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Integrated Inventory-Transportation Problem in Vendor-Managed Inventory System

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ABSTRACT The paper presents a two-echelon inventory-transportation problem in Vendor Managed Inventory (VMI) system. We consider a distribution system composed with single supplier, single distribution center and multiple retailers. Single kind of products are required to deliver from the manufacturer through distribution center to the retailers within soft time window. The objective of the problem is to minimize total logistics cost in the distribution network, including inventory cost, distribution cost, and time penalty cost. The upper echelon model focuses on minimizing inventory cost while the lower echelon model on vehicle routing problem. A mixed algorithm is designed to solve the problem with simulated annealing and ant colony with local search. The solution of upper and lower echelon model are substituted into each other based on the mixed algorithm step by step to get the optimization solutions. Computational experiments are executed to compare the performance of independent and integrated inventory-transportation optimization from the dimension of to verify the effectiveness of the model and the algorithms.

INDEX TERMS Integrated inventory-transportation, vendor-managed inventory, stimulated annealing, ant colony, local search.

I. INTRODUCTION

Vendor-Managed Inventory (VMI) has been a widely and successfully practiced strategy in supply chain management, aiming at reducing cost and adding value. Suppliers manages the inventory for the retailers including the decisions of replenishing, delivering and inventory management for supply chains implementing VMI (Lee and Seungjin [1]; Simchi-Levi *et al.* [2]). From the viewpoint of operating, VMI system needs to be practically executed by many specific ways in inventory-transportation problem. Integrated transportation-inventory problem incorporates transportation and inventory together by total cost minimization of supply chain by balancing the trade-off between the cost characteristics between transportation and inventory.

Implementing VMI could effectively help reduce the stock level of downstream enterprises in the supply chain and operating cost, however, at the same time, VMI requires a higher expectations for suppliers. In the operational process of VMI, vendors would transfer inventory and transportation to the suppliers as much as possible, thus increasing the depth and lasting time of suppliers confronting with the risks of inventory and transportation. With regard to inventory, if there exists stock-out in downstream vendors, suppliers would confront risks of delivery delay and profit loss. In respective of transport, suppliers need to determine a proper transportation strategy to deal with diversified products, small-batch, and different destinations of the orders. The objective of the vendor is to satisfy the requirements of the retailers as much as possible with minimum total cost.

Early models of integrated transportation-inventory models almost originated by Baumol and Vinod [3]. They proposed one of the earliest integrated transportation-inventory models. The demand of transportation could be derived by incorporated demand and inventory. Das [4] studied on supplier implementing on-time delivery by investing in inventory. It is an extension of the model of Baumol and Vinod [3] by another way of estimation models in forecasting customers' demand. Blumenfeld *et al.* [5] investigated the optimization of routing and shipment size problem considering cost of transportation and inventory. They firstly analyzed how transportation affects inventory.

There have been a handful of review papers on integrated transportation-inventory problem. Williams and Tokar [6] proposed a review on inventory integrated with multiple elements of logistics decisions and inventory management.

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They maintained that collaborative models to be important tool for solving stochastic problems. Bartolacci *et al.* [7] reviewed on collaboration in logistics in different levels and business aspects among different parties of the supply chain based on many applications such as collaborative planning forecasting and replenishment, vendor managed inventory and collaborative development chain management. Engebrethsen and Dauzère-Pérèsab [8] provided a review on inventory models considering different transportation modes.

The necessity of optimizing inventory and transportation process simultaneously under different demand distribution has been highlighted in previous papers. Yano and Gerchak [9] proposed joint optimization of inventory and transportation considering safety stock level and scheduling. Henig *et al.* [10] modeled simultaneous determination of inventory policy and inbound transportation. Ernst and Pyke [11] studied on a two-level system consisted of single warehouse and single retailer with random demand. They regarded different inventory policies, truck capacity, and delivery frequency. The stock out demand for warehouse and transportation would be satisfied by leasing vehicles. Geunes and Zeng [12] extended the research of Ernst and Pyke [11], modeling how warehouse management demonstrating how inventory impact on transportation cost and how to set stock level to lower total cost of inventory and transportation, concluding that the relationship of inventory and transportation cost could help to determine how to deal with demand shortage from the customers. Tempelmeier and Bantel [13] developed a multi-objective model composed of single supplier and multiple customers optimizing inventory and transportation jointly under stochastic demand. They discussed the conditions of both inhouse and outsourcing transportation. The innovation of their study was that they considered backorder in with previous ones are that backorder is considered both in inventory and transportation.

There are also a number of studies concerning with replenishment and delivery simultaneously. Cachon [14] concentrated on the problem of balancing transportation, warehouse and inventory costs under multi-products and stochastic demand. The authors came up with three strategies for truck scheduling including one continuous policy and two periodic policies. An EOQ based heuristic method was used to choose from the three different policies. The study derived that continuous policy led to lower cost than the other two periodic policies especially when lead time is short. Gürbüz *et al.* [15] developed the problem consisting of single supplier with single kind of items, and multiple retailers under stochastic demand. They introduced a new mode of replenishment which is different from traditional way that retailers separately ordering from the supplier by setting a virtual warehouse to deal with orders from different retailers and coordinated the demand. They also built a framework of jointly determining inventory and transportation with capacity limitation of transport. Tanrikulu *et al.* [16] developed stochastic joint replenishment problem with multiple products under stochastic demand. They considered a particular cost structure with additional vehicles in delivery process, and they also proposed a new policy that could make different items replenishing independently. Zhao *et al.* [17] studied on one vendor delivered to multiple customers, proposing a VMI system by building up a central warehouse. Markov decision process was used to construct integrated order delivery problem. The most significant innovation of this research is that a lot more real life conditions such as vehicle capacities, transportation cost, and penalty cost.

Integrated transportation inventory problem with routing is one of the most commonly studied aspects. The general objective is minimization of total cost by considering both inventory and vehicle routing problems. Integrated inventory –routing problem was originated by Bell *et al.* [18]. Comparatively early studies related ranged from single period (Federgruen and Zipkin [19]; Gallego and Simchi-Levi [20]) to multi-period (Anily and Federgruen, [21], [22]). Baita *et al.* [23] reviewed on dynamic inventory-routing problems under distribution logistics. Recent reviews on inventory routing problem could be found in Andersson *et al.* [24], Bertazzi and Speranza [25], Coelho *et al.* [26], and Archetti *et al.* [27]. The earliest branch-and-cut algorithm was implemented by Archetti *et al.* [28] to solve mixed-integer programming model of vendor-managed inventory-routing problem with single vehicle. The model of Archetti *et al.* was then improved by Avella [29] that they involved multi-product from single supplier to multiple customers. The model was made up of two parts, a lot-sizing problem for every customer and a VRP (vehicle routing problem) for each period. They reformulated the problem by taking constant demand and the upper bounds into account. Zhao *et al.* [30] extended classical VRP considering the factor of vehicle's travel speed. Such bi-objective problem has also been studied on integrated optimization on transportation industry [31]–[34]. There are also other further extensions related to inventory routing problem with pickups and deliveries (Iassinovskaia *et al.* [35]; Archetti *et al.* [36]), and environmental factors [37]–[39].

There are gap on previous research that simultaneously considering the factor of distribution period, delivery bunch and vehicle routing in single supplier and multiple customers. The contributions of our work is that we consider the Firstly, time period i.e. delivery frequency. If span of each period is known, the supplier needs to determine the time to deliver; if the span of each period is unknown, the supplier needs to determine the frequency to deliver. Secondly, replenishment quantity, the quantity that the supplier needs to determine at each time of deliver. Thirdly, vehicle routing, as time/frequency and replenishment quantity are already derived from the above two points, the supplier needs to determine VRP.

The remainder of the paper is organized as follows. Section II describes the problem and brings out the assumptions. In Section III, we present mathematical formulation of the two-echelon model of inventory-transportation problem.

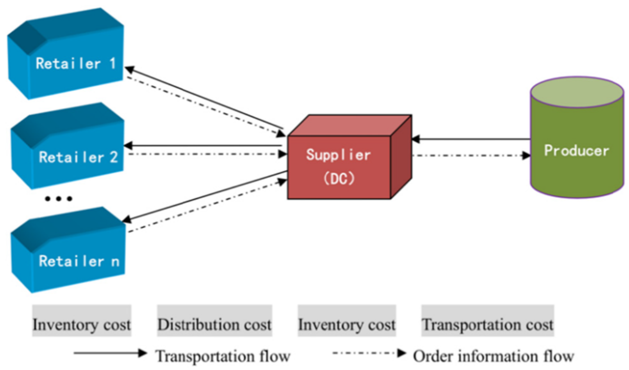

FIGURE 1. Logistics system structure under VMI mode.

An improved heuristic algorithm is proposed in Section IV. A numerical experiment is followed in Section V. Conclusions are presented in Section VI.

II. PROBLEM DESCRIPTION AND ASSUMPTIONS

We consider a VMI system composed with single vendor and multiple retailers. The demand of the retailers are determinant and only one kind of product. The logistics structure of the problem is descripted in Fig. 1. As in VMI system, the supervision of inventory and transportation are controlled by the retailer. Then the optimization of integrated inventory-transportation problem is operated from the perspective of the retailer, aiming at delivering at the right time with the proper replenishment quantity with minimization of total operating cost. In addition, we also consider the restriction of soft time window, which is reflected as penalty cost for failing to deliver on-time. The longer time delayed, the higher penalty cost are confronted by the retailer.

Based on the analysis above, the problem of integrated inventory-transportation optimization could be concluded as follows: under VMI system and infinite planned time period, the supplier offers transportation service for n number of retailers. The requirement quantity of retailer i is d_i. There are m number of vehicles with the maximum load capacity of W in the system that could be dispatched. The time window of receiving the replenishment is $[ET_i, LT_i]$. If the replenishment vehicle arrives out of scope of requested time window, retailers would still be able to receive the products, but the retailers would pay time penalty cost. The optimization objective of the problem is the total cost minimum of the system.

In addition to based set up of the problem, other assumptions considered are as follows:

- Replenishment system is composed with single product, single vendor and multiple retailers.
- Replenishment quantity for each retailer is determinant, and no stock-out is accepted by the retailers.
- Planned periodic time of the system is determinant.
- Inventory holding cost of the vendor and order cost are not considered.
- Inventory storage cost per unit of is the same for each retailer.
- Lead time of replenishment is zero.
- Vehicles for transportation are of the same type, and the vehicle numbers are limited.
- Replenishment quantity each time for each retailer is within the capacity of single vehicle.
- Soft windows are considered.

III. MATHEMATICAL FORMULATION

The following parameters and variables listed are used in the definitions and formulations of the model.

n: number of retailers.

i: index of retailer, $i = \{1, 2, 3 \ldots, n\}$. $i = 0$ indicates the supplier.

d_i: demand quantity of retailer i in each period.

D: total demand quantity for all retailers, and it is denoted by $D = \sum_{i=1}^{n} d_i$.

h: inventory holding cost per unit product and each time period for each retailer.

Q_i: replenishment quantity for retail i each time.

Q: total replenishment quantity of the supplier each deliver period.

c_w: fixed cost of each route.

m: number of vehicles.

w: loading capacity of each vehicle.

l_k: distance from the manufacturer to the distribution center.

c_k: fixed cost from the manufacturer to each retailer.

l_{ij}: distance of each retailer to the distribution center or the distance between every retailer.

c_{ij}: transportation cost from retailer i to retailer j. Transportation cost from the supplier to the retailer is c_{i0} or c_{0j}, where transportation cost is in proportion to distance with coefficient of 1, i.e. $l_k = c_k$; $l_{ij} = c_{ij}$.

c_u: fixed cost of delivering from the manufacturers to the supplier each time.

c_v: fixed cost of supplier's replenishment each time.

s_{uv}: transportation cost from the manufacturer to the supplier per unit product.

x_{ijk}: when the vehicle k travels from retailer i to retailer j, it equals to 1; otherwise, it equals to zero.

y_{ik}: when the cargo task of retailer i is accomplished by vehicle k, it equals to 1; otherwise, it equals to 0.

a_i: the beginning of time window of retailer i.

b_i: the ending of time window of retailer i.

s_i: the moment that delivering vehicle arrives to retailer i.

t_i: vehicle's waiting time after arriving at retailer i.

t_{ij}: travel time from retailer i to retailer j.

p: unit opportunity waiting cost for early arrival.

q: unit delay cost for late arrival.

$J(Q)$: distribution cost function for accomplishing once from the supplier to every retailer (including vehicle cost and unloading cost).

$P_1(Q)$: transshipment transportation cost function from the manufacturer to the supplier.

$P_2(Q)$: total cost function of the distribution.

$TC(Q)$: total cost function of integrated inventory-transportation in VMI system in a planned period.

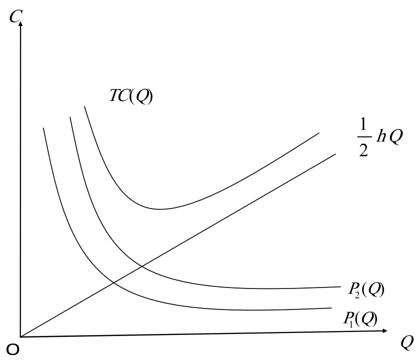

FIGURE 2. **Total cost function.**

We then establish an optimization model and solve the problem with two-echelon programming to determine the delivery frequency in planned period, order quantity, distribution quantity and the vehicle routing of each retailer.. The objective of the optimization is the minimum of total cost (transshipment cost, inventory cost and distribution cost). The relationship of different kinds of costs are shown in Fig. 2, indicating that the total cost consisted of transshipment cost from the manufacturer to the supplier and distribution cost firstly decreased with quantity and then increases.

As shown in Fig. 2 that $TC(Q)$ denotes the total cost of logistics system in a period. $hQ/2$, $P_1(Q)$, and $P_2(Q)$ representatively represent inventory cost, transshipment transportation cost from manufacturer to the supplier, and distribution cost. Thus, total logistics cost of the supplier is

$$TC(Q) = P_1(Q) + P_2(Q) + hQ/2$$

Transhipment cost is

$$P_1(Q) = (c_u + c_k)\frac{D}{Q} + s_{uv}D$$

Distribution cost is

$$P_2(Q) = c_v\frac{D}{Q} + J(Q)\frac{D}{Q}$$

Total cost in a period is

$$J(Q) = F + c_w k$$

where

$$F = \min \sum_{k=1}^{m}\sum_{j=0}^{n}\sum_{i=0}^{n} c_{ij}x_{ijk}$$

$k = \lceil Q/w \rceil$, that symbol $\lceil \ \rceil$ denotes round up to an integer.

Meanwhile, we introduce penalty function to add the restriction of soft time window with the formulation

$$P_j(s_j) = \begin{cases} p\left(a_j - s_j\right), & s_j < a_j \\ 0, & a_j \le s_j \le b_j \\ q\left(s_j - b_j\right), & s_j > b_j \end{cases}$$

The piecewise function could also be expressed as

$$P_j(s_j) = p\max\left(a_j - s_j, 0\right) + q\left(s_j - b_j, 0\right)$$

When vehicle k arrives at retailer j earlier than the beginning of time window a_j, the vehicle would wait at expense of opportunity cost of $p(a_j - s_j)$. When vehicle k arrives at retailer j later than the end of time window b_j, the distribution service is delayed at expense of delay cost $q(s_j - b_j)$. When vehicle k arrives at retailer in the interval of time window $[a_j, b_j]$, time penalty cost is zero.

Therefore, the two-echelon inventory routing problem with soft time window could be fomulated as follows.

$$U : \min\left[(c_u + c_k + c_v)D/Q + s_{uv}D + hQ/2 + J(Q)D/Q\right]$$

$$L : \min\Big[\sum_{k=1}^{m}\sum_{j=0}^{n}\sum_{i=0}^{n} c_{ij}x_{ijk} + p\sum_{j=0}^{n}\max(a_j - s_j, 0) + q\sum_{j=0}^{n}\max(s_j - b_j, 0) + c_w k \tag{1}$$

$$\lceil Q/w \rceil \le k \le k\_\max \tag{2}$$

$$1 \le k \le (D/w) \tag{3}$$

$$\sum_{i=1}^{n} d_i y_{ik} \le w \tag{4}$$

$$\sum_{j=1}^{n} x_{0jk} = \sum_{j=1}^{n} x_{j0k} = 1 \tag{5}$$

$$\sum_{k=1}^{m} y_{ki} = 1 \tag{6}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{kj} \tag{7}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ki} \tag{8}$$

$$s_0 = 0 \tag{9}$$

$$s_i + t_i + t_{ij} = s_j \tag{10}$$

$$a_j \le s_j \le b_j \tag{11}$$

$$t_i = \max\{a_i - s_i, 0\} \tag{12}$$

$$i, j = 0, 1, \cdots n; \quad i \ne j \tag{13}$$

$$k = 1, 2, \ldots, m \tag{14}$$

The role of constraint would be illustrated as follows. Constraint (2) ensures the value range of the number k that could be dispatched, and defines the mathematical relationship of k and distribution quantity Q. Constraint (3) restricts the number of vehicle that dispatching from the supplier. Capacity restriction is depicted by (4). Constraint (5) regulates that all vehicles start from and return to the distribution center. Constrain (6) guarantees that one retailer is only served by one vehicle each time. Constrains (7) and (8) specify that if retailer is served by vehicle k, the vehicle would travel from i to j. Constrain (9) indicates that the starting moment of the supplier is 0. Equation (10) represents the temporal relationship of vehicle traveling directly from retailer i to retailer j. Constrain (11) defines the time window limitation. Equation (12) defines the time of the vehicle waiting at retailer i. (13) and (14) respectively indicate the value range of i, j, and k.

Upper echelon model U is the supplier's inventory control strategy with minimization of integrated inventory-transportation system's total cost. Lower echelon L is the transportation strategy under condition of inventory in upper echelon, aiming at total distribution cost minimization from the supplier to retailers. Upper echelon model needs to be solved firstly to get the initial solution of inventory strategy, and then the initial solution would be substituted to lower echelon model to get the initial solution to transportation strategy. The initial solution of lower echelon has a feedback effect on the inventory strategy in the upper echelon, then we substitute the initial solution to lower echelon into upper echelon. Such mutual iterating process is repeated to make the whole integrated inventory-transportation system reach the optimum state.

IV. ALGORITHM ANALYSIS

A. UPPER ECHELON: A SIMULATED ANNEALING ALGORITHM

Simulated Annealing (SA) is a probabilistic technique to approximate the global optimum for the objective function. It has been studied in works such as Kirkpatrick *et al.* [40], Laarhoven and Aarts [41], Otten and Ginneken [42], and Ingber [43]. SA is based on what happens to the structure of the atomic when metal is rapidly cooled down. Suddenly decreasing the temperature would lead the non-symmetric change of the structure, while after a while cooling would lead the system get stuck or freeze. It is different with other gradient optimization methodologies, as it would not fall into local minimum. SA has been widely used in many optimization works, such as graph theory, Very Large Scale Integration (VLSI), neural networks, (Coughlin and Baran [44]), and Travelling Salesman Problem (TSP) (Cook [45]; Applegate *et al.* [46]) etc.

The procedures of solving upper echelon model of integrated inventory-transportation optimization are as follows.

Step 1: Initialization. Derive the best distribution quantity Q^1 based on Economic Order Quantity (EOQ) leaving out distribution cost, and substitute the initial solution to the lower echelon model. We could get vehicle routing solution and distribution cost $J(Q^1)$ under distribution quantity Q^1 through improved ant colony algorithm. Then we substitute distribution cost $J(Q^1)$ into upper echelon model to get total cost of the system $TC(Q^1)$. The iteration number k would be determined according to the given initial tempreture T_0, ending tempreture T_f, and annealing velocity.

Step 2: In regard to temperature T_n, the following procedures would be conducted.

a. Make iteration $k = 1$.
b. New state appears, i.e. another distribution quantity Q^2 is formated randomly from the neighbour of the initial solution.

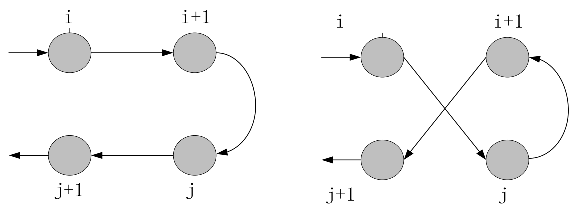

FIGURE 3. Principle of 2-opt.

c. Substitute Q^2 into lower echelon model, deriving the distribution cost $J(Q^2)$ under distribution quantity Q^2. Then substitute $J(Q^2)$ into upper echelon model, calculating total cost of the system $TC(Q^2)$.
d. Make $\Delta E = TC(Q^2) - TC(Q^1)$. If $\Delta E < 0$, distribution quantity Q^2 is accepted. If $\Delta E > 0$, whether to accept Q^2 depends on probibility, namely, a random $\varepsilon \in[0, 1)$, calculate a new accept probability $\exp(-\Delta E/kT)$, where T is present tempreture. When $\exp(-\Delta E/kT) > \varepsilon$, new solution Q^2 would be accepted.
e. If $k = m$, turn to Step 3; or $k = k + 1$, return to Step 2.

Step 3: Determine whether to terminate. If $T_n < T_f$, the algorithm would be terminated and minimization of the system's total cost is derived by optimum distribution quantity Q^*. Otherwise, cooling would be conducted by tempreture function $T_k = aT_{k-1}$, where $a \in[0.5, 0.99]$, the tempreture is cooling down by T_n. Then let $k = k + 1$, and rturn to Step 2.

B. LOWER ECHELON: AN IMPROVED ANT COLONY ALGORITHM

Lower echelon model is actually the problem of vehicle routing. As VRP is NP-hard problem and it is combinatorial optimization problem (Huang and Lin [47]; Yu and Yang [48]; Huang *et al.* [49]). Ant colony algorithm (ACO) heuristic has been widely used to solve optimization problem effectively and efficiently. However, basic ant colony algorithm has the disadvantages of falling into local optimum and slow rate of coverage. In our study, local search strategy is added to ACO algorithm to improve the structure of solution to every generation, aiming at to fold the length of routing and speed up the convergence rate. According to the characteristics of the lower echelon model, 2-opt local optimum is adopted. It is used to link two noncontiguous networks, deleting both the front end of the first network and the backend of the second network. If the new route is better than the previous one, new route would be chosen; otherwise, the new route is rejected. The principle of 2-opt is shown in Fig. 3.

The procedures of ACO with local search are as follows.

Step 1: Initialization. Figure out the locations of distribution center and retailers. Assume that the density of information pheromones in each route is the same, denoted by constraint $\tau_{\max}$. Put numbers of ant k into the distribution center, and set the number of iteration as $t = 0$.

Step 2: Judge the nodes' total transportation quantity in $allowed_k$ table. If it is lower than or equal to the ant's

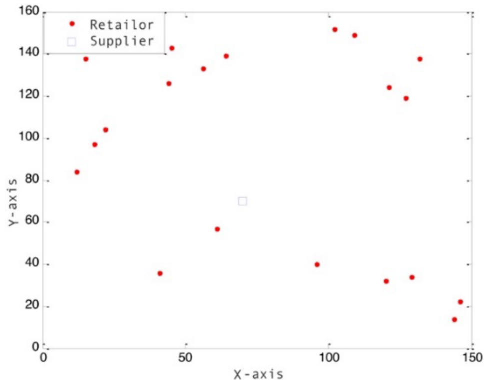

FIGURE 4. Geographical distribution information of retailers and the supplier.

total quantity, we formulate it as $sum(g) \leq w$ (w is the loading capacity of the ant), then make them as alternative nodes.

Step 3: Choose the routes. When the ants choose the routes, they calculate the state transition probability. Then max(P) and node j (the next node i) would be derived as

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\tau_{ij}^{\alpha}(t)\,\eta_{ij}^{\beta}(t)}{\sum\limits_{s \in allowed_k} \tau_{is}^{\alpha}(t)\,\eta_{is}^{\beta}(t)}, & j \in allowed_k \\ 0, & \text{otherwise} \end{cases}$$

Step 4: The chosen nodes are calculated into tabu search table $tabu_k$, and then refresh the load of the ants and $allowed_k$ table. If all nodes' demand quantity g_i equal to or higher than the rest of current carrying capacity, the ant would return to the distribution center. Otherwise, return to Step 2 and search for the next node until allowed k equals to zero, then the kth ant accomplishes the task.

Step 5: Make 2-opt local search to the solution and record it as the best solution in this iteration.

Step 6: If solution of this iteration is better than the current best solution, substitute it as the updated best solution.

Step 7: When kth ant finishes the task, let $t = t+1$, and use Equations (9), (10), and (11) to calculate the refresh information pheromone and increment.

$$\tau_{ij}(t+n) = \rho_1 \cdot \tau_{ij}(t) + \Delta\tau_{ij}(t, t+n)$$

$$\Delta\tau_{ij}(t, t+n) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t, t+n)$$

$$\Delta\tau_{ij}^{k}(t, t+1) = \begin{cases} \dfrac{Q}{L_k}, & \text{arc } (i,j) \text{ is passed by the } k\text{th ant} \\ 0, & \text{otherwise} \end{cases}$$

Step 8: When the number of iterations arrives at t_{max}, the iteration comes to an end, then we calculate the length of the shortest routes and corresponding. Otherwise, clear the tabu table and repeat Step 2.

Step 9: Output the optimum routes and corresponding cost.

TABLE 1. Basic information on retailers.

No.	X-axis	Y-axis	Demand	Service time	Time window
1	127	119	2.4	0.2	[0.5, 3.5]
2	109	149	0.8	0.2	[1.5, 5.0]
3	120	32	1.9	0.2	[2.0, 6.5]
4	18	97	1.8	0.4	[3.0, 10.0]
5	146	22	3.2	0.2	[2.2, 7.0]
6	15	138	1.4	0.5	[2.5, 8.0]
7	96	40	0.7	0.4	[0.6, 4.5]
8	121	124	2.1	0.2	[3.5, 8.5]
9	61	57	1.1	0.2	[1.4, 6.5]
10	45	143	2.3	0.2	[0.8, 5.0]
11	129	34	1.8	0.4	[2.4, 9.0]
12	12	84	1.0	0.4	[4.0, 12.0]
13	44	126	2.7	0.2	[1.5, 7.0]
14	102	152	1.5	0.2	[0.5, 5.0]
15	41	36	1.3	0.2	[5.0, 11.0]
16	132	138	2.4	0.4	[2.6, 9.0]
17	64	139	0.6	0.2	[4.2, 12.5]
18	144	14	1.3	0.4	[0.4, 4.0]
19	22	104	1.1	0.4	[1.6, 5.5]
20	56	133	0.7	0.4	[2.5, 9.0]

TABLE 2. Parameter settings of the model.

Parameter	Value	Parameter	Value
c_u	210	h	320
c_k	250	w	8
s_{uv}	180	K	6
c_w	280	p	100
c_v	160	q	100

V. COMPUTATIONAL EXPERIMENTS

Take retailers in region S as an example. A daily chemical product of a manufacturer would be delivered from the manufacturer to the distribution center in region S, and then distribute the products to 20 retailers downstream. Coordinates of location, requirement quantity, service time and time window of the retailers are listed in Table 1. Coordinates of distribution center is (70, 70). The locations of the retailers are shown in Fig.4.

Parameters are set in Table 2.

A. COMPUTATIONAL RESULTS

Based on the algorithm designed in Section IV, we use MATLAB to implement the solving of the computational experiment. The results are shown in Fig. 5 and Fig. 6. Upper echelon and lower echelon models are respectively solved by simulated annealing and ant colony with local search algorithms.

The results are shown that the minimum logistics cost is achieved in the 100th iteration, then we could get the optimum decision variables and routing. The lowest total cost $TC(Q)$ is 11794.90, the optimum total distribution quantity Q^* is

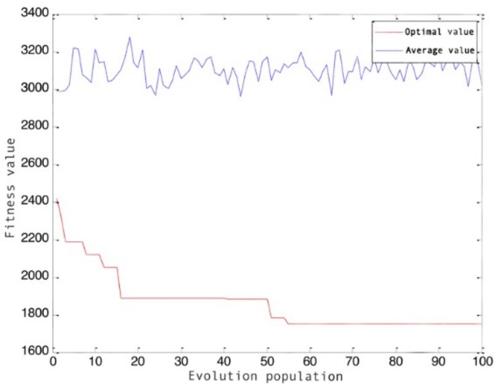

FIGURE 5. Ant colony algorithm with local search for evolutionary curves of lower echelon programming model.

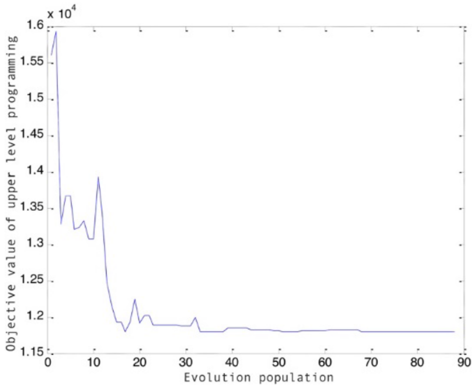

FIGURE 6. Evolutionary curves of simulated annealing algorithms for upper echelon programming model.

22.0 tons, distribution frequency is 1 time, and the specific distribution quantity of each retailer is shown in Table 3.

Optimum routing results are listed below.

- Route 1: Distribution center–Retailer7–Retailer18–Retailer5–Retailer11–Retailer3–Retailer8–Distribution center
- Route 2: Distribution center–Retailer1–Retailer14–Retailer2–Retailer16–Retailer17–Retailer20–Retailer13–Distribution center
- Route 3: Distribution center–Retailer10–Retailer6–Retailer19–Retailer4–Retailer12–Retailer9–Retailer15–Distribution center

The distributing routes are depicted in Fig. 7.

B. RESULTS ANALYSIS

We compare the results under integrated optimization and independent optimization of inventory and transportation. For simplification, independently optimized solution is simplified by "Scheme 1", and integrated optimized solution is denoted by "Scheme 2". Results of two solutions are respectively shown in Table 4 and Table 5.

We could see that compared with Scheme 1, the cost savings of Scheme 2 is 11.2%. Specifically, distribution frequency of Scheme 2 is added from 1 time in Scheme to

TABLE 3. Retailer demand and distribution information in integrated inventory-transportation problem.

No.	Demand	One-time delivery quantity	No.	Demand	One-time Delivery quantity
1	2.4	1.6	11	1.8	1.2
2	0.8	0.5	12	1.0	0.7
3	1.9	1.3	13	2.7	1.8
4	1.8	1.2	14	1.5	1.0
5	3.2	2.2	15	1.3	0.9
6	1.4	1.0	16	2.4	1.6
7	0.7	0.5	17	0.6	0.4
8	2.1	1.4	18	1.3	0.9
9	1.1	0.8	19	1.1	0.8
10	2.3	1.6	20	0.7	0.5
			Total	32.1	22.0

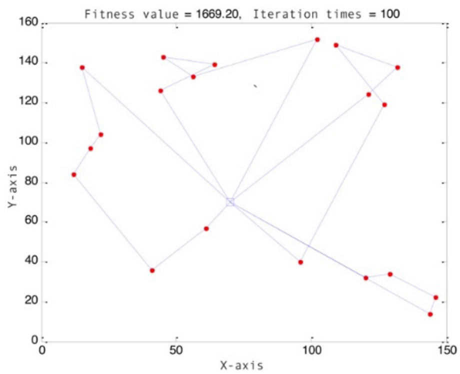

FIGURE 7. Optimal distribution routes.

TABLE 4. System cost details under independent optimization (scheme 1).

Indicators	Value
Number of vehicles required	5
Transportation cost from producer to supplier	5275.00
Total distribution cost	2866.80
Inventory cost	5136.00
Total cost	13277.80
Distribution frequency	1.00

TABLE 5. System cost details under integrated optimization (scheme 2).

Indicators	Value
Number of vehicles required	3
Transportation cost from producer to supplier	5486.90
Total distribution cost	2791.80
Inventory cost	3516.20
Total cost	11794.90
Distribution frequency	1.46

1.46 times, resulting in transshipment cost in Scheme 2 is 4% higher than Scheme 1. However, as the optimization of distribution, total distribution cost and transportation cost decrease sharply, for example, integrated optimization makes

TABLE 6. Results of routing under independent optimization (scheme 1).

No.	Route	Distance	Penalty cost	Vehicle Fixed cost	One-time delivery cost	Load	Vehicle utilization rate
1	0-10-13-20-0	172.6	0.0	280.0	452.6	5.7	71.3
2	0-6-19-4-12-15-9-0	245.4	75.1	280.0	600.5	7.7	96.3
3	0-7-3-11-5-0	184.9	61.6	280.0	526.5	7.6	95.0
4	0-18-8-0	279.5	0.0	280.0	559.5	3.4	42.5
5	0-1-16-2-14-17-0	237.3	50.4	280.0	567.7	7.7	96.3
Total		1119.8	187.0	1400.0	2706.8	32.1	80.3

TABLE 7. Results of routing under integrated optimization (scheme 2).

No.	Route	Distance	Penalty cost	Vehicle Fixed cost	One-time delivery cost	Load	Vehicle utilization rate
1	0-7-18-5-11-3-8-0	298.8	0.0	280	578.8	7.5	94.2%
2	0-1-14-2-16-17-20-13-0	303.3	0.0	280	583.3	7.6	95.0%
3	0-10-6-19-4-12-9-15-0	294.3	14.8	280	589.2	6.8	85.6%
Total		896.5	14.8	840	1751.3	22.0	91.6%

TABLE 8. Comparisons of key decision variables of two schemes.

	Scheme 1	Scheme 2
One-time delivery amount	32.1	22
Distribution frequency	1	1.46
Number of distribution routes	5	3
Route	0-14-2-16-8-0 0-1-7-3-17-0 0-18-5-11-20-0 0-13-10-0 0-6-19-4-12-15-9-0	0-7-18-5-11-3-8-0 0-1-14-2-16-17-20-13-0 0-10-6-19-4-12-9-15-0
Total cost	13277.8	11794.9

the inventory cost decrease by 31.5%. Although integrated optimization leads to a modest increase of transportation cost from manufacturer to the supplier, distribution cost and inventory cost decrease significantly, thus saving a lot of cost in the inventory-transportation system.

Distribution routing results of two schemes are respectively shown in Table 6 and Table 7. We could see from the results that except for loading quantity, other indicators of Scheme 2 are optimized compared with Scheme 1. Although the loading quantity of Scheme 2 is lower than that of Scheme 1, the increasing of distribution frequency makes distribution cost, inventory cost and total cost decrease obviously. In addition, the vehicle and loading quantity arrangement in Scheme 2 could increase the efficiency of the vehicles (average utilization rate of vehicles increase by 11.6%). In respect to time punishing cost, Scheme 2 could decrease the opportunity time cost and improve the timeliness.

Furthermore, performance of integrated optimization are compared with independent optimization from the aspects of distribution frequency, quantity and routing. The comparisons results are shown in Table 8. As the trade-off between inventory and transportation cost in one distribution system, Scheme 2 could result in the number of vehicle decreasing and distribution frequency increasing. Thus realizing the decreasing of distribution cost and inventory cost with the increasing of transshipment cost.

Through comparing independent optimization and integrated optimization of inventory and transportation, the two-echelon model proposed in Section III and the algorithm in Section IV could effectively solve the inventory-transportation optimization problem.

VI. CONCLUSION

The problem considered in this study combines two familiar problems in supply chain management of inventory management and transportation optimization under VMI system. We model a two-echelon model to formulate the integrated inventory-transportation optimization problem under VMI system with soft time window. It is a solution to inventory controlling and transportation scheduling integrated optimization from the perspective of the supplier. The supplier's aim is to explore a best distribution quantity, time, frequency and routing to minimize the total cost in the logistics system. As the problem of vehicle routing is NP-hard problem, we design combining methodologies to solve the two-echelon model. We combine heuristic and intelligence algorithm to solve the upper and lower echelon in the two-echelon problem, respectively. Then we implement computational experiments to compare performance of the two-echelon model and the cost saving results in the integrated inventory-transportation optimization problem. Resulting in the model and methodology proposed could help make the whole logistics system operate more effectively and efficiency compared to independent optimization of inventory and transportation.

In future research we plan to firstly take stochastic demand, more kinds of products, and multiple distribution centers into consideration. Secondly, algorithms of the solving the models could be added and compared with other methods such as multi-criterial optimization. Thirdly, environmental effect of the inventory-routing problem could also be considered in the problem as carbon emission in transportation is increasing rapidly.

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