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Stabilization of Markovian Jump Systems Via a Partially Mode-Dependent Controller of Dwell Times

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ABSTRACT In this paper, the stabilization problem of discrete-time Markovian jump systems (DMJSs) with partially mode-dependent controllers of dwell times is studied. Firstly, a kind of partially mode-dependent controller (PMC) experiencing dwell times is proposed, whose stability problem is transformed into a similar one about another DMJS. Secondly, by exploiting a switched quadratic Lyapunov function (SQLF), sufficient conditions for the designed controller are given in terms of linear matrix inequalities (LMIs). Moreover, more extensions about stabilization realized by fault-tolerant and disordered controllers are considered. Finally, two practical examples are used to show the effectiveness and practicability of the proposed methods.

INDEX TERMS Markovian jump systems, partially mode-dependent controllers, dwell times, fault-tolerant controllers, disordered controllers, linear matrix inequalities.

I. INTRODUCTION

In recent years, the hybrid dynamic system has become a hot topic in various fields [1], [2]. Decision makers can add random variables to the system according to actual experience and statistical information. As a result, abnormal operating conditions caused by disturbances and faults in dynamic systems can be better coped with. In hybrid systems, the Markov jump system (MJS) [3] is the most typical one. It can model and analyze reasons for different changes in a system, including the internal structure, the working node and the related matrix parameters change suddenly due to the influence of working environment. Up till now, it has been widely used in social economics [4]–[7], cytogenetics [8], [9], biomedical [10], aerospace [11] and various types of networked control [12]–[14]. Among the theoretical research results, stability and stabilization problems are the priorities to be concerned, such as [15]–[18]. When the transition rate or probability matrix is uncertain or partially unknown, some results were obtained in [19]–[21]. These innovative studies have not only enriched the theoretical system of the MJSs, but also contributed significantly to a variety of practical applications.

On the other hand, compared with open-loop systems [22], [23], the closed-loop system has better calming effect when output of system and disturbance emerge in practical applications. Thus, the study of closed-loop MJSs [24]–[26] is indispensable and important. Naturally, the design of a more compliant controller becomes a key issue. Over the past decades, a lot of research results about MJSs used mode-dependent controllers had been obtained in [27]–[31]. There, the modes of controller and subsystem should match with others at any time. To the contrary, mode-independent controller [32] does not need any mode information. It is an uniform controller which could stabilize any subsystem. Since the mode of the controller is totally ignored, it is said to be an absolute method and less conservative than mode-dependent methods. Furthermore, there are some existing research results for the situation where the modes of controllers do not match the modes of systems (e.g., the study of asynchronous controllers [33], [34]). In literature [19], the author introduced a detector to detect the subsystems with partial information in the modes of the system, so as to achieve the so-called effect of mode-dependent controller. In [28], mismatches between the modes of systems and

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the modes of controllers were described by using a hidden Markov model. Particularly, in [35], a kind of partially mode-dependent controller was designed by introducing Bernoulli variables. It was found that the mode-independent controller is instantaneous. However, in actual productions and manufacturing processes, due to the aging of internal components or structural failures, the mode of subsystem has been unable to be real-time monitored for a certain period of time. In this case, the mode-dependent, mode-independent and partially mode-dependent controllers above are not suitable to be applied. All the observations motivated us to conduct the current research.

In this paper, the stabilization problem of DMJSs by a partially mode-dependent controller of dwell times is studied. The main contributions of this paper are summarized as follows: 1) The model of a PMC with dwell times is presented, which is more suitable in practical applications; 2) Without considering its stability directly, it is analyzed by studying another DMJS, whose energy function of each subsystem during the dwell time is not necessarily decreased strictly; 3) Based on the key idea of PMC, two additional cases about controller with fault or disorder are investigated respectively; 4) All the existing conditions of controllers are given with LMI forms, which are convenient to be computed and extended to other cases.

Notation: $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{p\times q}$ represents the real matrices with $p \times q$ dimension. $\mathbb{Z}$ denotes the set of integers and $\mathbb{N}$ is the natural numbers set. For Probability space $(\Omega, \mathcal{F}, \mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\|\cdot\|$ represents the Euclidean vector norm or spectral matrix norm. $\lambda(\cdot)$ refers to the eigenvalue of matrix. $\mathscr{E}\{\cdot\}$ indicates expectation of random variable. And $*$ expresses the transpose of corresponding position in symmetric matrices.

II. PROBLEM FORMULATION

Consider a kind of discrete-time linear MJSs on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ as

$$\begin{aligned} x(k+1) &= A_{\eta(k)}x(k) + B_{\eta(k)}u(k) \\ x(0) &= x_0, \quad \eta(0) = \eta_0 \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state vector and $u(k) \in \mathbb{R}^p$ is the control input. $A_{\eta(k)}$ is the system matrix where $A_{\eta(k)} \in \mathbb{R}^{n\times n}$ and $B_{\eta(k)}$ represents input matrix where $B_{\eta(k)} \in \mathbb{R}^{n\times p}$. $\{\eta(k), k \in \mathbb{Z}\}$ is a Markov chain which takes value in a finite set $\mathbb{N} \triangleq \{1, 2, \ldots N\}$. $\Pi = (\pi_{ij})_{N\times N}$ is defined as the transition probability matrix composed of transition probability π_{ij} which satisfies

$$\Pr\{\eta(k+1) = j|\eta(k) = i\} = \pi_{ij} \tag{2}$$

where $\forall i, j \in \mathbb{N}$, it has $\pi_{ij} \geq 0$ and $\sum_{j=1}^{N} \pi_{ij} = 1$.

In addition, we define

$$\Pr\{\eta(k+n) = j|\eta(k) = i\} = \pi_{ij}^{(n)} \tag{3}$$

is the n-step transition probability of Markov chain $\{\eta(k), k \in \mathbb{Z}\}$. By the characteristic of the Markov chain, the operator $\Pi^{(n)}$ is so-called the n-step transition probability matrix and it satisfies $\Pi^{(n)} = \underbrace{\Pi \times \Pi \times \cdots \times \Pi}_{n}$. In order to conveniently represent the elements in the matrix $\Pi^{(n)}$, we define $\Theta \triangleq (\theta_{ij})_{N\times N} = \underbrace{\Pi \times \Pi \times \cdots \times \Pi}_{n}$ which consists of element θ_{ij}, where $\theta_{ij} \triangleq \pi_{ij}^{(n)}$. And for any $i, j \in \mathbb{N}$, it has $\theta_{ij} \geq 0$ and $\sum_{j=1}^{N} \theta_{ij} = 1$.

A kind of state feedback partial mode-dependent controller is described as

$$u(k) = \begin{cases} Kx(k), & k \in \Gamma_{1,n} \\ K_{\eta(k)}x(k), & k \in \Gamma_{2,n} \end{cases} \tag{4}$$

where K and $K_{\eta(k)}$ denote controller gains. $\Gamma_{1,n} \triangleq [l_{2n}, l_{2n+1})$, $\Gamma_{2,n} \triangleq [l_{2n+1}, l_{2n+2})$ and $l_0 = 0$. Then we define lengths of $\Gamma_{1,n}$ and $\Gamma_{2,n}$ are constants and $l_{2n+1} - l_{2n} = \kappa_1$, $l_{2n+2} - l_{2n+1} = \kappa_2$. A key assumption is that the values of κ_1 and κ_2 are given and fixed. It means that when $k \in \Gamma_{1,n}$, the mode signal is unknown or cannot be detected. The controller is mode-independent. When $k \in \Gamma_{2,n}$, the mode signal is known or can be detected. The controller is mode-dependent. So by detecting the modal signal $\eta(k)$ at time k, the length of dwell time can be obtained according to its value available or not. Particularly, by re-mentioning $\theta_{ij} \triangleq \pi_{ij}^{(n)}$, we further define $\theta_{ij}^{[1]} = \pi_{ij}^{(\kappa_1)}$ and $\theta_{ij}^{[2]} = \pi_{ij}^{(\kappa_2)}$.

By substituting (4) into (1), it has two sub-systems which could be written as

$$x(k+1) = \begin{cases} \bar{A}_{\eta(k)}x(k), & k \in \Gamma_{1,n} \\ \tilde{A}_{\eta(k)}x(k), & k \in \Gamma_{2,n} \end{cases} \tag{5}$$

where $\bar{A}_{\eta(k)} = A_{\eta(k)} + B_{\eta(k)}K$ and $\tilde{A}_{\eta(k)} = A_{\eta(k)} + B_{\eta(k)}K_{\eta(k)}$.

Our aim in this paper is to study the stabilization problem of system (5), and we assume that the value of κ_1 and κ_2 can be determined by detections. One possible way is to detect the mode of controller by applying a mode detector, while a time instant is used to indicate whether or not the mode is detected. However, there is another case in practical application, that is, the value of κ_1 and κ_2 can not be determined by detection because the information of system mode is not available. If someone wants to do further research in this case, it may be necessary to get the information of parameters firstly, which could be solved by using the method purposed by [19], and then one could obtain values of κ_1 and κ_2.

Remark 1: By summarizing a large number of existing literatures, it is found that most state feedback controllers of MJSs are mode-dependent ones, see, e.g., [36]–[39]. In other words, the corresponding operation mode should be assumed to be available on time. In order to remove this ideal assumption, a mode-independent controller [32] was usually designed, by ignoring its mode information totally. It was actually an absolute method. In order to bridge the above two methods, a kind of partially mode-dependent controller is proposed in [35]. However, it is said that the introduced Bernoulli variable was a traditional one, whose two states occur instantaneously. It is quite different from PMC (4) whose two states could last a period. Compare with these literatures, PMC (4) is more general and suitable. Since the above dwell times are considered, its stability analysis will be complicated, especially the controller design will be very difficult. For example, because of both mode-dependent and mode-independent cases contained, how to analyse them by using a mode-dependent Lyapunov function is the first one to be considered; Secondly, but not the last, due to dwell times play important roles, how to make the existence condition with solvable forms is another difficulty to be handled.

Definition 1: The system (5) is stochastically stable, if it holds

$$\mathscr{E}\{\sum_{k=0}^{\infty} \|x(k)\|^2 | x_0, \eta_0\} \leq \infty$$

for any initial conditions $x_0 \in \mathbb{R}^n$ and $\eta_0 \in \mathbb{N}$.

III. MAIN RESULTS

Theorem 1: System (5) with integers $\kappa_1 > 0$, $\kappa_2 > 0$ and controller (4) is stochastically stable, if for given parameters $1 > \alpha_1 > 0$ and $1 > \alpha_2 > 0$, there exist matrices $P_{1i} > 0$, $P_{2i} > 0$ for all $i, j \in \mathbb{N}$, such that

$$\begin{bmatrix} -\alpha_1 I & A_i + B_i K \\ * & -I \end{bmatrix} \leq 0 \quad (6)$$

$$\begin{bmatrix} -\alpha_2 I & A_i + B_i K_i \\ * & -I \end{bmatrix} \leq 0 \quad (7)$$

$$\alpha_1^{\kappa_1} \sum_{j=1}^{N} \theta_{ij}^{[1]} P_{2j} - P_{1i} < 0 \quad (8)$$

$$\alpha_2^{\kappa_2} \sum_{j=1}^{N} \theta_{ij}^{[2]} P_{1j} - P_{2i} < 0 \quad (9)$$

where $P_{1i} = \rho_{1i} I \in \mathbb{R}^{n\times n}$ and $P_{2i} = \rho_{2i} I \in \mathbb{R}^{n\times n}$.

Proof: First, the stochastic stability of system (5) will be proved. In fact, $k = l_{2n}$ and $k = l_{2n+1}$ can be seen as switching moments of two sub-systems. By the analysis of switching moments, solutions of the state equation can be obtained as

$$\begin{aligned} x(l_{2n+1}) &= \bar{A}_{\eta(l_{2n+1}-1)} \ldots \bar{A}_{\eta(l_{2n})} x(l_{2n}) \\ x(l_{2n+2}) &= \tilde{A}_{\eta(l_{2n+2}-1)} \ldots \tilde{A}_{\eta(l_{2n+1})} x(l_{2n+1}) \end{aligned} \quad (10)$$

For ease of notation, we will write $\Phi_{1\tilde{\eta}(l_{2n})} = \bar{A}_{\eta(l_{2n+1}-1)} \cdots \bar{A}_{\eta(l_{2n}+1)} \bar{A}_{\eta(l_{2n})}$ and $\Phi_{2\tilde{\eta}(l_{2n+1})} = \tilde{A}_{\eta(l_{2n+2}-1)} \cdots \tilde{A}_{\eta(l_{2n+1}+1)} \tilde{A}_{\eta(l_{2n+1})}$ with $\tilde{\eta}(l_{2n}) = [\eta(l_{2n+1} - 1), \ldots, \eta(l_{2n})]$ and $\tilde{\eta}(l_{2n+1}) = [\eta(l_{2n+2} - 1), \ldots, \eta(l_{2n+1})]$ in what follows. The state equation which only considers the switching moment can be obtained by (10) as follows

$$\mathcal{X}(\bar{k}+1) = \begin{cases} \Phi_{1\tilde{\eta}(l_{2n})} \mathcal{X}(\bar{k}), & \bar{k} = 2n \\ \Phi_{2\tilde{\eta}(l_{2n+1})} \mathcal{X}(\bar{k}), & \bar{k} = 2n+1 \end{cases} \quad (11)$$

It can be seen that $\mathcal{X}(2n)$ is actually equal to $x(l_{2n})$ and $x(l_{2n+1})$ is represented by $\mathcal{X}(2n+1)$.

Then, a switched quadratic Lyapunov function (SQLF) is introduced for (11) as follows

$$V(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k}) = \begin{cases} \mathcal{X}^T(\bar{k}) P_{1\eta(\bar{k})} \mathcal{X}(\bar{k}), & \bar{k} = 2n \\ \mathcal{X}^T(\bar{k}) P_{2\eta(\bar{k})} \mathcal{X}(\bar{k}), & \bar{k} = 2n+1 \end{cases} \quad (12)$$

Thus, when $\bar{k} = 2n$, it can be gotten that

$$\begin{aligned} &\Delta V_1(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k}) \\ &= \mathscr{E}\{V(\mathcal{X}(\bar{k}+1), \eta(\bar{k}+1), \bar{k}+1 | \mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k})\} \\ &\quad - V(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k}) \\ &= \mathscr{E}\{\mathcal{X}^T(\bar{k}+1) P_{2\eta(\bar{k}+1)} \mathcal{X}(\bar{k}+1)\} \\ &\quad - \mathcal{X}^T(\bar{k}) P_{1\eta(\bar{k})} \mathcal{X}(\bar{k}) \\ &= \mathscr{E}\{\mathcal{X}^T(\bar{k}) \Phi^T_{1\tilde{\eta}(l_{2n})} P_{2\eta(\bar{k}+1)} \Phi_{1\tilde{\eta}(l_{2n})} \mathcal{X}(\bar{k})\} \\ &\quad - \mathcal{X}^T(\bar{k}) P_{1\eta(\bar{k})} \mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k}) [\mathscr{E}\{\Phi^T_{1\tilde{\eta}(l_{2n})} P_{2\eta(\bar{k}+1)} \Phi_{1\tilde{\eta}(l_{2n})}\} - P_{1\eta(\bar{k})}] \mathcal{X}(\bar{k}) \\ &< 0 \end{aligned} \quad (13)$$

which could be guaranteed by

$$\mathscr{E}\{(\Phi^T_{1\tilde{\eta}(l_{2n})} P_{2\eta(\bar{k}+1)} \Phi_{1\tilde{\eta}(l_{2n})}\} - P_{1\eta(\bar{k})} < 0 \quad (14)$$

Further more, when $\bar{k} = 2n+1$, it has

$$\begin{aligned} &\Delta V_2(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k}) \\ &= \mathscr{E}\{V(\mathcal{X}(\bar{k}+1), \eta(\bar{k}+1), \bar{k}+1 | \mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k})\} \\ &\quad - V(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k}) \\ &= \mathscr{E}\{\mathcal{X}^T(\bar{k}+1) P_{1\eta(\bar{k}+1)} \mathcal{X}(\bar{k}+1)\} \\ &\quad - \mathcal{X}^T(\bar{k}) P_{2\eta(\bar{k})} \mathcal{X}(\bar{k}) \\ &= \mathscr{E}\{\mathcal{X}^T(\bar{k}) \Phi^T_{2\tilde{\eta}(l_{2n+1})} P_{1\eta(\bar{k}+1)} \Phi_{2\tilde{\eta}(l_{2n+1})} \mathcal{X}(\bar{k})\} \\ &\quad - \mathcal{X}^T(\bar{k}) P_{2\eta(\bar{k})} \mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k}) [\mathscr{E}\{\Phi^T_{2\tilde{\eta}(l_{2n+1})} P_{1\eta(\bar{k}+1)} \Phi_{2\tilde{\eta}(l_{2n+1})}\} - P_{2\eta(\bar{k})}] \mathcal{X}(\bar{k}) \\ &< 0 \end{aligned} \quad (15)$$

which could be guaranteed by

$$\mathscr{E}\{(\Phi^T_{2\tilde{\eta}(l_{2n+1})} P_{1\eta(\bar{k}+1)} \Phi_{2\tilde{\eta}(l_{2n+1})}\} - P_{2\eta(\bar{k})} < 0 \quad (16)$$

Particularly, we define that $\forall i \in \mathbb{N}$, there exists two positive constants $1 > \alpha_1 > 0$ and $1 > \alpha_2 > 0$ which satisfies

$$\|\bar{A}_i\|^2 \leq \alpha_1 I, \quad \|\tilde{A}_i\|^2 \leq \alpha_2 I \quad (17)$$

And it is worth mentioning that smaller values of α_1 and α_2 selected will lead to a greater decay rate of the state norm of system (5).

Then, it could be gotten that

$$\begin{aligned} \|\Phi_{1\tilde{\eta}(l_{2n})}\|^2 &= \|\bar{A}_{\eta(l_{2n+1}-1)} \ldots \bar{A}_{\eta(l_{2n})}\|^2 \\ &\leq \|\bar{A}_{\eta(l_{2n+1}-1)}\|^2 \ldots \|\bar{A}_{\eta(l_{2n})}\|^2 \\ &\leq \alpha_1^{\kappa_1} I \end{aligned} \quad (18)$$

$$\begin{aligned} \|\Phi_{2\tilde{\eta}(l_{2n+1})}\|^2 &= \|\tilde{A}_{\eta(l_{2n+2}-1)} \ldots \tilde{A}_{\eta(l_{2n+1})}) \\ &\leq \|\tilde{A}_{\eta(l_{2n+2}-1)}\|^2 \ldots \|\tilde{A}_{\eta(l_{2n+1})}\|^2 \\ &\leq \alpha_2^{\kappa_2} I \end{aligned} \quad (19)$$

So by using $\Lambda^T \mu \Lambda \leq \|\Lambda\|^2 \mu$ where $\mu \geq 0$ and (18), (19). (14) and (16) can be guaranteed by

$$\mathscr{E}\{P_{2j}\}\|\Phi_{1\tilde{\eta}(l_{2n})}\|^2 - P_{1i} \leq \alpha_1^{\kappa_1} \sum_{j=1}^{N} \theta_{ij}^{[1]} P_{2j} - P_{1i} < 0 \quad (20)$$

and

$$\mathscr{E}\{P_{1j}\}\|\Phi_{2\tilde{\eta}(l_{2n+1})}\|^2 - P_{2i} \leq \alpha_2^{\kappa_2} \sum_{j=1}^{N} \theta_{ij}^{[2]} P_{1j} - P_{2i} < 0 \quad (21)$$

where $P_{1i} = \rho_{1i} I \geq 0$ and $P_{2i} = \rho_{2i} I \geq 0$.

It is worth noting that (20) and (21) can only guarantee the Lyapunov functions of systems (12) are monotonically decreasing at the moment of switching and it can be gotten that $\lim_{\bar{k}\to\infty} V(\mathcal{X}(\bar{k}), \bar{k}) = 0$ which implies $\lim_{\bar{k}\to\infty} \|\mathcal{X}(\bar{k})\|^2 = 0$. Then it is obviously that

$$\begin{aligned}
&\sum_{\bar{k}=0}^{\infty} (\mathscr{E}\{V(\mathcal{X}(\bar{k}+1), \eta(\bar{k}+1), \bar{k}+1 | \mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k})\} \\
&\qquad - V(\mathcal{X}(\bar{k}), \eta(\bar{k}), \bar{k})) \\
&\quad = \mathscr{E}\{V(\mathcal{X}(\infty), \eta_\infty, \infty)\} - V(\mathcal{X}(0), \eta_0, 0) \\
&\quad \leq 0 \qquad (22)
\end{aligned}$$

Clearly, as $\mathcal{X}(0) = x(0)$, $\forall k \in \Gamma_{1,n}$, it has

$$\|x(k)\| \leq \alpha_1^{\kappa_1} \|\mathcal{X}(2n)\| \quad (23)$$

and $\forall k \in \Gamma_{2,n}$, it has

$$\|x(k)\| \leq \alpha_2^{\kappa_2} \|\mathcal{X}(2n+1)\| \quad (24)$$

It can be seen $\forall k \in [l_{2n}, l_{2n+2})$, one has $\|x(k)\| \leq \max\{\alpha_1, \alpha_2\}^{(\kappa_1+\kappa_2)} \|\mathcal{X}(2n)\|$. By defining $\bar{\varepsilon} = \max\{\alpha_1, \alpha_2\}^{(\kappa_1+\kappa_2)}$, there exists $\varrho = \frac{\varepsilon}{\bar{\varepsilon}} > 0$ such that $\|x(k)\| < \varepsilon$, $\forall k = 0, 1, 2, \ldots$ whenever $\|x(0)\| = \|\mathcal{X}(0)\| < \varrho$. Then, it can be concluded that the system (5) is stable. As $\bar{k} \to \infty$, it can be obtained

$$\begin{aligned}
&\mathscr{E}\{V(x(\infty), \eta_\infty, \infty | x_0, \eta_0, 0)\} - V(x_0, \eta_0, 0) \\
&\quad \leq \mathscr{E}\{V(\mathcal{X}(\infty), \eta_\infty, \infty | \mathcal{X}(0), \eta_0, 0)\} - V(\mathcal{X}(0), \eta_0, 0) \\
&\quad \leq -\rho \sum_{k=0}^{\infty} \mathscr{E}\{\|x(k)\|^2 | x_0, \eta_0\} \qquad (25)
\end{aligned}$$

which implies

$$\begin{aligned}
\mathscr{E}\{\sum_{k=0}^{\infty} \|x(k)\|^2 | x_0, \eta_0\} &\leq \sum_{k=0}^{\infty} \mathscr{E}\{\|x(k)\|^2 | x_0, \eta_0\} \\
&\leq \frac{1}{\rho} V(x_0, \eta_0, 0) < \infty \qquad (26)
\end{aligned}$$

Then the system (5) is stochastically stable. In addition, by (17), the controller can be calculated in terms of LMIs (6) and (7). This completes the proof.

Remark 2: Compare with the existing partially mode-dependent method [35], the dwell times are included and play important roles, which could lead to less conservative results. Moreover, it also contains mode-dependent and mode-independent cases special ones, which is analyzed by a mode-dependent Lyapunov function. Different from the method used in [40], a switched Lyapunov function is only selected to system (11) instead of (5). It has the advantage that the energy function is not required to satisfy the condition $\Delta V = \mathscr{E}\{V(x(k+1))\} - V(x(k)) < 0$ within the dwell time. It means that only stabilizing system (11) could guarantee system (5) stochastically stable. Then, it is less restrictive than traditional methods.

Remark 3: It is seen from (4) that when $k \in \Gamma_{2,n}$, the state feedback controller is designed as $u(k) = K_{\eta(k)} x(k)$. In other words, it is assumed that the active mode of system (1) is matched with the controller when $k \in \Gamma_{2,n}$. However, there is another situation that a mismatch $\eta(k)$ between $K_{\eta(k)}$ and system matrices $A_{\eta(k)}$ and $B_{\eta(k)}$ occurs. In this case, the method proposed in Theorem 1 cannot be applied directly. Fortunately, it is said that the above general problem could be studied by combining Theorem 1 and existing methods. For example, based on a robust method [41], one remodel controller (4) whose mode is a unmatched with the active mode. Then, similar to the proof of this theorem, one could get the corresponding results. However, some additional problems will be encountered and should be carefully considered. For example, how to make the obtained results with solvable forms is a first but important problem, since both uncertainty and partially mode-dependent property are considered. Moreover but not the last, an improved Lyapunov function should be constructed to get less conservative results. How to select a suitable from is also necessary to be studied.

Then, we are interested in considering two special cases for system (4). The first special case is that there exists the controller completely failure when $k \in \Gamma_{1,n}$, so that parameters in the system (4) are designed to correspond to the form, which means

$$x(k+1) = \begin{cases} \bar{A}_{\eta(k)} x(k), & k \in \Gamma_{1,n} \\ \tilde{A}_{\eta(k)} x(k), & k \in \Gamma_{2,n} \end{cases} \quad (27)$$

where $\bar{A}_{\eta(k)} = A_{\eta(k)}$ and $\tilde{A}_{\eta(k)} = A_{\eta(k)} + B_{\eta(k)} K_{\eta(k)}$.

Corollary 1: System (27) with integers $\kappa_1 > 0$, $\kappa_2 > 0$ and controller $K_{\eta(k)}$ is stochastically stable, if for given parameters $\gamma_1 > 0$ and $1 > \gamma_2 > 0$, there exist matrices $P_{1i} > 0$ and $P_{2i} > 0$ for all $i, j \in \mathbb{N}$, such that

$$\begin{bmatrix} -\gamma_1 I & A_i \\ * & q - I \end{bmatrix} \leq 0 \quad (28)$$

$$\begin{bmatrix} -\gamma_2 I & A_i + B_i K_i \\ * & -I \end{bmatrix} \leq 0 \quad (29)$$

$$\gamma_1^{\kappa_1} \sum_{j=1}^{N} \theta_{ij}^{[1]} P_{2j} - P_{1i} < 0 \quad (30)$$

$$\gamma_2^{\kappa_2} \sum_{j=1}^{N} \theta_{ij}^{[2]} P_{1j} - P_{2i} < 0 \quad (31)$$

where $P_{1i} = \rho_{1i} I \in \mathbb{R}^{n\times n}$ and $P_{2i} = \rho_{2i} I \in \mathbb{R}^{n\times n}$.

Proof: The proof of stochastically stability of system (27) is analogous to Theorem 1. The difference between it and Theorem 1 is that since $k \in \Gamma_{1,n}$, $K = 0$, so $\bar{A}_{\eta(k)} = A_{\eta(k)}$. It means that $\Phi_{1\tilde{\eta}(l_{2n})}$ is actually a known matrix which consists of $A_{\eta(k)}$ when $k \in \Gamma_{1,n}$. So the condition (6) in Theorem 1 is not required in this case. Similar to (17), one could define that $\forall i \in \mathbb{N}$, there exists two positive constants $\gamma_1 > 0$ and $1 > \gamma_2 > 0$ which satisfies $\|\bar{A}_i\|^2 \leq \gamma_1 I$ and $\|\tilde{A}_i\|^2 \leq \gamma_2 I$, by using the similar method as Theorem 1. The mode-dependence controller $K_{\eta(k)}$ can be calculated by the LMI of condition (29). This completes the proof.

Remark 4: It can be seen that when the eigenvalue of the system matrix satisfies $\bar{\lambda} = \max_{i\in\mathbb{N}} \Re e(\lambda(A_i)) > 1$, the increment of the energy function can not be guaranteed to be less than zero during the dwell time of systems with controllers failure. It is difficult to obtain sufficient conditions for the stability of system (27) by analyzing difference equations of the Lyapunov functions at two switch instants in the entire time domain, or the sufficient conditions obtained have largely conservative. In other words, by analyzing the reconstruction system consisting of switching instants of system (27), even if there is a case where the energy function increment is greater than zero in the dwell time of systems with controllers failure, the stabilization of system can be guaranteed by the constraints of the conditions (30) and (31).

Another special case is that the disordering between controllers is taken into account and without loss of generality, $\eta(k)$ here is considered as $\eta(k) \in \mathbb{N} = \{1, 2\}$. System (5) can actually be written as

$$x(k+1) = \begin{cases} (A_1 + B_1(K_1 + \Delta K_1))x(k), & k \in \Gamma_{1,n} \\ (A_2 + B_2(K_2 + \Delta K_2))x(k), & k \in \Gamma_{2,n} \end{cases} \quad (32)$$

where $\Delta K_1 = K_2 - K_1$ and $\Delta K_2 = K_1 - K_2$. For ease of notation, we would write system (32) as follows and the same representation will be used in what follows

$$x(k+1) = \begin{cases} (\bar{A}_1 + \Delta\bar{A}_1)x(k), & k \in \Gamma_{1,n} \\ (\bar{A}_2 + \Delta\bar{A}_2)x(k), & k \in \Gamma_{2,n} \end{cases} \quad (33)$$

where $\bar{A}_1 = A_1 + B_1K_1$, $\bar{A}_2 = A_2 + B_2K_2$, $\Delta\bar{A}_1 = B_1\Delta K_1$ and $\Delta\bar{A}_2 = B_2\Delta K_2$. It can be seen that system (32) could be regarded as a deterministic hybrid system because one subsystem must switch to another subsystem at time $k = l_{2n}$ and time $k = l_{2n+1}$.

Theorem 2: System (33) with integers $\kappa_1 > 0$, $\kappa_2 > 0$ and controller K_1, K_2 is asymptotically stable, if given parameters $1 > \zeta_1 > 0$, $1 > \zeta_2 > 0$, $\frac{\zeta_1^2}{2} > \omega_1 > 0$ and $\frac{\zeta_2^2}{2} > \omega_2 > 0$, there exist matrices $P_1 > 0$ and $P_2 > 0$, such that

$$\begin{bmatrix} -\omega_1 I & K_2^T B_1^T - K_1^T B_1^T \\ * & -I \end{bmatrix} \leq 0 \quad (34)$$

$$\begin{bmatrix} -\omega_2 I & K_1^T B_2^T - K_2^T B_2^T \\ * & -I \end{bmatrix} \leq 0 \quad (35)$$

$$\begin{bmatrix} -(\frac{\zeta_1^2}{2} - \omega_1)I & A_1^T + K_1^T B_1^T \\ * & -I \end{bmatrix} \leq 0 \quad (36)$$

$$\begin{bmatrix} -(\frac{\zeta_2^2}{2} - \omega_2)I & A_2^T + K_2^T B_2^T \\ * & -I \end{bmatrix} \leq 0 \quad (37)$$

$$\zeta_1^{2\kappa_1} P_2 - P_1 < 0 \quad (38)$$

$$\zeta_2^{2\kappa_2} P_1 - P_2 < 0 \quad (39)$$

where $P_1 = \rho_1 I \in \mathbb{R}^{n\times n}$ and $P_2 = \rho_2 I \in \mathbb{R}^{n\times n}$.

Proof: It can be seen that due to the disordered controller, system (33) has two modes during operation. Then rebuilding the system only considers the switching moments as follows

$$\mathcal{X}(\bar{k}+1) = \begin{cases} \Psi_1\mathcal{X}(\bar{k}), & \bar{k} = 2n \\ \Psi_2\mathcal{X}(\bar{k}), & \bar{k} = 2n+1 \end{cases} \quad (40)$$

where $\Psi_1 = (\bar{A}_1 + \Delta\bar{A}_1)^{\kappa_1}$ and $\Psi_2 = (\bar{A}_2 + \Delta\bar{A}_2)^{\kappa_2}$. For system (40), SQLF is introduced as

$$V(\mathcal{X}(\bar{k}), \bar{k}) = \begin{cases} \mathcal{X}^T(\bar{k})P_1\mathcal{X}(\bar{k}), & \bar{k} = 2n \\ \mathcal{X}^T(\bar{k})P_2\mathcal{X}(\bar{k}), & \bar{k} = 2n+1 \end{cases} \quad (41)$$

Based on the iterative law of (40) and formula (41), when $\bar{k} = 2n$, it has

$$\begin{aligned} \Delta V_1(\mathcal{X}(\bar{k}), \bar{k}) &= \mathcal{X}^T(\bar{k}+1)P_2\mathcal{X}(\bar{k}+1) - \mathcal{X}^T(\bar{k})P_1\mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k})\Psi_1^T P_2\Psi_1\mathcal{X}(\bar{k}) - \mathcal{X}^T(\bar{k})P_1\mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k})(\Psi_1^T P_2\Psi_1 - P_1)\mathcal{X}(\bar{k}) < 0 \end{aligned} \quad (42)$$

It is equivalent to

$$\Psi_1^T P_2\Psi_1 - P_1 < 0 \quad (43)$$

When $\bar{k} = 2n+1$, it has

$$\begin{aligned} \Delta V_2(\mathcal{X}(\bar{k}), \bar{k}) &= \mathcal{X}^T(\bar{k}+1)P_1\mathcal{X}(\bar{k}+1) - \mathcal{X}^T(\bar{k})P_2\mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k})\Psi_2^T P_1\Psi_2\mathcal{X}(\bar{k}) - \mathcal{X}^T(\bar{k})P_2\mathcal{X}(\bar{k}) \\ &= \mathcal{X}^T(\bar{k})(\Psi_2^T P_1\Psi_2 - P_2)\mathcal{X}(\bar{k}) < 0 \end{aligned} \quad (44)$$

It is equivalent to

$$\Psi_2^T P_1\Psi_2 - P_2 < 0 \quad (45)$$

It is known that (43) and (45) can only guarantee the Lyapunov functions of system (40) are monotonically decreasing at the moment of switching and it can be gotten that $\lim_{\bar{k}\to\infty} V(\mathcal{X}(\bar{k}), \bar{k}) = 0$ which implies $\lim_{\bar{k}\to\infty} \|\mathcal{X}(\bar{k})\|^2 = 0$. Clearly, as $\mathcal{X}(0) = x(0)$, there exists $1 > \zeta_1 > 0$ and $1 > \zeta_2 > 0$ that $\forall k \in [l_{2n}, l_{2n+1})$,

$$\begin{aligned} \|x(k)\| &\leq \|\bar{A}_1 + \Delta\bar{A}_1\|^{(k-l_{2n})}\|x(l_{2n})\| \\ &\leq \zeta_1^{(k-l_{2n})}\|x(l_{2n})\| \\ &= \zeta_1^{(k-l_{2n})}\|\mathcal{X}(2n)\| \\ &\leq \zeta_1^{\kappa_1}\|\mathcal{X}(2n)\| \end{aligned} \quad (46)$$

$\forall k \in [l_{2n+1}, l_{2n+2})$, it has

$$\begin{aligned}\|x(k)\| &\leq \|\bar{A}_2 + \Delta\bar{A}_2\|^{(k-l_{2n+1})}\|x(l_{2n+1})\| \\ &\leq \zeta_2^{(k-l_{2n+1})}\|x(l_{2n+1})\| \\ &= \zeta_2^{(k-l_{2n+1})}\|\mathcal{X}(2n+1)\| \\ &\leq \zeta_2^{\kappa_2}\|\mathcal{X}(2n+1)\| \end{aligned} \tag{47}$$

It can be seen $\forall k \in [l_{2n}, l_{2n+2})$, it has $\|x(k)\| \leq \max\{\zeta_1, \zeta_2\}^{(\kappa_1+\kappa_2)}\|x(l_{2n})\|$. By defining $\bar{\varepsilon} = \max\{\zeta_1, \zeta_2\}^{(\kappa_1+\kappa_2)}$, $\forall \varepsilon > 0$, there exists $\varrho = \frac{\varepsilon}{\bar{\varepsilon}} > 0$ such that $\|x(k)\| < \varepsilon$, $\forall k = 0, 1, 2, \ldots$ whenever $\|x(0)\| = \|\mathcal{X}(0)\| < \varrho$. Then, the system (33) is stable and it satisfies $\lim_{k\to\infty}\|x(k)\|^2 = 0$. In summary, by making system (40) asymptotically stable, it is equivalent to that the system (33) is asymptotic stability. And it is seen that smaller the values of ζ_1 and ζ_2 selected, which will lead to a greater decay rate of the state norm of system (33).

Clearly, by using $\Lambda^T\mu\Lambda \leq \|\Lambda\|^2\mu$ where $\mu \geq 0$ and (43), it has

$$\begin{aligned}\Psi_1^T P_2 \Psi_1 &= ((\bar{A}_1 + \Delta\bar{A}_1)^T)^{\kappa_1} P_2 (\bar{A}_1 + \Delta\bar{A}_1)^{\kappa_1} \\ &\leq \|\bar{A}_1 + \Delta\bar{A}_1\|^{2\kappa_1} P_2 \\ &\leq \zeta_1^{2\kappa_1} P_2 \end{aligned} \tag{48}$$

where $P_2 = \rho_2 I \geq 0$.

By substituting (48) into (43), it can be easily obtained that (43) could be guaranteed by condition (38).

Similarly, one has

$$\begin{aligned}\Psi_2^T P_1 \Psi_2 &= ((\bar{A}_2 + \Delta\bar{A}_2)^T)^{\kappa_2} P_1 (\bar{A}_2 + \Delta\bar{A}_2)^{\kappa_2} \\ &\leq \|\bar{A}_2 + \Delta\bar{A}_2\|^{2\kappa_2} P_1 \\ &\leq \zeta_2^{2\kappa_2} P_1 \end{aligned} \tag{49}$$

where $P_1 = \rho_1 I \geq 0$.

Then by substituting (49) into (45), it can be guaranteed by condition (39) obviously.

Particularly, (48) and (49) can be guaranteed by

$$\|\bar{A}_1 + \Delta\bar{A}_1\|^2 \leq \zeta_1^2 \tag{50}$$

and

$$\|\bar{A}_2 + \Delta\bar{A}_2\|^2 \leq \zeta_2^2 \tag{51}$$

Then, inequality (50) is implied by

$$(\bar{A}_1 + \Delta\bar{A}_1)^T(\bar{A}_1 + \Delta\bar{A}_1) \leq \zeta_1^2 I \tag{52}$$

which can be guaranteed by

$$2\bar{A}_1^T\bar{A}_1 + 2\Delta\bar{A}_1^T\Delta\bar{A}_1 \leq \zeta_1^2 I \tag{53}$$

Here the premise is required as

$$\Delta\bar{A}_1^T\Delta\bar{A}_1 \leq \omega_1 I \tag{54}$$

where $0 < \omega_1 < \frac{\zeta_1{}^2}{2}$.

Then by Schur complement lemma, one has

$$\begin{bmatrix} -\omega_1 I & B_1^T\Delta K_1^T \\ * & -I \end{bmatrix} \leq 0 \tag{55}$$

which implies condition (34).

By (53) and (54), it can be obtained that

$$\bar{A}_1^T\bar{A}_1 + \omega_1 I \leq \frac{\zeta_1^2}{2} I \tag{56}$$

which implies condition (36).

Similarly, by rementioned (51), it has

$$\begin{aligned}(\bar{A}_2 + \Delta\bar{A}_2)^T(\bar{A}_2+\Delta\bar{A}_2) - \zeta_2^2 I &\leq 2\bar{A}_2^T\bar{A}_2 + 2\Delta\bar{A}_2^T\Delta\bar{A}_2 - \zeta_2^2 I \\ &\leq 0 \end{aligned} \tag{57}$$

Here the premise is required as

$$\Delta\bar{A}_2^T\Delta\bar{A}_2 \leq \omega_2 I \tag{58}$$

where $0 < \omega_2 < \frac{\zeta_2{}^2}{2}$.

By (58), one can obtain

$$\begin{bmatrix} -\omega_2 I & B_2^T\Delta K_2^T \\ * & -I \end{bmatrix} \leq 0 \tag{59}$$

and it implies condition (35).

By (58) and (59), one has

$$\bar{A}_2^T\bar{A}_2 + \omega_2 I \leq \frac{\zeta_2^2}{2} I \tag{60}$$

which implies condition (37).

Then disordered controllers can be obtained by LMIs (37), (38), (39) and (40). This completes the proof.

Remark 5: On the one hand, compared to [41], the model here yields a more realistic result by introducing a dwell time. On the other hand, different from the SQLS established by Theorem 1, the SQLS established for system (32) can be understood as a disordered-dependent Lyapunov function. Since this paper considers a special case with two modes, so there are only two positive definite matrices P_1 and P_2. It is worth mentioning that the results above are equally applicable to the more general case. When the case of multiple modes ($N > 2$) is considered, more complex results similar to Theorem 2 can be obtained.

Remark 6: It can be seen that all sufficient conditions in this paper are given in the form of LMIs. This makes it convenient to solve and calculate. However, in the process of obtaining these conditions, the conservatism of the system also needs to be noticed. On the one hand, in the process of calculation, some problems are dealt with using robust ideas, which leads to the increase of system conservatism. On the other hand, when calculating the norm of the transfer matrix of the system, it can be seen in (18), (19), (48) and (49) that the method used will lead to more conservatism with the increase of dwell times. In addition, considering the existence of P_{1i} and P_{2i} as positive definite symmetric matrices rather than numbers, the result may be less conservative, but such consideration is contradictory to the derivation process and difficult to implement.

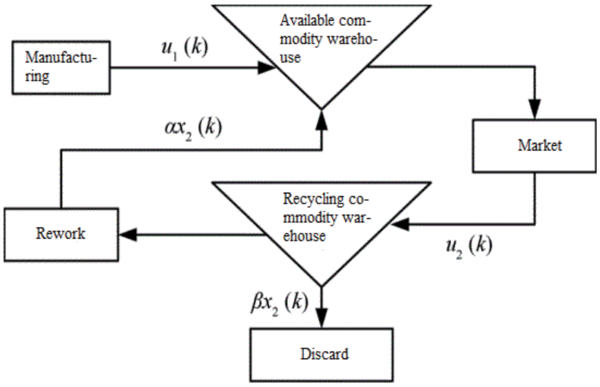

FIGURE 1. The structure of CLSC.

IV. NUMERICAL EXAMPLE

The following examples of simulations are aimed to show the applicability of theories presented in this paper.

Example 1: Consider a supply chain control (CLSC) system partially cited from [42]. Its structure is shown in Fig. 1. It is assumed here that the remanufactured products have the same product specifications and quality as the original finished products. Let $x_1(k) \in \mathbb{R}^n$ denotes the inventory levels of available commodity warehouses and $x_2(k) \in \mathbb{R}^n$ represents recycled commodity warehouses. $u_1(k) \in \mathbb{R}^p$ indicates the manufacturing rate of the manufacturing equipment at the time k and $u_2(k) \in \mathbb{R}^p$ is the recovery rate of the used goods at the time k.

Assumption 1: The model is based on the following assumptions:

a) The quantity of recycled product is determined by the manufacturer. That is, the market has a sufficient number of products to satisfy the recycling needs, and the manufacturer only needs to recycle the quantity it needs;

b) It is a fact that the value of a commodity will decay at the rate ρ. Here, ρ_1 and ρ_2 correspond to the decay rates of available warehouses and recycled warehouses respectively;

c) Parameters β_1 and β_2 denote the remanufacturing rate and abandonment rate, where $0 \le \beta_1 \le 1$, $0 \le \beta_2 \le 1$ and $0 < \beta_1 + \beta_2 \le 1$. C_1 and C_2 represent the maximum available warehouse storage and the maximum recoverable warehouse storage, respectively.

The closed-loop supply chain system considered here will use inventory levels as state variables. When $0 < x_1(k) < C_1$, the closed-loop supply chain system could be written as

$$x_1(k+1) = (1-\rho_1)x_1(k) + \beta_1 x_2(k) + u_1(k) \quad (61)$$

When $x_1(k) \le 0$, it means that there is a shortage of supply. One obtains

$$x_1(k+1) = x_1(k) + \beta_1 x_2(k) + u_1(k) \quad (62)$$

Similarly, when $0 < x_2(k) < C_2$, one has

$$x_2(k+1) = (1-\rho_2)x_2(k) - \beta_1 x_2(k) - \beta_2 x_2(k) + u_2(k) \quad (63)$$

When $x_2(k) \le 0$, one gets

$$x_2(k+1) = x_2(k) + u_2(k) \quad (64)$$

Combining (61) and (63), it is obtained that

$$\bar{x}(k+1) = A_1\bar{x}(k) + B_1\bar{u}(k) \quad (65)$$

Considering (61) and (64), it is got that

$$\bar{x}(k+1) = A_2\bar{x}(k) + B_2\bar{u}(k) \quad (66)$$

Taking into account (62) and (63), it is written to be

$$\bar{x}(k+1) = A_3\bar{x}(k) + B_3\bar{u}(k) \quad (67)$$

Under (62) and (64), it is obtained that

$$\bar{x}(k+1) = A_4\bar{x}(k) + B_4\bar{u}(k) \quad (68)$$

where

$$\bar{x}(k) = \begin{bmatrix} x_1^T(k) & x_2^T(k) \end{bmatrix}^T, \quad \bar{u}(k) = \begin{bmatrix} u_1^T(k) & u_2^T(k) \end{bmatrix}^T$$
$$A_1 = \begin{bmatrix} 1-\rho_1 & \beta_1 \\ 0 & 1-\rho_2-\beta_1-\beta_2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1-\rho_1 & \beta_1 \\ 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & \beta_1 \\ 0 & 1-\rho_2-\beta_1-\beta_2 \end{bmatrix},$$
$$A_4 = \begin{bmatrix} 1 & \beta_1 \\ 0 & 1 \end{bmatrix}$$

and $\bar{x}(0) = \bar{x}_0$. Thus, systems (65), (66), (67) and (68) can be regarded as system (1). It is well known that the supply chain has multiple levels and changes in real time depending on market conditions. Since $\bar{u}(k)$ depends on $\bar{x}(k)$, a state feedback controller is considered in this paper. A key assumption here is that the inventory information of the commodity in a certain interval cannot be detected, so $\bar{u}(k)$ is designed as a partial mode-dependent state feedback controller and its structure is as

$$\bar{u}(k) = \begin{cases} K\bar{x}(k), & k \in \Gamma_{1,n} \\ K_{\eta(k)}\bar{x}(k), & k \in \Gamma_{2,n} \end{cases}$$

It can be seen the controller designed here is similar to (4) and it means this kind of problem is suitable for solving the method used in the Theorem 1 of this article.

Next, a historical data of scrap steel recycling in a domestic steel company [43] will be combined, and an example of a closed-loop supply chain system is given. According to the actual situation and enterprise historical data, the model parameters are set as follows: The decay rate of the available commodity warehouse is $\rho_1 = 0.07$, and the decay rate of the recovered commodity warehouse is $\rho_2 = 0.09$. The initial value is set to $x_1(0) = 10$, $x_2(0) = 5$ (unit: 10^6 tons). Remanufacturing rate $\beta_1 = 0.56$ and rejection rate $\beta_2 = 0.15$. Therefore, we have the following system parameters

$$A_1 = \begin{bmatrix} 0.9300 & 0.5600 \\ 0 & 0.2000 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9300 & 0.5600 \\ 0 & 1.0000 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1.0000 & 0.5600 \\ 0 & 0.2000 \end{bmatrix},\ A_4 = \begin{bmatrix} 1.0000 & 0.5600 \\ 0 & 1.0000 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\quad B_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix},\quad B_3 = B_4 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where TRM is considered as

$$\Pi = \begin{bmatrix} 0.200 & 0.300 & 0.100 & 0.400 \\ 0.400 & 0.100 & 0.400 & 0.100 \\ 0.100 & 0.500 & 0.200 & 0.200 \\ 0.500 & 0.200 & 0.100 & 0.200 \end{bmatrix}$$

Then, by the characteristic of the Markov chain,

$$\Theta^{[1]} = \Pi^3 = \begin{bmatrix} 0.2870 & 0.2750 & 0.1860 & 0.2520 \\ 0.3340 & 0.2370 & 0.2220 & 0.2070 \\ 0.2680 & 0.2930 & 0.1930 & 0.2460 \\ 0.3190 & 0.2540 & 0.1950 & 0.2320 \end{bmatrix}$$

and

$$\Theta^{[2]} = \Pi^2 = \begin{bmatrix} 0.3700 & 0.2200 & 0.2000 & 0.2100 \\ 0.2100 & 0.3500 & 0.1700 & 0.2700 \\ 0.3400 & 0.2200 & 0.2700 & 0.1700 \\ 0.2900 & 0.2600 & 0.1700 & 0.2800 \end{bmatrix}$$

By condition (6) and (7) in Theorem 1, the mode-independent controller gain can be gotten as

$$K = \begin{bmatrix} -0.3057 & -0.2615 \end{bmatrix}$$

and mode-dependent controller gains can be obtained as

$$K_1 = \begin{bmatrix} -0.3720 & -0.2640 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} -0.2188 & -0.1906 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} -0.5000 & -0.3800 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} -0.5000 & -0.7800 \end{bmatrix}$$

The positive definite symmetric matrices P_{1j} and P_{2j} calculated are as follows

$$P_{11} = \begin{bmatrix} 16.7527 & 0 \\ 0 & 16.7527 \end{bmatrix} P_{12} = \begin{bmatrix} 17.1915 & 0 \\ 0 & 17.1915 \end{bmatrix}$$
$$P_{13} = \begin{bmatrix} 17.8905 & 0 \\ 0 & 17.8905 \end{bmatrix} P_{14} = \begin{bmatrix} 17.5226 & 0 \\ 0 & 17.5226 \end{bmatrix}$$
$$P_{21} = \begin{bmatrix} 15.8007 & 0 \\ 0 & 15.8007 \end{bmatrix} P_{22} = \begin{bmatrix} 16.3073 & 0 \\ 0 & 16.3073 \end{bmatrix}$$
$$P_{23} = \begin{bmatrix} 17.1379 & 0 \\ 0 & 17.1379 \end{bmatrix} P_{24} = \begin{bmatrix} 16.6830 & 0 \\ 0 & 16.6830 \end{bmatrix}$$

It can be seen that $\rho_{11} = 16.7527$, $\rho_{12} = 17.1915$, $\rho_{13} = 17.8905$, $\rho_{14} = 17.5226$, $\rho_{21} = 15.8007$, $\rho_{22} = 16.3073$, $\rho_{23} = 17.1379$, $\rho_{24} = 16.6830$.

Here we assume $\alpha_1 = 0.9$, $\alpha_2 = 0.8$, $\kappa_1 = 3$ and $\kappa_2 = 2$, it means that mode-independent controller acts on $k \in [nT, nT+3)$ and mode-dependent controller acts on $k \in [nT+3, nT+5)$, where the period $T = \kappa_1 + \kappa_2 = 5$. The mode switching simulation of system (5) is shown in Fig. 2 and the state response is shown in Fig. 3 under the initial condition $\bar{x}_0 = [10 \ \ 5]^T$. Meanwhile, Fig. 4 illustrates the input

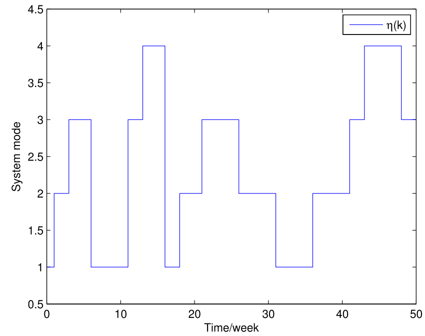

FIGURE 2. Simulations of operation modes.

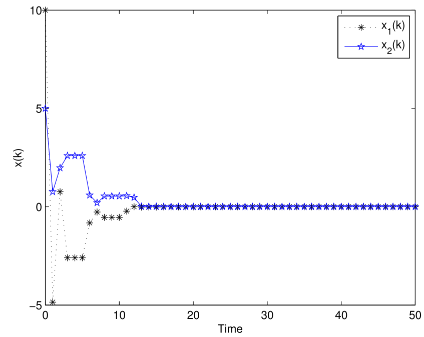

FIGURE 3. State response of the closed-loop system.

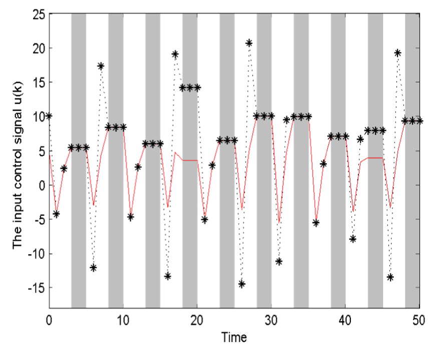

FIGURE 4. Control input subjects to mode-dependent and mode-independent.

control signal $u(k)$ where the shaded area implies the mode-dependent controller and unshaded area implies the mode-independent controller. As it shown in Fig. 3, the system is stochastically stable through Theorem 1.

Example 2: Consider the the problem of disordered controller for an industrial continuous-stirred tank reactor system, where chemical species A react to form species B. Fig. 5 shows the cross-sectional diagram of continuous flow stirred-tank reactor and Fig. 6 illustrates the physical structure of the system, where C_{Ai}, C_A, T, T_C are, respectively, the input concentration of a key reactant A, the output concentration of chemical species A, the reaction temperature and the cooling medium temperature. When modeling the industrial continuous-stirred tank reactor system, since the system is in

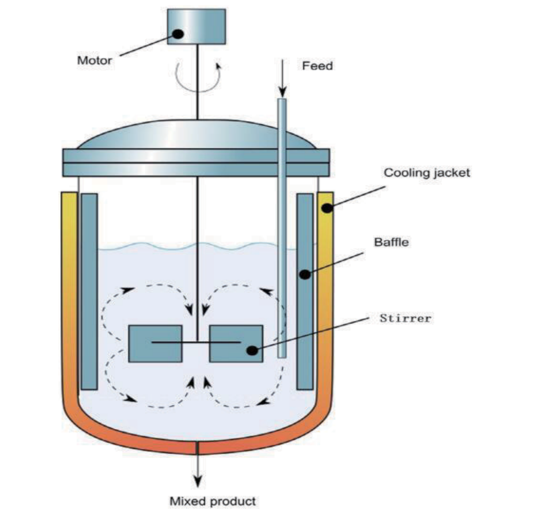

FIGURE 5. Cross-sectional diagram of continuous flow stirred-tank reactor.

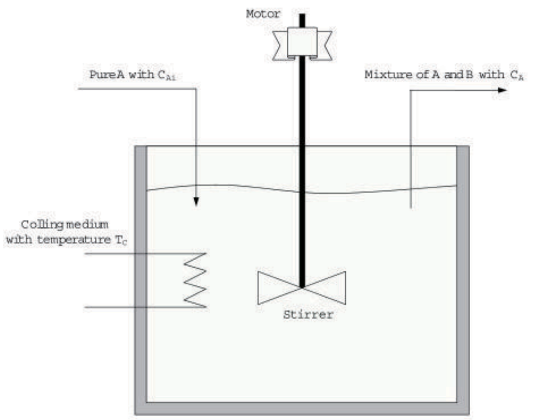

FIGURE 6. A continuous-stirred tank reactor model.

a network environment, in the process of data transmission, due to the complexity of the transmission path and node conflicts, the phenomenon of disordering is inevitable between the controllers. By selecting the state and input variables as $x = \left[C_A^T \; T^T \right]^T$, $u = \left[T_C^T \; C_{Ai}^T \right]^T$. A discrete-space model is obtained as the form of (31), where system matrix A_1, A_2 and the control matrix B_1, B_2 are taken from the linearized model of an industrial continuous-stirred tank reactor system in [44] as

$$A_1 = \begin{bmatrix} 1.0219 & -0.0987 \\ 0.1340 & 0.8628 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0839 & 0.0232 \\ 0.0761 & 0.4144 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.9719 & 0.1013 \\ 0.0340 & 0.8828 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0839 & 0.0232 \\ 0.0761 & 0.4144 \end{bmatrix}$$

By selected $\omega_1 = 0.3$, $\omega_2 = 0.1$, $\zeta_1 = 0.8$ and $\zeta_1 = 0.6$, then the gains of disordered controllers can be obtain by (34)-(37) as

$$K_1 = \begin{bmatrix} -12.5842 & 1.1598 \\ 2.0539 & -2.3083 \end{bmatrix}$$

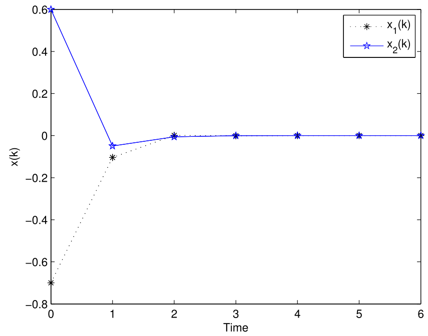

FIGURE 7. State response of system closed by a disordered controller.

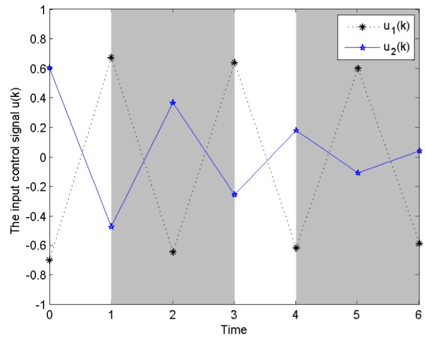

FIGURE 8. Control input subjects to disordered.

$$K_2 = \begin{bmatrix} -12.5717 & 1.1040 \\ 2.0569 & -2.2991 \end{bmatrix}$$

The positive definite symmetric matrices P_1 and P_2 calculated are as follows

$$P_1 = \begin{bmatrix} 49.4187 & 0 \\ 0 & 49.4187 \end{bmatrix} \; P_2 = \begin{bmatrix} 39.9579 & 0 \\ 0 & 39.9579 \end{bmatrix}$$

It can be seen that $\rho_1 = 49.4187$, $\rho_2 = 39.9579$.

The initial condition is selected as $x_0 = \left[-0.7 \; 0.6 \right]^T$. Here we assume $\kappa_1 = 1$ and $\kappa_2 = 2$, it means that disordering occurs in mode 1 of subsystem when $k \in [nT, nT + 2)$ and disordering occurs in mode 2 of subsystem when $k \in [nT + 2, nT + 3)$, where the period $T = \kappa_1 + \kappa_2 = 3$. Fig. 7 depicts the state evolution $x(k)$ and Fig. 8 illustrates the input control signal $u(k)$ where the shaded area implies that mode 1 of subsystem subjects to the disordered controller and unshaded area implies that mode 2 of subsystem subjects to the disordered controller. It can be seen from Fig. 7 that under the constraints of Theorem 2, the system with controller failure is progressively stable.

V. CONCLUSION

In this paper, the stabilization problem of discrete-time Markovian jump systems has been realized by a partially mode-dependent controller. More importantly, the dwell times of such a controller are not instantaneous but two constants. Instead of investigating the resulting closed-loop system directly, another DMJS has been constructed and studied by applying an SQLF. Moreover, the proposed methods have been further extended to other stabilization problems whose controllers are fault-tolerant and disordered. All the conditions have been given within LMI framework. Finally, two simulations have been used to illustrate the practicability and applicability of the proposed methods.

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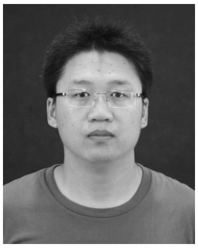

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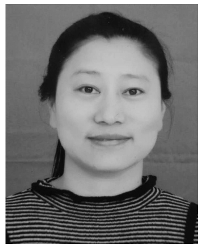

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