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Dual-Period Repetitive Control for Nonparametric Uncertain Systems With Deadzone Input

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ABSTRACT In this paper, a dual-period repetitive control scheme is proposed to accomplish the periodic trajectory tracking task for a class of nonparametric uncertain systems with deadzone nonlinear input, and there is no common multiple between the period length of periodic disturbance and that of reference signal. A dual-period repetitive controller is designed based on the Lyapunov synthesis. The nonparametric uncertainties, deadzone nonlinearity and periodic disturbance are compensated by using dual-period repetitive control and robust control, combinedly. The fully saturated learning strategy is applied to estimate the unknown periodic disturbance. As the repetitive cycle increases, the system error converges to zero. In the end, two illustrative examples are provided to demonstrate the effectiveness of the proposed dual-period repetitive control scheme.

INDEX TERMS Dual-period repetitive control, nonparametric systems, deadzone, adaptive control.

I. INTRODUCTION

Repetitive manufacturing tasks widely exist in industrial applications, such as welding, drilling, welding, drilling, and painting automobile body parts on an assembly line. As a result, tracking periodic trajectories and rejecting periodic disturbances are the common control problems in such actual projects. For dealing with such control tasks, in the late 1970s, Uchiyama [1] and Omata *et al.* [2] have originally proposed repetitive control (RC) schemes to solve the trajectory-tracking problem for robot manipulators. In RC systems, good perfect control performances may achieved by repetitively updating the control input according to the information of pervious periods, without an accurate modeling in prior. Nowadays, RC has been widely used in the accurate control for many industrial equipments such as permanent-magnet synchronous motor [3], hard disk drives [4], and so on.

In early studies on RC, frequency domain analysis method has often adopted to the controller design for linear time-invariant systems [5]. Since the beginning of this century, the RC algorithm design for nonlinear dynamics systems has attracted much attention. In [7], a Lyapunov-based RC scheme was developed for a class of nonlinear systems with an periodic exogenous disturbance. In [8], a robust repetitive learning controller was designed for a class of nonparametric uncertain systems. Reference [9] addressed the adaptive asymptotic rejection of unmatched periodic disturbances in nonlinear systems, with the output-feedback controller designed by using repetitive control technique. In [10], a suboptimal repetitive learning control algorithm was proposed for a class of nonlinear systems, which converge faster than non-optimal repetitive learning control systems. In [11], the observer-based repetitive learning control was developed for a class of nonlinear systems with non-parametric uncertainties. In [12], an adaptive backstepping repetitive learning algorithm was proposed to attenuate periodic uncertainties in nonlinear discrete-time systems.

Up to now, most existing RC algorithm results belong to single-period RC approaches. However, for the systems that there is no common multiple among the period lengths of parametric uncertainties and periodic disturbances, or the common multiple is very difficult to be founded even if it exists, single-period RC is not suitable for controller design; as a replacement, multi-period RC may be applied for

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these cases. So far, the academic concern little with multi-period repetitive control, and the relevant research results are few. Among these few achievements, most of them considered the control algorithms for linear systems [13]–[15], and the ones considering the control for nonparametric uncertain systems are very few [16]. It is really a significant job to carry out further researches on multi-period repetitive control for nonparametric uncertain systems.

On the other hand, as a class of important non-smooth nonlinearities, deadzones lie in the actuators of many industrial processes such as valves, DC servo motors and etc. The deadzone nonlinearities degrade the control performance, more or less. They may lead to the instability of control systems in serious cases. The studies on how to deal with deadzone nonlinearities have been carried out for long [17]. Tao and Kokotovic proposed a direct adaptive compensating method, by constructing an adaptive deadzone inverse to compensate deadzones [18]. Later on, in order to easily implement adaptive control algorithms for nonlinear system with deadzone input, Zhou et al. made further efforts on direct compensating deadzone, by constructing a smooth adaptive deadzone inverse [19]. As an alternative approach, a robust adaptive compensating technique was investigated to deal with the symmetric deadzone nonlinearity in [20]. In detail, the deadzone nonlinearity was decomposed into a linear parametric uncertain term and a disturbance, which bring convenience for the implementation of adaptive control and robust control. Reference [21] investigate the robust adaptive control for systems with unknown nonsymmetric deadzones, which are more general than the ones in [20]. Besides, intelligent control methods using neural networks [22] or fuzzy systems [23] were developed to handle deadzone nonlinearities. Up to now, the controller designs for systems with nonlinearity input have still been an interesting topic. For latest relevant results, see [24]–[27].

Motivated by the above discussions, this work focuses on the trajectory-tracking problem for a class of nonparametric uncertain systems with deadzone iput. There is no common multiple between the period length and that of time-varying period disturbance, or the common multiple is difficult to be obtained even if it exists. Compared to the existing results, the main contributions of this work mainly lie in as follows:

(1) Our proposed dual-period RC scheme is suitable to solve the trajectory-tracking problem for the cases that no common multiple between the period length of periodic disturbance and the one of reference signal, whereas the traditional single-period RC is not suitable for the controller design in these cases.

(2) The nonparametric uncertainty satisfied Lipschitz continuous condition and uncertain deadzone are handled by using robust control technique and repetitive learning control technique, combinedly.

(3) Full saturation learning strategy is used to estimate the unknown parametric uncertainties or the bound of bounded disturbance. Through rigorous analysis, all signals in the closed loop system are guaranteed to be bounded.

The rest of this paper is organized as follows. Problem formulation is presented in Section 2. In Section 3, by using Lyapunov synthesis, the dual-period repetitive control law and adaptive repetitive learning laws are developed. In Section 4, the uniform convergence of the closed loop system is proved. To demonstrate the effectiveness of the proposed dual-period repetitive control scheme, two illustrated examples are shown in Section 5, followed by Section 6 which concludes the work.

II. PROBLEM FORMULATION

Consider the following class of nonlinear systems

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t), \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_n(t) = f(\boldsymbol{x}(t)) + gu(v) + w(t), \end{cases} \tag{1}$$

in which, $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $g \in \mathbb{R}$ is an unknown positive constant, $f(\cdot)$ is an unknown smooth function such that

$$|f(\boldsymbol{\xi}_1) - f(\boldsymbol{\xi}_2)| \le \alpha(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)\|\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2\|, \quad \forall \boldsymbol{\xi}_1, \boldsymbol{\xi}_2 \in \mathbb{R}^n \tag{2}$$

with $\alpha(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)$ being a known smooth function, $w(t)$ is a periodic disturbance of known period T_w, and there is no common multiple between T_d and T_w; $u(v)$ and $v(t)$ are respectively the input and the output of an unknown deadzone defined as follows:

$$u(v) = \begin{cases} m_r(v - b_r) & v \ge b_r \\ 0 & b_l \le v < b_r \\ m_l(v - b_l) & v < b_l \end{cases} \tag{3}$$

The deadzone nonlinearity considered in this paper are similar to the one has been investigated in [20]:

(A1) The deadzone output $u(v)$ is not available for measurement.

(A2) The deadzone slopes in positive and negative region are same, i.e. $m_r = m_l = m$.

(A3) The deadzone parameters b_r, b_l and m are unknown, but their signs are known: $b_r > 0, b_l < 0, m > 0$.

The control objective is to design a dual-period repetitive control law for $v(t)$ to let $\boldsymbol{x}$ track the desired trajectory $\boldsymbol{x}_d = [x_{d,1}, x_{d,2}, \ldots, x_{d,n}]^{\mathrm{T}} = [x_{d,1}, \dot{x}_{d,1}, \ldots, x_{d,1}^{(n-1)}]^{\mathrm{T}}$ in the sense that $\lim_{t\to\infty}(\boldsymbol{x}(t) - \boldsymbol{x}_d(t)) = \boldsymbol{0}$.

For brevity, in this paper, the function argument t will be sometimes omitted while no confusion occurs.

Remark 1: In many iterative learning control and RC results, a common assumption is that system uncertainties are parameterizable, which means that the structure of system uncertainties is known but the parameters are unknown, or the uncertainties can be divided into known iteration-dependent/repetition-dependent and unknown iteration-independent/repetition-independent functions. An example of this sort may be seen in [6]: $d(\boldsymbol{x}, t) = \Theta(t)\xi(\boldsymbol{x}, t)$, with $\Theta(t)$ being an unknown continuous time varying parameter matrix and $\xi(\boldsymbol{x}, t)$ being a known vector function. Hence, the proposed algorithm in the above results may not be practical in some real applications with unparameterizable uncertainties [28]. In this work, we explore the dual-period repetitive control for nonlinear systems with nonparametric uncertainties. In (1), $f(\boldsymbol{x}(t))$ is a lumped, nonparameterizable, and local Lipschitzian nonlinear function, for example [8], $f(\boldsymbol{x}(t)) = \frac{x_2}{x_1^2+cos(t)+2}$.

III. CONTROL DESIGN

Let us define $\boldsymbol{e}(t) = [e_1(t), e_2(t), \ldots, e_n(t)]^{\mathrm{T}} = \boldsymbol{x} - \boldsymbol{x}_d$. It follows from (1) that

$$\begin{cases} \dot{e}_i = e_{i+1}, \quad i = 1, 2, \cdots, n-1 \\ \dot{e}_n = f(\boldsymbol{x}) + gu(v) + w - \dot{x}_{d,n}. \end{cases} \tag{4}$$

(4) can be rewritten in vector form as

$$\dot{\boldsymbol{e}} = A\boldsymbol{e} + \boldsymbol{b}(\boldsymbol{c}^{\mathrm{T}}\boldsymbol{e} + f(\boldsymbol{x}) + gu(v) + w - \dot{x}_{d,n}), \tag{5}$$

where,

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -c_1 & -c_2 & -c_3 & \cdots & -c_n \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix},$$

$\boldsymbol{c} = [c_1, c_2, \cdots, c_n]^{\mathrm{T}}$. $c_1, c_2, \cdots, c_n$ are the coefficients of Hurwitz Polynomial $p(s) = s^n + c_n s^{n-1} + \cdots c_2\, s + c_1$. Since A is Hurwitz, there exist positive positive definite symmetry matrices $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times n}$, which satisfy

$$PA + A^{\mathrm{T}}P = -Q. \tag{6}$$

Let us choose a Lyapunov function candidate as

$$V_1 = \frac{1}{2gm}\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{e}. \tag{7}$$

Taking the time derivative of V_1 yields

$$\begin{aligned} \dot{V}_1 = &-\frac{1}{2gm}\boldsymbol{e}^{\mathrm{T}}Q\boldsymbol{e} + \frac{1}{gm}\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\,(f(\boldsymbol{x}) - f(\boldsymbol{x}_d)) \\ &+ \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}[u(v) + \frac{1}{gm}\left(f(\boldsymbol{x}_d) + w + \boldsymbol{c}^{\mathrm{T}}\boldsymbol{e} - \dot{x}_{d,n}\right)] \\ &+ |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}| \cdot \max(b_r, |b_l|). \end{aligned} \tag{8}$$

From (2), we have

$$\begin{aligned} &\frac{1}{gm}\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}(f(\boldsymbol{x}) - f(\boldsymbol{x}_d)) \\ &\quad\le \frac{1}{gm}|\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\alpha_f(\boldsymbol{x}, \boldsymbol{x}_d)\|\boldsymbol{e}\| \\ &\quad\le \frac{1}{4gm}\lambda_Q\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \frac{4}{gm}\alpha_f^2(\boldsymbol{x}, \boldsymbol{x}_d)(\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b})^2. \end{aligned} \tag{9}$$

Substituting (9) into (8), we obtain

$$\begin{aligned} \dot{V}_1 \le &-\frac{1}{4gm}\lambda_Q\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \frac{4}{gm}\alpha_f^2(\boldsymbol{x}, \boldsymbol{x}_d)(\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b})^2 \\ &+ \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}[u + \frac{1}{gm}\left(f(\boldsymbol{x}_d) + w + \boldsymbol{c}^{\mathrm{T}}\boldsymbol{e} - \dot{x}_{d,n}\right)] \\ &+ |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}| \cdot \max(b_r, |b_l|) \\ \le &-\frac{1}{4gm}\lambda_Q\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}[u(v) + \frac{4}{gm}\alpha_f^2(\boldsymbol{x}, \boldsymbol{x}_d)\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b} \\ &+ \frac{1}{gm}\boldsymbol{c}^{\mathrm{T}}\boldsymbol{e} + \frac{1}{gm}(f(\boldsymbol{x}_d) - \dot{x}_{d,n}) + \frac{1}{gm}w)] \\ &+ |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|b_m. \end{aligned} \tag{10}$$

With the notations $\eta(t) = \frac{1}{gm}w(t)$, $\rho = \max(b_r, |b_l|)$, $\boldsymbol{\theta} = [\frac{4}{gm}, \frac{1}{gm}, \frac{1}{gm}(f(\boldsymbol{x}_d) - \dot{x}_{d,n})]^T$ and $\boldsymbol{\varphi}(\boldsymbol{x}) = [\alpha_f^2(\boldsymbol{x}, \boldsymbol{x}_d)\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}, \boldsymbol{c}^{\mathrm{T}}\boldsymbol{e}, 1]^T$, we rewrite (9) as

$$\dot{V}_1 \le -\frac{1}{4gm}\lambda_Q\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}[u(v) + \boldsymbol{\theta}^T\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{x}_d) + \eta] + |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\rho. \tag{11}$$

Note that the period length of $\eta(t)$ is T_w, the period length of $\boldsymbol{\theta}$ is T_d, and $T_w \ne T_d$. By virtue of (11), the control law and learning laws may be designed as follows:

$$u(v) = -\hat{\boldsymbol{\theta}}^T(t)\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{x}_d) - \hat{\eta}(t) - \hat{\rho}(t)\varpi_f - \mu_4\boldsymbol{e}^{\mathrm{T}}(t)P\boldsymbol{b}, \tag{12}$$

$$\begin{cases} \hat{\boldsymbol{\theta}}(t) = \mathrm{sat}(\hat{\boldsymbol{\theta}}^*(t)), \\ \hat{\boldsymbol{\theta}}^*(t) = \mathrm{sat}(\hat{\boldsymbol{\theta}}^*(t - T_d)) + \mu_1\gamma(t)\boldsymbol{\varphi}(\boldsymbol{x}, \boldsymbol{x}_d)\boldsymbol{e}^{\mathrm{T}}(t)P\boldsymbol{b}, \end{cases} \tag{13}$$

$$\begin{cases} \hat{\rho}(t) = \mathrm{sat}(\hat{\rho}^*(t)), \\ \hat{\rho}^*(t) = \mathrm{sat}(\hat{\rho}^*(t - T_d)) + \mu_2\gamma(t)|\boldsymbol{e}^{\mathrm{T}}(t)P\boldsymbol{b}| \end{cases} \tag{14}$$

and

$$\begin{cases} \hat{\eta}(t) = \mathrm{sat}(\hat{\eta}^*(t)), \\ \hat{\eta}^*(t) = \mathrm{sat}(\hat{\eta}^*(t - T_w)) + \mu_3\gamma(t)\boldsymbol{e}^{\mathrm{T}}(t)P\boldsymbol{b}, \end{cases} \tag{15}$$

where $\mu_1 > 0, \mu_2 > 0, \mu_3 > 0, \mu_4 > 0$; $\hat{\boldsymbol{\theta}}(t) = 0, \hat{\rho}^*(t) = 0, t \in [-T_d, 0]$; $\hat{\eta}^*(t) = 0, t \in [-T_w, 0]$,

$$\varpi_f = \begin{cases} \tanh(\dfrac{\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\hat{\rho}(t)}{\varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t}}), & \varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t} > 0, \\ 0, & \varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t} = 0, \end{cases} \tag{16}$$

$\varepsilon_1 > 0, \varepsilon_2 > 0$. In (13)-(15), $\gamma(t)$ is defined as

$$\gamma(t) = \begin{cases} 0, & t \le 0 \\ \omega(t), & 0 < t \le T_{\mathrm{m}} \\ 1, & t > T_{\mathrm{m}}, \end{cases} \tag{17}$$

where, $T_{\mathrm{m}} \triangleq \min(T_w, T_d)$ and

$$\omega(t) = 1 - \frac{10(T_{\mathrm{m}} - t)^3}{T_{\mathrm{m}}^3} + \frac{15(T_{\mathrm{m}} - t)^4}{T_{\mathrm{m}}^4} - \frac{6(T_{\mathrm{m}} - t)^5}{T_{\mathrm{m}}^5}, \quad t \in [0, T_{\mathrm{m}}]. \tag{18}$$

Remark 2: Theoretically, μ_1, μ_2, μ_3 and μ_4 may be any bounded positive numbers. In practical applications, the recommended ranges of design parameters are given as follows: $5 \le \mu_4 \le 15$, $0.5 \le \mu_1 \le 10$, $0.5 \le \mu_3 \le 10$, $0.01 \le \mu_2 \le 1$, $0.001 \le \varepsilon_1 \le 0.1$, $0.5 \le \varepsilon_2 \le 10$.

Note that in (12)-(15), dual-period repetitive learning strategy is adopted for control design. Since there is no common multiple between T_d and T_w, the single-period RC strategy is not suitable for the learning laws design in such an occasion.

For the cases that the minimum common multiple between T_d and T_w is much larger than $\max(T_d, T_w)$, the dual-period repetitive control closed-loop systems converge faster than the single-periodic repetitive control ones.

Remark 3: The relationship among T_d, T_w and their minimum common multiple T_{cm} may be summarized as follows:

Case 1. T_{cm} does not exist, for example, $T_d = 4$, $T_w = 2\pi$, or $T_d = \sqrt{2}$, $T_w = \mathrm{e}$.

Case 2. $T_{cm} \gg \max(T_d, T_w)$, for example, $T_d = 9$, $T_w = 11$, $T_{cm} = 99$.

Case 3. T_{cm} and $\max(T_d, T_w)$ are equal, for example, $T_d = 1.5$, $T_w = 3$,$T_{cm} = 3$.

Case 4. T_{cm} is close to $\max(T_d, T_w)$, for example, $T_d = 1.5$, $T_w = 2$, $T_{cm} = 3$.

For case 1, $f(\boldsymbol{x}_d) + w(t)$ can not be compensated by using single-period learning approach. For case 2, both dual-period learning approach and single-periodic learning approach can be used to design learning laws, and dual-period learning approach is recommended for higher convergence speed. For case 3 and 4, both dual-period learning approach and single-period learning approach may be used to design learning laws for compensating $f(\boldsymbol{x}_d) + w(t)$, and the error convergence speed of the two approaches is close to each other. The comparison between dual-period RC and single-period RC is summarized in Table 1.

TABLE 1. Comparison between dual-period RC and single-period RC.

	Dual-period RC	Single-period RC
Case 1	applicable	non-applicable
Case 2	converge fast	converge slowly
Case 3,4	applicable	applicable

IV. STABILITY ANALYSIS

The stability of the closed-loop system described by (1) and (12)-(15) is established in the following theorem:

Theorem 1: For the closed-loop system consisting of the plant (1), and the repetitive control given by (12)-(15), all system variables are guaranteed to be bounded for $t \in [0, +\infty)$, and the closed-loop system is stable in the sense that

$$\lim_{t\to+\infty} \boldsymbol{e}(t) = 0. \tag{19}$$

Proof: Substituting (12) into (11), we get

$$\dot{V}_1 \le -\frac{1}{4gm}\lambda_Q \boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta} + |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\rho - \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\hat{\rho}\varpi_f - \mu_4(\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b})^2, \tag{20}$$

where, $\tilde{\eta}(t) = \eta(t) - \hat{\eta}(t)$, $\tilde{\boldsymbol{\theta}}(t) = \boldsymbol{\theta}(t) - \hat{\boldsymbol{\theta}}(t)$.

Note that the hyperbolic tangent function $\tanh(\cdot)$ has the following property:

$$0 \le |\varrho| - \varrho\tanh(\frac{\varrho}{h}) < 0.2785h, \tag{21}$$

where $h > 0$ and $\varrho \in \mathbb{R}$. Then,

$$\begin{aligned}
&|\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\rho - \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\hat{\rho}\varpi_f \\
&\quad = |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\rho - |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\hat{\rho} + |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\hat{\rho} - \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\hat{\rho}\varpi_f \\
&\quad = |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\tilde{\rho} + 0.2785\varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t}
\end{aligned} \tag{22}$$

holds, with $\tilde{\rho} = \rho - \hat{\rho}$. By virtue of (20) and (22), we have

$$\dot{V}_1 \le -\frac{1}{4gm}\lambda_Q \boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta} + 0.2785\varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t}. \tag{23}$$

The following Lyapunov functional is introduced to facilitate the analysis

$$V_2 = V_1 + \frac{1}{2\mu_1}\int_{t-T_d}^{t}\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}}d\tau + \frac{1}{2\mu_2}\int_{t-T_w}^{t}\tilde{\eta}^2(\tau)d\tau. + \frac{1}{2\mu_3}\int_{t-T_d}^{t}\tilde{\rho}^2(\tau)d\tau \tag{24}$$

By virtue of (23), the time derivative of V_2 is

$$\begin{aligned}
\dot{V}_2 \le& -\frac{1}{4gm}\lambda_Q \boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta} \\
&+ 0.2785\varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t} + \frac{1}{2\mu_1}\big[\tilde{\boldsymbol{\theta}}^T(t)\tilde{\boldsymbol{\theta}}(t) - \\
&\tilde{\boldsymbol{\theta}}^T(t-T_d)\tilde{\boldsymbol{\theta}}(t-T_d)\big] + \frac{1}{2\mu_2}(\tilde{\eta}^2(t) - \tilde{\eta}^2(t-T_w)) \\
&+ \frac{1}{2\mu_3}(\tilde{\rho}^2(t) - \tilde{\rho}^2(t-T_d)).
\end{aligned} \tag{25}$$

Note that $w(t)= w(t - T_w)$ and while $t \ge T_{\mathrm{m}}$, $\gamma(t) = 1$. According to (15), for $t \ge T_{\mathrm{m}}$,

$$\begin{aligned}
&\frac{1}{2\mu_2}(\tilde{\eta}^2(t) - \tilde{\eta}^2(t-T_w)) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta}(t) \\
&\quad = \frac{1}{2\mu_2}(2\eta(t) - 2\hat{\eta}(t) + \hat{\eta}(t) - \hat{\eta}(t-T_w)) \\
&\qquad \times(\hat{\eta}(t-T_w) - \hat{\eta}(t)) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta}(t) \\
&\quad \le \frac{1}{\mu_2}\tilde{\eta}(t)(\hat{\eta}(t-T_w) - \hat{\eta}(t)) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\eta}(t) \\
&\quad = \frac{1}{\mu_2}[\eta(t) - \mathrm{sat}(\hat{\eta}^*(t))][\hat{\eta}^*(t) - \mathrm{sat}(\hat{\eta}^*(t))] \\
&\quad \le 0.
\end{aligned} \tag{26}$$

Similarly, while $t \ge T_{\mathrm{m}}$, by using (13) and (14), respectively, we obtain

$$\begin{aligned}
&\frac{1}{2\mu_1}(\tilde{\boldsymbol{\theta}}^T(t)\tilde{\boldsymbol{\theta}}(t) - \tilde{\boldsymbol{\theta}}^T(t-T_d)\tilde{\boldsymbol{\theta}}(t-T_d)) \\
&\quad + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T(t)\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) \\
&= \frac{1}{2\mu_1}[2\boldsymbol{\theta}(t) - 2\hat{\boldsymbol{\theta}}(t) + \hat{\boldsymbol{\theta}}(t) - \hat{\boldsymbol{\theta}}(t-T_d)]^T[\hat{\boldsymbol{\theta}}(t-T_d) \\
&\quad - \hat{\boldsymbol{\theta}}(t)] + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T(t)\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) \\
&\le \frac{1}{\mu_1}\tilde{\boldsymbol{\theta}}^T(t)(\hat{\boldsymbol{\theta}}(t-T_d) - \hat{\boldsymbol{\theta}}(t)) + \boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}\tilde{\boldsymbol{\theta}}^T(t)\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{x}_d) \\
&\le \frac{1}{\mu_1}[\boldsymbol{\theta} - \mathrm{sat}(\hat{\boldsymbol{\theta}}^*(t))]^T[\hat{\boldsymbol{\theta}}^*(t) - \mathrm{sat}(\hat{\boldsymbol{\theta}}^*(t))] \\
&\le 0
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
&\frac{1}{2\mu_3}(\tilde{\rho}^2(t) - \tilde{\rho}^2(t - T_d)) + |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\tilde{\rho}(t) \\
&\quad\le \frac{1}{\mu_3}\tilde{\rho}(t)(\hat{\rho}(t - T_d) - \hat{\rho}(t)) + |\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|\tilde{\rho}(t) \\
&\quad\le \frac{1}{\mu_3}(\rho - \hat{\rho}(t))(\hat{\rho}(t - T_d) - \hat{\rho}(t) + \mu_3|\boldsymbol{e}^{\mathrm{T}}P\boldsymbol{b}|) \\
&\quad= \frac{1}{\mu_2}[\rho - \mathrm{sat}(\hat{\rho}^*(t))][\hat{\rho}^*(t) - \mathrm{sat}(\hat{\rho}^*(t))] \\
&\quad= 0. \qquad (28)
\end{aligned}$$

Substituting (26)-(28) into (25) yields

$$\dot{V}_2 \le -\frac{1}{4gm}\lambda_Q\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + 0.2785\varepsilon_1\,\mathrm{e}^{-\varepsilon_2 t}, \quad \forall t \ge T_{\mathrm{m}}. \quad (29)$$

Then, from (29), we deduce that

$$\begin{aligned}
V_2(t) &\le V_2(T_{\mathrm{m}}) - \frac{1}{2g}\lambda_m(Q)\int_{T_{\mathrm{m}}}^{t}\|\boldsymbol{e}(\tau)\|^2 d\tau \\
&\quad + 0.2785\varepsilon_1\int_{T_m}^{t}\mathrm{e}^{-\varepsilon_2 t}d\tau \\
&\le V_2(T_{\mathrm{m}}) - \frac{1}{2g}\lambda_m(Q)\int_{T_{\mathrm{m}}}^{t}\|\boldsymbol{e}(\tau)\|^2 d\tau \\
&\quad + 0.2785\frac{\varepsilon_1}{\varepsilon_2}\,\mathrm{e}^{-\varepsilon_2 T_m} \qquad (30)
\end{aligned}$$

holds for $t \ge T_{\mathrm{m}}$, where $\lambda_m(Q)$ represents the minimum eigenvalue of matrix Q.

From (30), we can assert that $\boldsymbol{e}(t)$ and $\int_{T_{\mathrm{m}}}^{t}\|\boldsymbol{e}(\tau)\|^2 d\tau$ are bounded. By the property of saturation function, the boundedness of $\dot{\boldsymbol{e}}$ can be concluded from (5). By Barbalet's lemma, we have

$$\lim_{t\to\infty}\boldsymbol{e}(t) = 0. \quad (31)$$

■

V. SIMULATION RESULTS

In this section, we will illustrate the efficiency of the proposed RC approach. Consider the following second oreder nonlinear systems as follows:

$$\begin{cases}\dot{x}_1 = x_2, \\ \dot{x}_2 = f(x_1, x_2) + w(t) + u(v)\end{cases} \quad (32)$$

where, $f(x_1, x_2) = -0.1x_2 - x_1^3$, $g = 1$, $[x_1(0),\ x_2(0)]^T = [0.7, 0.2]^T$, the parameters of the deadzone are $b_r = 0.5$, $b_l = -0.6$, $m = 1$. The control objective is to make $\boldsymbol{x} = [x_1, x_2]^T$ track $\boldsymbol{x}_d = [x_{d,1}, x_{d,2}]^T$.

Example 1: Consider Case 3 in Remark 1. Let $w(t) = 12\cos(t)$, $\boldsymbol{x}_d = [\cos(\pi t), -\pi\sin(\pi t)]^{\mathrm{T}}$, $T_d = 2$ and $T_w = 2\pi$. T_{cm} does not exist. (12)-(15) are applied as the repetitive control law and adaptive learning laws, and the control parameters are chosen as $\mu_1 = 6$, $\mu_2 = 6$, $\mu_3 = 0.1$, $\mu_4 = 10$, $\varepsilon_1 = 0.01$, $\varepsilon_2 = 1$, $\boldsymbol{c} = [1, 1]^T$,

$$P = \begin{pmatrix} 12 & 4 \\ 4 & 8 \end{pmatrix}.$$

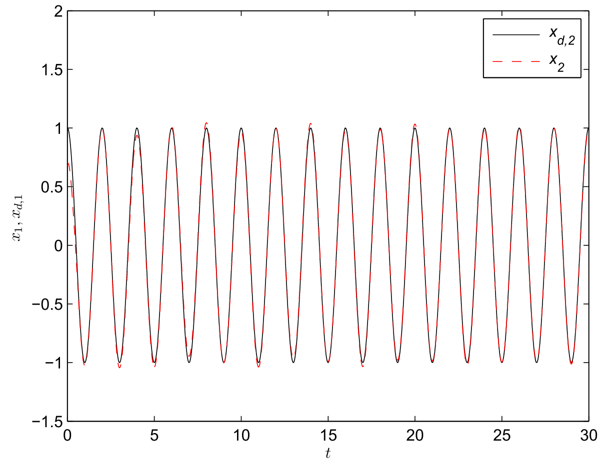

FIGURE 1. $\boldsymbol{x_1}$ **and** $\boldsymbol{x_{d,1}}$ **(Example 1).**

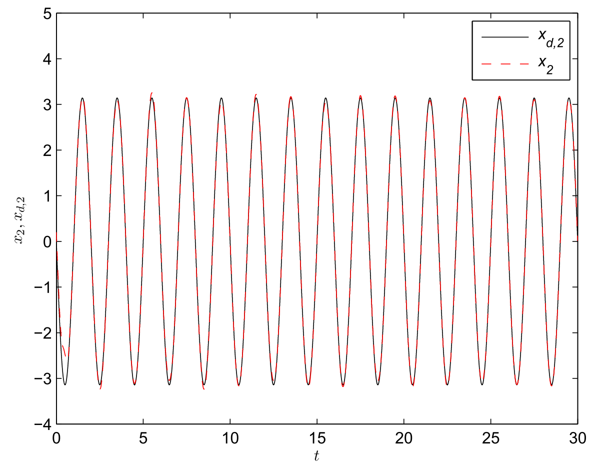

FIGURE 2. $\boldsymbol{x_2}$ **and** $\boldsymbol{x_{d,2}}$ **(Example 1).**

$\alpha(\boldsymbol{x}, \boldsymbol{x}_d)$ is chosen as $\sqrt{(3(x_1^2 + x_{d,1}^2)^2 + 0.01}$, which satisfies the condition given in (2). Simulation results are shown in Figs. 1-5. Figs. 1-2 show the tracking performance of x_1 and x_2 for $t \in [0, 30]$. Figs. 3-4 show the state tracking error for $t \in [0, 80]$. Fig. 5 shows the input control signal of deadzone $v(t)$. From Figs. 1-4, we can see the dual-periodic RC method is effective to solve trajectory-tracking problem for Case 1 in Remark 3, whereas the single-periodic RC approach is unsuitable to be adopted for this case.

Remark 4: In Example 1, $T_d = 2$ and $T_w = 2\pi$ and their common multiple T_{cm} does not exist. Thus, dual-period RC is suitable to design control law for the controlled system in this example, whereas traditional single-period RC can not be adopted here.

Example 2: In this example, we will compare the control performance between dual-period repetitive control and single-period repetitive control for Case 2 in Remark 3. $\boldsymbol{x}_d = [\cos(\frac{2\pi t}{3.1}), -\frac{2\pi}{3.1}\sin(\frac{2\pi t}{3.1})]^{\mathrm{T}}$, $w(t) = 12\cos(\pi t)$, $T_d = 3.1$, $T_w = 2$ and $T_{cm} = 62$. It is easy to see $T_{cm} \gg \max(T_d, T_w)$. (12)-(15) are applied as the repetitive control law and adaptive learning laws, with the control parameters and $\alpha(\boldsymbol{x}, \boldsymbol{x}_d)$ chosen as the same as the ones in Example 1.

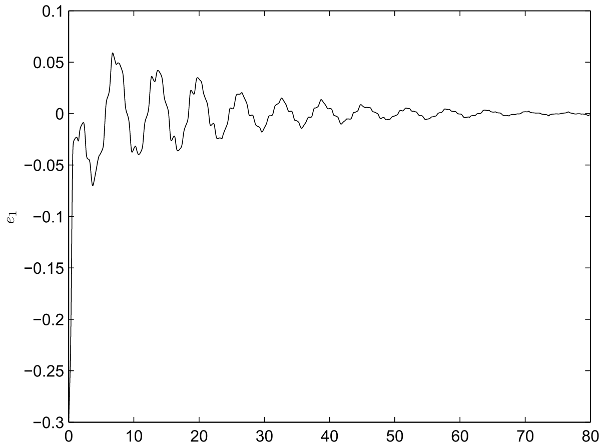

FIGURE 3. The error e_1 (Example 1).

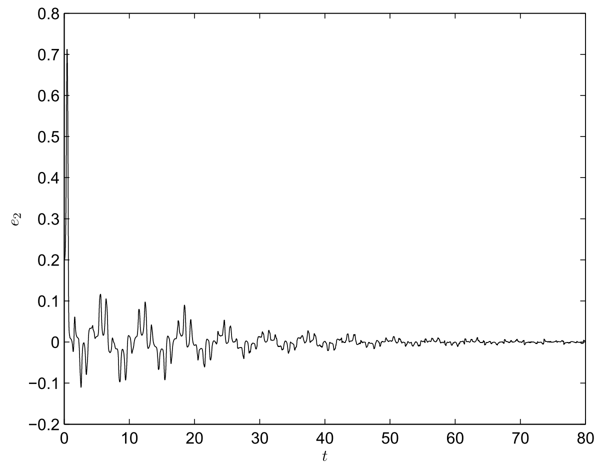

FIGURE 4. The error e_2 (Example 1).

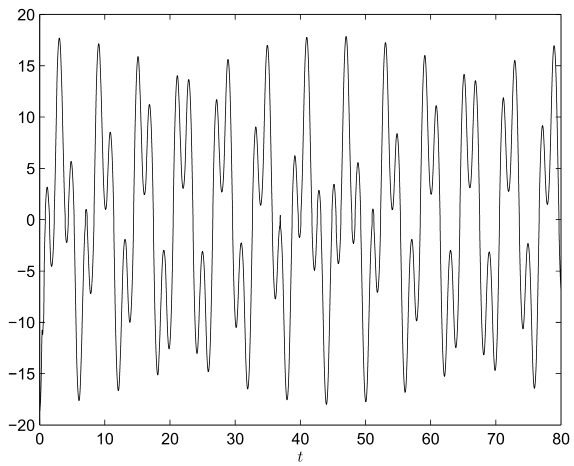

FIGURE 5. Control input v (Example 1).

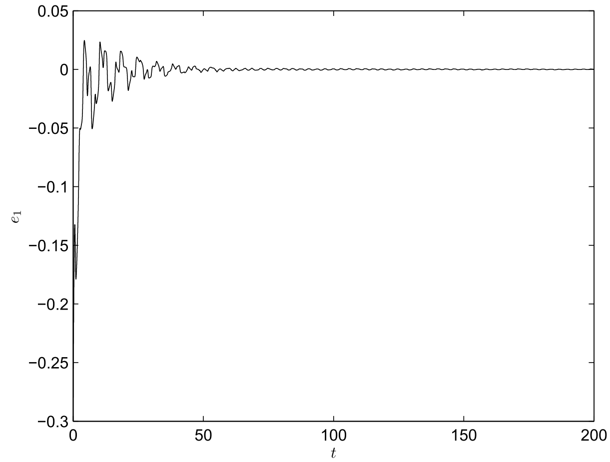

FIGURE 6. The error e_1 (Example 2, dual period).

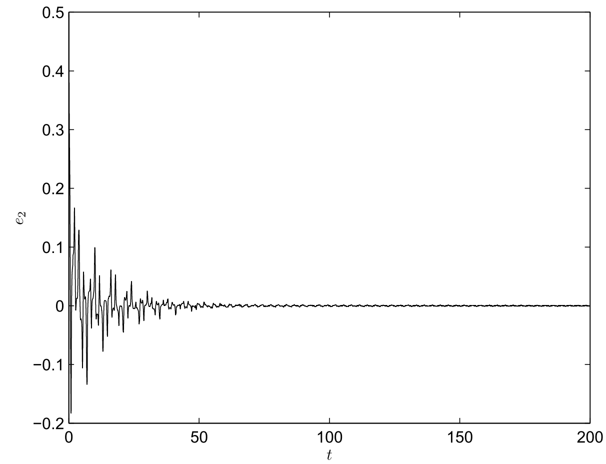

FIGURE 7. The error e_2 (Example 2, dual period).

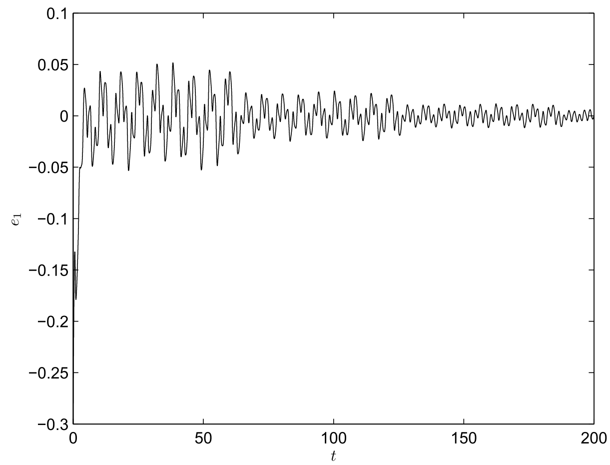

FIGURE 8. The error e_1 (Example 2, single period).

The state error profiles in the dual-period RC system are given in Figs. 6-7. Figs. 6-7 illustrate that the system error in the dual-period RC system converges to zero as the time increases. For comparison, the corresponding state error convergence results in the single-periodic RC system are given in Figs. 8-9. Figs. 8-9 illustrate that the system error in the dual-period RC system also converges to zero as time increases. Therefore, both dual-period RC and single-period RC are efficient to solve the trajectory-tracking problem for Case 2 in Remark 1. Comparing Fig. 6 with Fig. 8, we can see that the $e_1(t)$ in the dual-period RC system converges much faster than the one in the single-period RC system. Similarly, comparing Fig. 7 with Fig. 9, we can see

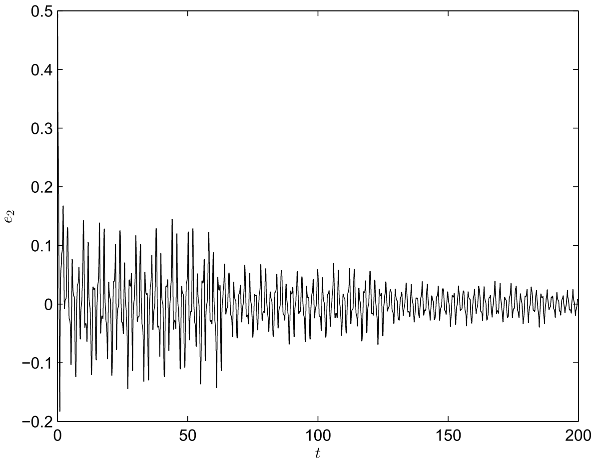

FIGURE 9. The error e_2 (Example 2, single period).

that the $e_2(t)$ in the dual-period RC system converges much faster than the one in the single-period RC system.

The results given in Example 1 and Example 2 verify the effectiveness of the proposed dual-single RC approach.

Remark 5: The results in Example 2 show that dual-period RC has higher convergence speed than traditional single-period RC for Case 2 In Remark 3. Actually, in some situations, the common multiple between T_d and T_d exists, but the common multiple is very difficult to be obtain obtained. Dual-period RC is more suitable to be adopted for such situations than single-period RC.

VI. CONCLUSION

In this work, we have proposed a dual-period RC scheme to solve periodic trajectory-tracking problem for a class of nonparametric uncertain systems with deadzone input. Since the common multiple between the period of periodic disturbance and that of the reference signal does not exist, the traditional single-period RC approach is not suitable to be adopted in controller design. The proposed dual-periodic RC scheme is also of significance for the cases that the minimum common multiple between the period length of reference signal and that of disturbance is much larger than the maximum of the two period lengths, which helps to achieve higher convergence speed than traditional single-periodic RC scheme. Two illustrative examples are provided to demonstrate the effectiveness of the proposed dual-period repetitive control scheme. The next step of this work is to investigate the multi-period RC for nonlinear systems with deadzone input.

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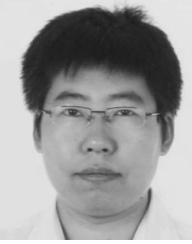

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