IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received October 27, 2019, accepted November 5, 2019, date of publication November 11, 2019, date of current version November 21, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2952900

Nonlinear Control of a Magnetic Levitation System Based on Coordinate Transformations

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This work was supported in part by the National Key Research and Development Program of China under Grant 2016YFB1200602, and in part by the Fundamental Research Funds for Central Universities under Grant 22120180572.

ABSTRACT In this paper, a novel concept of nonlinear controller design is proposed, which is applicable to nonlinear systems subject to external periodic disturbances. At first, by employing a pioneering nonlinear coordinate transformation, a new nonlinear system model with expected dynamic characteristics is derived. Then, a corresponding linear system is achieved by means of linearization techniques at equilibrium points. Next, based on the results of classical linear control theory, a linear quadratic regulator is applied to the resulted linear system. Finally, the nonlinear controller can be obtained through the inverse nonlinear coordinate transformation of the linear feedback controller. The performance of the proposed method is validated on a magnetic levitation system and is compared with the classical linearization feedback control. Simulation results show that the proposed controller design assures enhanced performance in terms of both system stabilization and disturbance attenuation.

INDEX TERMS Magnetic levitation system, coordinate transformation, linear quadratic regulator, nonlinear feedback, disturbance attenuation.

I. INTRODUCTION

In recent years, magnetic levitation systems (MLSs) have witnessed a growing trend of technological advancement and industrial application in diverse fields, e.g., vibration isolation systems, turbomolecular pumps, high-precision manufacturing facilities, flywheel systems, maglev trains, and so on [1]. Owing to the merits of MLSs, the MLS-based devices have the advantage of high operation speed, high reliability, energy efficiency, or reduced maintenance cost.

When an MLS is in operation, the target mass in the magnetic field is actuated by its constant gravity and the magnetic attracting force. Like most physical systems, the MLSs behave in a nonlinear fashion. Due to the inherent nonlinearity and open-loop instability associated with the MLSs, the controller design becomes a thriving research area, and thus has attracted substantial effort and attention from disparate scientific disciplines worldwide [2].

The generic idea to design a nonlinear controller is to simplify the nonlinear control system such that existing control theory or stability theorem is ready for use. Further, the general approach to carry out a nonlinear controller design is the generation of the control action as a function of system states or outputs [3]. In line with the approach used to deduce the essential function, controller design of nonlinear systems might be categorized into three approaches [4]:

1) linear methods, such as the linear control and the gain scheduled linear control. This approach attempts to expand the region of linear control operation by linearizing the dynamics about one or multiple operating points at first, and then designing linear controllers for each point through a variety of techniques. However, such designs have a limited operating region where the linear approximation is valid.

2) Lyapunov stability criteria-based methods, such as the sliding mode control and adaptive control. In principle, these methods are applicable to essentially all dynamic systems, whether in small or large motion. However, it suffers from the common difficulty of finding a Lyapunov function.

3) transformation methods, such as Input-Output feedback linearization, Input-State feedback linearization, approximate feedback linearization. The main concept in this approach is the use of transformations in the state and control variables to alter the nonlinear dynamics to a nearly linear

The associate editor coordinating the review of this manuscript and approving it for publication was Bohui Wang.

form such that the remaining nonlinearities can be cancelled by the designed feedback. Due to the requirements of specific linearization, this approach in most cases can only be applied to certain types of nonlinear system.

The methodology of nonlinear controller design has been developed for many years. In spite of significant gains, it is still a challenging task to address the controller design for nonlinear systems in a general manner. Therefore, research and development of nonlinear control techniques that are applicable to specific systems such as the MLSs is desired and of practical importance.

A great number of works under the above framework of controller design have been documented in literatures. The traditional proportional–differential (PD), proportional–integral–differential (PID) control were widely used 5], [6]; a more general PID controller was constructed by Morales et al. in [7], [8]; moreover, the PID controller with self-tuning gains had been reported in [9] for the request of faster transient dynamics, to account for the tracking or regulating objectives of the MLSs. Although the PID controller and its derivatives have many advantages, such as simple control structure and convenient gain scheduling, the capability and performance are always hindered by the intricate nonlinear dynamics of the physical system.

Consequently, there is a strong incentive to develop methods of nonlinear controller design. To mention a few, the control strategy of designing stabilizing fractional order controllers was developed by C.I. Muresan et al. in [10]; a fast nonlinear model predictive control (MPC) scheme was presented in [11]; the transverse feedback linearization has been adopted in [12]; a kind of fault tolerance control was implemented in [13]; the analysis about active suspension system was given in [14]; and a data-driven approach based on the state-dependent ARX model was presented in [15].

It is noticed that most of the above-mentioned results on the controller design of the MLSs have not considered the influence of creeping disturbances. However, disturbances caused by the physical system of concern or the operational environment are widely present. As such, much effort has been directed toward the development of high-performance controllers for the MLSs subject to external disturbances. For instance, a learning-based controller to asymptotically regulate an MLS while compensating for the periodic, exogenous disturbances was introduced by Costic et al. in [16]; an interconnection and damping assignment passivity-based controller was proposed in [17]; a set of linear bounded-input-bounded-output (BIBO) filters to facilitate an adaptive compensation for disturbances were designed in [18]; a state estimate observer in a backstepping fashion to achieve the asymptotic disturbance rejection was utilized in [19]; the robust of integral of sign of error control (RISE), was proposed in [20] and further developed in [21].

Besides, a variety of other methods have been developed to address the disturbance attenuation issue, such as robust dynamic surface control [22], neural network control [23], and sliding mode control [24]. Among the advanced control methods [25]–[28], the disturbance observer or alike has provided a flexible framework in dealing with disturbance attenuation within the MLSs [29]. By appropriate using of the observer or its derivatives, the robustness of an existing MLS against disturbances could be significantly improved.

The main purpose of approaches in the classical control theory intends to minimize the difference between the output and a given reference, at the earliest possible time or at the least cost. Driven by the ever-increasing sophisticated application scenario or stringent performance specification, modern nonlinear control theory has been expanded to overcome some of the impediments to designing advanced controllers for the MLSs. As stated above, both nonlinear and linear control schemes have been widely applied to the robust suspension within the MLSs, based on the feedback linearization, backstepping, sliding modes, neural networks, adaptive control, and other design methodologies. However, in practical applications, most of the available control schemes for the MLSs might be impeded by the limited operating ranges of the simplified linear model, or the demand of extra efforts to implement estimators.

The objective of this paper is to come up with a concept of controller design for the MLSs from a new perspective, in which the controller can be tuned to be effective within the operating range for specific applications. To the best of the authors' knowledge, very few controller design methods reported in the literature have took advantage of both linear method and nonlinear coordinate transformation techniques to address the control problem for the MLSs in the presence of extraneous disturbances. In this context, a novel concept of the controller design in which linearization technique, nonlinear coordinate transformation and linear quadratic regulator (LQR) cooperate with each other for stabilizing a levitation system is proposed herein. Simulation results show that this approach can provide satisfying performance in terms of both stabilization and disturbance attenuation.

The rest of this paper is organized as follows: Section II introduces the test setup and the mathematical model of the nonlinear magnetic levitation system. Section III introduces the proposed framework of controller design including the nonlinear coordinate transformation and the LQR with state feedback. Section IV validates feasibility and effectiveness of the proposed controller design through simulation results. Finally, the conclusions and discussions of this paper are provided in Section V.

II. PROBLEM FORMULATION

In this paper, an MLS is used to levitate an iron ball in air by the electromagnetic force provided by an electromagnet. The levitation system consists of an iron ball, an electromagnet and an electro-optical sensor. A schematic diagram of this system with input and output variables is shown in Figure 1.

The balance between the magnetic force F_m and the gravity force F_y allows the ball to be sustained at a given position. With the attraction of the controlled electromagnet, the iron ball would not fall down by gravity. The position

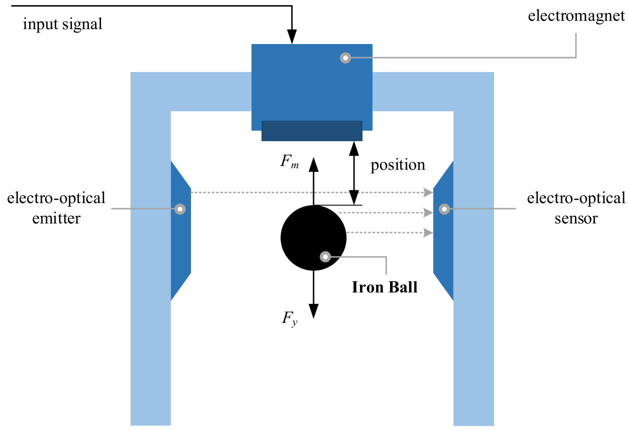

FIGURE 1. Schematic diagram of the magnetic levitation system.

of the ball can be detected by a sensor system (including the electro-optical emitter and sensor) and be regulated by the input signal. Further, the coil current in the electromagnet is measured to design the controller which aims at keeping the ball in a desired distance from the electromagnet. Note that, the coil current is adjusted to control the ball's position in the mechanical system, while the coil voltage is varied to change the coil current in the electrical system [30].

By taking the Kirchoff's voltage law and Newton's second law, the dynamic equations of the magnetic levitation system as shown in Figure 1 can be represented by (1), on the premise that the vertical upward direction is regarded as the positive direction [31]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{F_m(x_1, x_3)}{m} + g \\ \dot{x}_3 = \dfrac{k_{i2}u - x_3}{k_{i1}} \end{cases} \tag{1}$$

in which the three state variables x_1, x_2, x_3 represent the ball's position below the magnet, the velocity of the ball, and the current in the electromagnet, respectively. In addition, m is the mass of the ball, g is the gravitational acceleration. Input signal u is the control signal, and output variable is the ball's position x_1. In (1), the first two equations stand for the equations of the mechanical system and the last equation describes the current dynamics, that is, the electrical system.

The magnetic force $F_{\mathrm{m}}(x_1, x_3)$ exerted by the electromagnet on the iron ball can be expressed as a nonlinear function:

$$F_m(x_1, x_3) = k_{m1}x_3^2 \exp(-k_{m2}x_1) \tag{2}$$

where $k_{\mathrm{i}1}$, $k_{\mathrm{i}2}$, $k_{\mathrm{m}1}$, $k_{\mathrm{m}2}$ are constant coefficients.

The target of the design process is to specify a control signal that will stabilize the iron ball at a desired position, in the presence of disturbance forces. The controller design is made more complicated by the fact that the magnetic levitation system is a nonlinear system that is inherently open-loop unstable, and there is no prior knowledge of the disturbance force.

III. NONLINEAR CONTROLLER DESIGN

As summarized in Section I, there is not general nonlinear control design methodologies. However, there are powerful strategies applicable to specific nonlinear systems. Among a magnitude of control schemes that can deal with the nonlinear MLSs, the most widely used approach is the adoption of coordinate transformation techniques that render the nonlinear dynamic equations amenable to existing results in classical control theory. For example, the concept of feedback linearization, where a change of coordinates in the state and control space coupled with feedback applied to the nonlinear system, results in a controllable linear system. Once this has been achieved, one can apply proven methods of linear controller design to the transformed linear system.

In this paper, the proposed nonlinear controller method adopts the concept of coordinate transformation as well, whereas taking a different process and purpose compared with linearizing transformation or feedback linearization. The procedure is explained as follows:

Step 1): Derive a new nonlinear system model that will enhance performances of the controller though a nonlinear coordinate transformation of the original system;

Step 2): Linearize the obtained nonlinear system model in Step 1) at an equilibrium point using traditional approaches, such as Taylor series expansion or Jacobian matrix;

Step 3): Design a linear feedback controller for the linearized system in Step 2) by means of the pole assignment or LQR which is devised to realize the desired control performances;

Step 4): Obtain the corresponding nonlinear controller of the original system through the inverse nonlinear coordinate transformation of the linear controller designed in Step 3).

Moreover, a framework of the proposed controller design and a comparison of three existing paradigms are illustrated in Figure 2. The first one is a standard linear state feedback controller whose design is based on a linear model found by perturbing the nonlinear system at an operating point. It is effective only in the vicinity of operating points. The second controller is based on exact linearization where a nonlinear state-space transformation together with a nonlinear state feedback is used. It is in a nonlinear form and valid in a wide range, thus not limited to near the equilibrium points. By taking advantage of the linear state feedback and nonlinear coordinate transformation, the third one can achieve desired performances, whereas it is not subjected to harsh terms and is thus easily conducted.

A. COORDINATE TRANSFORMATION

The general target of feedback linearizing controllers is the derivation of coordinate transformations which converts the original nonlinear system into a simpler system in the sense that the controller synthesis is readily to be implemented. The novelty of the proposed controller design in this paper lies in implementation of deliberate nonlinear coordinate transformations, which can effectively improve the

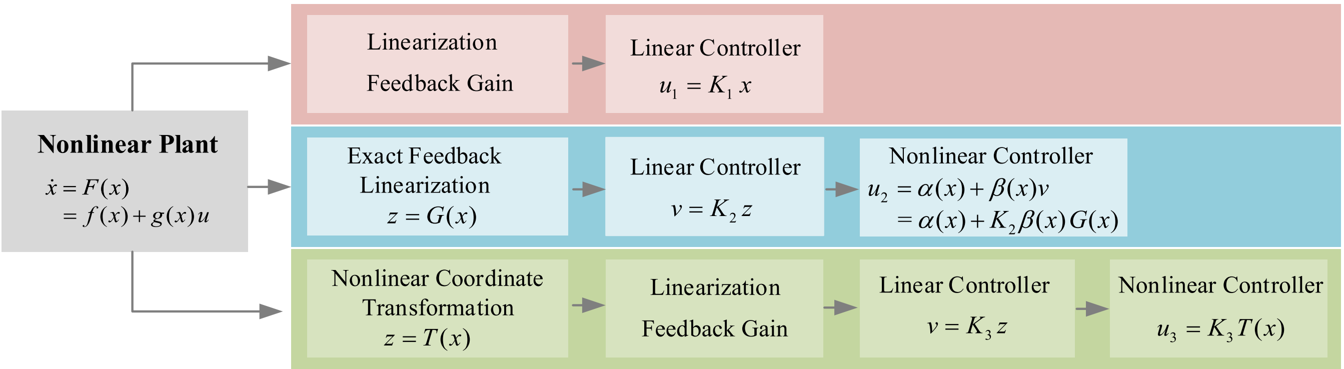

FIGURE 2. Frameworks of three controller designs for the nonlinear system.

TABLE 1. Parameter list of magnetic levitation system.

Symbol	Quantity	Unit
m	0.0571	kg
g	9.81	m/s^2
k_{m1}	3.01	H/m
k_{m2}	171.73	1/m
k_{i1}	0.025	S
k_{i2}	2.32	A

control performance in the aspect of robustness to disturbance force.

The heuristic method to find coordinate transformations is based on the following facts:

1) From the perspective of physical significance, the values of variable for gap and current are always positive, rather infinite intervals;

2) Because of saturation of magnetic flux, the region of gap, where magnetic force can be larger than the force of gravity, is limited.

As above, the region of interest for an individual case and candidate coordination transformations can be defined. For illustration, 3 to 18mm for the gap and 0 to 2.5A for the current in system (1) with parameter listed in Table 1 are always concerned in practice, that is, $0.003 < x_1 < 0.018$, $x_2 \in R$, $0 < x_3 < 2.5$. Correspondingly, three changes of coordinates can be introduced as follows:

$$T_1(\mathbf{x}) : \{z_1 = x_1\ ;\ \ z_2 = x_2;\ \ z_3 = x_3\} \tag{3}$$

$$T_2(\mathbf{x}) : \begin{cases} z_1 = x_1 + \dfrac{c_1}{1 - x_1/0.018} - \dfrac{c_1}{1 - 0.01/0.018} \\ z_2 = x_2; \quad z_3 = x_3 \end{cases} \tag{4}$$

$$T_3(\mathbf{x}) : \begin{cases} z_1 = x_1 + \dfrac{c_1}{1 - x_1/0.018} - \dfrac{c_1}{1 - 0.01/0.018} \\ \quad + \dfrac{c_2}{1 - x_1/0.003} - \dfrac{c_2}{1 - 0.01/0.003} \\ z_2 = x_2 \\ z_3 = x_3 \end{cases} \tag{5}$$

Note that transformations (3)-(5) result in three new state variables z_1, z_2, z_3, being the position of ball, velocity of the ball, and current in the electromagnet, respectively.

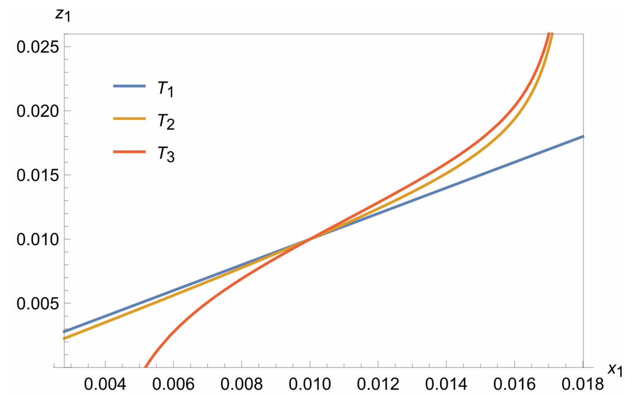

FIGURE 3. Illustration of the nonlinear coordinate transformations.

For comparison, T_1 represents the original system without any change of coordinates.

By taking $c_1 = 0.00005$, $c_2 = 0.0005$, the three coordinate transformations with respect to x_1 are illustrated in Figure 3.

It is found that the new state z_1 obtained from coordinate transformations T_2 and T_3 exhibit expected features over the considered range of x_1. That is, at end points of the interval of x_1, z_1 shows inward shrinkage. In addition to the change in x_1, the coordinate transformation can work on the other two states x_2, x_3 as well, according to the demand.

B. LINEAR QUADRATIC REGULATOR

The LQR is a widely used optimal controller design that deals with linear systems and minimizes quadratic objective functions with performance index determinations. It provides optimally controlled feedback gains to enable the closed-loop stable and high-performance design of systems.

For a continuous-time linear system (6), the LQR can be applied, with a quadratic cost function as defined in (7):

$$\dot{x} = Ax + Bu \tag{6}$$

$$J = \frac{1}{2}\int_0^{\infty} [x^{\mathrm{T}}Qx + u^{\mathrm{T}}Ru]dt \tag{7}$$

where Q is a positive semi-definite matrix, R is a positive definite symmetric matrix. Note that matrix Q defines the weights on the states while matrix R defines the weights on the control input in the cost function.

The feedback law that minimizes the value of cost reads,

$$u = -Kx \tag{8}$$

where K is given by,

$$K = R^{-1}(B^{\mathrm{T}}P + N^{\mathrm{T}}) \tag{9}$$

in which P is a positive symmetric matrix, and is achieved by solving the Algebraic Riccati Equation (ARE) as follows:

$$A^{\mathrm{T}}P + PA - (PB + N)R^{-1}(B^{\mathrm{T}}P + N^{\mathrm{T}}) + Q = 0 \tag{10}$$

In fact, the performance index J in (7) can be interpreted as an energy function, such that a smaller value of J means a lower total energy of the closed-loop system.

When it comes to robustness, the criterion for nonlinear systems is always in connection with certain performances. Therefore, robustness performances of different controllers in this paper can be compared by the maximum amplitude of an external periodic disturbance at a given frequency, with which system responses are still within the boundary.

IV. SIMULATIONS AND RESULTS

In this section, numerical simulations are carried out on the studied MLS in the presence of sinusoidal disturbances to validate the feasibility and performance of the proposed controller design with Wolfram Mathematica 11.2.

A. SIMULATION SETUP

The nonlinear system as shown in Figure 1 is parameterized as listed in Table 1. In addition, the target position of the iron ball is set to 0.01m. In the presence of sinusoidal disturbance force $a\sin(2\pi ft)$, the dynamics are given by equation (11).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_{m1}x_3^2\exp(-k_{m2}x_1)/m + g + a\sin(2\pi ft) \\ \dot{x}_3 = (k_{i2}u - x_3)/k_{i1} \end{cases} \tag{11}$$

For demonstration, different coordinate transformations i, ii and iii as represented by equations (12)-(14) are used to compare their effectiveness in terms of the ability against sinusoidal disturbances in this section. Transformation i in (12) is an identical transformation between coordinates $\mathbf{x}$ and $\mathbf{z}$. Transformation ii in (13) only considers the change of x_1 which can confine the state on the x_1-axis to an interval (0.003, 0.018). Furthermore, transformation iii in (14) intends to adjust feasible regions on both the x_1-axis and the x_3-axis. Note that, coordinate transformation on the x_2-axis is not taken into account since it is non-essential to limit the value of the ball's velocity in practice.

In addition, constants c_1, c_2, c_3 are employed as tuning parameters to achieve desired transformation process. For illustration, by taking $c_1 = 0.00005$, $c_2 = 0.0005$, $c_3 = 0.01$, the resulting nonlinear systems via coordinate transformations i, ii and iii are denoted by A, B and C, respectively.

$$i: \ \{z_1 = x_1; \quad z_2 = x_2; \quad z_3 = x_3\} \tag{12}$$

$$ii: \begin{cases} z_1 = x_1 + \dfrac{c_1}{1 - x_1/0.018} - \dfrac{c_1}{1 - 0.01/0.018} \\ \quad + \dfrac{c_2}{1 - x_1/0.003} - \dfrac{c_2}{1 - 0.01/0.003} \\ z_2 = x_2 \\ z_3 = x_3 \end{cases} \tag{13}$$

$$iii: \begin{cases} z_1 = x_1 + \dfrac{c_1}{1 - x_1/0.018} - \dfrac{c_1}{1 - 0.01/0.018} \\ \quad + \dfrac{c_2}{1 - x_1/0.003} - \dfrac{c_2}{1 - 0.01/0.003} \\ z_2 = x_2 \\ z_3 = \dfrac{x_3 - \sqrt{x_3^2 + 2x_3(-2 + c_3) + (-2 + c_3)^2}}{2} \\ \quad + \dfrac{\sqrt{1.018 + 6.036c_3 + c_3^2} + 1.018}{2} \end{cases} \tag{14}$$

B. SIMULATION RESULTS

For the nonlinear system affected by sinusoidal disturbances with given frequency, measurable amplitude and initial phase, the proposed controller design is simulated.

To implement the proposed controller, an operating point of the linearization is selected at $(x_1^*, x_2^*, x_3^*) = (0.01, 0, 1.018)^{\mathrm{T}}$ and $u^* = 0.439$, then the linearized system of (11) can be obtained as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1684.67 & 0 & -19.2718 \\ 0 & 0 & -40 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -40 \end{bmatrix} u \tag{15}$$

The design parameters of LQR are determined by setting matrix Q, R as:

$$Q = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \quad R = 100 \tag{16}$$

Through (9)-(10), the feedback gain K is then solved as:

$$(k_1, k_2, k_3) = (-52.7070, -3.7205, 0.8847) \tag{17}$$

With feedback gain K, the eigenvalues of linear system (15) are calculated as [– 39.5674, – 41.2666 ± 0.7862i], which indicate the stability of system (15). Furthermore, three controllers designed for nonlinear system (11) via coordinate transformations i, ii and iii can be achieved, denoted by u_{A}, u_{B} and u_{C}, as expressed in (18)-(20).

$$u_{\mathrm{A}} = k_1(x_1 - x_1^*) + k_2(x_2 - x_2^*) + k_3(x_3 - x_3^*) + u^* \tag{18}$$

$$\begin{aligned} u_{\mathrm{B}} = k_1(x_1 + &\frac{1/2000}{1 - x_1/0.018} + \frac{1/200}{1 - x_1/0.003} \\ &+ 0.001 - x_1^*) + k_2(x_2 - x_2^*) + k_3(x_3 - x_3^*) + u^* \end{aligned} \tag{19}$$

$$\begin{aligned} u_{\mathrm{C}} = k_1(x_1 + &\frac{1/2000}{1 - x_1/0.018} + \frac{1/200}{1 - x_1/0.003} \\ &+ 0.001 - x_1^*) + k_2(x_2 - x_2^*) \\ &+ k_3(\frac{x_3 + 2.27415 - \sqrt{x_3^2 - 3.8x_3 + 4.41}}{2} - x_3^*) + u^* \end{aligned} \tag{20}$$

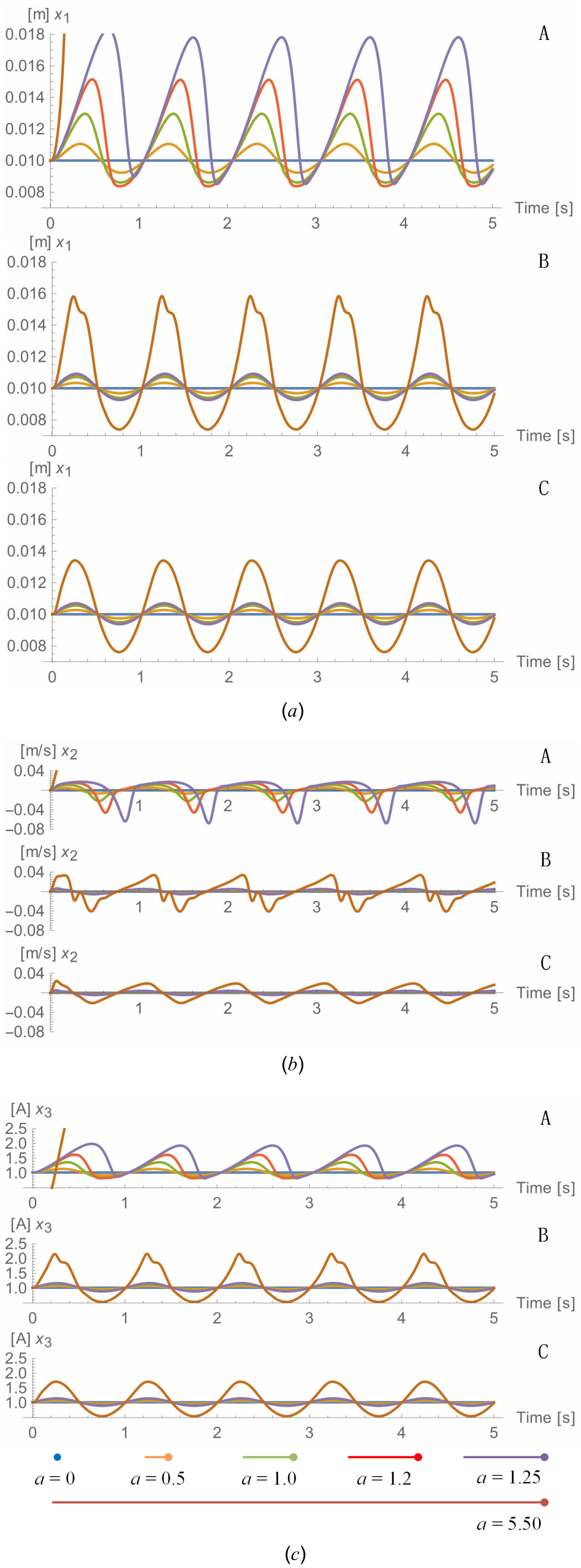

FIGURE 4. Simulation results with coordinate transformations at $f = 1$.

The frequency and initial phase of disturbance force are assigned by 1 and 0 in the first case study. To investigate the influence of amplitude on the performance of controllers, the amplitude a varies from 0 to 5.50. The performance of the proposed controllers and illustration of sinusoidal disturbance force $a\sin(2\pi ft)$ are shown in Figure 4 and 5, respectively.

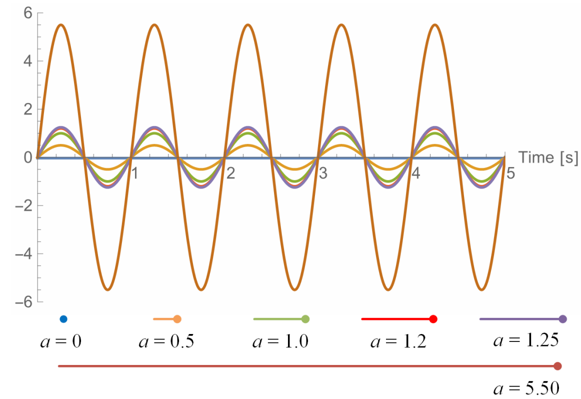

FIGURE 5. Demonstration of sinusoidal disturbance force at $f = 1$.

In Figure 4(a), it can be found that all the three controllers are able to track the defined reference position 0.01m very well when $a = 0$. Considering x_1 is regarded as the output y, it is concluded that under assumed ideal condition, the LQR in conjunction with nonlinear coordinate transformations *ii* and *iii* exhibit similar performances as with *i* which is equivalent to the linearization feedback control. However, along with the gradual increase of disturbance's amplitude, merits of the controller in conjunction with *ii*, *iii* come into effect.

As mentioned in Section III, the robustness of different controllers can be quantified by the maximum amplitude of external periodic disturbances at given frequencies, with which system responses still locate in the region of interest. It is found that system C exhibits the best robustness against consistent periodic disturbances. Specifically speaking, at $f = 1$, $a_{\max} \approx 1.25$ for linear feedback u_{A} and $a_{\max} \approx 5.5$ by using nonlinear control u_{C}, therefore, controller u_{C} has a better robustness than u_{A}.

In dynamic system theory, the phase-space is a space in which all possible states of a system under consideration are represented, with each possible state corresponding to one unique point in the phase-space. In comparison with the time-history plot as shown in Figure 4, a phase-space trajectory can provide additional valuable and unique insight into the dynamics of the systems of interest. The phase-space trajectory which consists of all possible values of the position, velocity and current of the control system (11) is plotted in Figure 6.

In such portraits, the three controllers are distinct in their responses to sinusoidal disturbances. It shows that system A (i.e., the proposed controller with *i*) always yields the largest phase-space volume throughout the four scenarios. On the contrary, system C remains the smallest trajectory. Through the characteristic phase-space portraits, it can be concluded that the LQR in conjunction with *ii* and *iii* are able to render a more stable closed-loop nonlinear system, due to the fact that a compact size of the phase-space portrait indicates that

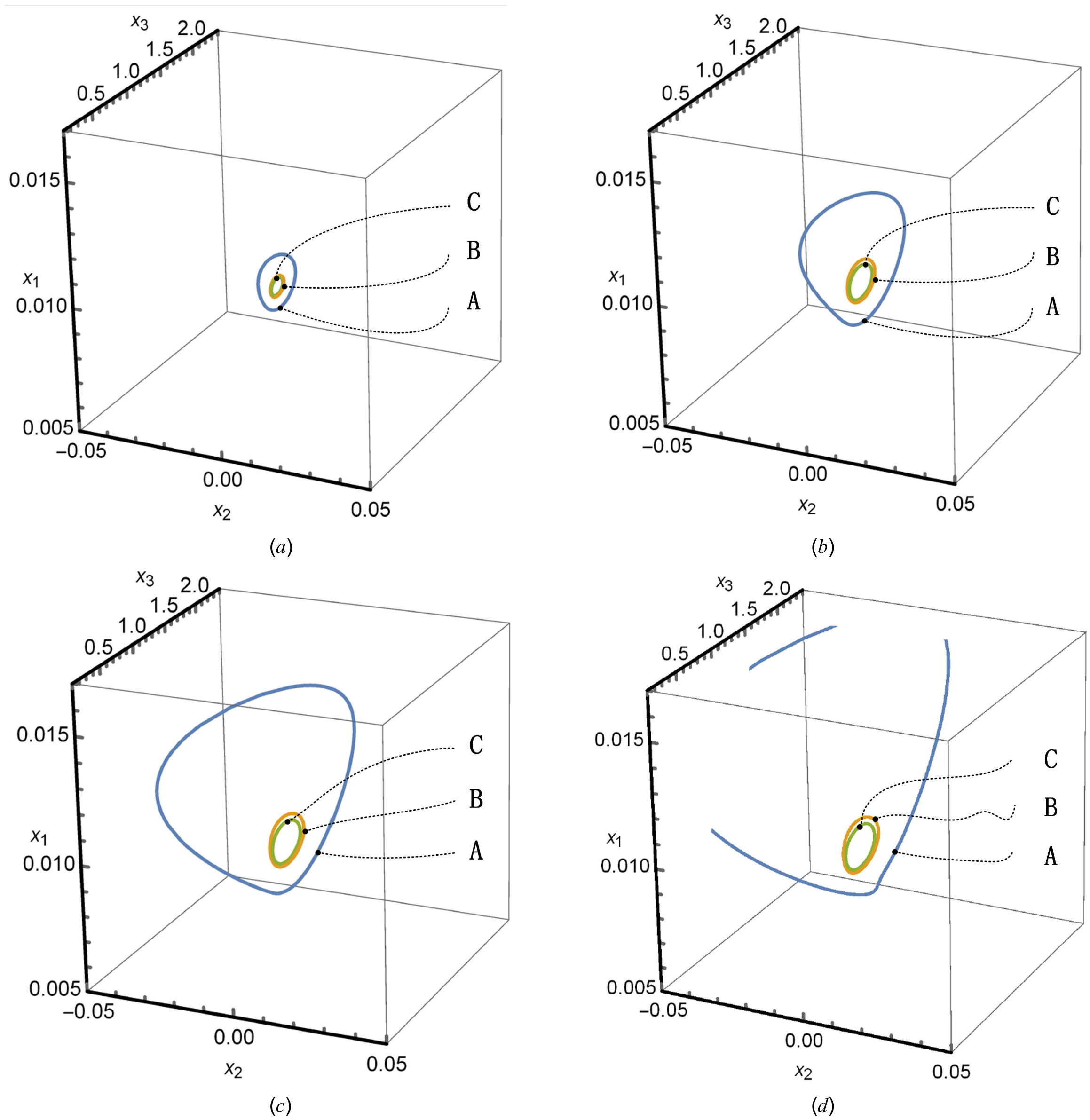

FIGURE 6. Phase-space trajectory of the closed-loop system with coordinate transformations at $f = 1$: (a) $a = 0.5$; (b) $a = 1.0$; (c) $a = 1.2$; (d) $a = 1.25$.

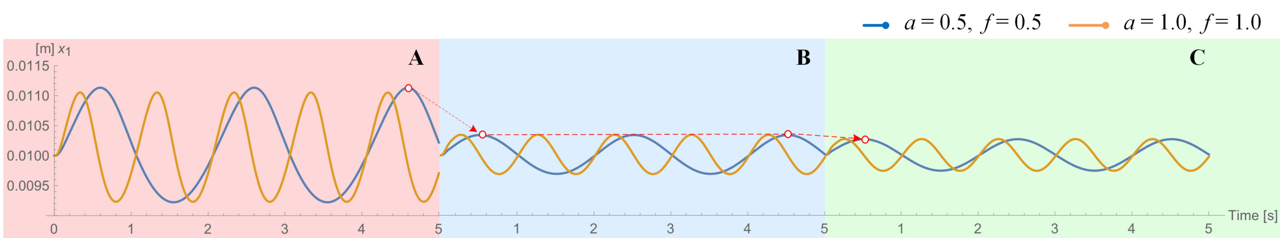

FIGURE 7. Simulation results with coordinate transformations at $a = 0.5$, $f = 0.5$ and $a = 1.0$, $f = 1.0$.

the states are closely surrounded the operating point. It is therefore concluded that robustness of the control system against sinusoidal disturbances can be adjusted and improved evidently, by means of the proposed nonlinear coordinate transformations.

Up to now, simulation results have represented responses of the control system affected by sinusoidal disturbances with different amplitudes. As the case stands, the performances of each controller also differ when the frequency varies. For illustration, the time evolutions of three control systems are

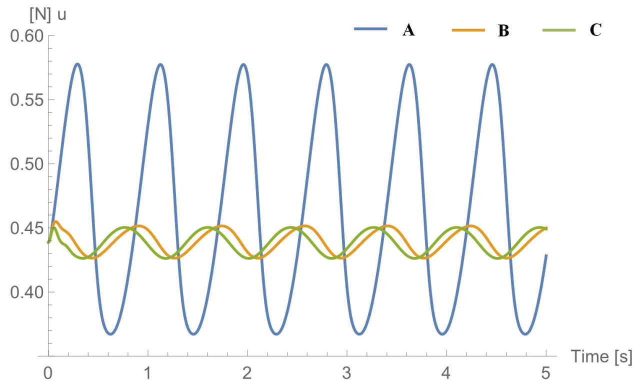

FIGURE 8. **Simulation results of input signal with coordinate transformations at $a = 1.0$, $f = 1.0$.**

TABLE 2. **Comparisons among different coordinate transformations.**

	a	f	$x_{1,\text{p-p}}$ [m]	$x_{2,\text{p-p}}$ [m]	$x_{3,\text{p-p}}$ [A]	$u_{\text{p-p}}$ [N]
A	0.5	0.5	0.0021	0.0063	0.2230	0.0974
	1.0	1.0	**0.0044**	**0.0328**	**0.5101**	**0.2284**
B	0.5	0.5	0.0005	0.0021	0.2221	0.0071
	1.0	1.0	0.0011	0.0083	0.5071	0.0231
C	0.5	0.5	**0.0004**	**0.0018**	**0.0966**	**0.0070**
	1.0	1.0	0.0011	0.0067	0.1987	0.0195

* subscript *p-p* represents the peak-to-peak value.

displayed in Figure 7. It is revealed that the controller with *iii* has an advantage over the others throughout respecting the capability of disturbance attenuation.

For most practical systems, a large fluctuation in the control signal is commonly undesirable concerning dynamics of the actuator. Hence, an in-depth analysis of the variations of three controllers are carried out. In Figure 8, input signal of the control system with three coordinate transformations under the conditions of $a = 1.0, f = 1.0$ are revealed. It can be found that three nonlinear controllers in conjunction with *ii* and *iii* behave in a like manner, while the controller with *i* exhibits a greater variation in the control signal.

In addition, the peak-to-peak value, which is the altitude of a signal from the crest to the trough, is used to quantify the variation of control signal, as listed in Table 2. It is noted that the peak-to-peak value of both states or control input increases with increment either in the amplitude or the frequency of the external sinusoidal disturbance. Being consistence with the conclusions above, the LQR coupled with coordinate transformation *iii* (i.e., system C) outputs the least peak-to-peak value.

V. CONCLUSIONS AND DISCUSSIONS

A. CONCLUSIONS

In this paper, a novel concept of nonlinear controller design targeted for the MLSs has been presented, which can regulate the position of an iron ball subject to sinusoidal disturbances. The key approach for implementing the proposed controller lies in the pioneering nonlinear coordinate transformations which is in line with the requirement of practical applications. In the absence of prior knowledge of periodic disturbances, the proposed controller can improve the control performance of the classical linearization feedback controller in the aspect of robustness to consistent sinusoidal disturbance forces, in spite of employing conventional linearization techniques. Simulation results have demonstrated the capability of the proposed controllers in terms of both stabilization of the iron ball and attenuating disturbances.

B. DISCUSSIONS

Despite satisfactory results have been obtained in this work, the proposed control method is still far from mature. From the perspective of the authors, there are several topics that require further research.

1) Due to conciseness of the framework and flexibility of the synthesis, the proposed concept in this work is highly compatible with other advanced control methods, such as the sliding mode control and predictive control, but not confined to the LQR. It is important to proceed in combination of the nonlinear transformation and other proven control theories.

2) The robustness of the proposed controller to a single periodic disturbance force with constant parameters have been validated. However, it is essential to consider a linear combination of sinusoids. Furthermore, the random distortion should be seriously considered as well.

3) The experimental results may better reveal feasibility and capability of the proposed controller design. Hence, it is a quite significant work to carry out experimental validations and comparations to other new research findings.

4) Although the system under consideration in this work focuses on the magnetic levitation control of an iron ball, the design concept is readily to be integrated into other industrial applications. There is a significant potential in applying the method in maglev bearing systems, mechanical arms, and other nonlinear systems.

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