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Exploiting Coefficient Symmetry in Conventional Polyphase FIR Filters

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ABSTRACT The conventional polyphase architecture for linear-phase finite impulse response (FIR) filter loses its coefficient symmetry property due to the inefficient arrangement of the filter coefficients among its subfilters. Although, existing polyphase structures can avail the benefits of coefficient symmetry property, at the cost of versatility and complex subfilters arrangement of the conventional polyphase structure. To address these issues, in this paper, we first present the mathematical expressions for inherent characteristics of the conventional polyphase structure. Thereafter, we use these expressions to develop a generalized mathematical framework which exploits coefficient symmetry by retaining the direct use of conventional FIR filter coefficients. Further, the transfer function expressions for the proposed Type-1/transposed Type-1 polyphase structures using coefficient symmetry are derived. The proposed structures can reduce the requirement of multiplier units in polyphase FIR filters by half. We also demonstrate the decimator design using the proposed Type-1 polyphase structure and the interpolator design using the proposed transposed Type-1 polyphase structure. Moreover, the phase and magnitude characteristics of the proposed Type-1/transposed Type-1 polyphase structures are presented. It is revealed via numerical examples that all subfilters of the proposed symmetric polyphase structure possess linear-phase characteristics.

INDEX TERMS Coefficient symmetry, frequency response, polyphase FIR structures, pre/post processing, sampling rate conversion.

I. INTRODUCTION

The communication technologies, such as Internet of Things (IoT), cognitive radio, and cooperative communication have gradually emerged; reflecting a large diversity of application domains and signal processing requirements. In such applications, the linear-phase finite impulse response (FIR) filter is widely used owing to its ability of inherent stability, no phase distortion, and coefficient symmetry property. For Nth order FIR filter, the coefficient symmetry implies that the nth coefficient is equal to its $(N-1-n)$th coefficient, which can be exploited to implement the filter using $\lceil \frac{N}{2} \rceil$ multiplier units instead of N, while keeping the adder and delay units same. This property has been fully exploited in the direct-form FIR structures, however, it is quite challenging for polyphase FIR structures. Moreover, the conventional polyphase structure is recognized as a major development towards efficient implementation of FIR filters.

A. THE CONVENTIONAL POLYPHASE STRUCTURE

The transfer function of Type-1 polyphase FIR filter can be mathematically represented as [1]–[3]

$$H(z) = \sum_{k=0}^{J-1} z^{-k} E_k(z^J), \tag{1}$$

where

$$E_k(z) = \sum_{l=0}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+k} z^{-l}, \tag{2}$$

J denotes the number of parallel connected subfilters, and $\lceil \frac{N}{J} \rceil$ represents the subfilters order of the polyphase structure. The Type-1 polyphase or transposed Type-1 polyphase

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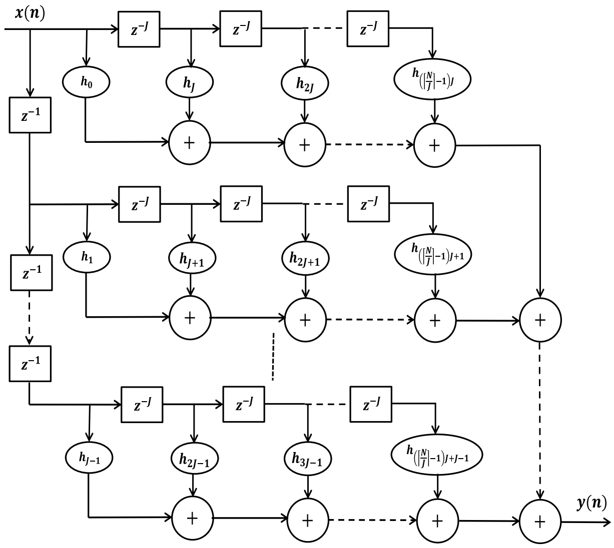


FIGURE 1. Conventional Type-1 polyphase structure.

structures[1] are referred to as conventional polyphase structures as defined in [1]–[3]. Using (1), Fig. 1 depicts the conventional Type-1 polyphase structure. Likewise, the conventional transposed Type-1 polyphase structure can be formed.

It is well known that the decimators and interpolators can be efficiently designed using these structures, which are basic building blocks of multirate applications, such as rational sampling rate converter (RSRC), filter banks, perfect reconstruction filter banks, etc. [1]–[6]. The authors in [7]–[9] have demonstrated the decimator and interpolator configurations by introducing the low sampling polyphase structures. Specifically, in [7], two separate low sampling structures for the cases $L > M$ and $L < M$ are introduced to reduce the computation rate by M, where L and M are the interpolator and decimator, respectively. Recently, a generalized low sampling RSRC polyphase FIR structure alongwith its mathematical expression for any arbitrary values of L and M has been proposed in [10]. Furthermore, the implementation of polyphase FIR filter is reported based on RSRC structures using fast cyclic convolution approach in [11], using minimal block processing approach in [12], and using constant matrix multiplication approach in [13].

The conventional polyphase based multirate systems can also find their applicability in designing of filter banks. The filter banks design involves analysis filters, downsampler, upsampler, and synthesis filters. In order to realize efficient filter banks, the structures should allow the movement of M towards the input side and L to the output side. Accordingly, all analysis and synthesis filter banks are represented in the conventional polyphase configuration, without any reduction in number of computational elements. For reducing the implementation complexity, a two-channel quadrature mirror filter banks has been proposed in [2], [3]. However, this approach is quite complex while generalizing the quadrature mirror filter banks over multiple channels, because of the requirements of various low-pass and high-pass filters.

Furthermore, the conventional polyphase FIR structured single-rate systems can provide benefits like increasing the throughput or decreasing the power consumption [14] over the direct-form FIR based single-rate approach. Replicating the conventional polyphase filter in parallel branches with lower complexity[2] using fast FIR algorithm (FFA) [15], and modifying the perfect reconstruction filter banks [16] are well known approaches for designing such systems. However, such approaches involve additional arrangement of the subfilters by incorporating extra adders over the conventional polyphase structure or pre-calculation of the filter coefficients. In this paper, we refer this additional processing as pre/post processing.

B. EFFICIENT POLYPHASE STRUCTURES FOR LINEAR-PHASE FIR FILTER

The parallel multirate structures with coefficient symmetry are presented in [17]–[23]. The structure reported in [17] can asymptotically reduce the adder complexity to $\frac{N}{2}$ as well as the multiplier requirement by half. This approach is extended for designing interpolators and decimators for the integer sampling rate conversion applications in [18]. Further, an efficient approach for implementing the linear-phase FIR filters for RSRC using coefficient symmetry has been reported in [19] and [20], wherein it was discussed that the multiplication complexity can approximately be halved if the length of the filter is very large. However, these approaches [17]–[20] cannot easily be generalized for arbitrary filter configurations. The implication of coefficient symmetry in the polyphase implementation of FIR filters by recasting the pairs of original polyphase components as sums or differences of auxiliary pairs of symmetric and anti-symmetric impulse response of FIR filters was presented in [21]. The multiple constant multiplication (MCM) based implementation of the polyphase structure is used to exploit coefficient symmetry by incorporating pre-processing of the original coefficients in [22]. However, the synthesis of symmetric FIR coefficients and adjustment of the transformational adders for different configurations are required to implement the symmetry, which is a major limitation of the MCM approach. The authors in [23] have proposed an approach (as depicted in Fig. 5.6 of [23]) for availing the benefits of coefficient symmetry over polyphase structures using pre and post processing. Moreover, the odd-order linear-phase FIR filters can be realized in parallel structures, as presented in [24], where half number of multipliers within a single subfilter block can be utilized for the multiplications of whole taps. The requirement of extra adders for the complicated arrangements

[1]The Type-2 configuration is just a permutation of the parallel subfilters of transposed Type-1 configuration.

[2]For example, 2-parallel fast FIR filtering structure requires about 25% less hardware complexity than the traditional 2 parallel branch implementation whereas 33% saving in 3 parallel branch implementation with 4 and 10 extra adders to arrange the subfilters, respectively [14].

TABLE 1. Related work on polyphase coefficient symmetry.

	Generalized structure	FFA single-rate systems	Multirate systems	Pre/Post processing
[18]	No	No	Yes	Yes
[19]	No	No	Yes	Yes
[20]	No	No	Yes	Yes
[21]	Yes	No	Yes	Yes
[22]	Yes	No	Yes	Yes
[23]	No	No	Yes	Yes
[24]	No	Yes	No	Yes
This Paper	Yes	Yes	Yes	No

of subfilters is a major drawback of FFA approach. The pre/post arrangement of subfilters makes the analysis complicated, and the resulting architecture is highly irregular for larger parallel branches. Thus, it is worth mentioning that the approaches reported in [17]–[23] are not easily extensible to realize the single-rate systems, whereas the approach reported in [24] cannot be used for multirate systems. Although in [25], the authors have proposed a method to convert the single-rate to multirate systems, at the cost of increased complexity. Recently, the authors in [26] have proposed a novel interpolation implementation approach to improve the performance of sample rate converters for aerospace applications.

C. MOTIVATION

It is clear from the previous discussions that the coefficient symmetry property should be maximally exploited over conventional polyphase structure for efficiently designing the decimators and interpolators, as it can bring several benefits in realizing FFA single-rate and multirate applications. In order to achieve this goal, several parallel structures have been developed, but all of them possess one or more of the following shortcomings.

1) Some desirable features of the conventional polyphase structures, such as filtering with original filter coefficients, versatility to be used in single-rate and multirate applications could not be retained.
2) It was argued that the multirate systems are not efficient in real-time switching applications [27], so the need for pre/post filtering associated with the existing structures further enhances the involved complications.
3) The subfilters of the conventional polyphase filter do not exhibit the linear-phase characteristics which may limit the available benefits. The linear-phase subfilters allow separate realization of each subfilter and combining their output samples at the end of the process. This feature may provide more flexibility to the designer in developing the applications like, coherent signal processing and demodulation, radar signal processing, and audio and image processing. Further, the equivalence between the linear-phase and coefficient symmetry in conventional FIR filters is discussed in [28]. However, such relationship between coefficient symmetry and linear-phase needs further discussion for conventional polyphase FIR filters.

These research gaps and open issues motivate us to develop the efficient symmetric polyphase structures, which can extract the maximum possible benefits of the coefficient symmetry for the conventional FIR filter.

D. CONTRIBUTIONS

Different from the existing work in the literature [18]–[24], whose characteristics are summarized in Table 1, in this paper, we propose a novel approach for exploiting the coefficient symmetry in the conventional polyphase structure. The major highlights of the proposed approach are as follows.

- We first present a relation between the symmetric coefficients, which is applicable to the polyphase FIR filter. We then provide some important properties, that help in locating these symmetric coefficients in the conventional polyphase filter. Using these properties, we show the symmetry among the coefficients of the polyphase filter by separating the lower and higher index coefficients[3] for all arbitrary positive values of N and J.
- By employing the coefficient symmetry, we eliminate all higher index coefficients and replace them with lower index coefficients. Thereafter, we deduce the new transfer functions of the Type-1/transposed Type-1 polyphase FIR filters for all arbitrary values of N and J. We also provide some numerical examples to show the modified polyphase FIR filter with coefficient symmetry. It is also revealed that the required number of multiplier units reduces to half. Furthermore, we demonstrate the coefficient symmetry in multirate systems.
- We also discuss the phase and magnitude characteristics of the proposed Type-1/transposed Type-1 polyphase FIR filters. With the aid of numerical examples, it is shown that the phase response of each subfilter of the proposed Type-1 polyphase FIR filter is linear, whereas in case of proposed transposed Type-1 polyphase FIR filter the subfilter coupled with its complementary subfilter makes the combined phase response linear. Moreover, the overall magnitude response of the proposed Type-1/transposed Type-1 polyphase FIR filter is the same as the magnitude response of conventional FIR filter.

E. OUTLINE

The rest of this paper is organized as follows. Section II describes the mathematical preliminaries pertaining to the conventional polyphase FIR filter. In particular, some inherent properties of conventional polyphase FIR filter are presented. Section III presents the mathematical framework and the design guidelines for Type-1/transposed Type-1 polyphase FIR structures. Moreover, we discuss the

[3] Note that the coefficients index less than or equal to the middle coefficient index (i.e., with index value $\lceil \frac{N}{2} \rceil - 1$) are considered as lower index coefficients, whereas the coefficients index greater than $\lceil \frac{N}{2} \rceil - 1$ are considered as higher index coefficients.

implementation of multirate systems. Section IV presents the overall phase and magnitude characteristics of the proposed structures. Section V presents the comparison between the proposed structures and the existing structures. Finally, the conclusions are drawn in Section VI.

Notations: In this paper, we consider $k, l, k', k'', L, M, N,$ and J as positive integers. $\lceil x \rceil$ represents the least integer that is greater than or equal to x, and $\lfloor x \rfloor$ represents the greatest integer that is less than or equal to x.

II. MATHEMATICAL PRELIMINARIES PERTAINING TO THE CONVENTIONAL POLYPHASE FIR FILTER

In order to build a robust mathematical framework which can lead to the development of symmetric polyphase FIR filter, we study various characteristics of conventional polyphase FIR filter and express their mathematical expressions in the sequel.

A. PROPERTIES OF POLYPHASE FIR FILTERS

The FIR filter with impulse response h_n can also be visualized in a 2-dimensional polyphase structure in terms of the parameters k and l, where k indicates the subfilter and l denotes the nth coefficient of h_n in this subfilter, and hence $n = Jl + k$, where $0 \le k \le J-1$ and $0 \le l \le \lceil \frac{N}{J} \rceil - 1$ (or $0 \le l \le \lceil \frac{N}{J} \rceil - 2$, depending on the subfilter length). Therefore, the coefficient symmetry of h_n i.e., $h_n = h_{N-1-n}$ can now be represented as

$$h_{Jl+k} = h_{N-(Jl+k)-1}. \tag{3}$$

Define k' (or k'') as the complementary subfilter of k subfilter, whose coefficients are the mirror image of k, and each value of k is associated with only one value of either k' or k''. Consequently, we can express the complementary subfilters with their z-transformation expressions using (1) as

$$H_{k'}(z) = \sum_{k'=0}^{p-1} z^{-(p-1-k')} E_{p-1-k'}(z^J), \tag{4}$$

$$H_{k''}(z) = \sum_{k''=p}^{J-1} z^{-(p+J-1-k'')} E_{p+J-1-k''}(z^J), \tag{5}$$

where

$$E_i(z) = \sum_{l=0}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+i} z^{-l}. \tag{6}$$

Moreover, $H(z)$ in (1) can be expressed in terms of the complementary subfilters using (4) and (5) as

$$H(z) = \begin{cases} H_{k'}(z) + H_{k''}(z), & \text{for } 1 \le p \le J-1 \\ H_{k'}(z), & \text{for } p = J. \end{cases} \tag{7}$$

Further, we have defined u as the index of respective symmetric coefficients of k' (or k'') subfilter, such that $N - (Jl + k) - 1 = Ju + k'$ (or $Ju + k''$). Moreover, in polyphase structures, the order of the subfilters can either be $\lceil \frac{N}{J} \rceil$ or $\lceil \frac{N}{J} \rceil - 1$. Therefore, without loss of generality, we classify the subfilters in two groups, viz., $G1$ with order $\lceil \frac{N}{J} \rceil$ and $G2$ with order $\lceil \frac{N}{J} \rceil - 1$. The complementary k' subfilters lie in $G1$, and the complementary k'' subfilters lie in $G2$. It may also be noted that, in some situations, the subfilter and its complementary subfilter may represent the same subfilter, i.e., $k = k'$ (or $k = k''$).

Based on the above discussion, the polyphase structure should exhibit following properties:

Property 1: The sum of index (l) and index (u) is either[4] $\lceil \frac{N}{J} \rceil - 1$ or $\lceil \frac{N}{J} \rceil - 2$, i.e., $u + l = \lceil \frac{N}{J} \rceil - 1$ or $u + l = \lceil \frac{N}{J} \rceil - 2$.

Property 2: The complementary subfilter pairs (k, k') and (k, k'') correspond to $G1$ and $G2$, and the relations between k and k', and k and k'', are given, respectively, as (see Appendix A for proof)

$$k' = p - 1 - k, \quad \text{for} \quad 0 \le k \le p-1, \tag{8}$$

$$k'' = p - 1 + J - k, \quad \text{for} \quad p \le k \le J-1, \tag{9}$$

where $p = N - J(\lceil N/J \rceil) + J$.

Property 3: In polyphase structure, the index of middle coefficient (i.e., (l, k)) can take two positions based on whether $\lceil \frac{N}{J} \rceil$ is even or odd (proof is given in Appendix B). Specifically, when $\lceil \frac{N}{J} \rceil$ is even, we have $(l, k) = \left(\frac{1}{2} \lceil \frac{N}{J} \rceil - 1, \lceil \frac{p+J}{2} \rceil - 1 \right)$, which lies in $G2$. Whereas, when $\lceil \frac{N}{J} \rceil$ is odd, we have $(l, k) = \left(\left\lfloor \frac{1}{2} \lceil \frac{N}{J} \rceil \right\rfloor, \lceil \frac{p}{2} \rceil - 1 \right)$, which lies in $G1$.

B. SEPARATION OF SUBFILTERS COEFFICIENTS OF THE POLYPHASE FIR FILTER

In this subsection, we separate the coefficients of each subfilter of the polyphase FIR filter into the lower index and higher index coefficients, using the properties discussed in previous subsection.

First, by using *Property 2* in (1), the modified expression of conventional polyphase FIR filter can be expressed as

$$H(z) = \underbrace{\sum_{k=0}^{p-1} z^{-k} E_k^p(z^J)}_{\triangleq H_{G1}(z)} + \underbrace{\sum_{k=p}^{J-1} z^{-k} E_k^q(z^J)}_{\triangleq H_{G2}(z)}, \tag{10}$$

where $E_k^p(z)$ and $E_k^q(z)$ represent the transfer functions of kth subfilter belong to $G1$ and $G2$, respectively, and can be

[4]Note that lth coefficient of the kth subfilter (i.e., h_{Jl+k}) is available as the uth coefficient of either k'th subfilter (i.e., $h_{Ju+k'}$) or k''th subfilter (i.e., $h_{Ju+k''}$). For example, if $l = 0$, $k = 0$, $N = 25$, and $J = 6$, then the coefficient $h_{Jl+k} = h_0$ is equivalent to $h_{Ju+k'} = h_{24}$ which lies in its complementary subfilter $k' = 0$ with $u = 4$. This indicates that $u + l = \lceil \frac{N}{J} \rceil - 1$. Further, for $l = 0$, $k = 1$, $N = 25$, and $J = 6$, we have $h_{Jl+k} = h_1$ and $h_{Ju+k''} = h_{23}$ with $u = 3$ and $k'' = 5$, which shows that h_1 lies in its complementary subfilter, and hence, $u + l = \lceil \frac{N}{J} \rceil - 2$ in this case. From this observation, we can deduce that for all pair of (k, k'), $u + l$ is equal to $\lceil \frac{N}{J} \rceil - 1$ and for (k, k''), $u + l$ is equal to $\lceil \frac{N}{J} \rceil - 2$.

mathematically given as

$$E_k^p(z) = \sum_{l=0}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+k} z^{-l}, \tag{11}$$

$$E_k^q(z) = \sum_{l=0}^{\lceil \frac{N}{J} \rceil - 2} h_{Jl+k} z^{-l}. \tag{12}$$

Then, using *Property 3*, (10) can be further modified under two scenarios: 1) Scenario 1: when $\lceil \frac{N}{J} \rceil$ is even, and 2) Scenario 2: when $\lceil \frac{N}{J} \rceil$ is odd.

1) SCENARIO 1 (WHEN $\lceil \frac{N}{J} \rceil$ IS EVEN)

For even $\lceil \frac{N}{J} \rceil$, the index of middle coefficient (i.e., (l, k)) lies in $G2$. Therefore, we can re-express (10) as

$$H(z) = \underbrace{\sum_{k=0}^{p-1} z^{-k} E_k^p(z^J)}_{\triangleq H_{G1}(z)} + \underbrace{\sum_{k=p}^{\lceil \frac{J+p}{2} \rceil - 1} z^{-k} E_k^{q_1}(z^J)}_{\triangleq H_{G2}^{(1)}(z)} + \underbrace{\sum_{k=\lceil \frac{J+p}{2} \rceil}^{J-1} z^{-k} E_k^{q_2}(z^J)}_{\triangleq H_{G2}^{(2)}(z)}, \tag{13}$$

where $E_k^p(z)$, $E_k^{q_1}(z)$, and $E_k^{q_2}(z)$ can be represented as

$$E_k^p(z) = \underbrace{\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^p(z)} + \underbrace{\sum_{l=\frac{1}{2}\lceil \frac{N}{J} \rceil}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^p(z)}, \tag{14}$$

$$E_k^{q_1}(z) = \underbrace{\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^{q_1}(z)} + \underbrace{\sum_{l=\frac{1}{2}\lceil \frac{N}{J} \rceil}^{\lceil \frac{N}{J} \rceil - 2} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^{q_1}(z)}, \tag{15}$$

$$E_k^{q_2}(z) = \underbrace{\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - 2} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^{q_2}} + \underbrace{\sum_{l=\frac{1}{2}\lceil \frac{N}{J} \rceil - 1}^{\lceil \frac{N}{J} \rceil - 2} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^{q_2}}, \tag{16}$$

respectively. The terms $E_{k,1}^p(z)$, $E_{k,1}^{q_1}(z)$, and $E_{k,1}^{q_2}$ contain the lower index coefficients of $E_k^p(z)$, $E_k^{q_1}(z)$, and $E_k^{q_2}(z)$, respectively, whereas $E_{k,2}^p(z)$, $E_{k,2}^{q_1}(z)$, and $E_{k,2}^{q_2}$ contain the higher index coefficients of $E_k^p(z)$, $E_k^{q_1}(z)$, and $E_k^{q_2}(z)$, respectively.

Furthermore, it is worthwhile to mention that the term $H_{G2}^{(2)}(z)$ of (13) becomes zero for the case when $p = J - 1$, and the terms $H_{G2}^{(1)}(z)$ and $H_{G2}^{(2)}(z)$ of (13) will be zero when $p = J$. Therefore, we can express (13) as

$$H(z) = \begin{cases} H_{G1}(z) + H_{G2}^{(1)}(z), & \text{for } p = J - 1 \\ H_{G1}(z), & \text{for } p = J \\ H_{G1}(z) + H_{G2}^{(1)}(z) + H_{G2}^{(2)}(z), & \text{o.w.} \end{cases} \tag{17}$$

2) SCENARIO 2 (WHEN $\lceil \frac{N}{J} \rceil$ IS ODD)

For odd value of $\lceil \frac{N}{J} \rceil$, the index of middle coefficient (i.e., (l, k)) lies in $G1$. Thus, we can re-express (10) as

$$H(z) = \underbrace{\sum_{k=0}^{\lceil \frac{p}{2} \rceil - 1} z^{-k} E_k^{p_1}(z^J)}_{\triangleq H_{G1}^{(1)}(z)} + \underbrace{\sum_{k=\lceil \frac{p}{2} \rceil}^{p-1} z^{-k} E_k^{p_2}(z^J)}_{\triangleq H_{G1}^{(2)}(z)} + \underbrace{\sum_{k=p}^{J-1} z^{-k} E_k^q(z^J)}_{\triangleq H_{G2}(z)}, \tag{18}$$

where $E_k^{p_1}(z)$, $E_k^{p_2}(z)$, and $E_k^q(z)$ can be given by

$$E_k^{p_1}(z) = \underbrace{\sum_{l=0}^{\left\lfloor \frac{\lceil \frac{N}{2J} \rceil}{2} \right\rfloor} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^{p_1}(z)} + \underbrace{\sum_{l=\left\lfloor \frac{\lceil N/J \rceil}{2} \right\rfloor + 1}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^{p_1}(z)}, \tag{19}$$

$$E_k^{p_2}(z) = \underbrace{\sum_{l=0}^{\left\lfloor \frac{\lceil N/J \rceil}{2} \right\rfloor - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^{p_2}(z)} + \underbrace{\sum_{l=\left\lfloor \frac{\lceil N/J \rceil}{2} \right\rfloor}^{\lceil \frac{N}{J} \rceil - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^{p_2}(z)}, \tag{20}$$

$$E_k^q(z) = \underbrace{\sum_{l=0}^{\left\lfloor \frac{\lceil N/J \rceil}{2} \right\rfloor - 1} h_{Jl+k} z^{-l}}_{\triangleq E_{k,1}^q(z)} + \underbrace{\sum_{l=\left\lfloor \frac{\lceil N/J \rceil}{2} \right\rfloor}^{\lceil \frac{N}{J} \rceil - 2} h_{Jl+k} z^{-l}}_{\triangleq E_{k,2}^q(z)}, \tag{21}$$

respectively. The terms $E_{k,1}^{p_1}(z)$, $E_{k,1}^{p_2}(z)$, and $E_{k,1}^q(z)$ contain the lower index coefficients of $E_k^{p_1}(z)$, $E_k^{p_2}(z)$, and $E_k^q(z)$, respectively, while the terms $E_{k,2}^{p_1}(z)$, $E_{k,2}^{p_2}(z)$, and $E_{k,2}^q(z)$ contain the higher index coefficients of $E_k^{p_1}(z)$, $E_k^{p_2}(z)$, and $E_k^q(z)$, respectively.

Moreover, it is noted that the term $H_{G1}^{(2)}(z)$ of (18) will be neglected when $p = 1$, and the term $H_{G2}(z)$ of (18) becomes zero for $p = J$. Therefore, we can express (18) as

$$H(z) = \begin{cases} H_{G1}^{(1)}(z) + H_{G2}(z), & \text{for } p = 1 \\ H_{G1}^{(1)}(z) + H_{G1}^{(2)}(z), & \text{for } p = J \\ H_{G1}^{(1)}(z) + H_{G1}^{(2)}(z) + H_{G2}(z), & \text{o.w.} \end{cases} \tag{22}$$

It can clearly be observed from the derived mathematical expressions that the higher and lower index coefficients are distinguished. Further, with the help of such analysis, we can exploit the coefficient symmetry in Type-1 polyphase FIR filter and transposed Type-1 polyphase FIR filter for any arbitrary positive integer values of J and N, which is demonstrated in what follows.

III. POLYPHASE FIR FILTERS WITH COEFFICIENT SYMMETRY

In this section, by exploiting the coefficient symmetry, we present Type-1/transposed Type-1 polyphase FIR filters and their transfer functions for all arbitrary positive integer values of N and J.

A. TYPE-1 POLYPHASE FIR FILTER WITH COEFFICIENT SYMMETRY

The overall transfer function of the Type-1 polyphase filter using coefficient symmetry under two cases: 1) when $\lceil \frac{N}{J} \rceil$ is even, and 2) when $\lceil \frac{N}{J} \rceil$ is odd, can be expressed as per the following theorem.

Theorem 1: The overall transfer function of Type-1 polyphase FIR filter for the case when $\lceil \frac{N}{J} \rceil$ is even can be expressed as

$$\begin{aligned} H(z) = & \sum_{k=0}^{p-1} z^{-k}\left(E^{p}_{k,1}(z^J) + z^{-J\frac{\lceil N/J\rceil}{2}} E^{p}_{k',1}(z^J)\right) \\ & + \sum_{k=p}^{\lceil \frac{J+p}{2}\rceil -1} z^{-k}\left(E^{q_1}_{k,1}(z^J) + z^{-J\frac{\lceil N/J\rceil}{2}} E^{q_1}_{k'',1}(z^J)\right) \\ & + \sum_{k=\lceil \frac{J+p}{2}\rceil}^{J-1} z^{-k}\left(E^{q_1}_{k,1}(z^J) + z^{-J\left(\frac{\lceil N/J\rceil}{2}-1\right)} E^{q_2}_{k'',1}(z^J)\right), \end{aligned} \tag{23}$$

and the overall transfer function of Type-1 polyphase FIR filter for the case when $\lceil \frac{N}{J} \rceil$ is odd can be expressed as

$$\begin{aligned} H(z) = & \sum_{k=0}^{\lceil \frac{p}{2}\rceil -1} z^{-k}\left(E^{p_1}_{k,1}(z^J) + z^{-J(\frac{\lceil N/J\rceil+1}{2})} E^{p_1}_{k',1}(z^J)\right) \\ & + \sum_{k=\lceil \frac{p}{2}\rceil}^{p-1} z^{-k}\left(E^{p_2}_{k,1}(z^J) + z^{-J(\frac{\lceil N/J\rceil-1}{2})} E^{p_2}_{k',1}(z^J)\right) \\ & + \sum_{k=p}^{J-1} z^{-k}\left(E^{q}_{k,1}(z^J) + z^{-J(\frac{\lceil N/J\rceil-1}{2})} E^{q}_{k'',1}(z^J)\right). \end{aligned} \tag{24}$$

Proof: The detailed proof is given in Appendix C. ■

It is evident from (23) and (24) that the proposed structures of Type-1 polyphase FIR filter for any arbitrary positive integer values of N and J can be designed.

Numerical Example: We consider a case when $\lceil \frac{N}{J} \rceil$ is even with $p = J$, $J = 4$, and $N = 16$ with all coefficient are positive. For fair comparison, when $J = 4$ and $N = 16$, we can easily design the conventional Type-1 polyphase FIR structure using Fig. 1, as shown in Fig. 2. Now, applying

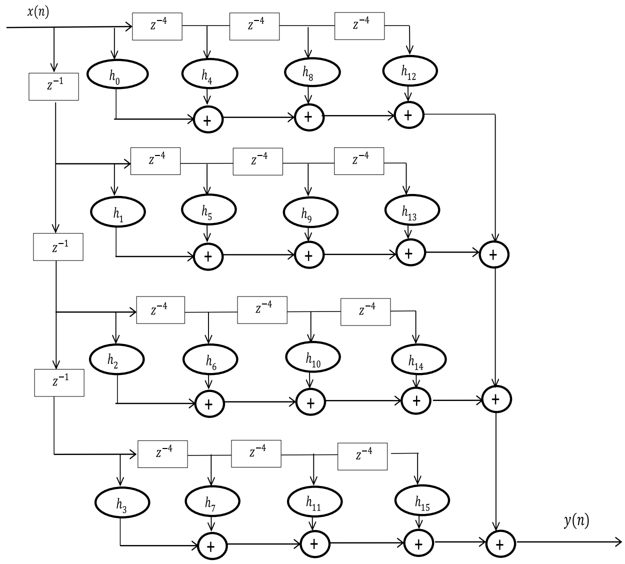


FIGURE 2. Fourth order conventional Type-1 polyphase structure.

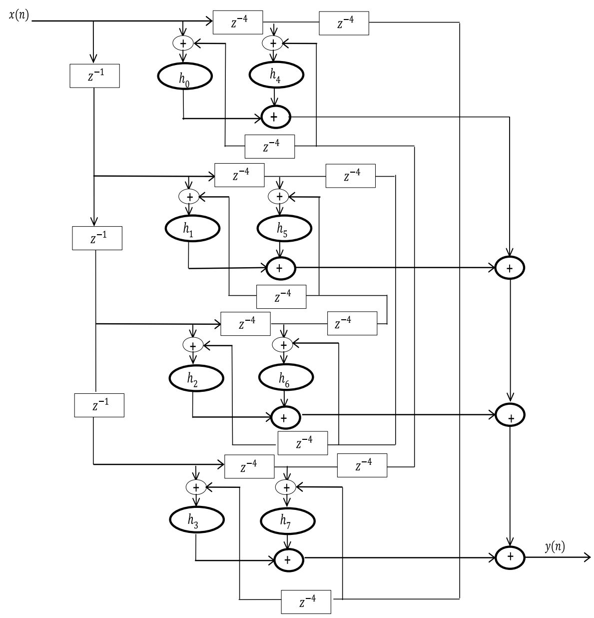


FIGURE 3. Fourth order proposed Type-1 polyphase structure.

$p = J$, $J = 4$, and $N = 16$ into (23), the transfer function of the proposed Type-1 polyphase FIR filter can be expressed as

$$H(z) = \sum_{k=0}^{3} z^{-k}\left(\sum_{l=0}^{1} h_{4l+k} z^{-4l} + z^{-8}\sum_{u=0}^{1} h_{4u+k'} z^{-4(1-u)}\right). \tag{25}$$

Using (25), the proposed Type-1 polyphase FIR structure can be designed, as shown in Fig. 3. It can be seen from Fig. 3 that the higher index coefficients in the subfilters of polyphase FIR filter (i.e., $h_8, h_{12}, h_9, h_{13}, h_{10}, h_{14}, h_{11}, h_{15}$, as shown in Fig. 2) are eliminated and the respective symmetric lower index coefficients (i.e., $h_7, h_3, h_6, h_2, h_5, h_1, h_4, h_0$)

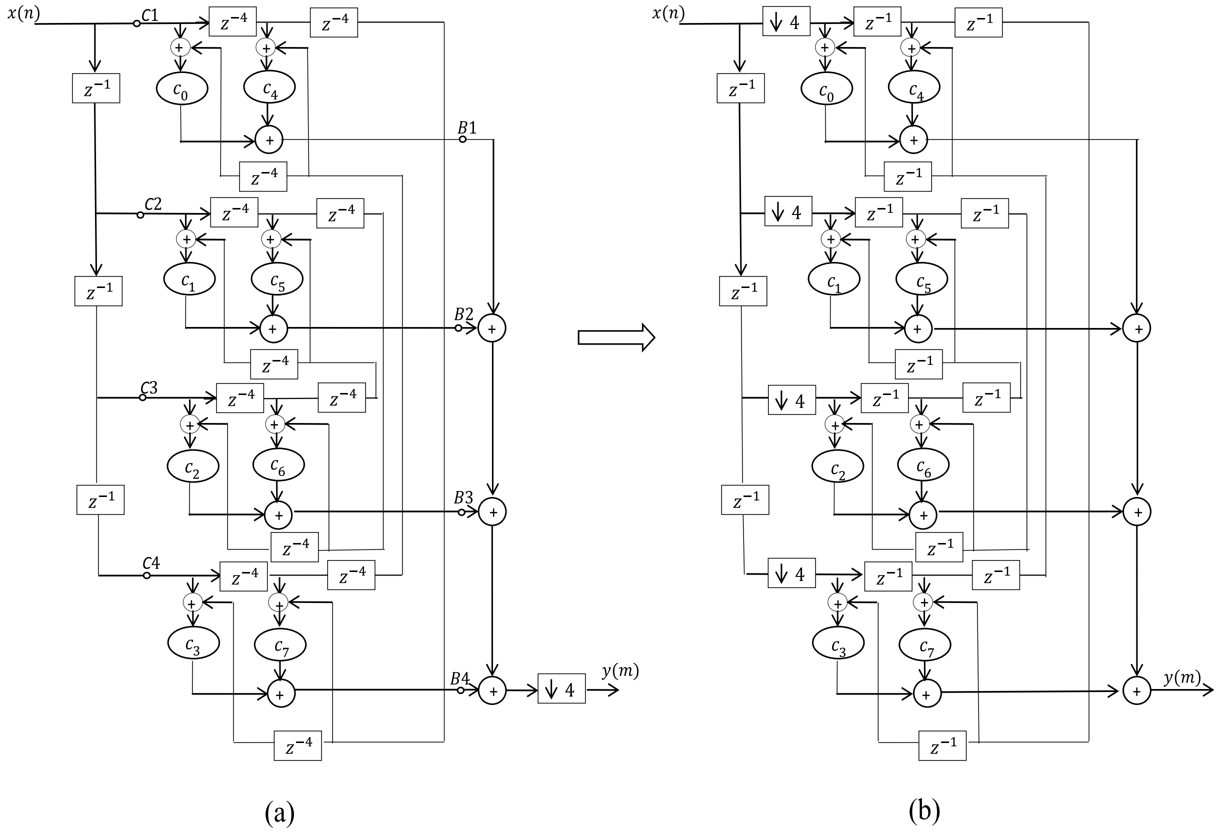


FIGURE 4. The Type-1 multirate structure, (a) Fourth order symmetric decimator Type-1 polyphase structure, (b) Fourth order symmetric decimator efficient Type-1 polyphase structure.

as available in the respective complementary subfilters of the proposed Type-1 polyphase FIR filter are utilized.

Moreover, it should be noted that in polyphase FIR filter, the multiplier units are N, whereas, it is $\lceil \frac{N}{2} \rceil$ in our proposed Type-1 polyphase FIR filter. However, the total adder units (i.e., $N - J$) in the proposed structure and conventional polyphase FIR filter are same.

Furthermore, it can easily be recognized that the proposed structures (i.e., for even $\lceil \frac{N}{J} \rceil$ in (23) and odd $\lceil \frac{N}{J} \rceil$ in (24)) are applicable for designing of multirate systems. To demonstrate that, without loss of generality, we consider the same example that is used to design the proposed Type-1 polyphase structure (as depicted in Fig. 3). Accordingly, a downsampler of $M = J = 4$ is inserted at the end of the filter output after choosing the suitable filter coefficients (i.e., for illustration, $c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}$, and c_{15}) as shown in Fig. 4(a). Hereby, such structure shall work as a decimator. However, it is not able to perform the filtering operation at the minimum sample rate i.e., $f_{in}/4$. After applying the noble identities, the downsampler which is inserted at the filter output can be equivalently replaced by four downsamplers at points $B1$, $B2$, $B3$, and $B4$ as marked in Fig. 4(a). Further, these four downsamplers can be equivalently shifted to points $C1$, $C2$, $C3$, and $C4$ by replacing all delay elements from z^{-4} to z^{-1} in Fig. 4(a). Therefore, the desired decimator can be realized at the minimum sample rate i.e., $f_{in}/4$, as shown in Fig. 4(b).

B. TRANSPOSED TYPE-1 POLYPHASE FIR FILTER WITH COEFFICIENT SYMMETRY

The overall transfer function of the proposed transposed Type-1 polyphase filter for all arbitrary positive integer values of N and J under two scenarios: 1) when $\lceil \frac{N}{J} \rceil$ is even, and 2) when $\lceil \frac{N}{J} \rceil$ is odd, can be given in the following theorem.

Theorem 2: The overall transfer function of transposed Type-1 polyphase FIR filter for the case when even $\lceil \frac{N}{J} \rceil$ can be given by

$$\begin{aligned} H(z) = &\sum_{k=0}^{p-1} z^{-k}\left(E_{k,1}^{p}(z^J) + E_{k',1}^{p}(z^J)\right) \\ &+ \sum_{k=p}^{\lceil \frac{J+p}{2} \rceil -1} z^{-k}\left(E_{k,1}^{q_1}(z^J) + E_{k'',1}^{q_1}(z^J)\right) \\ &+ \sum_{k=\lceil \frac{J+p}{2} \rceil}^{J-1} z^{-k}\left(E_{k,1}^{q_1}(z^J) + E_{k'',1}^{q_2}(z^J)\right). \end{aligned} \tag{26}$$

and the overall transfer function of transposed Type-1 polyphase FIR filter for the case when $\lceil \frac{N}{J} \rceil$ is odd can be expressed as

$$\begin{aligned} H(z) = & \sum_{k=0}^{\lceil \frac{p}{2} \rceil -1} z^{-k}\left(E^{p_1}_{k,1}(z^J) + E^{p_1}_{k',1}(z^J)\right) \\ & + \sum_{k=\lceil \frac{p}{2} \rceil}^{p-1} z^{-k}\left(E^{p_2}_{k,1}(z^J) + E^{p_2}_{k',1}(z^J)\right) \\ & + \sum_{k=p}^{J-1} z^{-k}\left(E^{q}_{k,1}(z^J) + E^{q}_{k'',1}(z^J)\right). \end{aligned} \tag{27}$$

Proof: See Appendix D for the detailed analysis. ■

It can be observed from (26) and (27) that the proposed structures of transposed Type-1 polyphase FIR filter for any arbitrary positive integer values of N and J can be designed.

Numerical Example: We consider a case when $\lceil \frac{N}{J} \rceil$ is even, $p = J$, $J = 4$, and $N = 16$ with all the coefficients are positive. By applying these into (26), the transfer function can be expressed as

$$H(z) = \sum_{k=0}^{3} z^{-k}\left(\sum_{l=0}^{1} h_{4l+k} z^{-4l} + \sum_{u=0}^{1} (h_{4u+k'} z^{-4u}) z^{-12+8u}\right). \tag{28}$$

By using (28), the proposed transposed Type-1 polyphase FIR structure can be realized, as shown in Fig. 5. Moreover, our proposed transposed Type-1 polyphase FIR filter requires only $\lceil \frac{N}{2} \rceil$ multiplier units compared to the conventional transposed polyphase FIR filter which requires N multiplier units. However, the total adder units are same in both the structures.

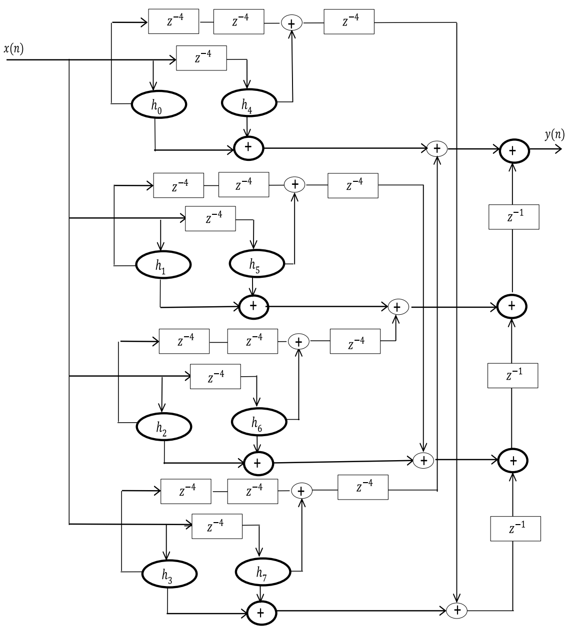


FIGURE 5. Fourth order proposed transposed Type-1 polyphase structure.

Furthermore, it is worth noted that the proposed structures (i.e., for even $\lceil \frac{N}{J} \rceil$ in (26) and odd $\lceil \frac{N}{J} \rceil$ in (27)) are applicable for designing of multirate systems. For the demonstration, we consider the same example which is used to design the proposed transposed Type-1 polyphase structure (depicted in Fig. 5). Accordingly, the upsampler of $L = J = 4$ is inserted at the input of the filter after choosing the suitable filter coefficients (i.e., for illustration, $c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9, c_{10}, c_{11}, c_{12}, c_{13}, c_{14}$, and c_{15}) as shown in Fig. 6(a), presented at the top of the next page. Hereby, such structure shall work as a interpolator. However, it performs the filtering operation at the higher sample rate i.e., $4f_{in}$. After applying the noble identities, the upsampler which is inserted at the filter input can be equivalently replaced by four upsampler at points $B1, B2, B3$, and $B4$ as marked in Fig. 6(a). Further, these four upsamplers can be equivalently shifted to points $C1$, $C2$, $C3$, and $C4$ by replacing all delay elements from z^{-4} to z^{-1} in Fig. 6(a). Therefore, the desired interpolator can be realized at the lower sample rate i.e., f_{in}, as shown in Fig. 6(b), depicted at the top of the next page.

From the above realizations of decimator and interpolator structures, it can easily be observed that neither pre processing nor post processing is required in our proposed polyphase structure based multirate system development. Since, the decimator and interpolator are used directly in various applications viz., FFA single-rate, multirate, and filter banks, these structures can be directly to applied to design a relatively less complex system.

IV. FREQUENCY RESPONSE ANALYSIS

In this section, we present the magnitude and phase characteristics of the proposed Type-1/transposed Type-1 polyphase FIR filters.

A. FREQUENCY RESPONSE OF PROPOSED TYPE-1 POLYPHASE FIR FILTER

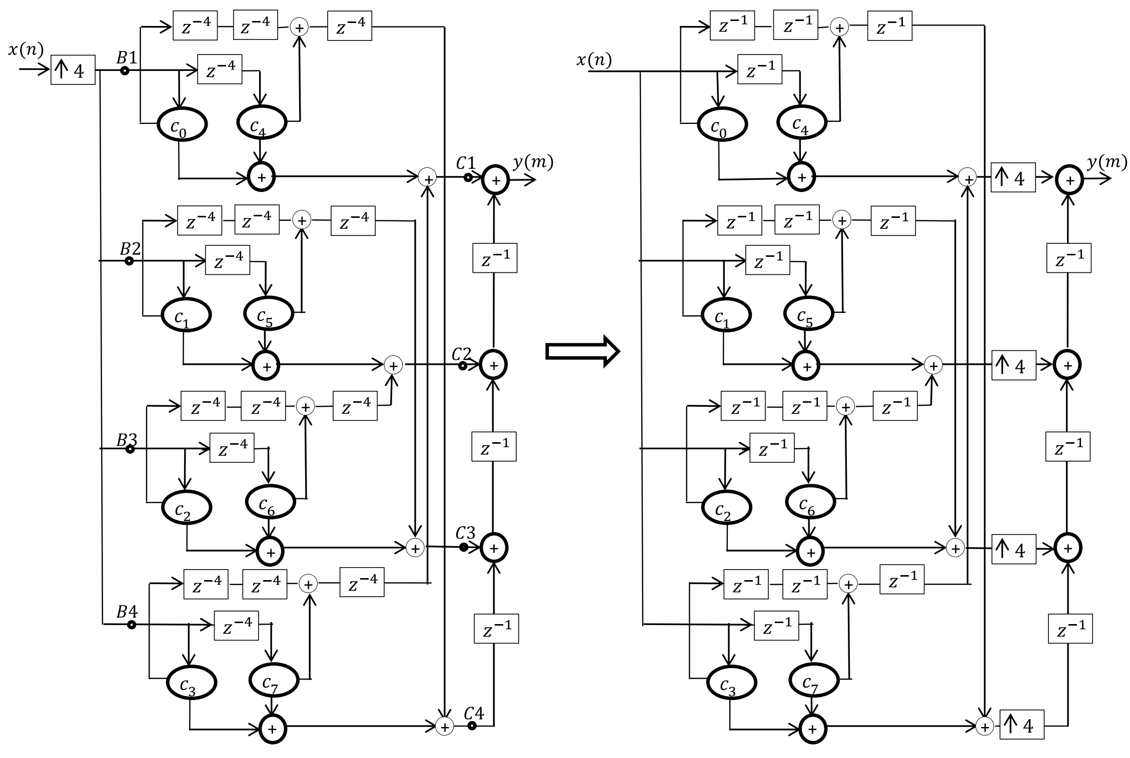
The frequency response of the proposed Type-1 polyphase FIR filter for the two cases,[5] i.e., $p = J$ with even $\lceil N/J \rceil$, and $p = J$ with odd $\lceil N/J \rceil$ can be expressed as per the following theorem.

Theorem 3: For the case when $p = J$ and $\lceil N/J \rceil$ is even, the frequency response of the proposed Type-1 polyphase FIR filter can be given as

$$H_k(\omega) = 2 \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} \cos\left(\omega\left(\frac{N-1}{2} - (Jl+k)\right)\right) \times e^{-j\omega(\frac{N-1}{2})}. \tag{29}$$

For the case when $p = J$ and $\lceil N/J \rceil$ is odd, the frequency response of the proposed Type-1 polyphase FIR filter can be

[5] The frequency response of the proposed Type-1 polyphase FIR filter can also be obtained for the case when $p \neq J$ and even and odd values of $\lceil N/J \rceil$.



(a)

(b)

FIGURE 6. **The transposed Type-1 multirate structure, (a) Fourth order proposed Type-1 polyphase structure, (b) Fourth order proposed interpolator efficient Type-1 polyphase structure.**

expressed as

$$H_k(\omega)=\begin{cases} 2\sum_{l=0}^{\lceil\frac{N}{2J}\rceil-\frac{1}{2}} h_{Jl+k}\cos\left(\omega\left(\frac{N-1}{2}-(Jl+k)\right)\right), \\ \times\, e^{-j\omega(\frac{N-1}{2})} \qquad \text{for } 0\le k\le\lceil\frac{J}{2}\rceil-1, \\ 2\sum_{l=0}^{\lceil\frac{N}{2J}\rceil-\frac{3}{2}} h_{Jl+k}\cos\left(\omega\left(\frac{N-1}{2}-(Jl+k)\right)\right), \\ \times\, e^{-j\omega(\frac{N-1}{2})} \qquad \text{for } \lceil\frac{J}{2}\rceil\le k\le J-1. \end{cases} \tag{30}$$

Proof: The detailed proof is given in Appendix E. ∎

From (29) and (30), it is worth mentioning that the phase of each subfilter after exploiting the coefficients symmetry becomes linear.

Numerical Example: To demonstrate the phase and magnitude characteristics and to verify the linear-phase property of proposed Type-1 polyphase FIR filter, we consider an example with $J = 4$ and $N = 16$. For fair comparison, we also plot the phase and magnitude characteristics of conventional FIR filter, as shown in Figs. 7(a) and 7(b) at the top of the next page. The filter coefficients in the proposed Type-1 polyphase FIR filter and conventional FIR filter are considered same.

For $J = 4$ and $N = 16$, we can express the frequency response of the proposed Type-1 polyphase FIR filter using (29) as

$$H_k(\omega)=2\sum_{l=0}^{1} h_{4l+k}\cos\left(\omega\left(7.5-(4l+k)\right)\right)e^{-j7.5\omega}. \tag{31}$$

Furthermore, (31) can be expressed for $k = 0, 1, 2, 3$, respectively, as

$$H_0(\omega) = 2\left(h_0\cos(7.5\omega)+h_4\cos(3.5\omega)\right)e^{-j7.5\omega}, \tag{32}$$
$$H_1(\omega) = 2\left(h_1\cos(6.5\omega)+h_5\cos(2.5\omega)\right)e^{-j7.5\omega}, \tag{33}$$
$$H_2(\omega) = 2\left(h_2\cos(5.5\omega)+h_6\cos(1.5\omega)\right)e^{-j7.5\omega}, \tag{34}$$
$$H_3(\omega) = 2\left(h_3\cos(4.5\omega)+h_7\cos(0.5\omega)\right)e^{-j7.5\omega}. \tag{35}$$

Then, using (32), (33), (34), and (35), we can plot the phase and magnitude responses of the proposed Type-1 polyphase FIR filter, as shown in Figs. 7(c) and 7(d), respectively, depicted at the top of the next page. It can be observed from Figs. 7(c) and 7(d) that by adding the magnitudes of $k = 0, 1, 2, 3$ subfilters with their respective phases, we can obtain the resultant magnitude equivalent to the magnitude of the conventional FIR filter (as shown in Fig. 7(b)). Moreover, we can also observe that the phase of each subfilter is linear.

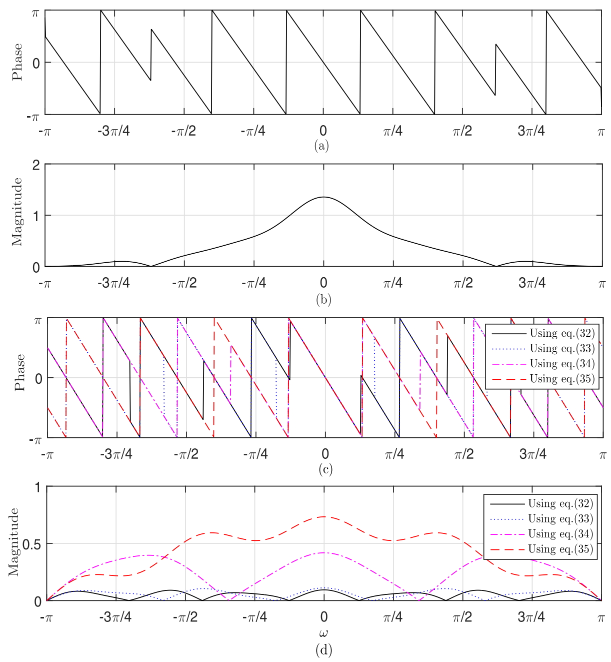


FIGURE 7. (a) Phase characteristic of conventional FIR filter, (b) Magnitude characteristic of conventional FIR filter, (c) Phase characteristic of proposed Type-1 polyphase FIR filter, and (d) Magnitude characteristic of proposed Type-1 polyphase FIR filter.

B. FREQUENCY RESPONSE OF PROPOSED TRANSPOSED TYPE-1 POLYPHASE FIR FILTER

To show the frequency response of the proposed transposed Type-1 polyphase FIR filter, we need to derive the composite transfer function that consists the pair of k and k' subfilters. For this, without loss of information, we consider[6] two cases, viz., $p = J$ with even $\lceil N/J \rceil$, and $p = J$ with odd $\lceil N/J \rceil$. The frequency response under these two cases can be given by the following theorem.

Theorem 4: For the case when $p = J$ and $\lceil N/J \rceil$ is even, the frequency response of the proposed transposed Type-1 polyphase FIR filter can be represented as

$$H_k^c(\omega) = 2\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - 1}\Big[h_{Jl+k}\cos\Big(\omega\Big(\frac{N-1}{2} - (Jl+k)\Big)\Big) + h_{Jl+J-1-k}\cos\Big(\omega\Big(\frac{N+1}{2} - J - (Jl-k)\Big)\Big)\Big] \times e^{-j\omega(\frac{N-1}{2})}. \tag{36}$$

For the case when $p = J$ and $\lceil N/J \rceil$ is odd, the frequency response of the proposed transposed Type-1 polyphase FIR filter can be expressed as

$$H_k^c(\omega) = 2\Big[\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - \frac{1}{2}} h_{Jl+k}\cos\Big(\omega\Big(\frac{N-1}{2} - (Jl+k)\Big)\Big) + \sum_{l=0}^{\lceil \frac{N}{2J} \rceil - \frac{3}{2}} h_{Jl+J-1-k}\cos\Big(\omega\Big(\frac{N+1}{2} - J - (Jl-k)\Big)\Big)\Big] \times e^{-j\omega(\frac{N-1}{2})}. \tag{37}$$

[6]Note that the transfer function of the proposed transposed Type-1 polyphase FIR filter can also be obtained for the case when $p \neq J$ with even and odd values of $\lceil N/J \rceil$.

Proof: See Appendix F for detailed proof. ■

From (36) and (37), it is important to note that the phase of pair of subfilters k and k' after exploiting the coefficient symmetry becomes linear.

Numerical Example: In this example, the magnitude and phase characteristics of a proposed transposed Type-1 polyphase FIR filter with $J = 4$ and $N = 16$ (i.e, $\lceil \frac{N}{J} \rceil$ is even) are studied. For a fair comparison, we have also included the phase and magnitude characteristics of the conventional FIR filter, as shown in Figs. 8(a) and 8(b), respectively. We also consider same filter coefficients in both the proposed transposed Type-1 polyphase FIR filter and conventional FIR filter.

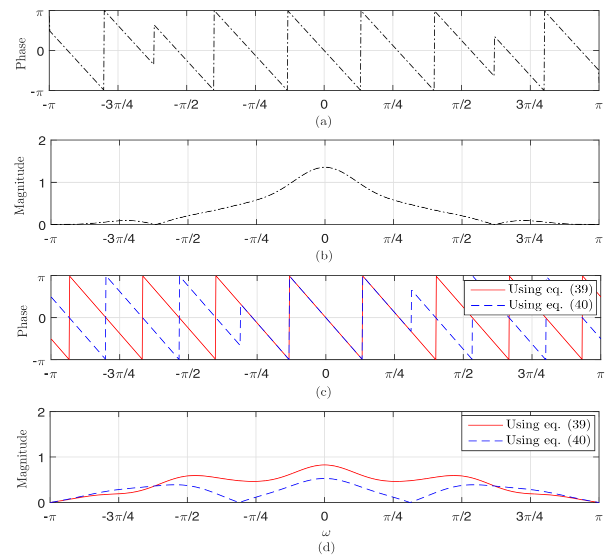


FIGURE 8. (a) Phase characteristic of conventional FIR filter, (b) Magnitude characteristic of conventional FIR filter, (c) Phase characteristic of proposed transposed Type-1 polyphase FIR filter, and (d) Magnitude characteristic of proposed transposed Type-1 polyphase FIR filter.

For $J = 4$ and $N = 16$, we can express the frequency response of the proposed system using (36) as

$$H_k^c(\omega) = \Big[2\sum_{l=0}^{1} h_{4l+k}\cos\big(\omega(7.5 - 4l - k)\big) + h_{4l+3-k}\cos\big(\omega(4.5 - 4l + k)\big)\Big]e^{-j7.5\omega}. \tag{38}$$

Now, invoking $k = 0$ into (38), we can re-express (38) as

$$H_0^c(\omega) = \Big[2h_0\cos\big(7.5\omega\big) + 2h_3\cos\big(4.5\omega\big) + 2h_4\cos\big(3.5\omega\big) + 2h_7\cos\big(0.5\omega\big)\Big]e^{-j7.5\omega}. \tag{39}$$

From (39), we can observe that it consists of the coefficients of $k = 0$th and $k = 3$rd subfilters. Moreover, by invoking $k = 3$ into (38), we have $H_3^c(\omega) = H_0^c(\omega)$, which implies that $k = 0$th subfilter and $k = 3$rd subfilter constitutes a pair of complementary subfilters.

Likewise, for $k = 1$ and $k = 2$, the frequency response can be expressed as

$$H_1^c(\omega) = \Big[2h_1 \cos\left(6.5\omega\right) + 2h_4 \cos\left(2.5\omega\right) + 2h_2 \cos\left(5.5\omega\right) + 2h_6 \cos\left(1.5\omega\right)\Big] e^{-j7.5\omega}, \quad (40)$$

$$H_2^c(\omega) = H_1^c(\omega). \quad (41)$$

This indicates that $k = 1$st subfilter and $k = 2$nd subfilter are complementary to each other.

Furthermore, with the help of (39) and (40), we plot the phase and magnitude characteristics of the proposed transposed Type-1 polyphase FIR filter, as shown in Figs. 8(c) and 8(d), respectively. It can be observed from Figs. 8(c) and 8(d) that the magnitudes of the subfilters (using (39) and (40)) will be added when the phase of the subfilters lies approximately in the range of $-\frac{\pi}{3}$ to $+\frac{\pi}{3}$, whereas the magnitudes subtract for rest of the range (as phases are with the difference of π). Consequently, the resultant magnitude of proposed transposed Type-1 polyphase FIR filter is equivalent to the magnitude response of conventional FIR filter (as shown in Fig. 8(b)). Moreover, the group delay of each subfilter is constant, which leads to the linear-phase of proposed transposed Type-1 polyphase FIR filter.

It is also worth noting that because of the fact that noble identities do not effect the phase characteristics of the system [3], linear-phase can be ensured in efficient decimator and interpolator structures. This linear-phase characteristics in efficient decimator and interpolator structures may brings the opportunity to process the signal on subfilters without the phase distortion.

V. COMPARISON OF PROPOSED STRUCTURES WITH EXISTING STRUCTURES

In this section, we present the comparison of our proposed structures with the structures presented in the literature under the coefficient symmetry property. The comparison is given in terms of the number of multipliers ($\mathcal{M}$) required to implement the multirate converters. Let L and M are the upsampler and downsampler, respectively, and N is the order of FIR filter.

In Table 2, we compare our proposed approach with [18] and [21] for decimator rate converter when $M = 4, N = 240$ and $M = 8, N = 96$. Here in this table, we have also shown the results of conventional polyphase structure (without coefficient symmetry) presented in [1] for reference. It can be seen from Table 2 that the conventional polyphase structure requires 240 and 96 multipliers when for $N = 240$ and $N = 96$, respectively. Whereas, by exploiting the coefficient symmetry the multipliers requirement almost reduce to half,

TABLE 2. Comparison of the proposed structure with [18] and [21] for decimator rate converters.

	M, N	$\mathcal{M}$	M, N	$\mathcal{M}$
[1]	4, 240	240	8, 96	96
[18]	4, 240	120	8, 96	48
[21]	4, 240	121	8, 96	49
Our Paper	4, 240	120	8, 96	48

as presented in [18], [21], and our proposed approach of Table 2. Furthermore, it can be observed that our proposed approach and the ones presented in [18] and [21] have almost same number of multiplier units for all values of M and N. However, our proposed approach is more efficient compared to [18] and [21], since it does not require pre/post processing and can also be applicable for FFA single-rate systems.

TABLE 3. Comparison of the proposed structure with [19] and [20] for rational rate converters.

	M, L, N	$\mathcal{M}$	M, L, N	$\mathcal{M}$
[1]	3,5,23	23	3,2,11	11
[19]	3,5,23	16	3,2,11	7
[20]	3,5,23	13	3,2,11	7
Our Paper	3,5,23	12	3,2,11	6

In Table 3, we present the comparison of our proposed approach with the ones presented in [19] and [20] for rational rate conversion when $M = 3, L = 5, N = 23$ and $M = 3, L = 2, N = 11$. Also, we present the results of conventional polyphase structure (without coefficient symmetry) presented in [1] for reference. It can readily be observed from Table 3 that the conventional polyphase structure requires the multiplier units same as the order of the filter. However, the multipliers requirement reduce when coefficient symmetry is exploited, as shown in [19], [20], and our proposed approach of Table 3. Further, it is worth mentioning that [19] and [20] can achieve half number of multiplier units in case of large N, whereas in our proposed approach the multiplier units will always be half, irrespective of the order of the filter. Moreover, the proposed structure is more efficient than that of the structures presented in [19] and [20] in various aspects; provides generalized structure; applicable in FFA single-rate systems, and no pre/post processing.

VI. CONCLUSION

In this paper, we have presented the efficient generalized polyphase structures for conventional Type-1/transposed Type-1 FIR filters, in which the coefficient symmetry is exploited. In particular, we have presented the mathematical expressions to distinguish the lower and higher index coefficients of polyphase FIR filter. Consequently, we have deduced the transfer function expressions for the proposed Type-1/transposed Type-1 polyphase FIR filters under coefficient symmetry. It is shown that the proposed structures can reduce the multiplication complexity by a factor of two, without performing any extra pre/post processing over FIR filter coefficients or subfilters. To show the applicability of the proposed structures in multirate systems, we have demonstrated the decimator design using the proposed Type-1 polyphase structure and the interpolator design using the proposed transposed Type-1 polyphase structure. Moreover, we have presented phase and magnitude characteristics of the proposed Type-1/transposed Type-1 polyphase structures, and it was observed that all subfilters of the proposed structures maintain the linear-phase characteristics. The numerical examples were considered to support and corroborate our proposed analysis. Finally, it can be concluded that the proposed symmetric structures are simple and efficient, which can be applicable in various applications of single-rate systems, multirate systems, and filter banks.

APPENDIX A
PROOF OF (8) AND (9)

By knowing the relation between symmetric coefficients, we have

$$Jl + k = N - (Ju + k') - 1, \tag{42}$$

which can be further simplified to get k' as

$$k' = N - (J(u + l) + k) - 1. \tag{43}$$

Likewise, we can also express k'' as

$$k'' = N - (J(u + l) + k) - 1. \tag{44}$$

Furthermore, using *Property 1*, by invoking $u+l = \lceil N/J \rceil - 1$ into (43) and $u + l = \lceil N/J \rceil - 2$ into (44), we can obtain (8) and (9), respectively.

APPENDIX B
PROOF OF PROPERTY 3

We know that $\left\lceil \frac{N}{2} \right\rceil = \frac{N}{2} + \frac{\mathrm{mod}(N,2)}{2}$, therefore, the index of middle coefficient (i.e., $\left\lceil \frac{N}{2} \right\rceil - 1$) of the conventional FIR filter can be represented by

$$\left\lceil \frac{N}{2} \right\rceil - 1 = \frac{N}{2} + \frac{\mathrm{mod}(N,2)}{2} - 1. \tag{45}$$

In order to obtain the index of middle coefficient of the polyphase FIR filter, we require to modify the right hand side (RHS) of (45). The RHS of (45) can be re-express by using the fact that $p = (N - J(\lceil N/J \rceil) + J)$ as

$$\Rightarrow J\left(\left\lceil \frac{N}{2J} \right\rceil - \frac{1}{2}\right) + \frac{\mathrm{mod}(N,2)}{2} + \frac{p}{2} - 1. \tag{46}$$

Further, for the case when $\left\lceil \frac{N}{J} \right\rceil$ is even, we can express (46) as

$$\Rightarrow J\left(\left\lceil \frac{N}{2J} \right\rceil - 1\right) + \frac{p+J}{2} - 1 + \frac{\mathrm{mod}(N,2)}{2}. \tag{47}$$

Then, using $\left\lceil \frac{p+J}{2} \right\rceil = \frac{p+J}{2} + \frac{\mathrm{mod}((p+J),2)}{2}$ into (47), we get

$$\Rightarrow J\left(\left\lceil \frac{N}{2J} \right\rceil - 1\right) + \left\lceil \frac{p+J}{2} \right\rceil - 1 + \underbrace{\left(\frac{\mathrm{mod}(N,2)}{2} - \frac{\mathrm{mod}((p+J),2)}{2}\right)}_{\triangleq s_1}. \tag{48}$$

Moreover, by performing the division of N by J, i.e., $N = p + JD$, where D is the quotient and p is the remainder, and invoking this into s_1 of (48), we can obtain s_1 as

$$s_1 = \frac{\mathrm{mod}((p+JD),2)}{2} - \frac{\mathrm{mod}((p+J),2)}{2}, \tag{49}$$

which can be further simplified by using the property $\mathrm{mod}((a + bc), 2) = \mathrm{mod}(\mathrm{mod}(a, 2) + \mathrm{mod}(\mathrm{mod}(b, 2) * \mathrm{mod}(c, 2), 2), 2)$, and knowing that $\mathrm{mod}(D, 2) = 1$ when D is odd (which is always odd until $\left\lceil \frac{N}{J} \right\rceil$ is even) as

$$s_1 = 0. \tag{50}$$

Now, invoking (50) into (48), the indices l and k of polyphase FIR filter can be expressed as $l = \left\lceil \frac{N}{2J} \right\rceil - 1$ and $k = \left\lceil \frac{p+J}{2} \right\rceil - 1$, respectively.

For the case when $\left\lceil \frac{N}{J} \right\rceil$ is odd, by following the same approach as used above, we can express the indices $l = \left\lfloor \frac{1}{2} \left\lceil \frac{N}{J} \right\rceil \right\rfloor$ and $k = \left\lceil \frac{p}{2} \right\rceil - 1$.

APPENDIX C
PROOF OF THEOREM 1

For even $\left\lceil \frac{N}{J} \right\rceil$, the higher index coefficients term $E^p_{k,2}(z)$ in (14) can be replaced by the symmetric lower index coefficients available in the respective complementary subfilter as

$$\begin{aligned} E^p_{k,2}(z) &= \sum_{l=\left\lceil \frac{N}{2J} \right\rceil}^{\left\lceil \frac{N}{J} \right\rceil - 1} h_{Jl+k} z^{-l} \\ &\overset{(a)}{=} \sum_{l=\left\lceil \frac{N}{2J} \right\rceil}^{\left\lceil \frac{N}{J} \right\rceil - 1} h_{N-(Jl+k)-1} z^{-l} \\ &\overset{(b)}{=} z^{-\frac{\lceil N/J \rceil}{2}} \underbrace{\sum_{u=0}^{\left\lceil \frac{N}{2J} \right\rceil - 1} h_{Ju+k'} z^{-\left(\frac{\lceil N/J \rceil}{2} - 1 - u\right)}}_{\triangleq E^p_{k',1}(z)}, \end{aligned} \tag{51}$$

where (a) is obtained by applying the coefficient symmetry from (3), i.e., $h_{Jl+k} = h_{N-(Jl+k)-1}$, and (b) is obtained by replacing $N - (Jl + k) - 1$ with $Ju + k'$ and using the relation

$l = \lceil N/J \rceil - u - 1$ from *Property 1*, and then reversing the limits of the summation (because k' is complementary or mirror image of k). $E^{p}_{k',1}(z)$ denotes the lower index coefficients of the complementary k' subfilter. From (51), we can see that the higher index coefficients of kth subfilter are eliminated and the respective symmetric coefficients available as mapped into the lower index coefficients of its complementary subfilter k' with a delay $z^{-\frac{\lceil N/J \rceil}{2}}$ are utilized, without affecting the input-output relation of the filter.

Now, applying (51) into (14), we can obtain

$$E^{p}_{k}(z) = E^{p}_{k,1}(z) + z^{-\frac{\lceil N/J \rceil}{2}} E^{p}_{k',1}(z). \tag{52}$$

Similarly, we can express $E^{q1}_{k}(z)$ in (15) and $E^{q2}_{k}(z)$ in (16), respectively, as

$$E^{q1}_{k}(z) = E^{q1}_{k,1}(z) + z^{-\frac{\lceil N/J \rceil}{2}} E^{q1}_{k'',1}(z), \tag{53}$$

$$E^{q2}_{k}(z) = E^{q1}_{k,1}(z) + z^{-\left(\frac{\lceil N/J \rceil}{2}-1\right)} E^{q2}_{k'',1}(z), \tag{54}$$

where $E^{q1}_{k'',1}(z)$ and $E^{q2}_{k'',1}(z)$ are the lower index coefficients of the complementary k'' subfilter, and can be expressed as

$$E^{q1}_{k'',1}(z) = \sum_{u=0}^{(\lceil \frac{N}{2J} \rceil)-2} h_{Ju+k''} z^{-(\frac{\lceil N/J \rceil}{2}-2-u)}, \tag{55}$$

$$E^{q2}_{k'',1}(z) = \sum_{u=0}^{(\lceil \frac{N}{2J} \rceil/2)-1} h_{Ju+k''} z^{-(\frac{\lceil N/J \rceil}{2}-1-u)}. \tag{56}$$

By invoking (52), (53), and (54) into (13), we can obtain the mathematical expression of the transfer function for the case when $\lceil \frac{N}{J} \rceil$ is even, as presented in (23).

For odd $\lceil \frac{N}{J} \rceil$, by following the similar approach as used to obtain (23) for even $\lceil \frac{N}{J} \rceil$, and by expressing the lower index coefficients of the complementary k' subfilter $E^{p1}_{k',1}(z)$ and $E^{p2}_{k',1}(z)$, and lower index coefficients of the complementary k'' subfilter $E^{q}_{k'',1}(z)$ as

$$E^{p1}_{k',1}(z) = \sum_{u=0}^{\lfloor \lceil N/J \rceil/2 \rfloor - 1} h_{Ju+k'} z^{-(\frac{\lceil N/J \rceil - 1}{2}-1-u)}, \tag{57}$$

$$E^{p2}_{k',1}(z) = \sum_{u=0}^{\lfloor \frac{\lceil N/J \rceil}{2} \rfloor} h_{Ju+k'} z^{-(\frac{\lceil N/J \rceil - 1}{2}-u)}, \tag{58}$$

$$E^{q}_{k'',1}(z) = \sum_{u=0}^{\lfloor \frac{\lceil N/J \rceil}{2} \rfloor - 1} h_{Ju+k''} z^{-(\frac{\lceil N/J \rceil - 1}{2}-1-u)}, \tag{59}$$

we can obtain the overall transfer function for the case when $\lceil \frac{N}{J} \rceil$ is odd, as presented in (24).

APPENDIX D
PROOF OF THEOREM 2

When $\lceil \frac{N}{J} \rceil$ is even, by applying the coefficient symmetry from (3), i.e., $h_{Jl+k} = h_{N-(Jl+k)-1}$, and by replacing $N-(Jl+k)-1$ with $Ju+k'$ and making use of the relation $l = \lceil N/J \rceil - u - 1$ from *Property 1*, and then reversing the limits of the summation (because k' is complementary or mirror image of k), we can replace the higher index coefficients term $E^{p}_{k,2}(z)$ in (14) in the symmetric lower index coefficients term $E^{p1}_{k'}(z)$ as

$$E^{p}_{k,2}(z) = \sum_{u=0}^{\lceil \frac{N}{2J} \rceil - 1} (h_{Ju+k'} z^{-u}) z^{-\left(\lceil N/J \rceil - 1 - 2u\right)} \triangleq E^{p1}_{k',1}(z). \tag{60}$$

In (60), it is noted that the input stream of k'th subfilter is first delayed by z^{-u} and then multiplied by $h_{Ju+k'}$, and the result is again delayed by $z^{-(\lceil N/J \rceil - 1 - 2u)}$. Further, invoking (60) into (14), the modified expression of kth subfilter with coefficient symmetry can be expressed as

$$E^{p}_{k}(z) = E^{p}_{k,1}(z) + E^{p1}_{k',1}(z). \tag{61}$$

Likewise, we can represent $E^{q1}_{k}(z)$ in (15) and $E^{q2}_{k}(z)$ in (16), respectively, as

$$E^{q1}_{k}(z) = E^{q1}_{k,1}(z) + E^{q1}_{k'',1}(z), \tag{62}$$

$$E^{q2}_{k}(z) = E^{q1}_{k,1}(z) + E^{q2}_{k'',1}(z), \tag{63}$$

where

$$E^{q1}_{k'',1}(z) = \sum_{u=0}^{\lceil \frac{N}{2J} \rceil - 2} (h_{Ju+k''} z^{-u}) z^{-(\lceil N/J \rceil - 2 - 2u)}, \tag{64}$$

$$E^{q2}_{k'',1}(z) = \sum_{u=0}^{\lceil \frac{N}{2J} \rceil - 1} (h_{Ju+k''} z^{-u}) z^{-(\lceil N/J \rceil - 1 - 2u)}. \tag{65}$$

By invoking (61), (62), and (63) into (13), we can get the mathematical expression of transfer function for the proposed transposed Type-1 FIR filter, as expressed in (26).

When $\lceil \frac{N}{J} \rceil$ is odd, by following the same steps as used above for even $\lceil \frac{N}{J} \rceil$, and by expressing the lower index coefficients of the complementary k' subfilter $E^{p1}_{k',1}(z)$ and $E^{p2}_{k',1}(z)$, and lower index coefficients of the complementary k'' subfilter $E^{q}_{k'',1}(z)$ as

$$E^{p1}_{k',1}(z) = \sum_{u=0}^{\lfloor \lceil N/J \rceil/2 \rfloor - 1} (h_{Ju+k'} z^{-u}) z^{-(\lceil N/J \rceil - 2 - 2u)}, \tag{66}$$

$$E^{p2}_{k',1}(z) = \sum_{u=0}^{\lfloor \frac{\lceil N/J \rceil}{2} \rfloor} (h_{Ju+k'} z^{-u}) z^{-(\lceil N/J \rceil - 1 - 2u)}, \tag{67}$$

$$E^{q}_{k'',1}(z) = \sum_{u=0}^{\lfloor \frac{\lceil N/J \rceil}{2} \rfloor - 1} (h_{Ju+k''} z^{-u}) z^{-(\lceil N/J \rceil - 2 - 2u)}. \tag{68}$$

the mathematical expression of the transfer function for this case can be expressed as (27).

APPENDIX E
PROOF OF THEOREM 3

For the case when $p = J$ and $\lceil N/J \rceil$ is even, the expression of transfer function of the symmetric kth subfilter, $0 \leq k \leq J-1$, can be expressed as

$$H(z)= \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} z^{-Jl-k} + \sum_{u=\lceil \frac{N}{2J} \rceil}^{\lceil \frac{N}{J} \rceil -1} h_{Ju+k'} z^{-Ju-k'}. \quad (69)$$

By invoking $u = \lceil \frac{N}{J} \rceil - 1 - l$ and $k' = J - 1 - k$ into (69) and using the relation $h_{Jl+k} = h_{N-(Jl+k)-1}$, we can re-express (69) after some manipulations as

$$H(z) = z^{-\frac{N-1}{2}} \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} \left(z^{(\frac{N-1}{2} - Jl - k)} + z^{-(\frac{N-1}{2} - Jl - k)} \right). \quad (70)$$

Substituting $z = e^{j\omega}$ into (70) and after some simplifications, the frequency response can be obtained, as expressed in (29).

For the case when $p = J$ and $\lceil N/J \rceil$ is odd, the expression of transfer function of the symmetric kth subfilter for the case when $p = J$ and odd $\lceil N/J \rceil$ can be given as

$$H(z) = \begin{cases} \displaystyle\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - \frac{1}{2}} h_{Jl+k} z^{-Jl-k} + \sum_{u=\lceil \frac{N}{2J} \rceil + \frac{1}{2}}^{\lceil \frac{N}{J} \rceil -1} h_{Ju+k'} z^{-Ju-k'}, \\ \qquad \text{for } 0 \leq k \leq \lceil \frac{J}{2} \rceil - 1, \\ \displaystyle\sum_{l=0}^{\lceil \frac{N}{2J} \rceil - \frac{3}{2}} h_{Jl+k} z^{-Jl-k} + \sum_{u=\lceil \frac{N}{2J} \rceil - \frac{1}{2}}^{\lceil \frac{N}{J} \rceil -1} h_{Ju+k'} z^{-Ju-k'}, \\ \qquad \text{for } \lceil \frac{J}{2} \rceil \leq k \leq J-1. \end{cases} \quad (71)$$

By invoking $u = \lceil \frac{N}{J} \rceil - 1 - l$ and $k' = J - 1 - k$ into (71) and using the relation $h_{Jl+k} = h_{N-(Jl+k)-1}$, and then substituting $z = e^{j\omega}$ into the result, the frequency response for this case can be obtained, as presented in (30).

APPENDIX F
PROOF OF THEOREM 4

For the case when $p = J$ and $\lceil N/J \rceil$ is even, the combined transfer function of the symmetric kth subfilter and its associated k'th subfilter can be expressed as

$$\begin{aligned} H_k^c(z) &= z^{-k} \left[\sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} z^{-Jl} + \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k'} z^{-(N-J-Jl)} \right] \\ &+ z^{-k'} \left[\sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k'} z^{-Jl} + \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} z^{-(N-J-Jl)} \right]. \end{aligned} \quad (72)$$

Further, by incorporating $k' = J - 1 - k$ and rearranging the terms, we can express (72) as

$$\begin{aligned} H_k^c(z) &= \sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} \Big[h_{Jl+k} (z^{-k-Jl} + z^{-(N-1-k-Jl)}) \\ &\quad + h_{Jl+J-1-k} (z^{(1-J-Jl+k)} + z^{-(N-J-Jl+k)}) \Big] \\ &= z^{-\frac{N-1}{2}} \Big[\sum_{l=0}^{\lceil \frac{N}{2J} \rceil -1} h_{Jl+k} (z^{(\frac{N-1}{2} - k - Jl)} + z^{-(\frac{N-1}{2} - k - Jl)}) \\ &\quad + h_{Jl+J-1-k} (z^{(\frac{N+1}{2} - J - Jl + k)} + z^{-(\frac{N+1}{2} - J - Jl + k)}) \Big]. \end{aligned} \quad (73)$$

Now, putting $z = e^{j\omega}$ into (73) and simplifying it, the frequency response can be expressed as (36).

For the case when $p = J$ and $\lceil N/J \rceil$ is odd, following the same steps as used above to obtain (36), the frequency response in this case can be expressed as (37).

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