feb. 2012.
[27] H. Zhang, H. Yan, and Q. Chen, "Stability and dissipative analysis for a class of stochastic system with time-delay," *J. Franklin Inst.*, vol. 347, no. 5, pp. 882–893, Jun. 2010.
[28] J. C. Willems, "Dissipative dynamical systems part I: General theory," *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 321–351, Jan. 1972.
[29] D. Hill and P. Moylan, "The stability of nonlinear dissipative systems," *IEEE Trans. Autom. Control*, vol. AC-21, no. 5, pp. 708–711, Oct. 1976.
[30] X. Chen and Q. Song, "Global stability of complex-valued neural networks with both leakage time delay and discrete time delay on time scales," *Neurocomputing*, vol. 121, no. 9, pp. 254–264, Dec. 2013.
[31] S. Ding, Z. Wang, and H. Zhang, "Dissipativity analysis for stochastic memristive neural networks with time-varying delays: A discrete-time case," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 3, pp. 618–630, Mar. 2018.
[32] J. Cao, K. Yuan, D. W. C. Ho, and J. Lam, "Global point dissipativity of neural networks with mixed time-varying delays," *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 16, no. 1, Apr. 2006, Art. no. 013105.
[33] Z. Zhang, X. Liu, D. Zhou, C. Lin, J. Chen, and H. Wang, "Finite-time stabilizability and instabilizability for complex-valued memristive neural networks with time delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 12, pp. 2371–2382, Dec. 2018.
[34] Z. Zhang, X. Liu, J. Chen, R. Guo, and S. Zhou, "Further stability analysis for delayed complex-valued recurrent neural networks," *Neurocomputing*, vol. 251, pp. 81–89, Aug. 2017.
[35] R. Guo, Z. Zhang, and M. Gao, "State estimation for complex-valued memristive neural networks with time-varying delays," *Adv. Difference Equ.*, Apr. 2018, Art. no. 118.
[36] Z. Zhang, R. Guo, X. Liu, and C. Lin, "Lagrange exponential stability of complex-valued BAM neural networks with time-varying delays," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2018.2840091.
[37] R. Guo, Z. Zhang, X. Liu, and C. Lin, "Existence, uniqueness, and exponential stability analysis for complex-valued memristor-based BAM neural networks with time delays," *Appl. Math. Comput.*, vol. 311, pp. 100–117, Oct. 2017.
[38] Z. Zhang, X. Liu, R. Guo, and C. Lin, "Finite-time stability for delayed complex-valued BAM neural networks," *Neural Process. Lett.*, vol. 48, no. 1, pp. 179–193, Aug. 2018.
[39] R. Guo, Z. Zhang, X. Liu, C. Lin, H. Wang, and J. Chen, "Exponential input-to-state stability for complex-valued memristor-based BAM neural networks with multiple time-varying delays," *Neurocomputing*, vol. 275, pp. 2041–2054, Jan. 2018.
[40] R. Guo, Z. Zhang, C. Lin, Y. Chu, and Y. Li, "Finite time state estimation of complex-valued BAM neutral-type neural networks with time-varying delays," *Int. J. Control Autom. Syst.*, vol. 17, no. 3, pp. 801–809, Mar. 2019.
[41] H. Ma and Y. Jia, "Stability analysis for stochastic differential equations with infinite Markovian switchings," *J. Math. Anal. Appl.*, vol. 435, no. 1, pp. 593–605, Mar. 2016.
[42] G. Li and M. Chen, "Infinite horizon linear quadratic optimal control for stochastic difference time-delay systems," *Adv. Difference Equ.*, Jan. 2015, Art. no. 14.
[43] X. Liu, Y. Li, and W. H. Zhang, "Stochastic linear quadratic optimal control with constraint for discrete-time systems," *Appl. Math. Comput.*, vol. 228, pp. 264–270, Feb. 2014.
[44] Y. Li, W. Zhang, and X. Liu, "Stability Of nonlinear stochastic discrete-time systems," *J. Appl. Math.*, Jul. 2013, Art. no. 356746, doi: 10.1155/2013/356746.
[45] Z. Wang, "A numerical method for delayed fractional-order differential equations," *J. Appl. Math.*, Apr. 2013, Art. no. 256071, doi: 10.1155/2013/256071.
[46] J. Chen, T. Zhang, Z. Zhang, C. Lin, and B. Chen, "Stability and output feedback control for singular Markovian jump delayed systems," *Math. Control Rel. Fields*, vol. 8, pp. 475–490, Jun. 2018.
[47] B.-S. Chen, X. Lin, W. Zhang, and T. Zhou, "On the system entropy and energy dissipativity of stochastic systems and their application in biological systems," *Complexity*, Dec. 2018, Art. no. 1628472, doi: 10.1155/2018/1628472.
[48] X. Wang, Z. Wang, and H. Shen, "Dynamical analysis of a discrete-time SIS epidemic model on complex networks," *Appl. Math. Lett.*, vol. 94, pp. 292–299, Aug. 2019.
[49] S. Jiao, H. Shen, Y. Wei, X. Huang, and Z. Wang, "Further results on dissipativity and stability analysis of Markov jump generalized neural networks with time-varying interval delays," *Appl. Math. Comput.*, vol. 336, pp. 338–350, Nov. 2018.
[50] X. Liang and R.-L. Wang, "Verification and validation of detonation modeling," *Def. Technol.*, vol. 15, no. 3, pp. 398–408, Jun. 2019.
[51] B. Liu and D. J. Hill, "Decomposable dissipativity and related stability for discrete-time switched systems," *IEEE Trans. Autom. Control*, vol. 56, no. 7, pp. 1666–1671, Jul. 2011.
[52] X. Wang, Z. Wang, Q. Song, H. Shen, and X. Huang, "A waiting-time-based event-triggered scheme for stabilization of complex-valued neural networks," *Neural Netw.*, vol. 121, pp. 329–338, Jan. 2020.
[53] J. Jia, X. Huang, Y. Li, J. Cao, and A. Alsaedi, "Global stabilization of fractional-order memristor-based neural networks with time delay," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/TNNLS.2019.2915353.
[54] Y. Zhu, W. X. Zheng, and D. Zhou, "Quasi-synchronization of discrete-time Lur'e-type switched systems with parameter mismatches and relaxed PDT constraints," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2019.2930945.

MEIJUAN LIU received the B.Sc. degree in mathematics from Qingdao University, Qingdao, China, in 1999, and the M.Sc. degree in mathematics from the Shandong University of Science and Technology, Qingdao, in 2005, where she is currently pursuing the Ph.D. degree with the College of Mathematics and Systems Science. Since 2005, she has been with the College of Mathematics and Systems Science, Shandong University of Science and Technology. Her current research interests include systems analysis, stochastic control, and stochastic neural networks.

XIANGRONG WANG received the Ph.D. degree in applied mathematics from Shandong University, Jinan, China, in 1998. He is currently a Professor with the Shandong University of Science and Technology. His research interests include stochastic control theory, forward and backward stochastic differential equations, and mathematics finance.

ZIYE ZHANG received the B.Sc. degree in mathematics from Yantai University, Yantai, China, in 2002, the M.Sc. degree in mathematics from Lanzhou University, Lanzhou, China, in 2005, and the Ph.D. degree from the Institute of Complexity Science, Qingdao University, Qingdao, China, in 2015. She was a Postdoctoral Fellow of Lakehead University, Thunder Bay, ON, Canada, from 2016 to 2017. Since 2005, she has been with the College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao. Her current research interests include systems analysis, fuzzy control, filter design, and neural networks.

ZHEN WANG received the M.S. degree in computational mathematics from the Ocean University of China, Qingdao, China, in 2004, and the Ph.D. degree from the School of Automation, Nanjing University of Science and Technology, Nanjing, China, in 2014. He has been an Associate Professor with the College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, since 2006. His current research interests include neural networks, Boolean networks, and fractional order systems.