Notice of Retraction

"Interpretable Facial Semantic Extractor Based on Axiomatic Fuzzy Set"
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After careful and considered review of the content of this paper by a duly constituted committee, this paper has been found to be in violation of IEEE's Publication Principles. IEEE hereby retracts the content of this paper.

It has been brought to the attention of the Editors that the data underlying this study are no longer accessible. Therefore, the data are not available for request by readers wishing to repeat the analyses presented in this work. Additionally, the Authors did not respond on whether the person depicted in the paper gave consent or the picture was in the public domain.

Due to the nature of this violation, reasonable effort should be made to remove all past references to this paper, and refrain from future references to this paper.

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Interpretable Facial Semantic Extractor Based on Axiomatic Fuzzy Set

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ABSTRACT The main purpose of this paper is to label one face to which it refers using a commonly accepted concept or linguistic term that corresponds to a given face class. Therefore, a facial semantic description method based on the axiomatic fuzzy set (AFS) theory is proposed. Firstly, the facial geometric features are defined based on the landmarks, such as distance features, angle features and scale features, etc. And then the salient features are selected via AFS feature selection method, and the fuzzy linguistic terms are defined the selected features. Thirdly, the facial descriptions which can characterize the salient characteristics are extracted via a clustering scheme in the framework of AFS theory. Multiple experiments are done on Chinese ethnic face database including *Korean*, *Uyghur* and *Zhuang*. The experimental results demonstrate that AFS feature selection can obtain higher accuracies via AFS clustering algorithm for each ethnic group than that via mRMR. Furthermore, compared with k-means, FCM and Minmax k-means, AFS clustering algorithm obtains the highest accuracies for each ethnic group, which verifies the efficacy of the proposed semantic extractor. Consequently, one comparative experimental results verify the consistency between the extracted semantic description method and the facial physical characteristics in physical anthropology. Also the facial semantics would be very useful to further describe some facial action units or facial expressions, which can be further used for affection analysis of images as well as semi-automatic action unit/facial expression annotation.

INDEX TERMS Semantic extraction, feature selection, axiomatic fuzzy set.

I. INTRODUCTION

With the Chinese development for thousands of years, due to the influence of history, geography, climate, culture and the most important genetic factors, all ethnic groups have their own characteristics in facial features. Meanwhile, ethnic information in the facial features included in the human face (such as age, gender, ethnicity, etc.) has high practical value in computer vision and anthropology. Therefore, the extraction of facial features from ethnic groups has an important position in face recognition.

In the multi-ethnic facial feature extraction, a Chinese multi-ethnic face database is established [1], [2], and conducted preliminary research on the facial feature extraction of some ethnic groups. The result shows that all ethnic faces are separable. Then, an ethnic face database [3] is created by extracting three types of features as ethnic representations. The discriminative power of the extracted features is demonstrated by learning the ethnicity manifold. However, the above researches focus on the low-level feature extraction.

Due to importance of ethic recognition and the semantic gap between low-level geometric features and high-level semantic concepts, how to extract the semantic descriptions for ethnic faces is a significant and attractive problem. Meanwhile, high-level semantics extraction has received much attention in face related applications. Recently, many authors endeavor to facial semantics extraction. Ren *et al.* [4] propose a facial semantic descriptor is extracted based on AFS theory. Nawaf *et al.* [5] present a semantic extraction in which the face attributes derived from face images automatically and

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it can be used for unconstrained human face identification. Liu *et al.* [6] propose a multistage KNN collaborative coding based Bag-of-Feature (MKCC-BoF) method in which the semantic gap between facial features and facial identification has been weakened to address SSPP problem. Reference [7] develop an eyebrow semantic description via clustering based on Axiomatic Fuzzy Set. References [8] and [9] propose the semantics extraction algorithms to obtain the shape characterizations for eyes and eyebrows, respectively. The facial semantic descriptor in [10] is proposed based on information granules to represent the facial semantics, the results shows that this descriptor not only can characterize the key semantics of facial components of data, but also can improve the semantic classification performance in comparison with human perception. Above researches have not focused on the extraction for multi-ethnic facial semantics.

In the ethnic face recognition, Zhang *et al.* [11] propose a multi-ethnic facial semantic network based on correlation of facial shape semantics, which proves differences in facial features of different ethnic groups. Fan *et al.* [12] propose a kind of "end-to-end" machine learning method to predict gender, age, glasses, ethnic and expression of a human facial image and achieved high performance which means the accuracy rates of nationality gender, age, glasses, and expression are high. A method of classifying a face image into an ethnic group is proposed in [13] by applying transfer learning from a previously trained classification network and yielded state-of-the-art success rates of African, Asian, Caucasian, and Indian. But the results have no interpretability for the race faces. In order to obtain the effective semantic description of each ethnic group, Wang *et al.* [14], [15] establish a Chinese national face database and used LPP [16], Isomap [17], LE [18], PCA [19] and LDA [20], etc. manifold methods to analyze the data set, revealing that the national face data can form a sub-manifold structure distributed by the ethnic semantics in the subspace, and enriches the ethnic face measurement index. Li *et al.* [21] apply the AFS clustering framework to construct the optimal classifier, so that the generated facial features of for each class (ethnic group) have own descriptions. Wang *et al.* [22] improve WM method to extract the facial semantics which can generate multi-ethnic facial semantic rules, and the semantic rules have good interpretability. However, these researches are supervised learning for facial semantics extraction, i.e., they need to provide the ethnic label of each face.

For the high-level semantic extraction of ethnic facial features by unsupervised learning, an ethnic facial semantic description extraction method is proposed based on AFS clustering algorithm (AFSC) [23]. First, the facial landmarks detection method [24] is used in order to extract facial components of frontal face images in Chinese ethnic face database. Second, the salient facial features will be selected via AFS feature selection (FS-AFS) or mRMR. Then the facial semantic descriptions are formed by clustering these selected facial features based on AFSC. Consequently, each ethnic cluster is labeled with some precise semantic descriptions. The main advantages of this work are semantic interpretability for ethnic faces and the clustering accuracies. Furthermore, this is the first work that an unsupervised clustering approach based on AFS theory is presented to extract the high-level semantic for ethnic face images. What's more, the experimental results on the database of three ethnic groups (*Korean*, *Uyghur* and *Zhuang*) will verify the consistency between the extracted semantics with the proposed method and the facial physical characteristics in physical anthropology.

The remainder of this paper is organized as follows: Section II briefly reviews AFS theory. Section III details the proposed ethnic facial semantic descriptor. Section IV reports the experimental results and the comparison analysis. Finally, we conclude this paper in Section V.

II. AFS THEORY

In this section, the basic symbols and some definitions of Axiomatic Fuzzy Sets (AFS) theory will be illustrated. In AFS theory, the definition of membership function and the logic operation of fuzzy set are automatically generated according to the distributions of raw data and the semantics of the fuzzy concepts [25]. As a suitable approach to semantic concept extraction and interpretations for multiple attributes, the AFS theory has been extended to many issues of pattern recognition and machine learning, including fuzzy decision trees [26], [27], fuzzy rough sets [28], [29], fuzzy classifiers [23], [30] and fuzzy clustering [31], [32]. Meanwhile, this motivated that AFS theory has been applied to many practical problems, such as semantic facial descriptor extraction [4], management strategic analysis [33], predicting readmissions in intensive care unit [34], time series analysis [35], mitigating the risk of customer churn [36], handwritten number recognition [37], etc.

In the following, we use one session of face data set in Chinese ethnic face database including *Korean*, *Uyghur* and *Zhuang* [14] as an illustrative example to show fuzzy sets in the framework of AFS theory. This ethnic database contains *Korean*, *Uyghur* and *Zhuang* with 300 face images. For each face image, 6 features "$f_1, f_2, \ldots, f_6$" are used to characterize this face shown in Table 1 and Figure 1, where the point "n" is the midpoint of the line between 20th landmark and 23th landmark. Frist, we define

Let $I_k = \{x_k^{f_1}, x_k^{f_2}, \ldots, x_k^{f_6}\}$, where $x_k^{f_i}$is the ith feature value of the kth face image in I_k, and $M = \{m_{i,j} | 1 \leq i \leq 6, 1 \leq j \leq 3\}$ is the set of fuzzy terms, where $m_{i,1}$, $m_{i,2}$, $m_{i,3}$ are fuzzy linguistic concept terms "*large*", "*small*" and "*medium*" associated with the feature f_i respectively. Then, for each set of fuzzy terms, $A \subseteq M$, $\prod_{m \in A} m$ represents a conjunction of the fuzzy terms in A. For instance, $A_1 = \{m_{2,2}, m_{4,2}, m_{5,2}\} \subseteq M$, the semantic of new fuzzy sets "$m_{2,2}$ and $m_{4,2}$ and $m_{5,2}$" can be interpreted as "*small ratio of right eye fissure height and nasal eye distance and small sum of the angle between right eye top and down point and right eye outer canthus with the angle between eyebrows centre and pupils and small sum of the angle between right*

TABLE 1. Definition for features.

f_i	Definition	Feature semantic description
f_1	$d(35,47)/d(23,51)$	Right eye fissure height / eyebrow nose distance
f_2	$d(31,35)/d(37,51)$	Right eye fissure height / nasal eye distance
f_3	$d(37,51)/d(16,24)$	Nasal eye distance / forehead height
f_4	$\angle(31,29,35)+\angle(45,n,46)$	The angle between right eye top and down point and right eye outer canthus + The angle between eyebrows centre and pupils
f_5	$\angle(31,29,35)+\angle((17,56,26)$	The angle between right eye top and down point and right eye outer canthus + The angle between outer eyebrows and nose apex
f_6	$\angle(51,25,59)$	The angle between left nasal alar top and down point and left eyebrow top point

eye top and down point and right eye outer canthus with the angle between outer eyebrows and nose apex", which are represented as "$\prod_{m\in A_1} m = m_{2,2}m_{4,2}m_{5,2}$". Let $A_2 = \{m_{4,2}, m_{6,2}\} \subseteq M$, $A_3 = \{m_{1,2}\} \subseteq M$, then a new fuzzy set as the disjunction of "$\prod_{m\in A_1} m$", "$\prod_{m\in A_2} m$", "$\prod_{m\in A_3} m$", i.e., "$m_{2,2}m_{4,2}m_{5,2}$" or "$m_{4,2}m_{6,2}$" or "$m_{1,2}$", can be represented as:

$$\sum_{u=1}^{3}(\prod_{m\in A_u} m) = \prod_{m\in A_1} m + \prod_{m\in A_2} m + \prod_{m\in A_3} m.$$

After above representation, the set EM^* can be defined as Eq.(1)

$$EM^* = \{\sum_{i=l}(\prod_{m\in A_i} m)|A_i \subseteq M, i \in I\} \tag{1}$$

where I is not an empty indexing set. Then, A binary relation R is defined on EM^*.

Definition 1 [41]: For $\sum_{i\in I}(\prod_{m\in A_i} m)$, $\sum_{j\in J}(\prod_{m\in B_j} m) \in EM^*$

$$[\sum_{i\in I}(\prod_{m\in A_i} m)]R[\sum_{j\in J}(\prod_{m\in B_j} m)]$$
$$\Leftrightarrow \begin{cases} (1)\, \forall i \in I, \exists k \in J, \text{ such that } A_i \supseteq B_k; \\ (2)\, \forall j \in J, \exists k \in I, \text{ such that } B_j \supseteq A_k. \end{cases} \tag{2}$$

With definition above, R is an equivalent relation on EM^*. Consequently, EM can be defined by EM^*/R [40]. In fact, it proves that each fuzzy set can be uniquely decomposed as

$$\zeta = \sum_{i\in I}(\prod_{m\in A_i} m) \tag{3}$$

where each A_i is a subset of M.

For the calculation of the membership function in the AFS framework an ordered relation is defined first. Let M be a set of fuzzy terms on a set X. From a linearly ordered relation "$\succcurlyeq$" for any $m \in A$ a collection can be defined as follow:

$$A^{\succcurlyeq} = \{y \in X | x \succcurlyeq_m y\} \subseteq X \tag{4}$$

where $A \subseteq M$, $x \in X$. $x \succcurlyeq_m y$ means that the degree of X belonging to M is greater than or equal to y, where $m \in M$. $A^{\succcurlyeq}(x)$ is a collection of all elements in X that belong to the set $\prod_{m\in A} m$ less than or equal to x, and which is determined by the semantics of the fuzzy terms in A and the probability distribution of the observed data sets. Next, a weight function can be defined as below.

Definition 2 (Liu and Pedrycz [40]): Let ν be a fuzzy terms on X, and $\rho_\nu\{\rho_\nu : X \to R^+ = [0,\infty)\}$ is a weight function of the fuzzy term if ρ_ν satisfies the condition as follow.

$$\begin{cases} \rho_\nu(x) = 0 \leftrightarrow x \not\succcurlyeq_m x, x \in X; \\ \rho_\nu(x) \geqslant \rho_\nu(y) \leftrightarrow x \not\succcurlyeq_m y, x, y \in X \end{cases} \tag{5}$$

Consequently, with above expressions, the coherence membership functions can be calculated as follow

$$\mu_\zeta(x) = \sup_{i\in I} \inf_{\gamma\in A_i} \frac{\sum_{u\in A_i^{\succcurlyeq}(x)} \rho_\gamma(u)N_u}{\sum_{u\in X} \rho_\gamma(u)N_u}, \quad \forall x \in X \tag{6}$$

where for any $\gamma \in A_i \subseteq M$, $\rho_\gamma(x) = 1$ means x belonging to γ at some degree, and $\rho_\gamma(x) = 0$ means x not belonging to γ, and $N_x = 1$ means that x is observed once. For detailed calculation, please refer to [40].

III. ETHNIC FACIAL SEMANTIC DESCRIPTOR

In this section, an ethnic facial semantic extractor will be shown in detail. There are three main steps to extract ethnic facial semantic descriptions. The first step is to detect the landmarks for facial components, such as eyebrows, eyes and nose, and extract the facial geometric features including distance, proportion and angle features based on facial landmarks. The second step is to select the salient features by feature selection, in order to remove the redundant features. The third step is to construct the semantic descriptions by AFSC. The key symbols are listed in Table 2.

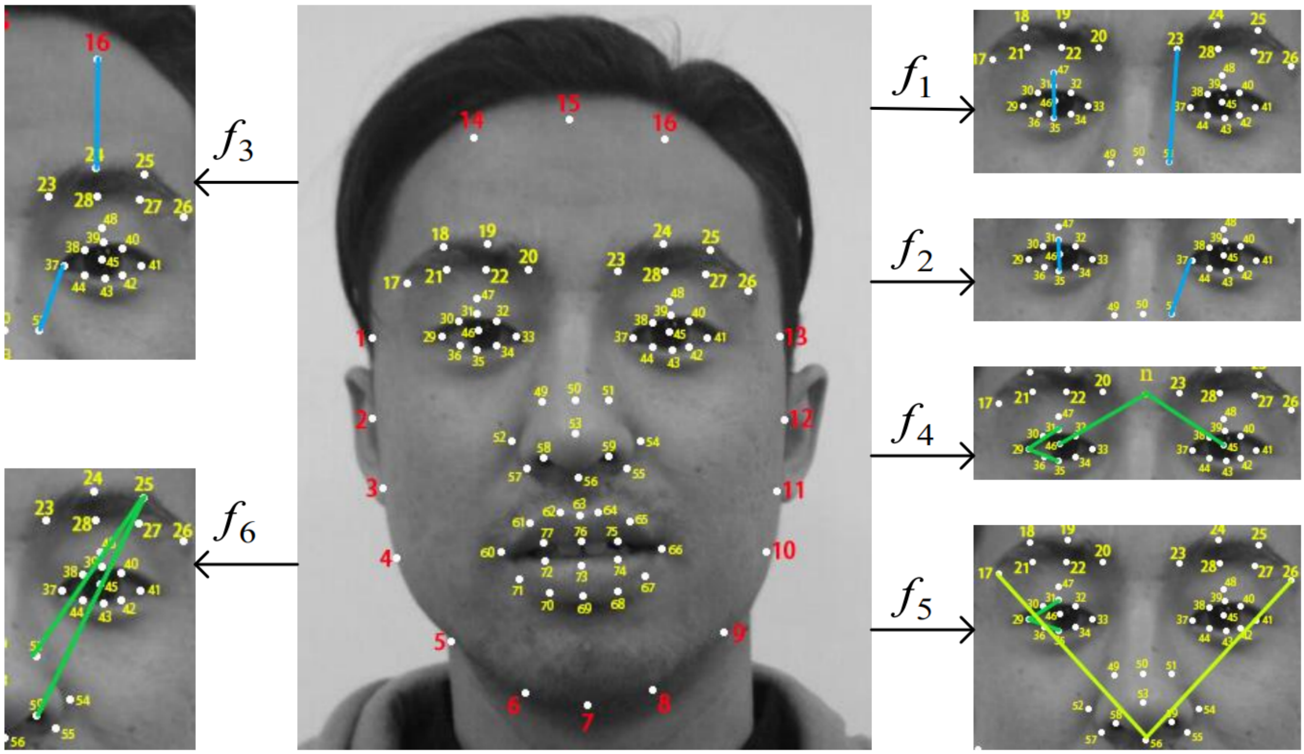

FIGURE 1. Feature extraction.

TABLE 2. Key symbols in this paper.

Symbol	Description of symbol
I^{face}	The set of frontal face images
F	The set of facial features
f_i	The ith facial feature in F
I_k	The kth face image
$x_k^{f_i}$	The ith feature value of I_k
$\mathbf{X}^{f_i}$	The feature vector associated with the f_i
$m_{i,j}$	The fuzzy linguistic term associated with the feature f_i in $F_{selected}$
M	The set of all fuzzy linguistic terms defined in $F_{selected}$
ζ_{I_k}	The characterization of I_k
r_{ij}	The correlation of two facial components I_i and I_j
I_α	The validation index
Γ_i	The set of fuzzy description
$\zeta_{\overline{C_i}}$	The semantic description of each face component cluster $\overline{C_i}$
$\mu_m(I_k)$	The membership degree of I_k belonging to fuzzy term m
$SI(\alpha,\beta)$	The similarity between α and β
$F_{selected}$	The set of selected features
$\mathbf{X}_{selected}$	The set of corresponding feature vectors of selected features

A. FEATURE EXTRACTION

The premise of extracting facial geometric features is to locate facial landmarks. The facial geometric features can be directly calculated via the facial landmarks. In this paper, the landmarks of facial components are extracted by Stand Active Shape Model (STASM) [24], which is an Active Shape Model (ASM) algorithm. Freely open the corresponding software source code under the Berkeley Software Distribution (BSD) style license.

Now 6 ethnic facial features are defined as shown in Table 1 and Figure 1. Let $I^{face} = \{I_1, I_2, \ldots, I_n\}$ be a set of frontal face images, where I_k is the kth face image in I^{face}, n is the number of face images. Then, 77 landmarks were obtained for each face image by this method, as shown in Figure 1. Let $I_k^{face} = \{l_1, l_2, \ldots, l_{77}\}$ be a set of 77 landmarks about the kth face, $l_i = (x_i, y_i)$, $i = 1, 2, \ldots, 77$ is the x-axis and y-axis of the ith landmark. $d(l_i, l_j), i, j = 1, 2, \ldots, 77$ denotes the Euclidean distance between the ith landmark and the jth landmark (Eq.(7)). $\angle(l_i, l_j, l_h)$, $i, j, h = 1, 2, \ldots, 77$ denotes the angle formed by edges ij and jh calculated by the cosine formula (Eq.(8)).

$$d(l_i, l_j) = \|l_i - l_j\|_2 = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{7}$$

$$\angle(l_i, l_j, l_h) = arccos\frac{d^2(l_i, l_j) + d^2(l_j, l_h) - d^2(l_i, l_h)}{2 * d(l_i, l_j) * d(l_j, l_h)} \tag{8}$$

In the following, let F be the set of facial features, and I_k be the kth face image of database.

$$F = \{f_1, f_2, \ldots, f_s\}, \quad I_k = \{x_k^{f_1}, x_k^{f_2}, \ldots, x_k^{f_s}\} \tag{9}$$

where $x_k^{f_i}$, $i = 1, 2, \ldots, s$ is the ith feature value of I_k, and s is the cardinality of facial features, i.e. in this paper $s = 6$ (refer to Table 1). Then, $\mathbf{X}^{f_i}$ is the feature vector associated with the f_i.

$$\mathbf{X}^{f_i} = (x_1^{f_i}, x_2^{f_i}, \ldots, x_n^{f_i})^T \tag{10}$$

B. FEATURE SELECTION

With defining the facial features f_i in F above, the most salient features can be selected as below.

Definition 3 (Liu and Ren [23]): Let $I^{face}=\{I_1, I_2, \ldots, I_n\}$ be a set of samples, $F = \{f_1, f_2, \ldots, f_s\}$ be a set of facial features. The similarity between $\alpha = f_i \in F$ and $\beta = f_j \in F$ $(i, j = 1, 2, \ldots, s)$ is defined as follow:

$$SI(\alpha, \beta) = \frac{\sum_{I_k \in I^{face}} \mu_{\alpha\wedge\beta}(I_k)}{\sum_{I_k \in I^{face}} \mu_{\alpha\vee\beta}(I_k)} \tag{11}$$

where the coherence membership functions $\mu_{\alpha\wedge\beta}(I_k)$ and $\mu_{\alpha\vee\beta}(I_k)$ are calculated by Eq.(6).

Now the salient features can be selected with the similaritisy obtained by Eq.(11). The process of feature selection is mainly divided into three steps.

Firstly, established an equivalent similarity matrix G on the feature set F. Let $H = (y_{ij})_{(s\times s)}$, where $y_{ij} = SI(f_i, f_j)$, $f_i, f_j \in F$, there must existence an integer r, which makes $(H^r)^2 = H^r$. Consequently, $G = H^r$ is a fuzzy equivalent similarity matrix.

Then, determined the initial clusters. $G = q_{ij}$ can generate partition trees with equivalence classes. If $q_{ij} \geq \alpha$, then f_i and f_j are in the same cluster under the threshold $\alpha \in [0, 1]$. The cluster with only one cell is discarded, then the initial clusters can be obtained $U_1^\alpha, U_2^\alpha, \ldots, U_d^\alpha$. So each cluster is a relatively similar set of characteristics.

Thirdly, find feature set according to the cluster obtained in the above step. Let P be the set of all the entries in G

$$P = \{p_1, p_2, \ldots, p_l\} = \{q_{ij} | 1 \leq i \leq s, 1 \leq j \leq s\} \tag{12}$$

in which $p_1 < p_2 \ldots < p_l$. Now a set B^{p_j} for the threshold p_j is defined.

$$B^{p_j} = F - U_{1\leq i\leq d} U_i^{p_j} \tag{13}$$

Each feature in B^{p_j} becomes a cluster by itself. In other words, comparing with other features, the similarity of features is too small to be combined with other features. Then the average entropy of the features in each B^{p_j} can be calculated and the smallest one can be selected as B^{p_j}. Consequently, the final set of selected features can be obtained. $U_{1\leq i\leq d} U_i^{p_j}(p(t) \in P)$.

In this part, features selected in the above way and their corresponding feature vectors are collected in $F_{selected}$ and $\mathbf{X}_{selected}$ respectively. In $F_{selected}$, each selected feature is given with three fuzzy linguistic terms "*large*", "*small*", "*medium*", and then all fuzzy linguistic terms defined in $F_{selected}$ are collected in set M. For detailed calculation, please refer to [23].

C. SEMANTIC DESCRIPTION EXTRACTION

To represent each cluster significantly, a set B_{I_k} of fuzzy terms $m \in M$ is defined as follows:

$$B_{I_k} = \{m \in M | \mu_m(I_k)\} = \mu_{\vee_{b\in M} b}(I_k) \tag{14}$$

where $\mu_m(I_k)$ is the membership degree of I_k belonging to fuzzy term m. Then, I_k is described by the combination of the salient characteristics selected above as:

$$\zeta_{I_k} = \wedge_{\beta\in B_{I_k}} \beta \tag{15}$$

where "$\wedge$" and "$\vee$" are the fuzzy logic operations in AFS algebra defined in [40], and the semantic logic expressions of "$\wedge$" and "$\vee$" are "or" and "and". The detailed computation for $\mu_m(I_k)$, B_{I_k} and ζ_{I_k} can be found in [23].

ζ_{I_k} is a characterization of I_k. Then the correlation of two facial components I_i and I_j can be defined as follow:

$$r_{ij} = min\{\mu_{\zeta_{I_i}\wedge\zeta_{I_j}}(I_i), \mu_{\zeta_{I_i}\wedge\zeta_{I_j}}(I_j)\} \tag{16}$$

Consequently, the larger r_{ij}, the more similar I_i and I_j.

Now, the optimal α for the partitions is decided by a validation index I_α [23]. The validation index I_α is defined as follow:

$$I_\alpha = \frac{\sum_{k=1,2,\ldots,n} \mu_{\zeta_{bou}}(I_k)}{\sum_{k=1,2,\ldots,n} \mu_{\zeta_{Total}}(I_k)} \tag{17}$$

where ζ_{bou}, ζ_{Total} be set of "the boundaries among different clusters which shows the clarity of the clusters" and "the overall characterization for all clusters using for normalization".

$$\zeta_{bou} = \vee_{1\leq i,j\leq l, i\neq j}(\zeta_{\overline{C_i}} \wedge \zeta_{\overline{C_j}}) \tag{18}$$

$$\zeta_{Total} = \vee_{1\leq i\leq l}\zeta_{\overline{C_i}}, \quad l \geq 2 \tag{19}$$

Consequently, the less I_α, the clearer the clustering. In practice, the threshold α is between the minimum and maximum values q_{ij} defined in a fuzzy relation matrix Q.

$$Q = R^t = (q_{ij})_{n\times n},\ R = (r_{ij})_{n\times n} \tag{20}$$

where t is an integer which makes $(R^t)^2 = R^t$. Then, let U be the set of all the entries in Q.

$$U = \{\alpha_1, \alpha_2, \ldots, \alpha_u\} = \{q_{ij} | 1 \leq i \leq n,\ 1 \leq j \leq n\} \tag{21}$$

where $\alpha_1 < \alpha_2 < \ldots < \alpha_u$. Thus, the best threshold α is selected from the matrix U.

Consequently, If $q_{ij} \geq \alpha$, then I_i and I_j are in the same cluster under the optimal threshold α. The clusters $\overline{C_1}, \overline{C_2}, \ldots, \overline{C_l}$, which have more than one face image (i.e., the cluster with one single face image is discarded), can be obtained under the comparison of the correlation r_{ij} and the optimal threshold α, $\alpha \in [0, 1]$.

After an initial clusters $\overline{C_i}$ has be obtained, the best ζ_{I_k} for constructing the semantic description of each cluster $\overline{C_i}$ can be selected as follows:

$$\Gamma_i = \{\zeta_{I_k} | \frac{|\{y | y \in \overline{C_i}, \mu_{\zeta_{I_k}}(y) \geq \lambda\}|}{|\overline{C_i}|} \geq \omega, I_k \in \overline{C_i}\} \tag{22}$$

where $i = 1, \ldots, l$ and l is the number of clusters. Γ_i is a set of fuzzy description, in which some representative descriptions ζ_{I_k} can be used to represent semantic descriptions of $\overline{C_i}$. Thus, if Γ_i is not ϕ the semantic description of each face component cluster can be defined as follow:

$$\zeta_{\overline{C_i}} = \wedge_{\gamma\in\Gamma_i}\gamma \tag{23}$$

TABLE 3. Facial features with mRMR scores.

f_i	f_1	f_2	f_3	f_4	f_5	f_6
Score	0.0121	0.3713	0.1132	0.1763	0.0107	-6.4069×10^{-17}

TABLE 4. Semantic descriptions obtained by feature selection mRMR and FS-AFS.

Method	Selected features	Descriptions $\zeta_{\overline{C_i}}$
mRMR	f_1, f_2, f_3, f_4, f_5	$\zeta_{\overline{C_1}} = m_{1,1}m_{2,1}m_{3,2}m_{4,1}m_{5,1}$ $\zeta_{\overline{C_2}} = m_{1,2}m_{2,2}m_{3,1}m_{4,2}m_{5,2}$ $\zeta_{\overline{C_3}} = m_{1,3}m_{2,3}m_{3,3}m_{4,3}$
FS-AFS	f_1, f_2, f_4, f_5, f_6	$\zeta_{\overline{C_1}} = m_{1,3}m_{4,3}m_{5,3}$ $\zeta_{\overline{C_2}} = m_{1,1}m_{2,1}m_{4,1}m_{5,1}m_{6,1}$ $\zeta_{\overline{C_3}} = m_{1,2}m_{2,2}m_{4,2}m_{5,2}$

Since the elements in Γ_i represent the salient features of each initial cluster $\overline{C_i}$. And the parameters λ and ω controls the universality and particularity of the semantic descriptions of $\overline{C_i}$. The effects of the variation of λ and ω are analyzed in Liu and Ren [23], where indicates that the algorithm is not sensitive to parameter settings if parameters are selected within a reasonable range.

In the nutshell, all semantic descriptions $\zeta_{\overline{C_i}}$ of each cluster $\overline{C_i}$ can be considered as a classifier. Then, a process called re-clustering process is proposed, which aims to modify the initial clusters and cluster the missing instances whose similarities r_{ij} with other examples is lower than the given α. Finally, the reclassification is decided by

$$I_k \in C_q, if\ q = arg \max_{1 \le i \le l} \{\mu_{\zeta_{\overline{C_i}}}(I_k)\} \tag{24}$$

In the next section IV, the experimental results on Chinese ethnic face database will be shown.

IV. EXPERIMENTAL RESULTS

In this section, an experiment on the Chinese ethnic face database is conducted via some well-known algorithms. Chinese ethnic face database including *Zhuang*, *Uyghur* and *Korean* was collected by Wang *et al.* [14], which includes frontal facial images of 100 people for each ethnic group (300 face images in total), half males and half females. And the image collection objects were all undergraduates of various nationalities aged between 18 and 22. Since agreements are not signed by the undergraduates involved in image acquisition, only the landmarks of each face are provided on http://zs.dlnu.edu.cn/minzu300face.rar.

To verify the efficacy of our method, the neutral faces of each individual are used. In experiment, three semantic concepts are specialized on each feature f_i. $m_{i,1}$, $m_{i,2}$, $m_{i,3}$ are three fuzzy linguistic concept terms, respectively, "*large*", "*small*", "*medium*" associated with the feature f_i in F. All membership degrees belonging to the fuzzy set of *EM* are determined by Eq.(6).

A. COMPARISONS BETWEEN FS-AFS AND MRMR VIA AFSC

In this section, two feature selection methods are utilized to select the salient features, that are feature selection via AFS (FS-AFS) and the minimal Redundancy Maximal Relevance (mRMR) [42]. FS-AFS selects the salient features by means of the similarities among the original features. And mRMR feature selection algorithm uses mutual information to measure the correlation and redundancy between features, and remove the redundancy while ensuring the maximum correlation. The original extracted features are listed in Table 1.

The six features in Table 1 are sorted by mRMR scores as follows, *score* as follows: $f_2, f_4, f_3, f_1, f_5, f_6$ as shown in Table 3. FS-AFS selects five features among the 6 features in Table 1, in order to facilitate the comparison, the first five features with higher weight *score* (*score* > 0) in the mRMR feature selection are used to cluster the face images. Then fuzzy semantic descriptions of each ethnic group can be obtained by AFSC via the two selected feature sets. The facial ethnic semantic descriptions extracted from each set are shown in Table 4. The accuracy of clustering results according to the ethnic labels via the salient features selected by FS-AFS and mRMR are shown in Table 5, where $Acc(K)$ means the accuracy for the ethnic group *Korean*, $Acc(U)$ for *Uyghur*, $Acc(Z)$ for *Zhuang*. And $Acc(Total)$ is the accuracy for all of the faces in the Chinese ethnic face database.

The detailed semantic descriptions via AFSC with mRMR,

- *Zhuang* ethnic group: $\zeta_{\overline{C_1}} = m_{1,1}m_{2,1}m_{3,2}m_{4,1}m_{5,1}$ with the fuzzy semantic descriptions "*The face in this cluster have large ratio of right eye fissure height and eyebrow nose distance, large ratio of right eye fissure height and nasal eye distance, small ratio of nasal eye distance and forehead height, large ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre and pupils, large sum of the angle between right eye top and down point and right eye outer canthus and the angle between outer eyebrows and nose apex*";
- *Korean* ethnic group: $\zeta_{\overline{C_2}} = m_{1,2}m_{2,2}m_{3,1}m_{4,2}m_{5,2}$, with the fuzzy semantic descriptions "*The face in this cluster have small ratio of right eye fissure height and eyebrow nose distance, small ratio of right eye fissure height and nasal eye distance, large ratio of nasal eye distance and forehead height, small ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre*

TABLE 5. The accuracy of clustering results via feature selection mRMR and FS-AFS.

Method	Selected features	$Acc(K)(\%)$	$Acc(U)(\%)$	$Acc(Z)(\%)$	$Acc(Total)(\%)$
mRMR	f_2, f_4, f_3, f_1, f_5	**98**	52	72	74
	f_2, f_4, f_3, f_1	98	45	**74**	72.3
	f_2, f_4, f_3	66	74	65	68.3
	f_2, f_4	97	55	66	72.7
FS-AFS	f_1, f_2, f_4, f_5, f_6	96	**77**	59	**77.3**

TABLE 6. The parameters setting for *k*-means, FCM, Minmax *k*-means and AFSC.

Method	Parameter	Setting
k-means	Number of clusters	3
FCM	Number of clusters	3
	Exponent for the fuzzy partition matrix U	2.0
	Maximum number of iterations	100
	Minimum improvement in objective function between two consecutive iterations	1e-5
Minmax k-means	Number of clusters	3
	Initial p (p_{init})	0.0
	Maximum p (p_{max})	0.5
	Step (p_{step})	0.01
	Maximum number of iterations (t_{max})	500
	Amount of memory for the weight updates (β)	0.3
	Number of MinMax k-means restarts ($Restarts$)	10
AFSC	Parameters λ and ω for controlling the universality and particularity of the fuzzy description	0.6 and 0.6

TABLE 7. The clustering accuracies for *k*-means, FCM, Minmax *k*-means and AFSC on Chinese ethnic face database.

Cluster	k-means	FCM	Minmax k-means	AFSC
$Acc(K)(\%)$	93	91	95	**96**
$Acc(U)(\%)$	84	79	**86**	77
$Acc(Z)(\%)$	52	54	51	**59**
$Acc(Total)(\%)$	76.3	74.6	77.3	**77.3**

and pupils, small sum of the angle between right eye top and down point and right eye outer canthus and the angle between outer eyebrows and nose apex'';

- *Uyghur* ethnic group: $\zeta_{\overline{C_3}} = m_{1,3}m_{2,3}m_{3,3}m_{4,3}$, with the fuzzy semantic descriptions ``*The face in this cluster have medium ratio of right eye fissure height and eyebrow nose distance, medium ratio of right eye fissure height and nasal eye distance, medium ratio of nasal eye distance and forehead height, medium ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre and pupils*''.

The detailed semantic descriptions via AFSC with FS-AFS,

- *Uyghur* ethnic group: $\zeta_{\overline{C_1}} = m_{1,3}m_{4,3}m_{5,3}$, with the fuzzy semantic descriptions ``*The face in this cluster have medium ratio of right eye fissure height and eyebrow nose distance, medium ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre and pupils, medium sum of the angle between right eye top and down point and right eye outer canthus and the angle between outer eyebrows and nose apex*'';
- *Zhuang* ethnic group: $\zeta_{\overline{C_2}} = m_{1,1}m_{2,1}m_{4,1}m_{5,1}m_{6,1}$ with the fuzzy semantic descriptions ``*The face in this cluster have large ratio of right eye fissure height and eyebrow nose distance, large ratio of right eye fissure height and nasal eye distance, large ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre and pupils, large sum of the angle between right eye top and down point and right eye outer canthus and the angle between outer eyebrows and nose apex, large angle between left nasal alar top and down point and left eyebrow top point*'';

TABLE 8. Facial physical characteristics of *Zhuang*, *Uyghur* and *Korean* in physical anthropology.

Ethnic group	Facial physical characteristics
Zhuang	Eye: wide rima oculi opening micro-displayed Mongolian fold Nose: convex in lower bridge, concave in upper Face shape: wide face
Uyghur	Eye: medium rima oculi opening wedge-shaped structure apparent upper eyelid folds Nose: medium high nasion, high tip of the nose Face shape: oval
Korean	Eye: narrow rima oculi open degree upper eyelid folds are poorly developed or absent lower eyelid is not swollen well developed Mongolia fold Face shape: oval

TABLE 9. Facial semantic description of *Zhuang, Uyghur* and *Korean* extracted by the ethnic facial semantic extractor.

Ethnic group	Semantics	The included physical characteristics
Zhuang	$\zeta = m_{1,1}m_{2,1}m_{4,1}m_{5,1}m_{6,1}$	Eye: wide rima oculi opening, wide pupil distance Noes: large nasal alar
Uyghur	$\zeta = m_{1,3}m_{4,3}m_{5,3}$	Eye: medium rima oculi opening
Korean	$\zeta = m_{1,2}m_{2,2}m_{4,2}m_{5,2}$	Eye: narrow rima oculi open degree, small pupil distance

- *Kearon* ethnic group: $\zeta_{\overline{C_3}} = m_{1,2}m_{2,2}m_{4,2}m_{5,2}$, with the fuzzy semantic descriptions "*The face in this cluster have small ratio of right eye fissure height and eyebrow nose distance, small ratio of right eye fissure height and nasal eye distance, small ratio of sum of the angle between right eye top and down point and right eye outer canthus and the angle between eyebrows centre and pupils, small sum of the angle between right eye top and down point and right eye outer canthus and the angle between outer eyebrows and nose apex*".

It is not difficult to see from Table 5, the accuracy of *Uyghur* obtained by FS-AFS is higher than that of mRMR, the accuracy of *Korean* and *Zhuang* obtained by FS-AFS are lower than that of mRMR. However, the accuracy of overall obtained by FS-AFS is higher than that of mRMR. Notably, mRMR feature selection algorithm is supervised which requires ethnic labels and the given number of selected features. But FS-AFS selects the salient features under the unsupervised conditions, which does not require manual definition of individual labels and the given number of selected features. In the nutshell, the clustering algorithm via FS-AFS is superior to that via mRMR.

B. PERFORMANCE COMPARISON WITH FCM, K-MEANS AND MINMAX K-MEANS

In order to verify the clustering ability of the ethnic facial semantic descriptor (i.e., the quality of the cluster), the ethnic facial semantic descriptor (based on FS-AFS) is compared with the traditional clustering algorithm FCM [43], k-means [44] and Minmax k-means [45] on the Chinese ethnic face database. Minmax k-means is one of the most popular clustering algorithms. Compared with k-means, it can suppress the appearance of large variance clusters, with less dependence on the initial cluster center and more adaptive to dataset [45]. The clustering results are shown in Table 7. All the parameters of above clustering algorithms are listed in Table 6, where the p exponent in Minmax k-means is a user specified constant that takes values in the range [0, 1), and please refer to [45] for more details. For the specific parameter setting for rules of each algorithm, please refer to [43], [44] and [45]. It is worth mentioning that the data is standardized before clustering.

It can be concluded from Table 7 that $Acc(K)$ and $Acc(Z)$ of AFSC are higher than that of k-means, FCM and Minmax k-means. In this case, $Acc(U)$ is lower than that of k-means, FCM and Minmax k-means. The reason may be that the accurate characterization for *Uyghur* also need 3D facial features, such as "*deep eye socket and high tip of the nose*", except for the selected facial features.

Notably, AFSC achieves the highest accuracy for three ethnic groups, which is equal to Minmax k-means. Furthermore, AFSC not only obtains the high clustering accuracy, but also extracts the interpretable semantics, which can be further used for affection analysis of images as well as semi-automatic action unit/facial expression annotation.

C. COMPARISON ON CONSISTENCY BETWEEN PHYSICAL ANTHROPOLOGY AND THE EXTRACTED SEMANTICS

As a multi-ethnic country, Chinese anthropologists have conducted researches on the physical characteristics of some ethnic groups in China, confirming the differences in facial features of Chinese ethnic groups To verify the consistency between the extracted semantics by the proposed descriptor and ethnic facial features in physical anthropology, a comparison between them is conducted in this section. Table 8 shows the differences in facial physical characteristics of *Zhuang*, *Uyghur* and *Korean* reported in [1], [46]–[48].

The semantic descriptions of the ethnic facial extracted in this paper can be summarized as Table 9, which is the parsimony ethnic facial features concluded from the semantics in Section IV-A. It can be included that the semantics of eyes in Table 8 and 9 have the same features "*wide rima oculi opening of Zhuang*", "*medium rima oculi opening of Uyghur*" and "*narrow rima oculi open degree of Korean*". Furthermore, because this paper only focus on the 2D geometric features of neutral faces, some facial physical characteristics in Table 8 such as convex in "*lower bridge of the nose of Zhuang*", "*high tip of the nose of Uyghur*" and "*well developed Mongolia fold of Korean*" have not been extracted. In the future, 3D facial features which can characterize the features of *Uyghur* well will be anticipated and introduced in the semantic extraction for ethnic groups.

In the nutshell, the ethnic semantic descriptor proposed in this paper not only can extract the facial semantics of the ethnic groups, but also be consistent with ethnic facial features in physical anthropology.

V. CONCLUSION

This paper presents an facial semantic extractor in the framework of AFS theory. The merits of this method are that it can extract the interpretable and understandable semantics for ethnic groups; it preserves the distinctive features among the ethnic groups (*Korean*, *Uyghur* and *Zhuang*); and the extracted semantics by this method is consistent with ethnic facial features in physical anthropology. Empirical tests on Chinese ethnic face database show that the facial semantic extractor obtains a significant efficacy on the representation for facial ethnic characteristics. Meanwhile, the comparisons via AFSC with FS-AFS and mRMR verifies the salient features selected by FS-AFS (unsupervised) is superior than that of mRMR (supervised). Meanwhile, the comparisons with FCM, k-means and Minmax k-means, the method obtains the highest semantic classification performance.

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