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Enhancing MR Image Reconstruction Using Block Dictionary Learning

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ABSTRACT While representing a class of signals in term of sparsifying transform, it is better to use a adapted learned dictionary instead of using a predefined dictionary as proposed in the recent literature. With this improved method, one can represent the sparsest representation for the given set of signals. In order to ease the approximation, atoms of the learned dictionary can further be grouped together to make blocks inside the dictionary that act as a union of small number of subspaces. The block structure of a dictionary can be learned by exploiting the latent structure of the desired signals. Such type of block dictionary leads to block sparse representation of the given signals which can be good for reconstruction of the medical images. In this article, we suggest a framework for MRI reconstruction based upon block sparsifying transform (dictionary). Our technique develops automatic detection of underlying block structure of MR images given maximum block sizes. This is done by iteratively alternating between updating the block structure of the sparsifying transform (dictionary) and block-sparse representation of the MR images. Empirically it is shown that block-sparse representation performs better for recovery of the given MR image with minimum errors.

INDEX TERMS Compressed sensing (CS), sparsifying transforms, magnetic resonance imaging (MRI), block k-singular value decomposition (BLKSVD).

I. INTRODUCTION

As per recent compressed sensing (CS) theory, a sparse signal can be accurately reconstructed from a small number of random measurements. Magnetic resonance (MR) images can be represented as sparse signals. Now a day, magnetic resonance imaging (MRI) is main source of reproducible diagnostic medical information because of no harmful radiations and accurate visualization of the anatomical skeleton. However, acquisition time is the bottle neck in MRI to get numerous image samples for good reconstruction. Patient must stay for long duration in MRI scanner. This problem can be addressed either by making changes in the hardware or software side. Development on software side by using efficient algorithm is easy rather than on hardware side. CS [1]–[9] theory provides an opportunity to make progress on those algorithms to recover the images accurately from reduced set of measurements. This is possible when given image is sparse in some transform domain. MRI [10]–[13] is well supported by CS theory for quality of reconstruction.

Numerous sparsifying transforms can be used for CS reconstruction of static MRI [10], [12], [14] as well as dynamic MRI (dMRI) [15]–[19]. These sparsifying transforms are either non-adaptive or adaptive. Non-adaptive sparsifying transforms are fixed such as total variation (TV), curvelet or wavelet etc. and are application specific. Now a days, adaptive dictionary learning is popular for image reconstruction which can give better sparse representation because dictionary is trained on a particular set of images [2], [20]. An adaptive patched based dictionary is trained via different algorithms from small number of k-space samples which results in a better reconstruction. These learning algorithms challenge to find a dictionary that leads to optimal sparse representations for the image and show impressive results for representations with arbitrary sparsity structures or blocks.

In image processing, dictionary learning has received significant popularity and has been extensively applied to image

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restoration [21], compression [22], image segmentation [23] and classification [24]. Recently, researchers categorized the dictionary learning techniques into unsupervised, supervised and semi supervised models. In unsupervised models, dictionaries are learned in such a way that are suitable for representing the data without any class information of samples, e.g. K-SVD [2]. The reconstruction error is minimized under the sparsity constraint for the given data and is not capable of producing optimized discriminative sparse code. This model is not usually considered good for classification [25]. Whereas in supervised and semi supervised models, also known as discriminative dictionary learning (DDL), a more discriminative sparse code is produced as compared to unsupervised models, resulting in better classification performance. Broadly speaking, DDL is further classified into three categories: (i) shared (ii) specific-class and (iii) combination of the two. Based on above categories, DDL anticipates to enhance the discriminative ability of sparse coding, which is then incorporated in design of classification techniques [25]–[29]. Examples of shared DDL are discriminative KSVD (D-KSVD) [30] and label consisting KSVD (LC-KSVD) [31]. In specific-class DDL, a structural dictionary is learnt by exploiting the reconstruction ability of atoms and then implements classification by means of representation residual of each class. Fisher discrimination dictionary learning (FDDL) [32] is considered as one of the renowned specific class DDL technique. In combination of shared specific-class category, DDL learns the specific-class dictionaries to acquire the discriminative features of each class and then shared dictionary is constructed by preserving the common features of all classes. Although yielding improved classification performance, these supervised dictionaries neither practice/use any block structure nor explicitly minimize the within-class redundancy. In addition to supervised dictionary, block-structured sparsifying transforms (dictionaries) have not only enhanced the reconstruction ability [33] but also its classification ability [34].

Another direction of recent research work for MRI reconstruction is on deep learning, particularly in convolutional neural networks (CNNs) [35]–[37], but the main bottle neck is the availability of high computation resources and large amount of data. In our proposed work, we conduct dictionary learning using a single image. This single image cannot be used for training a deep learning network. Hence, we have restricted our work to a dictionary learning based method.

We are interested in those medical MR signals that are known to be extracted from union of a small number of subspaces [2], [38], [39]. Dictionary atoms are categorized into underlying subspaces in such kind of signals which lead to sparse representation for block sparse structure [33]. There are many methods, such as block basic pursuit (BBP) [38]–[40] block orthogonal matching pursuit (BOMP) [41] and group Lasso [42], [43] that have been suggested to get the benefit of this structure in recovering the block sparse representation. Predetermined dictionaries and known block structures are supposed to be used in these methods.

In this article we suggest a technique to design an adaptive block-sparsifying transform or a dictionary for a given set of MR signals. Our goal is to construct such a dictionary that offers block-sparse representations which is suitable for the given set of MR signals. While taking benefit of the block structure by means of block-sparse representation methods, it is essential to know the block structure of the dictionary. It is not supposed to be known a priori. Instead, we only suppose all cluster (or blocks) have a known maximal size and gather the block structure from the data accordingly while adapting the dictionary. If we were not constraining the maximal block size, we would eventually end up with one block which contains all the dictionary atoms.

Motivation behind learning a block structured dictionary is to exploit any cluster or structure that is implanted in the MR signals for generating a more effective sparse representation. The proposed block structured dictionary learning method involves two steps: update sparsifying transform (dictionary) along with related sparse coefficients and estimate the block structure. In this way, after finding the sparse representations of the training signals, dictionary atoms are progressively merged according to the similarity of the set of MR signals they represent. We present a dictionary learning frame work for MR signal, where dictionary blocks based on sparsity pattern are sequentially updated, rather than updating atom by atom as in the case of K-SVD [2], to minimize the representation error at each iteration. Atoms of the sparsifying transforms are sequentially updated block by block based on the location of the non-zero entries of sparse coefficients. The proposed method is an extension of K-SVD where instead of learning and updating single atoms one-by-one, blocks of atoms are learned and updated iteratively. This method is called block K-SVD (BLKSVD) [33].

We found empirically that proposed method achieve high performance and improved MR image reconstruction as compared to the dictionary learning based MRI (DLMRI) method by Sai *et al.* [14] where K-SVD is applied. The performance is confirmed empirically by various sampling ratios and k-space under-sampling values. Additionally, experimental results demonstrate that the K-SVD may not be able to recover the underlying block-structure. This is a difference to our proposed framework which succeeds in identifying the underlying structures in the input data and learns them as blocks of dictionary. Moreover, BLKSVD leads to smaller representation error and converges more rapidly during the update steps due to simultaneous optimization of the atoms belonging to the same block. This also makes the proposed frame-work computationally faster than the KSVD.

BLKSVD is introduced on patch-based dictionary learning in our framework for reconstruction of MR image. The shift from the global image sparsity to patch-based sparsity is added feature in block-structured dictionary learning which shows better results in MRI reconstruction. Because block patch-based dictionaries can capture the local image features efficiently, and have a potential to eliminate the noise and aliasing artifacts without compromising on resolution.

The use of block-structured dictionary learning on patch based MR image has generated an added averaging effect that eliminates the noise. Moreover, a single MR image can be decomposed into many overlapping patches to train a sparsifying transforms. We suggest a unique framework for simultaneously learning the transform and reconstructing the MR image from undersampled k-space data.

Learning block- structured dictionary is applied for image denoising and inpainting during the recovery of specific image. These images are usually in pixel domain or image domain. So learning of transform (dictionary) from a fraction of the image pixels has been explored in the framework of image inpainting, that is, filling-in missing or poorly corrupted samples in an image [44], [45]. Unlike inpainting, the partial data in MRI is available in k-space rather than in an image domain. This is a basic difference between MRI and rest of non-MR image recovery data. In this framework we learn block-structured image-patched dictionary learning from a small number of k-space samples. The proposed method combines the advantage of block structure with patch-based adaptive dictionary learning making it possible for the enhanced MR image reconstruction.

For the sparse coding step of dictionary learning, block-OMP (BOMP) has been used due to block based sparsity. The blocks in a block-based dictionary can be determined in different ways. However, in our application where we are using block dictionary for MRI reconstruction, blocks in the dictionary are determined using block sparse coding technique, i.e. block-OMP (BOMP). Unlike simple OMP, BOMP seeks block of atoms in the dictionary in a greedy way that highly correlate with the input data and calculates coefficients for that block. The blocks selected in the block-sparse coding steps are further refined in the dictionary update step. This alternate two-step process continues iteratively until a refined block dictionary is learned for MRI data.

We summarize the contributions for this framework as follows:

(1) For the first time, we formulate MRI reconstruction problem as learning block structure of the MRI that can lead to block dictionary learning. This block dictionary learning helps in finding robust MRI reconstruction.

(2) Based upon block dictionary learning, MRI representation is formulated in terms of block sparsity instead of simply sparsity that leads to better reconstruction.

(3) The block structure exploitation of the MRI for block sparse representation is tested for different sampling schemes to validate MRI reconstruction based upon block sparsity.

(4) We have initialized the dictionary by extracting left singular vectors from the training data for block dictionary learning in our framework and then normalized each atom of the dictionary. We have observed that by initializing dictionary in this form, convergence becomes faster as compared with randomly initialized dictionary.

Rest of the paper is compiled as follows: In Section II, compressed sensing on MR image, learning sparsifying transform and block dictionary learning are discussed as prior work. Problem formulation for MR image reconstruction on adaptive patch based dictionary learning is discussed in Section III. Suggested algorithm is described in Section IV. Section V provides the empirical performance of our algorithm with several examples, using diverse sampling schemes. The conclusion is drawn in Section VI.

II. BACKGROUND AND RELATED WORK

A. COMPRESSED SENSING OF MR IMAGES

A sparse signal is one that has many zeros and few nonzero coefficients. The aim of the sparse approximation is to symbolize a given signal or measurement vector as a linear combination of a small number of sparsifying transform vectors, $\boldsymbol{\psi}_i$.

MR image acquisition can be modelled as an undersampled measurement of MR image in k-space with the help of measurement matrix $\boldsymbol{\Phi}_u$. Let MR image $\boldsymbol{x} \in \mathbb{C}^q$ is encoded to a measurement vector $\boldsymbol{z} \in \mathbb{C}^m$ such that

$$\boldsymbol{z} = \boldsymbol{\Phi}_u \boldsymbol{x} \tag{1}$$

where $\boldsymbol{\Phi}_u \in \mathbb{C}^{m\times q}$ is under-sampled Fourier encoding or measurement matrix. Whenever the number of unknowns is greater than the number of k-space samples ($q > m$), it is called under-sampling.

Compressed sensing (CS) provides a promising way of reconstructing $\boldsymbol{x}$ from its undersampled measurements $\boldsymbol{z}$ provided $\boldsymbol{x}$ is sparse in some sparsifying transform domain , $\boldsymbol{\Psi}$, also known as the dictionary. The recovery of $\boldsymbol{x}$ can be formulated as l_0 minimization of sparse signal $\boldsymbol{\Psi x}$ [46], [47];

$$\min_{\boldsymbol{x}} \|\boldsymbol{\Psi x}\|_0 \quad s.t.\ \boldsymbol{\Phi}_u \boldsymbol{x} = \boldsymbol{z} \tag{2}$$

$\boldsymbol{\Psi} \in \mathbb{C}^{t\times q}$ is a sparsifying transform such as wavelet or DCT or any other learned dictionary.

The disadvantage of the model in (2) is that the sparse coding step is NP (Nondeterministic Polynomial-time) hard because the algorithm involves l_0 norm. The non-convex formulation of (2) can be transformed to a convex problem using l_1 norm, [48] i.e.,

$$\min_{\boldsymbol{x}} \|\boldsymbol{\Phi}_u \boldsymbol{x} - \boldsymbol{z}\|_2^2 + \gamma \|\boldsymbol{\Psi x}\|_1 \tag{3}$$

where γ is the Lagrangian multiplier. This problem can be solved using many algorithms such as orthogonal matching pursuit (OMP) [49]–[51], basic pursuit (BP) [39], focal under-determined system solver (FOCUSS) [52] and least-angle regressions (LARS) [53].

B. LEARNING SPARSIFYING TRANSFORM

The quality of the reconstructed image mainly relies on the sparsifying transform. The undersampling constraint in nonadaptive compressed sensing can be solved efficiently using adaptive dictionary updates. In our framework, we use adaptive patch based dictionary learning. From a given signal $\boldsymbol{x} \in \mathbb{C}^q$, a patch of the signal is extracted as $\boldsymbol{x}_{ij} \in \mathbb{C}^n$ of square 2D image with the dimension of patch $(\sqrt{\text{n}} \times \sqrt{\text{n}})$ pixels, marked by the position (i, j) to its top left corner of the image. $\mathbf{D} \in \mathbb{C}^{n\times K}$ denotes the patch based dictionary having

K number of atoms and $\boldsymbol{\theta}_{ij} \in \mathbb{C}^K$ is the sparse representation of $\boldsymbol{x}_{ij}$ patch with respect to **D.** Mathematically $\boldsymbol{x}_{ij}$ can be expressed as follows.

$$\boldsymbol{x}_{ij} = \mathbf{W}_{ij}\boldsymbol{x} \tag{4}$$

where $\mathbf{W}_{ij} \in \mathbb{R}^{n\times q}$ matrix acts as an operator that brings out the patch $\boldsymbol{x_{ij}}$ from a given signal $\boldsymbol{x}$. Following optimization problem solves the dictionary learning as

$$\min_{\mathbf{D},\boldsymbol{\theta}} \sum_{ij} \left\|\mathbf{W}_{ij}\boldsymbol{x} - \mathbf{D}\boldsymbol{\theta}_{ij}\right\|_2^2 \quad s.t. \ \left\|\boldsymbol{\theta}_{ij}\right\|_0 \leq \tau_0 \quad \forall i, j \tag{5}$$

where τ_0 is the required sparsity. Combining all patches $\boldsymbol{x}_{ij}$ column wise in a matrix $\tilde{\mathbf{X}} \in \mathbb{C}^{n\times L}$, where L is the total number of patches acquired form of $\boldsymbol{x}$,we get.

$$\min_{\mathbf{D},\boldsymbol{\Theta}} \left\|\tilde{\mathbf{X}} - \mathbf{D}\Theta\right\|_F \quad s.t. \ \|\boldsymbol{\theta}\|_0 \leq \tau_0 \tag{6}$$

where $\boldsymbol{\Theta} \in \mathbb{R}^{K\times L}$ belongs to sparse representation. Optimization formulation used in (5) and (6) are NP hard for the fixed **D** and can be solved from many algorithms such as MOD and K-SVD. Such kind of algorithms normally alternate between finding the dictionary **D** and sparse representations $\boldsymbol{\Theta}$. Many researchers use the K-SVD to learn the dictionary where the atoms of the dictionary are updated one-by-one i.e. K -times SVD which increases the computation time. Prasad *et al.* [14] performed tremendous work on MR image reconstruction from highly under-sampled k-space data using the K-SVD technique to learn the dictionary and a popular greedy method orthogonal matching pursuit (OMP) for updating sparse coefficients. His work showed noticeable improvements in the reconstruction of different medical images along with other performance parameters mainly SNR and high frequency error numbers (HFEN). He compared/reviewed his results with Lusting *et al.* [10] (denoted by LDP).The CS framework exploits the sparsity of $\boldsymbol{\theta}$ in order to facilitate recovery. With proper chosen **D**, recovery is possible irrespectively of the location of the nonzero values of $\boldsymbol{\theta}$. This outcome has caused to generate a lot of recovery algorithms. A lot of recent work has been done to find an adaptive structure (block) dictionary that leads to optimal sparse representations for an impressive signal reconstruction.

C. BLOCK DICTIONARY LEARNING

Block structured dictionaries are learned to utilize any embedded structure in order to produce a more effective sparse representation. Block-structured dictionary method involves two steps: update the dictionary block and then finding the sparse coefficients according to block structure in the dictionary. We have used BOMP (instead of OMP) for updating the sparse representation matrix and BLKSVD [33] to learn the dictionary as discussed in Section IV. Algorithm BOMP selects the dictionary blocks sequentially that are best suited to the input signals. The key characteristic of BLKSVD is to update the atoms in blocks and corresponding non-zero coefficients simultaneously.

For a given set of MR signals, we want to find the dictionary whose atoms are sorted into blocks and provide the most accurate representation vectors. In our proposed framework, we make the assumption to have known maximal block size but the association of dictionary atoms into blocks is not known a priori. Let each block is assigned an index number and $\boldsymbol{b} \in \mathbb{R}^K$ be the vector of block assignments for the atoms of **D**. In other words, $\boldsymbol{b}[k]$ is the block index of the atom $\mathbf{D}_k$. A vector $\boldsymbol{\theta} \in \mathbb{C}^K$ is s-block sparse over $\boldsymbol{b}$ if its nonzero values are concentrated in s blocks only. We express this in following manner

$$\|\boldsymbol{\theta}\|_{0,b} = s \tag{7}$$

where $\|\boldsymbol{\theta}\|_{0,b}$ is the l_0 norm over block $\boldsymbol{b}$ and computes the number of non-zero blocks as defined by $\boldsymbol{b}$. Our objective is to learn the block dictionary **D** along with its block structure $\boldsymbol{b}$ having a maximum block size of s that leads to optimal τ_0-block sparse representation $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_p\}_{p=1}^{p=L}$. For block-dictionary learning, (6) can be expressed as.

$$\min_{\mathbf{D},\boldsymbol{\Theta}} \left\|\tilde{\mathbf{X}} - \mathbf{D}\Theta\right\|_F$$
$$s.t. \left\|\boldsymbol{\Theta}_p\right\|_{0,b} \leq \tau_0, \quad p = 1\ldots .L, \left|b_j\right| \leq s, \ j \in [1, N] \tag{8}$$

where N indicates the number of blocks and b_j is the set of indices represent the list of dictionary atoms in block j and can be expressed as

$$b_j = \{k \in 1, 2, 3, \ldots\ldots\ldots .K|b[k] = j\} \tag{9}$$

III. PROBLEM FORMULATION

Reconstructed compressively sampled biomedical MR images typically suffer from numerous artifacts during under sampling of k-space and noise in samples. These are two main causes of artifacts. So a decent dictionary must be capable of minimizing the artifacts which are noticed into zero filled Fourier reconstruction and be consistent to produce reconstructed images by available k-space data. In MRI, the available data is in k-space rather than in an image domain. Our formulation has a capability of both designing an adaptive dictionary learning, and also using it to reconstruct the underlying MR image. This is achieved by means of only the under-sampled k-space measurements, z. The problem formulation, for reconstructing the MR image $\boldsymbol{x}$**,** then becomes:

$$\min_{\mathbf{D},\boldsymbol{\theta},\boldsymbol{x}} \sum_{ij} \left\|\boldsymbol{x}_{ij} - \mathbf{D}\boldsymbol{\theta}_{ij}\right\|_2^2 + \eta\left\|\boldsymbol{\Phi}_u\boldsymbol{x} - z\right\|_2^2$$
$$s.t. \ \left\|\boldsymbol{\theta}_{ij}\right\|_{0,b} \leq \tau_0 \quad \forall i, j \tag{10}$$

In (10), the 1$^{\text{st}}$ term is responsible for the quality of the sparse approximation of the patch images with respect to the dictionary **D** whereas 2$^{\text{nd}}$ term enforces data consistency in k-space. Parameter η depends on standard deviation σ of measurement noise such as $\eta = (\frac{\lambda}{\sigma})$ and λ is taken as a positive constant.

IV. MR IMAGE RECONSTRCUTION

Adaptive learning of sparsifying transform in (10) is a non-convex problem and is computationally expensive. Typically the problem in (10) is solved in two steps. (i) dictionary learning and sparse coding are updated alternately keeping the estimated signal $\boldsymbol{x}$ fixed, while in second step (ii) update the estimated MR signal $\boldsymbol{x}$ to satisfy the data fidelity while keeping dictionary and sparse representation fixed.

A. UPDATING BLOCK DICTIONARY AND SPARSE CODING

Since MR signal $\boldsymbol{x}$ is fixed, the objective function in (10) becomes

$$\min_{\mathbf{D},\boldsymbol{\theta}} \sum_{ij} \left\|\boldsymbol{x}_{ij} - \mathbf{D}\boldsymbol{\theta}_{ij}\right\|_2^2 \quad s.t.\ \left\|\boldsymbol{\theta}_{ij}\right\|_{0,b} \leq \tau_0 \quad \forall i,j \tag{11}$$

Combining all patches in (11) into matrix $\tilde{\mathbf{X}}$ as described in (8). Since the learning sparsifying transform process involves two steps, i.e., block dictionary learning $\mathbf{D}$ and update the sparse coefficient matrix $\mathbf{\Theta}$, so we formulate the objective function as follows for given block $\boldsymbol{b}$ for l^{th} iteration:

$$[\mathbf{D}^l, \mathbf{\Theta}^l]_F = \min_{\mathbf{D},\mathbf{\Theta}} \left\|\tilde{\mathbf{X}} - \mathbf{D}\mathbf{\Theta}\right\|_F \quad s.t. \left\|\mathbf{\Theta}_p\right\|_{0,b^l} \leq \tau_0, \quad p=1\ldots\ldots\ldots L \tag{12}$$

Now applying block KSVD (BLKSVD) algorithm to recover the $\mathbf{D}$ and $\mathbf{\Theta}$ by optimizing (12) on given block structure $\boldsymbol{b}$. At every l^{th}- iteration, we fix dictionary $\mathbf{D}^{l-1}$ in first step and use the BOMP to solve (12) which optimizes as

$$\mathbf{\Theta}^l = \min_{\Theta} \left\|\tilde{\mathbf{X}} - \mathbf{D}^{l-1}\mathbf{\Theta}\right\|_{\mathrm{F}} \quad s.t. \left\|\mathbf{\Theta}_p\right\|_{0,b} \leq \tau_0, \quad p = 1\ldots.L \tag{13}$$

In second step, we obtain $\mathbf{D}^l$ while fixing $\mathbf{\Theta}^l$,$\boldsymbol{b}$ and $\tilde{\mathbf{X}}$.

$$\mathbf{D}^l = \min_{\mathbf{D}} \left\|\tilde{\mathbf{X}} - \mathbf{D}\mathbf{\Theta}^k\right\|_F \tag{14}$$

Motivated by KSVD algorithm, the blocks are updated sequentially in $\mathbf{D}^{l-1}$ along with corresponding non-zero sparse coefficients in $\mathbf{\Theta}^l$. Details for every block $j \in [1, N]$ are discussed as follows:

Let $\mathbf{E}_{\alpha_j}$ be the error matrix of the signals $\tilde{\mathbf{X}}_{\alpha_j}$ excluding the contribution of the jth block. We express it as follows:

$$\mathbf{E}_{\alpha_j} = \tilde{\mathbf{X}}_{\alpha_j} - \sum_{k\neq j} \mathbf{D}_{b_k}\left(\mathbf{\Theta}^{b_k}\right)_{\alpha_j} \tag{15}$$

where α_j is the set of indices corresponding to columns in sparse matrix $\mathbf{\Theta}^l$ that use the atom $\mathbf{D}_k$.The representative error of signal with indices α_j can be defined as follows.

$$\alpha_j \triangleq \left\|\mathbf{E}_{\alpha_j} - \mathbf{D}_{b_k}\mathbf{\Theta}_{\alpha_j}^{b_k}\right\|_{\mathrm{F}} \tag{16}$$

By taking the singular value decomposition (SVD) of error matrix $\mathbf{E}_{\alpha_j}$

$$\mathbf{E}_{\alpha_j} = \boldsymbol{USV}^{\mathrm{T}} \tag{17}$$

The dictionary and sparse representation matrix is updated as follows:

$$\mathbf{D}_{b_k} = \left[U_1, \ldots\ldots\ldots\ldots\ldots.U_{|b_k|}\right] \tag{18}$$

$$\mathbf{\Theta}_{\alpha_j}^{b_j} = \left[S_1^1\mathrm{V}_1, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.S_{|b_j|}^{|b_j|}V_{|b_j|}\right]^{\mathrm{T}} \tag{19}$$

Iteratively, dictionary and its blocks are continuously updated till the convergence or any predefined number of iterations. When block size is one then both BLKSVD and KSVD become identical.

B. UPDATING THE ESTIMATED IMAGE(S) FOR RECONSTRUCTION

To update the reconstruction MR image $\boldsymbol{x}$, keep the dictionary and the sparse representation constant then the sub-problem for our cost function in (10) can also be written as follows:

$$\min_{\boldsymbol{x}} \sum_{ij} \left\|\mathbf{W}_{ij}\boldsymbol{x} - \mathbf{D}\boldsymbol{\theta}_{ij}\right\|_2^2 + \eta\|\mathbf{\Phi}_u\boldsymbol{x} - z)\|_2^2 \tag{20}$$

The formulation in (20) is the least squares problem and detailed solution is in appendix-A. The solution is as follows:

$$\mathbf{\Phi}\boldsymbol{x}\left(k_x, k_y\right) = \begin{cases} \mathcal{N}\left(k_x, k_y\right), & \left(k_x, k_y\right) \notin \mho \\ \dfrac{\mathcal{N}\left(k_x, k_y\right) + \eta\mathcal{N}_0\left(k_x, k_y\right)}{1+\eta}, & \left(k_x, k_y\right) \in \mho \end{cases} \tag{21}$$

Here $\boldsymbol{x}$ is reconstructed by taking the IFFT of $\mathbf{\Phi}\boldsymbol{x}$. From (ix) in appendix-A

$$\mathcal{N} = \mathbf{\Phi} \sum_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{D}\boldsymbol{\theta}_{\mathbf{ij}}\frac{\mathbf{1}}{\boldsymbol{\alpha}} \tag{22}$$

(22) is called the "patch averaged result" in Fourier domain and $\mathbf{\Phi}\boldsymbol{x}\left(k_x, k_y\right)$ represents the updated value on location $\left(k_x, k_y\right)$ of the k-space. $\mathcal{N}_0 = \mathbf{\Phi}\mathbf{\Phi}_u^{\mathrm{H}}\boldsymbol{z}$ represents zero filled k-space measurement and denotes the subset of k-space that has been sampled.

A more extensive pseudocode is presented in Appendix-B

Algorithm 1

Goal: Reconstruction of undersampled MR image using block dictionary
Input:z = training signal(s) in k-space measurements, s for block size, and τ_0 for sparsity
Output: $\boldsymbol{x}$: Reconstructed MR image
Initialization: $\boldsymbol{x} = \boldsymbol{x_0} = \mathbf{\Phi}_u^{\mathrm{H}}z$
Main Iteration:

1. Alternately learn sparsifying transform (dictionary) by BLKSVD and BOMP for sparse coding.
2. Update $\hat{\boldsymbol{x}}$: Every pixel value attained by averaging the impact of patches that covering it
3. $\mathcal{N} \leftarrow \mathcal{FFT}(\boldsymbol{x})$
4. Restore sampled frequency to update the $\mathcal{N}$ as per (21)
5. $\hat{\boldsymbol{x}} \leftarrow$ IFFT($\mathcal{N}$)

V. EMPIRICAL TEST & RESULTS

The performance of the proposed algorithm is validated with two cases i.e. noiseless and noisy for the reconstruction of MR Image. We have used real world image single-slice (axial T2-wieghted reference brain image) dataset of size (512×512), vivo MR scans from American Radiology Services as used by Parasad *et al.* [14]. These images are converted to over lapping patches of size ($\sqrt{n} \times \sqrt{n}$), in our case $n = 36$. These patches are used for dictionary learning.

Different undersampling factors (2.5 folds to 4 folds) and undersampling schemes (like center dense and Cartesian including 2D random sampling) are used in both noiseless and noisy cases. We have compared our reconstructed images with leading DLMRI [14] method which had already outperformed other CSMRI [10] techniques.

All the experiments are performed on Intel Core i5-7200U 7th generation (2cores—4threads) in Matlab 9.2.0.5338062 (R2017a). We fixed some values for our experiments such as: maximum number of iterations = 15, sparsity = 6, dictionary size (36×36) and empirically set the block size $s = 3$.

To evaluate the performance of our experiments, we have computed power signal to noise ratio (PSNR) and high frequency error norm (HFEN). The PSNR is used as a quality measure for reconstructed images described in decibels and computes peak intensity signal-to-noise ratio between the original and a compressed image. The HFEN describes the fine features of the reconstructed images at edges and is computed as l_2 norm of the result acquired by Laplacian of Gaussian (LoG) filtering of the difference between reconstructed and reference images. Some quantitative measurements like correlation and similarity index (SSIM: Structural SIMilarity) are also computed for image comparison in both noiseless and noisy cases.

A. NOISELESS CASE

We performed our proposed method on noiseless scenario first and compared with DLMRI. 2D random center dense and Cartesian sampling schemes with different undersampling factors (2.5 – 4) folds are used to reconstruct the image without adding noise in k-space. The dictionary learning scheme by BLKSVD reconstructed the images free from artifacts and aliasing effect in all different sampling schemes. The recovered images are clear with fine edges with fast convergence than DLMRI. In all cases, as shown in Figures 1-2, our proposed method outperformed the DLMRI in term of PSNR and HFEN.

We have also compared DLMRI and our proposed frame work through statistical data with relative difference in the tables for the correlation and similarity index (SSIM). There is slight improvement of correlation and SSIM for center dense mask. It is observed that the correlation in Cartesian schemes on different sampling factors have same results but SSIM has improved as per Table 1.

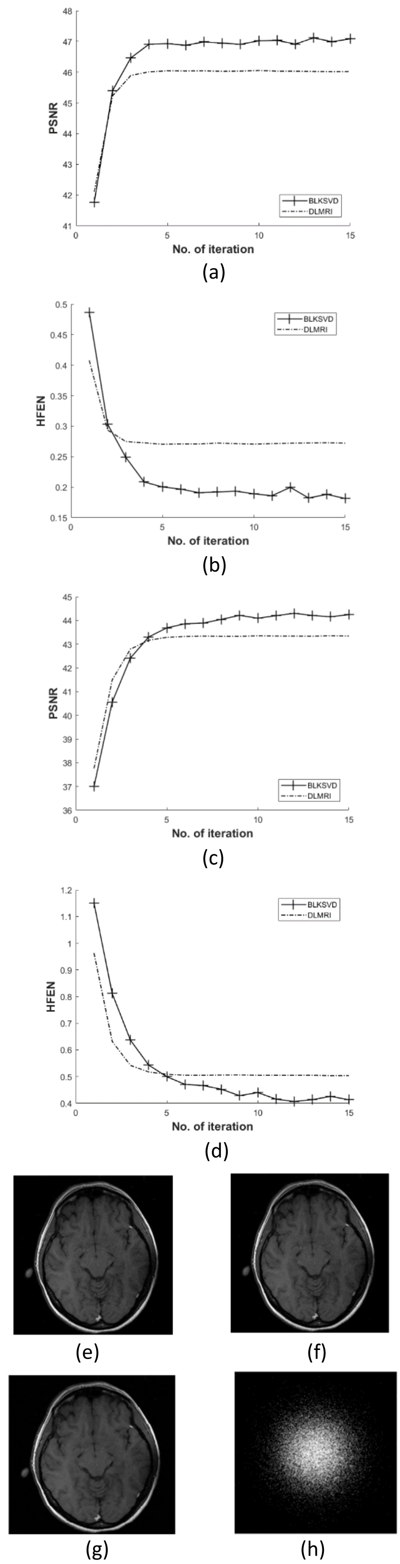

FIGURE 1. For Noiseless case: Center dense sampling scheme for reconstruction of brain image (a) PSNR vs iterations with sampling factors 2.5 folds (b) HFEN vs iterations with sampling factors 2.5 folds (c) PSNR vs iterations with sampling factors 4 folds (d) HFEN vs iterations with sampling factors 4 folds. (e) Recovered image with sampling factors 2.5 folds (f) Recovered image with sampling factors 4 fold (g) Reference image (h) Center dense mask.

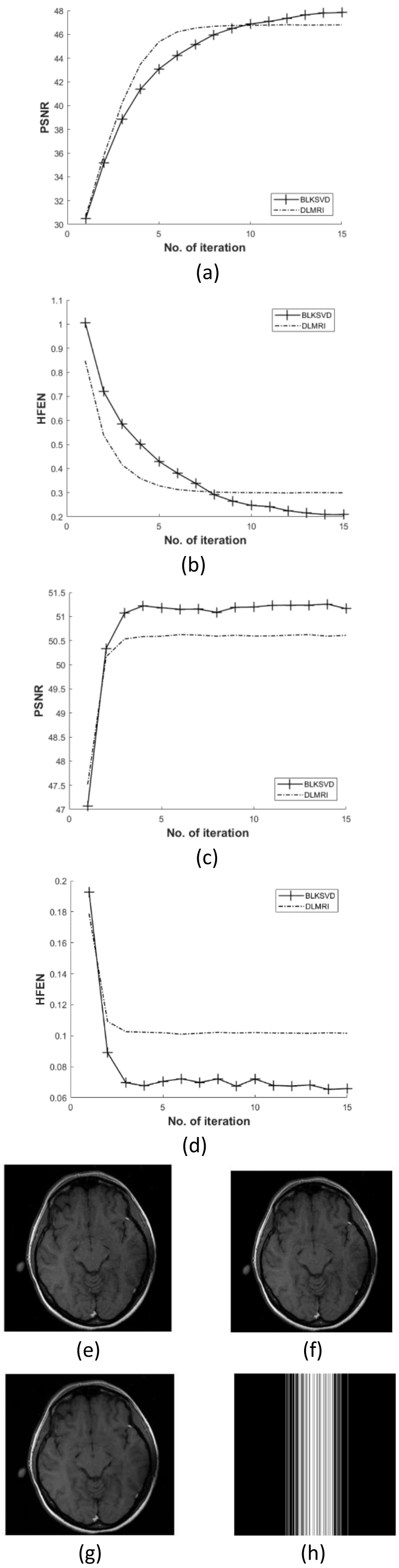

FIGURE 2. For Noiseless case: Cartesian sampling scheme for reconstruction of brain image (a) PSNR vs iterations with sampling factors 2.5 folds (b) HFEN vs iterations with sampling factors 2.5 folds (c) PSNR vs iterations with sampling factors 4 folds (d) HFEN vs iterations with sampling factors 4 folds. (e) Recovered image with sampling factors 2.5 folds (f) Recovered image with sampling factors 4 folds (g) Reference image (h) Cartesian mask.

TABLE 1. Performance parameter (BLKSVD vs DLMRI) of algorithm with noiseless case for brain image.

Center Dense Undersampling 2.5 Folds					
No	Parameters	DLMRI	BLKSVD	Difference	Remarks
1	**Correlation**	0.9995	0.9996	0.00010	Improved
2	**Similarity Index (SSIM)**	0.9516	0.9703	0.0187	Improved
Center Dense Undersampling 4 Folds					
No	Parameters	DLMRI	BLKSVD	Difference	Remarks
1	**Correlation**	0.9991	0.9992	0.00010	Improved
2	**Similarity Index (SSIM)**	0.9275	0.9569	0.0294	Improved
Cartesian Undersampling 2.5 Folds					
No	Parameters	DLMRI	BLKSVD	Difference	Remarks
1	**Correlation**	0.9996	0.9996	0.00000	same
2	**Similarity Index (SSIM)**	0.9587	0.9738	0.0151	Improved
Cartesian Undersampling 4 Folds					
No	Parameters	DLMRI	BLKSVD	Difference	Remarks
1	**Correlation**	0.9998	0.9998	0.00000	same
2	**Similarity Index (SSIM)**	0.9836	0.9883	0.0047	Improved

TABLE 2. Performance parameter (BLKSVD vs DLMRI) of algorithm with noisy case for brain image.

Center Dense Undersampling 2.5 Folds (Noisy)					
No.	Parameters	DLMRI	BKSVD	Difference	Remarks
1	**Correlation**	0.9994	0.9994	0	same
2	**Similarity Index (SSIM)**	0.9208	0.9364	0.0156	Improved

Cartesian Undersampling 2.5 Folds (Noisy)					
No.	Parameters	DLMRI	BKSVD	Difference	Remarks
1	**Correlation**	0.9994	0.9994	0	same
2	**Similarity Index (SSIM)**	0.9068	0.9175	0.0107	Improved

B. NOISY CASE

We have performed experiment by adding zero mean white Gaussian noise of standard deviation $\sigma = 3$ in k-space in all cases of center dense and Cartesian sampling at 2.5 fold undersampling. Our proposed method performed better as shown in Figure 3 than that of DLMRI in noisy case keeping same parameters and reference brain image and undersampled mask as in noiseless case.

The quantitative measurement of SIMM in noisy case has shown slight improvement but correlation is same on both cases of centered dense and Cartesian sampling schemes as observed in Table 2.

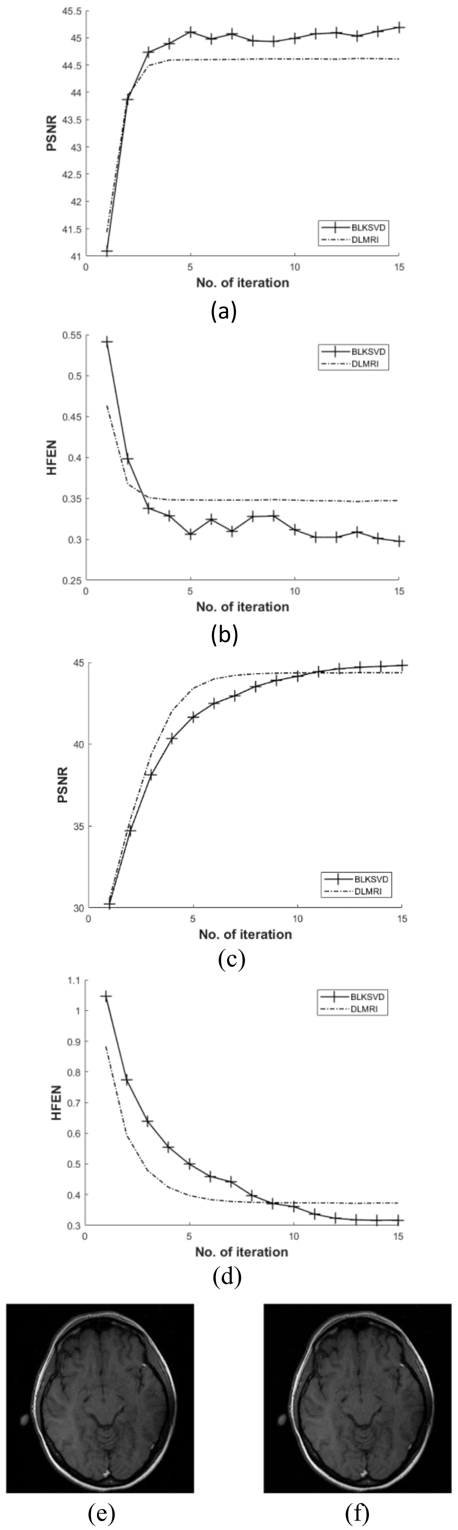

FIGURE 3. For Noisy case: Reconstruction of brain image (a) PSNR vs iterations with center dense sampling scheme at sampling factors 2.5 folds (b) HFEN vs iterations with center dense sampling scheme at sampling factors 2.5 folds (c) PSNR vs iterations with Cartesian sampling scheme at sampling factors 2.5 folds (d) HFEN vs iterations with Cartesian sampling scheme at sampling factors 2.5 folds. (e) Recovered image center dense sampling scheme at sampling factors 2.5 folds (f) Recovered image center dense sampling scheme at sampling factors 2.5 folds.

VI. CONCLUSION

In this paper, adaptive patch-based block-structured dictionary learning framework has been introduced for reconstruction of MR images. This proposed method has shown improved performance over other dictionary learning based methods such as DLMRI, for both noisy and noiseless cases. The performance is validated by using a diversity of sampling trails and k-space under sampling ratios. The designed algorithm may be implemented on the other medical signal processing problems which we may consider for future research work.

APPENDIX A

To update the reconstruction image $\boldsymbol{x}$, keep the dictionary and the sparse representation constant then the sub-problem for our cost function in (10) become as follows:

$$\min_{\boldsymbol{x}} \sum\nolimits_{ij} \left\| \mathbf{W}_{ij}\boldsymbol{x} - \mathbf{D}\boldsymbol{\theta}_{\mathbf{ij}} \right\|_2^2 + \eta \left\| \boldsymbol{\Phi}_u \boldsymbol{x} - z \right\|_2^2 \qquad \text{(i)}$$

Differentiate (i) w.r.t $\boldsymbol{x}$ and equal to zero, $\mathbf{W} \in \mathbb{R}$ and $\boldsymbol{\Phi}_u \in \mathbb{C}$

$$\left(\frac{\partial}{\partial x}\right)\left(\sum\nolimits_{ij} \left\| \mathbf{W}_{ij}\boldsymbol{x} - \mathbf{D}\boldsymbol{\theta}_{\mathbf{ij}} \right\|_2^2 + \eta \left\| \boldsymbol{\Phi}_u \boldsymbol{x} - z \right\|_2^2\right) = 0 \qquad \text{(ii)}$$

$$2\sum\nolimits_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\left(\mathbf{W}_{ij}\boldsymbol{x} - \mathbf{D}\boldsymbol{\theta}_{\mathbf{ij}}\right) + 2\eta\boldsymbol{\Phi}_u^{\mathrm{H}}(\boldsymbol{\Phi}_u\boldsymbol{x} - z) = 0 \qquad \text{(iii)}$$

The subscript H and T represents Hermitian transpose operation and real operand respectively. Separating the term belonging to $\boldsymbol{x}$ on left side to find it.

$$\left(\sum_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{W}_{ij} + \eta\boldsymbol{\Phi}_u^{\mathrm{H}}\boldsymbol{\Phi}_u\right)\boldsymbol{x} = \sum_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{D}\boldsymbol{\theta}_{ij} + \eta\boldsymbol{\Phi}_u^{\mathrm{H}}z \qquad \text{(iv)}$$

In (iv) the 1$^{\text{st}}$ term of left side $\sum_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{W}_{ij} \in \mathbb{R}^{q\times q}$ is diagonal matrix and corresponds to image pixel position. These entries are equal to the number of overlapped patches contributing on those pixels. Its diagonal values turn out to be all equal and denoted as

$$\sum\nolimits_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{W}_{ij} = \alpha\mathbf{I}_q \quad \text{where } \mathbf{I}_q \in \mathbb{R}^{q\times q} \qquad \text{(v)}$$

In our case, where diagonal values correspond to place of the image pixel and are equal to the number of overlapped patches contributing those pixel places.

Putting overlap stride r = 1 for the patches and $\alpha = n$ let $\boldsymbol{\Phi} \in \mathbb{C}^{\mathrm{qxq}}$ represent the complete Fourier encoding matrix normalized as $\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{\Phi} = \mathbf{I}_q$. Now $\boldsymbol{\Phi x}$ denote full rank data for k-space and putting in (iv) we get,

$$\left(\sum\nolimits_{ij} \boldsymbol{\Phi}\,\mathbf{W}_{ij}^{\mathrm{T}}\mathbf{W}_{ij}\boldsymbol{\Phi}^{H} + \eta\boldsymbol{\Phi}\boldsymbol{\Phi}_u^{\mathrm{H}}\boldsymbol{\Phi}_u\boldsymbol{\Phi}^{H}\right)\boldsymbol{\Phi x} = \boldsymbol{\Phi}\sum\nolimits_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{D}\boldsymbol{\theta}_{\mathbf{ij}} + \eta\boldsymbol{\Phi}\boldsymbol{\Phi}_u^{\mathrm{H}}z \qquad \text{(vi)}$$

In (vi), the 2$^{\text{nd}}$ term on left side represents the diagonal matrix with scaling η times comprising of zeros and ones. All those ones at diagonal values are related to sampled position k-space. $\boldsymbol{\Phi}\boldsymbol{\Phi}_u^{\mathrm{H}}z$ denotes the zero filled Fourier measurements vector and

$$\boldsymbol{\Phi}\sum\nolimits_{ij} \mathbf{W}_{ij}^{\mathrm{T}}\mathbf{W}_{ij}\boldsymbol{\Phi}^{H} = \alpha\boldsymbol{I}_q \qquad \text{(vii)}$$

Then (vi) becomes

$$\left(\alpha \boldsymbol{I}_q + \boldsymbol{I}\right) \boldsymbol{\Phi x} = \alpha \mathcal{N} + \eta \mathcal{N}_0 \quad \text{(viii)}$$

where:

$$\mathcal{N} \triangleq \boldsymbol{\Phi} \sum\nolimits_{ij} \mathbf{W}_{ij}^{\mathrm{T}} \mathbf{D} \boldsymbol{\theta}_{\mathbf{ij}} \frac{\mathbf{1}}{\boldsymbol{\alpha}} \quad \text{and } \mathcal{N}_0 \triangleq \boldsymbol{\Phi \Phi}_u^{\mathrm{H}} \boldsymbol{z} \quad \text{(ix)}$$

Dividing (viii) by α both sides we get

$$\left(\boldsymbol{I}_q + \left(\frac{\boldsymbol{I}}{\boldsymbol{\alpha}}\right)\right) \boldsymbol{\Phi x} = \mathcal{N} + \left(\frac{\eta}{\boldsymbol{\alpha}}\right) \mathcal{N}_0 \quad \text{(x)}$$

Here absorbing $\left(\frac{\eta}{\alpha}\right)$ is constant into η then solution of (vi) is as follows:

$$\boldsymbol{\Phi x}\left(k_x, k_y\right) = \begin{cases} \mathcal{N}\left(k_x, k_y\right), & \left(k_x, k_y\right) \notin \mho \\ \dfrac{\mathcal{N}\left(k_x, k_y\right) + \eta \mathcal{N}_0\left(k_x, k_y\right)}{1+\eta}, & \left(k_x, k_y\right) \in \mho \end{cases} \quad \text{(xi)}$$

APPENDIX B

Algorithm Pseudocode Enhancing MR Image Reconstruction Using Block Dictionary

Inputs: Noisy or noiseless MR Image
Output: Reconstructed MR image from under- sampled data

BLKSVD Parameters Initialization,
$\boldsymbol{x}$ = FFT (Input MR Image),
Add noise in k-space in noisy case,
Applying under-sampling mask,
  While number of maximum iterations:
    For $i = 1{:}\ n$ **do**
      Create image patches
        For $j = 1{:}\ m$ **do**
          Learn block dictionary
          Learn block sparse codes
        End
      Computing sparse representations of all patches
    Summing up the patch approximation
  End
$\hat{x}$ = Unmasked and Inverse FFT of k-Space
Compute various performances metric
End

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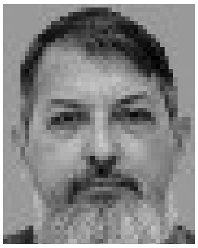

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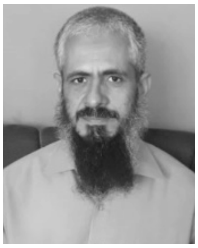

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