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Comprehensive Optimization of a Metro Timetable Considering Passenger Waiting Time and Energy Efficiency

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ABSTRACT Reducing passenger waiting time and energy consumption through train scheduling can have a great impact on improving the quality of service and energy efficiency from an operational view. However, there is a lack of studies of timetable optimization for connecting different modes of rail transit while considering the passenger transfer demand. This research considered the connection between a one-direction metro timetable and a railway timetable. Based on the spatiotemporal distribution of the demand of passenger flow, the optimal metro timetable for transfer passengers in different planning horizons was determined by optimizing and adjusting the following headway, running time of sections, and dwell time at stations. We proposed a multi-objective programming model consisting of three objective functions to minimize energy consumption, passenger waiting time at stations and the waiting time of transfer passengers at transfer stations. We used a fuzzy multi-objective optimization algorithm to solve the multi-objective programming model and added different weight factors to the three objective functions to obtain three optimal timetables, namely, timetables for energy-savings (E-timetable), passenger waiting time (WT-timetable), and transfer passenger waiting time (TWT-timetable); these timetables maximize energy efficiency, the quality of passenger service, and the transfer efficiency respectively. Finally, two practical cases based on real-world operational data were used to demonstrate the performance of the proposed models. The results showed that the three optimized timetables met the different requirements of the decision-makers; the combined use of the three optimized timetables can be used to guide actual operations.

INDEX TERMS Metro system, railway system, train timetable scheduling, energy efficiency, dynamic transfer passenger demand, passenger waiting time.

I. INTRODUCTION

With the expansion of rail transit (RT) networks in various cities, the number of nodes (transfer stations) between different modes has been increasing and the links between various types of RT have increased, resulting in a large number of cross-mode transfer passengers (passengers transferring from one mode of RT to another mode of RT); this mainly occurs for long-distance travel when the metro or railway is only a part of the journey. However, due to the poor organization of the transfer station and the poor connection of the timetables between different modes of RT, many problems occur in the transfer process, as shown in Figure 1. As shown in Figure 1(a), if the headway between metro trains is so long that some of the passengers cannot arrive at the transfer station before the departure of the railway train, they cannot take the subway and use other modes of transportation to the railway transfer station. In the case of Figure 1(b), if the headway between trains is so small that most transfer passengers will arrive at the transfer station by metro early, a large

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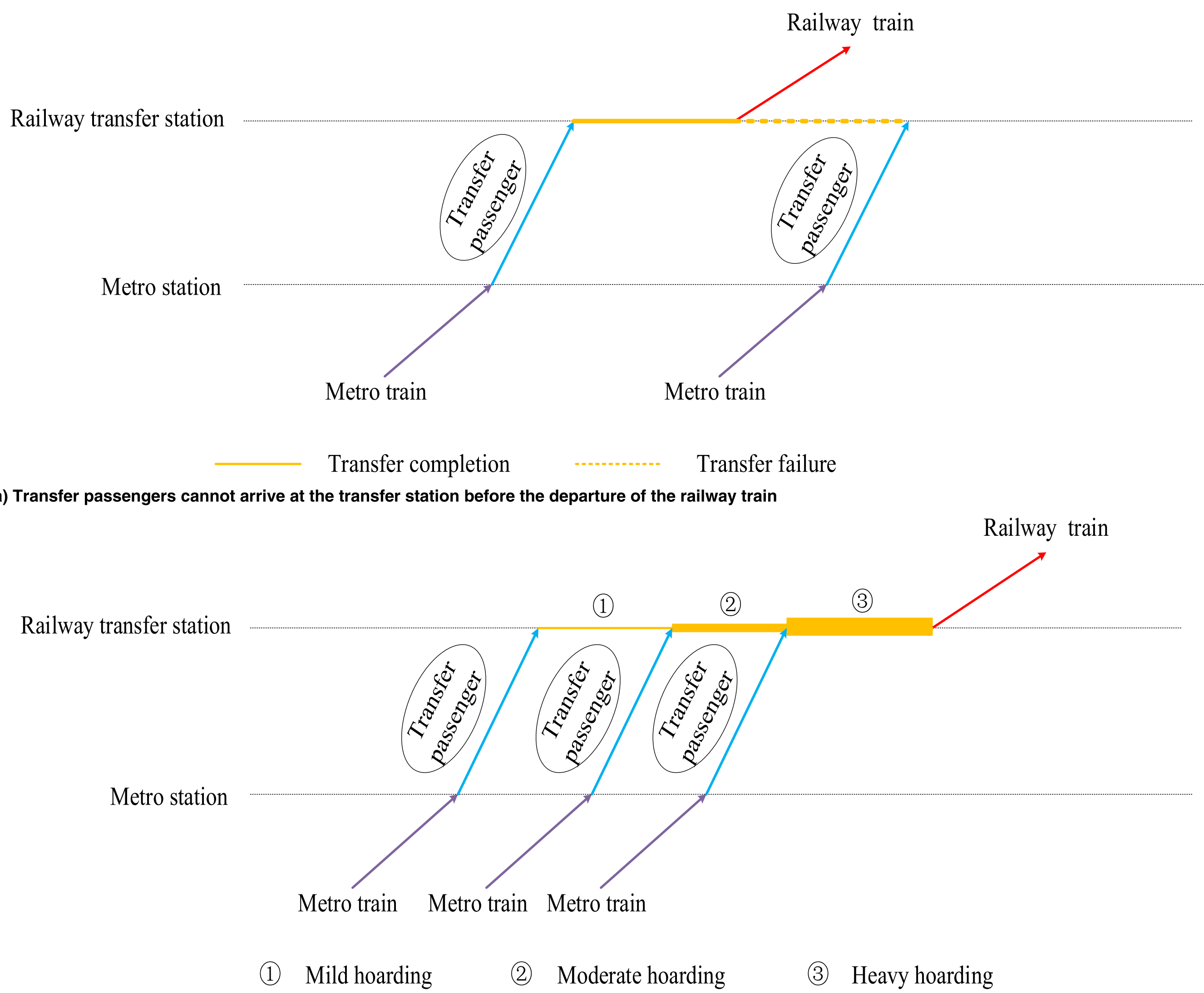


(a) Transfer passengers cannot arrive at the transfer station before the departure of the railway train

(b) Transfer passenger numbers increase at transfer stations

FIGURE 1. **Transfer problems in urban rail transit.**

number of passengers will accumulate at the transfer station, which increases the waiting time of the transfer passengers. The accumulation of passengers is particularly apparent at transfer stations with the same platform, e.g., the Chengdu Xipu transfer station. In summary, these problems are responsible for the poor connection of the train operation schemes (timetables) between metro trains and railway trains; as a result, the quality of passenger service is rather poor.

To bridge this gap, we consider the historical data of the passenger spatiotemporal transport demand and investigate the issue of metro train timetable optimization by taking into account the dynamic transfer demand of passengers from railways. We optimize the connection between the one-direction metro timetable and the railway timetable and adjust the timetable of the metro train by changing the headway between trains, the running time, and the dwell time. This ensures the convenience of the passenger transfer process and improves transfer efficiency, which is the priority of RT development.

Furthermore, improving transport services and reducing energy consumption has become a trend in RT development. Therefore, we should also consider the quality of passenger services and energy efficiency, and we need to consider the factors of energy-savings and the waiting time of transfer passengers in the entire metro network when we optimize the connection between the one-direction metro timetable and the railway timetable and adjust the timetable of metro trains.

The objective of timetable optimization [1]–[3] is a timetable that is suitable for specific tasks and practical applications; this topic is a very active research field in railway transportation. Cacchiani et al. (2016) [4] considered the train timetabling problem in a highly congested railway station and developed conflict-free timetables during a given planning time horizon while meeting all possible practical constraints. Xu et al. (2015) [5] developed a high-efficiency train timetabling approach for double-track railway lines and proposed three integrated train timetabling approaches, including a non-switchable policy, switchable policy, and improved switchable policy to prove the effectiveness of the proposed approaches.

Besides, some scholars mainly study timetable connection to minimize passenger waiting time and energy consumption. Yin et al. (2017) [6] proposed an integrated approach for the train scheduling problem to minimize the energy consumption and passenger waiting time. Which is rigorously formulated into two optimization models. The first model is an integer programming model that jointly minimizes energy consumption and passenger waiting time. The second model is formulated as a mixed-integer programming model. Considering the computational complexity of these two models, especially for large-scale real-world instances, they developed a Lagrangian relaxation (LR)-based heuristic algorithm to solve the problems. Wang et al. (2014) [7] adopted a stop-skipping strategy to reduce the passenger travel time and the energy consumption and proposed an efficient bi-level optimization approach to solve this train scheduling problem. The simulation results show that the efficient bi-level approach can effectively solve such problems. Yin et al. (2016) [8] proposed an integrated approach and an approximate dynamic programming approach for train scheduling problems in an urban metro line to minimize energy consumption and passenger waiting time.

Furthermore, D'ariano et al. (2007) [9] studied train scheduling and timetable optimization conditions faced by railway infrastructure managers in real-time traffic control. In practical cases, the branch and bound method was used to solve the proposed optimization model. Corman et al. (2012) [10] established a bi-objective programming model to minimize the train delay time. A detailed alternative graph model and two heuristic algorithms were developed to compute the Pareto front of non-dominated schedules. The Dutch railway was selected as a case study, and the results showed that the two algorithms accurately approximated the Pareto front and required low computation time. Meng and Zhou (2014) [11] developed an integer programming model of train scheduling using the Big-M method and Lagrangian relaxation solution framework to decompose the original complex rerouting and rescheduling problem into a sequence of single train optimization subproblems. Numerical experiments were used to demonstrate the performance of the optimal train scheduling approaches. Yang et al. (2014) [12] proposed a fuzzy optimization framework to reschedule trains by using a space-time network to represent the train trajectories; the authors formulated a two-stage 0-1 integer optimization model to find the optimal solution. The optimization model was solved by using GAMS optimization software. Mu and Dessouky (2013) [13] proposed a switchable dispatch policy for a double-track segment and the queueing theory was used to derive the delay functions of this policy. In practical cases, the switchable policy reduced the delay by up to 30% with fast train knock-on and the delay was reduced by up to 65% under crossovers at the middle of the double-track segment. Yang et al. (2015) [14] developed a scheduling approach to minimize the total energy consumption and maximize the utilization rate of the regenerative braking energy by optimizing the dwell time of trains at stations.

The mentioned studies show that the current solution methodologies to solve rail train scheduling problem can be divided into three categories, i.e., commercial optimization software [6], [12], heuristic algorithms [4], [11], and simulation methods [5], [7], [13]. In particular, the optimization objectives of these studies mostly focused on the train delay time [9]–[10], [13], passenger waiting time [4], [6]–[8], and total energy consumption of train operation [6]–[8], [14].

Due to an increased focus on energy conservation, emission reduction, and sustainable development, energy-saving operation of RT is becoming a research area of broad and current interest to reduce the operating costs of RT companies and enhance the competitiveness of RT in the field of transportation. Albrecht et al. (2016) [15] presented an optimization model to determine the characteristic optimal control modes and the allowable control transitions; the optimal switching points were investigated and the optimal driving strategies under the given speed limits were considered. Wang and Goverde (2019) [16] proposed a novel approach for energy-efficient timetabling by adjusting the running time allocation of given timetables using train trajectory optimization. Albrecht et al. (2011) [17] presented a new approach to control the operation time of trains based on dynamic programming; an optimal combination of headway and synchronization time was used to reduce energy consumption, and a fast and efficient numerical algorithm was employed to solve a key local energy minimization problem and find the optimal switching points. Miyatake and Ko (2010) [18] introduced three methods for solving the formulation, i.e., dynamic programming (DP), the gradient method, and sequential quadratic programming (SQP) to minimize train energy consumption. Li and Lo (2014) [19] established optimal models and used efficient mathematical algorithms to seek optimal driving strategies. Li et al. (2018) [20] presented a bi-objective timetable optimization model to maximize the operating revenue of the railway company while lowering the passengers' average travel cost. Tuyttens et al. (2013) [21] designed a genetic algorithm and a new method to solve a complex optimization model. Wang et al. (2016) [22] developed an optimization model that considered time and speed constraints derived from a practical timetable to calculate the minimum energy consumption of trains that were delayed. Bocharnikov et al. (2010) [23] proposed a single train speed profile optimization model that considered both the traction energy consumption and the utilization of regenerative braking energy.

Furthermore, the authors performed a simulation experiment to prove that their optimization model was efficient for minimizing energy consumption. Rodrigo et al. (2013) [24] designed a mathematics algorithm to solve problems in an energy-saving driving optimization model to improve the utilization ratio of regenerative braking energy.

TABLE 1. Recent publications on train scheduling and energy-saving in comparison with our work.

Publication	Decision variable		Objective			Model		Solution methods
	Timetable	Speed profiles	Energy - saving	Conventional passenger waiting time	Transfer passenger waiting time	Nonlinear	Linear	
Cacchiani et al. (2016)	√			√			√	Heuristic algorithm
Yin et al. (2016)	√		√	√			√	A Lagrangian relaxation-based heuristic algorithm
Yin et al. (2017)	√		√	√		√		A Lagrangian relaxation-based heuristic algorithm
Wang et al. (2016)		√	√			√		Pseudospectral method
Albrecht et al. (2016)		√	√			√		Mathematical deduction method
Albrecht et al. (2011)		√	√			√		Numerical algorithm
Miyatake and Ko (2010)		√	√			√		Dynamic programming
Rodrigo et al. (2013)		√	√			√		Langrage multipliers method
Li et al. (2018)		√	√			√		GA
Li and Lo (2014)		√	√			√		GA
Yang et al. (2015)	√		√			√		GA
Wang and Goverde (2019)		√	√			√		Pseudospectral method
This paper	√		√	√	√	√		Fuzzy multi-objective optimization algorithm

Scheepmaker et al. (2017) [25] conducted an extensive literature review on energy-efficient train control and the related topic of energy-efficient train timetabling; this subject includes advanced models and algorithms from the last decade that deal with varying gradients, speed limits, and regenerative braking along with optimal energy-efficient driving strategies for trains under different conditions. Ye and Liu (2017) [26] proposed a novel and effective approach to solving several complex train control problems, including optimal control for a fleet of interacting trains and optimal train control involving scheduling.

The mentioned studies mainly focused on improving the utilization ratio of regenerative braking energy [12], [23]–[24] and reducing the total energy consumption of train operations. The optimization objectives of these studies included strategies for energy-savings in trains [15]–[16], [19]–[20] and energy-saving timetable optimization [14], [26].

In order to clarify the contribution of this study, we listed recent major publications on train scheduling and energy-savings to compare the results of these studies with our research results; we mainly focused on the detailed characteristics, including the decision variables, objectives, models, and solution methods (Table 1).

In summary, although many studies listed in Table 1 have focused on train scheduling and energy-saving optimization models, the research objects were specific RT systems. In addition, the optimization of train timetables was conducted by focusing on the dynamic passenger demand, which only considers the passenger demand for a certain mode of rail transit. To date, few studies have contributed to research on the optimization of a train timetable that connects different modes of RT and considers the dynamic transfer passenger demand.

In this work, we investigate the connection between a one-direction metro timetable and a railway timetable; the main objectives and concepts of this study are as follows:

1) The main objectives are to minimize the total waiting time of transfer passengers at transfer stations and ensure the transfer efficiency. Based on the spatiotemporal distribution of the demand of passenger flow, the optimal metro timetable for transfer passengers in the planning horizons is found by optimizing and adjusting the following headway, running time of the sections, and dwell time at stations.
2) Considering that most passengers transported by the metro are conventional metro passengers, they do not need to transfer to the railway. Therefore, in order to ensure the quality of passenger service, we optimize the timetable of the metro train to reduce the total waiting time of all passengers at metro stations.
3) Due to the rising concerns about carbon emission and environmental problems, we also consider energy

efficiency by optimizing the train operation strategies and timetable to reduce traction energy consumption and improve regenerative braking energy utilization to achieve energy savings.

Using the proposed methods, we obtain three optimal timetables, namely, timetables for energy-saving (E-timetable), passenger waiting time (WT-timetable), and transfer passenger waiting time (TWT-timetable) to maximize energy efficiency, the quality of passenger service, and transfer efficiency respectively. Decision-makers can choose the appropriate optimal timetable to guide the operation of metro trains based on their preferences and demand.

In summary, in order to better present the purpose and content of this study, the research ideas of this paper are added in this version, as shown in Figure 2. This figure introduces the research background and the target of this study, which explains why this paper considers the connection between a metro timetable and a railway timetable. Then, it gives the relationship between the three objective functions. Moreover, it also explains how the three objective functions constructed in this paper can solve practical engineering problems to meet the needs of different preferences of decision-makers. Finally, it gives the expected results of this study.

The rest of this paper is organized as follows. In Section II, we give a detailed description of the problems to be solved. In Section III, a multi-objective programming model consisting of three optimization models for energy efficiency, service quality, and transfer efficiency are established based on actual constraints. Then, we propose the fuzzy multi-objective optimization algorithm to solve the multi-objective programming model in Section IV. In Section V, we describe and discuss two practical cases based on the operational data of the Chengdu Metro line 2 and the Xipu-Qingchengshan fast railway line to demonstrate the effectiveness of the proposed models and solution approaches. Finally, the conclusions and plans for further studies are presented in Section VI.

II. PROBLEM DESCRIPTION

We consider the optimal connection of timetables from a one-direction metro line with I_{m} stations to a one-direction railway with I_{r} stations, as shown in Figure 3. The total number of stations on a metro line and railway line are denoted by $\mathbf{I}_{\mathrm{m}} = \{1, 2, \ldots, i_{\mathrm{m}} \ldots, I_{\mathrm{m}}\}$ and $\mathbf{I}_{\mathrm{r}} = \{1, 2, \ldots, i_{\mathrm{r}} \ldots, I_{\mathrm{r}}\}$, respectively. The set of trains operating on the metro line is denoted by $\mathbf{K}_{\mathrm{m}} = \{1, 2, \ldots K_{\mathrm{m}}\}$ and the set of trains operating on the railway line is denoted by $\mathbf{K}_{\mathrm{r}} = \{1, 2, \ldots K_{\mathrm{r}}\}$.

The objective of this study is to find the optimal metro timetable for transfer passengers in the planning horizons $(t_o, t_d]$; this timetable should minimize the total waiting time of the transfer passengers at transfer stations by optimizing and adjusting the following headway, running time of sections, and dwell time at stations to ensure the transfer efficiency of the transfer passengers.

Figure 4 shows a diagram of the connection between the metro timetable and the railway timetable. There are nine metro trains serving metro passengers in the planning time horizons $(t_o, t_d]$ and two railway trains serving the railway passengers (including transfer passengers at transfer stations) in the planning time horizons $(t'_o, t'_d]$. Considering that the transfer passengers will transfer to the railway train after they get off at the transfer station, the time for the transfer passengers to take the metro train is limited, as shown by $(t_o^{\mathrm{transfer}}, t_d^{\mathrm{transfer}}]$ in Figure 3. That is to say, the transfer passengers will not be able to catch the transfer train by taking the metro train after this time period $(t_o^{\mathrm{transfer}}, t_d^{\mathrm{transfer}}]$.

In this case, the transfer passenger has only two choices: (1) abandon the transfer to the railway train, change the ticket, or re-purchase a railway ticket for another train to the final destination; (2) get to the transfer station by taking other means of transportation, such as a car. Both choices will reduce the number of RT passengers traveling in the planning horizons $(t_o, t_d]$. The above-mentioned outcomes are due to the poor connection between the metro timetable and the railway timetable. Furthermore, if the connection between the metro timetable and the railway timetable is poor, the passengers may be transported to the transfer station early, which lengthens the waiting time of the passengers at the transfer station and reduces the quality of passenger service in rail transit. Therefore, in order to ensure that all passengers arrive at the transfer station at a suitable time before the departure of the transfer railway train, we must optimize the metro train timetable in conjunction with the railway train timetable. Of course, in this optimization process, we also need to consider train driving strategies to reduce energy consumption and improve energy efficiency to achieve the purpose of energy savings.

In order to formulate the mathematical models for energy consumption and passenger waiting time, which are described in Section III, we first make the following assumptions:

Assumption 1: We do not consider the waiting time of railway passengers arriving at railway stations by other modes of transport. Most railway passengers make their trip plans according to the given timetable (railway timetable) and

Assumption 2: Considering that the objective of this research is to improve the transfer efficiency of passengers and shorten the waiting time of transfer passengers, we consider the flow of transfer passengers as being dynamic in time, whereas the flow of conventional metro passengers is defined as being non-dynamic in space and time.

Assumption 3: Considering that the optimal application of the connection between Metro and railway timetables studied in this paper will be more meaningful and effective in solving the passenger transfer problem on the same platform, therefore, the object of this paper is the phenomenon of passenger transfer on the same platform.

III. MATHEMATICAL FORMULATION

In this section, we describe the establishment of the energy consumption model of the metro and railway train, the dynamic spatiotemporal passenger waiting time

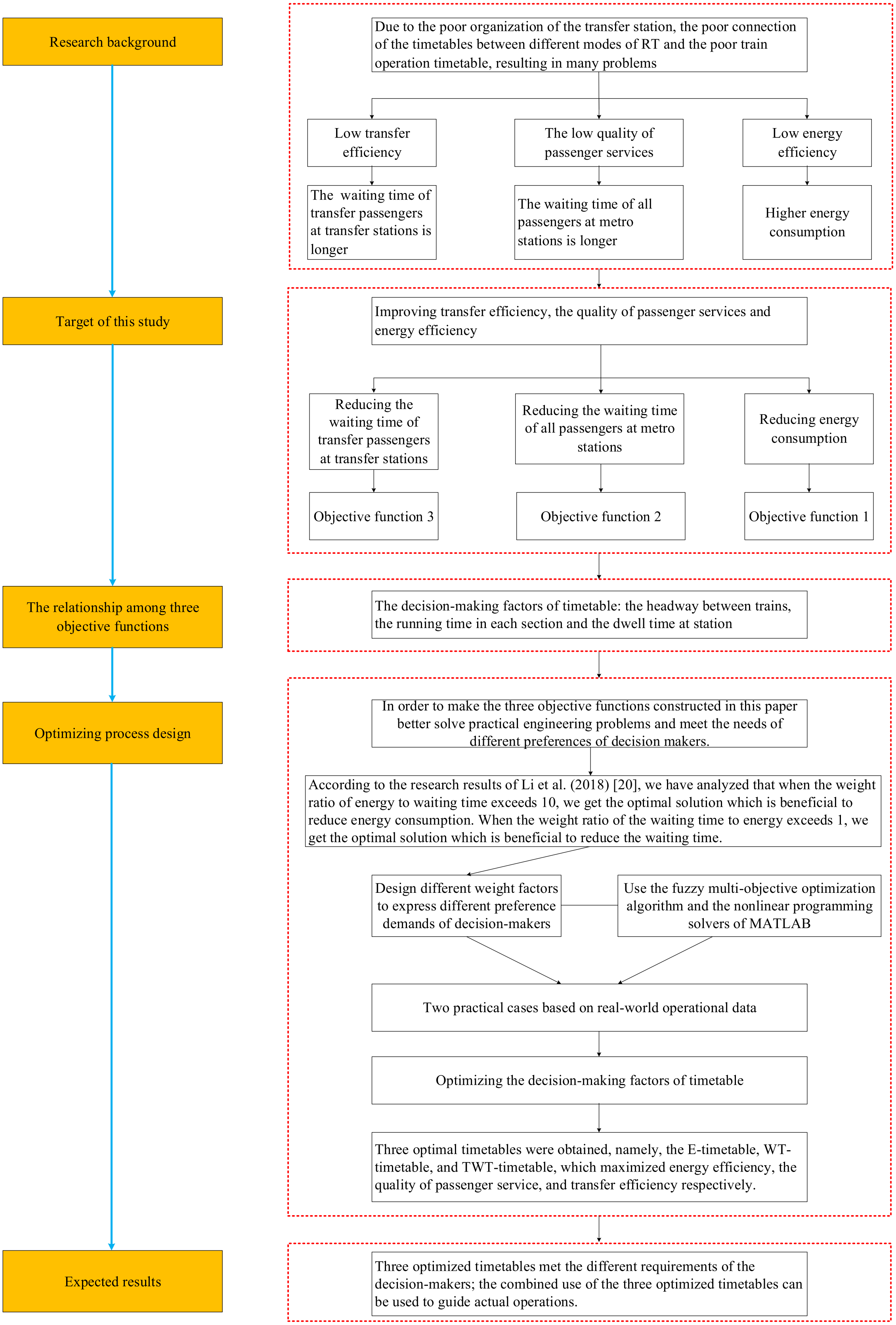


FIGURE 2. Diagram of research ideas in this paper.

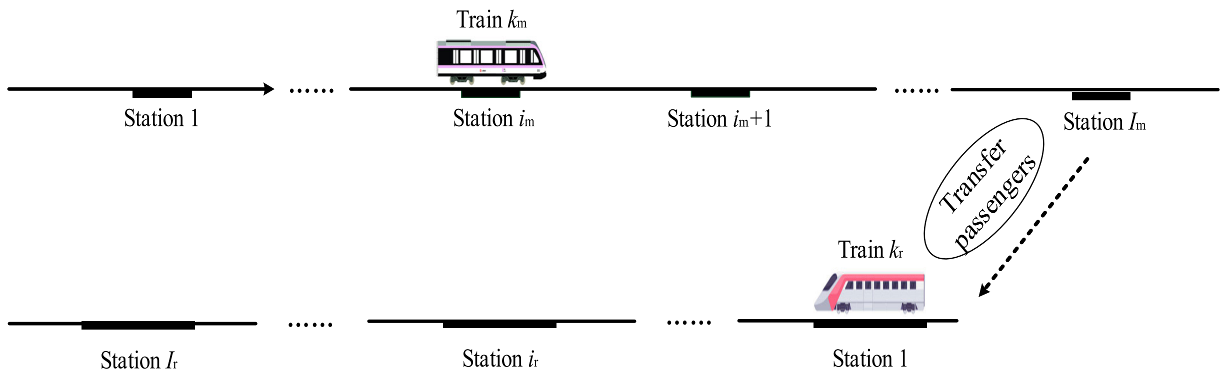


FIGURE 3. Diagram of the transfer process from the metro line to the railway line.

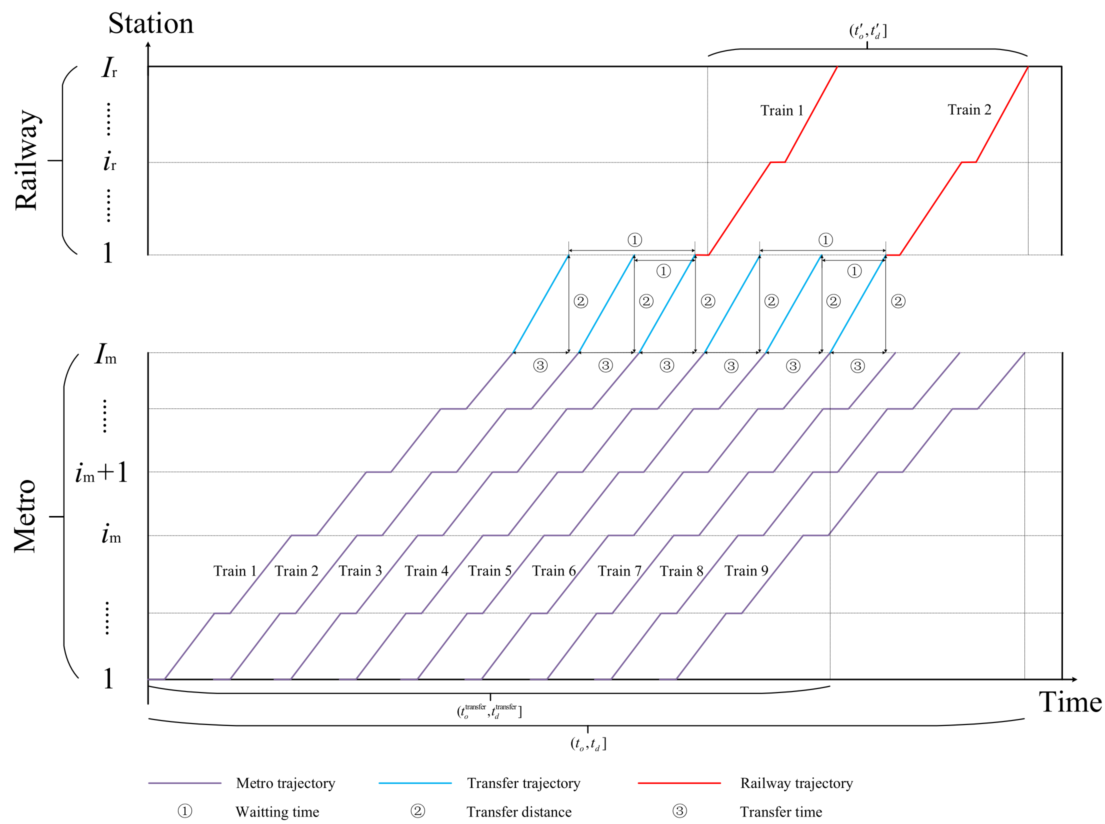


FIGURE 4. Timetable connection between metro and railway.

optimization model, the objective functions, and the systematic constraints.

A. NOTATIONS

To describe the proposed mathematical model, we list the symbols and parameters in Table 2 and the decision variables in Table 3.

B. ENERGY CONSUMPTION MODEL

a: ENERGY-SAVING DRIVING MODEL OF METRO TRAINS

Considering that the metro system has the characteristics of a short running time and small distance between adjacent stations, Howlett et al. (1994) [27] proposed that the optimal energy-saving driving strategy for a single metro train running in a section consists of three phases, i.e., the accelerating phase, coasting phase, and braking phase (Figure 5); the purpose was to ensure that the train can pass through the section with minimum energy consumption. Metro trains run on a track; the forces acting on the train include the traction force, basic line resistance, and braking force. In order to accelerate the train to a reasonable speed, it is necessary to provide sufficient kinetic energy for the train, which is provided by traction work in the accelerating phase.

TABLE 2. Symbols and parameters.

Symbols and parameters	Descriptions
$\mathbf{S}_{\mathrm{m}}=\{1,2...,s_{\mathrm{m}},...,S_{\mathrm{m}}\}$	Set of metro sections index
$\mathbf{S}_{\mathrm{r}}=\{1,2...,s_{\mathrm{r}},...,S_{\mathrm{r}}\}$	Set of railway sections index
$\mathbf{K}_{\mathrm{m}}=\{1,2...,K_{\mathrm{m}}\}$	Set of metro trains index
$\mathbf{K}_{\mathrm{r}}=\{1,2...,K_{\mathrm{r}}\}$	Set of railway trains index
$\mathbf{I}_{\mathrm{m}}=\{1,2...,i_{\mathrm{m}},...,I_{\mathrm{m}}\}$	Set of metro stations index
$\mathbf{I}_{\mathrm{r}}=\{1,2...,i_{\mathrm{r}},...,I_{\mathrm{r}}\}$	Set of railway stations index
$\mathbf{P}_{\mathrm{m}}=\{1,2...,P_{\mathrm{m}}\}$	Set of metro train driving phases
$\mathbf{P}_{\mathrm{r}}=\{1,2...,P_{\mathrm{r}}\}$	Set of railway train driving phases
$\mathbf{t}=\{t_o,...t,...t_d\}$	Index of time, t_o denotes the beginning time and t_d denotes the end time, t denotes an arbitrary intermediate moment, $t'>t$, s
t',t'',t'''	Index of time, t',t'',t''' is a tiny time, it satisfying $t'>t$, $t''>t$, $t'''>t$, s
τ,τ'	Index of time, τ,τ' is a tiny time, s
(t,t')	Index of time period from t to t', s
$i,j,i_{\mathrm{m}},j_{\mathrm{m}},i_{\mathrm{r}},j_{\mathrm{r}}$	Index of stations, $i_{\mathrm{m}},j_{\mathrm{m}}\in\mathbf{I}_{\mathrm{m}}$ and $i_{\mathrm{r}},j_{\mathrm{r}}\in\mathbf{I}_{\mathrm{r}}$
$(i_{\mathrm{m}},j_{\mathrm{m}}),(i_{\mathrm{r}},j_{\mathrm{r}})$	Index of directed links
$p_{\mathrm{m}},p_{\mathrm{r}}$	Index of driving phases, $p_{\mathrm{m}}\in\mathbf{P}_{\mathrm{m}}$ and $p_{\mathrm{r}}\in\mathbf{P}_{\mathrm{r}}$
$s_{\mathrm{m}},s_{\mathrm{r}}$	Index of sections, $s_{\mathrm{m}}\in\mathbf{S}_{\mathrm{m}}$ and $s_{\mathrm{r}}\in\mathbf{S}_{\mathrm{r}}$
$k,k_{\mathrm{m}},k_{\mathrm{r}}$	Index of trains, $k_{\mathrm{m}}\in\mathbf{K}_{\mathrm{m}}$ and $k_{\mathrm{r}}\in\mathbf{K}_{\mathrm{r}}$
$v_{k_{\mathrm{m}}s_{\mathrm{m}}p_{\mathrm{m}}}$	velocity of train k_{m} at the end of phase p_{m} in the metro section s_{m}, m/s
$v_{k_{\mathrm{r}}s_{\mathrm{r}}p_{\mathrm{r}}}$	velocity of train k_{r} at the end of phase p_{r} in the railway section s_{r}, m/s
$v_{\mathrm{m}}^{\max},v_{\mathrm{r}}^{\max}$	Maximum running velocity in the section of the metro train and railway train, m/s
$l_{k_{\mathrm{m}}s_{\mathrm{m}}p_{\mathrm{m}}}$	Running distance of train k_{m} in phase p_{m} of metro section s_{m}, m
$l_{k_{\mathrm{r}}s_{\mathrm{r}}p_{\mathrm{r}}}$	Running distance of train k_{r} in phase p_{r} of railway section s_{r}, m
$l_{s_{\mathrm{m}}}$	Running distance of train in section s_{m}, m
$l_{s_{\mathrm{r}}}$	Running distance of train in section s_{r}, m
$t_{k_{\mathrm{m}}s_{\mathrm{m}}}$	Running time of train k_{m} in section s_{m}, s
$t_{s_{\mathrm{m}}}^{\min},t_{s_{\mathrm{m}}}^{\max}$	The upper and lower bounds of running time in section s_{m}, s
$f_{\mathrm{m}}(v),f_{\mathrm{r}}(v)$	The traction force characteristic curve of the metro train and railway train, kN
$b_{\mathrm{m}}(v),b_{\mathrm{r}}(v)$	The braking force characteristic curve of the metro train and railway train, kN
$r_{\mathrm{m}}(v),r_{\mathrm{r}}(v)$	The basic resistance characteristic curve of the metro train and railway train, kN
$H^{\min},H^{\max}$	The upper and lower bounds of following headway between the metro trains, s
$D_{k_{\mathrm{m}}i_{\mathrm{m}}},D_{k_{\mathrm{r}}i_{\mathrm{r}}}$	Dwell time of train k_{m}, k_{r} at station i_{m}, i_{r}, s
$D_{i_{\mathrm{m}}}^{\min},D_{i_{\mathrm{m}}}^{\max}$	The upper and lower bounds of dwell time at station i_{m}, s

TABLE 2. ***(Continued.)*** **Symbols and parameters.**

$w_{k_m i_m}$, $w_{k_r i_r}$	Total mass of train k_m, k_r depart from station i_m, i_r, kg
w_m, w_r	The mass of the empty train (metro train, railway train), kg
$Q_{k_m i_m}$, $Q_{k_r i_r}$	Passenger volume of train k_m or k_r depart from station i_m or i_r, respectively
Q_m^{max}, Q_r^{max}	Maximum passenger volume of per metro train or railway train, respectively
$od_{i_m j_m tt'}$, $od_{i_r j_r tt'}$	Origin-destination demands of passengers from station i_m or i_r to the station j_m or j_r in time interval $(t, t']$, respectively
$od^{transfer}_{i_m i_r tt'}$	Origin-destination demands of transfer passengers from station i_m to the station i_r in time interval $(t, t']$
Q_m	Total number of passengers transported by Metro in the planning horizon $(t_o, t_d]$
$Q_m^{transfer}$	Total number of transfer passengers transported by Metro in the planning horizon $(t_o, t_d]$

TABLE 3. **Decision variables.**

Decision variables	Descriptions
$n_{i_m t(t+\tau)}$	The numbers of waiting passengers at timestamp t
$A^{transfer}_{k_m I_m}$	Transfer passengers arriving at metro station I_m by train k_m
$D_{k_m i_m}$	Dwell time of train k_m at station i_m, s
$t_{k_m s_m p_m}$	Running time of train k_m in phase p_m of metro section s_m, s
$t_{k_r s_r p_r}$	Running time of train k_r in phase p_r of railway section s_r, s
$H_{k_m(k_m+1)}$	Following headway between the metro trains, s

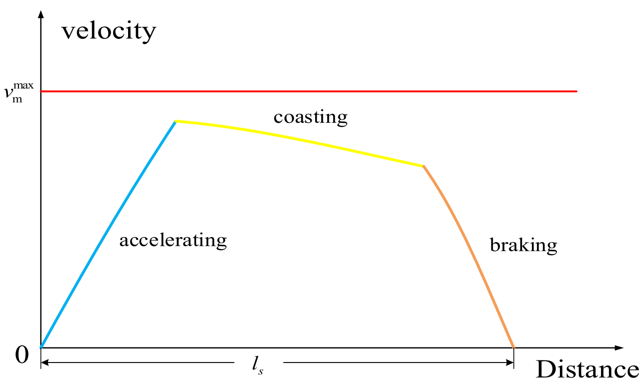


FIGURE 5. **Velocity-distance profile of a metro train operating in a section.**

The total traction energy consumption of metro trains operating on metro lines is defined in equation (1). Equations (2~6) provide the dynamic constraints and timetable constraints of the train operation:

$$TC_m^{total} = \sum_{k_m=1}^{K_m} \sum_{s_m=1}^{S_m} \sum_{p_m=1}^{P_m} \int_0^{l_{k_m s_m p_m}} f_m(v) \cdot l dl \quad (1)$$

Subject to

$$0 \le f_m(v) \le F_m^{max}$$
$$0 \le b_m(v) \le B_m^{max}$$
$$0 \le v \le v_m^{max} \quad (2)$$
$$l_{k_m s_m p_m} = \int_{v_{k_m s_m (p_m-1)}}^{v_{k_m s_m p_m}} \frac{w_{k_m i_m} v}{\theta_1 f_m(v) - \theta_2 b_m(v) - r_m(v)} dv$$
$$k_m \in K_m, s_m \in S_m, p_m \in P_m \quad (3)$$
$$l_{s_m} = \sum_{p_m=1}^{P_m} l_{k_m s_m p_m} k_m \in K_m, s_m \in S_m \quad (4)$$
$$0 \le \frac{f_m(v) - r_m(v)}{w_{k_m i_m}} \le \hat{a}_m^{acc} \quad (5)$$
$$0 \le \frac{b_m(v) + r_m(v)}{w_{k_m i_m}} \le \left|\hat{a}_m^{dec}\right| \quad (6)$$

where F_m^{max} is the maximum traction force of a metro train, kN; B_m^{max} is the maximum braking force of a metro train, kN; $\hat{a}_m^{acc}$ is the maximum acceleration of a metro train, m/s^2; $\hat{a}_m^{dec}$ is the maximum deceleration of a metro train, m/s^2; $\theta_1, \theta_2 \in [0, 1]$, $\theta_1 = 1, \theta_2 = 0$ in the accelerating phase,

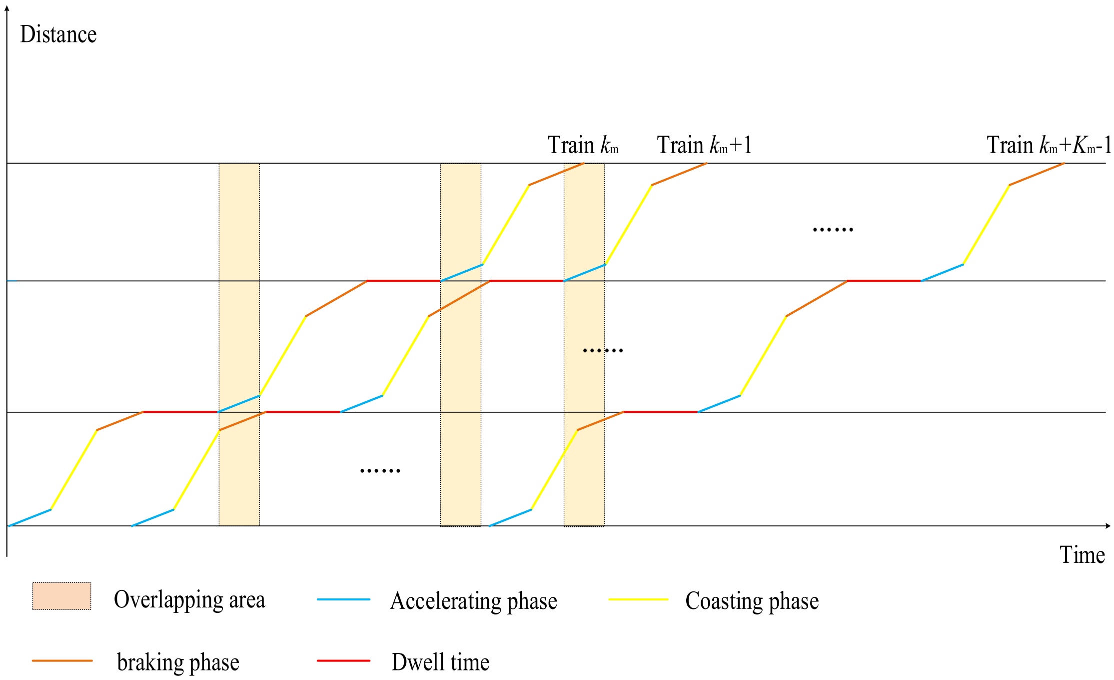


FIGURE 6. Utilization of regenerative braking energy.

$\theta_1 = 0, \theta_2 = 0$ in the coasting phase, $\theta_1 = 0, \theta_2 = 1$ in the braking phase.

- Utilization of regenerative braking energy

Regenerative braking is an energy-saving method, in which kinetic energy is converted into electric energy (regenerative braking energy) during the braking phase of a train; the electric energy is transferred to the train in the acceleration phase. When multiple trains $(k_{\mathrm{m}}, k_{\mathrm{m}}+1\ldots, k_{\mathrm{m}}+K_{\mathrm{m}}-1)$ run consecutively in the sections served by the same power supply substation, there exist many areas of overlap, i.e., overlaps between the braking phase of subsequent trains and the acceleration phase of preceding trains, as shown in Figure 6. Therefore, the regenerative braking energy that can be used by train k_{m} can be calculated using equation (7). The length of overlap time, which is defined in equation (8), determines the utilization of regenerative braking energy:

$$E_{k_{\mathrm{m}}}^{\mathrm{reg}} = \sum_{k=k_{\mathrm{m}}+1}^{K_{\mathrm{m}}} \sum_{s_{\mathrm{m}}=1}^{S_{\mathrm{m}}} \sum_{p_{\mathrm{m}}=1}^{P_{\mathrm{m}}} \int_0^{l_{ks_{\mathrm{m}}p_{\mathrm{m}}}} [(b_{\mathrm{m}}(v)\cdot l\mathrm{d}l)\cdot \eta_{s_{\mathrm{m}}}\cdot \xi] \tag{7}$$

$$\eta_{s_{\mathrm{m}}} = \sum_{k=k_{\mathrm{m}}+1}^{K_{\mathrm{m}}} \frac{t_{(k_{\mathrm{m}}\cap k)}^{overlap}}{t_{ks_{\mathrm{m}}3}} \tag{8}$$

where $E_{k_{\mathrm{m}}}^{\mathrm{reg}}$ is the regenerative braking energy that can be used by train k_{m}, kWh; $\eta_{s_{\mathrm{m}}}$ is the utilization ratio of the regenerative braking capacity of train k_{m} in section S_{m}; ξ is the regenerative braking energy conversion rate; $t_{(k_{\mathrm{m}}\bigcap k)}^{overlap}$ is the overlap time duration between other trains in the braking phase and train k_{m} in the accelerating phase, s; $t_{ks_{\mathrm{m}}^3}$ is the braking phase time duration of train k, s.

By combining equation (1) and equation (7), we obtain the total energy consumption of metro trains operating on metro lines, as defined in equation (9):

$$\begin{aligned} E_{\mathrm{m}}^{\mathrm{total}} = &\sum_{k_{\mathrm{m}}=1}^{K_{\mathrm{m}}} \sum_{s_{\mathrm{m}}=1}^{S_{\mathrm{m}}} \sum_{p_{\mathrm{m}}=1}^{P_{\mathrm{m}}} \int_0^{l_{k_{\mathrm{m}}s_{\mathrm{m}}p_{\mathrm{m}}}} f_{\mathrm{m}}(v)\cdot l\mathrm{d}l \\ &- \sum_{k_{\mathrm{m}}=1}^{K_{\mathrm{m}}} \sum_{k=k_{\mathrm{m}}+1}^{K_{\mathrm{m}}} \sum_{s_{\mathrm{m}}=1}^{S_{\mathrm{m}}} \sum_{p_{\mathrm{m}}=1}^{P_{\mathrm{m}}} \int_0^{l_{ks_{\mathrm{m}}p_{\mathrm{m}}}} \\ &\times [(b_{\mathrm{m}}(v)\cdot l\mathrm{d}l)\cdot \eta_{s_{\mathrm{m}}}\cdot \xi] \end{aligned} \tag{9}$$

b: ENERGY-SAVING DRIVING MODEL FOR RAILWAY TRAIN

Khmelnitsky (2000) [28] described the train operation process by using a differential equation with non-linear constraints and used the maximum principle to solve the model. It was concluded that the strategies for energy-savings in railway trains in a given section consisted of four phases (Figure 7), i.e., the accelerating phase, cruising phase,

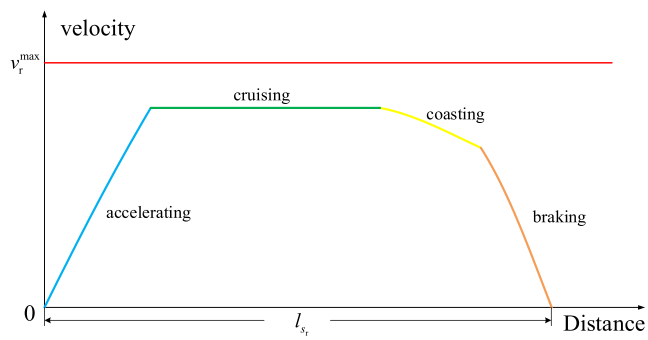


FIGURE 7. Velocity-distance profiles of a railway train operating in a section.

coasting phase, and braking phase. Railway trains run on lines and are affected by the traction force, basic line resistance, braking force, and air resistance during operation. Because of the heavy mass of the railway train, the traction or braking force required for the train to start or brake in a section is relatively large. In addition, a high velocity of the train in a section leads to greater air resistance, which cannot be ignored.

Equation (10) is the mathematical formula for calculating the air resistance of a railway train (Model: CRH3) during its operation. The total energy consumption of the railway trains operating on the lines is defined in equation (11). Equations (12∼16) provide the relevant constraints of the train operation:

$$A_r(v) = 0.0064 w_{k_r i_r} + 0.13 n_r + 0.00014 w_{k_r i_r} v + (0.000046 + 0.000065(N_r - 1)) A v^2 \quad (10)$$

$$E_r^{total} = \sum_{k_r=1}^{K_r} \sum_{s_r=1}^{S_r} \sum_{p_r=1}^{P_r} \int_0^{l_{k_r s_r p_r}} \theta_1 f_r(v) \cdot l \mathrm{d}l \quad (11)$$

Subject to

$$\begin{aligned} &0 \le f_r(v) \le F_r^{max} \\ &0 \le b_r(v) \le B_r^{max} \\ &0 \le v \le v_r^{max} \end{aligned} \quad (12)$$

$$l_{k_r s_r p_r} = \int_{v_{k_r s_r (p_r-1)}}^{v_{k_r s_r p_r}} \frac{w_{k_r i_r} v}{\theta_1 f_r(v) - \theta_2 b_r(v) - r_r(v) - A_r(v)} \mathrm{d}v$$
$$k_r \in \mathrm{K}_r, s_r \in \mathrm{S}_r, p_r \in \mathrm{P}_r \quad (13)$$

$$l_{s_r} = \sum_{p_r=1}^{P_r} l_{k_r s_r p_r} k_r \in \mathrm{K}_r, s_r \in \mathrm{S}_r \quad (14)$$

$$0 \le \frac{\theta_1 f_r(v) - A_r(v) - r_r(v)}{w_{k_r i_r}} \le \hat{a}_r^{acc} \quad (15)$$

$$0 \le \frac{b_r(v) + r_r(v) + A_r(v)}{w_{k_r i_r}} \le \left| \hat{a}_r^{dec} \right| \quad (16)$$

where $A_r(v)$ is the air resistance of a railway train, kN; n_r is the number of axles on a train; N_r is the number of rolling stocks on a train; A is the frontal section area of the headstock, m^2; $\theta_1, \theta_2 \in [0, 1]$, $\theta_1 = 1, \theta_2 = 0$ in the accelerating phase, $\theta_1 \epsilon (0, 1), \theta_2 = 0$ in the cruising phase, $\theta_1 = 0, \theta_2 = 0$ in the coasting phase, $\theta_1 = 0, \theta_2 = 1$ in the braking phase; F_r^{max} is the maximum traction force of a railway train, kN; B_r^{max} is the maximum braking force of a railway train, kN; $\hat{a}_r^{acc}$ is the maximum acceleration of a railway train, m/s^2; $\hat{a}_r^{dec}$ is the maximum deceleration of a railway train, m/s^2.

C. PASSENGER WAITING TIME OPTIMIZATION MODEL

a: TOTAL WAITING TIME OF PASSENGERS AT METRO STATIONS

With the wide application of smart card technology, we can easily obtain the data of spatiotemporal passenger flow in metro lines to analyze the spatiotemporal demand of passengers in a planning horizon. The spatiotemporal passenger flow includes conventional metro passengers (that only take the metro) and transfer passengers (from the metro to railway) and when both types of dynamic passengers are present simultaneously, the passenger waiting time optimization model will be very complex and difficult to solve in the prescribed time. In addition, the main research emphasis of this study is the connection between the metro timetable and railway timetable to improve the transfer efficiency of passengers and shorten the waiting time of passengers. Therefore, we only consider the flow of transfer passengers as being dynamic in space and time, whereas the flow of conventional metro passengers is defined as being non-dynamic in space and time.

We define $od_{i_m j_m t(t+\tau)}$ as the conventional metro passenger demand ratio from station i_m to j_m in time interval $(t, t+\tau]$, $od_{i_m i_r t(t+\tau)}^{transfer}$ indicates the transfer passenger demand ratio from station i_m to j_r in time interval $(t, t+\tau]$, $od_{i_m t(t+\tau)}$ indicates the passenger arrival rates at station i_m in time interval $(t, t+\tau]$; the relationship among these three variables is expressed as equation (17). The passenger demand in time interval $(t, t']$ is shown in equations (18∼19).

$$od_{i_m t(t+\tau)} = \sum_{j_m=i_m+1}^{I_m} od_{i_m j_m t(t+\tau)} + od_{i_m i_r t(t+\tau)} j_m > i_m \quad (17)$$

$$\begin{aligned} od_{i_m j_m tt'} &= (t'-t) od_{i_m j_m t(t+\tau)} \\ od_{i_m j_m t(t+\tau)} &= od_{i_m j_m (t+\tau)(t+2\tau)} = \ldots = od_{i_m j_m (t'-\tau)t'} \end{aligned} \quad (18)$$

$$\begin{aligned} od_{i_m i_r tt'}^{transfer} &= od_{i_m i_r t(t+\tau)}^{transfer} + od_{i_m i_r (t+\tau)(t+2\tau)}^{transfer} + \ldots + od_{i_m i_r (t'-\tau)t'}^{transfer} \\ od_{i_m i_r t(t+\tau)}^{transfer} &\ne od_{i_m i_r (t+\tau)(t+2\tau)}^{transfer} \ne \ldots \ne od_{i_m i_r (t'-\tau)t'}^{transfer} \end{aligned} \quad (19)$$

When the train k_m arrives at station i_m, it will remain on the platform for a short period of time $D_{k_m i_m}$ to ensure that the passengers can board and alight, as shown in Figure 8. $A_{k_m i_{m tt''}}$ indicates the passenger volume of train k_m alighting at metro station i_m in time interval $(t, t'']$, $Bk_m i_{m tt''}$ indicates the passenger volume of train k_m boarding at metro station i_m in time interval $(t, t'']$, and $Q_{k_m i_m}$ indicates the passenger volume of train k_m departing from station i_m; we obtain equations (20∼22) from equations (17∼19). If we assume that the passenger boarding rate at station i_m in time interval $(t, t+\tau]$ is $b_{k_m i_{m t(t+\tau)}}$ and the passenger alighting rate at station i_m in time interval $(t, t+\tau]$ is $a_{k_m i_{m t(t+\tau)}}$, we can obtain equation (24). The detailed expansions of equation (20) are shown in Proof. 1 of Appendix A.

$$Q_{k_m i_m} = Q_{k_m i_m - 1} + B_{k_m i_m tt''} - A_{k_m i_m tt''} \quad (20)$$

$$B_{k_m i_m tt''} = \begin{cases} \sum_{j_m=i_m+1}^{I_m} od_{i_m j_m tt'} + od_{i_m i_r tt'} i_m \in [1, I_m) \\ 0 \; i_m = I_m \end{cases} j_m > i_m \quad (21)$$

$$A_{k_m i_m tt''} = \begin{cases} 0 \; i_m = 1 \\ \sum_{i=1}^{i_m - 1} od_{i i_m tt'} i_m \in (1, I_m] \end{cases} i_m > i \quad (22)$$

$$\begin{aligned} t' &= t + H_{(k_m-1)k_m} \\ t'' &= t + D_{k_m i_m} \end{aligned} \quad (23)$$

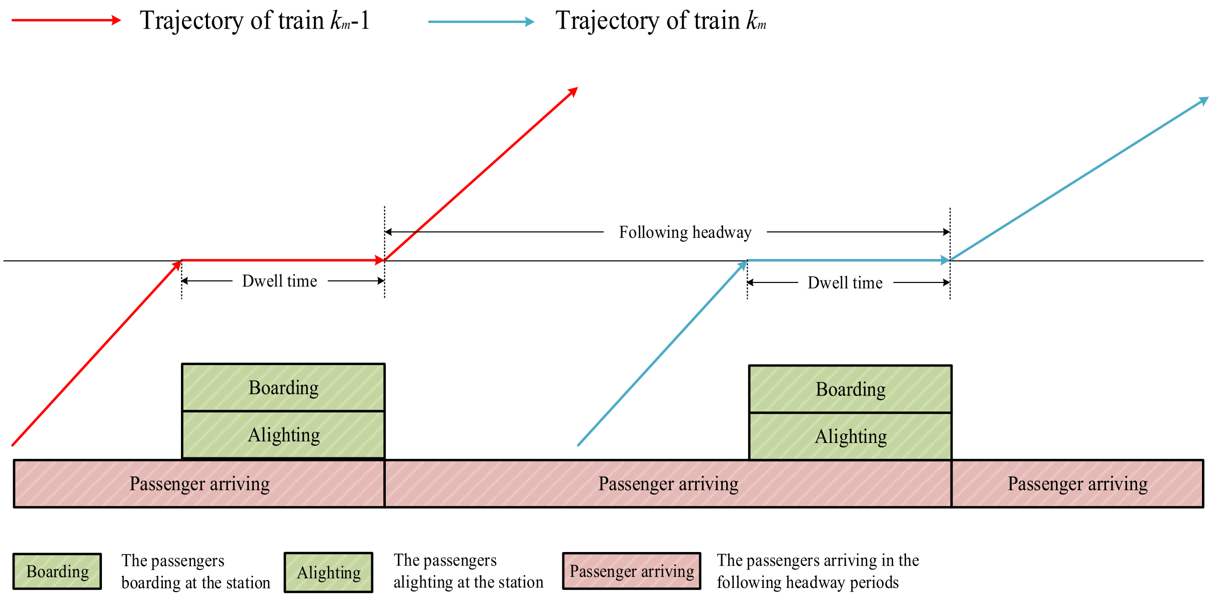


FIGURE 8. Boarding and alighting process at a metro station.

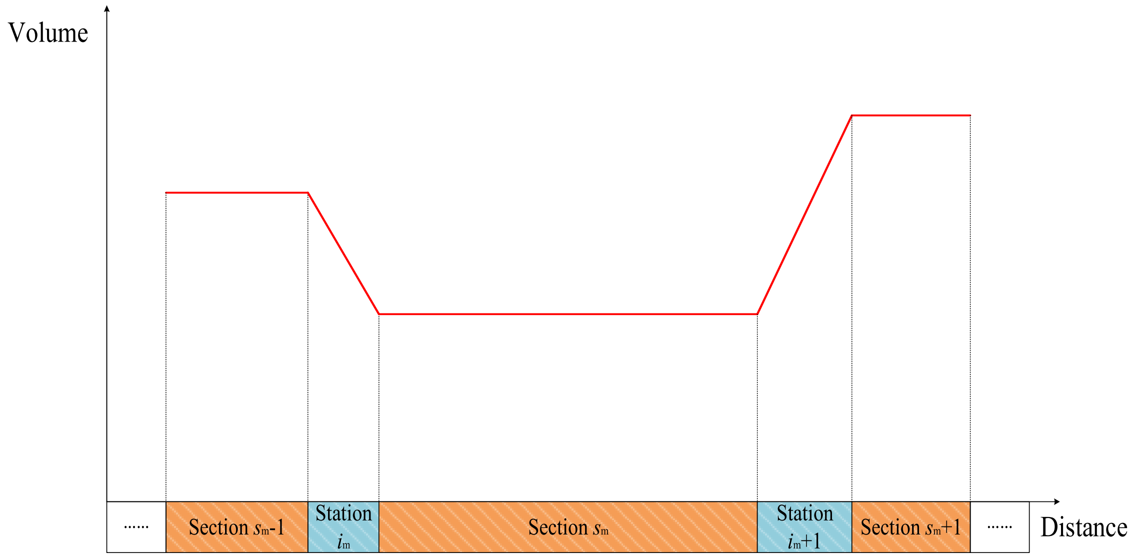


FIGURE 9. Passenger volume at the metro station.

$$B_{k_m i_m t t'} = \sum_{t \in (t,t']}^{t'} b_{k_m i_m t(t+\tau)}$$

$$A_{k_m i_m t t'} = \sum_{t \in (t,t']}^{t'} a_{k_m i_m t(t+\tau)} \tag{24}$$

Figure 9 shows the passenger volume of the train departing from the station; if the average weight of the passengers is 60 kg, we can calculate $w_{k_m i_m}$ as shown in equation (25).

$$w_{k_m i_m} = \frac{60 Q_{k_m i_m}}{1000} + w_m \tag{25}$$

We have now analyzed the whole process of passenger boarding and alighting at the station. Based on the research results of Barrena et al. (2014) [7], we derive the variable $n_{i_m t(t+\tau)}$, which represents the number of waiting passengers at station i_m at timestamp t; assuming that $n_{i_m t_o}$ is a constant, the total waiting time of passengers at metro stations in a planning horizon $(t_o, t_d]$ is calculated using equation (26):

$$WT_m^{\text{total}} = \sum_{i_m=1}^{I_m} \sum_{t=t_o}^{t_d} \tau \cdot n_{i_m t(t+\tau)}$$
$$(t, t+\tau] \in [(t_o, t_o+\tau], \ldots (t, t+\tau] \ldots, (t_d-\tau, t_d]] \tag{26}$$

$$n_{i_m t(t+\tau)} = n_{i_m (t-\tau) t} + od_{i_m t(t+\tau)} - b_{k_m i_m t(t+\tau)} \tag{27}$$

In equation (27), $n_{i_m (t-\tau) t}$ is the number of waiting passengers at station i_m at timestamp $t-\tau$; $od_{i_m t(t+\tau)}$ is the passenger arrival rate, which represents the number of arriving passengers in time interval $(t, t+\tau]$; $b_{k_m i_m t(t+\tau)}$ is the passenger boarding rates, which represents the number of boarding passengers in time interval $(t, t+\tau]$.

Considering that passengers can board only when a train is at the station, we obtain $b_{k_m i_m t(t+\tau)} > 0$. Otherwise, when there is no train the station, no passengers can board and we obtain $b_{k_m i_m t(t+\tau)} = 0$.

b: WAITING TIME OF TRANSFER PASSENGERS AT TRANSFER STATIONS

Unlike conventional subway passengers, the purpose of transfer passengers taking the metro is to catch the railway train; therefore, transfer passengers make their trip timetable according to the given railway timetable and this trip timetable is smaller than the planning horizon. We use $(t_o^{\text{transfer}}, t_d^{\text{transfer}}]$ to represent this trip timetable and it satisfies the following constraint (equation (28)). Therefore, we can obtain the waiting time of transfer passengers at metro stations, as shown in equation (29). Equation (31) represents the waiting time of transfer passengers at railway stations (transfer stations) and equation (32) presents the constraint. The equation indicates that transfer passengers traveling on a railway train k_r must arrive at the platform of the transfer station before the railway train leaves. The detailed expansions of equation (30) are shown in Proof. 2 of Appendix A.

$$(t_o^{\text{transfer}}, t_d^{\text{transfer}}] \subset (t_o, t_d]$$
$$t_d^{\text{transfer}} < T_{K_r}^d - T_{K_r}^c - T^{\text{transfer}} \tag{28}$$

$$WT_m^{\text{transfer}} = \sum_{i_m=1}^{I_m} \sum_{t=t_o^{\text{transfer}}}^{t_d^{\text{transfer}}} \tau \cdot n_{i_m t(t+\tau)}^{\text{transfer}} \tag{29}$$

$$n_{i_m t(t+\tau)}^{\text{transfer}} = n_{i_m (t-\tau) t}^{\text{transfer}} + od_{i_m t(t+\tau)}^{\text{transfer}} - b_{k_m i_m t(t+\tau)}^{\text{transfer}} \tag{30}$$

$$WT_r^{\text{transfer}} = \sum_{k_r=1}^{K_r} \sum_{k_m=1}^{K_m} A_{k_m I_m}^{\text{transfer}} (T_{k_r}^d - T_{k_m I_m}^a - T_{k_r}^c - T^{\text{transfer}}) \tag{31}$$

$$T_{k_r}^d - T_{k_m I_m}^a - T_{k_r}^c - T^{\text{transfer}} \geq 0 \tag{32}$$

where t_o^{transfer} is the timestamp of the first transfer passenger arriving at the metro station, s; t_d^{transfer} is the timestamp of the last transfer passenger departing from the metro station, s; $T_{K_r}^d$ is the departure timestamp of the last railway train K_r, s; $T_{K_r}^c$ is the departure timestamp of the last railway train K_r minus the timestamp of stopping ticket checking, s; T^{transfer} is the transfer time, s; $n_{i_m t(t+\tau)}^{\text{transfer}}$ is the number of waiting transfer passengers at timestamp t; $od_{i_m t(t+\tau)}^{\text{transfer}}$ is the transfer passenger arrival rates at station i_m in time interval $(t, t+\tau]$; $b_{k_m i_m t(t+\tau)}^{\text{transfer}}$ is the transfer passenger boarding rate at station i_m in time interval $(t, t+\tau]$; $A_{k_m I_m}^{\text{transfer}}$ is the number of transfer passengers arriving at metro station I_m by train k_m; $T_{k_r}^d$ is the departure timestamp of railway train k_r, s; $T_{k_r}^c$ is the departure timestamp of the last railway train k_r minus the timestamp of stopping ticket checking, s; $T_{k_m I_m}^a$ is the timestamp of the metro train k_m arriving at metro station I_m, s.

D. OBJECTIVE FUNCTIONS AND SYSTEMATIC CONSTRAINTS

The objective of this study is to optimize both the metro timetable and railway timetable to minimize the total waiting time of all passengers and the total waiting time of the transfer passengers at the transfer stations. In addition, timetable scheduling will cause changes in energy consumption; therefore, we also consider energy efficiency in the optimization process. We propose a multi-objective optimization model (MOOM) to minimize energy consumption and passenger waiting time, as shown in equations (33∼35). It consists of three objective functions, the three objective functions proposed in this paper are not isolated, they are actually related, and their connection lies in the decision-making factors of the timetable: the headway between trains, the running time in each section and the dwell time at stations. These objectives can be optimized at the same time, and the different needs of decision-makers determine the different optimization results (Three types of optimal timetables in this paper).

Equation (33) represents the total energy consumption of the trains (TEC), which is equal to the total energy consumption of metro trains in the planning horizon $(t_o, t_d]$ plus the total energy consumption of railway trains in the planning horizon $(t_o', t_d']$. Equation (34) represents the total waiting time for all passengers in the metro stations (TWTP). Equation (35) represents the total waiting time for transfer passengers at transfer stations (TWTTP).

$$\begin{aligned} Model1: E^{\text{total}} &= E_m^{\text{total}}(t_0, t_d) + E_r^{\text{total}}(t_0', t_d') \\ &= \sum_{k_m=1}^{K_m} \sum_{s_m=1}^{S_m} \sum_{p_m=1}^{P_m} \int_0^{l_{s_m p_m}} f_m(v) \cdot ldl \\ &\quad - \sum_{k_m=1}^{K_m} \sum_{k_m+1}^{K_m} \sum_{s_m=1}^{S_m} \sum_{p_m=1}^{P_m} \int_0^{l_{k s_m p_m}} \\ &\quad \times [(b_m(v) \cdot ldl) \cdot \eta_{s_m} \cdot \xi] \\ &\quad + \sum_{k_r=1}^{K_r} \sum_{s_r=1}^{S_r} \sum_{p_r=1}^{P_r} \int_0^{l_{k_T s_T p_r}} \theta_1 f_r(v) \cdot ldl \end{aligned} \tag{33}$$

$$Model2: WT_m^{\text{total}} = \sum_{i_m=1}^{I_m} \sum_{t=t_o}^{t_d} \tau \cdot n_{i_m t(t+\tau)} \tag{34}$$

$$\begin{aligned} Model3: WT_r^{\text{transfer}} &= \sum_{k_r=1}^{K_r} \sum_{k_m=1}^{K_m} A_{k_m I_m}^{\text{transfer}} (T_{k_r}^d - T_{k_m I_m}^a \\ &\quad - T_{k_r}^c - T^{\text{transfer}}) \end{aligned} \tag{35}$$

Constraints of Model 1:

$$\text{s.t.} \quad \text{equation}(2 \sim 6, 12 \sim 16, 25) \tag{36}$$

$$\sum_{k_m=1}^{K_m-1} H_{k_m(k_m+1)} + \sum_{s_m=1}^{S_m} \sum_{p_m=1}^{P_m} t_{K_m s_m p_m} + \sum_{i_m=1}^{I_m-1} D_{K_m i_m} = t_d - t_o \tag{37}$$

$$D_{i_m}^{\min} \le D_{k_m i_m} \le D_{i_m}^{\max} k_m \in K_m, i_m \in I_m \quad (38)$$
$$H^{\min} \le H_{k_m(k_m+1)} \le H^{\max} k_m \in K_m$$
$$T_{K_r}^a - T_1^d = t_d' - t_o' \quad (39)$$

Constraints of Model 2:

$$\text{s.t.} \quad \text{equation}(20 \sim 24, 37, 38) \quad (40)$$
$$0 \le b_{k_m i_m t(t+\tau)} \le b_{k_m i_m t(t+\tau)}^{\max} k_m \in K_m, i_m \in I_m, t \in (t_o, t_d] \quad (41)$$
$$0 \le Q_{k_m i_m} \le Q_{k_m i_m}^{\max} k_m \in K_m, i_m \in I_m \quad (42)$$

Constraints of Model 3:

$$\text{s.t.} \quad \text{equation}(20 \sim 24, 28, 32) \quad (43)$$
$$\sum_{k_m=k_o^t}^{k_d^t} H_{k_m(k_m+1)} + \sum_{s_m=1}^{S_m} \sum_{p_m=1}^{P_m} t_{k_d^t s_m p_m} + \sum_{i_m=1}^{I_m-1} D_{k_d^t i_m} = k_d^{\text{transfer}} - k_o^{\text{transfer}} \quad (44)$$
$$D_{i_m}^{\min} \le D_{k_m i_m} \le D_{i_m}^{\max} k_m \in [k_o^t, k_o^t+1, \ldots k_d^t], i_m \in I_m$$
$$H^{\min} \le H_{k_m(k_m+1)} \le H^{\max} k_m \in [k_o^t, k_o^t+1, \ldots k_d^t] \quad (45)$$

Key constraints of three models:

$$od_{i_m t t'''} = \begin{cases} (t'''-t) od_{i_m t(t+\tau)} (t, t'''] \subseteq (t_i^o, t_i^d] \\ \qquad (t_i^o, t_i^d] \subset (t_o, t_d] \\ 0 \quad \text{else} \end{cases} \quad (46)$$
$$t_{i_m}^d - t_{i_m}^o = \sum_{k_m=1}^{K_m-1} H_{k_m(k_m+1)} i_m \in I_m$$
$$od_{i_m t t'''}^{\text{transfer}} = \begin{cases} \sum_{t=0}^{t'''} od_{i_m t(t+\tau)}^{\text{transfer}} (t, t'''] \subseteq (t_o^{\text{transfer}}, t_d^{\text{transfer}}] \\ 0 \quad \text{else} \end{cases}$$
$$(t_o^{\text{transfer}}, t_d^{\text{transfer}}] \subset (t_o, t_d] \quad (47)$$

Equations (2) and (12) are the dynamical constraints of the trains and describe the maximum values of the traction and braking force. Equations (3∼6) and (13∼16) are kinematic constraints. Equation (25) describes the relationship between total mass and passenger capacity during train operation. Equations (37) and (39) define the duration of the planning horizon of $(t_o, t_d]$ and $(t_o', t_d']$; $T_{K_r}^a$ is the timestamp of the last railway train $K_r$arriving at the terminal station, T_1^d is the departure timestamp of the first railway train. Equation (38) defines the upper and lower bounds of the dwell time and following headway of the metro train. Equations (20∼24) are the calculations for the model of $Q_{k_m i_m}$, which represents the passenger volume of train $k_m$departing from station i_m. Equation (41) provides the maximum number of passengers who board the train per unit time. Equation (42) represents the upper bound of passengers on the metro train. Equation (28) expresses the relation between $(t_o, t_d]$ and $(t_o^{\text{transfer}}, t_d^{\text{transfer}}]$. Equation (32) is the critical condition for transfer passengers to complete the transfer from the metro to the railway to ensure that transfer passengers can board the train before the transfer train leaves. Equation (44) defines the duration of the planning horizon of $(t_o^{\text{transfer}}, t_d^{\text{transfer}}]$. Equation (45) represents the upper and lower bounds of the dwell time and following headway of the metro train with transfer passengers on board, k_o^t is the first metro train with transfer passengers on board and k_d^t is the last metro train with transfer passengers on board. Equation (46) gives the total number of metro passengers to be transported in the planning horizon and $(t_{i_m}^o, t_{i_m}^d]$ indicates that passengers arriving at station i_m should be transported by metro trains during the time period. To ensure that the total number of metro passengers transported by different optimized timetables in the planning horizon $(t_o, t_d]$ is the same, it is necessary to maintain the same length of time period $(t_i^o, t_i^d]$ for each metro station. Equation (47) ensures that all transfer passengers can be transported within a given planning horizon $(t_o^{\text{transfer}}, t_d^{\text{transfer}}]$.

IV. SOLUTION APPROACHES

A. THE SELECTION OF THE OBJECTIVE FUNCTIONS AND WEIGHT FACTORS

The proposed multi-objective programming model consists of three objective functions, i.e., E^{total}, WT_m^{total}, and WT_r^{transfer}. Different objective functions represent different optimizing objectives, e.g., E^{total} represents the total energy consumption of the trains (TEC), WT_m^{total} represents the total waiting time for all passengers in the metro stations (TWTP), and WT_r^{transfer} represents the total waiting time for transfer passengers at the transfer stations (TWTTP). We use different weight factors to express different preference demands of decision-makers and we can obtain different optimization results. In order to discuss the different optimization results of the different objective functions, we define E-timetable, WT-timetable, and TWT-timetable.

1. *E-timetable*: This is the energy-saving timetable; it represents the optimized results of the three objective functions with priority $E^{\text{total}} > WT_m^{\text{total}} > WT_r^{\text{transfer}}$ in the optimization process.

2. *WT-timetable*: This is the passenger waiting time timetable; it represents the optimized results of the three objective functions with priority $WT_m^{\text{total}} > WT_r^{\text{transfer}} > E^{\text{total}}$ in the optimization process.

3. *TWT-timetable*: This is the transfer passenger waiting time timetable; it represents the optimized results of the three objective functions with $WT_r^{\text{transfer}} > WT_m^{\text{total}} > E^{\text{total}}$ in the optimization process.

According to the research results of Li et al. (2018) [20], we have analyzed that when the weight ratio of energy to waiting time exceeds 10, we get the optimal solution which is beneficial to reduce energy consumption. When the weight ratio of the waiting time to energy exceeds 1, we get the optimal solution which is beneficial to reduce the waiting time. So, if decision-makers prefer to improve energy efficiency, the weight factors are $\lambda_1 = 0.9$, $\lambda_2 = 0.09$, $\lambda_3 = 0.01$ and the optimized timetable is called the E-timetable; if decision-makers prefer to improve the quality of passenger service, the weight factors are $\lambda_1 = 0.01, \lambda_2 = 0.9, \lambda_3 = 0.09$ and the optimized timetable is called the WT-timetable; if the preference of the decision-makers is to improve transfer efficiency, the weight factors are $\lambda_1 = 0.01, \lambda_2 = 0.09$, $\lambda_3 = 0.9$, and the optimized timetable is called the TWT-timetable.

B. FUZZY MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

The fuzzy multi-objective optimization algorithm models each target as a fuzzy set, and its membership function represents the satisfaction degree of the target. It is assumed that the membership increases linearly from 0 (for the least satisfactory value) to 1 (for the most satisfactory value). It is an efficient approach to solve multi-objective optimization problems and was first used by Zimmermann in 1978 [29] to aggregate fuzzy objectives to reach a compromise decision. Li et al. (2013) [30] proposed a multi-objective train scheduling model by minimizing the energy and carbon emission cost as well as the total passenger-time. Then, they adopt a fuzzy multi-objective optimization algorithm to solve the model. Yang et al. (2007) [31] proposed the location optimization model and aimed to determine the optimal location of fire station facilities. Then, a fuzzy multi-objective programming was adopted to solve this model. It can be seen that the fuzzy multi-objective optimization algorithm is a simple and effective algorithm for solving multi-objective programming, and from the existing references, the fuzzy multi-objective optimization algorithm has strong practical value. Therefore, this paper uses the fuzzy multi-objective optimization algorithm to solve the optimization model. The steps of the fuzzy multi-objective optimization algorithm are listed as follows.

Step 1. It is easy to calculate the range for each objective base for single-objective optimization methods, e.g., genetic algorithm, exterior point method, etc. In this study, we use $\underline{E}^{\text{total}}$ and $\bar{E}^{\text{total}}$ to denote the minimum and maximum TEC, $\underline{WT}_{\text{m}}^{\text{total}}$ and $\bar{WT}_{\text{m}}^{\text{total}}$ to denote the minimum and maximum TWTP, and $\underline{WT}_{\text{r}}^{\text{transfer}}$ and $\bar{WT}_{\text{r}}^{\text{transfer}}$ to denote the minimum and maximum TWTTP.

Step 2. We create the minimized ideal value vector $G^{\min}$ and maximized inverse ideal value vector $G^{\max}$ of the three objective functions E^{total}, $WT_{\text{m}}^{\text{total}}$, and $WT_{\text{r}}^{\text{transfer}}$ while satisfying their respective constraints, as shown in equation (48).

$$\begin{aligned} G^{\min} &= (\underline{E}^{\text{total}}, \underline{WT}_{\text{m}}^{\text{total}}, \underline{WT}_{\text{r}}^{\text{transfer}}) \\ G^{\max} &= (\overline{E}^{\text{total}}, \underline{WT}_{\text{m}}^{\text{total}}, \underline{WT}_{\text{r}}^{\text{transfer}}) \end{aligned} \tag{48}$$

Step 3. We establish the membership functions Ψ_1, Ψ_2, and Ψ_3 of the three objective functions E^{total}, $WT_{\text{m}}^{\text{total}}$, and $WT_{\text{r}}^{\text{transfer}}$, as shown in equations (49), (50), and (51).

$$\Psi_1 = \begin{cases} 1 & E^{\text{total}} \le \underline{E}^{\text{total}} \\ \dfrac{\bar{E}^{\text{total}} - E^{\text{total}}}{\bar{E}^{\text{total}} - \underline{E}^{\text{total}}} & \underline{E}^{\text{total}} < E^{\text{total}} < \bar{E}^{\text{total}} \\ 0 & E^{\text{total}} \ge \bar{E}^{\text{total}} \end{cases} \tag{49}$$

$$\Psi_2 = \begin{cases} 1 & WT_{\text{m}}^{\text{total}} \le \underline{WT}_{\text{m}}^{\text{total}} \\ \dfrac{\bar{WT}_{\text{m}}^{\text{total}} - WT_{\text{m}}^{\text{total}}}{\bar{WT}_{\text{m}}^{\text{total}} - \underline{WT}_{\text{m}}^{\text{total}}} & \underline{WT}_{\text{m}}^{\text{total}} < WT_{\text{m}}^{\text{total}} < \bar{WT}_{\text{m}}^{\text{total}} \\ 0 & WT_{\text{m}}^{\text{total}} \ge \bar{WT}_{\text{m}}^{\text{total}} \end{cases} \tag{50}$$

$$\Psi_3 = \begin{cases} 1 & WT_{\text{r}}^{\text{transfer}} \le \underline{WT}_{\text{r}}^{\text{transfer}} \\ \dfrac{\bar{WT}_{\text{r}}^{\text{transfer}} - WT_{\text{r}}^{\text{transfer}}}{\bar{WT}_{\text{r}}^{\text{transfer}} - \underline{WT}_{\text{r}}^{\text{transfer}}} & \underline{WT}_{\text{r}}^{\text{transfer}} < WT_{\text{r}}^{\text{transfer}} < \bar{WT}_{\text{r}}^{\text{transfer}} \\ 0 & WT_{\text{m}}^{\text{total}} \ge \bar{WT}_{\text{r}}^{\text{transfer}} \end{cases} \tag{51}$$

Step 4. We transform the multi-objective optimization problem into the following single-objective optimization problem, as shown in Equation (52). $\lambda_i (i = 1, 2, 3) \ge 0$ are the weight factors of the original objective functions E^{total}, $WT_{\text{m}}^{\text{total}}$, and $WT_{\text{r}}^{\text{transfer}}$, and they satisfy $\sum_{i=1}^{3}$; p is the distance parameter.

$$\Delta = \max[(\lambda_1\psi_1)^p + (\lambda_2\psi_2)^p + (\lambda_3\psi_3)^p]^{1/p} \tag{52}$$

Step 5. We select the appropriate weight factors $\lambda_i (i = 1, 2, 3)$ of the original objective functions E^{total}, $WT_{\text{m}}^{\text{total}}$, and $WT_{\text{r}}^{\text{transfer}}$ to represent different decision preferences to obtain different optimization timetables. Furthermore, we select the distance models by changing the value of p. The Manhattan distance model [32] is used and $p = 1$.

Step 6. We use the nonlinear programming solvers of MATLAB with a notebook computer (CPU: Intel (R) Core (TM) i5-6100@3.7GHz; 8GB memory) to solve the single-objective model, which is shown in equation (52).

In summary, the solution process is shown in Figure 10.

V. NUMERICAL EXPERIMENTS

In this section, we conduct a real-world case study using the east line of the Chengdu Metro line 2 to the Xipu-Qingchengshan fast railway line, as shown in Figure 11. This represents a typical transfer system connecting a metro line and railway line; it consists of three parts, i.e., the metro line, transfer coridor, and railway line.

A. DATA SETTINGS FOR THE DYNAMIC PASSENGER FLOW DEMAND

In practice, our metro passenger flow data are derived from historical operation data, including dynamic passenger flow demand data and train operation schedule data. For example, our metro passenger flow data is derived from the Automatic Fare Collection (AFC) data provided by the Chengdu Metro Company and the railway passenger flow data are obtained from statistics provided by the Chengdu Railway Administration Passenger Transport Center. Because the railway timetable is influenced by many factors, it cannot be easily changed; therefore, the planning time horizons of the metro should be determined according to the railway timetable of the Xipu-Qingchengshan fast railway line and the actual

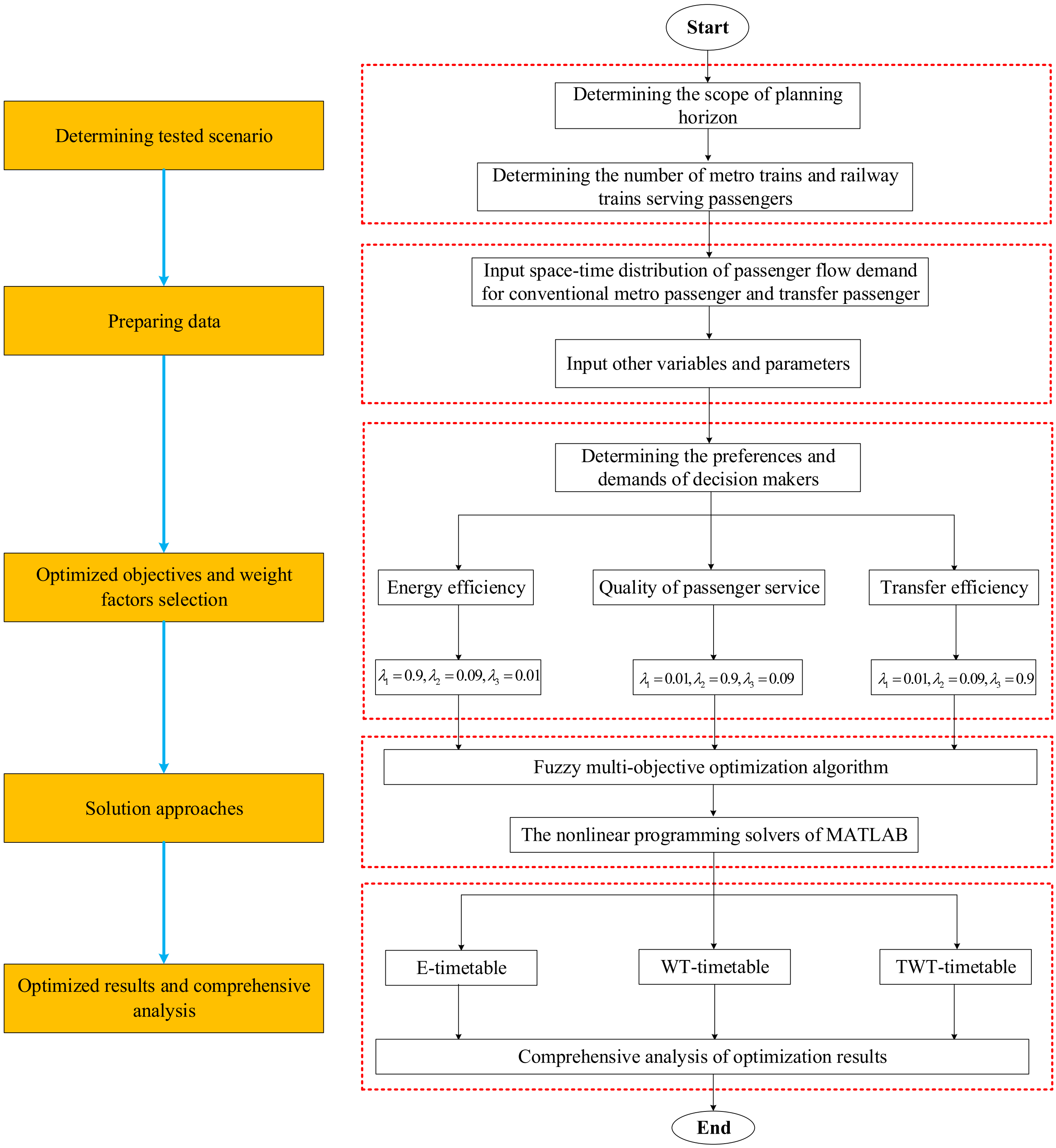


FIGURE 10. The solution process of the optimization model.

travel time range of passengers. In addition, in order to ensure that the case study is representative, we investigate two practical cases to validate the MOOM model; the planning time horizons for the two cases are the non-rush hours and the rush hours of the metro, respectively, as shown in Table 4.

Figure 12 shows the origin-destination (OD) passenger flow demand of Metro line 2 during the non-rush hours (Figure 12(a)) and during the rush hours (Figure 12(b)). Figure 13 presents the average passenger arrival rates at the Metro stations in different planning time horizons.

FIGURE 11. Illustration of the east line of the Chengdu Metro line 2 to Xipu-Qingchengshan fast railway line (Source: Baidu map).

TABLE 4. Planning time horizons settings.

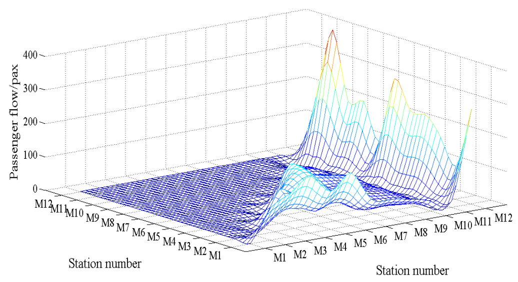
	Planning time horizons	
	$(t_o, t_d]$	$(t_o^{\text{transfer}}, t_d^{\text{transfer}}]$
Case A	(10:20,11:30] in the non-rush hours	(10:20,11:21]
Case B	(17:40,18:50] in the rush hours	(17:40,18:40]

We note that, since the east line of the Chengdu Metro line 2 is only a part of the Chengdu Metro line 2, there should be a certain number of passengers on the train when it arrives at the station M1. We assume that the number of passengers $Q_{k_m i_m}(i_m = 0)$ already on the train is 800 during non-rush hours and 1000 during rush hours. In addition, we assume that the terminal for this group of passengers is station M12; the existence of this group of passengers does not affect the calculation of the passenger waiting time and only affects the total mass of the train. Therefore, the calculated total mass of the train is more in line with the actual situation.

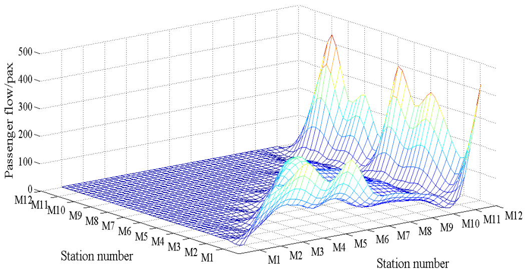
As passengers transfer from the Xipu Metro station (terminal station of Metro line 2) to the Xipu Express railway station (initial Station of the Xipu-Qingchengshan fast railway line) using the same platform, which is shown in Figure 14, the transfer passengers can enter the railway platform only by passing through two gates after getting off the metro train. Therefore, the transfer time for transfer passengers will be very short. Figure 15 shows the transfer passenger arrival rates at the Metro stations during non-rush hours (Figure 15(a)) and during rush hours (Figure 15(b)).

B. TIMETABLE DATA AND PARAMETER SETTINGS

The east line of the Chengdu Metro line 2 connects the Metro station of Zhongyida with the Xipu Metro station. The length of the line is 14.947 kilometers. There are 12 stations and 11 sections in this line. The basic line network data and the current timetable information are shown in Table 5. The Metro line 2 operates from 6:10. to 22:45, the rush hours are (7:30, 9:30] and (17:00, 19:00], and the other periods are non-rush hours. The current following headway of the train during non-rush hours is 410 s and that during rush hours is 280 s. Figure 16 shows the traction, basic resistance, and braking force of the metro train. The Xipu-Qingchengshan Express Railway Line consists of three main stations: Xipu, Dujiangyan, and Qingchengshan. The length of the line is 50 kilometers. The train information and timetable data in the planning time horizons are shown in Table 6. Figure 17 gives



(a) Origin-destination passengers flow demand of Metro line 2 from 10:20 to 11:30



(b) Origin-destination passengers flow demand of Metro line 2 from 17:40 to 18:50

FIGURE 12. Origin-destination passenger flow demands of Metro line 2 in different planning time horizons.

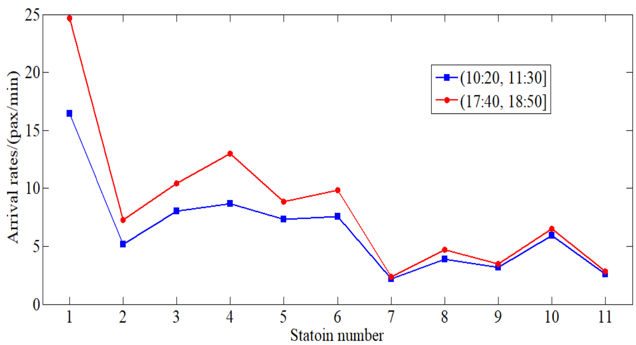


FIGURE 13. Average passenger arrival rates at Metro stations in different planning time horizons.

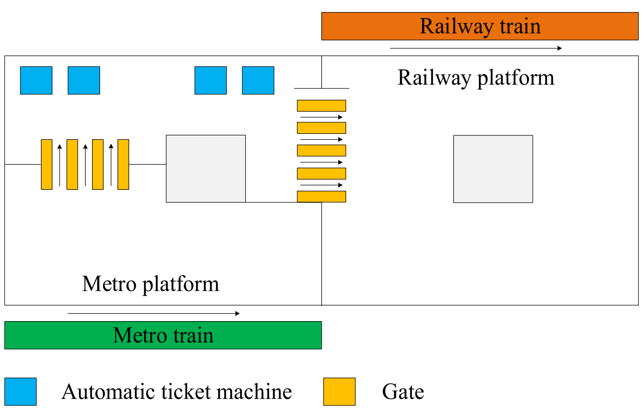
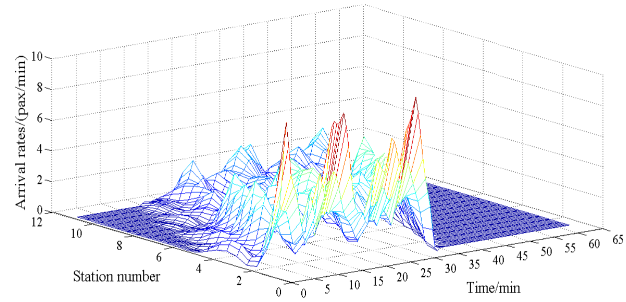
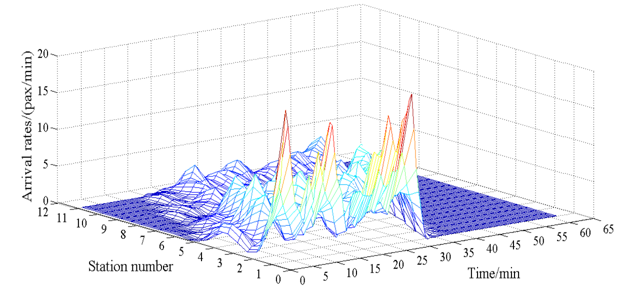


FIGURE 14. Illustration of passengers transferring from Xipu Metro station to Xipu Express railway station.



(a) Transfer passenger arrival rates at Metro stations from 10:20 to 11:22



(b) Transfer passenger arrival rates at Metro stations from 17:40 to 18:41

FIGURE 15. Transfer passenger arrival rates at Metro stations in different planning time horizons.

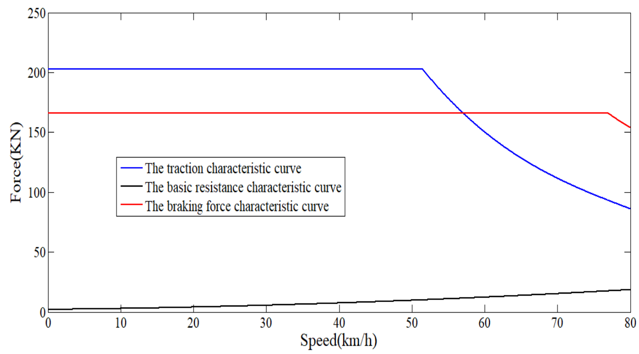


FIGURE 16. The traction, basic resistance, and braking force of the metro train.

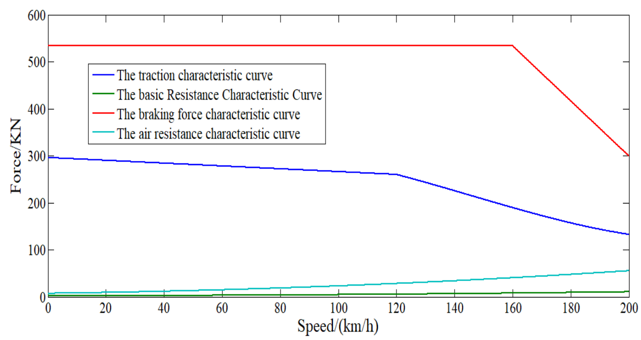


FIGURE 17. The traction, basic resistance, air resistance, and braking force of the railway train.

the traction, basic resistance, air resistance, and braking force of the railway train. The values of the other main parameters are shown in Table 7.

C. COMPUTATIONAL RESULTS

We use the main variables of the optimized timetables in the different cases to compare the performance of the timetables, i.e., the energy consumption (EC), number of

TABLE 5. The basic data of the east line of metro line 2.

Station ID	Station name	Length /m	Running time/s Current	Maximum	Minimum	Dwell time/s Current	Maximum	Minimum
M1	Zhongyida	1345	91	109	82	30	40	20
M2	Baiguolin	1161	78	94	70	25	40	20
M3	Shuhanlu dong	1170	81	97	73	25	40	20
M4	Yiping tianxia	900	67	80	60	30	40	20
M5	Yangxi lijiao	1540	113	136	102	25	40	20
M6	Cha dianzi	992	72	86	65	30	40	20
M7	Yingbin dadao	1113	83	100	75	25	40	20
M8	Jinke beilu	1110	81	97	73	25	40	20
M9	Jinzhou lu	2190	143	172	129	25	40	20
M10	Baicao lu	1766	121	145	109	25	40	20
M11	Tianhe lu	1660	107	128	96	25	40	20
M12	Xipu					30	40	20

TABLE 6. The train information and timetable data of xipu-qingchengshan express railway line.

Train index	Xipu Departure time	Dujiangyan Arrival time	Dujiangyan Departure time	Qingchengshan Arrival time	Planning time horizons $(t'_o, t'_d]$
Railway 1	10:54	11:13	11:16	11:24	
Railway 2	11:07	11:31	11:33	11:41	(10:54, 11:56]
Railway 3	11:22	11:46	11:48	11:56	
Railway 1	18:12	18:32	18:36	18:44	
Railway 2	18:23	18:47	18:49	18:57	(18:12, 19:11]
Railway 3	18:41	19:01	19:03	19:11	
Maximum speed/(km/h)	200		120		
Length/km	42		8		

passengers transported (NPT), and number of transfer passengers transported (NTPT). Table 8 presents the results for the E-timetable, WT-timetable, and TWT-timetable for Case A and Table 9 presents the results for the timetables for Case B.

The results in the two tables indicate that an appropriate following headway, running time in the section, and dwell time at the station results in a higher utilization rate of the regenerative braking energy of the E–timetable; this translates into a reduction in total energy consumption and indicates that the E-timetable is the most energy-efficient timetable. Since the conventional metro passengers are defined as non-dynamic passenger flow in this study, the NPT mainly depends on the arrival rate of passengers and the following headway because most metro passengers are conventional metro passengers. In order to reduce the total waiting time of all metro passengers, the WT-timetable ensures that the number of metro passengers on each train is equal by adjusting the following headway of the adjacent trains. The NTPT values of the TWT-timetables in Tables 8 and 9 indicate that the NTPT by the metro train that is most closely connected to the railway train should theoretically be highest, such as the NTPT values of Metros 2 and 4 in Table 8 and Metro 2, 5, and 9 in Table 9. Our guess is that this optimization results may make the waiting time of most transfer passengers equal to 0, thus reducing the total waiting time of all transfer passengers.

To illustrate the results, we present the E-timetable, WT-timetable, and TWT-timetable of Case A in Figs. 18, 19, and 20 and the timetables of Case B in Figs. 21, 22, and 23, respectively. It is evident that the E-timetables of Case A and Case B (Figs. 18 and 21) are not the most energy-saving solutions because the E-timetable must meet the requirement of the constraint of equation (46) and ensure that the total number of metro passengers transported by different optimized timetables is the same in the planning horizons. In addition, this E-timetable also ensures that all transfer passengers arrive at the designated railway platform before

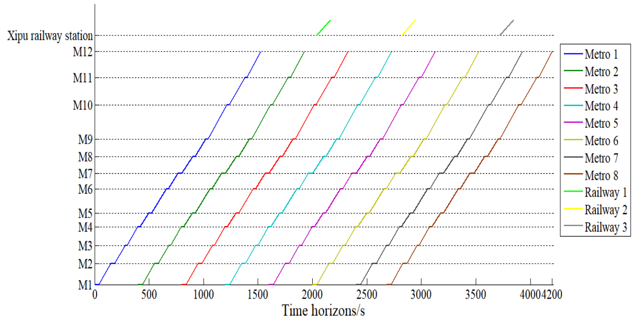


FIGURE 18. The E-timetable for Case A.

TABLE 7. The values of other main parameters.

Parameters	Symbol	Value
Maximum running velocity in section of metro train, m/s	$v_{\mathrm{m}}^{\max}$	80
The regenerative braking energy conversion rate	ξ	0.9
The lower bound of the following headway between the metro trains, s	$H^{\min}$	180
The upper bound of the following headway between the metro trains, s	$H^{\max}$	460
The mass of the empty metro train, kg	w_{m}	196000
The mass of the empty railway train, kg	w_{r}	479360
Maximum passenger volume per metro train	$Q_{\mathrm{m}}^{\max}$	1468
Maximum passenger volume per railway train	$Q_{\mathrm{r}}^{\max}$	601
Maximum acceleration of a metro train, m/s^2	$\hat{a}_{\mathrm{m}}^{\mathrm{acc}}$	1
Maximum deceleration of a metro train, m/s^2	$\hat{a}_{\mathrm{m}}^{\mathrm{dec}}$	1
Maximum acceleration of a railway train, m/s^2	$\hat{a}_{\mathrm{r}}^{\mathrm{acc}}$	0.5
Maximum deceleration of a railway train, m/s^2	$\hat{a}_{\mathrm{r}}^{\mathrm{dec}}$	1
Maximum traction force of a metro train, kN	$F_{\mathrm{m}}^{\max}$	203
Maximum braking force of a metro train, kN	$B_{\mathrm{m}}^{\max}$	166
Maximum traction force of a railway train, kN	$F_{\mathrm{r}}^{\max}$	296
Maximum braking force of a railway train, kN	$B_{\mathrm{r}}^{\max}$	534
Maximum number of passengers boarding in unit time, (person/min)	$b_{k_{\mathrm{m}} i_{\mathrm{m}} t(t+\tau)}^{\max}$	300
The numbers of waiting passengers at station i_{m} at timestamp t_i^o	$n_{i_{\mathrm{m}} t_i^o}$	0
Transfer time, s	T^{transfer}	20
The departure timestamp of last railway train minus the timestamp of stopping ticket checking, s	$T_{K_{\mathrm{r}}}^{c}$	30

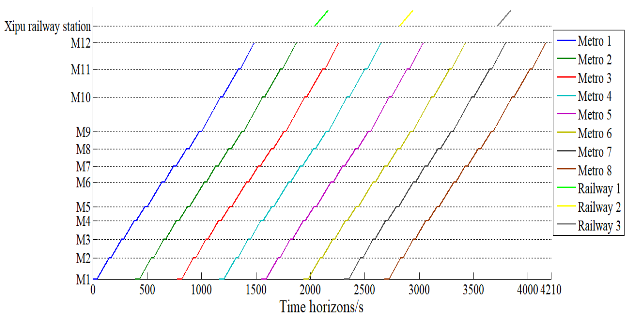


FIGURE 19. The WT-timetable for Case A.

the departure of the railway train they transfer to (satisfying the constraint of equation (47)). Therefore, the E-timetable is only the most energy-saving timetable for the optimization model and the constraints proposed in this study. The WT-timetables of Case A and Case B are shown in Figs. 19 and 22. It is apparent that the following headways of the adjacent trains are basically the same. We assume this

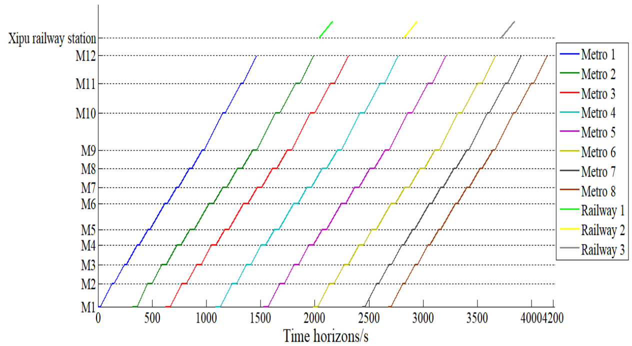


FIGURE 20. The TWT-timetable for Case A.

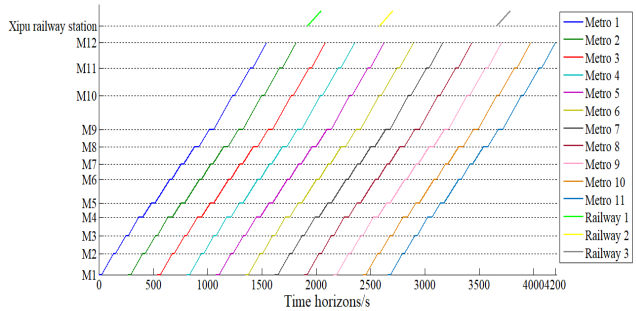


FIGURE 21. The E-timetable for Case B.

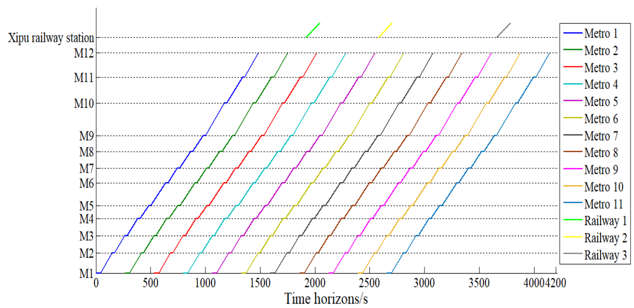


FIGURE 22. The WT-timetable for Case B.

occurred because the conventional metro passenger flow is assumed to be non-dynamic; as a result, the waiting numbers of the metro passengers at the station are determined by the arrival rate of the metro passengers at the station and the following headway. Therefore, in order to minimize the total waiting time of all passengers after meeting the constraints of equations (46) and (47), the following headway of any adjacent trains should be equal. Figures 20 and 23 display the TWT-timetables of Case A and Case B; it is evident that in Figure 20, there are 6 trains (Metro 1∼6) to transport transfer passengers to the transfer station and there are 9 trains (Metro 1∼9) to transport transfer passengers to the transfer station in Figure 23. This ensures that all transfer passengers arrive at the transfer platform before the departure of the railway train they transfer to. In addition, each TWT-timetable has 3 metro trains that are connected with 3 railway trains to achieve a perfect connection (the waiting time of transfer

TABLE 8. Comparisons of calculation results of different optimized timetables for Case A.

Train index	E-timetable			WT-timetable			TWT-timetable		
	EC/KJ	NPT	NTPT	EC/KJ	NPT	NTPT	EC/KJ	NPT	NTPT
Metro 1	$2.44{\cdot}10^5$	66	44	$2.81{\cdot}10^5$	64	40	$2.63{\cdot}10^5$	64	35
Metro 2	$2.44{\cdot}10^5$	473	123	$2.81{\cdot}10^5$	459	127	$2.98{\cdot}10^5$	476	132
Metro 3	$2.44{\cdot}10^5$	473	209	$2.81{\cdot}10^5$	459	183	$2.86{\cdot}10^5$	375	144
Metro 4	$2.44{\cdot}10^5$	473	101	$2.81{\cdot}10^5$	459	127	$2.85{\cdot}10^5$	544	166
Metro 5	$2.44{\cdot}10^5$	473	225	$2.81{\cdot}10^5$	459	196	$2.86{\cdot}10^5$	521	209
Metro 6	$2.44{\cdot}10^5$	473	149	$2.81{\cdot}10^5$	459	178	$2.86{\cdot}10^5$	544	165
Metro 7	$2.24{\cdot}10^5$	473	0	$2.84{\cdot}10^5$	438	0	$2.87{\cdot}10^5$	427	0
Metro 8	$2.98{\cdot}10^5$	331	0	$2.98{\cdot}10^5$	438	0	$2.98{\cdot}10^5$	284	0
Railway 1	$4.97{\cdot}10^5$	481	167	$4.97{\cdot}10^5$	481	167	$4.97{\cdot}10^5$	481	167
Railway 2	$3.29{\cdot}10^5$	481	310	$3.29{\cdot}10^5$	481	310	$3.29{\cdot}10^5$	481	310
Railway 3	$3.29{\cdot}10^5$	481	374	$3.29{\cdot}10^5$	481	374	$3.29{\cdot}10^5$	481	374

TABLE 9. Comparisons of calculation results of different optimization timetables for Case B.

Train index	E-timetable			WT-timetable			TWT-timetable		
	EC/KJ	NPT/s	NTPT/s	EC/KJ	NPT/s	NTPT/s	EC/KJ	NPT/s	NTPT/s
Metro 1	$2.40{\cdot}10^5$	118	85	$3.06{\cdot}10^5$	116	87	$2.95{\cdot}10^5$	116	70
Metro 2	$2.40{\cdot}10^5$	423	164	$3.06{\cdot}10^5$	416	162	$2.55{\cdot}10^5$	433	179
Metro 3	$2.40{\cdot}10^5$	423	205	$3.06{\cdot}10^5$	416	198	$2.55{\cdot}10^5$	282	127
Metro 4	$2.40{\cdot}10^5$	423	215	$3.06{\cdot}10^5$	416	222	$3.13{\cdot}10^5$	282	127
Metro 5	$2.40{\cdot}10^5$	423	108	$3.06{\cdot}10^5$	416	93	$2.55{\cdot}10^5$	464	166
Metro 6	$2.40{\cdot}10^5$	423	180	$3.06{\cdot}10^5$	416	190	$2.55{\cdot}10^5$	282	120
Metro 7	$2.40{\cdot}10^5$	423	238	$3.06{\cdot}10^5$	416	195	$3.01{\cdot}10^5$	282	111
Metro 8	$2.40{\cdot}10^5$	423	193	$3.06{\cdot}10^5$	416	240	$3.00{\cdot}10^5$	407	194
Metro 9	$2.40{\cdot}10^5$	423	0	$3.06{\cdot}10^5$	416	0	$3.02{\cdot}10^5$	720	294
Metro 10	$3.13{\cdot}10^5$	423	0	$3.06{\cdot}10^5$	416	0	$3.02{\cdot}10^5$	571	0
Metro 11	$3.13{\cdot}10^5$	355	0	$3.13{\cdot}10^5$	416	0	$3.13{\cdot}10^5$	437	0
Railway 1	$4.51{\cdot}10^5$	481	249	$4.51{\cdot}10^5$	481	249	$4.51{\cdot}10^5$	481	249
Railway 2	$3.29{\cdot}10^5$	481	420	$3.29{\cdot}10^5$	481	420	$3.29{\cdot}10^5$	481	420
Railway 3	$4.51{\cdot}10^5$	481	719	$4.51{\cdot}10^5$	481	719	$4.51{\cdot}10^5$	481	719

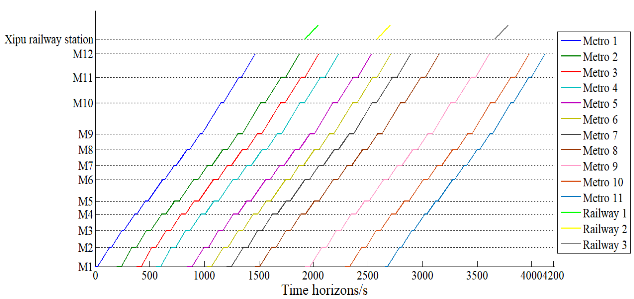


FIGURE 23. The TWT-timetable for Case B.

passengers at the transfer station is equal to 0); this ensures that the total waiting time of all transfer passengers at the transfer station is minimized.

D. COMPREHENSIVE ANALYSIS OF OPTIMIZATION RESULTS

Table 10 lists the performance results of the E-timetable, WT-timetable, and TWT-timetable for the Case A and Case B. In order to satisfy the key constraints (equations (46) and (47)), the values of Q_m and $Q_\text{m}^\text{transfer}$ are the same for the three optimized timetables for the different scenarios, for example, $Q_\text{m} = 3235$, $Q_\text{m}^\text{transfer} = 851$ in Case A and $Q_\text{m} = 4276$, $Q_\text{m}^\text{transfer} = 1388$ in Case B.

In case A, the TEC is lowest for the E-timetable and is 8.18% and 8.71% lower than the TEC of the WT-timetable and TWT-timetable, respectively. The TWTP is lowest for the WT-timetable and is 1.3% and 4.10% lower than the TWTP of the E-timetable and TWT-timetable respectively. The TWTTP is lowest for the TWT-timetable and is 33.51% and 42.13% lower than the TWTTP of the E-timetable and WT-timetable respectively.

In case B, the TEC is lowest for the E-timetable and is 12.63% and 8.20% lower than the TEC of the WT-timetable and TWT-timetable respectively. The TWTP is lowest for the WT-timetable and is 0.71% and 10.56% lower than the TWTP of the E-timetable and TWT-timetable. The TWTTP is lowest for the TWT-timetable and is by 25.61% and 35.75% lower than the TWTTP of the E-timetable and WT-timetable respectively.

It is evident that the difference in the TEC of the three optimized timetables is small for the two scenarios. The reason is that the TEC value calculated in this study is the total energy consumption of all trains in the planning horizons, including the total energy consumption of the metro trains

TABLE 10. Performance comparison of E-Timetable, WT-Timetable and TWT-Timetable for different cases.

Case index	Timetable types	Δ	Optimization objectives			CPU time/min
			TEC (KJ)	TWTP (s)	TWTTP (s)	
Case A 10:20~11:30 K_m=8 K_r=3 Q_m=3235 $Q_m^{transfer}$=851	E-timetable	$2.887\cdot10^6$	$3.143\cdot10^6$	$6.211\cdot10^5$	$2.716\cdot10^5$	<5
	WT-timetable	$6.142\cdot10^5$	$3.423\cdot10^6$	$6.132\cdot10^5$	$3.121\cdot10^5$	<4
	TWT-timetable	$2.545\cdot10^5$	$3.443\cdot10^6$	$6.394\cdot10^5$	$1.806\cdot10^5$	<5
Case B 17:40~18:50 K_m=11 K_r=3 Q_m=4276 $Q_m^{transfer}$=1388	E-timetable	$3.673\cdot10^6$	$4.019\cdot10^6$	$5.639\cdot10^5$	$5.428\cdot10^5$	<9
	WT-timetable	$6.065\cdot10^5$	$4.600\cdot10^6$	$5.599\cdot10^5$	$6.285\cdot10^5$	<7
	TWT-timetable	$4.635\cdot10^5$	$4.378\cdot10^6$	$6.260\cdot10^5$	$4.038\cdot10^5$	<8

and railway trains. However, our E-timetable only aims at the optimization of the Metro timetable and considering that the preparation of a railway timetable is affected by many factors, we did not optimize the railway timetable; therefore, the total energy consumption of the railway trains is the same for the three optimized timetables and only the total energy consumption values of the metro trains are different.

Furthermore, the deviation of the TWTP of the three optimized timetables is small in the two scenarios. We believe the reason is that the conventional metro passenger flow is assumed to be non-dynamic and the Q_m values of the three optimized timetables are the same. Therefore, when the timetable changes, the impact on the TWTP value is relatively small.

In contrast to the TEC and TWTP, the difference in the TWTTP values for the three optimized timetables is large. The possible reason is that the transfer passenger flow is assumed to be a dynamic passenger flow, which means that there is a spatiotemporal aspect to the transfer passenger flow. Therefore, when the following headway, running time, and dwell time change slightly, the impact on the TWTTP value is rather large.

VI. CONCLUSION

In this study, we investigated the optimization of timetables for the connection of a one-direction metro line and railway line and considered the spatiotemporal distribution of the demand of the dynamic passenger flow to minimize the total waiting time of passengers at stations and energy consumption of train operation. We developed a multi-objective programming model consisting of three objective functions. Considering different preferences of decision-makers, we added different weight factors to the three objective functions and used a fuzzy multi-objective optimization algorithm to solve the multi-objective programming model. Three optimal timetables were obtained, namely, the E-timetable, WT-timetable, and TWT-timetable, which maximized energy efficiency, the quality of passenger service, and transfer efficiency respectively. Two practical cases based on the real-world operational data of the Chengdu Metro line 2 and Xipu-Qingchengshan fast railway line were used to demonstrate the effectiveness of the proposed models and solution approaches. The results showed that the three optimized timetables met the different requirements of the decision-makers and the combined use of three optimized timetables was suitable for guiding actual operations. In summary, we suggest that the Chengdu Metro uses the TWT-timetable to schedule train operations during the transfer peak period. In the non-transfer peak period, if the conventional metro passenger flow is large, it is suggested that the Chengdu Metro use the WT-timetable because it shortens the total waiting time of passengers and improves the quality of passenger service. Furthermore, it is suggested that the Chengdu Metro adopt the E-timetable during the low-peak period of passenger travel at night because the passenger travel volume is low and the focus of optimization can be shifted from improving the quality of passenger service to improving energy efficiency.

In the future, our research will focus on the following major aspects. (1) Considering the passenger endurance factor, when the waiting time exceeds a specified time, passengers will give up traveling by RT and take other modes of transportation. (2) We hope to obtain more actual operational data and study transfer behavior in a larger-scale RT network. (3) We plan to investigate and design more advanced algorithms to improve the solution efficiency.

APPENDIX

Proof 1: This proof aims to find an equivalent relationship between $Q_{k_m I_m}$ and $B_{k_m i_m t t''}$. We can list a sequence of equations for any $i_m \in I_m$.

$$\begin{aligned} Q_{k_m 2} &= Q_{k_m 1} + B_{k_m 1 tt''} - A_{k_m 1 tt''} \\ Q_{k_m 3} &= Q_{k_m 2} + B_{k_m 2 tt''} - A_{k_m 2 tt''} \\ &\vdots \\ Q_{k_m I_m} &= Q_{k_m (I_m-1)} + B_{k_m (I_m-1) tt''} - A_{k_m (I_m-1) tt''} \end{aligned}$$

By summing up the sequence of equations, we can get

$$Q_{k_m I_m} = Q_{k_m 1} + \sum_{i_m=1}^{I_m-1} B_{k_m i_m tt''} - \sum_{i_m=1}^{I_m-1} A_{k_m i_m tt''}$$

Proof 2: This proof aims to find an equivalent relationship between $n^{\text{transfer}}_{i_m t_o^{\text{transfer}} t_d^{\text{transfer}}}$ and $od^{\text{transfer}}_{i_m t_o^{\text{transfer}} t_d^{\text{transfer}}}$. We can list a sequence of equations for any $t \in (t_o^{\text{transfer}}, t_d^{\text{transfer}}]$.

$$\begin{aligned} n^{\text{transfer}}_{i_m t_o^{\text{transfer}} (t_o^{\text{transfer}}+\tau)} &= n^{\text{transfer}}_{i_m (t_o^{\text{transfer}}-\tau) t_o^{\text{transfer}}} \\ &\quad + od^{\text{transfer}}_{i_m t_o^{\text{transfer}} (t_o^{\text{transfer}}+\tau)} \\ &\quad - b^{\text{transfer}}_{k_m i_m t_o^{\text{transfer}} (t_o^{\text{transfer}}+\tau)} \\ n^{\text{transfer}}_{i_m (t_o^{\text{transfer}}+\tau)(t_o^{\text{transfer}}+2\tau)} &= n^{\text{transfer}}_{i_m t_o^{\text{transfer}} (t_o^{\text{transfer}}+\tau)} \\ &\quad + od^{\text{transfer}}_{i_m (t_o^{\text{transfer}}+\tau)(t_o^{\text{transfer}}+2\tau)} \\ &\quad - b^{\text{transfer}}_{k_m i_m (t_o^{\text{transfer}}+\tau)(t_o^{\text{transfer}}+2\tau)} \\ &\vdots \\ n^{\text{transfer}}_{i_m (t_d^{\text{transfer}}-\tau) t_d^{\text{transfer}}} &= n^{\text{transfer}}_{i_m (t_d^{\text{transfer}}-2\tau)(t_d^{\text{transfer}}-\tau)} \\ &\quad + od^{\text{transfer}}_{i_m (t_d^{\text{transfer}}-\tau) t_d^{\text{transfer}}} \\ &\quad - b^{\text{transfer}}_{k_m i_m (t_d^{\text{transfer}}-\tau) t_d^{\text{transfer}}} \end{aligned}$$

By summing up the sequence of equations, we can get

$$\begin{aligned} n^{\text{transfer}}_{i_m t_o^{\text{transfer}} t_d^{\text{transfer}}} &= n^{\text{transfer}}_{i_m (t_o^{\text{transfer}}-\tau) t_o^{\text{transfer}}} + od^{\text{transfer}}_{i_m t_o^{\text{transfer}} t_d^{\text{transfer}}} \\ &\quad - b^{\text{transfer}}_{k_m i_m t_o^{\text{transfer}} t_d^{\text{transfer}}}. \end{aligned}$$

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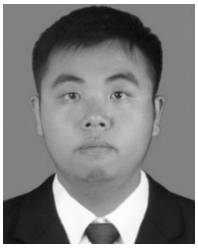

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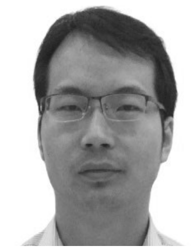

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