IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received September 29, 2019, accepted October 17, 2019, date of publication October 28, 2019, date of current version November 14, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2949923

Convergence Analysis of Iterative Learning Control Systems Over Networks With Successive Input Data Compensation in Iteration Domain

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This work was supported in part by the National Natural Science Foundation of China under Grant 61302118, Grant 61771432, Grant 61901418, Grant 61671011, Grant 61973104, and Grant U1604151, in part by the Scientific and Technological Project of Henan Educational Committee under Grant 17A510005, and in part by the Doctor Fund of Zhengzhou University of Light Industry under Grant 2013BSJJ049.

ABSTRACT This paper analyzes the convergence of iterative learning control for a class of discrete-time systems over networks with successive input data compensation. Specifically, the successively dropped input data in current iterations are compensated by the one actuator received correctly with the same time instant label in the latest iteration. Through analyzing the variation of elements in the transition matrices of the input errors at the controller side, the convergence of output errors is addressed. The analysis shows the selection range of learning gain is determined by the maximum number of input data dropped successively. Moreover, the convergence of system with successive input data compensation is guaranteed by trading the convergence speed, and the more input data are successively compensated in iteration domain, the more convergence speed of the system is reduced. Finally, numerical experiments are given to corroborate the theoretical analysis.

INDEX TERMS Iterative learning control, convergence, data dropouts, compensation.

I. INTRODUCTION

Recently, due to fast development of communication and network techniques, research on analysis and design of systems controlled over communication networks has attracted much attentions. Compared with traditional control systems, the networked control systems (NCSs) are closed via a real communication network, and focused on using information transmitted over the network to achieve some performance objectives with various physical limitations, which usually affect the control performance significantly [1].

When a desired trajectory is given and tracked repeatedly, controller adopting iterative learning control (ILC) strategy is useful in NCSs. In this case, the tracking error and input in previous iterations are used to adjust the input for the current iteration. If learning gain is selected to satisfy some conditions, the tracking errors converge in iteration domain. Since this strategy was first introduced by Arimoto *et al.* [2], many topics have been studied including stochastic noise [3], monotonic convergence [4], initial error [5], interval uncertainty [6], parameter optimization [7] and others surveyed in [8].

When a system is controlled by exploring the ILC strategy over networks, however, since the data needs to be transmitted via a communication network, there is a complex interconnection between the controller and the platform. As a result, analysis and design of such a system pose some new challenges including channel noise [9], [10], quantization error [11], time delay and data dropouts [12].

Over the last few years, we have witnessed some attempts about networked ILC systems with data dropouts. Based on a Kalman filtering approach, in [13], Ahn presented a

The associate editor coordinating the review of this manuscript and approving it for publication was Jun Shen.

mathematical formulation of robust ILC design when the output data vector is subject to data dropouts, and designed a method to select the learning gain optimally such that the system eventually converges to a desired trajectory as long as there is not complete data dropouts. In [14], they further considered the case where each component in the output data vector was dropped independently. After that, convergence conditions were established for the networked ILC systems in presence of data dropouts and delays in both input and output [15].

From a compensation standpoint, Pan contributed an earlier attempt on guaranteeing the convergence of the networked ILC systems with data dropouts [16]. In this paper, with regard to the effect of data dropouts on the system convergence, three different scenarios were discussed including single input data dropout, single output data dropout and general situations with multiple data dropouts. In the discussion, the dropped data at any time instant is replaced with the data received at the last time instant in the same iteration, which means the scheme is implemented in time domain. However, the method only guarantees the convergence of input error at the controller side. Inspired by the nature that the input of the networked ILC systems converges in iteration domain, Huang considered the ILC with random input data dropouts [17]. In this work, the data dropout was viewed as a binary switching sequence obeying the Bernoulli distribution, and the dropped data was compensated using the data in the last iteration at the same time instant with the lost one. That is to say, the compensation was applied in iteration domain, whereas the method only can be applied to deal with single dropped input data, and the author did not analyze the convergence speed of system with the proposed method. Liu applied ILC to remote control systems with random output data dropouts and delays [18], and proved that ILC can achieve asymptotical convergence along the iteration axis as far as the probabilities of the data dropouts and time delays are known.

The works of Ruan also discussed the convergence of networked ILC systems with incomplete information. For a class of repetitive discrete-time SISO nonlinear systems with stochastic data communication delays in one operation duration, the authors proposed a proportional-type ILC scheme to guarantee the convergence of system [19]. For another class of nonlinear systems with input and output data delays, two compensation methods were proposed [20]. In the two schemes, the delayed input data was replaced by the synchronous input data utilized at the previous iteration. For the delayed output data, the one scheme substituted it by the synchronous desired trajectory, while the other substituted it by the synchronous output data in the previous operation. For the systems described in [19] with output data dropouts, the two compensation method proposed in [20] were adopted to deal with the effect of output data dropouts [21]. For the systems described in [20] with both input and output data dropouts, mathematical expectation of the stochastic tracking errors was guaranteed by mending the dropped output data with the synchronous desired output in time domain, and driving the plant by refreshing the dropped input data with the one at the same time instant in previous iteration [22].

Bu studied networked ILC systems using super-vector formulation or Roesser model. In [23], using the super-vector formulation, the system was formulated as a linear discrete-time stochastic system in the iteration domain, and then a sufficient condition was presented, which guaranteed both stability of the ILC process and the desired $H\infty$ performance. It should be noted that the formulation fails to construct the super-vector form when the system is nonlinear. In [24], the design of ILC law with the effect of output data dropouts was transformed into the stabilization of a two-dimensional stochastic system described by the Roesser model. A sufficient condition for mean-square asymptotic stability was established by means of a linear matrix inequality technique, and formulas were given for the control law design. Furthermore, the result was extended to more general cases where the system matrices also contain uncertain parameters. In [25], authors proposed a robust ILC design method for uncertain linear systems with time-varying delays and random packet dropouts, and the ILC design was transformed into robust stability for a system described by the Rosser model.

Shen addressed the convergence of networked ILC systems from different angles. In [26] and [27], the authors modeled the random data dropouts as an arbitrary stochastic sequence with bounded length requirement, and then the almost sure convergence of system with the proposed method was proved. Specifically, a P-type control update algorithm was proposed in [26] for the SISO affine nonlinear system with random output data losses and unknown control direction, and a simple P-type update law was used in [27] for both linear and nonlinear cases based on stochastic approximation. In [28], authors first reviewed the recent progress on the networked ILC systems with data dropouts from the perspective of data dropout model, data dropout position and convergence meaning, respectively. After that, a general framework was proposed for the convergence analysis of three different data dropout models, namely, the stochastic sequence model, the Bernoulli variable model and the Markov chain model. In [29], the authors addressed networked ILC for stochastic linear systems with random output data dropouts, and provided two updating schemes. The intermittent updating scheme only updated its input when data is successfully transmitted, while the successive updating scheme continuously updated its input with the latest available data whether the output information of the last iteration is successfully received or lost. In [30], the authors discussed ILC under general data dropouts at both measurement and actuator sides for different systems. In these papers, a simple compensation mechanism was proposed which allows successive data dropouts in both time and iteration domains, while this mechanism needed the input data that the actuator received to be transmitted with output data to the controller simultaneously.

Based on these works about the convergence of ILC systems with data dropouts, some results could be summarized and listed as follows:

- According to the position where data dropouts occur, the works can be divided into two different categories: input data dropouts and output data dropouts. Most of the reported works only considered a special case that the dropout problem occurs in output data;
- By making appropriate assumptions, the proposed methods guaranteed the convergence of networked ILC systems with data dropouts, while literatures related to convergence speed of networked ILC systems with data compensation are very rare.

The above-mentioned considerations motivate us to address the convergence speed of networked ILC systems with successive input data compensation in iteration domain. Specifically, the main contributions of this paper are:

- Founding the selection range of learning gain to guarantee the convergence of networked ILC systems with general successive input data compensation;
- Indicating the convergence of output errors with successive input data compensation is guaranteed by trading the convergence speed.

The remainder of this paper is organized as follows. The networked ILC systems with data dropouts taken into account is formulated in Section 2. In Section 3, by assuming there are two input data dropped successively in iteration domain, the convergence speed of output errors is analyzed theoretically, and then the discuss is extend to the condition with the general successive input data dropouts in iteration domain. Numerical examples are given to corroborate the theoretical analysis in Section 4. In Section 5, some conclusions wrap up this paper.

II. PROBLEM FORMULATION

Consider a discrete-time, linear and time-invariant system defined as follows

$$\begin{cases} x_k(t+1) = Ax_k(t) + Bu_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{1}$$

where $x_k(t) \in R^n$, $u_k(t) \in R^m$ and $y_k(t) \in R^l$ denote the state, input and output vectors, respectively, $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{l\times n}$ are the known system matrices. $k = 0, 1, \cdots$ denotes iteration number and $t \in [1, 2, \cdots, T]$ is discrete time for each iteration of the system. The control objective is to track the known desired trajectory $y_d(t)$, which satisfies the following formulation

$$\begin{cases} x_d(t+1) = Ax_d(t) + Bu_d(t) \\ y_d(t) = Cx_d(t) \end{cases} \tag{2}$$

where $u_d(t)$ and $x_d(t)$ are desired input and state. In order to track $y_d(t)$ accurately, various ILC schemes have been proposed, and the P-style one can be expressed as

$$u_{k+1}(t) = u_k(t) + \Gamma(t)e_k(t+1) \tag{3}$$

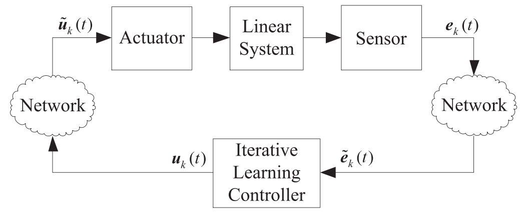

FIGURE 1. Diagram of the networked ILC system.

where $\Gamma(t) \in R^{r\times l}$ is learning gain, and the output error is $e_k(t) = y_d(t) - y_k(t)$.

The ILC system with output data transmitted from the sensor to the controller and input data transmitted from the controller to the actuator through a wired or wireless network is illustrated in Fig. 1. During the data interactions between the controller and the platform, some issues would occur due to the unreliability of the introduced network. Then, the networked ILC system can be represented as

$$\begin{cases} x_k(t+1) = Ax_k(t) + B\tilde{u}_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \tag{4}$$

$$u_{k+1}(t) = u_k(t) + \Gamma(t)\tilde{e}_k(t+1) \tag{5}$$

where $\tilde{u}_k(t)$ is the input data received at the actuator side and $\tilde{e}_k(t+1)$ is the output data received at the controller side. Taking the data dropout into account, $\tilde{u}_k(t)$ and $\tilde{e}_k(t+1)$ can be expressed as

$$\tilde{u}_k(t) = \xi_k(t)u_k(t) \tag{6}$$

$$\tilde{e}_k(t+1) = \eta_k(t)e_k(t+1) \tag{7}$$

where $\xi_k(t)$ and $\eta_k(t)$ are two scalar Bernoulli distributed random variables taking 0 or 1 (i.e., $\xi_k(t), \eta_k(t) \in \{0, 1\}, \forall k, t$). $\xi_k(t)$ is uncorrelated with $\eta_k(t)$. That is, if the variable takes value 0, then the data is dropped correspondingly, otherwise the data is received correctly.

Data dropouts introduced by the network affect the convergence of networked ILC systems, and some compensation methods were proposed to deal with this issue, while literatures related to convergence speed of networked ILC systems with data compensation are very rare. So, in next section, we want to focus our attention on discussing the convergence speed of the output errors when the successively dropped input data are compensated in iteration domain.

III. CONVERGENCE SPEED ANALYSIS OF THE SYSTEM WITH SUCCESSIVE INPUT DATA COMPENSATION

In [28], authors indicated there are two kinds of methods to guarantee the convergence of ILC systems with data dropouts: Kalman filtering-based method and data compensation method. As to data compensation, this approach can be further divided into time domain compensation and iteration domain compensation. In [31], authors compared the convergence of networked ILC systems with input data compensation in time and iteration domains, and pointed out that compensation in iteration domain guarantees the convergence of input errors at both the controller and the actuator side, while compensation in time domain only guarantees the convergence of input errors at the controller side. In this section, we would further analyze the convergence speed of networked ILC systems with successive input data compensation in iteration domain. In order to simplify the analysis, we assume $x_k(0) = x_d(0), \forall k$.

The iteration domain compensation methods adopted in [17], [20], [22] and [29] could be embraced as following:

$$\tilde{u}_k(t) = \xi_k(t)u_k(t) + (1 - \xi_k(t))\,\tilde{u}_{k-1}(t) \tag{8}$$

To see this, we rewrite (8) as

$$\begin{aligned}\tilde{u}_k(t) = {} & \xi_k(t)u_k(t) + (1 - \xi_k(t))\,\xi_k(t-1)u_k(t-1) + \cdots \\ & + (1 - \xi_k(t))\,(1 - \xi_k(t-1))\cdots \\ & \times (1 - \xi_k(t-i+1))\,\xi_k(t-i)u_k(t-i) + \cdots \end{aligned} \tag{9}$$

Note that the method allows successive input data dropouts in both time and iteration domains. In order to explain the reason, $u_k(t)$ is assumed to be dropped, and then four input data in adjacent iteration or time domains are used to describe the four dropout cases:

Case 1 Non-successive dropout:
$\xi_k(t) = 0$ and $\xi_k(t-1) = \xi_k(t+1) = \xi_{k-1}(t) = \xi_{k+1}(t) = 1$, which means only $u_k(t)$ is dropped and compensated by $u_{k-1}(t)$.

Case 2 Successive dropout in iteration domain:
$\xi_k(t) = \xi_{k-1}(t) = 0$ and $\xi_k(t-1) = \xi_k(t+1) = \xi_{k+1}(t) = 1$, which means $u_k(t)$ and $u_{k-1}(t)$ are dropped successively in iteration domain and compensated using $u_{k-2}(t)$ consecutively.

Case 3 Successive dropout in time domain:
$\xi_k(t) = \xi_k(t-1) = 0$ and $\xi_k(t+1) = \xi_{k-1}(t) = \xi_{k+1}(t) = 1$, which means $u_k(t)$ and $u_k(t-1)$ are dropped successively in time domain and compensated by $u_{k-1}(t)$ and $u_{k-1}(t-1)$ respectively.

Case 4 Successive dropout in iteration and time domain simultaneously:
$\xi_k(t) = \xi_k(t-1) = \xi_{k-1}(t) = 0$ and $\xi_k(t+1) = \xi_{k+1}(t) = 1$, which means $u_k(t)$, $u_k(t-1)$ and $u_{k-1}(t)$ are dropped successively in both iteration and time domains. Then $u_{k-2}(t)$ is used to replace the dropped $u_k(t)$ and $u_{k-1}(t)$, and $u_{k-1}(t-1)$ is used to replace the dropped $u_k(t-1)$.

It can be easily seen that Case 2 is a mix of Case 1 at the time t in $(k-1)-th$ and $(k)-th$ iterations, Case 3 can be seen as a mix of Case 1 at the time t and $t-1$ in $(k)-th$ iteration, and Case 4 can be seen as a mix of Case 2 and 3. Based on the relations among the four different cases, the convergence analysis of output errors under Case 1, 3 and 4 is similar with that under Case 2.

Next, the convergence speed of output errors is analyzed firstly by assuming there are two input data dropped successively at time t in $k-th$ and $(k-1)-th$ iterations, and then the analysis is extended to the general successive data dropouts in iteration domain. In the analysis, the transition matrices of input error at the controller side would be derived, and the variation of eigenvalues in the matrices would be discussed, which determine the convergence speed of networked ILC systems. Additionally, it can be seen that the convergence of output errors $e_{k+1}(i)$, $(t+1 \le i \le T)$ are affected by the dropped input data, so the convergence speed of the ILC system with successive input data compensation in iteration domain would be analyzed from the following two parts.

A. CONVERGENCE SPEED ANALYSIS OF OUTPUT ERROR AT TIME $t+1$

According to $e_k(t) = y_d(t) - y_k(t)$, the output error $e_k(t+1)$ can be represented as

$$\begin{aligned} e_k(t+1) &= y_d(t+1) - y_k(t+1) \\ &= C\delta x_k(t+1) \end{aligned} \tag{10}$$

Using (2) and (4), the state error $\delta x_k(t)$ can be expressed as

$$\begin{aligned} &\delta x_k(t+1) \\ &\quad = x_d(t+1) - x_k(t+1) \\ &\quad = A\delta x_k(t) + B\left(\xi_k(t)\delta u_k(t) + (1-\xi_k(t))\,\delta\tilde{u}_{k-1}(t)\right) \\ &\quad = \sum_{i=0}^{t} A^{t-i}B\left(\xi_k(i)\delta u_k(i) + (1-\xi_k(i))\,\delta\tilde{u}_{k-1}(i)\right) \end{aligned} \tag{11}$$

From (5) and (11), the input error $\delta u_{k+1}(t)$ can be represented as

$$\begin{aligned} \delta u_{k+1}(t) &= u_d(t) - u_{k+1}(t) \\ &= u_d(t) - u_k(t) - \Gamma(t)e_k(t+1) \\ &= \delta u_k(t) - \Gamma(t)\sum_{i=0}^{t} CA^{t-i}B \\ &\quad \times \left(\xi_k(i)\delta u_k(i) + (1-\xi_k(i))\,\delta\tilde{u}_{k-1}(i)\right) \end{aligned} \tag{12}$$

According to (8), the dropped input data $u_k(t)$ is compensated by $\tilde{u}_{k-1}(t)$. For this condition, (12) would be changed into

$$\begin{aligned} \delta u_{k+1}(t) &= \delta u_k(t) - \Gamma(t)CB\delta\tilde{u}_{k-1}(t) \\ &\quad - \Gamma(t)\sum_{i=0}^{t-1} CA^{t-i}B\delta u_k(i) \\ &= H_{k+1,k}(t)\cdot\varphi_k(t) - \Gamma(t)CB\delta\tilde{u}_{k-1}(t) \end{aligned} \tag{13}$$

where

$$H_{k+1,k}(t) = \begin{bmatrix} -\Gamma(t)CA^tB & \cdots & -\Gamma(t)CAB & I \end{bmatrix} \tag{14}$$

$$\varphi_k(t) = \begin{bmatrix} \delta u_k(0) & \delta u_k(1) & \cdots & \delta u_k(t) \end{bmatrix}^T \tag{15}$$

Further, if the input data $u_{k-1}(t)$ is dropped successively and compensated by $\tilde{u}_{k-2}(t)$, we would have

$$\delta u_k(j) = \delta u_{k-1}(j) - \Gamma(j)CB\delta u_{k-1}(j) - \Gamma(t)\sum_{j=0}^{t-1} CA^{t-j}B\delta u_{k-1}(j) \tag{16}$$

where $0 \le j \le t-1$, and

$$\begin{aligned} \delta u_k(t) &= \delta u_{k-1}(t) - \Gamma(t)CB\delta\tilde{u}_{k-2}(t) \\ &\quad - \Gamma(t)\sum_{i=0}^{t-1} CA^{t-i}B\delta u_{k-1}(i) \end{aligned} \tag{17}$$

In matrix form, we can rewrite (16) and (17) as

$$\varphi_k(t) = H_{k,k-1}(t)\cdot\varphi_{k-1}(t) + \begin{bmatrix} 0 & \cdots & 0 & -\Gamma(t)CB \end{bmatrix}^T \cdot \delta\tilde{u}_{k-2}(t) \quad (18)$$

where

$$H_{k,k-1}(t) = \begin{bmatrix} I-\Gamma(0)CB & 0 & \cdots & \cdots & 0 \\ -\Gamma(1)CAB & \ddots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -\Gamma(t-1)CA^{t-1}B & \cdots & -\Gamma(t-1)CAB & I-\Gamma(t-1)CB & 0 \\ -\Gamma(t)CA^tB & \cdots & \cdots & -\Gamma(t)CAB & I \end{bmatrix} \quad (19)$$

$$\varphi_{k-1}(t) = \begin{bmatrix} \delta u_{k-1}(0) & \delta u_{k-1}(1) & \cdots & \delta u_{k-1}(t) \end{bmatrix}^T \quad (20)$$

If the input data $u_{k-2}(t)$ is received correctly, we have $\delta\tilde{u}_{k-1}(t) = \delta\tilde{u}_{k-2}(t) = \delta u_{k-2}(t)$, and $\varphi_{k-1}(t)$ can be expressed using $\varphi_{k-2}(t)$ in matrix form as

$$\varphi_{k-1}(t) = H_{k-1,k-2}(t)\cdot\varphi_{k-2}(t) \quad (21)$$

where

$$H_{k-1,k-2}(t) = \begin{bmatrix} I-\Gamma(0)CB & 0 \cdots & \cdots & 0 \\ -\Gamma(1)CAB & \ddots\ \ddots & & \vdots \\ \vdots & \vdots\ \ddots & \ddots & \vdots \\ -\Gamma(t-1)CA^{t-1}B & \cdots\cdots & I-\Gamma(t-1)CB & 0 \\ -\Gamma(t)CA^tB & \cdots\cdots & -\Gamma(t)CAB & I-3\Gamma(t)CB \end{bmatrix} \quad (22)$$

$$\varphi_{k-2}(t) = \begin{bmatrix} \delta u_{k-2}(0) & \delta u_{k-2}(1) & \cdots & \delta u_{k-2}(t) \end{bmatrix}^T \quad (23)$$

Thus, $\delta u_{k+1}(t)$ can be represented using $\varphi_{k-2}(t)$ as

$$\delta u_{k+1}(t) = H_{k+1,k}(t)\cdot H_{k,k-1}(t)\cdot H_{k-1,k-2}(t)\cdot\varphi_{k-2}(t) \quad (24)$$

Compared with ideal conditions, it can be easily found that some element values in transition matrices are changed when successively dropped $u_k(t)$ and $u_{k-1}(t)$ are replaced with $u_{k-2}(t)$. First, the element at $(1, t+1)$ in $H_{k+1,k}(t)$ and the element at $(t+1, t+1)$ in $H_{k,k-1}(t)$ are all changed from '$I - \Gamma(t)CB$' to 'I'. Additionally, the eigenvalue at $(t+1, t+1)$ in $H_{k-1,k-2}(t)$ is changed from '$I - \Gamma(t)CB$' to '$I - 3\Gamma(t)CB$'. It can be easily seen that $H_{k,k-1}(t)$ and $H_{k-1,k-2}(t)$ are lower triangular matrices, and all of its eigenvalues are the diagonal elements. If the learning gain $\Gamma(t)$ is selected to satisfy $0 < \|I - \Gamma(t)CB\| < 1$ and $0 < \|I - 3\Gamma(t)CB\| < 1$, we could have $\lambda_m\left(\prod_{i=0}^{\infty}\|H_{i+1,i}(t)\|\right) \to 0, m = 0, \cdots, t$, which guarantees the input error at the controller side satisfying $\lim_{k\to\infty}\|\delta u_{k+1}(t)\| = 0$. Furthermore, the received input $\tilde{u}_k(t)$ at the actuator could also converge because $\lim_{k\to\infty}\|\delta\tilde{u}_k(t)\| = \lim_{k\to\infty}\|u_d(t) - u_{k-2}(t)\| = 0$. Due to the output error $\|e_k(t+1)\|$ is a function of input error $\|\delta\tilde{u}_k(i)\|, i \in [0, t]$, so the convergence of $\|\delta\tilde{u}_k(t)\|$ indicates $\lim_{k\to\infty}\|e_k(t+1)\| = 0$.

It should be noted that although the convergence of output error at the time $t+1$ is guaranteed, the convergence speed of which is reduced. The reason is that the element at $(1, t+1)$ in $H_{k+1,k}(t)$ and the element at $(t+1, t+1)$ in $H_{k,k-1}(t)$ are all increased from '$I - \Gamma(t)CB$' to 'I', and the increment makes a slowdown in the convergence speed of output error at the time $t+1$.

Next, we want extend the analysis to the condition with the general successive input data compensation. In other words, $u_k(t)$, $u_{k-1}(t)$,..., $u_{k-n+1}(t)$ are dropped successively and all replaced with $u_{k-n}(t)$, where n is the number of input data dropped successively in iteration domain at the same time t. By analogy, $u_{k+1}(t)$ can by rewritten using $\varphi_{k-n}(t) = [u_{k-n}(0), \ldots, u_{k-n}(t)]^T$ in matrix form as

$$\delta u_{k+1}(t) = \prod_{i=0}^{n} H_{k+1-i,k-i}(t)\cdot\varphi_{k-n}(t) \quad (25)$$

where $H_{k-j+1,k-j}(t) = H_{k,k-1}(t), 2 \le j \le n-1$, and $H_{k-1,k-2}(t)$ is given in (26), as shown at the bottom of this page.

On the one hand, it can be seen that the more input data are dropped successively and compensated using the method given in (8), the more eigenvalues at $(t+1, t+1)$ in the transition matrices $H_{k-j+1,k-j}(t), 2 \le j \le n-1$ are changed from '$I - \Gamma(t)CB$' to 'I', and then the more convergence speed of $e_k(t+1)$ is reduced. On the other hand, it can also be seen that the eigenvalue at $(t+1, t+1)$ in $H_{k-n+1,k-n}(t)$ is changed from '$I - \Gamma(t)CB$' to '$I - (n+1)\Gamma(t)CB$'. Due to the data dropout is a random process, it is hard to establish the connection between data dropout probability and the number of consecutive data dropouts, while the maximum number of input data dropped successively could be expressed as $n_{max} = T * P_d$,

$$H_{k-n+1,k-n}(t) = \begin{bmatrix} I-\Gamma(0)CB & 0 & \cdots & \cdots & 0 \\ -\Gamma(1)CAB & \ddots & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -\Gamma(t-1)CA^{t-1}B & \cdots & \cdots & I-\Gamma(t-1)CB & 0 \\ -\Gamma(t)CA^tB & \cdots & \cdots & -\Gamma(t)CAB & I-(n+1)\Gamma(t)CB \end{bmatrix} \quad (26)$$

where P_d is the input data dropout probability. Obviously, if $0 < \Gamma(t) < 2/(n_{max}+1)CB$ holds, the condition $0 < \|I-(n_{max}+1)\Gamma(t)CB\| < 1$ is satisfied, and $0 < \|I-(n+1)\Gamma(t)CB\| < 1$ is established for all values of n, so the convergence of $e_k(t+1)$ is guaranteed. Otherwise, some eigenvalues at $(t+1,t+1)$ in the transition matrices are equal or greater than one. Moreover, the higher input data drop probability is, the more input data are dropped successively in iteration domain, the more eigenvalues $I-(n+1)\Gamma(t)CB$ are equal or greater than one. And then, the convergence of $e_k(t+1)$ cannot be guaranteed.

B. CONVERGENCE SPEED ANALYSIS OF OUTPUT ERRORS AT TIME $t+1+i$, $1 \le i \le T-t-1$

In this part, we continue to analyze the convergence of $\|e_k(t+1+i)\|$, $1 \le i \le T-t-1$. Due to the similarity in the analysis process, the convergence of $\|e_k(t+2)\|$ would be proved, and then the convergence of $\|e_k(t+1+i)\|$, $2 \le i \le T-t-1$ would be proved by analogy.

The output error $e_k(t+2)$ can be represented as

$$\begin{aligned} e_k(t+2) &= y_d(t+2) - y_k(t+2) \\ &= C\sum_{i=0}^{t+1} A^{t+1-i}B\left(\xi_k(i)\delta u_k(i) + (1-\xi_k(i))\,\delta\tilde{u}_{k-1}(i)\right) \end{aligned} \tag{27}$$

According to (5) and (27), $\delta u_{k+1}(t+1)$ can be written as

$$\begin{aligned} \delta u_{k+1}(t+1) &= u_d(t+1) - u_{k+1}(t+1) \\ &= u_d(t+1) - u_k(t+1) - \Gamma(t+1)e_k(t+2) \\ &= \delta u_k(t+1) - \Gamma(t+1)\sum_{i=0}^{t+1} CA^{t+1-i}B \\ &\quad \times \left(\xi_k(i)\delta u_k(i) + (1-\xi_k(i))\,\delta\tilde{u}_{k-1}(i)\right) \end{aligned} \tag{28}$$

When the input data $u_k(t)$ is dropped and compensated by $\tilde{u}_{k-1}(t)$, (28) can be rewritten as

$$\begin{aligned} \delta u_{k+1}(t+1) &= \delta u_k(t+1) - \Gamma(t+1)CB\delta u_k(t+1) \\ &\quad - \Gamma(t)CAB\delta\tilde{u}_{k-1}(t) - \sum_{i=0}^{t-1} CA^{t+1-i}B\delta u_k(i) \\ &= H_{k+1,k}(t+1)\cdot\varphi_k(t+1) - \Gamma(t)CAB\delta\tilde{u}_{k-1}(t) \end{aligned} \tag{29}$$

where

$$\begin{aligned} &H_{k+1,k}(t+1) \\ &= \left[-\Gamma(t+1)CA^{t+1}B \;\cdots\; -\Gamma(t+1)CA^2B \;\; 0 \;\; I-\Gamma(t+1)CB\right] \end{aligned} \tag{30}$$

$$\begin{aligned} &\varphi_k(t+1) \\ &= \left[\delta u_k(0) \;\; \delta u_k(1) \;\; \cdots \;\; \delta u_k(t+1)\right]^T \end{aligned} \tag{31}$$

If the input data input data $u_{k-1}(t)$ is dropped successively and compensated by $\tilde{u}_{k-2}(t)$, $u_k(t+1)$ can be represented as

$$\begin{aligned} \delta u_k(t+1) &= \delta u_k(t) - \Gamma(t+1)CB\delta u_k(t+1) \\ &\quad - \Gamma(t)CAB\delta\tilde{u}_{k-2}(t) - \sum_{i=0}^{t-1} CA^{t+1-i}B\delta u_{k-1}(i) \end{aligned} \tag{32}$$

Because $\delta u_k(t) = \delta\tilde{u}_{k-1}(t) = \delta\tilde{u}_{k-2}(t)$, and

$$\begin{aligned} \delta u_k(j) &= \delta u_{k-1}(j) - \Gamma(j)CB\delta u_{k-1}(j) \\ &\quad - \Gamma(t)\sum_{i=0}^{t-1} CA^{t-j}B\delta u_{k-1}(j) \end{aligned} \tag{33}$$

where $0 \le j \le t-1$, $\varphi_k(t+1)$ can be rewritten as

$$\begin{aligned} \varphi_k(t+1) &= H_{k,k-1}(t+1)\cdot\varphi_{k-1}(t+1) \\ &\quad + \left[0 \;\; \cdots \;\; 0 \;\; 1 \;\; -\Gamma(t)CAB\right]^T\cdot\delta\tilde{u}_{k-2}(t) \end{aligned} \tag{34}$$

where $H_{k,k-1}(t+1)$ and $\varphi_{k-1}(t+1)$ are given in (35) and (36), as shown at the bottom of this page.

If the input data input data $\tilde{u}_{k-2}(t)$ is received correctly, $\delta\tilde{u}_{k-1}(t) = \delta\tilde{u}_{k-2}(t) = \delta u_{k-2}(t)$, and then $\varphi_{k-1}(t+1)$ can be expressed using $\varphi_{k-2}(t+1)$ in matrix form as

$$\varphi_{k-1}(t+1) = H_{k-1,k-2}(t+1)\cdot\varphi_{k-2}(t+1) \tag{37}$$

where $H_{k-1,k-2}(t+1)$ and $\varphi_{k-2}(t+1)$ are given in (38) and (39), as shown at the bottom of the next page.

Thus, $\delta u_{k+1}(t+1)$ can be represented using $\varphi_{k-2}(t+1)$ as

$$\begin{aligned} \delta u_{k+1}(t+1) &= H_{k+1,k}(t+1)\cdot H_{k,k-1}(t+1) \\ &\quad \cdot H_{k-1,k-2}(t+1)\cdot\varphi_{k-2}(t+1) \end{aligned} \tag{40}$$

Due to successively dropped input data $u_k(t)$ and $u_{k-1}(t)$ are replaced by $u_{k-2}(t)$, elements in $(t+1)-th$ row of $H_{k+1,k}(t+1)$, $H_{k,k-1}(t+1)$ and $H_{k-1,k-2}(t+1)$ are

$$H_{k,k-1}(t+1) = \begin{bmatrix} I-\Gamma(0)CB & 0 & \cdots & \cdots & \cdots & 0 \\ -\Gamma(1)CAB & \ddots & \ddots & & & \vdots \\ \vdots & & \ddots & \ddots & & \vdots \\ -\Gamma(t-1)CA^{t-1}B & \cdots & \cdots & I-\Gamma(t-1)CB & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & I & 0 \\ -\Gamma(t+1)CA^{t+1}B & \cdots & \cdots & -\Gamma(t+1)CAB & 0 & I-\Gamma(t+1)CB \end{bmatrix} \tag{35}$$

$$\varphi_{k-1}(t+1) = \left[\delta u_{k-1}(0) \quad \delta u_{k-1}(1) \quad \cdots \quad \delta u_{k-1}(t+1)\right]^T \tag{36}$$

changed. Specially, the eigenvalues in $(t+1)-th$ row of $H_{k,k-1}(t+1)$ and $H_{k-1,k-2}(t+1)$ are all changed from '$I-\Gamma(t)CB$' to 'I'. Since $H_{k,k-1}(t+1)$ and $H_{k-1,k-2}(t+1)$ are lower triangular matrices, and all of these eigenvalues are the diagonal elements, it can be easily derived that $\lambda_m\left(\prod_{i=0}^{\infty}\|H_{i+1,i}(t+1)\|\right)\rightarrow 0, m=0,1,\cdots t+1$, then $\lim_{k\rightarrow\infty}\|\delta u_{k+1}(t+1)\|=0$ is guaranteed.

With regard to $\|\delta u_{k+1}(t+i)\|, 2\leq i\leq T-t-1$, according to proving the convergence of $\|\delta u_{k+1}(t+1)\|$, it can be easily seen that eigenvalues in $(t+1)-th$ row of $H_{k,k-1}(t+i)$ and $H_{k-1,k-2}(t+i)$ would be changed to 'I'. For this condition, we can also have $\lambda_m\left(\prod_{i=0}^{\infty}\|H_{i+1,i}(j)\|\right)\rightarrow 0, m\in[0,j], j\in[t+2,T-1]$, then $\lim_{k\rightarrow\infty}\|\delta u_{k+1}(i)\|=0, i\in[t+2,T-1]$ is guaranteed.

Correspondingly, it can be easily conclude that $\lim_{k\rightarrow\infty}\|\delta\tilde{u}_{k+1}(i)\|=0, i\in[t+2,T-1]$ as $\lim_{k\rightarrow\infty}u_k(i)=\lim_{k\rightarrow\infty}u_{k-2}(i)$. Because $\|e_k(t+1+i)\|, i\in[1,T-t-1]$ is a function of $\|\delta\tilde{u}_k(j)\|, j\in[0,t+i]$, the convergence of $\|\delta\tilde{u}_k(j)\|, j\in[0,t+i]$ indicates $\lim_{k\rightarrow\infty}\|e_k(t+1+i)\|$, $i\in[1,T-t-1]$.

It is noteworthy that the convergence of output errors at the time $t+1+i, i\in[1,T-t-1]$ is guaranteed by trading the convergence speed. The reason is that eigenvalues in $(t+1)-th$ row of $H_{k,k-1}(t+1)$ and $H_{k-1,k-2}(t+1)$ are increased from '$I-\Gamma(t)CB$' to 'I', and then the convergence speed of $\|\delta u_{k+1}(t+1)\|$ is reduced, which further makes a slowdown in the convergence speed of output errors at the time $t+1+i, i\in[1,T-t-1]$.

Further, we continue to analyze the convergence of output errors with general successive input data compensation. If $u_k(t)$, $u_{k-1}(t)$,..., $u_{k-n+1}(t)$ are dropped successively and all replaced by $u_{k-n}(t)$, by analogy, $u_{k+1}(t+1)$ can by rewritten using $\varphi_{k-n}(t+1)=[u_{k-n}(0),\ldots,u_{k-n}(t+1)]^T$ in matrix form as

$$\delta u_{k+1}(t+1)=\prod_{j=0}^{n}H_{k-j+1,k-j}(t+1)\cdot\varphi_{k-n}(t+1) \tag{41}$$

where $H_{k-j+1,k-j}(t+1)=H_{k,k-1}(t+1), 2\leq j\leq n-1$, and $H_{k-n+1,k-n}(t+1)$ is given in (42), as shown at the bottom of this page.

It can be easily seen that the higher input data drop probability is, the more input data are dropped successively and compensated using the method given in (8), the more eigenvalues at $(t+1,t+1)$ in the transition matrices are increased from '$I-\Gamma(t)CB$' to 'I', the more convergence speed of output errors at the time $t+i, i\in[2,T-t]$ is reduced.

Based on the discoveries in last two subsections, it is easy to indicate that $0<\|I-(n+1)\Gamma(t)CB\|<1$ is the condition to guarantee the convergence of networked ILC systems with n input data compensated successively in iteration domain, which can be established when $0<\Gamma(t)<2/(n_{max}+1)CB$ holds. Moreover, the convergence of system is guaranteed by trading its convergence speed. The more input data are dropped successively and compensated in iteration, the more convergence speed of the system is reduced.

IV. SIMULATION RESULTS

In this section, some numerical results are given to corroborate the theoretical analysis about convergence speed of networked ILC systems with successive input data compensation. Consider the system (4) with matrices given by

$$\begin{aligned}x_k(t+1)&=\begin{bmatrix}-0.5&0&0\\1&1.24&-0.87\\0&0.87&0\end{bmatrix}x_k(t)+\begin{bmatrix}1\\0\\0\end{bmatrix}\tilde{u}_k(t)\\ y_k(t)&=\begin{bmatrix}2&2.6&-2.8\end{bmatrix}x_k(t)\end{aligned} \tag{43}$$

$$H_{k-1,k-2}(t+1)=\begin{bmatrix}I-\Gamma(0)CB&0&\cdots&\cdots&0\\-\Gamma(1)CAB&\ddots&&&\vdots\\\vdots&&\ddots&&\vdots\\\vdots&\cdots&\ddots&\ddots&\vdots\\0&\cdots&0&I&0\\-\Gamma(t+1)CA^{t+1}B&\cdots&\cdots&-3\Gamma(t+1)CAB&I-\Gamma(t+1)CB\end{bmatrix} \tag{38}$$

$$\varphi_{k-2}(t+1)=\begin{bmatrix}\delta u_{k-2}(0)&\delta u_{k-2}(1)&\cdots&\delta u_{k-2}(t+1)\end{bmatrix}^T \tag{39}$$

$$H_{k-n+1,k-n}(t+1)=\begin{bmatrix}I-\Gamma(0)CB&0&\cdots&\cdots&0\\-\Gamma(1)CAB&\ddots&&&\vdots\\\vdots&&\ddots&&\vdots\\\vdots&\cdots&\ddots&\ddots&\vdots\\0&\cdots&0&I&0\\-\Gamma(t+1)CA^{t+1}B&\cdots&\cdots&-(n+1)\Gamma(t+1)CAB&I-\Gamma(t+1)CB\end{bmatrix} \tag{42}$$

The desired trajectory is

$$y_d(t) = 5\sin\left[8\left(t-1\right)/T\right] \tag{44}$$

The P-style ILC method is described in (5). Initial state error $\delta x_k(0)$ and initial input $u_0(t)$ are 0, respectively. $T = 200$, $Prob\{\xi_k(t)=0\} = E\{\xi_k(t)=0\} = \alpha$ is the input data dropout probability. In this example, the mean of output errors is used to demonstrate the convergence of ILC systems with successive input data compensation, and three data dropout cases are simulated, that is, $\alpha = 6\%$, 12% and 18%. Correspondingly, the maximum numbers of input data dropped successively are 12, 24 and 36, and the maximum values of learning gain are 0.385, 0.2 and 0.136. In order to demonstrate the effect of the selected of learning gain on the convergence of ILC systems with successive input data compensation, three learning gains are used including $\Gamma = 0.12$, 0.20 and 0.28.

Fig. 2 shows the convergence of output errors when $\Gamma = 0.12$. Because 0.12 is less than the maximum of learning gain in the three data dropout cases, it can be easily seen that the mean of output errors all converges to zero. Fig. 3 shows the convergence of system when $\Gamma = 0.2$. It can be easily seen that the mean of output errors converges to zero when $\alpha = 6\%$ and 12%. For the case $\alpha = 6\%$, the convergence of output errors' mean is guaranteed because 0.2 is less than 0.385. For the case $\alpha = 12\%$, the used learning gain equal to the maximum value of learning gain. The convergence of output errors' mean is still guaranteed because the chance of this worst-case scenario that n_{max} input data are all dropped successively in each iteration is extremely remote. For the case $\alpha = 18\%$, the convergence of output errors' mean cannot be guaranteed because 0.2 is far larger than 0.136. Fig. 4 shows the convergence of system when $\Gamma = 0.28$. Because 0.28 is less than 0.385 and larger than 0.2 and 0.136, The convergence of output errors' mean is guaranteed when $\alpha = 6\%$, and not guaranteed when $\alpha = 12\%$ and $\alpha = 18\%$. Fig. 5-7 show the ILC system outputs with successive input data compensation in 50-th iteration with three input data dropout probabilities when $\Gamma = 0.2$, which further corroborates the theoretical analysis about the convergence of ILC systems with successive input data compensation in iteration domain.

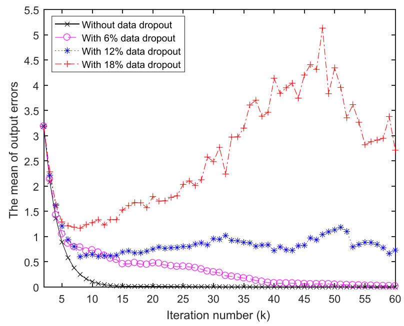

FIGURE 4. The mean of output errors with three input data dropout probabilities when $\Gamma(t) = 0.28$.

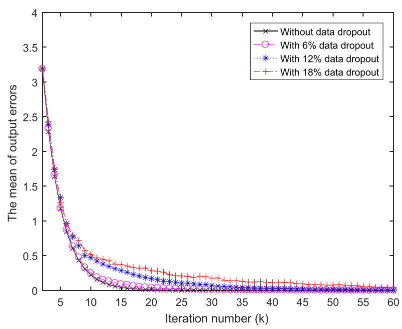

FIGURE 2. The mean of output errors with three input data dropout probabilities when $\Gamma(t) = 0.12$.

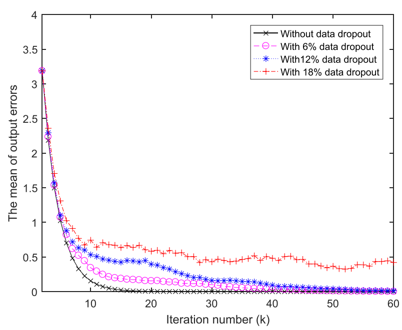

FIGURE 3. The mean of output errors with three input data dropout probabilities when $\Gamma(t) = 0.2$.

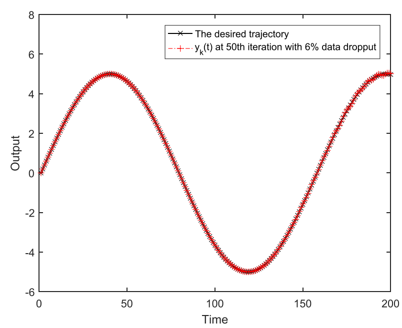

FIGURE 5. The output with successive input data compensation at 50-th iteration when $\alpha = 6\%$ and $\Gamma(t) = 0.2$.

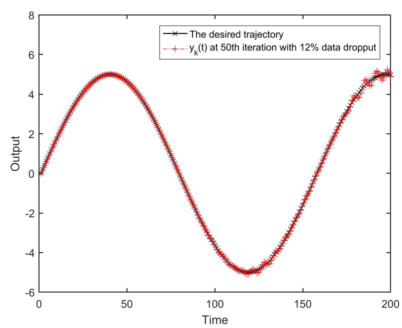

FIGURE 6. The output with successive input data compensation at 50-th iteration when $\alpha = 12\%$ and $\Gamma(t) = 0.2$.

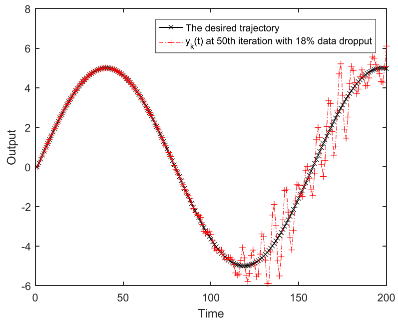

FIGURE 7. The output with successive input data compensation at 50-th iteration when $\alpha = 18\%$ and $\Gamma(t) = 0.2$.

V. CONCLUSION

In this paper, we addressed the convergence of networked ILC systems with successive input data compensation, which uses the data actuator received correctly in the latest iteration to replace the lost ones with the same time instant label in current iterations. Assuming there are two input data dropped successively in iteration domain, the convergence of output errors is addressed through analyzing the variation of elements in the transition matrices of the input error at the controller side. After that, the discuss is extend to the condition with the general successive input data dropouts in iteration domain. The analysis reveals that the convergence of networked ILC systems with successive compensation in iteration domain is guaranteed by trading the convergence speed. Additionally, the selection range of learning gain is found to guarantee the convergence of ILC systems with successive input data compensation. Theoretical analysis and simulation results corroborates the correctness of our conclusion.

In order to accelerate the convergence of networked ILC systems with successive input data compensation in iteration domain, we would design new learning strategies or process the received input data at the actuator side in the future study.

REFERENCES

[1] K. You, Z. Li, D. E. Quevedo, and F. L. Lewis, "Recent developments in networked control and estimation," *IET Control Theory Appl.*, vol. 8, no. 18, pp. 2123–2125, 2014.

[2] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *J. Robot. Syst.*, vol. 1, no. 2, pp. 123–140, 1984.

[3] S. S. Saab, "Optimal selection of the forgetting matrix into an iterative learning control algorithm," *IEEE Trans. Autom. Control*, vol. 50, no. 12, pp. 2039–2043, Dec. 2005.

[4] T. D. Son, G. Pipeleers, and J. Swevers, "Robust monotonic convergent iterative learning control," *IEEE Trans. Autom. Control*, vol. 61, no. 4, pp. 1063–1068, Apr. 2016.

[5] K.-H. Park and Z. Bien, "A generalized iterative learning controller against initial state error," *Int. J. Control*, vol. 73, no. 10, pp. 871–881, 2000.

[6] H.-S. Ahn, K. L. Moore, and Y. Chen, "Stability analysis of discrete-time iterative learning control systems with interval uncertainty," *Automatica*, vol. 43, no. 5, pp. 892–902, May 2007.

[7] D. H. Nguyen and D. Banjerdpongchai, "A convex optimization design of robust iterative learning control for linear systems with iteration-varying parametric uncertainties," *Asian J. Control*, vol. 13, no. 1, pp. 75–84, 2011.

[8] D. A. Bristow, M. Tharayil, and A. G. Alleyne, "A survey of iterative learning control," *IEEE Control Syst. Mag.*, vol. 26, no. 3, pp. 96–114, Jun. 2006.

[9] L.-X. Huang, Y. Fang, and T. Wang, "Method to improve convergence performance of iterative learning control systems over wireless networks in presence of channel noise," *IET Control Theory Appl.*, vol. 8, no. 3, pp. 175–182, Feb. 2014.

[10] L. X. Huang, Y. Fang, and T. Wang, "Convergence analysis of wireless remote iterative learning control systems with channel noise," *Asian J. Control*, vol. 17, no. 6, pp. 2374–2381, 2015.

[11] X. Bu, T. Wang, Z. Hou, and R. Chi, "Iterative learning control for discrete-time systems with quantised measurements," *IET Control Theory Appl.*, vol. 9, no. 9, pp. 1455–1460, 2015.

[12] D. Shen, "Iterative learning control with incomplete information: A survey," *IEEE/CAA J. Autom. Sinica*, vol. 5, no. 5, pp. 885–901, Jul. 2018.

[13] H.-S. Ahn, Y. Q. Chen, and K. L. Moore, "Intermittent iterative learning control," in *Proc. IEEE Int. Symp. Intell. Control*, Oct. 2006, pp. 832–837.

[14] H.-S. Ahn, Y. Chen, and K. L. Moore, "Discrete-time intermittent iterative learning controller with independent data dropout," *IFAC Proc. Volumes*, vol. 41, no. 2, pp. 12442–12447, 2008.

[15] H.-S. Ahn, Y. Chen, and K. L. Moore, "Stability of discrete-time iterative learning control with random data dropouts and delayed controlled signals in networked control systems," in *Proc. 10th Int. Conf. Control, Automat., Robot. Vis.*, Dec. 2008, pp. 757–762.

[16] Y. J. Pan, H. J. Marquez, T. W. Chen, and L. Sheng, "Effects of network communications on a class of learning controlled non-linear systems," *Int. J. Syst. Sci.*, vol. 40, no. 7, pp. 757–767, 2009.

[17] L. Huang and Y. Fang, "Convergence analysis of wireless remote iterative learning control systems with dropout compensation," *Math. Problems Eng.*, vol. 2013, Mar. 2013, Art. no. 609284.

[18] C. Liu, J. Xu, and J. Wu, "Iterative learning control for remote control systems with communication delay and data dropout," *Math. Problems Eng.*, vol. 2012, Jan. 2012, Art. no. 705474.

[19] J. Liu and X. Ruan, "Networked iterative learning control approach for nonlinear systems with random communication delay," *Int. J. Syst. Sci.*, vol. 47, no. 16, pp. 3960–3969, 2016.

[20] J. Liu and X. Ruan, "Networked iterative learning control design for discrete-time systems with stochastic communication delay in input and output channels," *Int. J. Syst. Sci.*, vol. 48, no. 9, pp. 1844–1855, 2017.

[21] J. Liu and X. Ruan, "Networked iterative learning control design for nonlinear systems with stochastic output packet dropouts," *Asian J. Control*, vol. 20, no. 3, pp. 1077–1087, May 2018.

[22] J. Liu and X. Ruan, "Networked iterative learning control for discrete-time systems with stochastic packet dropouts in input and output channels," *Adv. Difference Equ.*, vol. 2017, 2017, Art. no. 53.

[23] X. Bu, Z. Hou, F. Yu, and F. Wang, "H_∞ iterative learning controller design for a class of discrete-time systems with data dropouts," *Int. J. Syst. Sci.*, vol. 45, no. 9, pp. 1902–1912, 2014.

[24] X. Bu, Z. Hou, S. Jin, and R. Chi, "An iterative learning control design approach for networked control systems with data dropouts," *Int. J. Robust Nonlinear Control*, vol. 26, no. 1, pp. 91–109, Jan. 2016.

[25] X. H. Bu, Z. W. Hou, Z. S. Hou, and J. Q. Yang, "Robust iterative learning control design for linear systems with time-varying delays and packet dropouts," *Adv. Difference Equ.*, vol. 2017, 2017, Art. no. 84.

[26] D. Shen and Y. Wang, "ILC for networked nonlinear systems with unknown control direction through random lossy channel," *Syst. Control Lett.*, vol. 77, pp. 30–39, Mar. 2015.

[27] D. Shen and Y. Wang, "Iterative learning control for networked stochastic systems with random packet losses," *Int. J. Control*, vol. 88, no. 5, pp. 959–968, 2015.

[28] D. Shen and J.-X. Xu, "A framework of iterative learning control under random data dropouts: Mean square and almost sure convergence," *Int. J. Adapt. Control Signal Process.*, vol. 31, no. 12, pp. 1825–1852, 2017.

[29] D. Shen, C. Zhang, and Y. Xu, "Two updating schemes of iterative learning control for networked control systems with random data dropouts," *Inf. Sci.*, vol. 381, pp. 352–370, Dec. 2016.

[30] D. Shen and J.-X. Xu, "A novel Markov chain based ILC analysis for linear stochastic systems under general data dropouts environments," *IEEE Trans. Autom. Control*, vol. 62, no. 11, pp. 5850–5857, Nov. 2017.

[31] L. Huang, H. Ding, Z. Zhang, Z. Zhang, and L. Sun, "A comparison of compensation methods for random input data dropouts in networked iterative learning control system," *Adv. Difference Equ.*, vol. 2019, 2019, Art. no. 68.

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