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Delay-Dependent Admissibility Analysis and Dissipative Control for T-S Fuzzy Time-Delay Descriptor Systems Subject to Actuator Saturation

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ABSTRACT In this paper, a delay-dependent admissibility analysis method and dissipative controller design are developed for a class of nonlinear time-delay descriptor systems subject to actuator saturation and L_2-disturbances via a Takagi-Sugeno (T-S) fuzzy model. A less conservative admissible condition is first derived under which the system is not only regular, impulse free but also stable under certain initial conditions. The method can eliminate the impulsive behavior of a descriptor system so as to ensure the existence and uniqueness of solutions. The estimate of attraction domain is also determined in which the admissible initial states converge asymptotically to the origin. The disturbance attenuation capability is studied by designing the dissipative fuzzy controller such that the closed-loop system is admissible and holds the dissipative performance for the prescribed disturbance attenuation level and L_2-disturbances. The method is more suitable for admissibility analysis and robust control synthesis. Moreover, H_∞ control processes can be achieved in the same design process, which shows that the cost and time may potentially be reduced when a controller is designed for an actual physical system. Simulations are performed to validate the proposed methods and illustrate the decrease in conservativeness for a classic nonlinear system based on the T-S fuzzy time-delay descriptor model under actuator saturation and L_2-disturbances. The study seeks to establish a foundation for investigating the control synthesis of T-S fuzzy time-delay descriptor systems subject to actuator saturation.

INDEX TERMS Delay-dependent, dissipative control, L_2-disturbances, actuator saturation, T-S fuzzy descriptor systems.

I. INTRODUCTION

As is well known, time-delays involving in-state or control variables inevitably occur in a variety of practical systems and may frequently lead to undesirable system instability and poor performance, which greatly increases the difficulty and complexity of stability analysis and controller design. Therefore, the stability analysis and control of such systems have been the focus of research in recent decades [1]–[15]. These investigations may be divided into two types: delay dependent and delay independent. Delay dependent research takes the size of delays into account. It is less conservative, especially when the delay is relatively small [1]. A vital problem in delay dependent control is the appearance of an integral term, which does not be expressed normally as linear matrix inequalities (LMIs). A large number of research methods and techniques have contributed to stabilize the time-delay system and deal with the integral term, such as Jensen's inequality, Park's inequality, Moon's inequality, Wirtinger's inequality, free-weighting-matrix approach and so on. Each method has its own technical characteristics, so it is necessary to choose an appropriate method so as to reduce the conservativeness.

Actuators are frequently involved in most practical control applications to follow control signals emitted from

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controllers. Unfortunately, in the operation of driving the actuator the physical limitation of the actuator saturation is often unavoidable [7]. Performance deterioration and even instability can occur in a closed-loop system if the effect of actuator saturation is not considered when designing a controller. Therefore, problems involving control synthesis on actuator saturation have received extensive attention during the past decades, either for state systems [16]–[24], or descriptor systems [25]–[30]. The stability or admissibility analysis and disturbance rejection ability is worthy of study for linear systems or descriptor systems subject to actuator saturation. Existing works on disturbance rejection ability for systems with actuator saturation are divided into two categories depending how disturbances enter the system [21] – inputs with disturbances and inputs without disturbances, such as [20] and [21], [26] respectively. Although many stabilization methods above have been proposed, some cannot be applied directly to time-delay systems with saturating actuators. Thus the research on time-delay systems has received more and more attention as of late [7]–[10], [12]–[15]. For example, [8] determines simultaneously a state feedback control law and an associated domain of safe admissible states for which the stability of the closed-loop system is guaranteed when control saturation effectively occurs; [9] derives a less conservative estimate of the domain of attraction based on the Lyapunov-Razumikin and Lyapunov-Krasovskii functional approaches, and the estimate is maximized over the choice of the feedback gains. Another approach [10], differing from existing techniques, represents the saturation non-linearity as the convex combination of state feedback and auxiliary time-delay feedback, and proposes improvements to the delay-dependent local stabilization conditions; in [12], a delay-range-dependent method is adopted and the existence conditions of the stabilizing controller are derived. An estimate for the domain of attraction of the origin is obtained for the systems with different time-delay ranges. [13] develops delay-range-dependent sufficient conditions such that the descriptor system with time-varying delays is regular, impulse free and α-stable, and also presents an estimate of the convergence rate of such stable systems and an iterative LMI (ILMI) algorithm to compute a static output feedback controller gains. [14] establishes less conservative sufficient conditions to ensure that closed-loop system is locally robust admissible, and determines a domain of attraction in which the admissible initial states are ensured to converge asymptotically to the origin. By adopting the idea of the cone complementarity algorithm, the minimization problem is solved efficiently.

Willems introduced the dissipative notion in [31], which unifies passivity and the small gain concept. From the application point of view, a lot of systems must be dissipative in order to achieve effective noise attenuation [32], [33]. In general, dissipativity means that the increase in energy storage in the system is no more than the supplied energy from outside the system [34]. However, it is very difficult to find the storage function for a nonlinear dissipative system. Therefore, some scholars are dedicated to finding a simple solution for dissipative control [32]–[40]. However, it is known as an undeniable fact that the analysis and synthesis of nonlinear systems is far more complex than linear ones. T-S fuzzy descriptor system model is introduced by Taniguchi *et al.* [41], which is an innovative and simple method that deals with the problems involving a class of nonlinear singular systems. Many nonlinear dynamic systems can be represented as T-S fuzzy systems, which can approximate a nonlinear system with any precision [36]–[43]. A number of excellent results have been reported, such as [42] and [43]. Two efficient robust fuzzy model predictive control algorithms are given in [42] for T-S fuzzy discrete systems with multiple time delays and bounded disturbances using Lyapunov-Razumikhin function methods. The model predictive control is investigated in [43] using Razumikhin methods for T-S fuzzy discrete systems subjected to bounded time-varying delay and persistent disturbances. Most recently, robust control issue related to LMI technique is reported in [46] and helpful to investigate the dissipative control for descriptor systems subject to actuator saturation.

In this paper, a delay-dependent admissibility analysis method and dissipative controller design are developed for a class of nonlinear time-delay descriptor system subject to actuator saturation and L_2-disturbances via a T-S fuzzy model. A less conservative admissible condition is first derived under which the system is admissible for certain initial conditions. The method can eliminate the impulsive behavior of a descriptor system so as to ensure the existence and uniqueness of solutions. The estimate of attraction domain is also determined in which the admissible initial states converge asymptotically to the origin. The disturbance attenuation capability is studied by designing the dissipative fuzzy controller such that the closed-loop system is admissible and holds the dissipative performance for the prescribed disturbance attenuation level and L_2-disturbances. The method is more suitable for admissibility analysis and robust control synthesis for the time-delay nonlinear descriptor systems in the presence of actuator saturation and L_2-disturbances. Moreover, H_∞ control processes can be achieved in the same design process, which shows that the cost and time could potentially be reduced when a controller is designed for an actual system. Simulations are performed to validate the proposed methods and illustrate the decrease in conservatism for a classic nonlinear system based on the T-S fuzzy time-delay descriptor model under actuator saturation and L_2-disturbances. The study seeks to establish a foundation for investigating the control synthesis of T-S fuzzy time-delay descriptor control systems subject to actuator saturation.

The main innovations of this paper are as follows:

- The time-delays involving in-state or control variables may greatly increase the difficulty and complexity of the admissibility analysis and controller design. Until recently, there has been a lack of research discussing delay-dependent admissibility analysis and dissipative control of T-S fuzzy time-delay descriptor systems with actuator saturation and L_2-disturbances. The studies on stabilization and dissipative control for such systems are still under way.

• The advantage of the method described in this paper is the ability to eliminate the impulsive behavior of a descriptor system which ensures the existence and uniqueness of solutions. That is, it is not necessary to assume that the systems under consideration are regular and impulse free.

• The proposed method is more suitable for the admissibility analysis and robust control synthesis for the time-delay nonlinear descriptor systems in the presence of actuator saturation and L_2-disturbances.

• Moreover, the fact that H_∞ control processes can be achieved in the same design process indicates that the cost and time required may be potentially be reduced when designing a controller for actual physical systems. The study seeks to establish a foundation for investigating the control synthesis of T-S fuzzy time-delay descriptor control systems subject to actuator saturation.

This paper is organized as follows. Section 2 gives the problem formulations and preliminaries. In Section 3, a delay-dependent admissible sufficient condition is first derived. The estimate of attraction domain is also determined. The disturbance attenuation capability is then studied by designing the dissipative fuzzy controller. In Section 4, a design example is given to show the advantages of developed results. Some conclusions are drawn in Section 5.

Notation: The following notations will be used throughout this paper: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote respectively the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript T stands for matrix transposition. And the notation $A < 0 (A \leq 0)$ means that A is real symmetric and negative definite matrix (negative semi-definite matrix). $\rho(A)$ represents the spectral radius of matrix A. $\lambda_{\min}(A)(\lambda_{\max}(A))$ denotes the minimum (maximum) eigenvalue of matrix A. $C_{n,d} = C([-d, 0], \mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping $[-d, 0]$ into $\mathbb{R}^n$. $x_t = x(t + \theta)\,(\theta \in [-d, 0], t \geq 0)$ denotes the function family defined on $[-d, 0]$ which is generated by a n-dimensional real vector valued continuous function $x(t)$ $(t \in [-d,\ +\infty))$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm,

$$\|\varphi\|_c = \sup_{-d\leq t\leq 0} \|\varphi(t)\|$$

stands for the norm of a function $\varphi \in C_{n,d}$. All matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

II. PROBLEM FORMULATIONS AND PRELIMINARIES

Consider a nonlinear dynamical time-delay descriptor system subject to actuator saturation and L_2-disturbances. It can be described by the following fuzzy IF-THEN rules:

Model rule i:
IF $\xi_1(t)$ is M_{i1} and $\xi_2(t)$ is $M_{i2} \cdots$ and $\xi_p(t)$ is M_{ip}, THEN

$$\begin{aligned} E\dot{x}(t) &= A_i x(t) + A_{id} x(t-d) + B_i \mathrm{sat}(u(t)) + W_i w(t), \\ z(t) &= C_i x(t) \\ x(t) &= \varphi(t), \quad t \in [-d, 0] \end{aligned} \tag{1}$$

where r is the number of IF-THEN rules and $M_{ij}(j = 1, 2, \cdots, p)$ are fuzzy sets $\xi_1(t), \xi_2(t), \cdots, \xi_p(t)$ are premise variables which are the functions of state variables, and let

$$\xi(t) = \big(\xi_1(t), \xi_2(t), \cdots, \xi_p(t)\big)^T$$

The scalar d is a time-delay of the system and $\varphi(t)$ is a continuous initial function A_i, A_{id}, B_i, C_i and W_i are known real constant matrices with appropriate dimensions. The matrix $E \in \mathbb{R}^{n\times n}$ is singular and rank $(E) = s < n$. $x(t) \in \mathbb{R}^n$ is the state; $z(t) \in \mathbb{R}^q$ is the controlled output; $w(t) \in \mathbb{R}^p$ is the exogenous disturbance signal in $L_2[0, \infty)$; $u(t) \in \mathbb{R}^m$ is control input; sat: $\mathbb{R}^m \to \mathbb{R}^m$ is the vector valued standard saturation function defined as

$$\begin{aligned} \mathrm{sat}(u) &= (\mathrm{sat}(u_1)\mathrm{sat}(u_2)\cdots\mathrm{sat}(u_m))^T, \\ \mathrm{sat}(u_i) &= \mathrm{sign}(u_i)\min\{1, |u_i|\}, \quad i = 1, 2, \cdots, m. \end{aligned}$$

Note that it is without loss of generality to assume unity saturation level. A non-unity saturation level can be absorbed into B_i and $u(t)$ [21]. By fuzzy blending, the final fuzzy sigular systems are inferred as follows

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r} h_i(\xi(t))(A_i x(t) + A_{id}x(t-d) \\ &\quad + B_i\mathrm{sat}(u(t)) + W_i w(t)), \\ z(t) &= \sum_{i=1}^{r} h_i(\xi(t)) C_i x(t), \\ x(t) &= \varphi(t), \quad t \in [-d, 0], \end{aligned} \tag{2}$$

where $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij}. It can be seen that

$$\beta_i(\xi(t)) = \prod_{j=1}^{p} M_{ij}(\xi_j(t)) \geq 0, \quad i = 1, 2, \cdots, r$$

which implies that

$$h_i(\xi(t)) = \beta_i(\xi(t)) / \sum_{i=1}^{r} \beta_i(\xi(t)) \geq 0, \sum_{i=1}^{r} h_i(\xi(t)) = 1$$

for all t.

Consider the following fuzzy controller via the parallel distributed compensation (PDC) for the fuzzy model (2)

$$u(t) = \sum_{j=1}^{r} h_j(\xi(t)) F_j x(t) \triangleq \bar{F}_l x(t). \tag{3}$$

For the matrices $F_j \in \mathbb{R}^{m\times n}$, we define

$$\begin{aligned} &\bar{L}(F_j) \triangleq \left\{x(t) \in \mathbb{R}^n \,\middle|\, |f_{ji}x(t)| \leq 1, i \in [1, m]\right\}, \\ &j = 1, 2, \cdots, r, \end{aligned}$$

where f_{ji} represents the ith row of F_j.

Lemma 1 [16]: Let $F_j, H_j \in \mathbb{R}^{m\times n}, j = 1, 2, \cdots, r$. Then, for any $x(t) \in \bar{L}(H_j)$

$$\begin{aligned} &\mathrm{sat}(F_jx(t)) \in \mathrm{co}\{\Lambda_k F_j x(t) + \Lambda_k^- H_j x(t)\}, \\ &\qquad k \in [1, 2^m], \quad j = 1, 2, \cdots, r, \end{aligned} \quad (4)$$

where co{·} denotes the convex hull of a set. Λ expresses the set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^m elements in the set, labeled as $\Lambda_k, k \in [1, 2^m]$. We also let $\Lambda_k^- = I - \Lambda_k$.

The following fact is needed about the convex hull of a set of points [17]. For a group of points $u^1, u^2, \cdots, u^\ell$, the convex hull of these points is defined as

$$\mathrm{co}\{u^i \,|i \in [1, l]\} \triangleq \left\{ \sum_{i=1}^{l} \alpha_i u^i \left| \sum_{i=1}^{l} \alpha_i = 1, \alpha_i \geq 0 \right.\right\}. \quad (5)$$

For a non-singular matrix

$$P = \begin{pmatrix} P_1 & 0 \\ P_3 & P_4 \end{pmatrix}$$

($P_1 \in \mathbb{R}^{s\times s}, P_1 > 0, P_4 \in \mathbb{R}^{(n-s)\times(n-s)}, |P_4| \neq 0$), we denote

$$\varepsilon(EP, \rho) = \{x(t) \in \mathbb{R}^n \left| x^T(t)EPx(t) \leq \rho \right.\} \quad (6)$$

and an ellipsoid

$$\varepsilon(P_1, \rho) = \{x_1(t) \in \mathbb{R}^s \left| x_1^T(t)P_1x_1(t) \leq \rho \right.\},$$

where $x(t) = (x_1^T(t), x_2^T(t))^T$ ($x_1(t) \in \mathbb{R}^s, x_2(t) \in \mathbb{R}^{n-s}$).

Lemma 2: $x(t) \in \varepsilon(EP, \rho)$ if and only if $x_1(t) \in \varepsilon(P_1, \rho)$.

Proof: Assume that $x \in \varepsilon(EP, \rho)$. Then $x_1(t) \in \varepsilon(P_1, \rho)$ by the inequality $|x_1^T(t)P_1x_1(t) = |x^T(t)EPx(t)| \leq \rho$. On the other hand, assume that $x_1(t) \in \varepsilon(P_1, \rho)$. Then that $x(t) \in \varepsilon(EP, \rho)$ can be obtained through the inequality $\left|x^T(t)EPx(t)\right| = \left|x_1^T(t)P_1x_1(t)\right| \leq \rho$. Thus, $x(t) \in \varepsilon(EP, \rho)$ if and only if $x_1(t) \in \varepsilon(P_1, \rho)$.

Lemma 3 [39]: For a positive scalar ρ,

$$\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j),$$

if and only if

$$\varepsilon(P_1, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_{j1})$$

where $H_j = [H_{j1}, 0], H_{j1} \in \mathbb{R}^{m\times s}, 0 \in \mathbb{R}^{m\times(n-s)}$.

Lemma 4: If $\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j)$, then

$$\varepsilon(EP, \rho) \subset \bar{L}(\sum_{j=1}^{r} h_j(\xi(t))H_j) = \bar{L}(\bar{H}_l).$$

Proof: For any $x(t) \in \varepsilon(EP, \rho)$, we have

$$x(t) \in \bar{L}(H_j)(j = 1, 2, \cdots, r),$$

under condition $\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j)$. Therefore, it can be derived that

$$\left|\sum_{j=1}^{r} h_j(\xi(t))h_{ji}x(t)\right| \leq \sum_{j=1}^{r} \left|h_j(\xi(t))\right| \left|h_{ji}x(t)\right| \leq 1,$$

where h_{ji} is the ith row of matrix H_j. The above inequality indicates that $\varepsilon(EP, \rho) \subset \bar{L}(\sum_{j=1}^{r} h_j(\xi(t))H_j) = \bar{L}(\bar{H}_l)$.

Remark 1: Lemmas 2-4 are crucial in that they allow us to determine the estimate of attraction domain in a manner resembling normal state systems while also playing an important role in admissibility analysis. Using Lemma 2-4 as a basis, Lemmas 5-8 can be applied to determine the feasibility conditions under which a controller can be designed via LMIs for a time-delay T-S fuzzy descriptor system subject to actuator saturation and L_2- disturbances.

Let $\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j)$. By Lemma 1 and Lemma 4, we have

$$\mathrm{sat}(\bar{F}_lx(t)) \in \mathrm{co}\{\Lambda_k\bar{F}_lx(t) + \Lambda_k^-\bar{H}_lx(t)\}, k \in [1, 2^m]. \quad (7)$$

Then it is obvious from the expression (4) that

$$\begin{aligned} \mathrm{sat}(u(t)) &= \mathrm{sat}(\sum_{j=1}^{r} h_j(\xi(t))F_jx(t)) \\ &= \mathrm{sat}\left(\bar{F}_lx\right) = \sum_{k=1}^{2^m} \alpha_k(\Lambda_k\bar{F}_lx(t) + \Lambda_k^-\bar{H}_lx(t)) \\ &= \sum_{k=1}^{2^m} \alpha_k \sum_{j=1}^{r} h_j(\xi(t))(\Lambda_kF_j + \Lambda_k^-H_j)x(t). \end{aligned} \quad (8)$$

Applying the above controller to system (2) results in the following closed-loop systems

$$\begin{aligned} E\dot{x}(t) &= \sum_{k=1}^{2^m} \alpha_k \sum_{i=1}^{r} h_i(\xi(t)) \sum_{j=1}^{r} h_j(\xi(t))\{(A_i + B_i\Lambda_kF_j \\ &\quad + B_i\Lambda_k^-H_j)x(t) + A_{id}x(t-d) + W_iw(t)\} \\ &\triangleq \bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d) + \bar{W}_hw(t), \\ z(t) &= \sum_{i=1}^{r} h_i(\xi(t))C_ix(t) \triangleq \bar{C}_hx(t), \\ x(t) &= \varphi(t), \quad t \in [-d, 0]. \end{aligned} \quad (9)$$

Note that rank $(E) = s < n$. Then without loss of generality [39], we can decompose matrices in system (9) with $w(t) = 0$ as following

$$\begin{aligned} E &= \begin{pmatrix} I_s & 0 \\ 0 & 0 \end{pmatrix}, \bar{A}_{hlk} = \begin{pmatrix} \bar{A}_{hlk11} & \bar{A}_{hlk12} \\ \bar{A}_{hlk21} & \bar{A}_{hlk22} \end{pmatrix}, \\ \bar{A}_{hd} &= \begin{pmatrix} \bar{A}_{hd11} & \bar{A}_{hd12} \\ \bar{A}_{hd21} & \bar{A}_{hd22} \end{pmatrix}, \end{aligned} \quad (10)$$

where $\bar{A}_{hlk11} \in \mathbb{R}^{s\times s}, \bar{A}_{hlk22} \in \mathbb{R}^{(n-s)\times(n-s)}, \bar{A}_{hd11} \in \mathbb{R}^{s\times s}, \bar{A}_{hd22} \in \mathbb{R}^{(n-s)\times(n-s)}$.

Definition 1 [1]: System (9) with $w(t) = 0$ is said to be regular if $\det(sE - \bar{A}_{hlk})$ is not identically 0; system (9) is said to be impulse-free if $\deg_s \det(sE - \bar{A}_{hlk}) = \text{rank}(E)$

Lemma 5 [1]: System (9) $w(t) = 0$ is impulse-free if and only if $\bar{A}_{hlk22}$ is non-singular.

Lemma 6 [2], [3]: Assume that $\bar{A}_{hlk22}$ is non-singular and $\rho(\bar{A}_{hlk22}^{-1}\bar{A}_{hd22}) < 1$, then system (9) with $w(t) = 0$ is stable if there exist positive numbers α, β, η and a continuous functional $V : C_n[-d, 0] \to \mathbb{R}$ such that

$$\beta \|x_1(t)\|^2 \leq V(x_t) \leq \eta \|x_t\|_c^2, \quad \dot{V}(x_t) \leq -\alpha \|x(t)\|^2, \tag{11}$$

where $x_t = x(t+\theta)$, $\theta \in [-d, 0]$, and

$$x(t) = \left(x_1^T(t)\ x_2^T(t)\right)^T (x_1(t) \in \mathbb{R}^s, x_2(t) \in \mathbb{R}^{n-s})$$

satisfying (9). System (9) with $w(t) = 0$ is said to be admissible if it is regular, impulse-free and stable.

Consider an augmented Lyapunov functional in the following form

$$\begin{aligned} V(x_t) &= x^T(t)E^TPx(t) + \int_{t-d}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^TZE\dot{x}(s)dsd\theta \\ &= x_1^T(t)P_1x_1(t) + \int_{t-d}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)E^TZE\dot{x}(s)dsd\theta, \end{aligned} \tag{12}$$

where $x_t = x(t+\theta)$, $\theta \in [-d, 0]$, and

$$x(t) = \left(x_1^T(t)\ x_2^T(t)\right)^T (x_1(t) \in \mathbb{R}^s, x_2(t) \in \mathbb{R}^{n-s})$$

satisfying (9).

Lemma 7 [6]: For any positive symmetric constant matrix $Z \in \mathbb{R}^{n\times n}$, scalars a, b, satisfying $a < b$, a vector function $x(t)$ in $[a, b] \to \mathbb{R}^n$ such that the integration concerned is well defined, then

$$\int_a^b x^T(t)Zx(t)dt \geq \frac{1}{b-a}\int_a^b x^T(t)dtZ\int_a^b x(t)dt. \tag{13}$$

Lemma 8 [44]: Given any real matrices X, Y and W with appropriate dimensions such that $Y > 0$. Then, we have

$$X^TYX + X^TW + W^TX + W^TY^{-1}W \geq 0. \tag{14}$$

III. ADMISSIBLE ANALYSIS AND DISSIPATIVE CONTROL

Theorem 1: Given scalar $d_0 > 0$, then for any delay $0 < d \leq d_0$ and a positive scalar ρ , system (9) with $w(t) = 0$ subject to actuator saturation and L_2-disturbances is delay-dependent admissible within the set $\varepsilon(EP, \rho)$, if there exists a common matrix

$$P = \begin{pmatrix} P_1 & 0 \\ P_3 & P_4 \end{pmatrix}$$

$(P_1 \in \mathbb{R}^{s\times s}, P_3 \in \mathbb{R}^{(n-s)\times s}, P_4 \in \mathbb{R}^{(n-s)\times(n-s)}, P_1 > 0, |P_4| \neq 0)$ and matrices

$$Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_2^T & Q_4 \end{pmatrix} \geq 0, Z = \begin{pmatrix} Z_1 & Z_2 \\ Z_2^T & Z_4 \end{pmatrix} > 0$$

F_i, H_i such that the following set of matrix inequalities hold

$$\Phi_{iik} = \begin{pmatrix} \Xi_{iik} & * & * \\ \Upsilon_i & \left(\begin{matrix}-Q- \\ \frac{1}{d}E^TZE\end{matrix}\right) & * \\ \Theta_{iik} & A_{id} & -\frac{1}{d}Z^{-1} \end{pmatrix} < 0, \quad i = 1, 2, \cdots, r, \tag{15}$$

$$\Phi_{ijk} + \Phi_{jik} = \begin{pmatrix} \Xi_{ijk} + \Xi_{jik} & * & * \\ \Upsilon_i + \Upsilon_j & \left(\begin{matrix}-2Q- \\ \frac{2}{d}E^TZE\end{matrix}\right) & * \\ \Theta_{ijk} + \Theta_{jik} & A_{id} + A_{jd} & -\frac{2}{d}Z^{-1} \end{pmatrix} < 0, \quad j < i = 1, 2, \cdots, r. \tag{16}$$

$$\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j) \tag{17}$$

$$[\lambda_{\max}(P_1) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^TZE) \|\dot{\varphi}\|_c^2 \leq \rho, \tag{18}$$

where

$$\begin{aligned} \Phi_{ijk} &= \begin{pmatrix} \Xi_{ijk} & * & * \\ \Upsilon_i & -Q - \frac{1}{d}E^TZE & * \\ \Theta_{ijk} & A_{id} & -\frac{1}{d}Z^{-1} \end{pmatrix}, \\ \Xi_{ijk} &= \left(A_i + B_i\Lambda_kF_j + B_i\Lambda_k^-H_j\right)^T P \\ &\quad + P^T\left(A_i + B_i\Lambda_kF_j + B_i\Lambda_k^-H_j\right) \\ &\quad + Q - \frac{1}{d}E^TZE, \\ \Upsilon_i &= A_{id}^TP + \frac{1}{d}E^TZE, \\ \Theta_{ijk} &= A_i + B_i\Lambda_kF_j + B_i\Lambda_k^-H_j, \end{aligned} \tag{19}$$

and

$$\|\varphi\|_c^2 = \sup_{-d\leq t\leq 0} \|\varphi(t)\|^2, \|\dot{\varphi}\|_c^2 = \sup_{-d\leq t\leq 0} \|\dot{\varphi}(t)\|^2,$$

The estimate of attraction domain is

$$\rho_1 = [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^TZE) \|\dot{\varphi}\|_c^2 \leq \rho. \tag{20}$$

The asterisk $*$ denotes the transpose of symmetric position elements in the matrix in the following discussion.

Proof: To derive the admissibility of system (9) with $w(t) = 0$, we first prove that the closed-loop system (9) $w(t) = 0$ is regular and impulse free within $\varepsilon(EP, \rho)$.

Consider the Lyapunov functional (12) for system (9). From the expression of P and E, it is easy to see that

$E^TP = P^TE \geq 0$. Following a method similar to one in [2], it can be derived that

$$\begin{aligned}
&\lambda_{\min}(P_1)\|x_1(t)\|^2 \\
&\leq V(x_t) \\
&\leq [\lambda_{\max}(P_1) + d\lambda_{\max}(Q) \\
&\quad + \frac{d^2}{2}(\lambda_1+\lambda_2+\lambda_3+\lambda_4)] \sup_{\theta\in[-2d,0]} \|x(t+\theta)\|^2 \\
&= [\lambda_{\max}(P_1) + d\lambda_{\max}(Q) \\
&\quad + \frac{d^2}{2}(\lambda_1+\lambda_2+\lambda_3+\lambda_4)]\|x_t\|_{c_1}^2, \qquad (21)
\end{aligned}$$

where $\sup_{\theta\in[-2d,0]} \|x(t+\theta)\|^2 = \|x_t\|_{c_1}^2$. The following fact can be deduced

$$\begin{aligned}
\bar{A}_{hlk}^T Z \bar{A}_{hlk} &= \sum_{k=1}^{2^m}\alpha_k \sum_{i=1}^{r} h_i(\xi(t)) \sum_{j=1}^{r} h_j(\xi(t)) \\
&\quad \cdot (A_i + B_i\Lambda_k F_j + B_i\Lambda_k^- H_j)^T Z \\
&\quad \cdot \sum_{l=1}^{2^m}\alpha_l \sum_{m=1}^{r} h_m(\xi(t)) \sum_{n=1}^{r} h_n(\xi(t)) \\
&\quad \cdot (A_m + B_m\Lambda_l F_n + B_m\Lambda_l^- H_n) \\
&= \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \\
&\quad \cdot h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t)) \\
&\quad (A_i + B_i\Lambda_k F_j + B_i\Lambda_k^- H_j)^T Z \\
&\quad (A_m + B_m\Lambda_l F_n + B_m\Lambda_l^- H_n) \\
&= \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \cdot \\
&\quad h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t)) \\
&\quad \left\{\begin{array}{l} A_i^T Z A_m + A_i^T Z B_m \Lambda_l F_n + \\ A_i^T Z B_m \Lambda_l^- H_n + F_j^T \Lambda_k B_i^T Z A_m + \\ F_j^T \Lambda_k B_i^T Z B_m \Lambda_l F_n + \\ F_j^T \Lambda_k B_i^T Z B_m \Lambda_l^- H_n + \\ H_j^T \Lambda_k^- B_i^T Z A_m + H_j^T \Lambda_k^- B_i^T Z B_m \Lambda_l F_n \\ + H_j^T \Lambda_k^- B_i^T Z B_m \Lambda_l^- H_n \end{array}\right\} \\
&\triangleq \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \\
&\quad \cdot h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t))\Omega_{klijmn}^1,
\end{aligned}$$

$$\begin{aligned}
\bar{A}_{hlk}^T \bar{A}_{hlk} &= \sum_{k=1}^{2^m}\alpha_k \sum_{i=1}^{r} h_i(\xi(t)) \sum_{j=1}^{r} h_j(\xi(t)) \\
&\quad \cdot (A_i + B_i\Lambda_k F_j + B_i\Lambda_k^- H_j)^T \\
&\quad \sum_{l=1}^{2^m}\alpha_l \sum_{m=1}^{r} h_m(\xi(t)) \sum_{n=1}^{r} h_n(\xi(t)) \cdot \\
&\quad (A_m + B_m\Lambda_l F_n + B_m\Lambda_l^- H_n) \\
&= \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \cdot \\
&\quad h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t)) \\
&\quad (A_i + B_i\Lambda_k F_j + B_i\Lambda_k^- H_j)^T \\
&\quad (A_m + B_m\Lambda_l F_n + B_m\Lambda_l^- H_n) \\
&= \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \cdot \\
&\quad h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t)) \\
&\quad \left\{\begin{array}{l} A_i^T A_m + A_i^T B_m \Lambda_l F_n + \\ A_i^T B_m \Lambda_l^- H_n + F_j^T \Lambda_k B_i^T A_m + \\ F_j^T \Lambda_k B_i^T B_m \Lambda_l F_n + \\ F_j^T \Lambda_k B_i^T B_m \Lambda_l^- H_n + \\ H_j^T \Lambda_k^- B_i^T A_m + H_j^T \Lambda_k^- B_i^T B_m \Lambda_l F_n \\ + H_j^T \Lambda_k^- B_i^T B_m \Lambda_l^- H_n \end{array}\right\} \\
&\triangleq \sum_{k=1}^{2^m}\sum_{l=1}^{2^m}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\alpha_k\alpha_l \cdot \\
&\quad h_i(\xi(t))h_j(\xi(t))h_m(\xi(t))h_n(\xi(t))\Omega_{klijmn}^2,
\end{aligned}$$

$$\begin{aligned}
&\bar{A}_{hd}^T Z^T Z \bar{A}_{hd} \\
&= \left(\sum_{i=1}^{r} h_i(\xi(t))A_{id}\right)^T Z^T Z \sum_{j=1}^{r} h_j(\xi(t))A_{jd} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))A_{id}^T Z^T Z A_{jd} \\
&\triangleq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\Omega_{ij}^3,
\end{aligned}$$

$$\begin{aligned}
&\bar{A}_{hd}^T Z \bar{A}_{hd} \\
&= \left(\sum_{i=1}^{r} h_i(\xi(t))A_{id}\right)^T Z \sum_{j=1}^{r} h_j(\xi(t))A_{jd} \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))A_{id}^T Z A_{jd} \\
&\triangleq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\Omega_{ij}^4,
\end{aligned}$$

$$\begin{aligned}
&\lambda_1 = \lambda_{\max}(\Omega_{klijmn}^1),\ \lambda_2 = \lambda_{\max}(\Omega_{klijmn}^2), \\
&\lambda_3 = \lambda_{\max}(\Omega_{ij}^3),\ \lambda_4 = \lambda_{\max}(\Omega_{ij}^4), \\
&\qquad i, j, m, n = 1, 2, \cdots, r, k \in [1, 2^m]. \qquad (22)
\end{aligned}$$

Note that

$$x \in \varepsilon(EP, \rho) \Leftrightarrow x \in \varepsilon(E(\frac{P}{\rho}), 1).$$

Then we have $H_{j2} = 0 (j = 1, 2, \cdots, r)$ by expression (17) [25], where$H_j = [H_{j1}, H_{j2}], H_{j1} \in R^{m\times s}, H_{j2} \in R^{m\times(n-s)}$. The following abbreviated form will be used from here on.

$$\sum_{j=1}^{r} h_j(\xi(t))H_j := \bar{H}_l = [\bar{H}_{l1}, 0],$$
$$H_{l1} \in R^{m\times s}, 0 \in R^{m\times(n-s)},$$
$$\sum_{j=1}^{r} h_j(\xi(t))F_j := \bar{F}_l = [\bar{F}_{l1}, \bar{F}_{l2}],$$
$$\bar{F}_{l1} \in R^{m\times s}, \bar{F}_{l2} \in R^{m\times(n-s)}. \quad (23)$$

It can be derived by inequalities (15) and (16) along with

$$\bar{\Phi}_{hlk} = \sum_{k=1}^{2^m} \alpha_k \sum_{i=1}^{r} h_i(\xi(t)h_i(\xi(t))\Phi_{iik} + \sum_{k=1}^{2^m} \alpha_k \sum_{j<i} h_i(\xi(t))h_j(\xi(t))(\Phi_{ijk} + \Phi_{jik})$$

that

$$\bar{\Phi}_{hlk} = \sum_{k=1}^{2^m} \alpha_k \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi(t))h_j(\xi(t))\Phi_{ijk} \triangleq \begin{pmatrix} \Xi_{hlk} & * & * \\ \Upsilon_h & \begin{pmatrix} -Q- \\ \frac{1}{d}E^TZE \end{pmatrix} & * \\ \bar{A}_{hlk} & \bar{A}_{hd} & -\frac{1}{d}Z^{-1} \end{pmatrix} < 0,$$
$$\forall x(t) \in \varepsilon(EP, \rho)\backslash 0 \quad (24)$$

where $\Xi_{hlk} = \bar{A}_{hlk}^T P + P^T\bar{A}_{hlk} + Q - \frac{1}{d}E^TZE$ and $\Upsilon_h = \bar{A}_{hd}^T P + \frac{1}{d}E^TZE$.

By Schur complement lemma, the above inequality (24) is equivalent to

$$\bar{G}_{hlk} = \sum_{k=1}^{2^m} \alpha_k \sum_{i=1}^{r} h_i(\xi(t)) \sum_{j=1}^{r} h_j(\xi(t)) \cdot \left(\begin{pmatrix} \Xi_{ijk} & * \\ \Upsilon_i & -Q - \frac{1}{d}E^TZE \end{pmatrix} + d\begin{pmatrix} \Theta_{ijk}^T \\ A_{id}^T \end{pmatrix} Z \begin{pmatrix} \Theta_{ijk} & A_{id} \end{pmatrix} \right)$$
$$\triangleq \sum_{k=1}^{2^m} \alpha_k \sum_{i=1}^{r} h_i(\xi(t)) \sum_{j=1}^{r} h_j(\xi(t))\bar{G}_{ijk} < 0,$$
$$\forall x(t) \in \varepsilon(EP, \rho)\backslash 0 \quad (25)$$

Noting expression (10) and inequality (24), then it turns out with some manipulations that

$$\bar{A}_{hlk}^T P + P^T\bar{A}_{hlk} + Q - \frac{1}{d}E^TZE = \begin{pmatrix} \oplus & \oplus \\ \oplus & (P_4^T\bar{A}_{hlk22} + \bar{A}_{hlk22}^T P_4 + Q_4) \end{pmatrix} < 0,$$
$$k \in [1, 2^m], \forall x \in \varepsilon(EP, \rho)\backslash 0 \quad (26)$$

where $\oplus$ denotes matrices which are not relevant in the discussion. It is obtained from the above inequality (26) that

$$P_4^T\bar{A}_{hlk22} + \bar{A}_{hlk22}^T P_4 + Q_4 < 0,$$
$$k \in [1, 2^m], \forall x(t) \in \varepsilon(EP, \rho)\backslash 0 \quad (27)$$

which indicates that $\bar{A}_{hlk22}$ is non-singular.

Taking the matrix inequality (24)and the expression (10) into account, it is not hard to see that

$$\begin{pmatrix} \begin{pmatrix} \bar{A}_{hlk}^T P + P^T\bar{A}_{hlk}+ \\ Q - \frac{1}{d}E^TZE \end{pmatrix} & * \\ \bar{A}_{hd}^T P + \frac{1}{d}E^TZE & -Q - \frac{1}{d}E^TZE \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \oplus & \oplus \\ \oplus & \begin{pmatrix} P_4^T\bar{A}_{hlk22}+ \\ \bar{A}_{hlk22}^T P_4 + Q_4 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \oplus & \oplus \\ \oplus & P_4\bar{A}_{hd22} \end{pmatrix} \\ \begin{pmatrix} \oplus & \oplus \\ \oplus & \bar{A}_{hd22}^T P_4 \end{pmatrix} & \begin{pmatrix} \oplus & \oplus \\ \oplus & -Q_4 \end{pmatrix} \end{pmatrix}$$
$$< 0, k \in [1, 2^m], \forall x(t) \in \varepsilon(EP, \rho)\backslash 0, \quad (28)$$

where $\bigoplus$ represents matrices that are not relevant in the following discussion. The inequality (28) implies that

$$\begin{pmatrix} P_4^T\bar{A}_{hlk22} + \bar{A}_{hlk22}^T P_4 + Q_4 & P_4^T\bar{A}_{hd22} \\ \bar{A}_{hd22}^T P_4 & -Q_4 \end{pmatrix} < 0,$$
$$k \in [1, 2^m], \forall x(t) \in \varepsilon(EP, \rho)\backslash 0. \quad (29)$$

Pre- and post-multiplying (29) by $\begin{pmatrix} -\bar{A}_{hd22}^T\bar{A}_{hlk22}^{-T} & I \end{pmatrix}$ and its transpose, respectively, it can be obtained that

$$\bar{A}_{hd22}^T\bar{A}_{hlk22}^{-T}Q_4\bar{A}_{hlk22}^{-1}\bar{A}_{hd22} - Q_4 < 0. \quad (30)$$

Then we have $\rho(\bar{A}_{hlk22}^{-1}\bar{A}_{hd22}) < 1$ by the above inequality (30).

Taking the time derivative of $V(x_t)$ along with the trajectory of the closed-loop system (9) with $w(t) = 0$ yields

$$\begin{aligned} \dot{V}(x_t) &= \left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right)^T Px(t) \\ &\quad + x^T(t)P^T\left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right) \\ &\quad + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) \\ &\quad + d\left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right)^T Z\left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right) \\ &\quad - \int_{-d}^{0} \dot{x}^T(t+\theta)E^TZE\dot{x}(t+\theta)d\theta, \end{aligned} \quad (31)$$

By Jensen's inequality and noting expression (31), it follows that

$$\begin{aligned} \dot{V}(x_t) &\le x^T(t)\bar{A}_{hlk}^T Px(t) + x^T(t-d)\bar{A}_{hd}^T Px(t) \\ &\quad + x^T(t)P^T\bar{A}_{hlk}x(t) + x^T(t)P^T\bar{A}_{hd}x(t-d) \\ &\quad + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) \\ &\quad + d\left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right)^T Z\left(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d)\right) \\ &\quad - \frac{1}{d}x^T(t)E^TZEx(t) + \frac{1}{d}x^T(t)E^TZEx(t-d) \\ &\quad + \frac{1}{d}x^T(t-d)E^TZEx(t) - \frac{1}{d}x^T(t-d)E^TZEx(t-d), \end{aligned}$$

$$= \varsigma_1^T(t)\begin{pmatrix}\begin{pmatrix}\bar{A}_{hlk}^T P+ \\ P^T\bar{A}_{hlk}+ \\ Q- \\ \frac{1}{d}E^T ZE\end{pmatrix} & * \\ \begin{pmatrix}\bar{A}_{hd}^T P+ \\ \frac{1}{d}E^T ZE\end{pmatrix} & \begin{pmatrix}-Q- \\ \frac{1}{d}E^T ZE\end{pmatrix}\end{pmatrix}\varsigma_1(t)$$
$$+ d\varsigma_1^T(t)\begin{pmatrix}\bar{A}_{hlk}^T \\ \bar{A}_{hd}^T\end{pmatrix}Z\begin{pmatrix}\bar{A}_{hlk} & \bar{A}_{hd}\end{pmatrix}\varsigma_1(t)$$
$$= \varsigma_1^T(t)\bar{G}_{hlk}\varsigma_1(t)$$
$$= \sum_{k=1}^{2^m}\alpha_k\sum_{i=1}^{r}h_i(\xi(t))\sum_{j=1}^{r}h_j(\xi(t))\varsigma_1^T(t)\bar{G}_{ijk}\varsigma_1(t), \quad (32)$$

where

$$\varsigma_1(t) = \begin{pmatrix} x(t) \\ x(t-d)\end{pmatrix}.$$

It is clear from the fact $\bar{G}_{ijk} < 0$, $\sum_{i=1}^{r} h_i(\xi(t)) = 1$ and $\sum_{k=1}^{2^m}\alpha_k = 1$ that

$$\dot{V}(x_t) \le -\alpha\|\varsigma_1(t)\|^2 \le -\alpha\|x(t)\|^2, \quad \forall x(t) \in \varepsilon(EP,\rho)\backslash 0, \quad (33)$$

where $\alpha = -\lambda_{\max\ 1\le i,j\le r,k\in[1,2^m]}(\bar{G}_{ijk})$.

From $\dot{V}(x_t) \le 0$, it follows that $V(x_t) \le V(x_0) = V(\varphi)$, $x_0 = \varphi(t)$, $t \in [-d, 0]$. Therefore, we have

$$\begin{aligned} x_1^T(t)P_1x_1(t) &= x^T(t)E^T Px(t) \le V(x_t) \\ &\le V(x_0) = V(\varphi) \\ &= \varphi_1^T(0)P_1\varphi_1(0) + \int_{-d}^{0}\varphi^T(s)Q\varphi(s)ds \\ &\quad + \int_{-d}^{0}\int_{\theta}^{0}\dot{\varphi}^T(s)E^T ZE\dot{\varphi}(s)dsd\theta \\ &\le [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)]\,\|\varphi\|_c^2 \\ &\quad + \frac{d^2}{2}\lambda_{\max}(E^T ZE)\,\|\dot{\varphi}\|_c^2 \le \rho, \end{aligned} \quad (34)$$

where

$$\|\varphi\|_c^2 = \sup_{-d\le t\le 0}\|\varphi(t)\|^2, \|\dot{\varphi}\|_c^2 = \sup_{-d\le t\le 0}\|\dot{\varphi}(t)\|^2.$$

Thus, the system is admissible as it is also regular and impulse-free. The estimate of attraction domain is

$$\rho_1 = [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)]\,\|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^T ZE)\,\|\dot{\varphi}\|_c^2 \le \rho.$$

Theorem 2: Given scalar $\delta > 0$ and $d_0 > 0$. Then for any delay $0 < d \le d_0$ and a positive scalar ρ, system (9) with $w(t) = 0$ subject to actuator saturation and L_2-disturbances is delay-dependent admissible within $\varepsilon(EX^{-1},\rho)$, if there exists a common matrix

$$X = P^{-1} = \begin{pmatrix} P_1^{-1} & 0 \\ -P_4^{-1}P_3P_1^{-1} & P_4^{-1}\end{pmatrix} \triangleq \begin{pmatrix} X_1 & 0 \\ X_3 & X_4\end{pmatrix}$$

($X_1 \in \mathbb{R}^{s\times s}, X_3 \in \mathbb{R}^{(n-s)\times s}, X_4 \in \mathbb{R}^{(n-s)\times(n-s)}, X_1 > 0, |X_4| \ne 0$), matrices

$$Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_2^T & Q_4\end{pmatrix} \ge 0,$$

$\bar{Z} = Z^{-1} > 0$, G_i, M_i such that the following LMIs hold

$$\widehat{\Phi}_{iik} = \begin{pmatrix} \bar{\Xi}_{iik} & * & * & * & * \\ A_{id}^T & -Q & * & * & * \\ \bar{\Theta}_{ijk} & A_{id} & -\frac{1}{d}\bar{Z} & * & * \\ X & 0 & 0 & -(2I-Q) & * \\ EX & 0 & 0 & 0 & -d\bar{Z}\end{pmatrix} < 0,$$
$$i = 1, 2, \cdots, r, k \in [1, 2^m], \quad (35)$$

$$\widehat{\Phi}_{ijk} + \widehat{\Phi}_{jik} = \begin{pmatrix} \begin{pmatrix}\bar{\Xi}_{ijk} \\ + \\ \bar{\Xi}_{jik}\end{pmatrix} & * & * & * & * \\ \begin{pmatrix}A_{id}^T \\ + \\ A_{jd}^T\end{pmatrix} & -2Q & * & * & * \\ \begin{pmatrix}\bar{\Theta}_{ijk} \\ + \\ \bar{\Theta}_{jik}\end{pmatrix} & \begin{pmatrix}A_{id} \\ + \\ A_{jd}\end{pmatrix} & -\frac{2}{d}\bar{Z} & * & * \\ X & 0 & 0 & -\frac{1}{2}(2I-Q) & * \\ EX & 0 & 0 & 0 & -\frac{d}{2}\bar{Z}\end{pmatrix}$$
$$< 0, j < i =, 1, 2, \cdots, r, k \in [1, 2^m], \quad (36)$$

$$\begin{pmatrix} \rho & \rho g_{j1i} \\ \rho g_{j1i}^T & X_1\end{pmatrix} \ge 0, i \in [1, m], j = 1, 2, \cdots, r, \quad (37)$$

$$[\lambda_{\max}(X_1^{-1}) + d\lambda_{\min}(Q)]\,\|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^T\bar{Z}^{-1}E)\,\|\dot{\varphi}\|_c^2 \le \rho. \quad (38)$$

where

$$\widehat{\Phi}_{ijk} = \begin{pmatrix} \bar{\Xi}_{ijk} & * & * & * & * \\ A_{id}^T & -Q & * & * & * \\ \bar{\Theta}_{ijk} & A_{id} & -\frac{1}{d}\bar{Z} & * & * \\ X & 0 & 0 & -(2I-Q) & * \\ EX & 0 & 0 & 0 & -d\bar{Z}\end{pmatrix}$$

$$\begin{aligned} \bar{\Xi}_{ijk} &= X^T A_i^T + M_j^T\Lambda_k B_i^T + G_j^T\Lambda_k^- B_i^T \\ &\quad + A_iX + B_i\Lambda_k M_j + B_i\Lambda_k^- G_j + \frac{1}{d}E^T\bar{Z}E \\ &\quad + \frac{1}{d}E^T EX + \frac{1}{d}X^T E^T E, \\ \bar{\Theta}_{ijk} &= A_iX + B_i\Lambda_k M_j + B_i\Lambda_k^- G_j, \\ G_j &= H_jX = \begin{pmatrix}H_{j1}X_1 & 0\end{pmatrix} \\ &\triangleq \begin{pmatrix}G_{j1} & 0\end{pmatrix}, F_iX = M_i, \end{aligned} \quad (39)$$

and g_{j1i} denotes the ith row of G_{j1}.

Proof: Pre- and post-multiplying inequality (15) by diag$\{P^{-T}, I, I\}$ and diag$\{P^{-1}, I, I\}$, respectively, and noting $X = P^{-1}, G_j = H_jX, M_i = F_iX$, it can be derived that

$$\begin{pmatrix} \breve{\Xi}_{ijk} & * & * \\ \breve{\Upsilon}_i & \begin{pmatrix} -Q- \\ \frac{1}{d}E^TZE \end{pmatrix} & * \\ \bar{\Theta}_{iik} & A_{id} & -\frac{1}{d}Z^{-1} \end{pmatrix} < 0 \tag{40}$$

where

$$\begin{aligned} \breve{\Xi}_{ijk} &= \left(X^TA_i^T + M_i^T\Lambda_kB_i^T + G_i^T\Lambda_k^-B_i^T\right) \\ &\quad + \left(A_iX + B_i\Lambda_kM_i + B_i\Lambda_k^-G_i\right) \\ &\quad + X^TQX - \frac{1}{d}X^TE^TZEX, \\ \breve{\Upsilon}_i &= A_{id}^T + \frac{1}{d}E^TZEX \end{aligned}$$

Note that inequalities

$$-\frac{1}{d}X^TE^TZEX \le \frac{1}{d}E^TZ^{-1}E + \frac{1}{d}E^TEX + \frac{1}{d}X^TE^TE$$

and

$$X^TZ + Z^TX \le Z^TY^{-1}Z + X^TYX, Y > 0.$$

From here, it is easy to see that the inequality (40) holds only if

$$\begin{pmatrix} \tilde{\Xi}_{iik} & * & * \\ A_{id}^T & -Q & * \\ \bar{\Theta}_{iik} & A_{id} & -\frac{1}{d}Z^{-1} \end{pmatrix} < 0, \tag{41}$$

where

$$\begin{aligned} \tilde{\Xi}_{iik} &= \left(X^TA_i^T + M_i^T\Lambda_kB_i^T + G_i^T\Lambda_k^-B_i^T\right) \\ &\quad + \left(A_iX + B_i\Lambda_kM_i + B_i\Lambda_k^-G_i\right) \\ &\quad + X^TQX + \frac{1}{d}E^TZ^{-1}E + \frac{1}{d}E^TEX \\ &\quad + \frac{1}{d}X^TE^TE + \frac{1}{d}X^TE^TZEX. \end{aligned}$$

By Schur complement lemma, the above inequality is equivalent to that

$$\begin{pmatrix} \hat{\Xi}_{iik} & * & * & * & * \\ A_{id}^T & -Q & * & * & * \\ \bar{\Theta}_{iik} & A_{id} & -\frac{1}{d}Z^{-1} & * & * \\ X & 0 & 0 & -Q^{-1} & * \\ EX & 0 & 0 & 0 & -dZ^{-1} \end{pmatrix} < 0, \tag{42}$$

where

$$\begin{aligned} \hat{\Xi}_{iik} &= X^TA_i^T + M_i^T\Lambda_kB_i^T + G_i^T\Lambda_k^-B_i^T \\ &\quad + A_iX + B_i\Lambda_kM_i + B_i\Lambda_k^-G_i \\ &\quad + \frac{1}{d}E^TZ^{-1}E + \frac{1}{d}E^TEX + \frac{1}{d}X^TE^TE. \end{aligned}$$

Let $\bar{Z} = Z^{-1}$. Noting the inequality $-Q^{-1} \le Q - 2I$ and the Schur complement lemma, we then have $\widehat{\Phi}_{iik} < 0$. We can in a manner similar to the one above prove that $\widehat{\Phi}_{ijk} + \widehat{\Phi}_{jik} < 0$ in (36). This proof is omitted as the proof would be a repeat of the above steps.

By Lemma 3,

$$\varepsilon(EP, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_j)$$

if and only if

$$\varepsilon(P_1, \rho) \subset \bigcap_{j=1}^{r} \bar{L}(H_{j1})$$

where $H_j = [H_{j1}, 0], H_{j1} \in \mathbb{R}^{m\times s}, 0 \in \mathbb{R}^{m\times(n-s)}$. Then the condition

$$\varepsilon(P_1, \rho) \subset \bigcap_{j=1}^{r} \mathcal{L}(H_{j1})$$

is equivalent to [16]

$$h_{j1i}P_1^{-1}h_{j1i}^T \le \frac{1}{\rho}, i \in [1, m], \quad j = 1, 2, \cdots, r. \tag{43}$$

By Schur complement equivalence, the above inequality (43) is equivalent to

$$\begin{pmatrix} \rho & h_{j1i}\rho P_1^{-1} \\ \rho P_1^{-1}h_{j1i}^T & P_1^{-1} \end{pmatrix} \ge 0, \quad i \in [1, m], \ j = 1, 2, \cdots, r, \tag{44}$$

where h_{j1i} is the ith row of H_{j1}. Note that g_{j1i} is the ith row of G_{j1} . Then inequality (44) is equivalent to

$$\begin{pmatrix} \rho & \rho g_{j1i} \\ \rho g_{j1i}^T & X_1 \end{pmatrix} \ge 0, \quad i \in [1, m], \ j = 1, 2, \cdots, r.$$

Note that

$$X = P^{-1} = \begin{pmatrix} P_1^{-1} & 0 \\ -P_4^{-1}P_3P_1^{-1} & P_4^{-1} \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ X_3 & X_4 \end{pmatrix},$$

$\bar{Z} = Z^{-1}$. Then the condition (18) is equivalent to

$$[\lambda_{\max}(X_1^{-1}) + d\lambda_{\min}(Q)] \, \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^T\bar{Z}^{-1}E) \, \|\dot{\varphi}\|_c^2 \le \rho.$$

In this section, we shall focus on the delay-dependant dissipative analysis and control problem for system (9) with time-delay.

Definite the quadratic energy supply function associated with system (9) by

$$s(w, z) = z^T(t)\bar{Q}z(t) + 2z^T(t)\bar{S}w(t) + w^T(t)\bar{R}w(t), \tag{45}$$

where $\bar{Q} = \bar{Q}^T, \bar{R} = \bar{R}^T$ and $\bar{S}$ are real matrices with appropriate dimensions.

We then introduce the following definitions.

Definition 2 [31]: Given matrices $\bar{Q} = \bar{Q}^T, \bar{R} = \bar{R}^T$ and $\bar{S}$, system (9) with energy supply function (45) is said to be quadratic dissipative, if for some real functional $\varpi(\cdot)$, $\varpi(0) = 0$ such that the following condition is satisfied:

$$\int_0^{\tau} [z^T(t)\bar{Q}z(t) + 2z^T(t)\bar{S}w(t) + w^T(t)\bar{R}w(t)]dt + \varpi(x_0) \ge 0, \forall\tau \ge 0. \tag{46}$$

Furthermore, system (9) is said to be strictly quadratic dissipative, if for some positive scalar δ, the following condition is satisfied:

$$\int_0^\tau [z^T(t)\bar{Q}z(t)+2z^T(t)\bar{S}w(t)+w^T(t)\bar{R}w(t)]dt + \varpi(x_0)-\delta\int_0^\tau w^T(t)w(t)dt \geq 0, \forall\tau\geq 0 \quad (47)$$

Without loss of generality, we always assume matrix $\bar{Q}\leq 0$ and $\bar{Q}_1=(-\bar{Q})^{\frac{1}{2}}$ in the following part.

Theorem 3: Given scalar $\delta>0$, $d_0>0$, matrices $\bar{Q}=\bar{Q}^T$, $\bar{R}=\bar{R}^T$ and $\bar{S}$. Then for any delay $0<d\leq d_0$ and a positive scalar ρ, system (9) subject to actuator saturation and L_2 -disturbances is delay-dependent admissible and strictly dissipative within $\varepsilon(EP,\rho)$, if there exists a common non-singular matrix

$$P=\begin{pmatrix} P_1 & 0\\ P_3 & P_4\end{pmatrix}$$

($P_1\in\mathbb{R}^{s\times s}$, $P_3\in\mathbb{R}^{(n-s)\times s}$, $P_4\in\mathbb{R}^{(n-s)\times(n-s)}$, $P_1>0$, $|P_4|>0$), matrices

$$Q=\begin{pmatrix} Q_1 & Q_2\\ Q_2^T & Q_4\end{pmatrix}\geq 0,\ Z=\begin{pmatrix} Z_1 & Z_2\\ Z_2^T & Z_4\end{pmatrix}>0,$$

H_i, F_i such that the following matrix inequalities hold

$$\Psi_{iik}=\begin{pmatrix}\Xi_{iik} & * & * & * & *\\ \Upsilon_i & -\frac{1}{2}\bar{\Omega}_{22} & * & * & *\\ \Delta_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & *\\ \Theta_{iik} & A_{id} & W_i & -\frac{1}{d}Z^{-1} & *\\ \bar{Q}_1C_i & 0 & 0 & 0 & -I\end{pmatrix} < 0,\ i=1,2,\cdots,r, \quad (48)$$

$$\Psi_{ijk}+\Psi_{jik}=\begin{pmatrix}\begin{pmatrix}\Xi_{ijk}+\\ \Xi_{jik}\end{pmatrix} & * & * & * & *\\ \begin{pmatrix}\Upsilon_i+\\ \Upsilon_j\end{pmatrix} & -\bar{\Omega}_{22} & * & * & *\\ \begin{pmatrix}\Delta_i+\\ \Delta_j\end{pmatrix} & 0 & -\bar{\Omega}_{33} & * & *\\ \begin{pmatrix}\Theta_{ijk}+\\ \Theta_{jik}\end{pmatrix} & \begin{pmatrix}A_{id}\\ +\\ A_{jd}\end{pmatrix} & \begin{pmatrix}W_i\\ +\\ W_j\end{pmatrix} & -\frac{2}{d}Z^{-1} & *\\ \begin{pmatrix}\Pi_i+\\ \Pi_j\end{pmatrix} & 0 & 0 & 0 & -2I\end{pmatrix} < 0,\ j<i=1,2,\cdots,r, \quad (49)$$

$$\varepsilon(EP,\rho)\subset\bigcap_{j=1}^{r}\bar{L}(H_j), \quad (50)$$

$$\rho_1=[\lambda_{\max}(P_1)+d\lambda_{\min}(Q)]\,\|\varphi\|_c^2+\frac{d^2}{2}\lambda_{\max}(Z_1)\,\|\dot{\varphi}\|_c^2\leq\rho, \quad (51)$$

where

$$\Psi_{ijk}=\begin{pmatrix}\Xi_{ijk} & * & * & * & *\\ \Upsilon_i & -\frac{1}{2}\bar{\Omega}_{22} & * & * & *\\ \Delta_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & *\\ \Theta_{ijk} & A_{id}^T & W_i & -\frac{1}{d}Z^{-1} & *\\ \Pi_i & 0 & 0 & 0 & -I\end{pmatrix},$$
$$\Delta_i=W_i^TP-\bar{S}^TC_i,$$
$$\Pi_i=\bar{Q}_1C_i,$$
$$\bar{\Omega}_{22}=2Q+\frac{2}{d}E^TZE,$$
$$\bar{\Omega}_{33}=2\bar{R}-2\delta I, \quad (52)$$

and symbols Ξ_{ijk}, Υ_i and Θ_{ijk} can be found in (19). The estimate of attraction domain is

$$\rho_1=[\lambda_{\max}(P_1)+d\lambda_{\min}(Q)]\,\|\varphi\|_c^2+\frac{d^2}{2}\lambda_{\max}(Z_1)\,\|\dot{\varphi}\|_c^2\leq\rho$$

where

$$\|\varphi\|_c^2=\sup_{-d\leq t\leq 0}\|\varphi(t)\|^2,\ \|\dot{\varphi}\|_c^2=\sup_{-d\leq t\leq 0}\|\dot{\varphi}(t)\|^2. \quad (53)$$

Proof: Firstly, we prove that closed-loop system (9) is admissible within $\varepsilon(EP,\rho)$. By inequalities (48) and (49) along with

$$\bar{\Psi}_{hlk}=\sum_{k=1}^{2^m}\alpha_k(\sum_{i=1}^{r}h_i(\xi(t))h_i(\xi(t))\Psi_{iik}+\sum_{j<i}h_i(\xi(t))h_j(\xi(t))(\Psi_{ijk}+\Psi_{jik})),$$

it can then be derived that

$$\bar{\Psi}_{hlk}=\sum_{k=1}^{2^m}\alpha_k\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\xi(t))h_j(\xi(t))\Psi_{ijk} \triangleq\begin{pmatrix}\underline{\Xi}_{hlk} & * & * & * & *\\ \underline{\Upsilon}_h & -\frac{1}{2}\bar{\Omega}_{22} & * & * & *\\ \underline{\Delta}_h & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & *\\ \bar{A}_{hlk} & \bar{A}_{hd} & \bar{W}_h & -\frac{1}{d}Z^{-1} & *\\ \underline{\Pi}_h & 0 & 0 & 0 & -I\end{pmatrix} < 0, \quad (54)$$

where $\underline{\Delta}_h=\bar{W}_h^TP-\bar{S}^T\bar{C}_h$ and $\underline{\Pi}_h=\bar{Q}_1\bar{C}_h$. Symbols $\underline{\Xi}_{hlk}$ and $\underline{\Upsilon}_h$ can be found in (24). From Schur complement lemma, the above inequality is equivalent to

$$\begin{pmatrix}\underline{\Xi}_{hlk} & * & *\\ \underline{\Upsilon}_h & -\frac{1}{2}\bar{\Omega}_{22} & *\\ \underline{\Delta}_h & 0 & -\frac{1}{2}\bar{\Omega}_{33}\end{pmatrix}+\begin{pmatrix}\underline{\Pi}_h^T\\ 0\\ 0\end{pmatrix}(\underline{\Pi}_h\ \ 0\ \ 0)+d\begin{pmatrix}\bar{A}_{hlk}^T\\ \bar{A}_{hd}^T\\ \bar{W}_h^T\end{pmatrix}Z(\bar{A}_{hlk}\bar{A}_{hd}\bar{W}_h)<0. \quad (55)$$

Then, it is obvious from inequality (55) and the proof of Theorem 1 that system (9) with $w(t) = 0$ is admissible within $\varepsilon(EP, \rho)$. And the estimate of attraction domain is

$$\rho_1 = [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(Z_1) \|\dot{\varphi}\|_c^2 \le \rho.$$

Next, it will be prove that the close-loop system (9) is strictly dissipative. To this end, we select the descriptor type Lyapunov-Krasovskii functional (9) as $\varpi(\cdot)$ of Definition 2. it can be derived that $E^TP = P^TE \ge 0$ from the expression of P and E. Then, the time-derivative of $V(x_t)$ along with the solution of closed-loop system (9) is given by

$$\begin{aligned}\dot{V}(x_t) = {}& (\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d) + \bar{W}_h w(t))^T Px(t) \\ & + x^T(t)P^T(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d) + \bar{W}_h w(t)) \\ & + x^T(t)Qx(t) - x^T(t-d)Qx(t-d) \\ & + d(\bar{A}_{hlk}x(t) + \bar{A}_{hd}x(t-d) + \bar{W}_h w(t))^T Z(\bar{A}_{hlk}x(t) \\ & + \bar{A}_{hd}x(t-d) + \bar{W}_h w(t)) \\ & - \int_{-d}^{0} \dot{x}^T(t+\theta)E^TZE\dot{x}(t+\theta)d\theta \end{aligned} \tag{56}$$

By Jensen's inequality and expressions (55) and (56), it follows that

$$\begin{aligned}&\dot{V}(x_t) - z^T(t)\bar{Q}z(t) - 2z^T(t)\bar{S}w(t) \\ &\quad - w^T(t)\bar{R}w(t) + \delta w^T(t)w(t) \\ &\le \bar{\varsigma}_1^T(t)\left\{ \begin{pmatrix} \underline{\Xi}_{hlk} & * & * \\ \underline{\Upsilon}_h & -\frac{1}{2}\bar{\Omega}_{22} & * \\ \underline{\Delta}_h & 0 & -\frac{1}{2}\bar{\Omega}_{33} \end{pmatrix} + \begin{pmatrix} \Pi_h^T \\ 0 \\ 0 \end{pmatrix} (\Pi_h \quad 0 \quad 0) + d\begin{pmatrix} \bar{A}_{hlk}^T \\ \bar{A}_{hd}^T \\ \bar{W}_h^T \end{pmatrix} Z(\bar{A}_{hlk}\bar{A}_{hd}\bar{W}_h) \right\} \bar{\varsigma}_1(t) < 0\end{aligned}$$

where

$$\bar{\varsigma}_1(t) = \begin{pmatrix} x(t) \\ x(t-d) \\ w(t) \end{pmatrix} \tag{57}$$

It follows from the inquality (57) that

$$\begin{aligned}&\dot{V}(x_t) - z^T(t)\bar{Q}z(t) - 2z^T(t)\bar{S}w(t) - w^T(t)\bar{R}w(t) \\ &e + \delta w^T(t)w(t) < 0. \end{aligned} \tag{58}$$

By integrating (58) over the period $[0, \tau]$, it can be derived that

$$\begin{aligned}V(x_\tau) \le {}& V(x_0) + \int_0^\tau [z^T(t)\bar{Q}z(t) + 2z^T(t)\bar{S}w(t) \\ & + w^T(t)\bar{R}w(t)]dt - \delta\int_0^\tau w^T(t)w(t)dt. \end{aligned} \tag{59}$$

This shows that system (9) is strictly dissipative by Definition 2. In a word, the system (9) subject to actuator saturation and is admissible and strictly dissipative within $\varepsilon(EP, \rho)$. Furthermore estimate of attraction domain is

$$\rho_1 = [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(Z_1) \|\dot{\varphi}\|_c^2 \le \rho.$$

We should note that matrix inequalities in (48)-(50) of Theorem 3 are not linear matrix inequalities. In the following, we propose a design method of the dissipative controller via LMIs.

Theorem 4: Given scalar $\delta > 0$, $d_0 > 0$, matrices $\bar{Q} = \bar{Q}^T$, $\bar{R} = \bar{R}^T$ and $\bar{S}$. Then for any delay $0 < d \le d_0$ and a positive scalar ρ, system (9) subject to actuator saturation and L_2-disturbances is delay-dependent admissible and strictly dissipative within $\varepsilon(EX^{-1}, \rho)$, if there exists a common matrix

$$X = P^{-1} = \begin{pmatrix} P_1^{-1} & 0 \\ -P_4^{-1}P_3P_1^{-1} & P_4^{-1} \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ X_3 & X_4 \end{pmatrix}$$

($X_1 \in \mathbb{R}^{s\times s}$, $X_4 \in \mathbb{R}^{(n-s)\times(n-s)}$, $X_1 > 0$, $|X_4| \neq 0$), matrices

$$Q = \begin{pmatrix} Q_1 & Q_2 \\ Q_2^T & Q_4 \end{pmatrix} \ge 0,$$

$\bar{Z} = Z^{-1} > 0$, G_i, M_i such that the following set LMIs hold

$$\begin{aligned}&\widehat{\Psi}_{iik} \\ &= \begin{pmatrix} \bar{\Xi}_{iik} & * & * & * & * & * & * \\ A_{id}^T & -Q & * & * & * & * & * \\ \underline{\Delta}_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & * & * & * \\ \bar{\Theta}_{ijk} & A_{id} & W_i & -\frac{1}{d}\bar{Z} & * & * & * \\ \underline{\Pi}_i & 0 & 0 & 0 & -I & * & * \\ X & 0 & 0 & 0 & 0 & -2\Omega_{66} & * \\ EX & 0 & 0 & 0 & 0 & 0 & -d\bar{Z} \end{pmatrix} \\ &< 0, i = 1, 2, \cdots, r, k \in [1, 2^m], \end{aligned} \tag{60}$$

$$\begin{aligned}&\widehat{\Psi}_{ijk} + \widehat{\Psi}_{jik} = \begin{pmatrix} \Gamma_1 & * \\ \Gamma_2 & \Gamma_3 \end{pmatrix} < 0, \\ &j < i = 1, 2, \cdots, r, k \in [1, 2^m], \end{aligned} \tag{61}$$

$$\begin{pmatrix} \rho & \rho g_{j1i} \\ \rho g_{j1i}^T & X_1 \end{pmatrix} \ge 0, i \in [1, m], j = 1, 2, \cdots, r, \tag{62}$$

$$\begin{aligned}&[\lambda_{\max}(X_1^{-1}) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 \\ &+ \frac{d^2}{2}\lambda_{\max}(E^T\bar{Z}^{-1}E) \|\dot{\varphi}\|_c^2 \le \rho, \end{aligned} \tag{63}$$

where

$$\widehat{\Psi}_{ijk} = \begin{pmatrix} \bar{\Xi}_{ijk} & * & * & * & * & * & * \\ A_{id}^T & -Q & * & * & * & * & * \\ \Delta_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & * & * & * \\ \bar{\Theta}_{ijk} & A_{id} & W_i & -\frac{1}{d}\bar{Z} & * & * & * \\ \underline{\Pi}_i & 0 & 0 & 0 & -I & * & * \\ X & 0 & 0 & 0 & 0 & -\Omega_{66} & * \\ EX & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}d\bar{Z} \end{pmatrix}$$

$$\begin{aligned}&\Omega_{66} = \frac{1}{2}(2I - Q), \\ &\underline{\Delta}_i = W_i^T - \bar{S}^TC_iX, \underline{\Pi}_i = \bar{Q}_1C_iX, \\ &G_j = H_jX = (H_{j1}X_1 \quad 0) \triangleq (G_{j1} \quad 0), F_iX = M_i,\end{aligned}$$

$$\Gamma_1 = \begin{pmatrix} \begin{pmatrix} \bar{\Xi}_{ijk} + \\ \bar{\Xi}_{jik} \end{pmatrix} & * & * \\ \begin{pmatrix} A_{id}^T + \\ A_{jd}^T \end{pmatrix} & -2Q & * \\ \begin{pmatrix} \underline{\Delta}_i + \\ \underline{\Delta}_j \end{pmatrix} & 0 & -\bar{\Omega}_{33} \end{pmatrix}$$

$$\Gamma_2 = \begin{pmatrix} \begin{pmatrix} \bar{\Theta}_{ijk} + \\ \bar{\Theta}_{jik} \end{pmatrix} & \begin{pmatrix} A_{id} + \\ A_{jd} \end{pmatrix} & \begin{pmatrix} W_i + \\ W_j \end{pmatrix} \\ \begin{pmatrix} \underline{\Pi}_i + \\ \underline{\Pi}_j \end{pmatrix} & 0 & 0 \\ X & 0 & 0 \\ EX & 0 & 0 \end{pmatrix}$$

$$\Gamma_3 = \begin{pmatrix} -\frac{2}{d}\bar{Z} & * & * & * \\ 0 & -2I & * & * \\ 0 & 0 & -\Omega_{66} & * \\ 0 & 0 & 0 & -\frac{d}{2}\bar{Z} \end{pmatrix} \tag{64}$$

and g_{j1i} denotes the ith row of G_{j1} The other symbols $\bar{\Xi}_{ijk}, \bar{\Theta}_{ijk}$ can be found in (39). and $\bar{\Omega}_{33}$ is defined in (52). The estimate of attraction domain is $\rho_1 = [\lambda_{\max}(X_1^{-1}) + d\lambda_{\min}(Q)] \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(E^T\bar{Z}^{-1}E)\|\dot{\varphi}\|_c^2 \leq \rho$.

Proof: Pre- and post-multiplying inequality (48) of Theorem 3 by diag$\{P^{-T}, I, I, I, I\}$ and $diag\{P^{-1}, I, I, I, I\}$, respectively, and noting $X = P^{-1}, G_j = H_jX, M_i = F_iX$, then the inequality $\Psi_{iik} < 0$ in (48) only if

$$\tilde{\Psi}_{iik} = \begin{pmatrix} \breve{\Xi}_{ijk} & * & * & * & * \\ \breve{\Upsilon}_i & -\frac{1}{2}\bar{\Omega}_{22} & * & * & * \\ \underline{\Delta}_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & * \\ \bar{\Theta}_{iik} & A_{id} & W_i & -\frac{1}{d}Z^{-1} & * \\ \underline{\Pi}_i & 0 & 0 & 0 & -I \end{pmatrix} < 0, \tag{65}$$

where symbols $\breve{\Xi}_{ijk}$ and $\breve{\Upsilon}_i$ are defined in (40), $\bar{\Theta}_{iik}$ and $\bar{\Omega}_{22}$ can be found in (39) and (52) respectively.

Noting

$$-\frac{1}{d}X^TE^TZEX \leq \frac{1}{d}E^TZ^{-1}E + \frac{1}{d}E^TEX + \frac{1}{d}X^TE^TE,$$
$$X^TZ + Z^TX \leq Z^TY^{-1}Z + X^TYX\ (Y > 0),$$

then it is easy to see that inequality (65) holds only if

$$\hat{\Psi}_{iik} = \begin{pmatrix} \tilde{\Xi}_{iik} & * & * & * & * \\ A_{id}^T & -Q & * & * & * \\ \underline{\Delta}_i & 0 & -\frac{1}{2}\bar{\Omega}_{33} & * & * \\ \bar{\Theta}_{iik} & A_{id} & W_i & -\frac{1}{d}Z^{-1} & * \\ \underline{\Pi}_i & 0 & 0 & 0 & -I \end{pmatrix} < 0 \tag{66}$$

where $\tilde{\Xi}_{iik}$ is defined in (41). Let $\bar{Z} = Z^{-1}$. Noting $-Q^{-1} \leq Q - 2I$ and Schur complement lemma, then $\hat{\Psi}_{iik} < 0$ in (66) only if $\widehat{\Psi}_{iik} < 0$ in (60).

Similarly, we can prove that $\Psi_{ijk} + \Psi_{jik} < 0$ in (49) only if $\widehat{\Psi}_{ijk} + \widehat{\Psi}_{jik} < 0$ in (61). See the Appendix for the full proof.

From the proof above of theorem 2, it can be seen that inequalities in (50) and (51) are respectively equivalent to inequalities (62) and (63).

Remark 2: Let $\bar{Q} = -\frac{1}{\gamma^2}I, \bar{R} - \delta I = I, \bar{S} = 0$, the dissipative control then degenerates into H_∞ control. This implies that the dissipative control is more general than H_∞ control. Once the dissipative controller is obtained, H_∞ controller is only a special case of the former, which could potentially reduce the time and cost spent when designing the controller for an actual system with time-delay in the presence of actuator saturation.

IV. DESIGN EXAMPLE

Example 1: The following nonlinear descriptor system (67) will be expressed as a T-S fuzzy model, and the dissipative controller under saturated control will be designed by solving the set of LMIs in Theorem 4 as validation for the proposed method. Consider a nonlinear time-delay system with actuator saturation [3]

$$(1 + a\cos\theta(t))\ddot{\theta}(t) = -b\dot{\theta}^3(t) + c\theta(t) + c_d\theta(t-d) + gw(t) + f\,\text{sat}(u), \tag{67}$$

where the range of $\dot{\theta}(t)$ is assumed to satisfy $|\dot{\theta}(t)| < \phi$.

When $|a| \geq 1$, the methods in [41] and [45] are not applicable. [3] uses the descriptor fuzzy control approach to achieve the stabilization of the system (67), in which the condition $|a| < 1$ is no longer needed, and reducing the terms and LMIs computations involved. We will validate our derivation by using the nonlinear system in the following section.

Let $x(t) = (x_1(t)\ x_2(t)\ x_3(t))^T$, where $x_1(t) = \theta(t), x_2(t) = \dot{\theta}(t), x_3(t) = \ddot{\theta}(t)$. Then nonlinear system (67) is exactly represented as the following T-S descriptor fuzzy model.

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{3} h_i(\xi(t))(A_ix(t) + A_{id}x(t-d)) \\ &\quad + \sum_{i=1}^{3} h_i(\xi(t))(B_isat(u(t)) + W_iw(t)), \\ z(t) &= \sum_{i=1}^{3} h_i(\xi(t))C_ix(t), \\ x(t) &= \varphi(t), \quad t \in [-d, 0], \end{aligned} \tag{68}$$

where

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & -b(\phi^2+2) & a-1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & 0 & -a-1-a\phi^2 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & 0 & a-1 \end{pmatrix},$$

$$
\begin{aligned}
B_1 &= B_2 = B_3 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \\
W_1 &= W_2 = W_3 = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}, \\
A_{1d} &= A_{2d} = A_{3d} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix}, \\
C_1 &= C_2 = C_3 = \begin{pmatrix} c_{11} & 0 & 0 \end{pmatrix}, \\
h_1 &= \frac{x_2^2(t)}{\phi^2+2}, h_2 = \frac{1+\cos x_1(t)}{\phi^2+2}, \\
h_3 &= \frac{\phi^2 - x_2^2(t) + 1 - \cos x_1(t)}{\phi^2+2}.
\end{aligned}
\tag{69}
$$

For simulation purposes, we select the following coefficients

$$
\begin{aligned}
&a = -0.0063,\ b = 0.0209,\ c = -0.00145,\ f = -0.0074, \\
&g = -0.00191, c_{11} = 1, c_d = 0.0267, \\
&\bar{Q} = -1, \bar{R} = 0.0100, \bar{S} = 0.0300, d = 0.0011, \rho = 1.5 \\
&\phi = \begin{pmatrix} 0.1000 & -0.1000 & -0.1000 \end{pmatrix}.
\end{aligned}
\tag{70}
$$

By solving the set of LMIs in (60)-(63), then we can obtain the feasible solutions of a fuzzy dissipative controller with dissipative degree $\delta = 2.8909$ as follows.

$$
\begin{aligned}
P &= \begin{pmatrix} 41.9650 & 2.4611 & 0 \\ 2.4611 & 12.6190 & 0 \\ 50.3500 & 25.0701 & 0.0040 \end{pmatrix}, \\
P_1 &= \begin{pmatrix} 41.9650 & 2.4611 \\ 2.4611 & 12.6190 \end{pmatrix}, \\
F_1 &= 10^3(0.0239\ 6.7993\ \ -0.0911), \\
F_2 &= 10^3(0.0239\ 7.7621\ 0.0039), \\
F_3 &= 10^3(0.0239\ 7.7621\ \ -0.0911), \\
H_1 &= 10^{-8}(0.3167\ \ -0.5293\ 0), \\
H_2 &= 10^{-8}(-0.3167\ 0.5293\ 0), \\
H_3 &= 10^{-8}(-0.3167\ 0.5293\ 0).
\end{aligned}
\tag{71}
$$

The simulation shows that system (67) under the saturated control is regular, impulse-free and stable when the following initial conditions are satisfied

$$
\begin{aligned}
\rho_1 &= [\lambda_{\max}(P_1) + d\lambda_{\min}(Q)]\, \|\varphi\|_c^2 + \frac{d^2}{2}\lambda_{\max}(Z_1)\, \|\dot{\varphi}\|_c^2 \\
&= 42.1707\, \|\varphi\|_c^2 + 6.0500e-007\, \|\dot{\varphi}\|_c^2 \le 1.5.
\end{aligned}
\tag{72}
$$

Let $\varphi = [0.1000 - 0.1000 - 0.1000]$, $\forall t \in [-0.00110]$. Figures 1-3 show state evolution under saturated control. Figure 4 indicates the saturated actuator output corresponding to the trajectory shown in Figures 1-3.

Remark 3: The example shows that the obtained method can stabilize the system and achieve dissipative performance for the prescribed disturbance attenuation level $\delta = 2.8909$ and disturbances $w(t) = e^{-t}\sin t$ in the presence of actuator saturation. An estimation of the attraction domain is $\rho_1 = 1.2651$ in simulation above.

Example 2: In many cases, it is more convenient to express practical models by the descriptor systems than ordinary ones due to the presence of algebraic constraint equations. The following is an example economic model expressed using a descriptor system instead of the ordinary

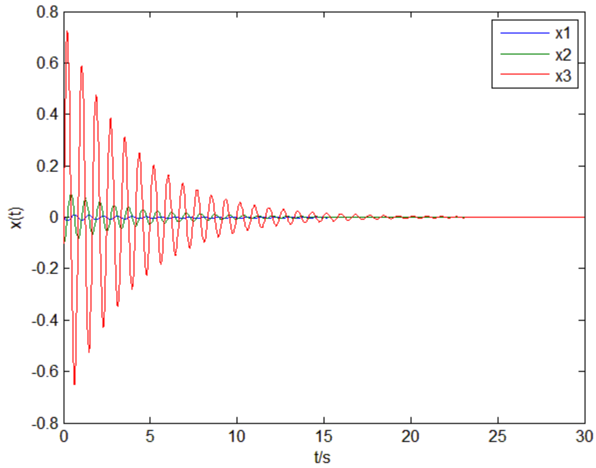

FIGURE 1. The state response curves.

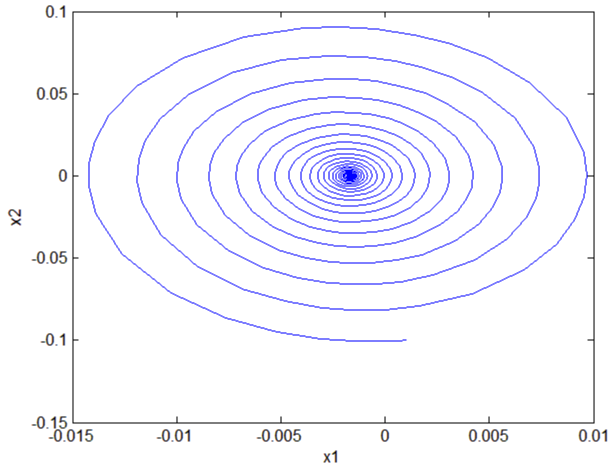

FIGURE 2. The phase plane curve.

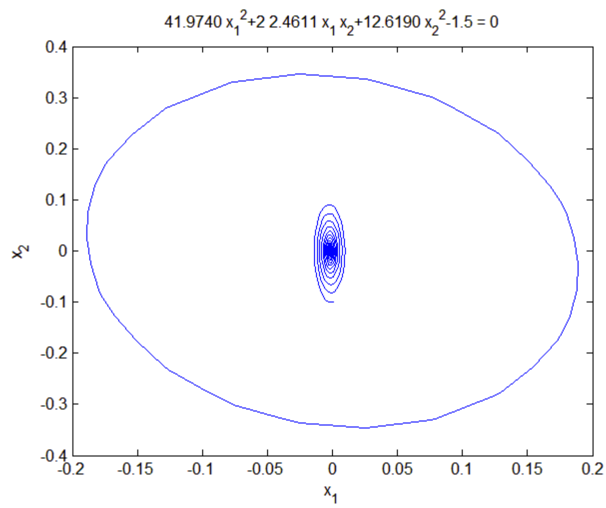

FIGURE 3. The ellipsoid $\varepsilon(EP, 1.5)$ and trajectory with disturbance.

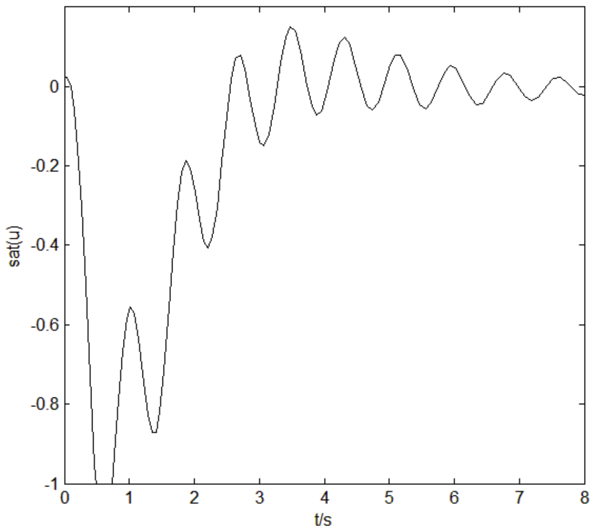

FIGURE 4. The actuator output corresponding to the trajectories shown in figures 1-3.

one [36].

$$\begin{aligned}
\dot{x}_1(t) &= \left(-\frac{\bar{\alpha}\bar{\beta}}{r_2} - \frac{\bar{\eta}c}{p}\right)x_1(t) + \bar{\alpha}x_2(t) - \frac{c}{p}x_3(t) - \bar{\eta}x_1^2(t) \\
&\quad - x_1(t)x_3(t) + \bar{\alpha}_d x_1(t-d) + d_{11}w(t), \\
\dot{x}_2(t) &= \bar{\beta}x_1(t) - r_2 x_2(t), \\
0 &= p\left(\frac{\bar{\alpha}\bar{\beta}}{r_2} - r_1 - \bar{\beta} - \frac{\bar{\eta}c}{p}\right)x_1(t) + px_1(t)x_3(t) \\
&\quad + b_{13}sat(u(t)), \\
z(t) &= c_{11}x_1(t) + c_{12}x_2(t) + c_{13}x_3(t).
\end{aligned}$$

where $u(t)$ represents the government's ability to develop and manage free resources. Excessive control exerted by the government can lead to an instability or even collapse of entire population systems. d_{11} indicates the disturbance factor for young populations, such as seasonal interference and disease interference. The meaning of the other coefficients and variables are described in [36]. The above non-linear economic model can be expressed by the following T-S fuzzy descriptor model.

$$\begin{aligned}
E\dot{x}(t) &= \sum_{i=1}^{2} h_i(x_1(t))(A_i x(t) + B_i sat(u(t)) \\
&\quad + A_{id}x(t-d) + W_i w(t)) \\
z(t) &= \sum_{i=1}^{2} h_i(x_1(t))C_i x(t),
\end{aligned}$$

where

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_1 = \begin{pmatrix} \left(-\frac{\bar{\alpha}\bar{\beta}}{r_2} - \frac{\bar{\eta}c}{p} + \bar{\eta}l\right) & \bar{\alpha} & -\frac{c}{p} + l \\ \bar{\beta} & -r_2 & 0 \\ p\left(\frac{\bar{\alpha}\bar{\beta}}{r_2} - r_1 - \bar{\beta} - \frac{\bar{\eta}c}{p}\right) & 0 & -pl \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \left(-\frac{\bar{\alpha}\bar{\beta}}{r_2} - \frac{\bar{\eta}c}{p} - \bar{\eta}l\right) & \bar{\alpha} & -\frac{c}{p} - l \\ \bar{\beta} & -r_2 & 0 \\ p\left(\frac{\bar{\alpha}\bar{\beta}}{r_2} - r_1 - \bar{\beta} - \frac{\bar{\eta}c}{p}\right) & 0 & pl \end{pmatrix},$$

$$A_{1d} = \begin{pmatrix} \bar{\alpha}_d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_{2d} = \begin{pmatrix} \bar{\alpha}_d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$W_1 = W_2 = \begin{pmatrix} d_{11} \\ 0 \\ 0 \end{pmatrix}, \quad B_1 = B_2 = \begin{pmatrix} 0 \\ 0 \\ b_{13} \end{pmatrix},$$

$$\begin{aligned}
C_1 &= C_2 = (c_{11}\; c_{12}\; c_{13}), \\
h_1(x_1(t)) &= \frac{1}{2}\left(1 - \frac{x_1(t)}{l}\right), \\
h_2(x_1(t)) &= \frac{1}{2}\left(1 + \frac{x_1(t)}{l}\right), \; |x_1(t)| < l, \, l > 0.
\end{aligned}$$

By following the steps prescribed by the method presented in Example 1, we are able to stabilize the above non-linear economic model in the presence disturbances and actuator saturation while achieving dissipative performance for the prescribed disturbance attenuation along with obtaining an estimate of the attraction domain. For the sake of brevity, we will not repeat the proof here.

Current research has shown that T-S fuzzy descriptor systems can have a wide range of applications in bio-economic systems. For example, in [47], the hepatitis B model is constructed using T-S fuzzy descriptor systems. The fuzzy controller is designed to inhibit the development of the disease while achieving system stability at free-disease equilibrium. In [37], a new SEIR (Susceptible, Exposed, Infectious, Recovered) infectious disease model was established based on a T-S fuzzy descriptor system. The state evolution diagram of the susceptible and infectious persons was given to show that the T-S fuzzy descriptor system can fit the SEIR infectious disease model. The dissipative controller is designed to have the susceptible population reach a stable state while effectively inhibiting the interference caused by seasonal influence. That is, by controlling the number of susceptible persons within a population, the purpose of controlling the spread of the disease is achieved. In turn, an outbreak of the infectious disease modeled may be avoided.

The applications of T-S fuzzy descriptor systems in real-life situations demonstrate that the analysis and control of the T-S fuzzy descriptor system is worth studying in depth.

V. CONCLUSION

A delay-dependent admissibility analysis method and a dissipative controller design scheme are developed for a class of nonlinear time-delay descriptor system subject to actuator saturation and L_2-disturbances via a T-S fuzzy model. The presented method can eliminate the impulsive behavior of a descriptor system which ensures the existence and uniqueness of solutions, and is more suitable for the admissibility analysis and robust control synthesis for the time-delay nonlinear descriptor systems in the presence of actuator saturation and L_2-disturbances. Moreover, H_∞ control processes can be achieved in the same design process which indicates that the cost and time should be potentially reduce when designing a controller for an actual physical system. The study may lay a foundation for investigating the control synthesis of T-S fuzzy time-delay descriptor control systems subject to actuator saturation.

APPENDIX

Proof of $\Psi_{ijk} + \Psi_{jik} < 0$ in Theorem 4:

Proof: Before and after multiplying the inequality (49) of Theorem 3 by $diag\{P^{-T}, I, I, I, I\}$ and $diag\{P^{-1}, I, I, I, I\}$ respectively, and noting $X = P^{-1}$, $G_j = H_jX$, $M_i = F_iX$, then the inequality $\Psi_{ijk} + \Psi_{jik} < 0$ only if

$$\begin{pmatrix} \begin{pmatrix} \breve{\Xi}_{ijk} \\ + \\ \breve{\Xi}_{jik} \end{pmatrix} & * & * & * & * \\ \begin{pmatrix} \breve{\Upsilon}_i \\ + \\ \breve{\Upsilon}_j \end{pmatrix} & -\bar{\Omega}_{22} & * & * & * \\ \begin{pmatrix} \underline{\Delta}_i \\ + \\ \underline{\Delta}_j \end{pmatrix} & 0 & -\bar{\Omega}_{33} & * & * \\ \begin{pmatrix} \bar{\Theta}_{ijk} \\ + \\ \bar{\Theta}_{jik} \end{pmatrix} & \begin{pmatrix} A_{id} \\ + \\ A_{jd} \end{pmatrix} & \begin{pmatrix} W_i \\ + \\ W_j \end{pmatrix} & -\frac{2}{d}Z^{-1} & * \\ \begin{pmatrix} \underline{\Pi}_i \\ + \\ \underline{\Pi}_j \end{pmatrix} & 0 & 0 & 0 & -2I \end{pmatrix} < 0, \quad (73)$$

where $\breve{\Xi}_{ijk}$ and $\breve{\Upsilon}_i$ are defined in (40), $\bar{\Theta}_{ijk}$ is defined in (39), $\bar{\Omega}_{22}$ and $\bar{\Omega}_{33}$ is defined in (52), $\underline{\Pi}_i$ and $\underline{\Delta}_i$ can be found in (64). Note

$$-\frac{1}{d}X^TE^TZEX \leq \frac{1}{d}E^TZ^{-1}E + \frac{1}{d}E^TEX + \frac{1}{d}X^TE^TE,$$
$$X^TZ + Z^TX \leq Z^TY^{-1}Z + X^TYX \; (Y > 0).$$

Then it can be seen that the above inequality (73) holds only if

$$\begin{pmatrix} \begin{pmatrix} \tilde{\Xi}_{ijk} \\ + \\ \tilde{\Xi}_{jik} \end{pmatrix} & * & * & * & * \\ \begin{pmatrix} A_{id}^T \\ + \\ A_{jd}^T \end{pmatrix} & -2Q & * & * & * \\ \begin{pmatrix} \underline{\Delta}_i \\ + \\ \underline{\Delta}_j \end{pmatrix} & 0 & -\bar{\Omega}_{33} & * & * \\ \begin{pmatrix} \bar{\Theta}_{ijk} \\ + \\ \bar{\Theta}_{jik} \end{pmatrix} & \begin{pmatrix} A_{id} \\ + \\ A_{jd} \end{pmatrix} & \begin{pmatrix} W_i \\ + \\ W_j \end{pmatrix} & -\frac{2}{d}Z^{-1} & * \\ \begin{pmatrix} \underline{\Pi}_i + \\ \underline{\Pi}_j \end{pmatrix} & 0 & 0 & 0 & -2I \end{pmatrix} < 0, \quad (74)$$

where $\tilde{\Xi}_{ijk}$ can be found in (41).

Let $\bar{Z} = Z^{-1}$. Noting the inequality $-Q^{-1} \leq Q - 2I$ and Schur complement lemma, then the inequality (74) holds only if $\widehat{\Psi}_{ijk} + \widehat{\Psi}_{jik} < 0$ in (61).

REFERENCES

[1] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems* (Lecture Notes in Control and Information Sciences), vol. 332. Berlin, Germany: Springer, 2006.

[2] S. Zhu, C. Zhang, Z. Cheng, and J. Feng, "Delay-dependent robust stability criteria for two classes of uncertain singular time-delay systems," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 880–885, May 2007.

[3] C. Lin, G. Wang, T. H. Lee, and Y. He, *LMI Approach to Analysis and Control of Takagi-Sugeno Fuzzy Systems With Time Delay*. Berlin, Germany: Springer, 2007.

[4] C. Lin, Q.-G. Wang, and T. H. Lee, "Stability and stabilization of a class of fuzzy time-delay descriptor systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 4, pp. 542–551, Aug. 2006.

[5] M. Kchaou, H. Gassara, and A. El-Hajjaji, "Robust observer-based control design for uncertain singular systems with time-delay," *Int. J. Adapt. Control Signal Process.*, vol. 28, no. 2, pp. 169–183, 2014.

[6] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston, MA, USA: Birkhäser, 2003.

[7] B.-S. Chen, S.-S. Wang, and H.-C. Lu, "Stabilization of time-delay systems containing saturating actuators," *Int. J. Control*, vol. 47, no. 3, pp. 867–881, 1988.

[8] S. Tarbouriech, Jr., and J. M. G. Da Silva, "Synthesis of controllers for continuous-time delay systems with saturating controls via LMIs," *IEEE Trans. Autom. Control*, vol. 45, no. 1, pp. 105–111, Jan. 2000.

[9] Y.-Y. Cao, Z. Lin, and T. Hu, "Stability analysis of linear time-delay systems subject to input saturation," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 49, no. 2, pp. 233–240, Feb. 2002.

[10] Y. Chen, S. Fei, and Y. Li, "Stabilization of neutral time-delay systems with actuator saturation via auxiliary time-delay feedback," *Automatica*, vol. 52, pp. 242–247, Feb. 2015.

[11] H. Li, J. Wang, L. Wu, H.-K. Lam, and Y. Gao, "Optimal guaranteed cost sliding-mode control of interval type-2 fuzzy time-delay systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 1, pp. 246–257, Feb. 2018.

[12] L. Zhang, E. K. Boukas, and A. Haidar, "Delay-range-dependent control synthesis for time-delay systems with actuator saturation," *Automatica*, vol. 44, no. 10, pp. 2691–2695, Oct. 2008.

[13] A. Haidar, E. K. Boukas, S. Xu, and J. Lam, "Exponential stability and static output feedback stabilisation of singular time-delay systems with saturating actuators," *IET Control Theory Appl.*, vol. 3, no. 9, pp. 1293–1305, Sep. 2009.

[14] H. Gassara, M. Kchaou, A. El Hajjaji, and M. Chaabane, "Control of time delay fuzzy descriptor systems with actuator saturation," *Circuits Syst. Signal Process.*, vol. 33, no. 12, pp. 3739–3756, 2014.

[15] Y. Ma, P. Yang, and Q. Zhang, "Robust H_∞ control for uncertain singular discrete T–S fuzzy time-delay systems with actuator saturation," *J. Franklin Inst.*, vol. 353, no. 13, pp. 3290–3311, 2016.

[16] T. Hu and Z. Lin, *Control Systems With Actuator Saturation: Analysis and Design*. Boston, MA, USA: Birkhäuser, 2001.

[17] T. Hu, Z. Lin, and B. M. Chen, "Analysis and design for discrete-time linear systems subject to actuator saturation," *Syst. Control Lett.*, vol. 45, no. 2, pp. 97–112, 2002.

[18] T. Hu, Z. Lin, and B. M. Chen, "An analysis and design method for linear systems subject to actuator saturation and disturbance," *Automatica*, vol. 38, no. 2, pp. 351–359, 2002.

[19] Y. Yu, H.-K. Lam, and K. Y. Chan, "T–S fuzzy-model-based output feedback tracking control with control input saturation," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3514–3528, Dec. 2018.

[20] Z. Lin, A. Saberi, and A. R. Teel, "The almost disturbance decoupling problem with internal stability for linear systems subject to input saturation—State feedback case," *Automatica*, vol. 32, no. 4, pp. 619–624, 1996.

[21] H. Fang, Z. Lin, and T. Hu, "Analysis of linear systems in the presence of actuator saturation and L_2-disturbances," *Automatica*, vol. 40, no. 7, pp. 1229–1238, 2004.

[22] Y.-Y. Cao and Z. Lin, "Stability analysis of discrete-time systems with actuator saturation by a saturation-dependent Lyapunov function," *Automatica*, vol. 39, no. 7, pp. 1235–1241, 2003.

[23] H. Liu, E.-K. Boukas, F. Sun, and D. W. C. Ho, "Controller design for Markov jumping systems subject to actuator saturation," *Automatica*, vol. 42, no. 3, pp. 459–465, 2006.

[24] Y.-F. Li, "Design of H_∞ static output feedback control for discrete-time systems with limited actuator," *Asian J. Control*, vol. 17, no. 1, pp. 284–296, Jan. 2015.

[25] Z. Lin and L. Lv, "Set invariance conditions for singular linear systems subject to actuator saturation," *IEEE Trans. Autom. Control*, vol. 52, no. 12, pp. 2351–2355, Dec. 2007.

[26] L. Lv and Z. Lin, "Analysis and design of singular linear systems under actuator saturation and L_2/L_∞ disturbances," *Syst. Control Lett.*, vol. 57, no. 11, pp. 904–912, 2008.

[27] W. Lan and J. Huang, "Semiglobal stabilization and output regulation of singular linear systems with input saturation," *IEEE Trans. Autom. Control*, vol. 48, no. 7, pp. 1274–1280, Jul. 2003.

[28] X. Ji, Y. Sun, and H. Su, "Analysis and design for singular discrete linear systems subject to actuator saturation," *Asian J. Control*, vol. 13, no. 2, pp. 350–355, 2011.

[29] S. Ma, C. Zhang, and S. Zhu, "Robust stability for discrete-time uncertain singular Markov jump systems with actuator saturation," *IET Control Theory Appl.*, vol. 5, no. 2, pp. 255–262, Jan. 2011.

[30] C. Yang, J. Sun, and X. Ma, "Stabilization bound of singularly perturbed systems subject to actuator saturation," *Automatica*, vol. 49, no. 2, pp. 457–462, 2013.

[31] J. C. Willems, "Dissipative dynamical systems Part I: General theory," *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 321–351, 1972.

[32] L. Wu, W. X. Zheng, and H. Gao, "Dissipativity-based sliding mode control of switched stochastic systems," *IEEE Trans. Autom. Control*, vol. 58, no. 3, pp. 785–791, Mar. 2013.

[33] L. Wu, X. Yang, and H.-K. Lam, "Dissipativity analysis and synthesis for discrete-time T–S fuzzy stochastic systemswith time-varying delay," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 2, pp. 380–394, Apr. 2014.

[34] J. Zhao and D. J. Hill, "Dissipativity theory for switched systems," *IEEE Trans. Autom. Control*, vol. 53, no. 4, pp. 941–953, May 2008.

[35] Z. Feng, J. Lam, and H. Gao, "α-Dissipativity analysis of singular time-delay systems," *Automatica*, vol. 47, no. 11, pp. 2548–2552, 2011.

[36] Y. Zhang, Q.-L. Zhang, and T.-Z. Zhang, "H_∞ control of descriptor bioeconomic systems," *J. Northeastern Univ. Natural Sci.*, vol. 32, no. 10, pp. 1369–1373, 2011.

[37] B. Y. Zhu, S. Y. Xing, and Q. L. Zhang, *Dissipative Analysis and Control for T-S Fuzzy Descriptor Systems*. Beijing, China: Tsinghua Univ. Press, 2016.

[38] Q. L. Zhang and B. Y. Zhu, *Analysis and Control for T-S Fuzzy Descriptor Systems*. Beijing, China: National Defense Industry Press, 2012.

[39] B. Zhu, J. Zhang, Q. Zhang, and D. Zhang, "Dissipative control for T–S fuzzy descriptor systems with actuator saturation and disturbances," *J. Franklin Inst.*, vol. 353, no. 18, pp. 4950–4978, 2016.

[40] C. Han, L. Wu, S. Peng, and Q. Zeng, "On dissipativity of Takagi–Sugeno fuzzy descriptor systems with time-delay," *J. Franklin Inst.*, vol. 349, no. 10, pp. 3170–3184, 2012.

[41] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 4, pp. 442–452, Aug. 2000.

[42] L. Teng, Y. Wang, W. Cai, and H. Li, "Efficient robust fuzzy model predictive control of discrete nonlinear time-delay systems via Razumikhin approach," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 2, pp. 262–272, Feb. 2019.

[43] L. Teng, Y. Wang, W. Cai, and H. Li, "Fuzzy model predictive control of discrete-time systems with time-varying delay and disturbances," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1192–1206, Jun. 2018.

[44] L. Xie and C. E. de Souza, "Robust H_∞ control for linear systems with norm-bounded time-varying uncertainty," *IEEE Trans. Autom. Control*, vol. 37, no. 8, pp. 1188–1191, Aug. 1992.

[45] K. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H_∞ control theory, and linear matrix inequalities," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 1–13, Feb. 1996.

[46] X. Zhang and Y. Chen, "Admissibility and robust stabilization of continuous linear singular fractional order systems with the fractional order α: The $0 < \alpha < 1$ case," *ISA Trans.*, vol. 82, pp. 42–50, Apr. 2017.

[47] C. L. Sun and L. C. Zhao, "T-S fuzzy control with hepatitis B model," (in Chinese), *J. Univ. Sci. Technol. Liaoning*, vol. 33, no. 4, pp. 344–348, 2010.

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