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# Modeling Vehicle Merging Position Selection Behaviors Based on a Finite Mixture of Linear Regression Models

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**ABSTRACT** Vehicle merging is a complex and tactical decision process. Merging position selection behavior has been largely ignored in microscopic traffic simulators. Driver heterogeneity has received substantial attention in recent years; however, few studies have focused on the heterogeneity in merging behaviors. To account for the heterogeneity among merging drivers during the merging process and to improve the accuracy of the merging model, a finite mixture of linear regression models was developed for describing the merging position selection model. BIC was used to determine the optimal number of classes, and Latent Gold 5.0 was used to estimate parameters. Based on the US101 data in the NGSIM project, which were provided by FHWA, a 3-class linear regression model was developed. The results demonstrate that the variables differ across the classes, and the sign of each variable may also differ among the classes; hence, the strategies that are used by drivers for merging position selection differ across the classes. Cooperative lane changing of the putative leading vehicle was found to have significant influence on the merging position selection behavior; thus, merging behavior is a two-dimensional behavior that may be influenced by both lateral and longitudinal factors. Compared with previous studies, the proposed model can naturally identify the heterogeneity among drivers and is much more accurate; therefore, the proposed model is a promising tool for microscopic traffic simulation and automatic driving systems or driver assistance systems.

**INDEX TERMS** Microscopic traffic simulation, merging position selection behavior, finite mixture of linear regression model, heterogeneity, cooperative lane change.

## I. INTRODUCTION

Merging is a type of typical mandatory lane change in which a vehicle must move from an on-ramp to the main road to continue following its route [1], [2]. Drivers must complete their merging manoeuvres in the merging area, which may result in traffic congestion and even breakdowns [3]–[8]. Traffic management strategies can be used to mitigate traffic congestion at merging areas. However, their performances should be evaluated via microscopic traffic simulation prior to implementation because transportation system field experiments are too expensive and complex in practice. Thus, it is important to build an accurate merging behavior model for improving the accuracy of the microscopic traffic simulation

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model and for more accurately simulating the real traffic conditions.

In previous studies, drivers are always assumed to be homogeneous and consistent during the driving process, which raises questions regarding the performance of microscopic traffic simulation models in representing reality [9]–[14]. Studies have demonstrated wide heterogeneity among drivers in terms of driving behavior [5], [6], [15], [16]. Different drivers may behave differently under the same traffic conditions and the same driver may even behave differently under varying traffic conditions. Heterogeneity among drivers has been studied in macroscopic traffic models in several studies [17]. Driver heterogeneity has attracted attention in studies of car-following models [12], [16], [18]–[20]. However, few studies have focused on heterogeneity in lane-changing models [5], [6], [21].

The merging process is a sequential decision process that reflects the dynamic optimization of drivers' merging strategies by considering changes in surrounding traffic conditions. Previous studies use a sequential two-step model to describe merging behaviors, which consists of the following steps: (i) acceptable gap searching and (ii) merging execution [22]–[24]. The adjacent gaps are continuously compared with the critical gap, which is defined as the minimum length of the acceptable gap, to determine whether the merging drivers would accept the current adjacent gap. The simplified continuous-comparison process was criticized for disrupting the continuity of merging tactics and for ignoring the real merging behavior that is observed in practice [22], [25], [26]. Thus, to better describe the merging behavior, the merging process can be modelled via three steps, as illustrated in Figure 1: (i) gap selection, (ii) merging position selection, and (iii) merging execution or lane changing. During the gap selection period, a merging driver assesses the available gaps and chooses to accept or reject these gaps. After accepting a gap, the merging driver chooses a suitable merging position and adjusts the speed and relative position according to the putative leading (PL) and putative following (PF) vehicles to realize to the chosen merging position. Then, the merging driver initiates the merging execution and begins moving laterally until the merging vehicle has fully entered the adjacent

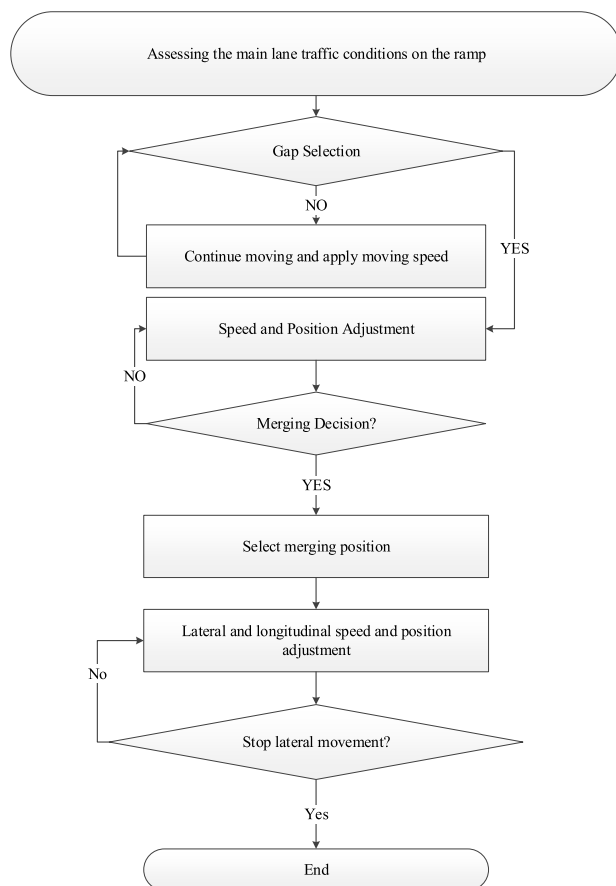


FIGURE 1. Merging process.

main lane, after which the driver terminates lateral movement. Vehicle merging is a tactical optimization process and the tactics that are used by merging vehicles will differ in terms of the steps that are executed according to the scenario [22].

Too much emphasis was placed on the first step of merging process, i.e., the merging decision, which was the combination of the gap selection process and merging position selection process. However, as a tactical process with several transitions, among which, the merging position selection process is one of the most important transition processes. The ignorance of merging position selection process would lead to a considerable amount of errors in traffic simulation outputs.

The limited existed analysis of merging tactics and merging driver heterogeneity motivated us to investigate the heterogeneity among merging drivers during the process of selecting the desired merging position. This paper targets on the second step of the merging process: merging position selection. The objective of this paper was to use a systematic method to build a merging decision model which not only can naturally incorporate and investigate the driver heterogeneity, but also can produce accurate prediction results that could contribute significantly to the microscopic traffic simulation. The main contribution can be summarized as below:

(1) Prove the existence of heterogeneity during among merging position selection behaviors.

(2) Identify different driving styles and attitudes during merging process.

(3) Build an accurate merging position selection model.

The remainder of this paper is organized as follows: The next section presents a literature review. Section 3 describes the data that are used in this paper. Section 4 describes the methodology that is used to build a finite mixture of linear regression models. The results are presented and discussed in Section 5. Finally, the conclusions are presented in Section 6.

## II. LITERATURE REVIEW

### A. FRAMEWORKS OF MERGING BEHAVIOR

Gap acceptance is one of the most important theories that are used in lane changing models. The most important assumption in gap acceptance theory is that a driver changes lanes when both the lead and the lag gaps in the target lane are larger than the critical gap. Gap acceptance theory was initially developed for estimating the capacity of signalized intersections [27], [28]. Gipps [29] was the first to use gap acceptance theory to develop a comprehensive framework for a lane-changing model. Gipps' framework has been widely used in freeway merging models [25], [30] and microscopic traffic simulation software [31]–[33]. Various definitions of the critical gap were used in these models and software.

The most prevalent commercial microscopic traffic simulation tools have been criticized for not accurately reproducing traffic behaviors near merge areas under congestion [34]. Thus, “forced” and “cooperative” lane change models were proposed for describing distinctive behaviors of vehicles under congestion [30], [35]. Choudhury *et al.* [36] proposed

a framework for merging behavior with latent plans, in which normal merging, merging with courtesy, and forced merging were considered. According to Marczak *et al.* [37], in this framework only accepted gaps were considered while rejected gaps were ignored and some of the estimated coefficients in the model were not significant.

Gap acceptance theory with critical gaps is most widely used in lane changing models, in which it is assumed that a driver will accept the adjacent gap only if both the lead and the lag gaps are larger than the critical gap [26], [36], [38]. However, this is often inconsistent with observations in practice that vehicles still take lane changing actions when only the lead or the lag gap, or neither, is larger than the critical gap [1], [37], [39]. To overcome this shortcoming, a binary logit model was built by Kita [40] for describing the probability of merging behavior. Weng and Meng [41] and Marczak, *et al.* [37] used the same model to predict merging decisions in short-term work zone merging areas and to compare the gap acceptance for merging decision between two sites, respectively. A mixed probit model was developed by Weng *et al.* [42] for describing merging behavior. To account for the effects of time, a time-varying mixed logit model was built for describing the vehicle merging behavior in work zone merging areas by Weng *et al.* [43]. Sun *et al.* [44] criticized gap acceptance theory for not fully describing merging behavior due to failure to consider the rejected gaps. Thus, multi-rejected gaps were considered in this study. Gu *et al.* [45] analysed the crash risk at merging areas based on a multilevel random-parameter logistic regression model. To facilitate understanding of human driving behavior, game theory was also applied in the development of merging models [46], [47]. Recently, data-driven methods, such as CART, Bayesian network and fuzzy logic models, were used to build merging models or lane changing models and satisfactory results were obtained [48]–[52].

However, these works are all based on the two-step sequential decision framework and regard the gap selection process and the merging position selection process as a single unified decision step, which is unrealistic, as discussed in Section 1. Failure to consider the merging position selection process will reduce the accuracy of microscopic traffic simulation models.

## B. MERGING POSITION MODELS

To investigate the merging position selection behaviors, studies used field data or driving simulators to analyse merging positions. Polus *et al.* [53] compared the merging positions of four acceleration lanes in Israel, which included both tapered and parallel acceleration lanes. Ahammed *et al.* [54] collected field data in Ottawa City, Canada and modelled the merging position. However, the data were collected only during off-peak hours. Recently, Weng and Meng [41] built a merging position model by treating the merging position as a random variable that followed a lognormal distribution and considering the traffic density and speed. However, the gap selection process and merging position selection process were not distinguished in these studies.

Researchers tried to build the merge position selection models for describing the relative positions between the merging vehicle and its PL and PF vehicles. Hidas [35] built a merge position model that considers (1) the location with the minimum gap distance to the PL vehicle; (2) the location with the minimum gap distance to the PF vehicle; and (3) the other locations in between the PL and PF vehicles. In this study, the merge position was determined by the approaching direction of the merging vehicle to its target gap. Choudhury *et al.* [24] proposed that the desired merge position (in terms of the distance between the merging vehicle and its PF vehicle) was linearly related to the gap distance between the merging vehicle's PL and PF vehicles. A similar linear model was presented by Wan *et al.* [22], in which the routing plans of surrounding vehicles were considered. In Wan *et al.* [22], the merging drivers were arbitrarily classified into original-gap-targeting vehicles and forward-gap-targeting vehicles based on whether the merging vehicles selected the original gap as their target gap. Speed synchronization, acceleration, and the final merging execution behaviors were also analysed. However, the arbitrary classification might be oversimplified because the choice between the original gap and forward gap depended on the surrounding traffic conditions instead of the drivers' characteristics and forward-gap-targeting drivers that had rejected several gaps might behave differently from those who had rejected fewer gaps. The routing plans of surrounding vehicles were found to affect the merging position selection behavior. However, they were observed by the researchers instead of the merging drivers. In addition, the traffic conditions, such as the traffic density, were not considered in this study. Lane changing vehicles interact with their surrounding vehicles in both the longitudinal and lateral directions. However, most previous studies focused on longitudinal factors, such as gaps and relative speeds, and none of these studies considered the effects of lateral factors, such as the lane changing behaviors of surrounding vehicles. In practice, two successive merging vehicles might select the same gap and initiate lane changing simultaneously. Vehicles in the adjacent main lane might change to the inner lane to make space for merging vehicles, which is called cooperative lane changing in previous studies [1], [25]. How these lateral behaviors of surrounding vehicles influence merging behaviors was not investigated in the literature. Thus, in this study, the lateral behaviors of surrounding vehicles were included in the merging position model.

In recent years, driver heterogeneity has been regarded as a key element in driver behavior. Heterogeneity has been considered in macroscopic traffic flow studies [55]–[57], car following studies [9]–[12], [58] and studies on bicyclists' running behavior [59]. However, driver heterogeneity has not been considered in lane changing models. Aggressiveness parameters were used in traffic simulation software packages [32], [33]. The main series of studies that incorporate driver heterogeneity was conducted by Ben-Akiva's group [23], [36], [60]–[62]. In these studies, a lane change decision framework that consists of latent (unobservable)

levels of decision hierarchy was proposed. Driver heterogeneity was considered by using an individual-specific latent variable. However, the driver heterogeneity in searching for a merging position has never been investigated.

Arbitrary classification and failure to consider driver heterogeneity will reduce the accuracy of microscopic traffic simulation models, which might cause decision-making mistakes in traffic management departments when using these traffic simulation models. Thus, a finite mixture of linear regression models was introduced in this paper for modelling the merging position selection behaviors during the merging process. The finite mixture of linear regression models utilizes two techniques: clustering and regression analysis. The unobserved heterogeneity is naturally incorporated into the merging position selection model and the drivers are automatically segmented into homogeneous populations. The proposed finite mixture of linear regression models can explain the strategies that are utilized in merging position selection behaviors. The main contributions of this study are threefold: First, this study develops a merge position selection model. Second, the heterogeneity in drivers' characteristics and merging tactics was considered in this study. Third, lateral factors, such as cooperative lane changes, are regarded as influencing factors, which have never been considered in previous studies.

### III. DATA PREPARATION

#### A. DATA DESCRIPTION

In this study, traffic data that were provided by the Federal Highway Administration's Next Generation SIMulation (NGSIM) project were used to model the merging position selection behavior. As an open-source dataset, the NGSIM dataset has been used in many previous studies and has proved to be reliable. In the NGSIM project, vehicle trajectory data were collected from both freeways and urban roads. In this paper, data that were collected on a segment of southbound U.S. Highway 101 (Hollywood Freeway) in Los Angeles, CA are selected [21]. The US-101 section was located between an on-ramp and an off-ramp, was 640 metres long, and had five main lanes and one auxiliary lane, as shown in Figure 2. The vehicle trajectories were collected from 7:50 A.M. to 8:35 A.M. on June 15, 2005. The road section was covered by eight cameras and the dataset was updated at a resolution of 10 frames per second (fps) [63]. The dataset has three data subsets, all of which were collected over 15 minutes. The collected videos were processed using an object tracking algorithm and the coordinates of every vehicle in the video were recorded. Thus, the NGSIM dataset provides the coordinates, speed and acceleration of any vehicle at any instant. To promote the development of traffic engineering, FHWA decided to make the NGSIM dataset available to the public, which has contributed substantially to the research of transportation science.

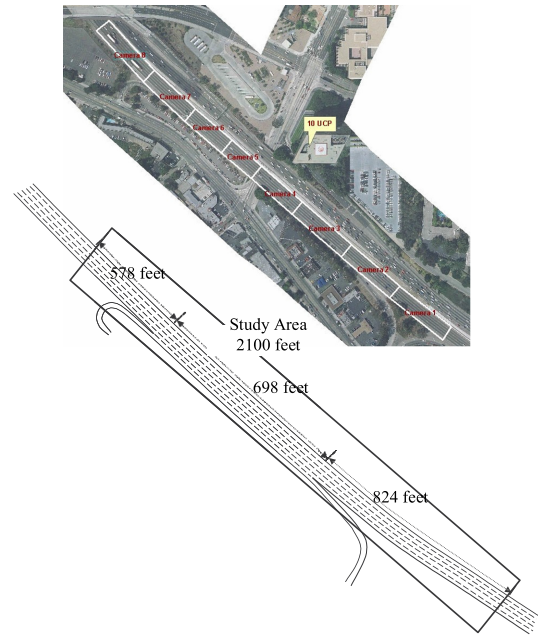


FIGURE 2. U.S. Highway 101 study corridor from NGSIM [63].

#### B. DATA PROCESSING

Although the NGSIM data has been proven highly accurate, it still contains random noise [64], [65]. Thus, the authors applied the data smoothing technique that was provided by Thiemann *et al.* [65] prior to further data analysis. First, the velocities and accelerations of the vehicles were directly estimated from the longitudinal positions. Then, the symmetric exponential moving average filter (sEMA), which was proposed by Thiemann *et al.* [65], was used to smooth the locations (both local lateral and longitudinal coordinates), velocities and accelerations.

In previous studies, the authors found that the local coordinates of three subsets of the US-101 datasets are inconsistent with one another. Linear regression was performed between local coordinates and global coordinates for each subset. Three linear relationships were obtained for each subset:

$$Localy_1 = 0.3209globalx_1 - 1.1326globaly_1 \quad (1)$$

$$Localy_2 = 0.3291globalx_2 - 1.1334globaly_2 \quad (2)$$

$$Localy_3 = 0.3209globalx_3 - 1.1333globaly_3 \quad (3)$$

The  $R^2$  values of the three linear relationships are 0.9996, 0.9997 and 0.9997, respectively; hence for the points with same global coordinates, the three subsets have same local x-coordinates but different local y-coordinates. In the local longitudinal coordinate, the upstream edge in dataset 1 is at 12.275 m in dataset 2 and 10.598 m in dataset 3. Thus, the three subsets were also unified by using the local coordinates of one of the subsets. For details on the data processing, one can refer to Ref. [5], [6], [66]

After smoothing, filtering and consistency checking, trajectories of 388 merging vehicles were extracted from the dataset.

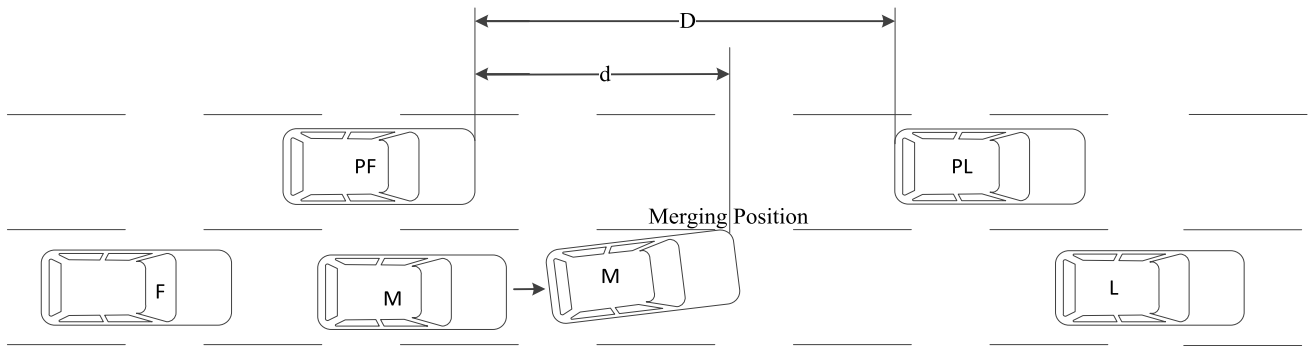


FIGURE 3. Definition of the merging position.

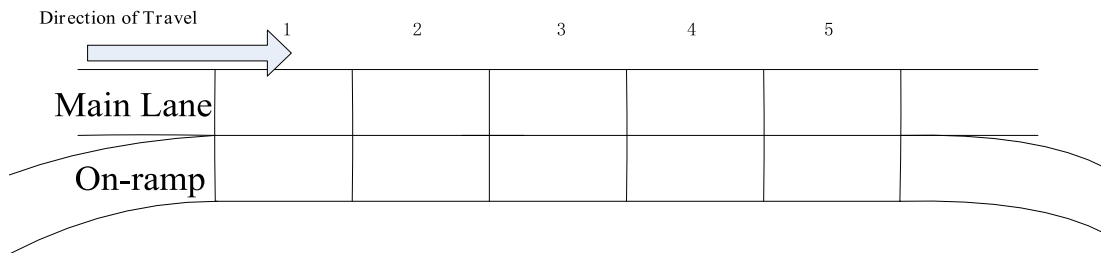


FIGURE 4. Space segmentation for the analysis of the traffic density.

### C. DATA EXTRACTION

In previous studies, the merging position was typically defined as the location where the subject merging vehicle initiates lateral movements without oscillation [24], [67]. To identify the merging position, the lateral position should be used. Most studies determined the merging position based on the curvature of the lateral position [24], [67]. However, it is too difficult and arbitrary to identify the correct point of the curvature. Fortunately, the NGSIM dataset provides not only the vehicle position information but also the length and width of the vehicle. Thus, the merging position is defined as the position where the left front bumper of the merging vehicle invades the adjacent main lane (as illustrated in Figure 3). After choosing an acceptable gap, a merging vehicle driver will select a merging position. The dependent variable  $d$  is defined as the distance from the merging vehicle's desired merging position to its PF vehicle (as illustrated in Figure 3), which is consistent with previous studies [24], [67].

It has been proved that the traffic density has a substantial influence on the merging behavior [66]. To investigate the influence of the traffic density on the selection of the merging position, a cell-based traffic density was introduced in reference [68]. The weaving section is segmented into  $N$  sections (cells) with equal length (see Figure 4). As described in Park *et al.* [69], the cell length should be set as the free-flow moving distance of vehicles over 1.5 sec, which is the driver's perception-reaction time [70]. The free-flow speed is set at 96 km/h, which is the speed limit of US-101. Thus, the cell length should be set at 40.0 m. For simplicity, the auxiliary lane and the adjacent lane are segmented into 5 cells with

the same length of 42.4 m (as illustrated in Figure 4). The traffic density within each cell is aggregated according to Equation (4):

$$k = 1000 \frac{N}{L} \quad (4)$$

In Equation (4),  $k$  (veh/lane/km) is the density of the influencing cell,  $N$  is the number of mainline vehicles on the adjacent lane within the influencing cell and  $L$  (m) is the length of each cell.

During the merging process, merging vehicles and their PL and PF vehicles are travelling separately in two lanes. A merging behavior is a two-dimensional behavior of vehicles, which differs from car following. However, most previous studies focused on the influence of longitudinal factors, such as gaps and relative speeds, on merging behaviors and none of them considered the influence of lateral factors, such as the lateral movements of surrounding vehicles. It has been observed in studies that adjacent main lane vehicles might change to the inside lane to make space for merging vehicles, which is called cooperative lane changing [1], [25]. However, the lateral interactions in the weaving sections are highly complicated. In addition to cooperative lane changing, other surrounding lane changes may influence the merging vehicle drivers' decisions. Figure 5 illustrates the 8 possible lane changing behaviors that may influence the decisions of merging vehicles, which are also described in Table 1. Eight dummy variables, namely,  $LC_i^j$  ( $i = PL, PF, j = cooperative, inside, offramp, merge$ ), are introduced to represent the possible surrounding lane changes in this paper.

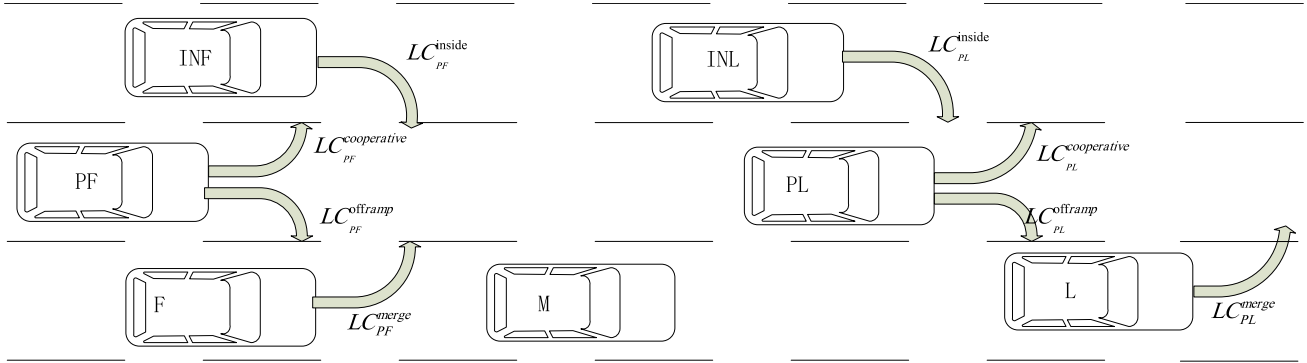


FIGURE 5. Surrounding lane changes during the merging process.

As a consecutive decision process, the gap selection process may influence the merging position selection behavior. According to previous studies, the merging decisions during the merging position selection period differ between original-gap-target vehicles and forward-gap-target vehicles. However, this classification is oversimplified, as the number of rejected gaps of forward-gap-target vehicles ranged from 1 to 16 in the US-101 dataset. Thus, a variable that represents the number of rejected gaps ( $NORG$ ) is also introduced in this paper.

Other factors that have been considered in previous studies [24], [67] are also considered in this study. The explanatory variables that may affect the merging position selection behaviors are listed in Table 1. In this paper, it is assumed that merging drivers would choose the desire merging position after they chose the accepted gap. Thus, the input explanatory variables were collected at the time that the merging drivers involved with final accepted gaps.

#### IV. METHODOLOGY

##### A. FINITE MIXTURE OF LINEAR REGRESSION MODELS

Finite mixture regression is based on the assumption that the observed data come from a population with several sub-populations or groups [71], [72]. The overall population is modelled as a mixture of the groups using finite mixture models.

Let  $\mathbf{Y}$  denote the response vector, which has values in  $\mathbf{R}^d$ , and  $\mathbf{X}$  denote the explanatory vector, which has values in  $\mathbf{R}^p$ . Given data with  $N$  observations  $(\mathbf{x}_i, \mathbf{y}_i)(i = 1, \dots, N)$ , a finite mixture of regression models with  $K$  classes has the form:

$$h(\mathbf{y}|\mathbf{x}, \psi) = \sum_{k=1}^K \pi_k f(\mathbf{y}|\mathbf{x}, \theta_k) \quad (5)$$

$$\pi_k > 0, \quad \sum_{k=1}^K \pi_k = 1 \quad (6)$$

where  $h(\mathbf{y}|\mathbf{x}, \psi)$  is the conditional density of  $y$  given  $\mathbf{X}$  and  $\theta_k$ ,  $\pi_k$  is the prior probability,  $\theta_k$  is the parameter vector for the density function  $f$  given class  $k$ , and  $f$  is

TABLE 1. Descriptions of explanatory variables.

Variable	Description
$D$ (m)	The size of the accepted gap.
$V$ (m/s)	The speed of the merging vehicle.
$\Delta V_{PL}$ (m/s)	The speed difference between the PL vehicle and the merging vehicle.
$\Delta V_{PF}$ (m/s)	The speed difference between the PF vehicle and the merging vehicle.
$\Delta V_{Lead}$ (m/s)	The speed difference between the merging vehicle and the leading vehicle in the auxiliary lane.
$D_{lead}$ (m)	The distance between the merging vehicle and the leading vehicle in the auxiliary lane.
$\Delta V_{Following}$ (m/s)	The speed difference between the merging vehicle and the lag vehicle in the auxiliary lane.
$D_{Following}$ (m)	The distance between the merging vehicle and the following vehicle in the auxiliary lane.
$RRD$	Ratio of the remaining distance from the vehicle to the end of the auxiliary lane to the length of the auxiliary lane (between 0 and 1).
$NORG$	Number of rejected gaps prior to merging.
$k_{main}$ (veh/lane/km)	The traffic density of the current adjacent cell in the adjacent main lane.
$k_{aux}$ (veh/lane/km)	The traffic density of the current cell in the auxiliary lane.
$LC_i^j$ (dummy variable)	Existence of surrounding vehicle lane changes ( $i = PL, PF, j = cooperative, inside, offramp, merge$ )
$Type_{PL}$ (dummy variable)	Type of PL vehicle: 0 represents a car and 1 represents a heavy vehicle.
$Type_{PF}$ (dummy variable)	Type of PF vehicle: 0 represents a car and 1 represents a heavy vehicle.

a univariate normal density with class-specific mean  $\beta'_k \mathbf{x}$  and variance  $\sigma_k^2$ . The vector of all parameters is defined as  $\psi = (\pi_1, \dots, \pi_K, \theta_1, \dots, \theta_K)$ , where  $\theta_k = (\beta'_k, \sigma_k^2)$ .

It is assumed in a finite mixture of linear regression models that individual drivers are distributed heterogeneously with a discrete distribution within the population. To satisfy the constraints in Equation (6), the probabilities are parameterized with a multinomial logit form [73]:

$$\pi_k = \frac{\exp(\alpha_k)}{\sum_{k=1}^K \exp(\alpha_k)}, \quad \alpha_K = 0 \quad (7)$$

Based on specified individual characteristics, the prior probabilities can be extended as follows [73]:

$$\pi_{ik} = \frac{\exp(\theta_k \mathbf{z}_i)}{\sum_{k=1}^K \exp(\theta_k \mathbf{z}_i)}, \quad \theta_K = 0 \quad (8)$$

where  $\theta_K$  is the vector of class-specific parameters and  $\mathbf{z}_i$  is the optional set of individual characteristics for observation  $i$ .

Data can be segmented according the posterior probability by assigning each observation to the class with the maximum posterior probability [74]. The posterior probability can be calculated as

$$P(j|\mathbf{x}_i, \mathbf{y}_i, \psi) = \frac{\pi_j f(\mathbf{y}_i|\mathbf{x}_i, \theta_j)}{\sum_{k=1}^K \pi_k f(\mathbf{y}_i|\mathbf{x}_i, \theta_k)} \quad (9)$$

The aim of this study is to build a model to predict the desired merging position. Hence, data collected at the time that the merging drivers involved with final accepted gaps were used as the input explanatory variables  $\mathbf{X}$  and the final merging position  $d$  defined in Figure 3 was used as the response variable  $\mathbf{Y}$ .

## B. MODEL PARAMETER ESTIMATION

Parameters of the finite mixture of linear regression models can be efficiently estimated via the expectation-maximization (EM) algorithm, which is an iterative method that alternates between the expectation and the maximization steps until the likelihood of improvement falls under a pre-specified threshold or a maximum number of iterations have been conducted [75]. However, the EM algorithm suffers from slow convergence and a long processing time. Thus, in this paper, Latent GOLD 5.0 is used to estimate the parameters. Latent GOLD 5.0 has the advantages of both the EM and Newton-Raphson algorithms. It uses EM iterations to get close to the final solution and subsequently switches to the Newton-Raphson method to complete the estimation [76].

It is not easy to determine the number of classes  $K$ , as  $K$  is not a parameter in a convex space and cannot be tested directly through hypotheses. Entropy-based criteria such as Akaike's information criterion (AIC) and the Bayesian information criterion (BIC), which are often used to estimate the relative quality of statistical models for a set of data, can be used to determine  $K$  [73], [77]–[79]. According to Allenby [80], the AIC tends to unduly favour a model with more classes for large data samples and the BIC corrects this bias because it considers the sample size. Thus, in this study,  $K$  is determined by using the BIC:

$$BIC_{\text{model}} = -2LL + \log(N) \gamma \quad (10)$$

where  $LL$  is the log-likelihood value,  $\gamma$  is the number of free parameters to be estimated, and  $N$  is the number of observations in the data. A lower BIC value corresponds to a more accurate model.

## V. RESULTS AND DISCUSSION

### A. RESULTS

The finite mixture of linear regression models is fitted with an increasing number of classes  $K = 1, \dots, 8$ . The BIC values of the resulting model for various numbers of classes are plotted in Figure 6. The finite mixture of linear regression models calculates the lowest BIC value at  $K = 3$ . Hence, the optimal number of classes for the model is 3 according to the BIC.

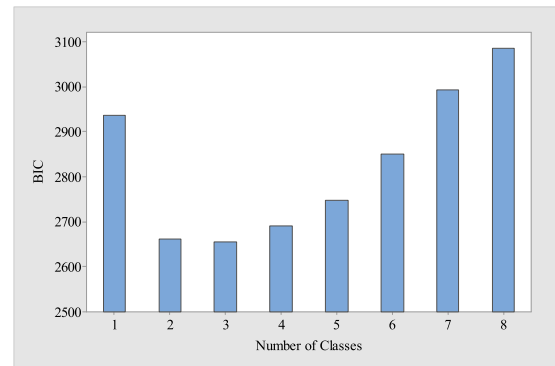


FIGURE 6. Determination of the number of classes.

The estimation results of the 3-class linear regression model are presented in Table 2. In this table and in the following tables, the symbol \* denotes that the parameters are significant at the 95% level. Not all the parameters are significant at the 95% level in Table 2. To select the model variables, the backward elimination method is adopted in this paper. It starts with all candidate variables and evaluates the model after the deletion of each variable using the overall Wald statistic, which reflects whether the variable contributes significantly to the model. This method deletes the variable with least significant Wald statistic and this process is repeated until all variables' Wald statistics are significant at the 95% level. The estimated explanatory variable parameters and the related statistical analysis results are presented in Table 3. According to Table 3,  $D$ ,  $V$ ,  $\Delta V_{PL}$ ,  $\Delta V_{PF}$ ,  $D_{lead}$ ,  $k_{main}$ ,  $Type_{PL}$ ,  $Type_{PF}$ ,  $LC_{PL}^{cooperative}$  and  $RRD$  are retained in the model.

The probabilities of a merging vehicle driver being assigned to the three classes, as indicated by the  $\pi$  values in Table 3, are 36.83%, 32.70% and 30.47%, respectively. Via Equation (9), each merging vehicle can be classified into a class and the posterior numbers of drivers in each class are 199, 89 and 100. According to Table 3, the parameters of NORG are not significant; hence, the number of rejected gaps does not significantly influence the choice of merging position. Therefore, the heterogeneity among the merging drivers is endogenous rather than due to the gap rejection behaviors.

The results on variable significance can be easily obtained from Table 3. The parameters of  $D$  are significant in all three classes; hence, a merging vehicle driver will increase the distance between the desired merging position and its

TABLE 2. Model estimation results of the 3-class linear regression model.

Variable (m/s)	Wald Test		Finite mixture of linear regression models ( $K = 3$ )		
			Class 1 ( $\pi_1 = 0.4643$ )	Class 2 ( $\pi_2 = 0.3762$ )	Class 3 ( $\pi_3 = 0.1595$ )
	Wald Statistic	P-value	Parameter	Parameter	Parameter
$D$	4588.914	0.0000	0.9919*	0.1209*	0.5275*
$V$	29.3077	0.0000	-0.4329*	0.5464*	-1.3012*
$\Delta V_{PL}$	119.0793	0.0000	-1.7819*	-0.5186*	-1.9662*
$\Delta V_{PF}$	27.8345	0.0000	-0.1958	-0.6156*	-2.3555*
$\Delta V_{Lead}$	15.1108	0.0017	-0.2745*	0.0327	1.2990*
$D_{lead}$	13.3494	0.0039	-0.0285*	0.0214	1.3898
$\Delta V_{Following}$	9.1003	0.028	0.2222*	-0.0932	0.7050*
$D_{Following}$	7.5382	0.057	-0.0206*	0.0054	-0.0154
$NORG$	7.8328	0.05	0.1276	0.1058	-1.6099*
$RRD$	9.9481	0.019	-2.9954*	1.4346	-5.5840
$k_{main}$	26.4911	0.0000	0.3759	0.6218	3.9564*
$k_{aux}$	5.6191	0.13	-0.7562	-0.6114	-1.0162
$Type_{PL}$	39.3839	0.0000	-8.8011*	1.5691	-3.7630
$Type_{PF}$	19.3869	0.00023	-1.1304	4.5536*	2.7790
$LC_{PL}^{cooperative}$	29.9231	0.0000	2.3396*	1.5545	-23.3737*
$LC_{PL}^{offramp}$	2.5749	0.46	0.3956	-3.0075	-3.2389
$LC_{PL}^{merge}$	4.2356	0.24	1.4414	1.0048	4.8065
$LC_{PL}^{inside}$	6.498	0.09	11.8368	1.8449	26.4535
$LC_{PF}^{cooperative}$	0.6773	0.88	-0.9101	1.3342	-4.006
$LC_{PF}^{offramp}$	0.8423	0.84	1.5714	-0.8322	2.9282
$LC_{PF}^{merge}$	1.1563	0.76	-2.1218	-2.4098	-4.5541
$LC_{PF}^{inside}$	0.4832	0.92	0.7110	0.1982	0.1982
Constant	9.6870	0.021	17.6759	-16.1646*	18.8122*
$R^2$	0.9771		0.9802	0.5993	0.9520

TABLE 3. Model estimation results after variable selection.

Variables	Wald Test		Finite mixture of linear regression models ( $C = 3$ )		
			Class 1 ( $\pi_1 = 0.3683$ )	Class 2 ( $\pi_2 = 0.3270$ )	Class 3 ( $\pi_3 = 0.3047$ )
	Wald Statistic	P-value	Parameter	Parameter	Parameter
$D$	4397.038	0.0000	0.4517*	0.0217*	0.2387*
$V$	16.5280	0.0009	-0.3876*	0.1239*	-0.2578
$\Delta V_{PL}$	73.1654	0.0000	-0.9085*	-0.4291*	-1.0581*
$\Delta V_{PF}$	8.3381	0.0420	-0.1820	0.1579*	0.1898
$RRD$	8.5785	0.0405	-2.7136*	-0.4168	0.0385
$Type_{PL}$	14.0916	0.0028	-7.6056*	1.3164	-1.6138
$Type_{PF}$	11.1164	0.0110	0.3306	3.7552*	2.4754*
$k_{main}$	11.0966	0.0110	0.0074	0.0915*	0.0820*
$LC_{PL}^{cooperative}$	11.2610	0.0110	0.3329	-2.5318*	2.2401*
Constant	7.9296	0.0480	-6.8895*	-3.0217*	-4.5133*
$R^2$	0.9663		0.9813	0.5346	0.9543

PF vehicle with the increase of the gap size. This is easy to understand and is consistent with previous studies [24], [67].  $V$ ,  $\Delta V_{PL}$ , and  $\Delta V_{PF}$  are significant in the calibrated model.

However, only  $\Delta V_{PF}$  is significant in all three classes. The negative sign of the parameters of  $\Delta V_{PL}$  in all three classes indicates that when a merging vehicle moves faster than its



PL vehicle, the merging vehicle driver will select a merging position near its PF vehicle. In contrast to previous studies [22], [67], [81],  $\Delta V_{PF}$  is significant in Class 2 and  $V$  is significant in Class 1 and Class 2; neither was considered in the final calibrated model in previous studies [22], [81]. Therefore, the proposed finite mixture of linear regression models can mine hidden information that may be ignored in the linear regression models. The negative sign of the parameter of  $\Delta V_{PF}$  in Class 2 indicates that drivers in Class 2 tend to merge near the PF vehicles when they are moving faster than their PF vehicles. The signs of the parameters of  $V$  differ between Class 1 and Class 2; hence, there is substantial heterogeneity among the merging drivers, which has never been demonstrated in previous studies. The negative sign in Class 1 and the positive sign in Class 2 indicate that with the increase of the speed, drivers in Class 1 tend to select merging positions that are near their PF vehicles, while drivers in Class 2 tend to choose merging positions that are near their PL vehicles.  $RRD$  is only significant in Class 1. The negative sign of the parameter indicates that drivers in Class 1 tend to select merging position that are further from their PF vehicles when they get closer to the end of the auxiliary lane. Merging vehicles that choose to merge late in the auxiliary lane are likely under congested traffic conditions and might accept smaller gaps. Compared to PF vehicles, PL vehicles are easier to be observe by merging vehicles. Thus, merging vehicle drivers must leave sufficient space for PF vehicles so that they can control the space and relative speed between the PL vehicles and themselves. The reason why  $RRD$  is only significant in Class 1 is likely that merging vehicles in Class 1 are under more congested traffic conditions and, thus, the accepted gaps are smaller.

$Type_{PL}$  is only significant in Class 1 and the negative sign of the parameter indicates that the merging position is closer to the PF vehicle if the PL vehicle is a heavy vehicle. The parameters of  $Type_{PF}$  are significant in Class 2 and Class 3 and have positive signs, which implies that the merging vehicle drivers will maintain larger distances from the PF vehicles to the desired merging positions. It has been proved that heavy vehicles impose heavier physical and psychological effects on the surrounding traffic due to their physical characteristics (e.g., length and size) and their operational characteristics (e.g., acceleration/deceleration and manoeuvrability) [82]–[84]. Thus, merging drivers tend to maintain larger distances from heavy vehicles for safety reasons.

The parameters of  $LC_{PL}^{cooperative}$  are significant in Class 2 and Class 3. However,  $LC_{PL}^{cooperative}$  has a negative sign in Class 2 but a positive sign in Class 3. Hence, when PL vehicles perform cooperative lane changes to make room for merging vehicles, drivers in Class 2 will select merging positions that are closer to their PF vehicles, while drivers in Class 3 will increase the distances between the desired merging positions and their PF vehicles. This finding again emphasizes the inner-heterogeneity among drivers. The effects of other surrounding lane changes are not significant in this model.

The traffic density of the adjacent main lane  $k_{main}$  is found to be significant in Class 2 and Class 3, with a positive sign in both classes. Hence, drivers in Class 2 and Class 3 tend to maintain larger distances from their desired merging positions to PF vehicles. Merging drivers must accept smaller gaps under congestion and the PL vehicles are easier for them to observe; thus, they tend to select merging positions that are further from PF vehicles for safety reasons.

The correlation coefficients  $R^2$  for the three classes and the overall model are 0.9813, 0.5346, 0.9543 and 0.9663, respectively, which are much higher than those in previous studies (0.664 for OGT vehicles and 0.584 for FGT vehicles in Wan *et al.* [22]). Therefore, the proposed finite mixture of linear regression models can explain most variances of the merging positions. The value of  $R^2$  for Class 2 is the lowest value among the three classes; hence, the scenario that is faced by drivers in Class 2 might be more complicated and varied than in the other two classes.

The mean values and standard errors of the attribute variables for each class are calculated and listed in Table 4 to further demonstrate the overall classification characteristics.

**TABLE 4.** Mean values and standard errors of related exploratory variables.

Variable	Class 1		Class 2		Class 3	
	Mean	Standard Error	Mean	Standard Error	Mean	Standard Error
$d$ (m)	16.51	17.25	10.23	6.36	14.36	13.42
$V$ (m/s)	11.52	3.02	13.93	3.34	12.979	3.13
$\Delta V_{PL}$ (m/s)	0.49	1.74	1.70	2.48	1.416	2.25
$\Delta V_{PF}$ (m/s)	1.06	1.73	2.47	2.50	1.981	2.02
$D$ (m)	29.15	18.72	44.42	31.12	34.08	24.10
$RRD$	0.69	0.33	0.67	0.36	0.5898	0.36
$k_{main}$ (veh/lane/km)	46.43	15.08	33.62	16.90	41.08	14.88

According to Table 4, the related variables in the merging position selection model differ substantially among the three classes. The average distance from the merging position to the PF vehicle in Class 1 is the largest among the three classes, while the average accepted gap in Class 1 is the smallest among the three classes. However, the average accepted gap in Class 2 is the largest among the three classes, while the average distance from the merging position to the PF vehicle is the smallest among the three classes. This finding differs from the finding of previous studies [24], [35] that the merging position is linearly related to the accepted gap. It also implies that the proposed finite mixture of linear regression models can extract the hidden heterogeneity among the merging drivers in merging position selection behaviors.

The average speed and the average speed difference have the largest values in Class 2 and the smallest values in Class 1. In addition, Class 1 has the highest average traffic density

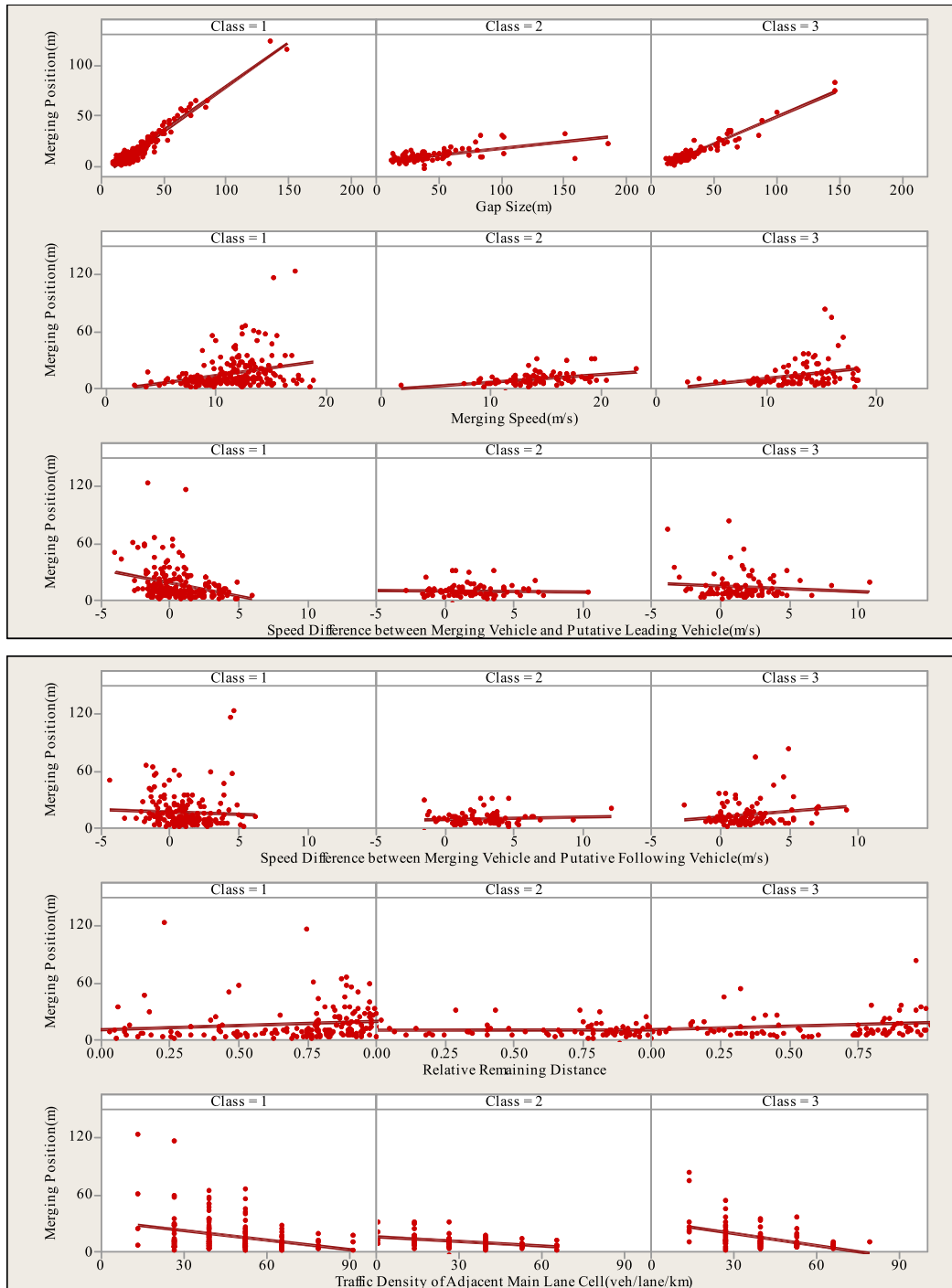


FIGURE 7. Relations between the merging position and related variables.

and Class 2 has the lowest average traffic density. Thus, the heterogeneity in the merging position selection behaviors is likely due to the varied traffic conditions instead of the gap choice behavior. This finding also supports the conjecture that *RRD* is only significant in Class 1 and it is of negative sign because the merging vehicles in Class 1 were under more congested traffic conditions and accepted the smallest gaps.

When selecting the merging position, they tend to leave more space for PF vehicles as PL vehicles are easier to observe. To better illustrate the differences among the three classes, Figure 7 plots the relations between the merging position and related variables in each class. The  $d$  values increase with the increase of the accepted gap in all three classes. However, the slopes differ substantially among the classes. Similar

heterogeneities can also be observed in the relations between the merging position and other related variables in each class. These figures clearly demonstrate the heterogeneity among merging vehicle drivers in terms of merging position selection behaviors and strategies.

## B. DISCUSSION

The finite mixture of linear regression models automatically classified the merging drivers into three classes. The identified three classes differ in terms of characteristics and merging conditions. The results that are presented above support the existence of heterogeneity among the merging drivers. It is demonstrated that the classes of models may differ in terms of their significant variables, and a variable may differ in sign among classes; thus, the strategies that drivers use in merging position selection may differ across classes.

Drivers in Class 1 were under the most congested traffic conditions and had the smallest accepted gaps and the lowest speeds. To safely and successfully merge into the main lane, drivers required more time to adjust their speeds according to the speeds of the PL and PF vehicles. Thus, drivers in Class 1 try to choose the merging position at the midpoint between the PL and PF vehicles and the merging speeds are similar to the speeds of the PL and PF vehicles. In contrast to Class 1, drivers in Class 2 were under the lowest traffic density and had the largest accepted gaps and the highest speeds. These largest accepted gaps gave the merging drivers more space for adjusting their positions and speeds after entering the adjacent main lane. Thus, drivers in Class 2 tend to merge close to the PF vehicle and to maintain the largest relative speeds among the three classes. It can be seen in Table 4 that the standard error of  $d$  in class 2 is only 6.36, which is much smaller than those in the other two classes (17.25 and 13.42). However the standard error of  $D$  in Class 2 is just the opposite. It means that drivers in Class 2 have a merging strategy that they are driving much faster than the main lane vehicles and keep close to the PF vehicle because they choose the largest accepted gaps. The high discreteness of the accepted gaps of Class 2 resulted in the the lowest  $R^2$  values among the three classes. The drivers in Class 3 fall in between. The accepted gaps of Class 3 are slightly larger than those of Class 1 and are much smaller than those of Class 2. However, the relative speeds of Class 3 are much larger than those of Class 1. Hence, drivers in Class 3 are much more aggressive than drivers in Class 1.

Three vehicles (ID 22386, ID 30676, and ID 11428) were randomly selected from the three classes, respectively. The basic variables are listed in Table 5. Figure 8 plots  $\Delta V_{PL}$  and  $\Delta V_{PF}$  during the merging process. According to Figure 8, the vehicle with ID 22386 had a reduced speed difference at the start of merging process because it is difficult to find acceptable gaps under congestion. The speed differences of the other two vehicles at the start of the merging process were both larger. However, the vehicle with ID 11428 reduced its speed substantially during the merging process, while the vehicle with ID 30676 maintained a large speed difference

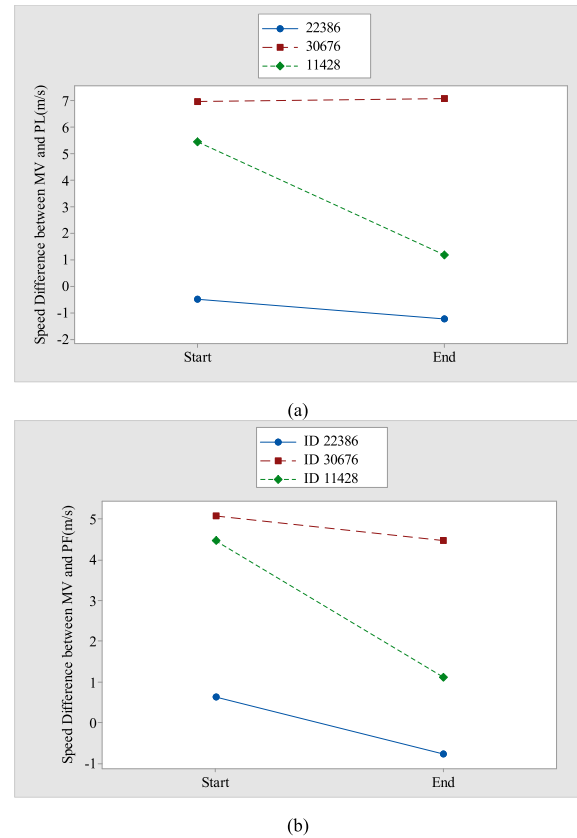


FIGURE 8. (a)  $\Delta V_{PL}$  and (b)  $\Delta V_{PF}$  during the merging processes of three selected vehicles.

TABLE 5. Variables of three randomly selected vehicles.

Variable	ID 22386	ID 30676	ID 11428
$d$ (m)	14.16	5.33	13.39
$V$ (m/s)	10.80	43.65	12.979
$\Delta V_{PL}$ (m/s)	1.22	7.07	1.19
$\Delta V_{PF}$ (m/s)	0.77	4.47	1.11
$D$ (m)	23.86	43.65	28.42
$RRD$	0.9806	0.2398	0.3205
$k_{main}$ (veh/lane/km)	52.49	26.25	39.37

throughout the merging process. According to Table 5 and Figure 8, the three vehicles exhibit distinct behaviors during the merging position selection process.

The underlying causes for the heterogeneous merging behaviors could be both external and internal. For external causes, the traffic condition is one of the most important ones, which has been proved in several studies [1], [37], [66], [85]. The significant difference of  $k_{main}$  among the three classes also provide certain degree of evidence. Driver characteristics such as aggressiveness, age, driving skill level, are all important internal causes. However, limited to the dataset, human factors cannot be collected in this study.

The results demonstrate that incorporating heterogeneity can dramatically improve the accuracy of the merging position selection model; hence, the proposed model is a promising microscopic traffic simulation tool. By collecting more data, the proposed model can also be used to recognize various driving styles, which can facilitate the development of a personalized autonomous driving system or driving assistance system.

## VI. CONCLUSION

To investigate the heterogeneity among the drivers in terms of merging position selection behaviors, the authors presented a model that is composed of a finite mixture of linear regression models. The proposed model can naturally incorporate the unobserved heterogeneity into the merging position selection model. BIC values are used to determine the optimal number of classes and the parameters were estimated by Latent GOLD 5.0. The US-101 dataset in the NGSIM dataset was used to calibrate the model.

Ultimately, a 3-class linear regression model was built and significant heterogeneity among merging vehicle drivers was identified. The results demonstrate that the variables differ among the classes and the sign of each variable may also differ among the classes; hence, the strategies that are used by drivers for merging position selection differ among the classes. Cooperative lane changing behavior by the PL vehicle as found to significantly influence the merging position selection behavior; thus, merging behavior is a two-dimension behavior and may be influenced by both lateral and longitudinal factors. The three classes can be regarded as conservative, normal and aggressive drivers, which can be helpful for the further study of driving behavior, microscopic traffic simulation and car insurance. This study presented a comprehensive analysis method that has the extensive applicability and practical significance. If we collect more data from more places, using this method could thoroughly dig up the driver heterogeneity.

In conclusion, the proposed model can naturally identify the heterogeneity among merging drivers and produce accurate predicted results of the desired merging position. Thus, the proposed model is a promising tool for microscopic traffic simulation and automatic driving systems or driver assistance systems. It can also be used to classify driving styles, which may attract attention from the insurance industries.

Nonetheless, this study only used the NGSIM data and did not consider the characteristics of drivers due to the lack of driver information. In the future, we will attempt to collect more experimental data and to build a more comprehensive model that connects the personal characteristics with the heterogeneity. We will also conduct a focus group study to investigate the influence of human factors on merging behaviors.

## REFERENCES

- [1] W. Daamen, M. Loot, and S. P. Hoogendoorn, "Empirical analysis of merging behavior at freeway on-ramp," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2188, no. 1, pp. 108–118, 2010.
- [2] W. Tang and D. M. Levinson, "Deviation between actual and shortest travel time paths for commuters," *J. Transp. Eng. A, Syst.*, vol. 144, no. 8, 2018, Art. no. 04018042.
- [3] L. Elefteriadou, R. P. Roess, and W. R. McShane, "Probabilistic nature of breakdown at freeway merge junctions," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1484, pp. 80–89, 1995.
- [4] H. Yi and T. E. Mulinazzi, "Urban freeway on-ramps: Invasive influences on main-line operations," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2023, no. 1, pp. 112–119, 2007.
- [5] G. Li, "Application of finite mixture of logistic regression for heterogeneous merging behavior analysis," *J. Adv. Transp.*, vol. 2018, Nov. 2018, Art. no. 1436521.
- [6] G. Li and L. Sun, "Characterizing heterogeneity in drivers' merging maneuvers using two-step cluster analysis," *J. Adv. Transp.*, vol. 2018, May 2018, Art. no. 5604375.
- [7] W. Lan, C. Xuewu, and L. Tao, "Additional riderships estimation under different configurations of bus rapid transit system," *Math. Problems Eng.*, vol. 2015, Feb. 2015, Art. no. 406545.
- [8] Y. Zhao, S. V. Ukkusuri, and J. Lu, "Multidimensional scaling-based data dimension reduction method for application in short-term traffic flow prediction for urban road network," *J. Adv. Transp.*, vol. 2018, Nov. 2018, Art. no. 3876841.
- [9] S. Ossen, S. P. Hoogendoorn, and B. G. H. Gorte, "Interdriver differences in car-following: A vehicle trajectory-based study," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1965, no. 1, pp. 121–129, 2006.
- [10] S. P. Hoogendoorn and R. Hoogendoorn, "Generic calibration framework for joint estimation of car-following models by using microscopic data," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2188, no. 1, pp. 37–45, 2010.
- [11] S. Ossen and S. Hoogendoorn, "Car-following behavior analysis from microscopic trajectory data," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1934, pp. 13–21, Jul. 2005.
- [12] S. Ossen and S. P. Hoogendoorn, "Heterogeneity in car-following behavior: Theory and empirics," *Transp. Res. C, Emerg. Technol.*, vol. 19, no. 2, pp. 182–195, 2011.
- [13] J. Kim and H. S. Mahmassani, "Correlated parameters in driving behavior models: Car-following example and implications for traffic microsimulation," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2249, no. 1, pp. 62–77, 2011.
- [14] Y. Pan, "Lagrangian relaxation for the multiple constrained robust shortest path problem," *Math. Problems Eng.*, vol. 2019, Jun. 2019, Art. no. 3987278.
- [15] N. Chiabaut, L. Leclercq, and C. Buisson, "From heterogeneous drivers to macroscopic patterns in congestion," *Transp. Res. B, Methodol.*, vol. 44, no. 2, pp. 299–308, 2010.
- [16] I. Kim, T. Kim, and K. Sohn, "Identifying driver heterogeneity in car-following based on a random coefficient model," *Transp. Res. C, Emerg. Technol.*, vol. 36, no. 11, pp. 35–44, Nov. 2013.
- [17] L. Sun, Y. Pan, and W. Gu, "Data mining using regularized adaptive B-splines regression with penalization for multi-regime traffic stream models," *J. Adv. Transp.*, vol. 48, no. 7, pp. 876–890, 2014.
- [18] S. Ossen and S. P. Hoogendoorn, "Driver heterogeneity in car following and its impact on modeling traffic dynamics," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1999, no. 1, pp. 95–103, 2007.
- [19] J. Taylor, X. Zhou, N. M. Roupail, and R. J. Porter, "Method for investigating intradriver heterogeneity using vehicle trajectory data: A dynamic time warping approach," *Transp. Res. B, Methodol.*, vol. 73, pp. 59–80, Mar. 2015.
- [20] H. Wang, W. Wang, J. Chen, and M. Jing, "Using trajectory data to analyze intradriver heterogeneity in car-following," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2188, no. 1, pp. 85–95, 2010.
- [21] V. Alexiadis, J. Colyar, J. Halkias, R. Hranac, and G. McHale, "The next generation simulation program," *ITE J. Inst. Transp. Eng.*, vol. 74, no. 8, p. 22, 2004.
- [22] X. Wan, P. J. Jin, H. Gu, X. Chen, and B. Ran, "Modeling freeway merging in a weaving section as a sequential decision-making process," *J. Transp. Eng. A, Syst.*, vol. 143, no. 5, 2017, Art. no. 05017002.
- [23] K. I. Ahmed, "Modeling drivers' acceleration and lane changing behavior," M.S. thesis, Massachusetts Inst. Technol., Cambridge, MA, USA, 1999.
- [24] C. F. Choudhury, V. Ramanujam, and M. Ben-Akiva, "Modeling acceleration decisions for freeway merges," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2124, no. 1, pp. 45–57, 2009.

- [25] J. Wang, R. Liu, and F. Montgomery, "A simulation model for motorway merging behaviour," in *Proc. 16th Int. Symp. Theory Traffic Flow Transp.*, 1971, pp. 281–301.
- [26] D. Sun and L. Elefteriadou, "A driver behavior-based lane-changing model for urban arterial streets," *Transp. Sci.*, vol. 48, no. 2, pp. 184–205, 2014.
- [27] D. R. Drew, L. R. LaMotte, J. Buhr, and J. Wattleworth, "Gap acceptance in the freeway merging process," *Highway Res. Rec.*, vol. 208, pp. 1–36, Jan. 1967.
- [28] R. Herman and G. Weiss, "Comments on the highway-crossing problem," *Oper. Res.*, vol. 9, no. 6, pp. 828–840, 1961.
- [29] P. G. Gipps, "A model for the structure of lane-changing decisions," *Transp. Res. B, Methodol.*, vol. 20, no. 5, pp. 403–414, Oct. 1986.
- [30] P. Hidas, "Modelling lane changing and merging in microscopic traffic simulation," *Transp. Res. C, Emerg. Technol.*, vol. 10, nos. 5–6, pp. 351–371, Oct./Dec. 2002.
- [31] L. Bloomberg and J. Dale, "A comparison of the VISSIM and CORSIM traffic simulation models," in *Proc. Inst. Transp. Eng. Annu. Meeting*, Washington, DC, USA, 2000, pp. 1–17.
- [32] *VISSIM 5.30-05 User Manual*, PTV-Vision, Karlsruhe, Germany, 2011.
- [33] *S-Paramics 2005 Reference Manual*, SIAS Ltd., Edinburgh, U.K., 2005.
- [34] M. Sarvi and M. Kuwahara, "Microsimulation of freeway ramp merging processes under congested traffic conditions," *IEEE Trans. Intell. Transp. Syst.*, vol. 8, no. 3, pp. 470–479, Sep. 2007.
- [35] P. Hidas, "Modelling vehicle interactions in microscopic simulation of merging and weaving," *Transp. Res. C, Emerg. Technol.*, vol. 13, no. 1, pp. 37–62, 2005.
- [36] C. F. Choudhury, M. E. Ben-Akiva, T. Toledo, G. Lee, and A. Rao, "Modeling cooperative lane changing and forced merging behavior," in *Proc. 86th Annu. Meeting Transp. Res. Board*, Washington, DC, USA, 2007, pp. 1–23.
- [37] F. Marczak, W. Daamen, and C. Buisson, "Merging behaviour: Empirical comparison between two sites and new theory development," *Transp. Res. C, Emerg. Technol.*, vol. 36, pp. 530–546, Nov. 2013.
- [38] Z. Zheng, "Recent developments and research needs in modeling lane changing," *Transp. Res. B, Methodol.*, vol. 60, pp. 16–32, Feb. 2014.
- [39] T. D. Chu, "A study on merging behavior at urban expressway merging sections," Ph.D. dissertation, Nagoya Univ., Nagoya, Japan, 2014.
- [40] H. Kita, "Effect of merging lane length on the merging behavior at expressway onramps," in *Proc. 12th Int. Symp. Theory Traffic Flow Transp.*, 1993, pp. 37–51.
- [41] J. Weng and Q. Meng, "Modeling speed-flow relationship and merging behavior in work zone merging areas," *Transp. Res. C, Emerg. Technol.*, vol. 19, no. 6, pp. 985–996, 2011.
- [42] J. Weng, S. Xue, Y. Yang, X. Yan, and X. Qu, "In-depth analysis of drivers' merging behavior and rear-end crash risks in work zone merging areas," *Accident Anal. Prevention*, vol. 77, pp. 51–61, Apr. 2015.
- [43] J. Weng, G. Du, D. Li, and Y. Yu, "Time-varying mixed logit model for vehicle merging behavior in work zone merging areas," *Accident Anal. Prevention*, vol. 117, pp. 328–339, Aug. 2018.
- [44] J. Sun, K. Zuo, S. Jiang, and Z. Zheng, "Modeling and predicting stochastic merging behaviors at freeway on-ramp bottlenecks," *J. Adv. Transp.*, vol. 2018, May 2018, Art. no. 9308580.
- [45] X. Gu, M. Abdel-Aty, Q. Xiang, Q. Cai, and J. Yuan, "Utilizing UAV video data for in-depth analysis of drivers' crash risk at interchange merging areas," *Accident Anal. Prevention*, vol. 123, pp. 159–169, Feb. 2019.
- [46] D. Arbis and V. V. Dixit, "Game theoretic model for lane changing: Incorporating conflict risks," *Accident Anal. Prevention*, vol. 125, pp. 158–164, Apr. 2019.
- [47] K. Kang and H. A. Rakha, "Modeling driver merging behavior: A repeated game theoretical approach," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2672, no. 20, pp. 144–153, 2018.
- [48] Q. Meng and J. Weng, "Classification and regression tree approach for predicting drivers' merging behavior in short-term work zone merging areas," *J. Transp. Eng.*, vol. 138, no. 8, pp. 1062–1070, Aug. 2012.
- [49] E. Balal, R. L. Cheu, and T. Sarkodie-Gyan, "A binary decision model for discretionary lane changing move based on fuzzy inference system," *Transp. Res. C, Emerg. Technol.*, vol. 67, pp. 47–61, Jun. 2016.
- [50] J. Tang, F. Liu, W. Zhang, R. Ke, and Y. Zou, "Lane-changes prediction based on adaptive fuzzy neural network," *Expert Syst. Appl.*, vol. 91, pp. 452–463, Jan. 2018.
- [51] Y. Hou, P. Edara, and C. Sun, "Modeling mandatory lane changing using Bayes classifier and decision trees," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 647–655, Apr. 2014.
- [52] S. Ramyar, A. Homaifar, A. Karimodini, and E. Tunstel, "Identification of anomalies in lane change behavior using one-class SVM," in *Proc. IEEE Int. Conf. Syst., Man, Cybern. (SMC)*, Oct. 2016, pp. 4405–4410.
- [53] A. Polus, M. Livneh, and J. Factor, "Vehicle flow characteristics on acceleration lanes," *J. Transp. Eng.*, vol. 111, no. 6, pp. 595–606, 1985.
- [54] M. A. Ahammed, Y. Hassan, and T. A. Sayed, "Modeling driver behavior and safety on freeway merging areas," *J. Transp. Eng.*, vol. 134, no. 9, pp. 370–377, Sep. 2008.
- [55] L. Sun, J. Yang, H. Mahmassani, W. Gu, and B.-J. Kim, "Data mining-based adaptive regression for developing equilibrium speed-density relationships," *Can. J. Civil Eng.*, vol. 37, no. 3, pp. 389–400, 2010.
- [56] L. Sun, H. Zhang, R. Gao, W. Gu, B. Xu, and L. Chen, "Gaussian mixture models for clustering and classifying traffic flow in real-time for traffic operation and management," *J. Southeast Univ.*, vol. 27, no. 2, pp. 174–179, 2011.
- [57] W. Zhang, T. V. Le, S. V. Ukkusuri, and R. Li, "Influencing factors and heterogeneity in ridership of traditional and app-based taxi systems," *Transportation*, vol. 2018, pp. 1–26, Oct. 2018.
- [58] Y. Li, Z. Li, H. Wang, W. Wang, and L. Xing, "Evaluating the safety impact of adaptive cruise control in traffic oscillations on freeways," *Accident Anal. Prevention*, vol. 104, pp. 137–145, Jul. 2017.
- [59] Y. Guo, Z. Li, Y. Wu, and C. Xu, "Exploring unobserved heterogeneity in bicyclists' red-light running behaviors at different crossing facilities," *Accident Anal. Prevention*, vol. 115, pp. 118–127, Jun. 2018.
- [60] T. Toledo, H. N. Koutsopoulos, and M. Ben-Akiva, "Estimation of an integrated driving behavior model," *Transp. Res. C, Emerg. Technol.*, vol. 17, no. 4, pp. 365–380, 2009.
- [61] G. Lee, "Modeling gap acceptance at freeway merges," M.S. thesis, Massachusetts Inst. Technol., Cambridge, MA, USA, 2006.
- [62] A. Rao, "Modeling anticipatory driving behavior," M.S. thesis, Dept. Civil Environ. Eng., Massachusetts Inst. Technol., Cambridge, MA, USA, 2006.
- [63] *NGSIM US 101 Data Analysis: Summary Report, Prepared for Federal Highway Administration*, Cambridge Syst., Cambridge, MA, USA, 2005.
- [64] V. Punzo, M. T. Borzacchiello, and B. Ciuffo, "On the assessment of vehicle trajectory data accuracy and application to the next generation simulation (NGSIM) program data," *Transp. Res. C, Emerg. Technol.*, vol. 19, pp. 1243–1262, Dec. 2011.
- [65] C. Thiemann, M. Treiber, and A. Kesting, "Estimating acceleration and lane-changing dynamics from next generation simulation trajectory data," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2088, no. 1, pp. 90–101, 2008.
- [66] G. Li and J. Cheng, "Exploring the effects of traffic density on merging behavior," *IEEE Access*, vol. 7, pp. 51608–51619, 2019.
- [67] X. Wan, P. J. Jin, F. Yang, and B. Ran, "Merging preparation behavior of drivers: How they choose and approach their merge positions at a congested weaving area," *J. Transp. Eng.*, vol. 142, no. 9, 2016, Art. no. 05016005.
- [68] J. A. Laval and C. F. Daganzo, "Multi-lane hybrid traffic flow model: A theory on the impacts of lane-changing maneuvers," presented at the 84th Annu. Meeting Transp. Res. Board, Washington, DC, USA, 2005.
- [69] M. Park, K. Jang, J. Lee, and H. Yeo, "Logistic regression model for discretionary lane changing under congested traffic," *Transportmetrica A, Transp. Sci.*, vol. 11, no. 4, pp. 333–344, 2015.
- [70] W. Hugemann, "Driver reaction times in road traffic," in *Proc. 6th EVU (Eur. Assoc. Accident Res. Accident Anal.) Annu. Meeting*, Portorož, Slovenia, vol. 32, 2002, pp. 1–12.
- [71] C. Fraley and A. E. Raftery, "Model-based clustering, discriminant analysis, and density estimation," *J. Amer. Statist. Assoc.*, vol. 97, no. 458, pp. 611–631, 2002.
- [72] G. McLachlan and D. Peel, *Finite Mixture Models*. Hoboken, NJ, USA: Wiley, 2004.
- [73] W. H. Greene and D. A. Hensher, "A latent class model for discrete choice analysis: Contrasts with mixed logit," *Transp. Res. B, Methodol.*, vol. 37, no. 8, pp. 681–698, 2003.
- [74] F. Leisch, "FlexMix: A general framework for finite mixture models and latent class regression in R," *J. Stat. Softw.*, vol. 11, no. 8, pp. 1–18, 2004.
- [75] A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *J. Roy. Stat. Soc. B, Methodol.*, vol. 39, no. 1, pp. 1–22, 1977.
- [76] J. K. Vermunt and J. Magidson, *Latent GOLD 5.0 Upgrade Manual*. Belmont, MA, USA: Statistical Innovations Inc., 2013.
- [77] H. Akaike, "A new look at the statistical model identification," *IEEE Trans. Autom. Control*, vol. AC-19, no. 6, pp. 716–723, Dec. 1974.
- [78] G. Schwarz, "Estimating the dimension of a model," *Ann. Statist.*, vol. 6, no. 2, pp. 461–464, 1978.

- [79] C. Biernacki, G. Celeux, and G. Govaert, "Assessing a mixture model for clustering with the integrated completed likelihood," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 22, no. 7, pp. 719–725, Jul. 2000.
- [80] G. M. Allenby, "Hypothesis testing with scanner data: The advantage of Bayesian methods," *J. Marketing Res.*, vol. 27, no. 4, pp. 379–389, 1990.
- [81] Q. Wang, Z. Li, and L. Li, "Investigation of discretionary lane-change characteristics using next-generation simulation data sets," *J. Intell. Transp. Syst.*, vol. 18, no. 3, pp. 246–253, 2014.
- [82] M. S. Uddin and S. A. Ardekani, "An observational study of lane changing on basic freeway segment," in *Proc. Transp. Res. Board Annu. Meeting*, Washington, DC, USA, 2002.
- [83] A. Al-Kaisy and F. Hall, "Guidelines for estimating capacity at freeway reconstruction zones," *J. Transp. Eng.*, vol. 129, no. 5, pp. 572–577, 2003.
- [84] S. Moridpour, E. Mazloumi, and M. Mesbah, "Impact of heavy vehicles on surrounding traffic characteristics," *J. Adv. Transp. Banner*, vol. 49, no. 4, pp. 535–552, 2015.
- [85] C. Hill, L. Elefteriadou, and A. Kondyli, "Exploratory analysis of lane changing on freeways based on driver behavior," *J. Transp. Eng.*, vol. 141, no. 4, 2014, Art. no. 04014090.



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