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Integral Backstepping and Synergetic Control of Magnetic Levitation System

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ABSTRACT Magnetic Levitation Systems are used to levitate a ferromagnetic object in the air. It has a wide area of applications because it eradicates energy losses that occur due to friction of the surface. In this paper, nonlinear controllers have been designed by using backstepping, integral backstepping and synergetic control techniques to obtain certain control objectives. Nonlinear controllers have been designed because of nonlinear dynamics present in the system model. It is required to generate a certain amount of flux by applying control input to the system. The magnetic flux is then used to levitate the body in air at a certain distance from the coil so that the movement of the body within that magnetic flux is negligible. The magnetic force provides an acceleration against the earth gravitational force to lift the body towards the coil. For each nonlinear controller, Lyapunov based theory has been used to check the global asymptotic stability of the system. MATLAB/Simulink environment is then used to analyze the system's performance for the proposed controllers. Moreover, a comparative analysis of proposed controllers has been given with linear (PI) controller.

INDEX TERMS Magnetic levitation (MAGLEV) system, nonlinear controller, integral backstepping (IBS) controller, synergetic controller, backstepping controller.

I. INTRODUCTION

In MAGLEV system an object is levitated in air without any kind of support using a magnetic force. This system has brought a way to overcome the problem of friction for the isolation of vibrations. So the friction loss in this system is negligible which usually affects the desired response and hence the optimum performance of the system [1]–[3].

The MAGLEV system is the future technology and has the vast area of applications. This system is popular due to its non-contact property and zero friction and have many application such as in high-speed transportation systems [1], [4]–[6], brushless DC motors [7]–[10], bio-medical devices [11], launching rockets [12], [13], levitation of molten metals in induction furnaces and that of heavy metal slabs during manufacturing etc. This also helps to increase system longevity as there is no wear and tear of moving parts.

It can be systematized into attractive and repulsive nature based on levitation forces. To stabilize this type of system, many control designers are trying to design different control algorithms. It has nonlinear dynamical behavior and is highly unstable which interpose additional difficulties to control these systems. So, for better performance of the system, a good dynamical model and controller are required.

In recent years, there has been active research done to enhance the performance of this system. A number of mathematical models using different parameters have been proposed in different papers having two or three states. A. Saberi [14] has proposed a standard control scheme for linear systems in which reference was generated by an exosystem. This technique has been used for the regulation of linear systems, so the performance of the system has been achieved only in the close vicinity of the equilibrium point. Later on, Ramos *et al.* [15] has extended this work for nonlinear dynamical models. A linear quadratic Gaussian (LQG) based PID controller has been proposed in [16], which uses the LQG controller along with PID for position estimation. Also, other types of controllers based on linear and nonlinear methods have been reported in the literature such as: nonlinear feedback linearization control [17], integral variable-structure grey control [18], robust linear control methods such as H-infinity optimal control [19] and Q-parametrization as discussed in [20]–[23]. A variable gradient technique has been used on the mathematical model of MAGLEV system

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in [24] and a controller has been designed. But the results have delayed convergence and overshoots/undershoots which may affect the system's performance.

The backstepping control algorithm is a recursive technique that uses the Lyapunov stability theory to design a controller for nonlinear systems. To design a controller by using the backstepping algorithm, we have to transform the system into strict feedback form and then divide it into subsystems. For each subsystem, the control law is designed and we have to ensure the stability of the system at each step. The IBS algorithm is a slight modification of the backstepping algorithm in which an integral term is added which helps the system for fast convergence of the states to the reference value and to reduce the steady state error. Synergetic controller uses a macro-variable function which takes into account all the error values involved in the system and helps to track the system references and convergence in finite time [18], [25].

In this paper, three nonlinear controllers have been proposed namely backstepping, IBS and synergetic controller for obtaining certain control objectives. The main control objective is to generate certain magnitude of magnetic flux so that the ball can levitate at desired distance from the center of the coil. The proposed nonlinear controllers are then compared with linear PI controller to show the difference in the performance of the system.

The paper has been arranged as follows: The electrical circuitry of the MAGLEV system has been discussed in section II. The nonlinear mathematical model of the system has been described in section III. The control design methodology has been discussed in section IV. Control design procedure for IBS controller has been discussed in subsection IV-A, backstepping controller in subsection IV-B and synergetic controller in subsection IV-C. The simulation results and comparative analysis of proposed controllers has been discussed in Section V and at last, the conclusion has been highlighted in Section VI.

II. ELECTRIC CIRCUITRY OF MAGLEV SYSTEM

The MAGLEV system is an electro-mechanical system which consists of an electrical and a mechanical part. The electrical one consists of a current-carrying coil having N number of turns called solenoid and controlled DC power source having a potential difference of u across it. A coil is wrapped around a fixed iron core or ferromagnetic material in which all atoms are aligned in the same direction in the presence of a magnetic field. The coil has been used to transform the electrical signal coming from DC power source into mechanical movement [26].

In the mechanical part, we can adjust the distance of the metallic ball from the center of the solenoid. This can be done by changing the electrical signals coming from the DC power source. When the applied voltage u of the coil changes, the current I passing through it also changes which results in changing the magnitude of magnetic flux λ and the magnetic force f_e acting on the metallic ball. The magnitude of f_e helps to maintain the distance of the ball from the center of the solenoid by pulling it against the gravitational force f_g.

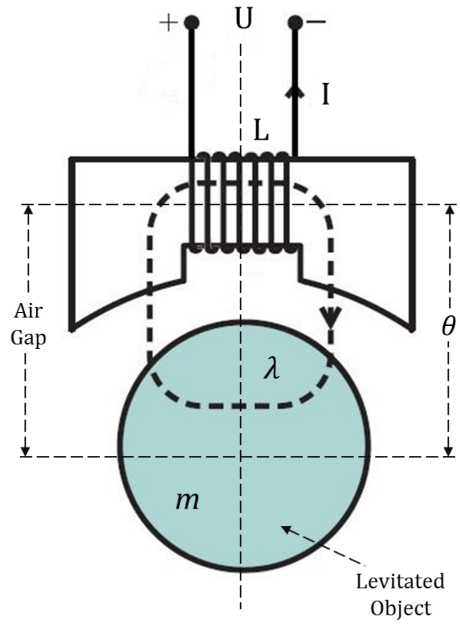

FIGURE 1. Electrical circuitry of magnetic levitation system [26].

The force f_g will pull the ball downward by an acceleration of $9.8ms^{-2}$ while the magnetic force f_e will pull the ball upward against gravity. The ball has been levitated in the air when f_e acting on the metallic ball will be equal to f_g and hence the net force f_{net} on the ball will be zero. So, if the magnitude of current I passing through the coil or solenoid increases with the increase of applied voltage u, then the strength of magnetic force f_e also increases which decreases the air gap and vice versa. In levitated position of the ball, we have:

$$f_{net} = f_e - f_g = 0 \tag{1}$$

III. MATHEMATICAL MODELLING OF THE SYSTEMS

To derive the mathematical model for a MAGLEV system, we have to analyze the dynamic behavior of the system by using its electrical and mechanical parts. By applying Kirchhoff's voltage law and Newton's second law on the MAGLEV system circuit shown in figure 1 as discussed in [20], [24], [26], we have:

$$\frac{d\lambda}{dt} = -Ri + u \tag{2}$$

$$m\frac{d^2\theta}{dt} = f_e - mg \tag{3}$$

The eq.(2) and eq.(3) are dynamical equations for the MAGLEV system in which θ shows the air gap between the metallic ball and center of the coil and λ is the magnitude of the magnetic flux produced by the current-carrying coil which is the function of the current I passing through the coil and air gap θ. The mass of the metallic ball has been represented by m and the earth's gravity that pulls the ball

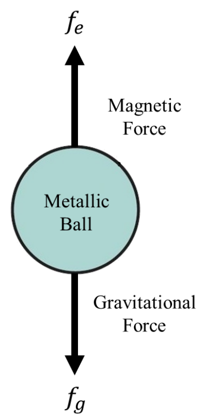

FIGURE 2. Levitation of metallic ball.

downward is g. If L is the inductance of the coil then flux produced by it is given as:

$$\lambda = Li \tag{4}$$

where;

$$L = \frac{k}{(1-\theta)} \tag{5}$$

where k is the positive constant whose value relies on the number of turns of coil [26]. By putting the value of L from eq.(5) in eq.(4), we have:

$$\lambda = \frac{k}{(1-\theta)} i \tag{6}$$

By re-arranging eq.(6) the amount of current flowing through the coil can be calculated in term of θ, λ and k as:

$$i = \frac{(1-\theta)\lambda}{k} \tag{7}$$

By putting the value of i from eq.(7) in eq.(2), we have:

$$\frac{d\lambda}{dt} = -R\frac{(1-\theta)\lambda}{k} + u \tag{8}$$

Now, the magnetic force f_e is proportional to the rate of change of coil inductance with respect to air gap given as:

$$f_e = \frac{1}{2}\frac{dL}{d\theta} i^2 \tag{9}$$

By putting the value of f_e from eq.(9) in eq.(3), we have:

$$m\frac{d^2\theta}{dt} = \frac{1}{2}\frac{dL}{d\theta} i^2 - mg \tag{10}$$

Representing the momentum ρ of the ball by the product of its mass m and velocity $\dot{\theta}$, we have:

$$\rho = m\dot{\theta} \tag{11}$$

By using eq.(11), we get:

$$\frac{d\theta}{dt} = \frac{\rho}{m} \tag{12}$$

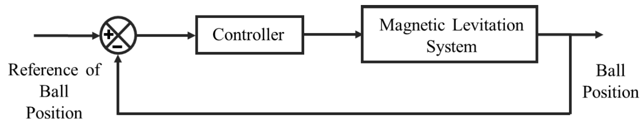

FIGURE 3. Control of MAGLEV system.

So, replacing $m\dot{\theta}$ by ρ in eq.(10), we get:

$$\dot{\rho} = \frac{1}{2}\frac{dL}{d\theta} i^2 - mg \tag{13}$$

Taking time derivative of eq.(5), we have:

$$\frac{dL}{d\theta} = \frac{k}{(1-\theta)^2} \tag{14}$$

From eq.(7), eq.(13) and eq.(14), we get:

$$\dot{\rho} = \frac{\lambda^2}{2k} - mg \tag{15}$$

From eq.(8), eq.(12) and eq.(15) a global mathematical model has been formed as:

$$\frac{d\lambda}{dt} = -R\frac{(1-\theta)}{k}\lambda + u \tag{16}$$

$$\frac{d\theta}{dt} = \frac{\rho}{m} \tag{17}$$

$$\frac{d\rho}{dt} = \frac{\lambda^2}{2k} - mg \tag{18}$$

Representing the air gap θ by y_1 and the momentum of the levitated body with in magnetic field ρ by y_2 and the magnetic flux λ by y_3, we get a simplified model of MAGLEV system [20], [24], [26], given as:

$$\dot{y}_1 = \frac{y_2}{m} \tag{19}$$

$$\dot{y}_2 = \frac{y_3^2}{2k} - mg \tag{20}$$

$$\dot{y}_3 = -R\frac{(1-y_1)}{k} y_3 + u \tag{21}$$

IV. CONTROLLER DESIGN METHODOLOGY

MAGLEV system eq.(19-21) is a nonlinear system because it has nonlinear dynamics due to y_3^2 and product of y_1 and y_3. For this, a good nonlinear controller is required to attain the control objectives of the system. Fig 3, shows the block diagram representation of closed loop system, actual ball position is used as a feedback for controller design. The difference between reference value and actual value of ball position has been taken in form of an error equation.

In this paper, three controllers have been designed by using nonlinear control techniques which include backstepping, IBS and synergetic control. These controllers have been designed in such a way that the following control objectives are achieved:

- Tracking the desired air gap between the metallic ball and the solenoid.
- Tracking of required magnetic flux to maintain the air gap.

- The momentum of the ball within the magnetic field should reach to zero.
- The whole system should be globally asymptotic stable.

A. INTEGRAL BACKSTEPPING CONTROLLER DESIGN PROCEDURE

To achieve our control objectives and to design a controller by using IBS technique, we introduce the error in momentum state y_2 of the system model as:

$$e_1 = y_2 - y_{2ref} \tag{22}$$

which is the difference between the actual momentum y_2 and its reference value y_{2ref}. To track the reference value perfectly, the error terms should converge to zero. So by taking the time derivative of e_1, we get:

$$\dot{e}_1 = \dot{y}_2 - \dot{y}_{2ref} \tag{23}$$

By putting the value of $\dot{y}_2$ from eq.(20) in eq.(23), we have:

$$\dot{e}_1 = \frac{y_3^2}{2k} - mg - \dot{y}_{2ref} \tag{24}$$

Now introducing an integral term ψ for fast convergence of the states to the reference values as:

$$\psi = \int_0^t (y_2 - y_{2ref})dt \tag{25}$$

Now by taking derivative of eq.(25) with respect to time, we get:

$$\dot{\psi} = y_2 - y_{2ref} \tag{26}$$

From eq.(22) and eq.(26), we get:

$$\dot{\psi} = e_1 \tag{27}$$

Consider a Lyapunov candidate function for the system as:

$$V_1 = \frac{1}{2}e_1^2 + \frac{\beta}{2}\psi^2 \tag{28}$$

By taking time derivative of eq.(28), we get:

$$\dot{V}_1 = e_1\dot{e}_1 + \beta\psi\dot{\psi} \tag{29}$$

Putting the value of $\dot{\psi}$ from eq.(27) in eq.(29) and simplifying, we get:

$$\dot{V}_1 = e_1(\dot{e}_1 + \beta\psi) \tag{30}$$

By putting the value of $\dot{e}_1$ from eq.(24) in eq.(30), we get:

$$\dot{V}_1 = e_1\left(\frac{y_3^2}{2k} - mg - \dot{y}_{2ref} + \beta\psi\right) \tag{31}$$

For the system to be stable, $\dot{V}_1$ should be negative definite. For this purpose, we take:

$$\frac{y_3^2}{2k} - mg - \dot{y}_{2ref} + \beta\psi = -c_1e_1 \tag{32}$$

where c_1 is the gain of the controller which is a positive constant. From eq.(31) and eq.(32), we get:

$$\dot{V}_1 = -c_1e_1^2 \leq 0 \tag{33}$$

Now we consider the state y_3 as virtual control law which is the desired reference for next state and denoting it by α and solving eq.(32) for it, we get:

$$\alpha = \left(2kmg + 2k\dot{y}_{2ref} - 2k\beta\psi - 2kc_1e_1\right)^{\frac{1}{2}} \tag{34}$$

For the system to be stable, y_3 should be equal to the virtual control input α. So the next error is taken to be:

$$e_2 = y_3 - \alpha \tag{35}$$

where e_2 is the difference between the actual magnetic flux y_3 and the virtual control input α. By putting the value of y_3 from eq.(35) in eq.(24), we have:

$$\dot{e}_1 = \frac{(e_2+\alpha)^2}{2k} - mg - \dot{y}_{2ref} \tag{36}$$

From eq.(36), we get:

$$\dot{e}_1 = \frac{e_2^2}{2k} + \frac{\alpha^2}{2k} + \frac{e_2\alpha}{k} - mg - \dot{y}_{2ref} \tag{37}$$

From eq.(34), we get:

$$\alpha^2 = 2kmg + 2k\dot{y}_{2ref} - 2k\beta\psi - 2kc_1e_1 \tag{38}$$

By putting the value of α^2 from eq.(38) in eq.(37), we have:

$$\dot{e}_1 = \frac{e_2^2}{2k} + \frac{e_2\alpha}{k} - c_1e_1 - \beta\psi \tag{39}$$

Taking time derivative of the eq.(35), we get:

$$\dot{e}_2 = \dot{y}_3 - \dot{\alpha} \tag{40}$$

To compute $\dot{\alpha}$, taking the time derivative of eq.(34), we get:

$$\dot{\alpha} = \frac{1}{2}\left(2kmg + 2k\dot{y}_{2ref} - 2k\beta\psi - 2kc_1e_1\right)^{\frac{-1}{2}} \times \left(2k\ddot{y}_{2ref} - 2k\beta e_1 - 2kc_1\dot{e}_1\right) \tag{41}$$

For simplification, let the R.H.S of eq.(41) $= \frac{1}{2}T(e_1, e_2)$. So eq.(41) becomes:

$$\dot{\alpha} = \frac{1}{2}\Big(T(e_1, e_2)\Big) \tag{42}$$

By putting the value of $\dot{y}_3$ from eq.(21) and $\dot{\alpha}$ from eq.(42) in eq.(40), we get:

$$\dot{e}_2 = -R\frac{(1-y_1)}{k}y_3 + u - \frac{1}{2}\Big(T(e_1, e_2)\Big) \tag{43}$$

For the system to be globally stable, we consider the combined Lyapunov candidate function as:

$$V_c = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \tag{44}$$

By taking time derivative of eq.(44), we have:

$$\dot{V}_c = e_1\dot{e}_1 + e_2\dot{e}_2 \tag{45}$$

Now putting the value of $\dot{e}_1$ from eq.(39) in eq.(45), we get:

$$\dot{V}_c = -c_1e_1^2 + e_2\left(\frac{e_1e_2}{2k} + \frac{e_1\alpha}{k} + \dot{e}_2\right) \tag{46}$$

For the system to be stable, $\dot{V}_c$ should be negative definite. For this purpose, we take:

$$\frac{e_1e_2}{2k} + \frac{e_1\alpha}{k} + \dot{e}_2 = -c_2e_2 \tag{47}$$

where c_2 is also the gain of the controller which is a positive constant. By re-arranging eq.(47), we get:

$$\dot{e}_2 = -\frac{e_1e_2}{2k} - \frac{e_1\alpha}{k} - c_2e_2 \tag{48}$$

So, eq.(46) becomes,

$$\dot{V}_c = -c_1e_1^2 - c_2e_2^2 \tag{49}$$

We can adjust c_1 and c_2 according to the system design requirements, we have:

$$\dot{V}_c \leq 0 \tag{50}$$

Comparing eq.(43) and eq.(48), we have:

$$-R\frac{(1-y_1)}{k}y_3 + u - \frac{1}{2}\Big(T(e_1,e_2)\Big) = -\frac{e_1e_2}{2k} - \frac{e_1\alpha}{k} - c_2e_2 \tag{51}$$

Solving eq.(51) for u, we have:

$$u = R\frac{(1-y_1)}{k}y_3 - \frac{e_1e_2}{2k} - \frac{e_1\alpha}{k} - c_2e_2 + \frac{1}{2}\Big(T(e_1,e_2)\Big) \tag{52}$$

which is the required control law by using IBS technique.

B. BACKSTEPPING CONTROLLER DESIGN PROCEDURE

To design a controller by using backstepping technique, we have to generate errors e_1 and e_2 in y_2 and y_3 states of the system. The only difference between backstepping and IBS technique is the integral term ψ. As there is no integral term used in backstepping technique, so a Lyapunov candidate function for the system becomes:

$$V_1 = \frac{1}{2}e_1^2 \tag{53}$$

By taking time derivative of eq.(53), we get:

$$\dot{V}_1 = e_1\dot{e}_1 \tag{54}$$

By putting the value of $\dot{e}_1$ from eq.(24) in eq.(54), we get:

$$\dot{V}_1 = e_1\left(\frac{y_3^2}{2k} - mg - \dot{y}_{2ref}\right) \tag{55}$$

For the system to be stable, $\dot{V}_1$ should be negative definite. For this purpose, we take:

$$\frac{y_3^2}{2k} - mg - \dot{y}_{2ref} = -c_1e_1 \tag{56}$$

So, eq.(55) becomes:

$$\dot{V}_1 = -c_1e_1^2 \leq 0 \tag{57}$$

So we consider the state y_3 as updated virtual control law which is the desired reference for next state and denoting it by α and solving eq.(56) for it, we get:

$$\alpha = \Big(2kmg + 2k\dot{y}_{2ref} - 2kc_1e_1\Big)^{\frac{1}{2}} \tag{58}$$

To compute $\dot{\alpha}$, taking the time derivative of eq.(58), we get:

$$\dot{\alpha} = \frac{1}{2}\Big(2kmg + 2k\dot{y}_{2ref} - 2kc_1e_1\Big)^{\frac{-1}{2}} \times \Big(2k\ddot{y}_{2ref} - 2kc_1\dot{e}_1\Big) \tag{59}$$

For simplification, let the R.H.S of eq.(59) $= \frac{1}{2}G(e_1, e_2)$. So eq.(59) becomes:

$$\dot{\alpha} = \frac{1}{2}\Big(G(e_1,e_2)\Big) \tag{60}$$

The updated α without an integral term derived in backstepping controller design procedure will directly change the control law u. All other equations are same as used in IBS controller design procedure.

$$u = R\frac{(1-y_1)}{k}y_3 - \frac{e_1e_2}{2k} - \frac{e_1\alpha}{k} - c_2e_2 + \frac{1}{2}\Big(G(e_1,e_2)\Big) \tag{61}$$

which is the required control law by using backstepping technique.

C. SYNERGETIC CONTROLLER DESIGN PROCEDURE

To derive the control law u by using synergetic control algorithm, we have to generate errors e_0, e_1 and e_2 in all states of the system as given in eq.(22), eq.(62) and eq.(63).

$$e_0 = y_1 - y_{1ref} \tag{62}$$

$$e_2 = y_3 - y_{3ref} \tag{63}$$

So by taking the time derivative of eq.(62) and eq.(63), we get:

$$\dot{e}_0 = \dot{y}_1 - \dot{y}_{1ref} \tag{64}$$

$$\dot{e}_2 = \dot{y}_3 - \dot{y}_{3ref} \tag{65}$$

In this technique, we introduce the macro variable which contains tracking error of all the three states. The number of macro variables depends upon the number of inputs to the system. In our case the number of inputs is one, so we introduce only one macro variable Ω as:

$$\Omega = w_0e_0 + w_1e_1 + w_2e_2 \tag{66}$$

where w_1, w_2 and w_3 are the positive gains of the system which are set manually by trial and error method. By taking time derivative of eq.(66), we have:

$$\dot{\Omega} = w_0\dot{e}_0 + w_1\dot{e}_1 + w_2\dot{e}_2 \tag{67}$$

Now, consider dynamic equation as follows:

$$T\dot{\Omega} + \Omega = 0 \tag{68}$$

where T is a positive constant which helps for fast convergence of states. By putting the value of $\dot{\Omega}$ from eq.(67) in eq.(68), we get:

$$T[w_0\dot{e_0} + w_1\dot{e}_1 + w_2\dot{e}_2] + \Omega = 0 \tag{69}$$

By putting the value of $\dot{e}_1$ from eq.(23), $\dot{e_0}$ from eq.(64) and $\dot{e}_2$ from eq.(65) in eq.(69), we get:

$$T\Big(w_0(\dot{y}_1 - \dot{y}_{1ref}) + w_1(\dot{y}_2 - \dot{y}_{2ref}) + w_2(\dot{y}_3 - \dot{y}_{3ref})\Big) + \Omega = 0 \tag{70}$$

By re-arranging eq.(70), we have:

$$w_0\dot{y}_1 + w_1\dot{y}_2 + w_2\dot{y}_3 = w_0\dot{y}_{1ref}w_1 + \dot{y}_{2ref} + w_2\dot{y}_{3ref} - \frac{\Omega}{T} \tag{71}$$

By putting the value of $\dot{y}_1$, $\dot{y}_2$ and $\dot{y}_3$ from eq.(19), eq.(20) and eq.(21) in eq.(71), we get:

$$w_0\left(\frac{y_2}{m}\right) + w_1\left(\frac{y_3^2}{2k} - mg\right) + w_2\left(-R\frac{(1-y_1)}{k}y_3 + u\right) = w_0\dot{y}_{1ref} + w_1\dot{y}_{2ref} + w_2\dot{y}_{3ref} - \frac{\Omega}{T} \tag{72}$$

By solving eq.(72) for u, we have:

$$u = \frac{1}{w_2}\left(w_0\dot{y}_{1ref} + w_1\dot{y}_{2ref} + w_2\dot{y}_{3ref} - \frac{\Omega}{T} - \frac{w_0 y_2}{m} + w_1 mg - \frac{w_1 x_3^2}{2k} + \frac{w_2 R(1-x_1)x_3}{k}\right) \tag{73}$$

which is the final control law by using synergetic control algorithm to track the desired references. For stability of system, Lyapunov candidate function can be taken as:

$$V_c = \frac{1}{2}\Omega^2 \tag{74}$$

Taking time derivative of eq.(74), we have:

$$\dot{V}_c = \Omega\dot{\Omega} \tag{75}$$

From eq.(68), the following expression can be obtained:

$$\dot{\Omega} = \frac{-\Omega}{T} \tag{76}$$

So eq.(75) becomes:

$$\dot{V}_c = -\frac{\Omega^2}{T} \leq 0 \tag{77}$$

It is clear from eq.(77) that the system is globally asymptotic stable by using Lyapunov theory.

TABLE 1. Component values of MAGLEV circuit.

Parameters	Values
Inductance of the Coil L	$0.01H$
Resistance of the Coil R	1.00Ω
Mass of the Metallic Ball m	$1.00g$
Coil Constant k	1.00
Earth Gravitational Constant g	$9.8ms^{-2}$

TABLE 2. Values of gain parameters.

Controller	Variable	Values
Backstepping controller	c_1 c_2	2 10.62
Integral Backstepping controller	c_1 c_2 β	7.865 3.92 0.003
Synergetic based controller	w_0 w_1 w_2 T	6 3.03 0.99 0.99

V. SIMULATION RESULTS OF PROPOSED CONTROLLERS

To verify the system's validity and performance, the outputs of all three system states (y_1, y_2, y_3) and control law u using proposed controllers have been simulated in MATLAB/Simulink environment. The specifications of all the circuit components have been given in the Table 1:

The design parameters of the proposed nonlinear controllers have been set by the trial and error method, while the parameters of linear PI controller have been obtained by using auto-tune tool available in MATLAB/Simulink. In this method, we manually assign the values to all the gains and then check the system's response. If the system states do not meet their references then we change that gain value. Other methods include the neural network in which the system assigns the weights to the parameters by using machine learning. All of the other methods are more complex. The parametric values of IBS, generic backstepping and synergetic controllers have been shown in Table 2:

The simulation result of air gap state of backstepping controller has been discussed in figure 4. It shows the tracking of air gap reference which is $2cm$. This shows that the metallic ball has been levitated in the air at a distance of $2cm$ away from solenoid. The initial condition $y_1(0)$ for air gap at time $t = 0$ is again $1.895cm$. The rise time and settling time are $0.1954s$ and $0.2863s$ respectively. Also, there is no overshoot and undershoot in this case. The peak value is $2.0002cm$ which is at peak time of $0.3893s$, which shows that the steady-state error is very small, i.e. $0.0002cm$.

To improve the simulation results and removing steady state error an integral term has been added in backstepping control algorithm. The simulation results of air gap state of

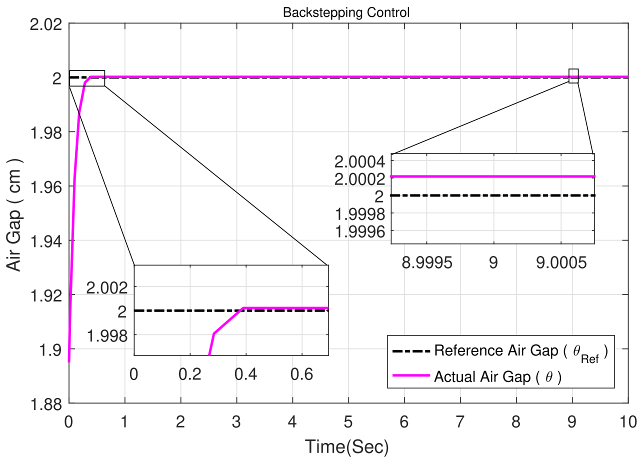

FIGURE 4. Tracking of Air Gap (θ) for backstepping controller.

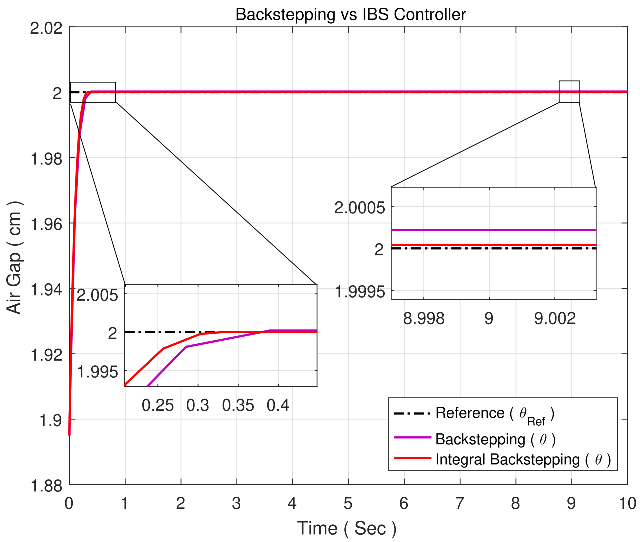

FIGURE 5. Compassion of Air Gap (θ) between Backstepping vs IBS controller.

IBS controller are then compared with backstepping controller and have been discussed in figure 5.

In case of IBS controller, figure 5 shows the tracking of air gap reference which is $2cm$. This shows that the metallic ball has been levitated in the air at a distance of $2cm$ away from solenoid. The initial condition $y_1(0)$ for air gap at time $t = 0$ is $1.895cm$. The rise time is $0.1756s$. Also, there is no overshoot and undershoot that may degrade the system's performance, whereas the peak value is $2cm$ which is at peak time of $0.3342s$. The settling time is $0.2587s$ after which the system tracks $2cm$, which shows that the steady-state error is almost zero. This has been shown that the response of IBS controller shows fast convergence and less steady state error as compared to backstepping controller.

Moreover, synergetic controller has also been proposed for comparative analysis of different nonlinear controllers. The simulation results of air gap state of synergetic controller are

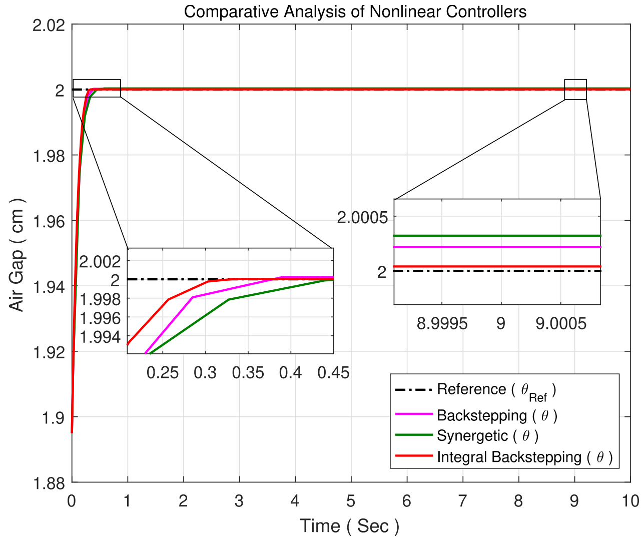

FIGURE 6. Comparative analysis of Air Gap (θ) for Backstepping, IBS and Synergetic controllers.

then compared with backstepping and IBS controller, which have been discussed in figure 6.

Figure 6 shows the tracking of air gap reference for synergetic control algorithm which is $2.0003cm$. This shows that the metallic ball has been levitated in the air at a distance of $2.0003cm$ away from solenoid. The initial condition $y_1(0)$ for air gap at time $t = 0$ is again $1.895cm$. The rise time and settling time are $0.2082s$ and $0.3477s$ respectively. The peak value is $2.0003cm$ which is at peak time of $0.5980s$, which shows that the steady-state error is very small, i.e. $0.0003cm$. Also, there is no overshoot and undershoot in this case. The results shows that the IBS controller still shows better results as compared to backstepping and synergetic control algorithms with respect to convergence and steady state error.

Figure 7 shows the tracking of air gap reference for PI controller which is $1.9998cm$. This shows that the metallic ball has been levitated in the air at a distance of $1.9998cm$ away from solenoid. The initial condition $y_1(0)$ for air gap at time $t = 0$ is again $1.895cm$. The rise and settling times are $0.1053s$ and $15.9026s$ respectively. The peak value is $2.3736cm$ which is at peak time of $0.9294s$. Also, there are undershoots and overshoots at different times in PI controller which are not good for system performance.

From figure 7 and table 3, this has been seen that the nonlinear controller's results are better than the linear PI controller. As the system has nonlinear dynamics, so the linear controller is not much suitable for implementation. For the air gap state, the IBS controller shows fast convergence as the settling time is $0.2587s$ and less steady-state error as compared to all other proposed controllers.

In figure 8, IBS controller shows fast convergence of momentum to its reference value as the settling time is $0.3043s$ and zero steady-state error. While the other controllers show late convergence and zero steady state error.

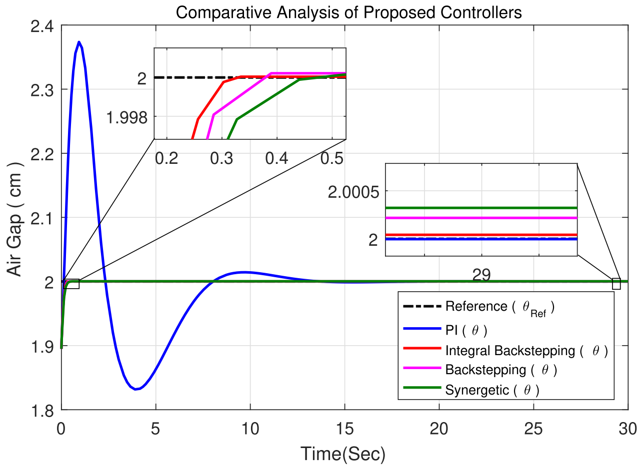

FIGURE 7. Comparative analysis of Air Gap for nonlinear and linear PI controllers.

TABLE 3. Response of proposed controllers.

Response	IBS	Backstepping	Synergetic	PI
Rise Time (s)	0.1756	0.1954	0.2082	0.1053
Settling Time (s)	0.2587	0.2863	0.3477	15.9026
Overshoot (cm)	0	0	0	0.3736
Undershoot (cm)	0	0	0	0.16
Steady State Error	0.00001	0.0002	0.0003	0.0001
Peak Time (s)	0.3342	0.3893	0.5980	0.9294
Peak Value (cm)	2.00001	2.0002	2.0003	2.3736

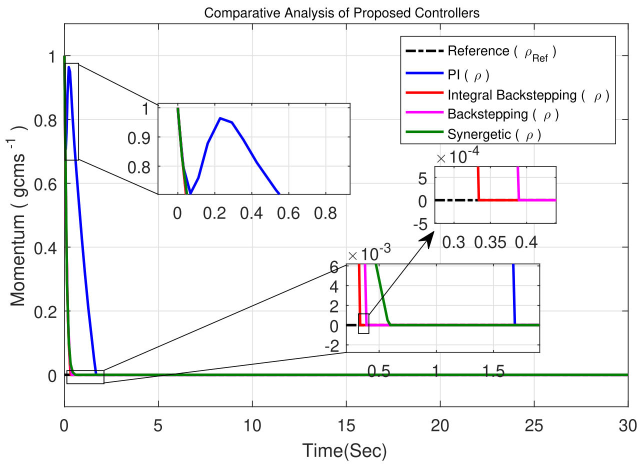

FIGURE 8. Comparative analysis of momentum for nonlinear and linear PI controllers.

In figure 9, IBS controller shows fast convergence of magnetic flux to its reference value as the settling time is $0.4252s$, zero steady-state error and no overshoots and undershoots. While the other controllers show late convergence and more steady state error. Also, the backstepping and synergetic controllers have no overshoot and undershoots while the PI controller has peak value of $5.0471Wb$ which comes at peak time of $0.1617s$. PI controller has overshoot as shown in zoomed area of figure.

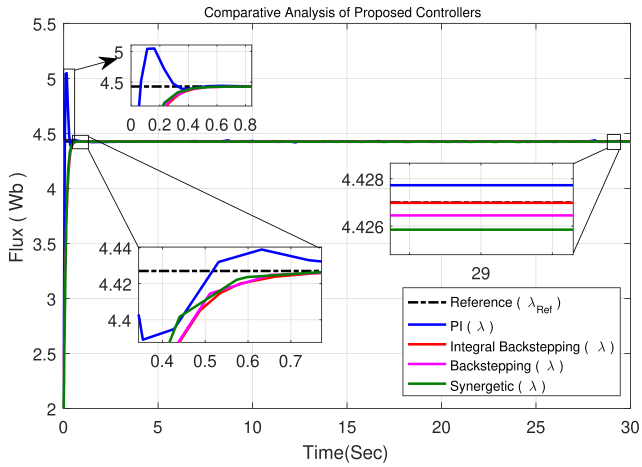

FIGURE 9. Comparative analysis of magnetic flux for nonlinear and linear PI controllers.

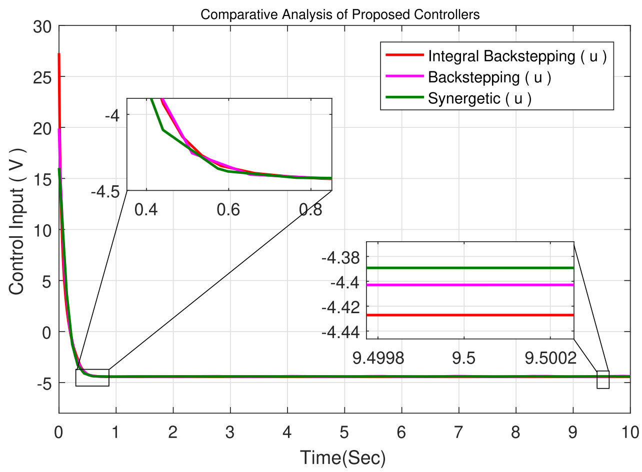

FIGURE 10. Comparative analysis of control inputs for proposed controllers.

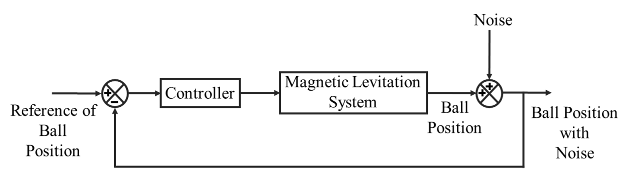

FIGURE 11. Tracking of Air Gap in case of noise.

The comparative analysis of control law u for the backstepping, IBS and synergetic controllers have been shown in figure 10.

The linear controller such as PI controller has a disadvantage of neglecting the nonlinear terms of the state space while the nonlinear controllers can achieve global stability of the system. Figure 11 shows the block diagram of tracking of air gap reference in case of noise.

Figure 12 shows the Gaussian noise of mean $= 0$ and variance $= 0.1$ with a sample time $= 0.1s$ in air gap state of the system model.

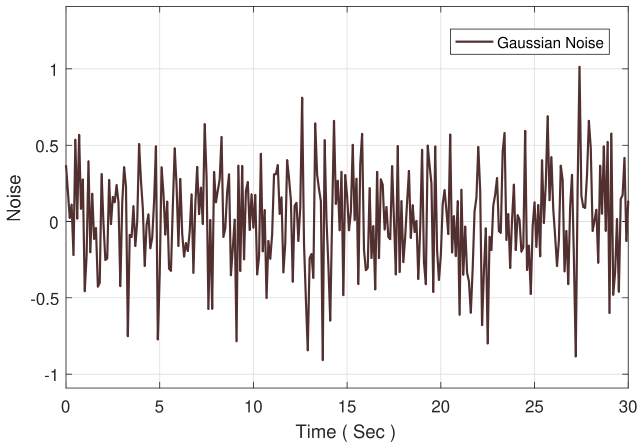

FIGURE 12. Gaussian noise.

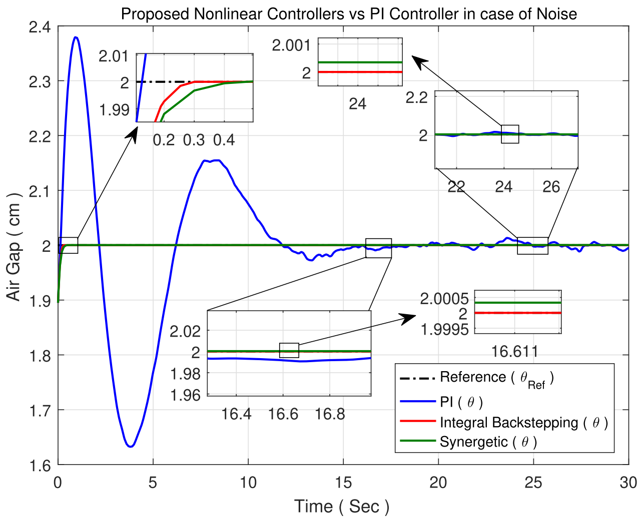

FIGURE 13. Tracking of Air Gap in case of noise.

The comparative analysis of proposed nonlinear and PI controllers in case of noise has been shown in figure 13. It shows that the IBS and synergetic controllers tracks the air gap reference properly and has very small steady state error while the PI controller is not able to track air gap reference properly. However, IBS controller shows little bit fast convergence and less steady state error as compared to synergetic controller.

VI. CONCLUSION

In this paper, a nonlinear dynamical model of the MAGLEV system has been discussed. The controller design of the system has been done by using three nonlinear control techniques namely backstepping, IBS and synergetic control. It has been shown that all the nonlinear controllers give satisfactory results as compared to the linear PI controller. The nonlinear controllers show fast convergence, less steady-state error and no overshoot and undershoot. While the linear controller shows the late convergence, more steady-state error and have undershoots and overshoots that is not good for the system performance. It has been analyzed that the IBS controller gives better results as compared to all other controllers. The comparative analysis of the IBS controller has been given with other two nonlinear controllers and the PI controller which shows that the IBS controller has fast convergence at $0.2687s$ and less steady state error of $0.00001cm$, while linear controller shows late convergence at $15.9026s$ and more steady state error of $0.0001cm$. So, it is better to use the IBS controller in the MAGLEV system rather than any other controller to enhance the system's performance.

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