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An Improved Method for Measuring the Complexity in Complex Networks Based on Structure Entropy

MINGLI LEI, LIRONG LIU, AND DAIJUN WEI
School of Science, Hubei Minzu University, Enshi 445000, China
Corresponding author: Daijun Wei (prof.wei@hotmail.com)

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ABSTRACT Although structure entropy is a useful method to measure the complexity of complex networks, there exist shortcomings, such as the limits of network scales and network types. By combining structure entropy and the absolute density of network, a method is improved to effectively measure the complexity of complex networks. For the improved measure, not only the topology of network is considered, but also the scales of network are considered, and the measurement of network complexity is not affected by the network scales and types. Moreover, the complexity of small-world networks, BA scale free networks, Sierpinski self-similarity networks, Erodös-Rényi (ER) random networks and six real networks (i.e., the 9/11 terrorist network, Celegans network, a USAir network, the USA Political blogs network, a collaboration network in science of networks (NetScience) and yeast protein interaction (YPI) network) are measured by employing the method. The results show that the improved method is effective and feasible to measure the complexity of complex networks.

INDEX TERMS Complex network, structure entropy, absolute density.

I. INTRODUCTION

With the extensive study on complex network, such as small-world network model [1], Newman and Watts proposed the NW network model [2], BA scale free network model [3], self-similarity network model [4] and ER random network model [5] etc., the complexity of complex network was researched by many scholars [6]–[10]. There is not a standard model for representing the complexity of network. However, some researchers were tried to define a measurement of the complexity of networks from different fields. In Ref. [11], the entropy of a canonical network ensembles was defined based on the log-likelihood function $\mathcal{L} = -\sum_{i<j} \log \pi_{ij}\left(a_{ij}\right)$ and the Shannon entropy of the ensemble was given on the basis of the entropy of canonical network ensembled to measure complexity of network. In addition to these two entropies, by considering the Laplacian matrix of network, the Neumann entropy of a network ensemble is also introduced to measure the complexity of network [11]. To accurately measure the structure heterogeneity of complex networks, an entropy was defined based on automorphism partition by considering the number of nodes in the vertex set [12]. Then, the degree sequence of in and out degrees of directed networks was considered to measure the complexity of network [13]. Moreover, the complexity of real networks were measured. For example, the complexity of public transport networks [14] and social network [15] were studied.

Network connectivity, as a measure of network stability and complexity, was applied to study the complexity of the real networks [16]–[18]. The complexity and stability of small-world networks have been studied based on the connectivity [17]. The connectivity in airline networks and complexity analysis of Lufthansa's network were studied by Reggiani *et al.* [18]. For the network connectivity, network minimum cutting edges were considered. In the same way, from the perspective of considering network edges, the total number edges of network were taken into account for network density [19].

Network density was a useful method to measure the complexity of network by considering the degree of network intensiveness, which has been widely studied for the real networks [20]–[22]. For example, analysis of Japanese water supply organizations based on network density revealed that the economies of network density existed [23]. The density

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of the mobile network was perceived, and the base station density adaptive algorithm was designed to improve the throughput of a mobile network [24]. In 2006, a density metric was proposed to measure the complexity of business process models on the basis of a social network [25]. The relationship between traffic density and spatial distribution in cellular networks has been studied [26]. Analysis of network complexity by density only considers the edges of network simply, and the density of different sizes network cannot be compared. The concept of absolute density was proposed in the social network [27], the problem of network densities of different sizes being incomparable was avoided. However, the network connectivity and absolute density only consider the complexity of network from the perspective of the network edges, except for the edges, some information contained in the topology of the network is ignored.

Besides the connectivity and density of network, entropy is a good tool for studying network information and topology uncertainty [28], which has been widely applied to measure the complexity of network [29]–[32], especially for the structure entropy. The degree and betweenness of network are considered in structure entropy. Based on the degree centrality, local structure entropy was used to identify the network centrality by Zhang *et al.* [33]. The degree was considered to define the structure entropy which was used to measure the heterogeneity and resilience of network [34]. At the same time, in order to measure the structure properties of the weighted network, the betweenness structure entropy has been proposed [35]. On the basis of Tsallis entropy [36], an improved structure entropy was applied to measure the complexity of network. In addition, for the Tsallis structure entropy, which was modified by considering the degree of nodes to measure the complexity [37]. Meanwhile, the Tsallis entropy was combined with fractal dimension to define new structure entropy for measuring network complexity [38]. The complexities of real networks were also discussed by structure entropy. The complexity of the associated domains of genes in cells can also be reflected by structure entropy [39]. The structure entropy of an automorphism partition was used to measure the heterogeneity of the network [12]. Only one indicator was considered in these entropies, therefore, a modified structure entropy is given based on nonextensive statistical mechanics, which combined the betweenness and degree [40].

The complexities of some special structure networks were effectively measured by the modified structure entropy in Ref. [40]. However, for the different density and no weighted spacial structure network, the complexities of these networks could be not measured by the modified structure entropy in Ref. [40]. Meanwhile, the network complexity of different scales cannot be measured by the existing structure entropy. An improved method is proposed to measure the complexity of network in this paper. For the proposed method, not only the structure entropy of network is considered, but also the absolute density of network is considered. The degree and betweenness are considered in the structure entropy. Using absolute density, the problem that network with different scales cannot be compared is avoided. For the proposed method, when the betweenness of network is equal to zero, the structure entropy degenerates into the degree structure entropy [34]. The structure entropy proposed in Ref. [40] is the special case of the proposed method. Moreover, seven constructed small-world networks, BA scale free networks, Sierpinski self-similarity networks, ER random networks, and six real networks are used to illustrate the proposed method. These results show that the proposed method is effective and feasible.

The paper is organized as follows: some basic concepts are introduced in Section 2. In Section 3, a new method is proposed to measure the complexity of complex networks. In Section 4, seven constructed small-world networks, BA scale free networks, Sierpinski self-similarity networks, ER random networks, and six real networks are used to illustrate the feasibility and effectiveness of the proposed method. Section 5 is the conclusion.

II. PRELIMINARIES

In this section, some basic concepts including the entropy and structure entropy of complex network are described.

A. ENTROPY OF NETWORK

Entropy is applied to measure the degree of uncertainty in an event. The value of the entropy increases, and the uncertainty of the event increases. The complexity of more and more events could be measured by the entropy. Some definitions of entropy are given to measure the complexity of complex networks.

1) TSALLIS ENTROPY

In 1988, based on the Boltzmann-Gibbs entropy [41] and Shannon entropy [42], a more general entropy was given by Tsallis [36]:

$$S_q = k \sum_{i=1}^{N} p_i \ln_q \frac{1}{p_i}, \tag{1}$$

where p_i is the probabilities associated with an event, q is the entropy index and denotes the different relationships among those subsystems. N is the total number of nodes, k represents the Boltzmann universal constant. The logarithmic function in equation (1) is expressed by

$$\ln_q p_i = \frac{p_i^{1-q} - 1}{1-q}, \tag{2}$$

or

$$\ln_q \frac{1}{p_i} = \frac{\frac{1}{p_i}^{1-q} - 1}{1-q} = \frac{p_i^{q-1} - 1}{1-q}. \tag{3}$$

where $p_i > 0$, $q \in \Re$ and the logarithmic function is $\ln_1 p_i = \ln p_i$ in equation (2) while q equal to 1. From equations (1) and (2), the Tsallis entropy was rewritten as [40]

$$S_q = k \sum_{i=1}^{N} p_i \frac{p_i^{q-1} - 1}{1 - q},$$
$$S_q = k \sum_{i=1}^{N} \frac{p_i^{q} - p_i}{1 - q},$$
$$S_q = k \frac{1 - \sum_{i=1}^{N} p_i^q}{q - 1}. \quad (4)$$

The second formation of equation (4) is improved by Zhang et al. and it will be introduced in the next subsection.

B. STRUCTURE ENTROPY

Inspired by entropy, structure entropy is applied to measure the complexity of network. In this subsection, the degree structure entropy [34], betweenness structure entropy [35] and an improved Tsallis structure entropy by considering nonextensive statistical mechanics [40] are introduced, respectively.

1) DEGREE STRUCTURE ENTROPY

The degree structure entropy was defined as follows [34],

$$E_{\text{deg}} = -k \sum_{i=1}^{N} p_i \log p_i, \quad (5)$$

where p_i is defined based on degree and given by [43],

$$p_i = \frac{\text{degree }(i)}{\sum_{i=1}^{N} \text{degree }(i)}. \quad (6)$$

2) BETWEENNESS STRUCTURE ENTROPY

Inspired by the degree structure entropy, the betweenness is also used to define the structure entropy. The betweenness structure entropy was defined as follows [35],

$$E_{\text{bet}} = -\sum_{i=1}^{N} p_i' \log p_i', \quad (7)$$

where p_i' is defined on the basis of betweenness $b(i)$ [44], the definitions of p_i' and $b(i)$ are given in the following, respectively.

$$p_i' = \frac{b(i)}{\sum_{i=1}^{N} b(i)}, \quad (8)$$

$$b_i = \sum_{\substack{1 \leq i \leq N \\ s \neq i \neq t}} \frac{\sigma_{st}(i)}{\sigma_{st}}, \quad (9)$$

where σ_{st} is the total shortest paths between node s and t, $\sigma_{st}(i)$ denotes the number of shortest path through node i.

3) STRUCTURE ENTROPY BASED ON DEGREE AND BETWEENNESS

Based on the Tsallis entropy, the degree entropy and betweenness entropy were combined with the nonextensive statistical mechanics, a structure entropy was defined by Zhang *et al.* [40]:

$$S_Q' = -k \sum_{i=1}^{N} \frac{p_i^{q_i} - p_i}{1 - q_i}, \quad (10)$$

where p_i is defined by equation (6),

$$q_i = 1 + (b(\max) - b(i)),$$
$$b(\max) = \max[b(i), (i = 1, 2, 3, \ldots, N)].$$

When all the q_i is equal to 1, i.e.,

$$q = \{q_1, q_2, \ldots, q_i, \ldots, q_N\} = \{1, 1, \ldots 1, \ldots, 1\},$$

That is the special case of equation and the base of the logarithmic function is 1, the entropy is degenerated to the degree structure entropy.

The structure entropy is used to measure the complexity of network, the values of S_Q' of different network are different. The entropy index q is defined by the betweenness, the degree is also considered in the entropy. The structure entropy could deal with the complexity of some special structure networks effective. However, there is a disadvantage of this entropy for measuring the complexity of network, which will be introduced in the next section.

C. ABSOLUTE DENSITY OF NETWORK

In order to describe the intensity of interconnection among nodes in network, an important concept, named as density, is introduced. The network density characterizes the complexity of edges connection in network. The density of network is generally defined as "the ratio of the number of edges actually present in network to the number of edges that are theoretically the most likely to be produced in network", the density of network $G(V, E)$ was defined as follows [45]

$$d(G) = \frac{2L}{N(N-1)}, \quad (11)$$

where L denotes the total number of edges of network.

Density is an important measure and easy to calculate. However, the simple definition of density makes it vulnerable to the size of network and properties of the relationship in network, it can not measure the density of different scale networks. Therefore, the absolute density by defined to make the comparison of network density unaffected by the network scale, which was denoted by [27]:

$$d'(G) = L/(4lR^3/3D), \quad (12)$$

where l, R, D are the network circumference, network radius, and diameter, respectively. Network circumference l is the maximum of all paths in the network. Network diameter D is the maximum value of the shortest path of network, the value of network radius R is half of network diameter D.

III. THE PROPOSED METHOD

In this section, a new method is proposed to measure the complexity of network. Not only the structure entropy of network is considered, but also the absolute density of network is considered.

A. THE DISADVANTAGE OF THE STRUCTURE ENTROPY S'_Q

The structure entropy of network combined with degree and betweenness was improved based on the Tsallis entropy [40]. The structure entropy has a disadvantage, i.e., it is not effective to measure the complexity of some networks. On the one hand, when the betweenness of the network is all equal to zero, that is, $q_i = 1$, the structure entropy degenerates into the degree structure entropy at this time. If the degree of each node of the network is the same as shown in figure 1, then the structure entropy is invalid, the complexity of the network is unable to be measured by this method. On the other hand, in reference [40], when the nodes had the same degree and betweenness, the complexities of the network are not also measured.

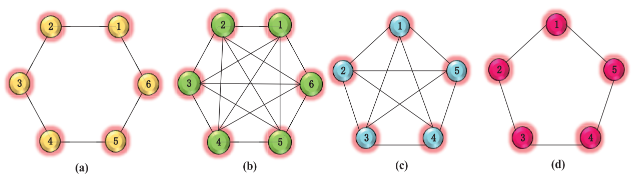


FIGURE 1. Four constructed networks. Networks (a) and (b) have 6 nodes, networks (c) and (d) have 5 nodes. All the nodes of networks (b) and (c) are connected, networks (a) and (d) are the ring network.

In figure 1, networks (a) and (d) have the same structure, which are the ring networks. The q_i of each node is equal in the same network. For the networks (b) and (c), they are globally coupled networks. The values of degree and betweenness of each node are also equal in the same network. Compared network (a) with (b), the structures of the two networks are different, the structures of the network (c) and (d) are also unlike. When the complexity of the four networks is measured by the structure entropy proposed in reference [40], the values of S'_Q of network (a) are equal to the network (b), network (c) equal to the network (d). The complexity of network is measured by equation (10), we have

$$S'_Q(a) = S'_Q(b) = 1.7918, S'_Q(c) = S'_Q(d) = 1.6094.$$

The results shows that the complexity of the four networks is

$$Network(a) = Network(b), Network(c) = Network(d).$$

From figure 1, the structure of the network (a) and network (b), network (c) and network (d) are not the same. Although the degree and betweenness of the four networks are different, the complexity of network (a) and network (b) are the same, the values of structure entropy of network (c) and network (d) are the same. For the situation the same as the four networks, the structure entropy of equation (10) can not measure the complexity of these networks.

B. A NEW METHOD TO MEASURE THE COMPLEXITY OF NETWORK

On the one hand, the structure entropy is proportional to the complexity of the network. The larger the network structure entropy S'_Q, the more complex the network. On the other hand, the density of network reflects the degree of association among nodes. The more connections among network nodes, the greater the density of the network. Absolute density not only reflects the connection of nodes within network but also the measurement is not limited by the size of the network. The structure entropy is proportional to the complexity of the network. In this section, a new method is proposed to measure the complexity of network. Not only the structure entropy of network is considered, but also the absolute density of network is taken into account. In the proposed method, the total number edges, radius R and circumference l of the network are used to calculated the absolute density of the network. Moreover, the degree and betweenness are considered in this method. The definition of network complexity is given as follows:

Definition 1: In arbitrary network $G(V, E)$, the complexity of network C is measured by the product of absolute density $d'(G)$ and structure entropy S. The absolute density $d'(G)$ of network is denoted by the D, R, l, and L. Where D, R is the diameter and radius of network, l is the circumference of network, and L is the total number of edges.

The mathematical expression is given by,

$$C = d'(G) \cdot S \tag{13}$$

where $d'(G)$, is the absolute density of network defined in equation (12) [27]. S denotes the structure entropy of network [40], that is $S = -k \sum_{i=1}^{N} \frac{p_i^{q_i} - p_i}{1-q_i}$. The p_i is defined in equation (6), and q_i is denoted in equation (10). k denotes the Boltzmann universal constant.

If the betweenness of all nodes are equal to zero, i.e.,

$$q = \{q_1, q_2, \ldots q_i \ldots q_N\} = \{1, 1, \ldots 1 \ldots 1\}.$$

The structure entropy degenerates degree structure entropy [34] and for the arbitrary one node the betweenness is equal to zero, its entropy value is also replaced by the degree structure entropy. There are the isolated nodes whose structure entropy is assumed to be 0.0001. The structure entropy in reference [40] is the special case of the proposed method, that is, the absolute density of network is equal to 1.

For example, using the proposed method, the complexity of four networks in figure 1 could be measured. For figure 1, using the proposed method and equation (10) to measure the complexity of the four networks, the results are shown in table 1.

From table 1, the values C of the networks are different, so the complexities of different networks can be measured. The complexity of network (a) and network (b) are different, the values of the complexity of network (c) and network (d) could be measured. The complexity of the four networks in figure 1 is not effectively measured by equation (10),

TABLE 1. The values of complexity of networks in figure 1.

Network	Network (a)	Network (b)	Network (c)	Network (d)
Nodes	6	6	5	5
Edges	6	15	10	5
Circumference l	5	5	4	4
Diameter	3	1	2	1
Complexity C	1.1945	10.7506	12.0708	3.0177
S'_Q	1.7918	1.7918	1.6094	1.6094

TABLE 2. The complexities of constructed small-networks.

Network	Network 1	Network 2	Network 3	Network 4	Network 5	Network 6	Network 7
Nodes	100	200	500	800	1000	1300	1500
Edges	548	3661	11872	9201	11004	11452	9732
Circumference l	99	199	499	799	999	1299	1499
Diameter	5	3	3	5	5	6	7
Complexity C	0.0491	0.0858	0.041	0.0098	0.0081	0.0045	0.0029
S'_Q	4.0529	5.1244	6.1338	6.536	6.7304	6.9865	7.072

while the complexity of the four networks can be effectively distinguished by the proposed method.

From figure 1 (a) and (b), their structures are different, while when using the equation (10) to measure their complexity, the results are the same. Similarly, for the networks (c) and (d) in figure 1 the structures are different, the values of structure entropy S'_q are also equal, that is,

$$S'_Q(a) = S'_Q(b) > S'_Q(c) = S'_Q(d).$$

However, by using the proposed method to measure the complexity, their complexity can be effectively identified, i.e.,

$$C(a) < C(b), \quad C(d) < C(c).$$

The complexity of networks (a) and (b), networks (c) and (d) in figure 1 are different, they are effectively distinguished for the networks which have the same nodes while the structure is different. In addition, for the networks with the same structure but the different number of nodes, the proposed method is also effective to measure the complexity. Form the first row of table 1, the values of the complexity of networks (a) and (d), networks (b) and (c) are different, that is

$$C(a) < C(d), \quad C(b) < C(c).$$

From figure 1, the edges of networks (b) and (c) are more than networks (a) and (d). The values of complexity of networks (b) and (c) are big. Because the absolute density of network is considered, the proposed method is effective to measure the complexity of network. Once a network is given, the proposed method could measure the complexity of the network which is not affected by the network scale and type.

IV. APPLICATION TO NETWORKS

In this section, the constructed small-world networks, BA scale free networks [3], self-similarity networks (Sierpinski self-similarity network [46]), ER random networks [5] and six real networks are used to illustrate the feasibility of the proposed method.

A. SMALL-WORD NETWORKS

With the further study of the network, the small-world property of complex networks has been discovered [1], and there exists small world property in the real network. The small-world network has short average distances and large clustering coefficients. The small-world network is very similar to the real network. In order to illustrate the effectiveness of the proposed method, seven small-world networks are constructed. The construction steps of the small-world network are as follows [1], [47],

1) Start with the regular network. Consider the nearest coupled network with N nodes, these nodes constituted a ring in which each node is connected to its neighboring $K/2$ nodes, K is an even.

2) Randomize reconnection. Each edge in the regular network is randomly reconnected with probability p, that is, one endpoint of the edge remains unchanged, and the other endpoint becomes a randomly selected one of the remaining $N - K - 1$ nodes in the network with probability p. Among them, there can be at most one edge between any two different nodes.

According to the constructed method, seven small-world networks are produced, one of them is shown in figure 2 (a). Using the proposed method to measure the complexity of seven small-world networks, the results are shown in table 2.

From table 2, although the seven networks are all the small-world networks, the values of C are different after the complexity is measured by the proposed method. And the values of the proposed measure are different from the structure entropy S'_Q, the reason is that not only the structure

TABLE 3. The complexity of constructed BA-networks.

Network	Network 1	Network 2	Network 3	Network 4	Network 5	Network 6	Network 7
Nodes	100	250	520	880	1000	1314	1500
Edges	235	1235	2056	1761	3992	3942	4500
Circumference l	14	27	22	878	995	1313	1498
Diameter	4	4	5	8	5	6	6
Complexity C	0.4196	0.2811	0.2361	0.0014	0.0025	0.0022	0.0027
S'_Q	3.1328	4.0485	4.3293	2.6247	4.3293	3.5441	4.7846

TABLE 4. The complexity of different Sierpinski self-similar networks.

Network	Network 1	Network 2	Network 3	Network 4	Network 5	Network 6	Network 7
Nodes	42	125	336	786	1095	1369	1561
Edges	81	243	729	390	2187	682	777
Circumference l	7	9	11	15	13	20	22
Diameter	6	8	10	8	12	10	15
Complexity C	0.1677	0.0808	0.0434	0.1947	0.0265	0.1369	0.0425
S'_Q	1.4091	1.5506	1.5928	6.2314	1.6557	7.0477	7.0077

entropy is considered in the proposed method, but also the network absolute density is taken into account.

From table 2, the values of complexities C of the seven constructed small-world networks are small, the interval of complexity values is smaller and the results are divided more finely. The values of structure entropy S'_Q increase with the increase of network nodes. In fact, for these networks, although the nodes of network increase, the edges of network is fluctuating. Thus, the complexity of these networks are fluctuate. For the proposed method, the complexity C of the seven networks are fluctuating. The measure is not affected by the network size and type. Once a network is given, the complexity can be measured by the proposed method.

B. BA SCALE FREE NETWORKS

To explain the power law generation mechanism, the BA scale-free model was proposed by Barabási and Albert [3]. The network is characterized by increasing network size and has heterogeneity (i.e., nodes generated in the network tend to connect to the big degree nodes). The BA network has similar characteristics of many practical social networks and can explain many practical social phenomena [48]. In order to verify the feasibility of the proposed method, we construct seven BA networks with different nodes and measure their complexity. The algorithm construction process is as follows [3], [49]:

1) Assume that the system has m_0 nodes, add a node with degree m and connect the m edges to the existing m_0 nodes.

2) The relationship between the connection probability $\Pi(k_i)$ of a new node with an existing node i and assumed that the connection probability $\Pi(k_i)$ is proportional to the degree k_i of the node i, where $\Pi(k_i) = k_i / \sum_j k_j$. After t time interval, a network with mt edges and $N = m_0 + t$ nodes is formed.

The nodes, edges and other information of seven constructed BA scale free networks are shown in table 3. The complexity of seven constructed BA scale free networks is measured, the results are shown in table 3. From table 3, the complexity C of different networks are different. Compared with the structure entropy S'_Q, because the absolute density is considered in the proposed measure, the values of complexities C are small and the interval is small, the result is more subdivided. For example, network 1, network 2 and network 3 have a high probability of edges connection, the values of complexity C of these networks are big.

C. SELF-SIMILARITY NETWORKS

With the study of self-similarity network, many networks with the self-similar property are found. In reality, although many network structures are still unclear, according to the self-similarity of network, the subnetwork is studied to estimate the characteristics of the whole network. In this section, to further verify the feasibility of the proposed method, two self-similar iterative networks are constructed. The initial networks with different numbers of nodes and average degree of these networks are simple after occurring different iterations.

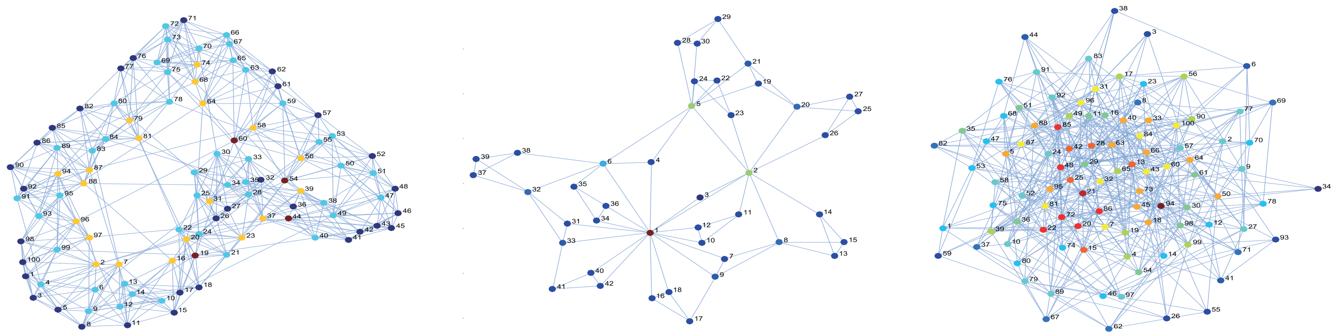
The iterative network is strictly self-similar network. By taking a given center and then iterating to generate nodes, the detailed steps are summarized in the following steps [50].

1) Select a basic graph as the iteration center and use it as the initial network.

2) Iterate the replication center network centered on each node of the central network.

3) Set the number of iterations m and the number of nodes to increase f, copy the center network graphics.

Using the iterative network generation step to make seven self-similarity networks, one of them is shown in figure 2 (b). The complexities of these seven networks are measured by



(a) (b) (c)

FIGURE 2. Four constructed networks, the degree of nodes is equal, the nodes have the same color. (a). An constructed small-world network with 100 nodes. (b). An constructed Sierpinski self-similar network with 44 nodes. (c). An constructed ER random network with 100 nodes and the probability of connected edges is 0.1.

TABLE 5. The complexity of constructed ER random networks.

Network	Network 1	Network 2	Network 3	Network 4	Network 5	Network 6	Network 7
Nodes	100	300	650	800	1000	1500	2000
Edges	508	2309	6372	63302	5016	11250	9963
Circumference l	99	199	499	799	999	1299	1499
Diameter	17	34	38	32	22	34	24
Complexity C	0.4633	0.2983	0.3114	0.3829	0.253	0.3958	0.258
S'_Q	4.2006	5.4091	6.3111	6.5351	6.6802	7.1771	7.43

the proposed method and equation (10). The corresponding results are shown in table 4.

From table 4, the values of complexity C are bigger than structure entropy S'_Q and the complexity of different networks is different. For the structure of the self-similarity networks is special, the heterogeneity is obvious. The connection probability between network nodes is small, resulting in the low absolute density of the network, thus values of complexities C of network are small. The reason is that not only the structure entropy is considered, but also the absolute density of network is considered in the proposed method. The values of complexity C of the different network sizes are different. Once a network is given, the complexity can be measured by the proposed method. The results shown that the proposed measure is effective and feasible identification network complexity.

D. ER RANDOM NETWORKS

ER model has been proposed by Erodös and Rényi, the ER random networks are widely studied. Due to the randomness of large scale networks with complex topologies and unknown connection rules, ER random networks are often used to study the properties of complex networks. In this section, seven constructed ER random networks are applied to test the feasibility of the proposed method, one of them is shown in figure 2 (c). The construction process is as follows [51],

1) Initialization. Give N nodes and the probability p of the connected edges.

2) Randomly connect edges. Select a pair of nodes without edges and generate a random r. If $r < p$ then the pair of nodes are connected, otherwise the edges are not connected.

3) Repeat the process of connecting edges until all node pairs have been selected.

For these constructed ER random networks, the complexities are measured by the proposed method and equation (10), the results are shown in table 5. From table 5, the values of complexity C are bigger than structure entropy S'_Q for the same network. The values of complexity C are not the same for different networks. For the ER random network, internally connected edges are random and do not increase with the increase of network nodes. Because both absolute density and structure entropy are considered, the measure of network complexity of the proposed method is not affected by the size of the network.

E. REAL NETWORKS

In order to verify the feasibility of the proposed method, the complexity of six real networks are respectively calculated by this method, the complexity results are shown in the table 6. The six networks that are the 9/11 terrorist network [52], Celegans network [1], a USAir network [53], the political blogs network [54], a collaboration network in science of networks (NetScience) [43] yeast protein interaction *YPI* network [53]. These networks include the social networks and biological networks, these networks are close to our lives and can reflect the practicality of the proposed method.

TABLE 6. The complexity of different real networks.

Network	9/11	*Celegans*	*USAir*	*Political blogs*	*NetScience*	*YPI*
Nodes	62	297	332	1490	1589	2375
Edges	304	2359	2126	16717	2742	11693
Circumference l	31	64	70	1010	19	70
Diameter	5	5	6	8	17	15
Complexity C	0.052	0.0472	0.0358	0.0024	0.035	0.008
S'_Q	1.3438	2.5168	3.0046	5.1841	6.4151	4.2152

From table 6, the values of the complexity C are different for the different networks. The number of nodes of the six actual networks is different, the structures of these networks are also different. Using the equation (10), when the network nodes increase the values of S'_Q are different. When using the proposed method to measure the complexity of these networks, the changes of the values of complexity C of the corresponding networks are different as the scale of networks are different. In addition, the values of complexity C are bigger than S'_Q for the same network. The reason is that not only the structure entropy is considered, but also the absolute density of network is taken into account, the results showed that the proposed method is effective. The proposed method is not affected by the size and types of networks. When the network is given, its complexity can be measured by the proposed method.

V. CONCLUSION

Measuring the complexity of complex network is an open issue. This paper proposed a method to measure the complexity of the network by improving structure entropy. For the proposed method, not only the structure entropy of network is considered, but also the absolute density of network are taken into account. In the structure entropy, the degree and betweenness are combined. The network radius and circumference are considered in the absolute density. In the proposed method, while the betweenness of network is equal to zero, the structure entropy degenerates into the degree structure entropy. When a network is given, the proposed method could be applied to measure the complexity of the network. The measure is not affected by network size and types. The proposed method is illustrated by seven constructed small-world networks, BA scale free networks, self-similarity networks and ER random networks. Moreover, six real networks are also used to verify the feasibility of the proposed method. The results show that the proposed method is feasible to measure the complexity of network.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

REFERENCES

[1] D. J. Watts and S. H. Strogatz, "Collective dynamics of small-world networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[2] M. E. J. Newman and D. J. Watts, "Renormalization group analysis of the small-world network model," *Phys. Lett. A*, vol. 263, nos. 4–6, pp. 341–346, Dec. 1999.

[3] A. L. Barabási and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[4] C. Song, S. Havlin, and H. A. Makse, "Self-similarity of complex networks," *Nature*, vol. 433, no. 7024, pp. 392–395, Jan. 2005.

[5] P. Erdős and A. Rényi, "On the evolution of random graphs," *Publ. Math. Inst. Hung. Acad. Sci.*, vol. 5, no. 1, pp. 17–60, 1960.

[6] N. R. Hall and S. Preiser, "Combined network complexity measures," *IBM J. Res. Develop.*, vol. 28, no. 1, pp. 15–27, Jan. 1984.

[7] C. Sun, Z. Fei, C. Cao, X. Wang, and D. Jia, "Low complexity polar decoder for 5G EMBB control channel," *IEEE Access*, vol. 7, pp. 50710–50717, 2019.

[8] S. Battiston, J. D. Farmer, A. Flache, D. Garlaschelli, A. G. Haldane, H. Heesterbeek, C. Hommes, C. Jaeger, R. May, and M. Scheffer, "Complexity theory and financial regulation," *Science*, vol. 351, no. 6275, pp. 818–819, Feb. 2016.

[9] D. Bonchev, 'Complexity analysis of yeast proteome network," *Chem. Biodiversity*, vol. 1, no. 2, pp. 312–326, 2004.

[10] D. Bonchev and G. A. Buck, "Quantitative measures of network complexity," in *Complexity in Chemistry, Biology, and Ecology*. Cham, Switzerland: Springer, 2005, pp. 191–235.

[11] K. Anand and G. Bianconi, "Entropy measures for networks: Toward an information theory of complex topologies," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 80, no. 2, pp. 045102-1–045102-4, Oct. 2009.

[12] Y.-H. Xiao, W.-T. Wu, H. Wang, M. Xiong, and W. Wang, "Symmetry-based structure entropy of complex networks," *Phys. A, Stat. Mech. Appl.*, vol. 387, no. 11, pp. 2611–2619, Apr. 2008.

[13] G. Bianconi, "The entropy of randomized network ensembles," *EPL (Europhys. Lett.)*, vol. 81, no. 2, 2007, Art. no. 28005.

[14] H. Lu and Y. Shi, "Complexity of public transport networks," *Tsinghua Sci. Technol.*, vol. 12, no. 2, pp. 204–213, Apr. 2007.

[15] J. A. Espinosa, G. Lee, and W. DeLone, "Global team boundary complexity: A social network perspective," in *Proc. 47th Hawaii Int. Conf. Syst. Sci.*, Jan. 2014, pp. 321–330.

[16] O. Sporns, G. Tononi, and G. M. Edelman, "Connectivity and complexity: The relationship between neuroanatomy and brain dynamics," *Neural Netw.*, vol. 13, nos. 8–9, pp. 909–922, Oct./Nov. 2000.

[17] S. Sinha, "Complexity vs. stability in small-world networks," *Phys. A, Stat. Mech. Appl.*, vol. 346, nos. 1–2, pp. 147–153, 2005.

[18] A. Reggiani, P. Nijkamp, and A. Cento, "Connectivity and concentration in airline networks: A complexity analysis of Lufthansa's network," *Eur. J. Inf. Syst.*, vol. 19, no. 4, pp. 449–461, 2010.

[19] A. K. Mackworth and E. C. Freuder, "The complexity of some polynomial network consistency algorithms for constraint satisfaction problems," *Artif. Intell.*, vol. 25, no. 1, pp. 65–74, Jan. 1985.

[20] M. Elsharief, M. A. A. El-Gawad, and H. Kim, "Density table-based synchronization for multi-hop wireless sensor networks," *IEEE Access*, vol. 6, pp. 1940–1953, 2018.

[21] J. Leskovec, J. Kleinberg, and C. Faloutsos, "Graphs over time: Densification laws, shrinking diameters and possible explanations," in *Proc. 11th ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining*, Aug. 2005, pp. 177–187.

[22] Z. Zou, X. Su, X. Qu, and P. Zhou, "DA-Net: Learning the fine-grained density distribution with deformation aggregation network," *IEEE Access*, vol. 6, pp. 60745–60756, 2018.

[23] F. Mizutani and T. Urakami, "Identifying network density and scale economies for Japanese water supply organizations," *Papers Regional Sci.*, vol. 80, no. 2, pp. 211–230, Apr. 2001.
[24] S. Mollahasani and E. Onur, "Density-aware, energy- and spectrum-efficient small cell scheduling," *IEEE Access*, vol. 7, pp. 65852–65869, 2019.
[25] J. Mendling, "Testing density as a complexity metric for EPCS," in *Proc. German EPC Workshop Density Process Models*, 2006, pp. 1–7.
[26] S. Zhou, D. Lee, B. Leng, X. Zhou, H. Zhang, and Z. Niu, "On the spatial distribution of base stations and its relation to the traffic density in cellular networks," *IEEE Access*, vol. 3, pp. 998–1010, 2015.
[27] J. Scott, "Social network analysis," *Sociology*, vol. 22, no. 1, pp. 109–127, 1988.
[28] A. Li and Y. Pan, "Structural information and dynamical complexity of networks," *IEEE Trans. Inf. Theory*, vol. 62, no. 6, pp. 3290–3339, Jun. 2016.
[29] G. E. Crooks and S. E. Brenner, "Protein secondary structure: Entropy, correlations and prediction," *Bioinformatics*, vol. 20, no. 10, pp. 1603–1611, Feb. 2004.
[30] I. Toranzo and J. Dehesa, "Rényi, Shannon and Tsallis entropies of Rydberg hydrogenic systems," *EPL (Europhys. Lett.)*, vol. 113, no. 4, 2016, Art. no. 48003.
[31] Q. Zhang, M. Li, and Y. Deng, "Measure the structure similarity of nodes in complex networks based on relative entropy," *Phys. A, Stat. Mech. Appl.*, vol. 491, pp. 749–763, Feb. 2018.
[32] Y. Al-Qudah and N. Hassan, "Complex multi-fuzzy soft set: Its entropy and similarity measure," *IEEE Access*, vol. 6, pp. 65002–65017, 2018.
[33] Q. Zhang, M. Li, Y. Du, Y. Deng, and S. Mahadevan, "Nonextensive analysis on the local structure entropy of complex networks," 2015, *arXiv:1502.00111*. [Online]. Available: https://arxiv.org/abs/1502.00111
[34] B. Wang, H. W. Tang, C. H. Guo, and Z. L. Xiu, "Entropy optimization of scale-free networks' robustness to random failures," *Phys. A, Stat. Mech. Appl.*, vol. 363, no. 2, pp. 591–596, May 2006.
[35] Q. Zhang, M. Li, and Y. Deng, "A betweenness structure entropy of complex networks," 2014, *arXiv:1407.0097*. [Online]. Available: https://arxiv.org/abs/1407.0097
[36] C. Tsallis, "Possible generalization of Boltzmann-Gibbs statistics," *J. Stat. Phys.*, vol. 52, nos. 1–2, pp. 479–487, 1988.
[37] Q. Zhang, C. Luo, M. Li, D. Yong, and S. Mahadevan, "Tsallis information dimension of complex networks," *Phys. A, Stat. Mech. Appl.*, vol. 419, pp. 707–717, Feb. 2015.
[38] T. Wen and W. Jiang, "Measuring the complexity of complex network by Tsallis entropy," *Phys. A, Stat. Mech. Appl.*, vol. 526, Jul. 2019, Art. no. 121054.
[39] A. S. Li, X. Yin, B. Xu, D. Wang, J. Han, Y. Wei, Y. Deng, Y. Xiong, and Z. Zhang, "Decoding topologically associating domains with ultra-low resolution Hi-C data by graph structural entropy," *Nature Commun.*, vol. 9, no. 1, p. 3265, Aug. 2018.
[40] Q. Zhang, M. Li, and Y. Deng, "A new structure entropy of complex networks based on nonextensive statistical mechanics," *Int. J. Mod. Phys. C*, vol. 27, no. 10, 2016, Art. no. 1650118.
[41] J. W. Gibbs, *Elementary Principles in Statistical Mechanics: Developed with Especial Reference to the Rational Foundation of Thermodynamics*. New York, NY, USA: Dover, 1902.
[42] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379–423, Jul./Oct. 1948.
[43] M. E. J. Newman, "The structure and function of complex networks," *SIAM Rev.*, vol. 45, no. 2, pp. 167–256, 2003.
[44] M. Barthélemy, "Betweenness centrality in large complex networks," *Eur. Phys. J. B*, vol. 38, no. 2, pp. 163–168, Mar. 2004.
[45] J. C. Mitchell, *Social Networks in Urban Situations: Analyses of Personal Relationships in Central African Towns*. Manchester, U.K.: Manchester Univ. Press, 1969.
[46] Z. Zhang, S. Zhou, L. Fang, J. Guan, and Y. Zhang, "Maximal planar scale-free Sierpinski networks with small-world effect and power law strength-degree correlation," *EPL (Europhys. Lett.)*, vol. 79, no. 3, 2007, Art. no. 38007.
[47] X. F. Wang and G. Chen, "Complex networks: Small-world, scale-free and beyond," *IEEE Circuits Syst. Mag.*, vol. 3, no. 1, pp. 6–20, Sep. 2003.
[48] A. Fronczak, J. A. Hołyst, M. Jedynak, and J. Sienkiewicz, "Higher order clustering coefficients in Barabási–Albert networks," *Phys. A, Stat. Mech. Appl.*, vol. 316, nos. 1–4, pp. 688–694, Dec. 2002.
[49] A.-L. Barabási, R. Albert, and H. Jeong, "Mean-field theory for scale-free random networks," *Phys. A, Stat. Mech. Appl.*, vol. 272, nos. 1–2, pp. 173–187, Oct. 1999.
[50] T. Carletti and S. Righi, "Weighted fractal networks," *Phys. A, Stat. Mech. Appl.*, vol. 389, no. 10, pp. 2134–2142, May 2010.
[51] P. Erdös and A. Rényi, "On the evolution of random graphs," *Publ. Math. Inst. Hung. Acad. Sci.*, vol. 5, no. 1, pp. 17–60, 1960.
[52] V. Krebs, "Mapping networks of terrorist cells," *Connections*, vol. 24, no. 3, pp. 43–52, 2002.
[53] N. Blagus, L. Šubelj, and M. Bajec, "Self-similar scaling of density in complex real-world networks," *Phys. A, Stat. Mech. Appl.*, vol. 391, no. 8, pp. 2794–2802, 2012.
[54] L. A. Adamic and N. Glance, "The political blogosphere and the 2004 U.S. election: Divided they blog," in *Proc. 3rd Int. Workshop Link Discovery*, Aug. 2005, pp. 36–43.

MINGLI LEI received the B.S. degree from the Department of Mathematics and Computer Science, Sichuan Minzu University, Kangding, China, in 2017. He is currently pursuing the master's degree with the School of Science, Hubei Minzu University. His research interests include complex networks and bio-inspired algorithms.

LIRONG LIU received the B.S. degree from the School of Information Engineering, Southwest Jiaotong University, Chongqing, China, in 2017. She is currently pursuing the master's degree with the School of Science, Hubei Minzu University. Her research interests include complex networks, biomathematics, and nonlinear switching dynamic systems.

DAIJUN WEI graduated from the Hubei University for Nationalities, Enshi, China, in 2001. He received the M.S. degree from Central China Normal University, Wuhan, China, in 2008, and the Ph.D. degree from Southwest University, Chongqing, China, in 2014. He is currently a Professor with the School of Science, Hubei Minzu University, Hubei, China. His current research interests include fuzzy systems, economic statistics, and complex networks.