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Multi-Colony Ant Colony Optimization Based on Generalized Jaccard Similarity Recommendation Strategy

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ABSTRACT Ant Colony Optimization has achieved good results in solving Traveling Salesman Problem (TSP), it has a tendency to fall into local optima and the convergence speed is limited. To address this problem, multi-colony ant colony optimization based on the generalized Jaccard similarity recommendation strategy (JCACO) is proposed. Firstly, two classical ant populations, Ant Colony System and Max-Min Ant System are selected to form heterogeneous multi-colony. Secondly, attribute-based collaborative filtering recommendation mechanism is proposed to balance the diversity and convergence of the algorithm, three strategies have been implemented under this recommendation mechanism: The attribute cross-learning strategy is used to highlight the effect of excellent attributes and improve the attribute comprehensive performance; According to the diversity results of the population measured by information entropy, the attribute recommendation learning strategy is used to enrich the diversity of the population adaptively; The pheromone reward strategy is implemented on the public path to accelerate the convergence speed; Among which, according to the generalized Jaccard similarity coefficient, the most suitable communication object is recommended in order to achieve the best learning efficiency. Finally, when the algorithm stagnates, the elite reverse learning mechanism is used to jump out of the local optimum. Experimental results show that JCACO has good performance and high stability in TSP instances, especially in large-scale TSP instances.

INDEX TERMS Attribute cross-learning, attribute recommendation learning, ant colony optimization, elite reverse learning, generalized Jaccard similarity coefficient, traveling salesman problem.

I. INTRODUCTION

TSP is one of the famous NP-hard problems, which refers to the shortest path problem that a traveler starts from a certain starting point, passes all the given demand points, and each demand point only passes once, finally returns to the starting point. Many algorithms can solve the TSP problem, include Particle Swarm Optimization (PSO) [1], [2], Genetic Algorithm (GA) [3], Simulated Annealing (SA) [4], Ant Colony Optimization (ACO) and so on. Every algorithm has advantages and disadvantages in solving TSP problems, Ant colony optimization is the main algorithm to solve the TSP problem.

Ant colony algorithm starts from ant foraging mechanism and it has the characteristics of positive feedback and strong robustness. Ant colony algorithm has been successfully applied in several fields, the most successful of which is used for combinatorial optimization problems, therefore, the ant colony optimization proposed in this paper adopts the TSP problem for experimental testing. In the future, we will use ant colony optimization to solve the robot path planning and task scheduling problems [5].

Ant Colony Optimization [6], [7] was originally proposed by Italian scholar M. Dorigo in 1996 based on the ant foraging mechanism to solve the traveling salesman and distributed optimization problems. The experimental results show that the algorithm can provide a better solution, but it will fall into the problem of local optimum and slow convergence when solving large scale TSP problem; Then, Dorigo proposed Ant Colony System (ACS) [8], combining local pheromone update with global pheromone update to improve the convergence speed of the algorithm. In 2000, T. Stutzle *et al.*

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proposed the Max-Min Ant System (MMAS) [9], by limiting the range of pheromones, the problem of premature stagnation in the algorithm is improved. The above are classical Ant Colony Optimization, they have efficient search ability and provide valuable experiences to further research, but there are still problems such as easy to fall into local optimum and slow convergence.

In order to balance the diversity and convergence of algorithm, some scholars have made different improvements to ACS. W. Deng *et al.* proposed an improved ant colony optimization algorithm based on multi-population strategy, coevolution mechanism, pheromone updating strategy and pheromone diffusion mechanism, which can improve the optimization performance of solving large-scale optimization problems by balancing the convergence rate and solving diversity [10]. J. Li *et al.* proposed a 2-opt domain search strategy to enhance the ability to build solutions and improve the quality of the solution [11]. L. Zhang *et al.* proposed combining the bacterial foraging algorithm with the ant colony algorithm to improve the slow convergence of the traditional algorithm [12]. X. Meng *et al.* proposed a new direction pheromone to describe the global information in the optimization process, which improved the global solution and accelerated the convergence of the algorithm [13]. The parameters setting of the ant colony optimization (ACO) have a profound impact on the experimental results, many researchers use various methods to optimize the parameters of the ACO [14], [15], they all used the PSO to optimize the parameters in ACO, which reduce the impact of parameter selection on the experiment and enhance the quality of solutions. F. Olivas *et a*l. used fuzzy control theory to make the parameters of ant colony algorithm achieve the dynamic adaptive effect, and the improved ant colony algorithm is applied to robot path planning [16]–[21].

With the improvement of single colony ant colony algorithm, researchers studied multiple ant colony optimization algorithms, hoping to achieve better performance through the collaborative work between multiple ant colonies. The concept of multiple ant colonies was first proposed by Gambardella to solve the vehicle routing problem with time windows [22]. Multiple ant colony optimization are divided into homogenous ant colony optimization and heterogeneous ant colony optimization, Chu S C *et al*. proposed a homogenous multiple colonies ant colony optimization with seven communication methods to update the pheromones based on the best route of all colonies [23]. M. Birattari *et al*. proposed a migration integration strategy for homogeneous ant colony communication [24]. The homogenous ant colony algorithms are relying on the basic ant colony algorithms to enhance the feature of single colony, to some extent, the heterogeneous ant colony algorithms, in which different ant colonies have different behaviors, are more likely to take full advantage of different ACO. M. Xu *et al.* proposed a heterogeneous double colonies ant colony algorithm based on heuristic information, by introducing exchange factors and carrying out information exchange regularly, the convergence and diversity of the algorithm are balanced in large scale problems, but the self-adaptability of the algorithm still needs to be improved [25]. T. Zheng used the adaptive migration rules based on population diversity to propose a parallel multi-group adaptive ant colony algorithm for automatic test case generation [26]. X. He *et al*. proposed a two-population ant colony optimization based on heterogeneous ant colony, which improved the diversity of solutions through heterogeneous evolution and information exchange, due to the exchange frequency is related to the number of iterations, the algorithm mode was relatively fixed [27].

There are three main problems in multi-colony ant colony algorithm: 1. How to communicate among populations? 2. What is exchanged between the populations? 3. When to communicate? According to the references, the optimal solution and pheromone matrix can be exchanged between populations, or the worst solution can be replaced by the optimal solution. P. Zhang *et al.* selected communication objects according to the similarity between populations, and then exchanged the optimal solution and pheromone between populations [28]. In the communication mechanism, different communication strategies can be adopted in different situations, X. Deng *et al*. proposed two neighborhood topologies for exchange between populations [29]. X. Chen proposed random weight, asynchronous change factor and population elimination strategy to increase the communication between populations [30].

The existing multi-colony algorithm balances the diversity and convergence of the algorithm and reflecting the advantages of the multi-colony algorithm, however, the interaction strategy between populations is relatively simple, and the adaptability of the algorithm needs to be improved. To solve these problems, some scholars have introduced the principle of recommendation system into ant colony optimization and adopted interdisciplinary methods to make the direction of the improvement more clear. There are two ways to combine recommendation algorithm with ant colony algorithm. One is to use ant colony algorithm to optimize the recommendation algorithm [31], [32], and the other is to use the recommendation algorithm to optimize the ant colony algorithm [33]. Experiments of both methods verify the effectiveness of the algorithm and optimize the overall performance of the algorithm.

This paper focuses on the diversity and convergence of the balance algorithm and the accuracy of the solution in large-scale TSP problems. Multi-colony ant colony optimization based on the generalized Jaccard similarity recommendation strategy is proposed, the collaborative filtering recommendation algorithm is adopted to optimize ant colony algorithm. The main contributions of this paper are as follows:

1. Multiple subpopulations of ACS and MMAS were selected to form heterogeneous multi-population ant colony optimization to balance the diversity and convergence speed of the algorithm.

2. Attribute-based collaborative filtering recommendation mechanism is proposed to exchange information and learn

between populations, there are three strategies for this mechanism: attribute cross-learning strategy, attribute recommendation learning strategy, public path reward strategy. In the attribute cross-learning strategy, the concept of attribute comprehensive performance of the population was proposed to measure whether a population is excellent or not, excellent populations exchange information through attribute cross-learning strategy, so that they can give full play to the role of excellent attributes and improve the attribute comprehensive performance of the population; In the attribute recommendation learning strategy, information entropy is used to measure diversity of population, when the information entropy of population is lower than the threshold, the fusion of pheromones through attribute recommendation learning strategy to improve the diversity of population; In the public path reward strategy, the public path between the current optimal path of the population with poor convergence and the historical optimal path was found, the pheromone reward is applied to the public path, which makes the algorithm directional and accelerating the convergence speed. These three strategies adjust the communication frequency adaptively according to the dynamic information feedback of the population, to balance between breadth optimization and depth exploration of the algorithm.

3. The attribute cross-learning strategy and attribute recommendation learning strategy mentioned in contribution 2 need appropriate communication objects when they are executed, the generalized Jaccard similarity coefficient is constructed to measure the similarity between populations, and the most suitable communication objects are recommended for the above strategies, so as to achieve the best learning effect between populations.

4. The strategy in contribution 2 may cause excessive accumulation of pheromones, which may cause the algorithm to stagnates, to avoid this situation, the elite reverse learning mechanism is proposed to jump out of the local optimum and obtain a more accurate solution.

This paper is organized as follows: Section II briefly introduces the ACS, MMAS, generalized Jaccard similarity coefficients and information entropy. Section III introduces the relevant content of the JCACO including the constructing the generalized Jaccard similarity coefficient, attribute-based collaborative filtering recommendation mechanism and elite reverse learning mechanism. Section IV verifies the effectiveness of the relevant strategies proposed in this paper through experiments, and compares JCACO with traditional ant colony algorithm and other optimization algorithms. Section V includes summary and future work.

II. RELATED WORK

A. ANT COLONY SYSTEM WITH TSP

In the early 1990s, scientists discovered through experiments that when ants are foraging, they will leave a similar chemical substance—pheromone on the path they walk through. The ants will choose the next path based on the pheromone concentration, the higher the concentration, the higher the probability of being selected, at the same time due to the volatile nature of pheromones, the pheromone on the poor path will be less and less, thus the optimal path can be selected.

1) CONSTRUCT THE SOLUTION

The selection formula for each ant in the ACS from city i to city j:

$$J = \begin{cases} \arg\max \left\{ \tau_{ij} \left[\eta_{ij} \right]^{\beta} \right\}, & q \le q_0 \\ s, & q > q_0 \end{cases} \tag{1}$$

where η_{ij} represents the reciprocal of the distance between city i and city j, τ_{ij} represents pheromone concentration between city i and city j, q_0 is a parameter which is a consistent value. q is a random variable subjected to a uniform distribution between 0 to 1. J is the next city to be selected. s is equal to Eq.(2). The Eq. (1) shows that cities with high pheromone or with relatively close distances are more likely to be selected. When $q \le q_0$, use Eq.(1); otherwise, use Eq. (2).

$$p_{ij}^{k} = \begin{cases} \dfrac{\left[\tau_{ij}\right]^{\alpha} \left[\eta_{ij}\right]^{\beta}}{\sum\limits_{l \in N_i^k} \left[\tau_{il}\right]^{\alpha} \left[\eta_{il}\right]^{\beta}}, & j \in N_i^k \\ 0, & j \notin N_i^k \end{cases} \tag{2}$$

where α is information heuristic factor; β is expectation heuristic facto; N_i^k is a collection of cities that ants can reach directly and have not visited yet. η_{ij} is heuristic function, its expression is formula (3).

$$\eta_{ij} = \frac{1}{d_{ij}} \tag{3}$$

2) PHEROMONE UPDATE

Local pheromone update rule: when the ant carries out path construction, it moves from the current city i to the next city j, and immediately updates the pheromone on the path, which can be expressed as equation (4).

$$\tau_{ij} \leftarrow (1-\rho)\, \tau_{ij} + \rho \tau_0 \tag{4}$$

where ρ is local pheromone evaporation coefficient whose rang is [0, 1]; τ_0 is pheromone initial value.

Global pheromone update rule: After all ants completed their tour, only the global optimal path can update the pheromone, which accelerates the convergence of the algorithm, and its expression is formula (5).

$$\tau_{ij} \leftarrow (1-\xi)\, \tau_{ij} + \xi \Delta \tau_{ij}^{bs} \tag{5}$$

$$\Delta \tau_{ij}^{bs} = \frac{1}{C^{bs}} \tag{6}$$

where ξ is global pheromone evaporation coefficient, C^{bs} is the length of the global optimal path; $\Delta \tau_{ij}^{bs}$ is the pheromone added to the global optimal path, its expression is formula (6).

B. MAX-MIN ANT SYSTEM

In order to avoid fast convergence and stagnation of the algorithm, the MMAS algorithm limits the size of pheromones: $[\tau_{\min}, \tau_{\max}]$. If $\tau_{ij} \leq \tau_{\min}$, we set $\tau_{ij} = \tau_{\min}$; If $\tau_{ij} \geq \tau_{\max}$, we set $\tau_{ij} = \tau_{\max}$. The value of $\tau_{\max}$ and $\tau_{\min}$ are in formula (7) and formula (8).

$$\tau_{\max} = (1/\rho) * (1/T^{gb}) \tag{7}$$

$$\tau_{\min} = \tau_{\max}/2n \tag{8}$$

where T^{gb} is global optimal path.

1) PHEROMONE UPDATE

In MMAS, only the pheromone on the best tour can be update in each iteration. The pheromone update rules are as shown in formula (9) and formula (10).

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}^{best} \tag{9}$$

$$\Delta\tau_{ij}^{best} = 1/f(s^{best}) \tag{10}$$

where $f(s^{best})$ is the best tour.

C. GENERALIZED JACCARD SIMILARITY COEFFICIENT

The Jaccard similarity coefficient is used to compare similarities and differences between finite sample sets. The larger the Jaccard coefficient value, the higher the sample similarity. The generalized Jaccard coefficient is an extension of the Jaccard coefficient, also known as the Tanimoto coefficient. Its expression is (11).

$$EJ(A,B) = \frac{A \cdot B}{\|A\|^2 + \|B\|^2 - A \cdot B} \tag{11}$$

where A and B are two n-dimensional vectors. $A \cdot B$ is vector product, $\|A\|^2$ is vector norm: $\|A\|^2 = \sqrt{\sum_{i=1}^{n} a_i^2}$.

D. INFORMATION ENTROPY

Information Entropy is a word borrowed from thermodynamics by C. E. Shannon in 1948 to solve the problem of quantitative measurement of information. It's also one of several ways to measure diversity. Entropy can be written explicitly:

$$H(X) = -\sum_{x \in X} P(x) lb(P(x)) \tag{12}$$

where b is the base of the logarithm. $P(x)$ is the probability mass function.

The entropy of the unknown result is maximized if each probability is fair. Therefore, this paper uses information entropy to measure the diversity of population [34], [35].

III. MULTI-COLONY ANT COLONY OPTIMIZATION BASED ON GENERALIZED JACCARD SIMILARITY RECOMMENDATION STRATEGY

The recommendation algorithm is a computer science algorithm that uses some user behavior to guess what the user might like through some mathematical algorithms. There are six main types of recommendation algorithms: content-based, collaborative filtering, association rules, utility-based, knowledge-based and combination recommendation. The recommendation algorithm based on collaborative filtering is the most commonly used and the effect is better. Therefore, this paper chooses a collaborative filtering recommendation algorithm.

A. ATTRIBUTE-BASED COLLABORATIVE FILTERING RECOMMENDATION MECHANISM

1) SIMILARITY MEASURE

The core of the recommendation algorithm is the measure of similarity of the use. Traditional ant colony optimization has poor diversity and slow convergence. Therefore, this paper uses the diversity factor and convergence factor of the population to form a 2-dimensional vector and uses the generalized Jaccard similarity coefficient to measure the similarity between populations. In this paper, we use information entropy to measure the diversity of populations and use formula (13) to measure the convergence of populations.

$$Con_i = \frac{iter_G}{iter_i} \tag{13}$$

where Con_i is the convergence of the population i, $iter_G$ is the iteration number of optimal convergence in history, $iter_i$ is the iteration number of current optimal convergence in population i.

$$JE(A,B) = \frac{A \cdot B}{\|A\|^2 + \|B\|^2 - A \cdot B} = \frac{a_1 b_1 + a_2 b_2}{\sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2} - (a_1 b_1 + a_2 b_2)} \tag{14}$$

We use formula (14) to measure the similarity between population A and B. Where $JE(A,B)$ is the similarity between population A and B, $A(a_1, a_2)$ is the vector describing population A, a_1 is the diversity of population A and the value of a_1 is ratio of current information entropy of population to maximum information entropy, a_2 is the convergence of population and the value of a_2 is obtained by Equation(13). The larger the value of $JE(A,B)$, the more similar the population A and B are.

2) ATTRIBUTE CROSS-LEARNING STRATEGY

After one iteration, the similarity matrix of each pair of populations is calculated according to the Equation (4), and the population with the best attribute comprehensive performance is found, the attribute comprehensive performance of the population is measured using the formula (15). Every M generation, the population with the best attribute comprehensive performance is communicated with its most similar population. There are two reasons: One is due to the positive feedback of pheromones, when the two population information exchanges too frequently, too many pheromones will cause both populations to fall into local optimum, so it is necessary to control the exchange frequency of the population. The other is according to formula (15), the attribute comprehensive performance of the population includes diversity, convergence, and quality of the solution, the population with the best attribute comprehensive performance does not necessarily have all the excellent attributes, therefore, choosing the most similar population for communication can help each other learn excellent attributes(As shown in FIGURE 1, the red box is the excellent attributes of the two population), play the role of excellent attributes, and improve the attribute comprehensive performance of excellent populations.

$$Per_i = Div_i \cdot Sol_i \cdot Con_i \tag{15}$$

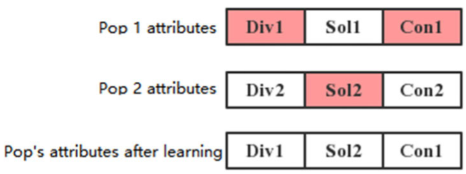


FIGURE 1. Attribute cross-learning strategy.

Formula (15) is the measurement of attribute comprehensive performance, where Per_i is the attribute comprehensive performance of the population i, Div_i is the ratio of the current information entropy of the population i to the global maximum information entropy, it reflects the diversity of the population; and Sol_i is the ratio of the standard optimal solution of the population i to the current optimal solution, it reflects the accuracy of the solution; Con_i from formula (13) and it reflects the convergence of the population.

$$Similarty_{iter} = \begin{bmatrix} JE_{1,1} & JE_{1,2} & \cdots & JE_{1,s1+s2} \\ JE_{2,1} & JE_{2,2} & \cdots & JE_{2,s1+s2} \\ \cdots & \cdots & \cdots & \cdots \\ JE_{s1+s2,1} & JE_{s1+s2,2} & \cdots & JE_{s1+s2,s1+s2} \end{bmatrix} \tag{16}$$

Formula (16) is the similarity matrix between the populations, where $JE_{i,j}$ is the similarity between population i and population j, if $i = j$, $JE_{i,j} = 0$; if $i \neq j$, $JE_{i,j}$ calculated according to formula (14), $s1$ is the number of ACS subpopulation, $s2$ is the number of MMAS subpopulation.

3) ATTRIBUTE RECOMMENDATION LEARNING STRATEGY

When the information entropy of a population is below the threshold, that is, the diversity of the population is too low, then the attribute recommendation learning strategy is executed. Firstly, k populations with a higher similarity with this population are filtered out according to formula (16). Secondly, the population with the highest information entropy of the k populations is recommended for learning. The reasons are as follows: One is, if we choose the best diversity subpopulation directly from the whole population for learning, the attribute comprehensive performance of these two populations may differ greatly, learning between them will lose the better solutions that have been found, which will cause the rate of convergence to slow down. The other is, the low diversity of the population will reduce the search performance of the algorithm, choosing the population with the highest information entropy among the k populations for learning will improve the diversity of the population and increase the search breadth of the algorithm. The learning rule is the fusion of pheromones. See formula (17).

$$P_{1\_new} = \frac{1}{2}(P_1 + P_2) \tag{17}$$

where P_{1_new} is a new pheromone matrix of the population whose information entropy is below the threshold, P_1 is the original pheromone matrix of the population whose information entropy is below the threshold, and p_2 is the pheromone matrix of the population whose communicating with the population P_1. The pheromone of the two populations is fused by formula (17), so that the excellent performance in the original population is preserved and the excellent performance of the exchange population is obtained.

4) PUBLIC PATH REWARD STRATEGY

This algorithm not only needs to maintain the diversity of the population, but also needs to improve the convergence of the algorithm. Here, the convergence of a population is measured by formula (13). When the convergence of the population is lower than the threshold, which is $Con < \omega$(ω is convergence threshol), to improve the convergence speed, the public path between the current optimal path of the population with poor convergence and the historical optimal path was found, and reward certain pheromone to the public path. The definition of public path is shown in FIGURE 2, if three or more nodes are identical in succession, they are regarded aspublic path. The reward rule is formula (18).

$$P_{new} = \left(1 + \frac{1}{n}e^{-iter}\right) P \tag{18}$$

where P is the original pheromone matrix of the public path, P_{new} is pheromone matrix of the public path after rewar, n is the number of cities. Pheromone rewards related to the number of iterations. In the early stage of the algorithm, a path is selected by both populations and is part of the current optimal path, it can be considered that there is a component of the optimal solution around the path or the path, reward this part with pheromones, this will make the algorithm more directional and speed up the convergence. To prevent the excessive accumulation of pheromone, then lead the algorithm to fall into local optimum at the later stage, we use formula (18) for rewards. As the number of iterations increases, fewer pheromones are rewarded.

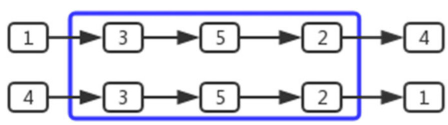


FIGURE 2. Public path.

B. ELITE REVERSE LEARNING MECHANISM

The traditional ant colony optimization algorithm often encounters the problem of local optimization in the later stage. When the population falls into local optimization, the appropriate learning object needs to be selected to jump out of local optimization. Here the selection of learning objects is the key. There are generally two options, option 1: the general population was randomly selected for learning; Option 2: select the population with the current optimal solution for learning. Although option 1 can maintain diversity, the optimal solution of the subpopulation may deviate from the standard optimal solution, leading to a decrease in convergence rate and solution accuracy. Although option 2 is the closest to the standard optimal solution, if the selected population also falls into the local optimal solution, the diversity and learning efficiency of the population after learning will be reduced. To balance the advantages and disadvantages of these two options, this paper proposes the concept of the Elite Mixed Knowledge Board (EMKB), which is used to store the elite status of the population, when the algorithm is stagnant, the population performs the elite reverse learning mechanism to achieve the goal of jumping out of the local optimum. The specific operation is as follows:

Step1: First build an EMKB to store elite status, EMKB structure as shown in TABLE 1.

TABLE 1. Elite mixed knowledge board.

Length of elite path	Information entropy	Pheromone matrix
L_1	E_1	P_1
....	....	....
L_{s1+s2}	E_{s1+s2}	P_{s1+s2}

Step2: Each subpopulation generates a current elite state with each iteration, the length of the elite path, information entropy, and pheromone matrix are stored in EMKB.

Step3: EMKB updates itself as the number of iterations changes. The update rules are: If the length of the elite path of the current iteration is better than the length of the worst elite path in the EMKB, the replacement is then performed, including the length of the current elite path, the information entropy, and the pheromone matrix.

Step4: When a subpopulation falls into local optimum, adds the length of the elite path obtained by the current population to EMKB and sort by the length of the elite path from small to large, if the length of this elite path is sorted, the sequence number is x, then its reverse sequence number is x^*, the calculation method is formula (19).

$$x^* = x_{\min} + x_{\max} - x \tag{19}$$

where $x_{\min}$ is the minimum sequence number of the elite path length, and $x_{\max}$ is the maximum sequence number of the elite path length. The reverse learning area of the population is $(\min(x, x^*), \max(x, x^*))$.

Step5: Select the elite state with the highest information entropy in the learning area to learn, learning the length of the current elite path, information entropy, and pheromone matrix.

C. ALGORITHM FRAMEWORK

The following is the execution process of JCACO:

Step1: Initialize parameters and pheromone matrices that appear in JCACO, calculate the distance between cities;

Step2: Iteration starts from here, path construction of each subpopulation of ACS and MMAS according to Eq. (1) and Eq. (2), local pheromone update of subpopulations according to Eq. (4), note the limitations of MMAS pheromone;

Step3: When all subpopulations complete an iteration, updating the global pheromone of each subpopulation according to Eq. (5) and Eq. (9), preserve the current optimal solution of the algorithm at this time, if the current optimal solution is better than the historical optimal solution, it is retained after replacing the historical optimal solution, otherwise, it is not replaced;

Step4: Calculate the information entropy of subpopulations according to Eq. (12), calculate the similarity between populations according to Eq. (14), measure the convergence of subpopulations according to Eq. (13);

Step5: Every M iterations, calculate the attributes comprehensive performance of the population according to Eq. (15), according to the similarity between populations, recommend appropriate populations and population with the highest attribute comprehensive performance execution attribute cross-learning strategy;

Step6: When the information entropy of the subpopulation is below the threshold, according to the similarity matrix, the most suitable population and the population with low entropy were selected to implement attribute recommendation learning strategy;

Step7: When the convergence of the subpopulation is below the threshold, the public path between the current optimal path of this subpopulation and the historical optimal path was found, the pheromone reward strategy is implemented on the public path;

Step8: If the current optimal path length of a subpopulation has not changed continuously for T iterations, then we think that the subpopulation may fall into local optimum, at this moment, the elite reverse learning mechanism is executed by this subpopulation;

Step9: The number of iterations increases, back to Step2;

Step10: When the maximum number of iterations is reached, the global optimal solution of the algorithm is output.

In this framework, the number of iteration is nc, the number of subpopulation ant is m, the number of city is n, all subpopulations run in parallel in the computer.

Through the analysis of the algorithm framework, we know that the time complexity of JCACO is $O(nc * m * (n-1))$, and the maximum time complexity is $O(nc * m * n)$. As we known, the maximum time complexity of ACS and MMAS

Algorithm 1 JCACO Algorithm for TSP

```
Initialize the pheromone and the parameters
Calculate the distance between cities
While termination condition is not satisfied do
    Construct ant solutions for ACS, MMAS with Eq.(1), Eq.(2)
    Update pheromone for MMAS, ACS with Eq.(4), Eq.(5), Eq.(9)
    Calculate information entropy with Eq.(12)
    Calculate Generalized Jaccard coefficient with Eq.(14)
    If nc % M ==0 then
        Executive attribute cross-learning strategy
    End-If
    If subpopulation information entropy below the threshold then
        Executive attribute recommendation learning strategy
    End-If
    If subpopulation convergence is below the threshold then
        Execute public path reward strategy
    End-If
    If subpopulations fall into local optimum then
        Executive elite reverse learning mechanism
    End-If
    nc = nc+1
End-While
```

is $O(nc * m * n)$. Therefore, the maximum time complexity of JCACO is the same as ACS and MMAS, indicating that this algorithm does not require additional time consumption.

IV. EXPERIMENT AND SIMULATION

A. SIMULATION ENVIRONMENT AND PARAMETER SETTINGS

The experiment was simulated in MATLAB R2016a environment in Windows 10. To verify the performance of JCACO, we selected TSP instances of various scales for experiments and compared them with ACS and MMAS. To enable JCACO to have better performance, three levels and four factors orthogonal experiments were used to determine the value of each parameter and the levels are based on pre-experiments, the optimum scheme of each factor is found out by orthogonal experiment. Each combination of the various parameters was tested 10 times, the average value was taken for analysis, take eil51 as an example to determine the parameter value (the internal operations of the ACS and MMAS subpopulations have not been changed, therefore, the parameters of ACS and MMAS can be determined separately to achieve the best effect of each).

Based on the above experiments, it can be known that: in MMAS, the best scheme of parameters is that α is equal to 1, β is equal to 3, and ρ is equal to 0.1; in ACS, the best scheme of parameters is that α is equal to 1, β is equal to 4, ρ is equal to 0.1, and ξ is equal to 0.2.

TABLE 2. Experimental factors and levels of MMAS.

	α	β	ρ
Level 1	1	1	0.1
Level 2	2	2	0.2
Level 3	3	3	0.3

Note: ρ is pheromone updating parameter of MMAS algorithm.

TABLE 3. Orthogonal test scheme and test results of MMAS.

no.	α	β	ρ	*results*
1	1	1	0.1	436.2
2	1	2	0.2	433.8
3	1	3	0.3	428.9
4	2	1	0.3	439.8
5	2	2	0.1	435.2
6	2	3	0.2	433.5
7	3	1	0.2	439.3
8	3	2	0.3	436.4
9	3	3	0.1	433.2

Note: The *no.* is the number of the test, and the *results* is the average value after 10 tests in each group.

TABLE 4. Analysis of test results of MMAS.

T	α	β	ρ
T_1	**1298.9**	1315.3	**1304.6**
T_2	1308.5	1305.4	1306.6
T_3	1308.9	**1295.6**	1305.1
t_1	**432.97**	438.43	**434.87**
t_2	436.17	435.13	435.53
t_3	436.30	**431.87**	435.03
max	436.30	438.43	435.53
min	432.97	431.87	434.87
range	3.33	6.56	0.66
scheme	**Level 1**	**Level 3**	**Level 1**

Note: T_i ($i=1,2,3$) are the sum of results. t_i ($i=1,2,3$) are the means of every level. *range* is the difference by the max minus the min, which will be applied to determine which one factor is important, and larger *range* is generally more important. And *scheme* is the project of every factor by orthogonal test to obtain the best result.

TABLE 5. Experimental factors and levels of ACS.

	Level 1	Level 2	Level 3
α	1	2	3
β	2	3	4
ρ	0.1	0.2	0.3
ξ	0.2	0.3	0.4

B. EXPERIMENT ANALYSIS

1) STRATEGY TESTING AND PERFORMANCE ANALYSIS

There are three strategies for the attribute-based collaborative filtering mechanism proposed in this paper: attribute cross-learning strategy, attribute recommendation learning strategy,

TABLE 6. Orthogonal test scheme and test results of ACS.

no.	α	β	ρ	ξ	results
1	1	2	0.1	0.2	429.9
2	1	3	0.2	0.3	428.9
3	1	4	0.3	0.4	427.9
4	2	2	0.2	0.4	430.6
5	2	3	0.3	0.2	428.5
6	2	4	0.1	0.3	427.7
7	3	2	0.3	0.3	431.5
8	3	3	0.1	0.4	429.5
9	3	4	0.2	0.2	427.9

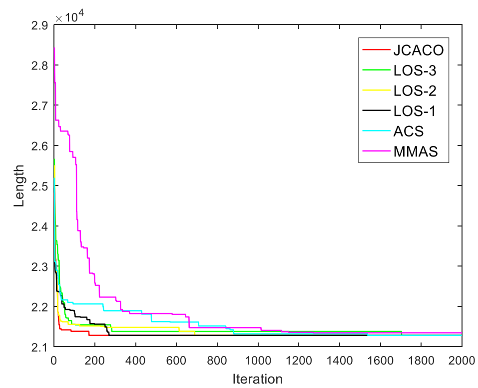
Note: ρ is local pheromone updating parameter of ACS algorithm, and ξ is global pheromone updating parameter of ACS algorithm.

TABLE 7. Analysis of test results of ACS.

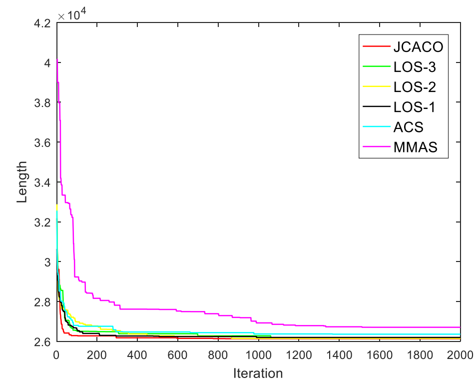
T	α	β	ρ	ξ
T_1	**1286.7**	1292.0	**1287.1**	**1286.3**
T_2	1286.8	1286.9	1287.4	1288.1
T_3	1288.9	**1283.5**	1287.9	1288.0
t_1	**428.90**	430.67	**429.03**	**428.77**
t_2	428.93	428.97	429.13	429.37
t_3	429.63	**427.83**	429.30	429.33
max	429.63	430.67	429.30	429.37
min	428.90	427.83	429.03	428.77
range	0.73	2.84	0.27	0.60
scheme	Level 1	Level 3	Level 1	Level 1

public path reward strategy. To verify the validity of the three strategies, we selected kroA100, kroB150 for the experiment, the selected TSP instances was tested 15 times, and each experiment was performed 2000 iterations. This experiment was analyzed from the following aspects: the optimal solution (Best), the worst solution (Worst), the average solution (Mean), iteration number of optimal solution (Convergence). Experimental data are shown in TABLE 8. Wherein, LOS-3 is an algorithm that has an attribute cross-learning strategy and an attribute recommendation learning strategy but lacks a public path reward strategy. LOS-2 is an algorithm that has an attribute cross-learning strategy and a public path reward strategy but lacks an attribute recommendation learning strategy. LOS-1 is an algorithm that has attribute recommendation learning strategy and publi cpath reward strategy but lacks anattribute cross-learning strategy.

Firstly, the optimization effects of the three strategies are analyzed. The comparison groups were ACS and MMAS, and the experimental group were JCACO, LOS-3, LOS-2 and LOS-1. As can be seen from TABLE 8, compared with the ACS and MMAS algorithms, the optimization effect was achieved in all four experimental groups. All the four improved algorithms and ACS can find the optimal solution in kroA100, while MMAS cannot find the optimal solution, only JCACO can find the optimal solution in kroB150. In TABLE 8, the quality of the worst solution and average solution of the four improved algorithms are better than ACS and MMAS. Therefore, the four improved algorithms formed by the three strategies have better stability and higher solution accuracy. It can be seen from FIGURE 3 that the convergence speed of JCACO, LOS-3, LOS-2 and LOS-1 algorithms is faster than ACS and MMAS.



(a) kroA100



(b) kroB150

FIGURE 3. Comparison of convergence rates of different algorithms.

Then, the respective effects of the three strategies were analyzed. The comparison group was JCACO, and the experimental group were LOS-3, LOS-2, and LOS-1. As can be seen from TABLE 8, JCACO found the optimal solution in both the kroA100 and kroB150, the optimal solution, the worst solution and the average solution of the LOS-1 and LOS-2 algorithms are worse than the other two algorithms, therefore, attribute cross-learning strategy and attribute recommendation learning strategy focus on improving the accuracy of the solution and the stability of the algorithm. It can be seen from the convergence columns in TABLE 8 and FIGURE 3 that compared with the other three algorithms, the convergence rate of the LOS-3 algorithm is the slowest. Therefore, the public path reward strategy focuses on improving the convergence speed of the algorithm.

TABLE 8. Performance analysis of algorithms composed of different strategies.

TSP Instances	algorithm	Best	Worst	Mean	Convergence
kroA100	JCACO	**21282**	21443	**21322**	**171**
	LOS-3	**21282**	**21389**	21323	1704
	LOS-2	**21282**	21440	21350	690
	LOS-1	**21282**	21470	21341	269
	ACS	**21282**	21926	21433	1538
	MMAS	21346	22075	21652	1272
kroB150	JCACO	**26130**	**26389**	**26233**	**836**
	LOS-3	26178	26655	26332	1061
	LOS-2	26146	26835	26421	867
	LOS-1	26194	26605	26431	990
	ACS	26358	26941	26498	923
	MMAS	26704	27887	27107	1559

Finally, these three strategies have different improvements to the algorithm, and after the composite use, the accuracy, convergence speed and stability of the solution are improved. They complement each other so that the improvement effects of each strategy play a better role in the combination of each other.

2) COMPARATIVE ANALYSIS OF JCACO AND TRADITIONAL ANT COLONY ALGORITHM

In order to compare the performance of ACS, MMAS with JCACO, this paper selects 18 TSP instances of different scales for experiments. This experiment was analyzed from the following aspects: the optimal solution (Best), the worst solution (Worst), the average solution (Mean), iteration number of optimal solution (Convergence), minimum error rat (Er) and Standard deviation (dev). Experimental data are shown in TABLE 9. The minimum error rate is expressed by the formula (20).

$$Er = \frac{L_{ACO} - L_{\min}}{L_{\min}} \times 100\% \tag{20}$$

where L_{ACO} is the optimal solution found for the algorithm, $L_{\min}$ is the standard optimal solution for the TSP instances.

$$dev = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(l_i - l_{avg})^2} \tag{21}$$

Eq. (21) is the standard deviation, where N is the number of times each TSP instance is tested (in this paper $N = 15$), l_i is the current optimal solution for each experiment, l_{avg} is the average solution of N experiments.

As can be seen from TABLE 9: The JCACO is superior to ACS and MMAS in the selected TSP Instances, whether it is the optimal solution, the worst solution, the average solution, and the error rate. In small scale TSP instances such as eil51, eil76, rat99, kroA100, and kroB100, JCACO can quickly find the standard optimal solution, due to the information exchange strategy among the populations and the public path reward strategy, JCACO has the fastest convergence rate and the highest solution accuracy compared with ACS and MMAS; In medium scale TSP instances such as ch130, kroA150, kroB150, ch150, kroA200, kroB200, the JCACO obtains the standard optimal solution in the kroB150 and ch150, although other instances do not achieve the standard optimal solution, the error rate remains within 1%. The elite reverse learning mechanism enables JCACO to jump out of the local optimal, but ACS and MMAS are easily trapped in the local optimal, for example, ACS stagnates in the 595 and 299 generations of ch150 and kroA200, respectively; In large scale TSP instances such as tsp225, a280, rand300, lin318 and fl417, it is difficult to find the optimal solution due to the large size of the city, although JCACO did not find the standard optimal solution, it still converges slightly faster than ACS and MMAS, the error rate of the optimal solution remains within 1%, and the solution accuracy is higher than ACS and MMAS.

In short: The JCACO algorithm improves the accuracy of the solutions, speeds up the convergence, and can jump out of the local optimum. The search ability greatly exceeds ACS and MMAS.

FIGURE 4 shows the convergence changes of JCACO, ACS and MMAS on 18 TSP instances. It can be seen from the figure that the JCACO algorithm retains a faster convergence rate than ACS and MMAS at the early stage, and makes the solution converge to the optimal solution nearby, no matter it is small TSP instance, medium TSP instance or large TSP instance. ACS and MMAS algorithms tend to fall into local optimum in the later stage, this algorithm adopts an elite reverse learning mechanism, which enables the algorithm to jump out of local optimum and improve the accuracy of the solutions.

This paper uses the standard deviation (Eq. (21)) to reflect the stability of the algorithm, FIGURE 5 shows the standard deviation of 18 different TSP instances participating in the experiment (15 tests per TSP instance). As can be seen from

TABLE 9. Performance comparison of JCACO, ACS, MMAS in different TSP instances.

TSP Instances	Opt	algorithm	Best	Worst	Mean	Er	dev	Convergence
eil51	426	JCACO	**426**	**428**	**427**	**0.00**	**1.067**	**279**
		ACS	**426**	435	428	**0.00**	6.867	1092
		MMAS	427	441	432	0.23	8.400	352
eil76	538	JCACO	**538**	**542**	**539**	**0.00**	**2.400**	**174**
		ACS	**538**	553	544	**0.00**	9.067	1528
		MMAS	543	574	555	0.93	18.590	437
rat99	1211	JCACO	**1211**	**1223**	**1214**	**0.00**	9.067	**80**
		ACS	1213	1228	1219	0.16	**8.800**	1987
		MMAS	1230	1275	1253	1.57	21.600	1193
kroA100	21282	JCACO	**21282**	**21443**	**21322**	**0.00**	**120.067**	**171**
		ACS	**21282**	21926	21433	**0.00**	492.333	1538
		MMAS	21346	22075	21652	0.30	422.533	1272
kroB100	22141	JCACO	**22141**	**22295**	**22248**	**0.00**	**46.467**	**461**
		ACS	22246	22358	22311	0.47	46.933	536
		MMAS	22274	22962	22648	0.60	313.467	1945
ch130	6110	JCACO	**6129**	**6197**	**6172**	**0.31**	**24.267**	**1040**
		ACS	6146	6370	6220	0.59	149.533	1096
		MMAS	6189	6367	6284	1.29	82.467	1387
kroA150	26524	JCACO	**26621**	**27074**	**26845**	**0.36**	**228.600**	1595
		ACS	26664	27447	27108	0.53	338.067	1434
		MMAS	26856	27906	27536	1.25	369.467	**880**
kroB150	26130	JCACO	**26130**	**26389**	**26233**	**0.00**	**155.600**	**836**
		ACS	26358	26941	26498	0.87	418.500	923
		MMAS	26704	27887	27107	2.20	780.067	1559
ch150	6528	JCACO	**6528**	**6584**	**6558**	**0.00**	**25.133**	1792
		ACS	6553	6671	6596	0.38	74.600	**595**
		MMAS	6616	6831	6727	1.35	103.333	1247
kroA200	29368	JCACO	**29406**	**29635**	**29542**	**0.12**	**92.200**	1821
		ACS	29486	29926	29604	0.40	521.267	**299**
		MMAS	30232	31532	30650	2.94	881.400	1541
kroB200	29437	JCACO	**29525**	**30270**	**29982**	**0.29**	**287.800**	1807
		ACS	29819	30888	30194	1.29	693.200	**1114**
		MMAS	30423	31514	31073	3.35	440.867	1708
tsp225	3916	JCACO	**3935**	**4048**	**4007**	**0.48**	**41.067**	**1711**
		ACS	3944	4145	4023	0.72	121.733	1880
		MMAS	4104	4298	4192	4.80	105.800	1986
a280	2579	JCACO	**2590**	**2681**	**2628**	**0.42**	**52.267**	**712**
		ACS	2605	2717	2642	1.00	74.400	1885
		MMAS	2722	2911	2784	5.54	126.867	1987
rand300	11865	JCACO	**11976**	**12377**	**12082**	**0.93**	294.400	**1626**
		ACS	12022	12387	12139	1.32	**247.667**	1721
		MMAS	12346	12932	12637	4.05	294.667	1985
lin318	42029	JCACO	**42399**	**43548**	**43163**	**0.88**	**384.800**	1858
		ACS	43203	44391	43626	2.79	764.800	**1583**
		MMAS	44794	46745	45285	6.57	528.067	1881
fl417	11861	JCACO	**11969**	**12431**	**12216**	**0.91**	**165.200**	**1353**
		ACS	12193	12584	12330	2.80	254.267	1989
		MMAS	12664	13987	13116	6.77	257.733	1890
pr439	107217	JCACO	**108375**	**112994**	110700	**1.08**	**2292.100**	**1793**
		ACS	109037	115188	**110650**	1.70	4540.900	1906
		MMAS	117104	129770	122830	9.22	6935.300	1808
att532	86729	JCACO	**88895**	93723	**90842**	**2.49**	**1382.200**	**1881**
		ACS	89652	**92531**	91044	3.37	1487.300	1986
		MMAS	93211	98259	95398	7.47	2861.000	1878

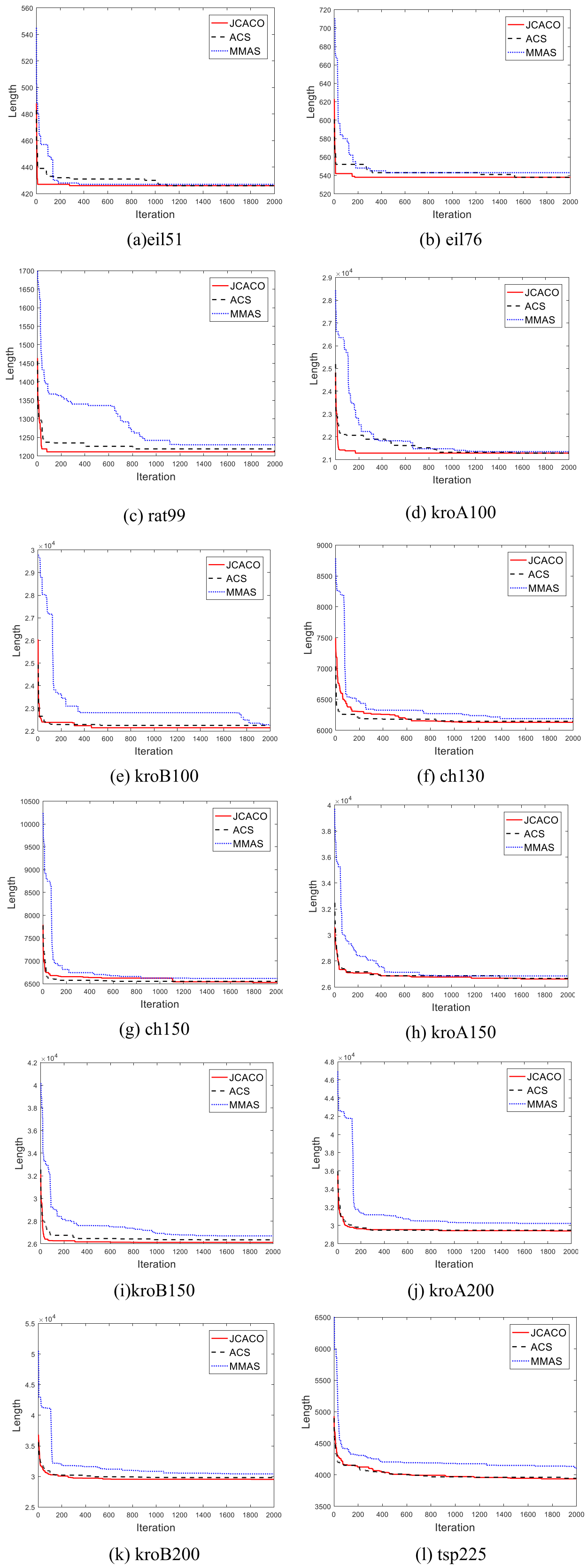


FIGURE 4. Comparison of convergence rates of different algorithms.

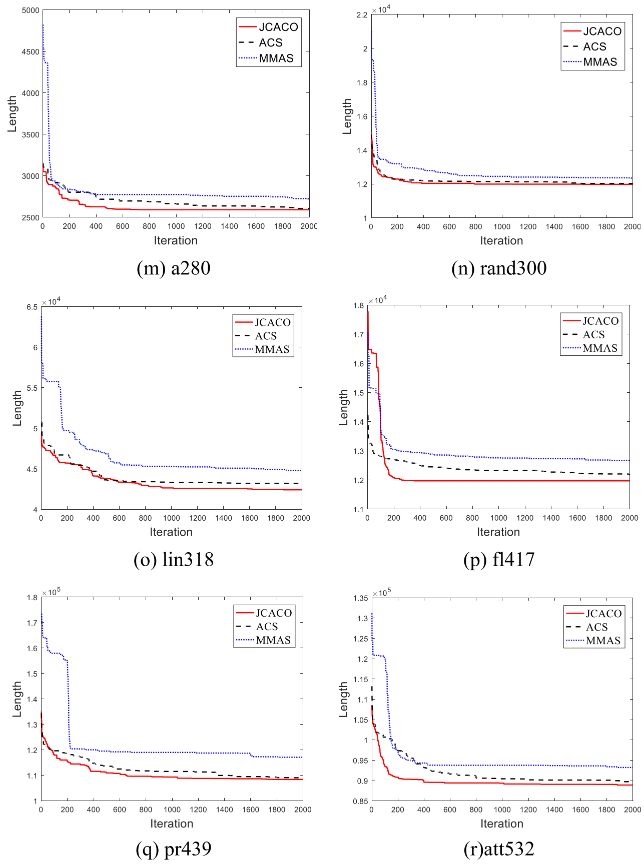


FIGURE 4. ***(Continued.)*** **Comparison of convergence rates of different algorithms.**

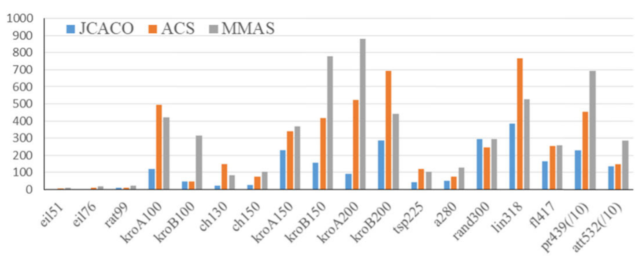


FIGURE 5. Comparison of stability of different algorithms.

FIGURE 5 and the *dev* column in TABLE 9, the standard deviation of JCACO is mostly lower than ACS and MMAS, this shows that the stability of the JCACO is better than ACS and MMAs.

FIGURE 6 shows the optimal solution path found by the JCACO algorithm for 18 TSP instances of different sizes participating in the experiment.

3) COMPARATIVE ANALYSIS OF JCACO AND OTHER OPTIMIZATION ALGORITHMS

The improved algorithm JCACO in this paper is also compared with other optimization algorithms to verify its

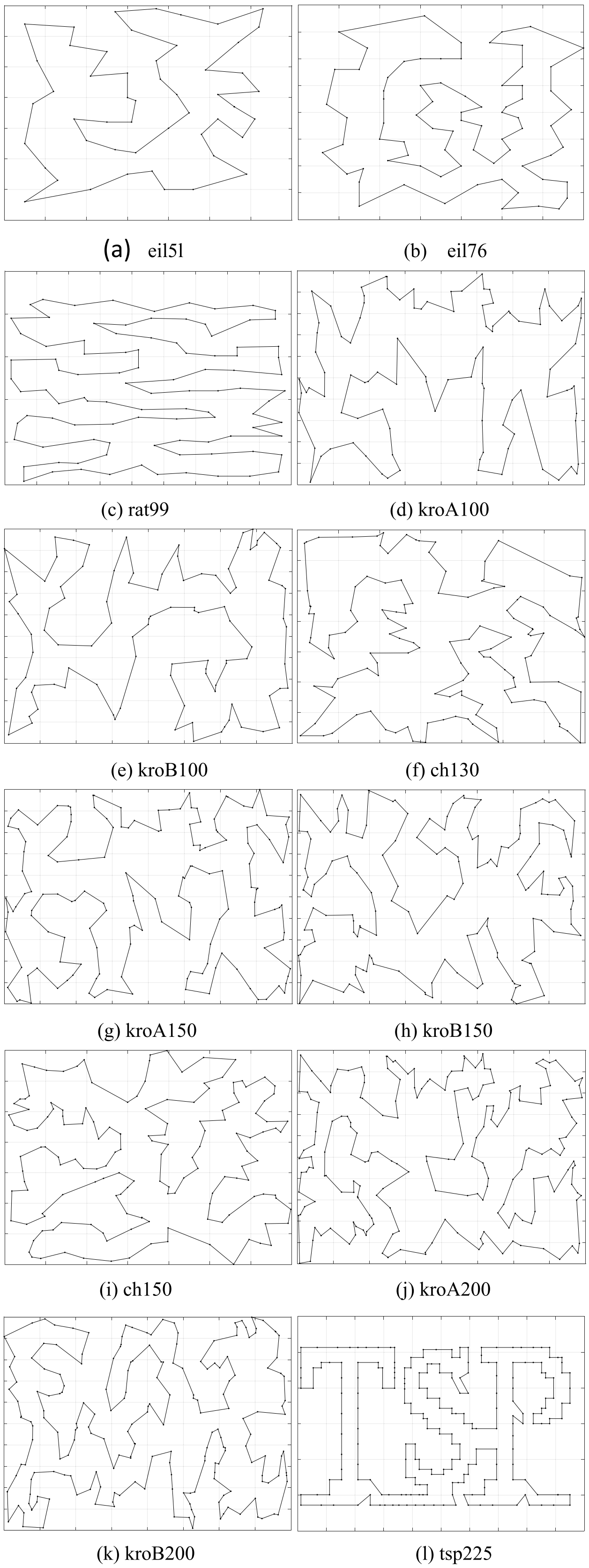


FIGURE 6. **Optimal solution path for each TSP instance found by JCACO.**

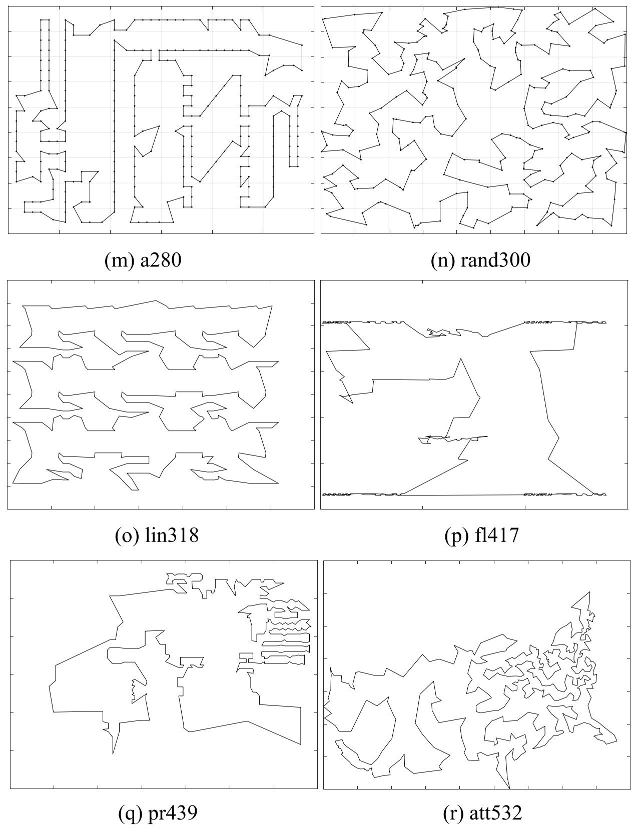


FIGURE 6. ***(Continued.)*** **Optimal solution path for each TSP instance found by JCACO.**

performance, as shown in TABLE 10, including improved multi-colony Ant Colony Optimization [15], [36], [37], Single colony Ant Colony Optimization [3], Particle Swarm Optimization [40], Genetic Algorithm and Artificial Bee Colony Algorithm [41]. The PCCACO algorithm data comes from [36], EDHACO algorithm data comes from [37], PACO-3opt algorithm data comes from [15], D-ACS algorithm data comes from [3], IACA algorithm data comes from [3], IMRGHPSO algorithm data comes from [40], GA and ABC algorithm data comes from [41].

It can be seen from TABLE 10 that JCACO has higher solution accuracy and faster convergence speed than intelligent algorithms such as single colony ant colony optimization (see TABLE 9, comparison with traditional ant colony algorithm); Compared with the improved multi-colony ant colony optimization, the algorithm can find the optimal solution in small-scale instances, and the quality of the solution obtained by JCACO is better in medium and large-scale instances, and the error rate of the optimal solution is kept within 1% (as shown in TABLE 9).

Through comprehensive analysis of experimental data, it can be seen that JCACO has certain advantages compared with traditional ant colony algorithm, improved ant colony optimization and other intelligent algorithms, and improves solution quality and convergence speed to some extent.

TABLE 10. Comparison of JCACO and other algorithms in TSP instances.

TSP Instances	JCACO	PCCACO	EDHACO	PACO-3opt	D-ACS	IACA	IMRGHPSO	GA	ABC
eil51	**426**	**426**	**426**	**426**	**426**	449	429	-	**426**
eil76	**538**	**538**	**538**	**538**	**538**	573	-	545	541
rat99	**1211**	-	-	1213	-	1339	1231	-	-
kroA100	**21282**	**21282**	**21282**	**21282**	**21282**	23190	21316	21292	21379
kroB100	**22141**	-	22237	-	22235	-	22338	-	-
kroA150	**26621**	26654	26727	-	26792	30312	-	-	-
kroB150	**26130**	**26130**	26328	-	26147	-	-	-	-
ch150	**6528**	-	-	6570	-	-	6652	6615	6533
kroA200	29406	**29391**	29694	29533	29539	34530	30189	-	-
kroB200	**29525**	29541	-	-	-	-	30175	-	-
tsp225	3935	3937	-	-	-	-	-	-	**3926**
a280	**2590**	-	-	-	-	3315	-	-	-
lin318	**42399**	42461	43291	-	-	-	-	-	-
fl417	**11969**	-	-	11972	-	-	-	-	-
pr439	**108375**	-	-	108482	-	-	-	-	-

V. CONCLUSION

This paper proposes a multi-colony ant colony optimization based on the generalized Jaccard similarity recommendation strategy, $s1$ ACS subpopulations and $s2$ MMAS subpopulations are selected to form heterogeneous multi-colony, the diversity and convergence speed of the algorithm are balanced. Introducing the generalized Jaccard coefficient to measure the similarity between two populations and using it in the attribute-based collaborative filtering recommendation mechanism. Three optimization strategies are proposed under this recommendation mechanism: Strategy 1 is an attribute cross-learning strategy, it is used between population with high attribute comprehensive performance and its most similar population to highlight the role of excellent attributes, and improve the performance of the algorithm; Strategy 2 is an attribute recommendation learning strategy, which is used between the population with poor diversity and the population with the best diversity among k similar populations, it is used to increase the diversity of population and enhance the breadth search ability of algorithm; Strategy 3 is a public path reward strategy, which is used in the population with poor convergence, to reward the public path of the current optimal path and the historical optimal path, making the algorithm directional and accelerating the convergence speed. Finally, an elite mixed knowledge board is proposed to store the elite state of subpopulations, when the algorithm is stagnant, the elite reverse learning mechanism is used to jump out of local optimal. Experimental results show that compared with other ant colony optimization, the proposed algorithm balances the diversity and convergence speed of the algorithm, and improves the quality of solutions.

Future research directions are:

1. We will continue to study the optimization effect of the ant colony algorithm combined with the recommendation system on the larger TSP instances, and an improved ant colony algorithm is applied to other problems (For example robot path planning problems).

2. Study the optimization effect of ant colony algorithm combined with other disciplines, for example, ant colony algorithm combined with game theory, and it will be tested in practical application.

3. The application of the algorithm in a multi-objective problem is further studied. For example, in robot path planning problems, the multi-objective problem that turning Angle is the most suitable and the moving path is the shortest is considered simultaneously.

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