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Consensus-Based Formation of Second-Order Multi-Agent Systems via Linear-Transformation-Based Partial Stability Approach

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ABSTRACT The paper studies the time-invariant formation problem of second-order multi-agent systems under a time-invariant directed communication topology. Extensions of the consensus protocol are introduced in the formation control. By choosing appropriate consensus states, the state-linear-transformation approach and the partial stability theory are adopted to analyze the formation problem. Sufficient and necessary algebraic criteria are derived for the formation regulation problem with or without velocity constrains and the formation tracking problem. They are expressed in terms of Hurwitz stability of matrices which are constructed from the gain matrices of formation control protocols.

INDEX TERMS Formation, second-order multi-agent systems, consensus, state-linear-transformation, partial stability.

I. INTRODUCTION

Formation control is one of the fundamental problems of multi-agent systems (MASs) where each agent maintains a desired geometric distance with its neighbors [1], [2]. It is beneficial for improving the capacity of highway transportation, increasing the efficiency of unknown environment exploration, or saving the fuel of flight. Lots of attention has been attracted in its research due to these various applications for satellites [3], [4], vehicles [5], [6], robots [7], [8], and so on.

A MAS is an interesting system whose model has three simple rules: separation, alignment and cohesion [9]. The separation rule avoids crowding neighboring agents. The alignment and cohesion rule gives an agent the ability to respectively align and cohere itself with other nearby agents. Several typical control strategies have been adopted in the formation control of MASs, e.g., leader-follower [10], [11], virtual structure [13], [14], and behavior-based [15] formation control strategies. For the leader-follower strategy, there is one or more agents acting as the group leaders while other agents follow the leaders according to the formation shape. These leaders play important roles but whose lack of feedback from its followers will lead to failure when a follower is perturbed. In a virtual structure, the formation is treated as a single rigid body, where each agent has a position that it embeds in the structure and the desired formation motion is translated into desired motions for each agent. In the behavior-based approach, the behavior of each agent is decomposed into a set of basic behaviors and their action are controlled by the weighted average of these basic behaviors. The desired behaviors can not be defined explicitly and its mathematical formalization is difficult. For the MAS itself, there are many situations to consider when a suitable control strategy is adopted. For example, no specific agent is designated as the leader in a leaderless MAS [16], the agent cannot update the control input in time due to communication delay in a time-delay MAS [17], and the control input subject to saturation owing to its maximum and minimum limits [12].

Consensus is another fundamental problems in coordinated control where every agent in the multi-agent system updates its state based on the local information exchange with its neighbors and the states of all the agents asymptotically

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achieve an agreement on a common value. Actually, it is closely related with formation control [18]. Through choosing appropriate consensus states, the consensus theory can be adopted in the formation control problem. Ren [19] introduced the consensus-based formation control strategies in the second-order multi-agent system to guarantee accurate formation maintenance in the general case of arbitrary information flow between agents. It proved that many existing leader-follower, virtual structure, and behavior-based formation control strategies can be unified in the general framework of consensus building and treated as special cases of consensus-based formation control strategies. However, the study only considered the formation maintenance without the formation form part. In this paper, the initial positions are arbitrary and the formation form part is considered. Xiao et al. [20] studied a finite-time formation control framework which assumed only a part of agents obtained the global formation information and navigated the whole team. It proposed a class of nonlinear consensus protocols and proved that the specified constant formation can achieve in finite time. Xie and Wang [21] dealt with the formation control problem of second-order MASs only with position information exchange under undirected communication topologies and proposed a sufficient condition to achieve a time-invariant formation. Lafferriere et al. [22] investigated formation control problems for a special second-order multi-agent systems and proved a necessary and sufficient condition, i.e., the communication digraph had a rooted directed spanning tree. The special mode structure made analysis much easier. Dong et al. [23] investigated formation control problems for high-order linear time-invariant MASs with time delays whose analysis relied on the Jordan normal form of Laplacian matrix. In this paper, the general MAS is considered where the spanning tree or the Jordan normal form are not necessary.

We have investigated consensus problems of MASs and proposed a linear-transformation-based partial stability approach in [24] to analyze them. In this framework, several problems, e.g., general consensus [24], output consensus [25], consensus with state observers [26], consensus with switching topologies [27], and consensus with strongly connected topologies [28], were analyzed. The approach is now extended to the study of the formation control of second-order MASs. In this paper, three points of view, i.e., formation regulation with velocity consensus, formation tracking and formation regulation without velocity constrains, are considered. First, a state translation based on the proposed formation representation is adopted to transform the formation control problem into a consensus problem. Second, the state-linear-transformation approach sets a bridge between the consensus of the MAS and the partial stability of a corresponding auxiliary system. Compared with the existing methods, they require the calculation of eigenvectors [19], eigenvalues [20], [22], or Jordan normal form [23] of the Laplacian matrix representing the communication topology, thus can not be applied to the case of the proposed formation protocol because there is not visible Laplacian matrix. The contributions of this paper are threefold. First, two constant formation representation methods are presented. It makes possible to analyze the formation problem with consensus theory. Second, the linear-transformation-based partial stability approach is extended to the formation field. These formation problems include formation forming and maintaining under a time-invariant directed communication topology. Third, according to the analysis, the formation regulation problem with velocity consensus and the formation tracking problem share the same formation criterion in theorem 1 with different formation control protocols. In the framework of proposed constant formation representation methods, every agent owns a fixed heading angle. Thus with the same formation control protocol, criteria of formations with or without velocity constrains are essentially equivalent to each other. The simulation verify the effectiveness of the result.

The remainder of the paper is organized as follows. In Section II, the formation representation is presented. In section III and IV, the formation regulation control with or without velocity constrains and the formation tracking problem are analyzed respectively. Numerical simulations are given to illustrate the effectiveness of the theoretical results in section V. Finally, the conclusion provides a summary and future work.

II. FORMATION REPRESENTATION

When the consensus theory is applied to the formation control problem, a consensus object should be found. From this point of view, we consider that there is a virtual reference point g in the formation, which can be arbitrary chosen. The relationship between the reference point and the agent is shown as follows:

$$r_{gi} = r_i - h_i, \quad i = 1, \cdots, N, \tag{1}$$

where $r_{gi} \in \mathbf{R}^n$ is the position of the reference point with respect to the ith agent, $r_i \in \mathbf{R}^n$ is the position of the ith agent, $h_i \in \mathbf{R}^n$ is the position offset between the reference point and the ith agent which can be called formation vector, $i = 1, \cdots, N$ is the index of agents.

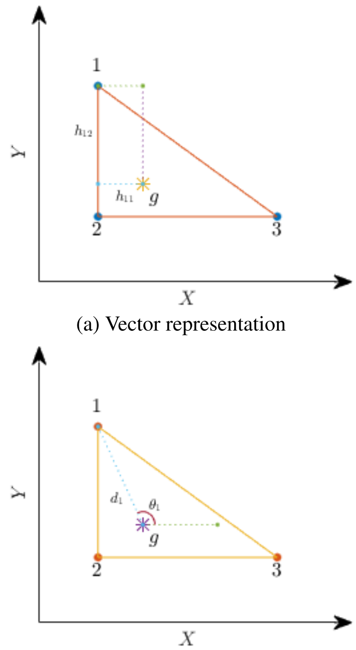
Definition 1: The position $r_{gi}(t)$, $i = 1, \cdots, N$, is said to achieve global consensus if for any initial $r_{gi}(0)$, it satisfies

$$\lim_{t \to +\infty} \|r_{gi}(t) - r_{gj}(t)\| = 0, \quad \forall i, j \in \{1, \cdots, N\}. \tag{2}$$

To better understand the relationship (2), a right-angled triangle formation which is formed by three agents in the two-dimensional plane is shown in Fig. 1. There are two methods to define h_i which are shown in Figure 1 (a) and (b), respectively.

$$h_i^a = \begin{bmatrix} h_{i1} \\ h_{i2} \end{bmatrix}, \quad h_i^b = \begin{bmatrix} d_i \cos\theta_i \\ d_i \sin\theta_i \end{bmatrix}, \tag{3}$$

where $[h_{i1}, h_{i2}]^T$ and d_i are the vector difference and the distance of the ith agent and the reference point, respectively.



(a) Vector representation

(b) Distance and angle representation

FIGURE 1. Formation example.

θ_i is the angle between the positive x-axis and the vector from the reference point to the ith agent.

Definition 2: Let $h = [h_1^T, h_2^T, \cdots, h_N^T]^T$, $r = [r_1^T, r_2^T, \cdots, r_N^T]^T$, the formation space Ξ is defined as follows:

$$\Xi = \{r | r - h = \mathbf{1}_N \otimes r_g\}, \tag{4}$$

where r_g is the position of reference point, $\mathbf{1}_N \in \mathbf{R}^N$ denote the all ones column vector of size N, the symbol $\otimes$ denotes the standard matrix Kronecker product. The vector h can be called the formation vector which describes a formation pattern.

III. CONSENSUS BASED FORMATION REGULATION

Consider a second-order MAS consisting of N agents and the dynamics is given as follows:

$$\begin{cases} \dot{r}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1, \cdots, N, \tag{5}$$

where $r_i \in \mathbf{R}^n$, $v_i \in \mathbf{R}^n$ and $u_i \in \mathbf{R}^n$ are the position, velocity and control input of the ith agent, respectively. Generally speaking, n can be 1, 2, and 3 which are corresponding to the formation in one-, two- or three-dimensional space respectively.

For convenience and tractability, the system (5) can be written in a general form as follows:

$$\dot{x}_i = Ax_i + Bu_i, \quad i = 1, \cdots, N, \tag{6}$$

where $x_i = [r_i^T, v_i^T]^T \in \mathbf{R}^{2n}$ is the state of the ith agent, the parameter matrices $A \in \mathbf{R}^{2n\times 2n}$ and $B \in \mathbf{R}^{2n\times n}$ are:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_n, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_n,$$

I_n is a $n \times n$ identity matrix.

According to the definition 1, every agent is desired to keep a specific distance with the virtual reference point and hold a consensus velocity. Thus the control input (formation protocol) u_i is constructed as follows:

$$u_i = K(x_i - \bar{h}_i) + \sum_{j\in N_i} W_{ij}((x_j - \bar{h}_j)) - (x_i - \bar{h}_i)), \tag{7}$$

where $K \in \mathbf{R}^{n\times 2n}$ and $W_{ij} \in \mathbf{R}^{n\times 2n}$ are the gain matrices, $\bar{h}_i = [h_i^T, \mathbf{0}]^T$ is the new formation vector which includes the velocity component, $\mathbf{0}$ is the zero matrix whose dimension depends on the context, the initial value of h_i^T is arbitrary and the above two formation representation (3) can be adopted here.

The graph $G = (V, E)$ in graph theory is a natural tool for the representation of the multi-agent system where a vertex set $V = \{v_1, \cdots, v_N\}$ represents N agents, a edge set $E \subseteq V \times V$ represents communication links. When the jth agent can send information to the ith agent, it is the neighbor of the ith agent. The set N_i is the index set of all neighbors of the ith agent and the set $\mathcal{N} = \{N_i : i = 1, \cdots, N\}$ represents the communication topology of the multi-agent system.

If the formation vector $\bar{h}_i = \mathbf{0}$, the formation problem is converted to a consensus problem where all the agents converge to a common point, and the formation protocol (7) is the same as the consensus protocol which was proposed in our previous work [24]. As we have said in [24], the first part of the protocol (5) with matrix K is a state feedback of the ith agent itself and it can be used to regulate the agent performance. The second part with matrices W_{ij} is the cooperation part which depends on the relative state information of the ith agent and its neighbors. Especially, matrices W_{ij} provide individual gains for each state component. The generalized gain matrices enable to set independently set weights for each component of the relative states. It will meet many practical requirements better. For instance, the position and velocity of a vehicle are measured by GPS and the wheel speed sensor respectively and they may be weighted separately in the design process.

Substituting the protocol (7) into the system (6), we can rewrite it in a compact form:

$$\dot{x} = M_1 x + R_1, \tag{8}$$

where $x = [x_1^T \cdots x_N^T]^T$ is the stacked state of agents, and

$$M_1 = I_N \otimes (A + BK) - (I_N \otimes B)L_W,$$
$$R_1 = (-I_N \otimes (BK) + (I_N \otimes B)L_W)\bar{h},$$

$\bar{h} = [\bar{h}_1^T, \ldots, \bar{h}_N^T]^T$ is the stacked formation vector, the Laplacian matrix L_W is defined as follows:

$$L_W = \begin{bmatrix} \sum_{j\in N_1} W_{1j} & -W_{12} & \cdots & -W_{1N} \\ -W_{21} & \sum_{j\in N_2} W_{2j} & \cdots & -W_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -W_{N1} & -W_{N2} & \cdots & \sum_{j\in N_N} W_{Nj} \end{bmatrix}.$$

Definition 3: Under the given communication topology $\mathcal{N}$, the second-order MAS (3) is said to form and maintain a formation via the protocol (7) if for any initial state $x_i(0)$, it satisfies

$$\lim_{t\to+\infty} \|(x_i(t) - \bar{h}_i) - (x_j(t) - \bar{h}_j)\| = 0, \quad i, j = 1, \cdots, N, \tag{9}$$

i.e., for any initial state $r_i(0)$ and $v_i(0)$, it satisfies

$$\begin{cases} \lim_{t\to+\infty} \|(r_i(t) - h_i) - (r_j(t) - h_j)\| = 0, \\ \lim_{t\to+\infty} \|v_i(t) - v_j(t)\| = 0, \quad i, j = 1, \cdots, N, \end{cases} \tag{10}$$

In this definition, all agents asymptotically form and maintain a desired formation with the same velocity.

Based on preceding state in section 2, a state translation $\bar{x} = x - \bar{h}$ is introduced to transform the formation problem into a consensus problem, and the system (8) is changed into:

$$\dot{\bar{x}} = M_1 \bar{x}, \tag{11}$$

Thus only the consensus problem of the state $\bar{x}$ needs to be considered. Using the state linear transformation proposed in [24], a bridge between the consensus problem of the MAS (5) and the partial stability problem of a corresponding system is set up. The state-linear-transformation is given as follows:

$$\hat{x} = P_1 \bar{x},\ P_1 = \begin{bmatrix} \tilde{P}_0 \\ \mathbf{1}_N^T \end{bmatrix} \otimes I_{2n}, \quad \tilde{P}_0 = \begin{bmatrix} e_1 - e_2 \\ \vdots \\ e_{N-1} - e_N \end{bmatrix} \tag{12}$$

where $e_i, i = 1, \cdots, N$, are the standard basis vectors with 1 in the ith column and 0 in the other columns.

Substituting the state linear transformation (12) into the system (11), we get

$$\dot{\hat{x}} = P_1 M_1 P_1^{-1} \hat{x}, \tag{13}$$

where the inverse matrix of the matrix P_1 is

$$P_1^{-1} = \begin{bmatrix} \hat{P}_0 & N^{-1}\mathbf{1}_N \end{bmatrix} \otimes I_{2n},$$

$$\hat{P}_0 = \frac{1}{N} \begin{bmatrix} N-1 & N-2 & \cdots & 1 \\ -1 & N-2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -2 & \cdots & 1 \\ -1 & -2 & \cdots & -(N-1) \end{bmatrix}.$$

According to the state-linear-transformation (12), the state $\hat{x}$ can be divided into two parts, i.e., $y = [\hat{x}_1^T, \cdots, \hat{x}_{N-1}^T]^T$ is a stacked vector of the error variables $\hat{x}_i = \bar{x}_i - \bar{x}_{i+1}$, and $z = \hat{x}_N$ is the sum of all $\bar{x}_i$. Thus the consensus problem of system (11) is equivalent to the stability problem of partial variables y (in short, y-stability problem). The system (13) can be further written in two equation form:

$$\begin{cases} \dot{y} = \bar{A}_1 y + \bar{B}_1 z, \\ \dot{z} = \bar{C}_1 y + \bar{D}_1 z, \end{cases} \tag{14}$$

where

$$\begin{aligned} \bar{A}_1 &= (\tilde{P}_0 \otimes I_{2n}) M (\hat{P}_0 \otimes I_{2n}) \\ &= I_{N-1} \otimes (A + BK) - (\tilde{P}_0 \otimes B) L_W (\hat{P}_0 \otimes I_{2n}), \\ \bar{B}_1 &= (\tilde{P}_0 \otimes I_{2n}) M (N^{-1}\mathbf{1}_N \otimes I_{2n}) = \mathbf{0}, \\ \bar{C}_1 &= (\mathbf{1}_N^T \otimes I_{2n}) M (\hat{P}_0 \otimes I_{2n}) \\ &= -(\mathbf{1}_N^T \otimes B) L_W (\hat{P}_0 \otimes I_{2n}), \\ \bar{D}_1 &= (\mathbf{1}_N^T \otimes I_{2n}) M (N^{-1}\mathbf{1}_N \otimes I_{2n}) \\ &= A + BK. \end{aligned}$$

Since $\bar{B}_1 = \mathbf{0}$, the stability of partial variables y is not affected by z. We get the following theorem 1.

Theorem 1: Under the given communication topology $\mathcal{N} = \{N_i : i = 1, \cdots, N\}$, the second-order MAS (5) is said to form and maintain a formation via the protocol (7) if and only if the matrix $\bar{A}_1$ in (14) is Hurwitz stable.

Obviously, the matirx $\bar{A}_1 \in \mathbf{R}^{2n(N-1)\times 2n(N-1)}$ has fewer dimensions than the entire system, which may greatly benefit the computation efficiency.

If we consider the formation tracking problem where a predefined trajectory f is given, the formation protocol is changed as follows:

$$u_i = f + K(x_i - \bar{h}_i) + \sum_{j\in N_i} W_{ij}((x_j - \bar{h}_j)) - (x_i - \bar{h}_i)), \tag{15}$$

According to the same analysis, we get

$$\begin{cases} \dot{y} = \bar{A}_1 y, \\ \dot{z} = \bar{C}_1 y + \bar{D}_1 z + NBf. \end{cases} \tag{16}$$

Obviously, theorem 1 still hold true for the formation tracking problem.

IV. CONSENSUS BASED FORMATION REGULATION WITHOUT VELOCITY CONSTRAINS

When we only consider the position state of formation, the formation protocol (7) can be rewritten as follows:

$$\begin{aligned} u_i = K_r(r_i - h_i) + K_v v_i + \sum_{j\in N_i} (a_{ij}((r_j - h_j) \\ -(r_i - h_i)) + b_{ij}(v_j - v_i)), \end{aligned} \tag{17}$$

where $[K_r, K_v] = K$, $[a_{ij}, b_{ij}] = W_{ij}$.

In this case, let $x = [r_1^T, \cdots, r_N^T, v_1^T, \cdots, v_N^T]^T$. The compact form of MAS is shown as follows:

$$\dot{x} = M_2 x + R_2, \tag{18}$$

where

$$M_2 = \begin{bmatrix} \mathbf{0} & I_{Nn} \\ I_N \otimes K_r - L_r & I_N \otimes K_v - L_v \end{bmatrix},$$

$$R_2 = \begin{bmatrix} \mathbf{0} \\ -(I_N \otimes K_r - L_r)h \end{bmatrix},$$

$$L_r = \begin{bmatrix} \sum_{j\in N_1} a_{1j} & -a_{12} & \cdots & -a_{1N} \\ -a_{21} & \sum_{j\in N_2} a_{2j} & \cdots & -a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{N1} & -a_{N2} & \cdots & \sum_{j\in N_N} a_{Nj} \end{bmatrix},$$

$$L_v = \begin{bmatrix} \sum_{j\in N_1} b_{1j} & -b_{12} & \cdots & -b_{1N} \\ -b_{21} & \sum_{j\in N_2} b_{2j} & \cdots & -b_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{N1} & -b_{N2} & \cdots & \sum_{j\in N_N} b_{Nj} \end{bmatrix}.$$

After the state translation $\bar{x} = x - [h^T, \mathbf{0}]^T$ is applied, the system (18) is changed to

$$\dot{\bar{x}} = M_2\bar{x}, \tag{19}$$

where $\bar{x} = [r_{g1}^T, \cdots, r_{gN}^T, v_1^T, \cdots, v_N^T]^T$.

The corresponding state-linear-transformation is given as follows:

$$\hat{x} = P_2\bar{x}, P_2 = \begin{bmatrix} \tilde{P}_0 & \mathbf{0} \\ \mathbf{1}_N^T & \mathbf{0} \\ \mathbf{0} & I_N \end{bmatrix} \otimes I_n, \tag{20}$$

The inverse matrix of P_2 is:

$$P_2^{-1} = \begin{bmatrix} \hat{P}_0 & N^{-1}\mathbf{1}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_N \end{bmatrix} \otimes I_n. \tag{21}$$

Substituting the state-linear-transformation (20) into the system (19), we get

$$\dot{\hat{x}} = P_2M_2P_2^{-1}\hat{x}. \tag{22}$$

According to the state-linear-transformation (20), the variable $\hat{x}$ can be divided into two parts, i.e., $\hat{x} = [y^T, z^T]^T$, $y = [r_{g1}^T - r_{g2}^T, \cdots, r_{g(N-1)}^T - r_{gN}^T]^T$, and $z = [\sum_{i=1}^N r_{gi}^T, v_1^T, \cdots, v_N^T]^T$. The system (22) is rewritten as follows:

$$\begin{cases} \dot{y} = \bar{A}_2y + \bar{B}_2z, \\ \dot{z} = \bar{C}_2y + \bar{D}_2z, \end{cases} \tag{23}$$

where

$$\bar{A}_2 = (\begin{bmatrix} \tilde{P}_0 & \mathbf{0} \end{bmatrix} \otimes I_n)M(\begin{bmatrix} \hat{P}_0 \\ \mathbf{0} \end{bmatrix} \otimes I_n) = \mathbf{0},$$

$$\begin{aligned} \bar{B}_2 &= (\begin{bmatrix} \tilde{P}_0 & \mathbf{0} \end{bmatrix} \otimes I_n)M(\begin{bmatrix} N^{-1}\mathbf{1}_N & \mathbf{0} \\ \mathbf{0} & I_N \end{bmatrix} \otimes I_n) \\ &= \begin{bmatrix} \mathbf{0} & \tilde{P}_0 \end{bmatrix} \otimes I_n, \end{aligned}$$

$$\begin{aligned} \bar{C}_2 &= (\begin{bmatrix} \mathbf{1}_N^T & \mathbf{0} \\ \mathbf{0} & I_N \end{bmatrix} \otimes I_n)M(\begin{bmatrix} \hat{P}_0 \\ \mathbf{0} \end{bmatrix} \otimes I_n) \\ &= \begin{bmatrix} \mathbf{0} \\ (I_N \otimes K_r - L_r)(\hat{P}_0 \otimes I_n) \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{D}_2 &= (\begin{bmatrix} \mathbf{1}_N^T & \mathbf{0} \\ \mathbf{0} & I_N \end{bmatrix} \otimes I_n)M(\begin{bmatrix} N^{-1}\mathbf{1}_N & \mathbf{0} \\ \mathbf{0} & I_N \end{bmatrix} \otimes I_n) \\ &= \begin{bmatrix} \mathbf{0} & \mathbf{1}_N^T \otimes I_n \\ N^{-1}\mathbf{1}_N \otimes K_r & I_N \otimes K_v - L_v \end{bmatrix}. \end{aligned}$$

It should be pointed out that $y = [r_{g1}^T - r_{g2}^T, \cdots, r_{g(N-1)}^T - r_{gN}^T]^T$. So the partial stability of the equilibrium point $\hat{x} = 0$ of the system (22) should be considered and the following key lemma is stated.

Lemma 1: The multi-agent system (5) forms and maintains a formation via the protocol (17) under the given information topology $\{N_i : i = 1, \cdots, N\}$ if and only if the equilibrium point $\hat{x} = 0$ of the system (22) is globally asymptotically stable with respect to the partial variables y.

Lemma 1 builds a bridge between the formation problem and the asymptotically partial stability problem. Thus the stability of the partial variable y relies on z. According to the partial stability theory [29], y-stability of the system (23) is equivalent to the stability of a auxiliary system. The corresponding auxiliary system which is derived form the result in [29] is given as follows:

$$\dot{\zeta} = \bar{M}\zeta, \tag{24}$$

where

$$\bar{M} = \begin{bmatrix} \mathbf{0} & \bar{B}_2L_3 \\ L_1\bar{C}_2 & L_1\bar{D}_2L_3 \end{bmatrix}.$$

The construction of above mentioned auxiliary system (24) depends on matrices $\bar{B}_2$ and $\bar{D}_2$. A matrix is constructed as follows:

$$V_p = \begin{bmatrix} \bar{B}_2^T & \bar{D}_2^T\bar{B}_2^T & \cdots & (\bar{D}_2^T)^{p-1}\bar{B}_2^T \end{bmatrix}^T \tag{25}$$

The constructive procedure is described by the following steps:

Step 1. calculate the number $s = \min\{k : rank V_k = rank V_{k+1}\}$, then denote a new variable $h = rank V_s$.

Step 2. construct $h \times p$ matrix L_1 whose rows are obtained via the operation on V_s successively from top to bottom removing the rows of the matrix V_s that linearly depend on these rows reserved via the former operation.

Step 3. construct $h \times h$ matrix L_2 from the linearly independent columns in L_1, for example, they are $i_1, \cdots, i_h$ columns in L_1.

Step 4. construct $n \times h$ matrix L_3, whose i_jth row is the jth row of $L_2^{-1}, j = 1, \cdots, h$, and other rows are $\mathbf{0}$.

Thus we get the following theorem:

Theorem 2: Under the given communication topology $\mathcal{N} = \{N_i : i = 1, \cdots, N\}$, the second-order MAS (5) is said to form and maintain a formation via the protocol (17) if and only if the matrix $\bar{M}$ in (24) is Hurwitz stable.

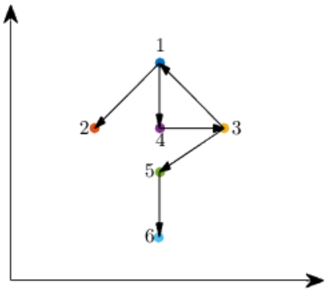
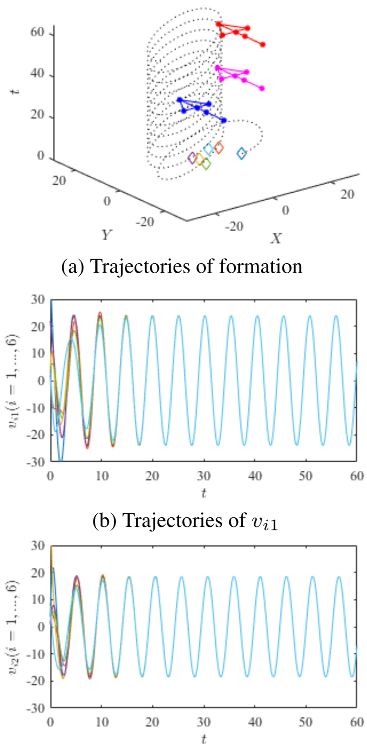


FIGURE 2. Communication topology.



(a) Trajectories of formation

(b) Trajectories of v_{i1}

(c) Trajectories of v_{i2}

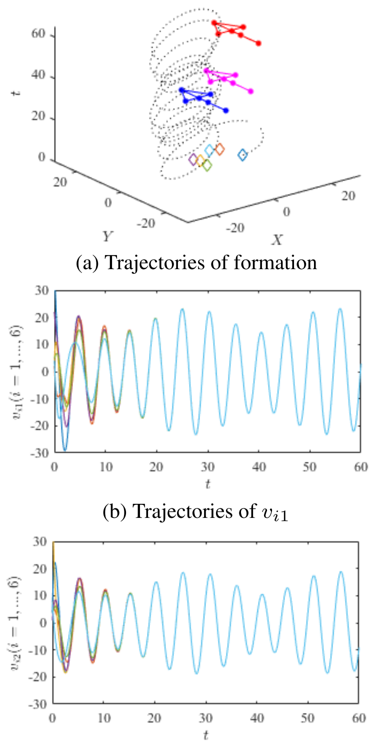
FIGURE 3. Trajectories of formation regulation.

As the analysis of the theorem 1, the dimension of matrix $\bar{M}$ is reduced. But the matrix $\bar{M}$ must constructed along the above procedure which is slightly more complicated.

V. SIMULATION EXAMPLES

Consider a second-order MAS (5) consisting of six agents in two-dimensional planar coordinates. It aims to form and maintain a arrow formation which is described as follows:

$$\begin{aligned} d_1 &= 10, \theta_1 = 90^\circ, \\ d_2 &= 5, \theta_2 = 150^\circ, \\ d_3 &= 5, \theta_3 = 30^\circ, \\ d_4 &= 2, \theta_4 = 90^\circ, \\ d_5 &= 2, \theta_3 = 270^\circ, \\ d_6 &= 10, \theta_4 = 270^\circ. \end{aligned} \tag{26}$$



(a) Trajectories of formation

(b) Trajectories of v_{i1}

(c) Trajectories of v_{i2}

FIGURE 4. Trajectories of formation tracking.

The gain matrix K in formation protocol (7) is given as follows:

$$K = \begin{bmatrix} -1.8 & -1.1 & -1.4 & 1.6 \\ 0.9 & -1.3 & 1.1 & -1.7 \end{bmatrix}, \tag{27}$$

The time-invariant communication topology $\mathcal{N}$ is given in Fig. 2. And the gain matrices are given as follows:

$$\begin{aligned} W_{13} &= \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.3 \\ 0.5 & 0.9 & 0.2 & 0.8 \end{bmatrix}, \\ W_{21} &= \begin{bmatrix} 0.6 & 0.8 & 0.1 & 0.3 \\ 0.3 & 0.6 & 0.2 & 0.5 \end{bmatrix}, \\ W_{34} &= \begin{bmatrix} 0.2 & 0.5 & 0.2 & 0.4 \\ 0.1 & 0.3 & 0.4 & 0.8 \end{bmatrix}, \\ W_{41} &= \begin{bmatrix} 0.7 & 0.6 & 0.5 & 0.2 \\ 0.1 & 0.2 & 0.6 & 0.3 \end{bmatrix}, \\ W_{53} &= \begin{bmatrix} 0.2 & 0.7 & 0.8 & 0.3 \\ 0.3 & 0.6 & 0.8 & 0.1 \end{bmatrix}, \\ W_{65} &= \begin{bmatrix} 0.2 & 0.4 & 0.2 & 0.3 \\ 0.2 & 0.3 & 0.7 & 0.1 \end{bmatrix}, \end{aligned} \tag{28}$$

The eigenvalues of $\bar{A}_1$ in theorem 1 are the same as that of $\bar{M}$ in theorem 2. They are -2.6149, $-0.6509 \pm 1.6867i$, $-0.2597 \pm 1.5506i$, $-0.2864 \pm 1.1874i$, $-0.4359 \pm 1.1485i$, $-0.5256 \pm 1.0172i$, -2.0626, -0.9139, -1.1179, $-1.6091 \pm 0.8006i$, $-1.5641 \pm 0.8873i$, $-1.4136 \pm 0.9221i$. It can be seen that the real parts of them are negative, and it means the matrix $\bar{A}_1$ in theorem 1 and $\bar{M}$ in theorem 2 are all Hurwitz stable.

Fig. 3 (a)-(c) show the trajectories of the agents starting from the initial states [14, 2.4, 40, 3.6], [12, 10, 2, 8], [2, 6, 3, 50], [1, 8, 22, 1], [1.5, 2, 4.9, 4],[9, 11, 4, 8], respectively. In Fig. 3(a), the diamond points indicate the initial positions of six agents and the three arrow formations are the positions of the MAS in 40s, 41s and 52s from bottom to the top respectively. To avoid confusion, only the trajectory of the 1th agent is shown by dotted line. The simulation illustrates the effectiveness of the formation criterion. Fig. 3(b)-(c) show the trajectories of v_{i1} and v_{i2}.

In the same setting, a predefined trajectory f of the protocol (15) is given as follows:

$$f = \begin{bmatrix} 5\sin(t) \\ \cos(t) \end{bmatrix}, \tag{29}$$

Thus the corresponding trajectories are shown in Fig. 4 (a)-(c).

VI. CONCLUSION

This paper presented consensus-based formation control protocols for second-order MASs under a time-invariant directed communication topology. Sufficient and necessary algebraic criteria are derived for the formation regulation problem with or without velocity constrains and the formation tracking problem. However, the angle variation of formation is not considered in the paper. It need to be studied in future work. And more complex formation cases will also be investigated, e.g., the time-varying formation control, the formation control in highway environment.

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