IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 28, 2019, accepted October 11, 2019, date of publication October 23, 2019, date of current version November 7, 2019.

*Digital Object Identifier 10.1109/ACCESS.2019.2949176*

# Distributed Adaptive Consensus Control and Disturbance Suppression of Unknown Nonlinear Multi-Agent Systems

**QIANGDE WANG, SHAONING LIU, AND CHUNLING WEI**
Department of Automation, Qufu Normal University, Shandong 276826, China

Corresponding author: Qiangde Wang (qdwang70@126.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61473171, Grant 61773237, and Grant 61873330, and in part by the Shandong Provincial Natural Science Foundation under Grant ZR2017MF063.

**ABSTRACT** This paper considers consensus control of unknown nonlinear multi-agent systems with unknown disturbance under undirected networks. The expected output signal of the leader is not available to all subsystems, only some subsystems can obtain it, and other subsystems will obtain the output regulation error through the network connections. The critical contribution of this paper is to develop a new distributed adaptive control protocol and disturbance observer based on relative output information to achieve the consensus control objective of the subsystems. Since only the relative output information is used, the adaptive control protocol we proposed is distributed. Also, a new lemma is proposed for the first time to analyze the stability of the subsystems. Comparing to current results, the challenge exists in this paper is that the external disturbance is unknown and does not have explicit expression. Based on this, designing a disturbance observer based on relative output information to achieve consensus output regulation is the motivation of this paper. Different from the existing disturbance suppression methods, only the relative output information is used for disturbance suppression, and only the part of disturbance that affects the common trajectory will be suppressed. The stability analysis of the systems is carried out by using algebraic graph theory, Lyapunov function, and Barbalat lemma. The outcome of the paper is that all variables of the systems are bounded and the output regulation errors of the subsystems converge to zero asymptotically. Finally, a numerical simulation is given to demonstrate the effectiveness of the proposed method.

**INDEX TERMS** Consensus control, disturbance suppression, nonlinear multi-agent systems.

## I. INTRODUCTION

As an important research direction in the field of artificial intelligence, multi-agent system has achieved remarkable achievements in economic, military, transportation, clinical medicine, architecture, aerospace and so on [1]–[7]. In simple terms, a multi-agent system refers to a certain number of physical or abstract conceptual individuals who present an orderly coordinated movement behavior on the collective level through mutual cooperation between each other and the self-organization of these individuals, so that the group composed by these individuals has some complex functions.

In recent years, more and more researchers are paying attention to the modeling problem of complex dynamic networks, that is, complex dynamic networks are regarded as multi-agent systems. In a network-connected multi-agent system, each subsystem has the same or similar dynamics and performs the same tasks. The practical application includes vehicle formation control, cooperative control and so on [8]. This type of control system is often referred to the consensus control system, because all subsystems can be controlled to achieve the same control objectives. The consensus problem, as the basis study of multi-agent systems, is mainly used to study how all individuals in a multi-agent system can achieve consensus under the condition of the limited local information exchange [9], whose main task is to design a control protocol based on local information exchange to make the states of all agents to achieve a common state value. There are three core elements in the study of consensus problems: the dynamic equations of agents, the topological

The associate editor coordinating the review of this manuscript and approving it for publication was Min Wang.

relationships between agents, and the rules for information exchange between agents. The significant difference between the consensus control and other control designs is the use of information gathered from adjacent subsystems [10], [11]. The structure of the network connection described as a connection graph is critical to consensus control design, which determines the success of any proposed consensus control design. The adjacency matrix or Laplacian matrix based on graph theory is used to describe the network connection, and some characteristics such as the eigenvalues of the Laplacian matrix are used for control design (e.g. [12]–[17]). In particular, some important properties of Laplace matrix related to consensus control are described in detail in [18]. The nonlinearity of the system is another challenge in consensus control, when the states of the subsystems or just the outputs are available, they can be used to resolve the nonlinearity of the state or output feedback linearization, or even to design the local controller to make the dynamics identical. There are many results on nonlinear and adaptive of nonlinear systems in the output feedback form when they stand alone [19]–[22]. In [23], the fully distributed adaptive consensus control of a class of high-order nonlinear systems with a directed topology and unknown control directions is studied under the condition that the smooth nonlinear functions are known. In addition, relative information, such as the relative output or state of a subsystem relate to other subsystems, is typically used for consensus control, rather than measurement of the actual output or state of the subsystem [24]. The motivation for using relative information is due to applications such as formation control, where each subsystem or agent is non-introspective [25].

Disturbance suppression has always been one of the basic problems in control design, and has been widely used in aerospace, automation, process engineering and so on. Its significant applications in different fields make it may appear in multifarious names such as disturbance attenuation, anti-disturbance, and output regulation except disturbance suppression [26]–[31]. In real production process, external disturbance exists generally, which is the main reason for the poor stability and performance of the system. In order to suppress deterministic disturbances such as constant and sinusoidal disturbances, the controller must be able to generate the required inputs to eliminate the disturbances. This desired control input can be generated by disturbance observer or an internal model, which depends on the disturbance suppression problem's form [32]–[34]. Many researchers have study the consensus of multi-agent systems with disturbance [35]–[38]. For more details, the disturbance suppression of a single-input single-output smooth nonlinear system with uncertain internal modes is studied, where the nonlinear function is known [27]. What is studied in [39] is the consensus disturbance suppression of a linear multi-agent system. The disturbance is generated by a known external system, and the designed disturbance observer is under the premise of the states of the system are completely known. In [40], the containment of a linear multi-agent system with bounded exogenous disturbances is studied, and the external disturbance is generated by exogenous system. In [41], the containment of a linear multi-agent system with disturbance generated from heterogeneous nonlinear external systems is studied under the condition that the nonlinear function is global Lipschitz. The consensus control of disturbance suppression based on disturbance observer for nonlinear multi-agent system lacks systematic study. The purpose of this paper is to bridge this gap.

In this paper, we consider distributed adaptive consensus control and disturbance suppression of unknown nonlinear multi-agent systems. The subsystem in this paper is general nonlinear dynamic, which can be used in real plants. Through relative output feedback, we present a new distributed adaptive protocol and a disturbance observer to achieve state consensus, which utilize the relative output information obtained from the neighboring subsystems and consider the high-order nonlinear terms. The gain of the observer depends on the network connectivity. When the connected graph of the system is undirected, the symmetry of Laplace matrix is used in the design of disturbance observer for the adaptive control of the network connected system. Since the proposed scheme is based on relative output information, it will not suppress the disturbance that does not cause the common trajectory difference of subsystems. The stability analysis of the subsystem is carried out using carefully selected Lyapunov function candidate.

The rest of this paper is described as follows. In Section 2, we give some useful preliminaries, problem statement and assumptions. In Section 3, we introduce the state transformation and some lemmas. In Section 4, based on relative output information, the distributed adaptive control protocol and disturbance observer are proposed. In Section 5, an example with the simulation results is given to demonstrate the effectiveness of the proposed method. Finally, in Section 6 we conclude this paper.

## II. PRELIMINARIES AND PROBLEM STATEMENT

### A. GRAPH THEORY

The communication topology among $N+1$ subsystems is represented by an undirected graph $G$ without self-loops. It consists of a finite nonempty set of nodes $v = \{0, 1, 2, \cdots, N\}$ and a set of edges $\varepsilon \subseteq v \times v$. Each node stands for a subsystem and each edge stands for a connection. If node $j$ is a neighbor of node $i$, then we denote it by an edge $(j, i) \in \varepsilon$, which represents that node $j$ can obtain information from node $i$ and vice versa, i.e., all the subsystems can obtain each other's information. The weighted adjacency matrix of a graph $G$ is a nonnegative matrix $A = (a_{ij}) \in R^{N\times N}$, which is defined as $a_{ij} = 1$ if $(j, i) \in \varepsilon$ and otherwise $a_{ij} = 0$. According to the connection is undirected, we can get the adjacency matrix is symmetric, i.e., $A = A^T$. We define the in-degree matrix $D$ as a diagonal matrix with $D = diag\left(\sum_{j=0, j\neq i}^{N} a_{ij}\right)$, then the Laplacian matrix of the undirected graph $G$ is defined as $L = D - A$, where diagonal elements $L_{ii} = \sum_{j=0, j\neq i}^{N} a_{ij}$ and $L_{ij} = -a_{ij}, i \neq j$ which implies that its row sum is zero. If the subsystem has no neighbor we call it leader, and we call it the follower if it has at least one neighbor.

### B. PROBLEM FORMULATION

In this paper, a multi-agent system consisted of $N+1$ coupled nonlinear subsystems is considered, of which the dynamics of the $i$th subsystem can be described as:

$$\begin{aligned} \dot{x}_i &= A_c x_i + \phi(y_i) + b(u_i + \omega_i), \\ y_i &= C x_i, \end{aligned} \tag{1}$$

with $A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in R^n$, $C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in R^n$, and $x_i \in R^n$, $u_i, y_i \in R$ $(i = 0, 1, \ldots, N)$ are the state, control input and output of the $i$th subsystem, respectively; $n$ represents the order of the subsystems and it is a known positive constant integer, with $b$ being an unknown Hurwitz vector of system (1), here we assume that it satisfies $b_1 \neq 0$ which indicates the relative degree of the system is 1, $\phi : R \times R^m \to R^n$ contains unknown nonlinear functions with each element being polynomials of its variables and satisfies $\phi(0) = 0$, $\omega_i$ is unknown exogenous disturbance of the subsystem and satisfies $\omega_0 = 0$, in addition, it contains slowly-time-varying functions, hence its derivative can be neglected.

In what follows, the multi-agent system we mainly considered in this paper contains one leader and $N$ followers. For notational convenience, we suppose that the subsystem indexed by 0 is leader, and by $1, 2, \ldots, N$ are followers, and for the input of the leader we set its value as 0, that is $u_0 = 0$, which implies that the leader is self-active and can move independently.

The output regulation error is defined as

$$e_i = y_i - y_0, i = 1, \ldots, N. \tag{2}$$

In our design, not every subsystem can obtain the information of $y_0$. But there exists at least one path from the leader to the follower. To describe the graph $G$ completely, we define a diagonal matrix $\Delta$ to represent the access to $y_0$, element $\delta_{ii} = 1$ denotes the $i$th subsystem can obtain the information of $y_0$ in the control design, otherwise $\delta_{ii} = 0$. To achieve the consensus tracking objective, subsystems that can't obtain the information of tracking signals depend on the network connections.

Consensus refers to that the states and outputs of subsystem track the state and output of the leader asymptotically. Through using the relative output information $y_i - y_j, i \neq j$ to design an adaptive control protocol and a disturbance observer, the distributed adaptive consensus output regulation and disturbance suppression problem considered in this paper is solved. The network connection between each subsystem provides the relative output information, which guarantees that the output regulation errors $e_i$ for $i = 1, \ldots, N$ in the state space convergence to zero asymptotically under any initial condition.

Three standard assumptions about the dynamics of the subsystems and network connections are given as follows.

*Assumption 1:* The sign of the unknown high-frequency gain $b_1$ is given in advance.

*Assumption 2:* The connection graph among the subsystems is undirected.

*Assumption 3:* For the nonlinear function $\phi$, there exists an unknown positive real number $\alpha_\phi$ and a known positive integer $p$, such that

$$\|\phi(y_i) - \phi(y_0)\|^2 \leq \alpha_\phi \left(e_i^2 + e_i^{2p}\right).$$

*Remark 1:* The subsystem (1) is in the standard output feedback form. In [20], the geometric conditions for transforming a nonlinear system into the output feedback form are given. In practice, for the mass-spring-damper systems with hardened or softened springs the nonlinear relations between the spring forces and the displacements can be modeled in the nonlinear output feedback form using polynomial nonlinearities output feedback. And the displacement is disturbed by the external disturbance.

*Remark 2:* Assumption 1 is essential for designing the control protocol and disturbance observer to achieve the objective of consensus control. Assumption 2 ensures all followers can obtain each other's information and the adjacency matrix is symmetric, i.e., $A = A^T$, and for more details on this assumption, please refer to [42]. For a general linear system with unknown parameters, assumption 3 holds. The assumption is also valid when the tracking trajectory $y_0$ is bounded and the polynomial order is $p$, in which the positive constant $p$ is a bounded constant. The similar assumption can be found in [43]. From a practical point of view, the leader's trajectory is bounded and the nonlinear functions can be approximated by polynomials under assumption 3. Examples for physical systems satisfied assumptions 1 and 3 are mass-spring-damper systems and van der Pol oscillators, where van der Pol oscillators describe the dynamics of a RLC circuit with a nonlinear resistor [19].

## III. MAIN RESULTS

To extract the internal dynamics of (1) with $\bar{z}_i \in R^{n-1}$, a state transformation is introduced for the subsystem $i$, and $\bar{z}_i \in R^{n-1}$ is given by

$$\bar{z}_i = x_{i,2:n} - \frac{b_{2:n}}{b_1} y_i, \tag{3}$$

where $(\cdot)_{2:n}$ denotes the 2nd row to the $n$th row of the vector or matrix. For further deduce, we introduce the coordinates

$(\bar{z}_i, y_i)$, then the system (1) is transformed to:

$$\begin{aligned}
\dot{\bar{z}}_i &= \dot{x}_{i,2:n} - \frac{b_{2:n}}{b_1}\dot{x}_{i,1} \\
&= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_{i,2} \\ x_{i,3} \\ \vdots \\ x_{i,n-1} \\ x_{i,n} \end{bmatrix} + \phi_{2:n}(y_i) \\
&\quad + \begin{bmatrix} b_2 \\ b_3 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} (u_i + \omega_i) \\
&\quad - \begin{bmatrix} \frac{b_2}{b_1} \\ \frac{b_3}{b_1} \\ \vdots \\ \frac{b_{n-1}}{b_1} \\ \frac{b_n}{b_1} \end{bmatrix} \cdot \left(x_{i,2} + \phi_1(y_i) + b_1(u_i + \omega_i)\right) \\
&= B\bar{z}_i + \bar{\phi}(y_i), \\
\dot{y}_i &= C\dot{x}_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \left( \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} x_i + \phi(y_i) \right. \\
&\quad \left. + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} (u_i + \omega_i) \right) \\
&= \bar{z}_{i,1} + \bar{\phi}_y(y_i) + b_1(u_i + \omega_i),
\end{aligned} \tag{4}$$

where $B$ is the left companion matrix of $b$ given by

$$B = \begin{bmatrix} -b_2/b_1 & 1 & 0 & \cdots & 0 \\ -b_3/b_1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -b_{n-1}/b_1 & 0 & 0 & \cdots & 1 \\ -b_n/b_1 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{5}$$

and

$$\begin{aligned}
\bar{\phi}(y_i) &= B\frac{b_{2:n}}{b_1}y_i + \phi_{2:n}(y_i) - \frac{b_{2:n}}{b_1}\phi_1(y_i), \\
\bar{\phi}_y(y_i) &= \frac{b_2}{b_1}y_i + \phi_1(y_i).
\end{aligned}$$

Denote $z_i = \bar{z}_i - \bar{z}_0$. Then we can obtain the dynamic of $(z_i, e_i)$ as

$$\begin{aligned}
\dot{z}_i &= Bz_i + \psi(y_i, y_0), \\
\dot{e}_i &= h^T z_i + \psi_y(y_i, y_0) + b_1(u_i + \omega_i),
\end{aligned} \tag{6}$$

where $h = [1, 0, \ldots, 0]^T \in R^{n-1}$, $\psi(y_i, y_0) = \bar{\phi}(y_i) - \bar{\phi}(y_0)$, and $\psi_y(y_i, y_0) = \bar{\phi}_y(y_i) - \bar{\phi}_y(y_0)$.

*Lemma 1 (Young's inequality):* General form: Suppose $m, n$ are non-negative real numbers,$p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$, then we have

$$mn \le \frac{m^p}{p} + \frac{n^q}{q}.$$

Generalized form: $mn \le \frac{\gamma^p}{p}|m|^p + \frac{1}{q\gamma^q}|n|^q$, where $\gamma$ is arbitrarily small.

The following lemma can be obtained from above Young's inequality, it will be used throughout the paper and plays a key role in dealing with polynomials.

*Lemma 2 [44]:* For a variable $m \in R > 0$, there exists positive integers $\chi_1$, $\chi_2$, and $\chi_3$ with $\chi_1 \ge 2$, $\chi_3 < \chi_1 < \chi_2$ and positive real constants $\beta_1$ and $\beta_2$, such that

$$m^{\chi_1} \le \beta_1 m^{\chi_2} + \beta_2 m^{\chi_3}. \tag{7}$$

Moreover, with given $\beta_1$, $\beta_2$ is a function of $\beta_1$ and $\chi_i$ for $i = 1, 2, 3$, and is independent on $m$.

*Proof:* Let $a, b > 0$, $a + b = \chi_1$, $p, q > 0$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then from Young's inequality, we can obtain that

$$\left(\beta m^a\right)\left(m^b\beta^{-1}\right) \le \beta^p\frac{m^{ap}}{p} + \beta^{-q}\frac{m^{bq}}{q}, \tag{8}$$

where $\beta$ is an any positive number. Setting $ap = \chi_2$ and $bq = \chi_3$, we have

$$\begin{aligned}
a &= \frac{\chi_2(\chi_1 - \chi_3)}{\chi_2 - \chi_3}, \quad b = \frac{\chi_3(\chi_2 - \chi_1)}{\chi_2 - \chi_3}, \\
p &= \frac{\chi_2 - \chi_3}{\chi_1 - \chi_3}, \quad q = \frac{\chi_2 - \chi_3}{\chi_2 - \chi_1}.
\end{aligned}$$

Then with $\beta^p/p = \beta_1$, we have

$$\beta_2 = \frac{\chi_2 - \chi_1}{\chi_2 - \chi_3}\left(\frac{\chi_2 - \chi_3}{\chi_1 - \chi_3}\beta_1\right)^{-\frac{\chi_1 - \chi_3}{\chi_2 - \chi_1}}. \tag{9}$$

◁

From Assumption 3 and Lemma 2, it is not hard to obtain below two inequalities:

$$\|\psi(y_i, y_0)\|^2 \le \alpha_\psi\left(e_i^2 + e_i^{2p}\right), \tag{10}$$

$$\psi_y^2(y_i, y_0) \le \alpha_y\left(e_i^2 + e_i^{2p}\right), \tag{11}$$

where $\alpha_\psi$ and $\alpha_y$ are some unknown positive real constants.

Before we introduce the control design, the following useful lemma is given in order to derive the expected results.

*Lemma 3:* According to the fact that there exists no neighbor of the leader, the Laplacian Matrix $L$ associated with graph $G$ can be partitioned as:

$$L = \begin{bmatrix} 0 & 0_{1\times N} \\ L_{oc} & \bar{L} \end{bmatrix},$$

where the matrix $\bar{L} \in R^{N\times N}$ is a positive definite symmetric matrix, then for a positive diagonal matrix $S = diag\{s_1, \ldots, s_N\}$, the following inequality holds:

$$S\bar{L} + \bar{L}^T S \ge \theta_0 I \tag{12}$$

for some positive constant $\theta_0$. Evidently, there has $\bar{l}_{ij} = l_{ij}$ for $i, j = 1, 2, \ldots, N$.

*Proof:* The above result has been applied in the literature of Li et al. [45]. From the structure of $L$ in [17], it can be easy seen that $\bar{L}$ is a non-singular $M$-matrix and positive definite symmetric matrix. The existence of $S$ for (12) is obtained by the theorem in [17]. ◁

For $i = 1, 2, \ldots, N$, we define

$$\zeta_i = \sum_{j=0}^{N} a_{ij}\left(y_i - y_j\right). \tag{13}$$

which implies that

$$\begin{aligned}\zeta_i &= \sum_{j=1}^{N} l_{ij}y_j + l_{i0}y_0 = \sum_{j=1}^{N} l_{ij}y_j + \sum_{j=0}^{N} l_{ij}y_0 - \sum_{j=1}^{N} l_{ij}y_0 \\ &= \sum_{j=1}^{N} l_{ij}y_j - \sum_{j=1}^{N} l_{ij}y_0 = \sum_{j=1}^{N} \bar{l}_{ij}e_j.\end{aligned} \tag{14}$$

Let $e = [e_1, e_2, \ldots, e_N]^T$ and $\zeta = [\zeta_1, \zeta_2, \ldots, \zeta_N]^T$. Then, from (14) we can get

$$\zeta = \bar{L}e. \tag{15}$$

Obviously, $\zeta_i$ is available to the $i$th subsystem's control design.

In order to facilitate the stability analysis of the subsystems, a new lemma about the relationship between $e$ and $\zeta$ is proposed.

*Lemma 4:* According to $\zeta = \bar{L}e$, for any positive integer $p$ we have the following inequality:

$$\sum_{i=1}^{N} e_i^{2p} \le N^{p-1}\lambda_{\min}^{-p}\left(\bar{L}^T\bar{L}\right)\sum_{i=1}^{N} \zeta_i^{2p} \tag{16}$$

where $\lambda_{\min}\left(\bar{L}\right)$ denotes the minimum eigenvalue of $\bar{L}$.

*Proof:* By direct calculation we can get

$$\begin{aligned}&\sum_{i=1}^{N} \zeta_i^{2p} \\ &= N\left(\left[\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^2\right)^p\right]^{1/p}\right)^p \ge N\left(\left[\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^2\right)\right]\right)^p \\ &= N^{1-p}\left(\|\zeta\|^2\right)^p = N^{1-p}\left\|\bar{L}e\right\|^{2p} = N^{1-p}\left(e^T\bar{L}^T\bar{L}e\right)^p \\ &\ge N^{1-p}\left[\lambda_{\min}\left(\bar{L}^T\bar{L}\right)e^Te\right]^p = N^{1-p}\lambda_{\min}^p\left(\bar{L}^T\bar{L}\right)\|e\|^{2p} \\ &= N^{1-p}\lambda_{\min}^p\left(\bar{L}^T\bar{L}\right)\left(\sum_{i=1}^{N} e_i^2\right)^p \\ &\ge N^{1-p}\lambda_{\min}^p\left(\bar{L}^T\bar{L}\right)\sum_{i=1}^{N} e_i^{2p}\end{aligned}$$

from which, (16) is obtained. ◁

Barbalat Lemma has been widely used in stability analysis of control systems since it was proposed. And the most common expression and corollary of this lemma are as follows:

*Lemma 5 [46]:* Barbalat Lemma: If a scalar function $f(t)$ is uniformly continuous such that $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ exists and is finite, then $\lim_{t\to\infty} f(t) = 0$.

*Corollary 1:* If $f(t) \in L_2 \cap L_\infty, \dot{f}(t) \in L_\infty$, then $\lim_{t\to\infty} f(t) = 0$.

## IV. CONTROL DESIGN

In this section, a distributed adaptive control protocol and a disturbance observer are proposed to solve the general nonlinear multi-agent systems' consensus output regulation and disturbance suppression problem, i.e., all the followers can be driven to follow the leader asymptotically.

In order to deal with the disturbance in the subsystem, a disturbance observer can be designed as

$$\dot{\hat{\omega}}_i = sign\left(b_1\right)s_i\left(k_i + \eta_i\right)\left(\zeta_i + \zeta_i^{2p-1}\right) \text{ with } \hat{\omega}_i(0) = 0 \tag{17}$$

$$\dot{k}_i = \alpha_k\left(\zeta_i^2 + \zeta_i^{2p}\right) \text{ with } k_i(0) = k_0 \tag{18}$$

where $\hat{\omega}_i$ is the estimate of $\omega_i$, $k_0$ is an any known positive constant, $\alpha_k$ is a positive design parameter and $\eta_i = \zeta_i^2$.

Design the control protocol using output information as follows:

$$u_i = -sign\left(b_1\right)\left(k_i + \eta_i\right)\left(\zeta_i + \zeta_i^{2p-1}\right) - \hat{\omega}_i. \tag{19}$$

*Theorem 1:* Suppose Assumptions 1-3 are satisfied. The consensus output regulation and disturbance suppression problem of network connected unknown nonlinear systems consisting of system (1) is solved by the disturbance observer(17), distributed control protocol (19) together with adaptive laws (18), which implies that the output regulation error $e_i$ converges to zero asymptotically for $i = 1, 2, \ldots, N$.

*Proof:* Firstly, to analyze the dynamics of $z_i$, let

$$V_z = \sum_{i=1}^{N} z_i^TPz_i.$$

Since $B$ is Hurwitz, there exists a positive define matrix $P$ such that

$$PB + B^TP = -3I.$$

From (6), we can obtain

$$\begin{aligned}&\dot{V}_z \\ &= 2\sum_{i=1}^{N} z_i^TP\dot{z}_i = 2\sum_{i=1}^{N} z_i^TP\left(Bz_i + \psi\left(y_i, y_0\right)\right) \\ &= -3\sum_{i=1}^{N}\left\|z_i^2\right\| + 2\sum_{i=1}^{N} z_i^TP\psi\left(y_i, y_0\right) \\ &\le -2\|z\|^2 + \|P\|^2\sum_{i=1}^{N}\left\|\psi\left(y_i, y_0\right)\right\|^2 \\ &\quad -\left(\|z\|^2 - 2\sum_{i=1}^{N} z_i^TP\psi\left(y_i, y_0\right) + \|P\|^2\sum_{i=1}^{N}\left\|\psi\left(y_i, y_0\right)\right\|^2\right)\end{aligned}$$

$$\begin{aligned}
&\leq -2\|z\|^2+\|P\|^2\sum_{i=1}^{N}\|\psi(y_i,y_0)\|^2 \\
&\quad -\left(\|z\|-\|P\|\sum_{i=1}^{N}\|\psi(y_i,y_0)\|\right)^2 \\
&\leq -2\|z\|^2+\|P\|^2\sum_{i=1}^{N}\|\psi(y_i,y_0)\|^2. \qquad (20)
\end{aligned}$$

Using (10) and Lemma 4, we have

$$\dot{V}_z \leq -2\|z\|^2+\|P\|^2\beta_\psi\sum_{i=1}^{N}\left(\zeta_i^2+\zeta_i^{2p}\right) \qquad (21)$$

where $\beta_\psi$ is a positive real constant.

Then, through submitting equation (19) into (6), we can obtain the subsystem dynamics of $e_i$ as

$$\begin{aligned}
\dot{e}_i &= h^T z_i+\psi_y(y_i,y_0)-b_1 sign(b_1)(k_i+\eta_i)\left(\zeta_i+\zeta_i^{2p-1}\right) \\
&\quad +b_1(\omega_i-\hat{\omega}_i) \\
&= h^T z_i+\psi_y(y_i,y_0)-|b_1|(k_i+\eta_i)\left(\zeta_i+\zeta_i^{2p-1}\right)+b_1\tilde{\omega}_i, \qquad (22)
\end{aligned}$$

where $\tilde{\omega}_i=\omega_i-\hat{\omega}_i$.

Next, to consider the stability of $\zeta$, $k_i$ and $\tilde{\omega}$, we define

$$\begin{aligned}
V_{\zeta,\tilde{\omega}} &= \sum_{i=1}^{N}2s_i\left(k_i\left(\frac{\zeta_i^2}{2}+\frac{\zeta_i^{2p}}{2p}\right)+\left(\frac{\zeta_i^4}{4}+\frac{\zeta_i^{2p+2}}{2p+2}\right)\right) \\
&\quad +\frac{1}{2\alpha_k}\sum_{i=1}^{N}(k_i-k^*)^2+|b_1|\tilde{\omega}^T\bar{L}\tilde{\omega} \qquad (23)
\end{aligned}$$

where $k^*$ is a design constant that will be determined later and $\tilde{\omega}=(\omega_1,\cdots,\omega_n)^T$. From (12), (15), (17), (18) and (22), we can obtain that

$$\begin{aligned}
&\dot{V}_{\zeta,\tilde{\omega}} \\
&= \sum_{i=1}^{N}s_i\left(\zeta_i^2+\frac{\zeta_i^{2p}}{p}\right)\dot{k}_i+\sum_{i=1}^{N}2s_ik_i\left(\zeta_i+\zeta_i^{2p-1}\right)\sum_{j=1}^{N}\bar{l}_{ij}\dot{e}_j \\
&\quad +\sum_{i=1}^{N}2s_i\left(\zeta_i^3+\zeta_i^{2p+1}\right)\sum_{j=1}^{N}\bar{l}_{ij}\dot{e}_j+\frac{1}{\alpha_k}\sum_{i=1}^{N}(k_i-k^*)\cdot\dot{k}_i \\
&\quad -2|b_1|\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\omega}_j\bar{l}_{ij}\dot{\hat{\omega}}_i \\
&= \sum_{i=1}^{N}2s_i(k_i+\eta_i)\left(\zeta_i+\zeta_i^{2p-1}\right)\sum_{j=1}^{N}\bar{l}_{ij}\dot{e}_j+\sum_{i=1}^{N}s_i\left(\zeta_i^2+\frac{\zeta_i^{2p}}{p}\right) \\
&\quad \cdot\dot{k}_i+\frac{1}{\alpha_k}\sum_{i=1}^{N}(k_i-k^*)\cdot\dot{k}_i-2|b_1|\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\omega}_j\bar{l}_{ij}\dot{\hat{\omega}}_i \\
&= -|b_1|\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)\left(S\bar{L}+\bar{L}^TS\right)\left(I_N+\eta^{p-1}\right) \\
&\quad \cdot(K+\eta)\zeta+2\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)S\bar{L}\left(\left(I_N\otimes h^T\right)z\right. \\
&\quad \left.+\Psi_y\right)+\alpha_k\zeta^T\left(\eta+\frac{\eta^p}{p}\right)S\left(I_N+\eta^{p-1}\right)\zeta \\
&\quad +\zeta^TK\left(I_N+\eta^{p-1}\right) \\
&\quad \cdot\zeta-k^*\sum_{i=1}^{N}\left(\zeta_i^2+\zeta_i^{2p}\right)+\sum_{i=1}^{N}2s_i(k_i+\eta_i)\left(\zeta_i+\zeta_i^{2p-1}\right) \\
&\quad \cdot\sum_{j=1}^{N}\bar{l}_{ij}b_1\tilde{\omega}_j-2|b_1|\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{\omega}_j\bar{l}_{ij}sign(b_1)s_i(k_i+\eta_i) \\
&\quad \cdot\left(\zeta_i+\zeta_i^{2p-1}\right) \\
&\leq -|b_1|\theta_0\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)\left(I_N+\eta^{p-1}\right)(K+\eta)\zeta \\
&\quad +2\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)S\bar{L}\left(\left(I_N\otimes h^T\right)z+\Psi_y\right) \\
&\quad +\alpha_k\zeta^T\left(\eta+\frac{\eta^p}{p}\right)S\left(I_N+\eta^{p-1}\right)\zeta+\zeta^TK\left(I_N+\eta^{p-1}\right)\zeta \\
&\quad -k^*\sum_{i=1}^{N}\left(\zeta_i^2+\zeta_i^{2p}\right) \qquad (24)
\end{aligned}$$

where $s_i$, $i=1,\ldots,N$, are the diagonal elements of $S$, and $S$ has been defined in (12), $z$ and $\Psi_y$ are vectors that are formed by stacking up their corresponding individual elements $z_i$ and $\psi_y(y_i,y_0)$ in the order from 1 to $N$, respectively, and $K=diag\{k_i\}$, $\eta=diag\{\eta_i\}$.

According to Lemma 1 and Lemma 2, we have

$$\begin{aligned}
&2\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)S\bar{L}\left(\left(I_N\otimes h^T\right)z+\Psi_y\right) \\
&= 2\zeta^T(K+\eta)\left(I_N+\eta^{p-1}\right)S\bar{L}\left(I_N\otimes h^T\right)z \\
&\quad +2\zeta^T(K+\eta)\cdot\left(I_N+\eta^{p-1}\right)S\bar{L}\Psi_y \\
&\leq 2\left[\frac{\theta}{16}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{4}{\theta}\left\|S\bar{L}z\right\|^2\right] \\
&\quad +2\left[\frac{\theta}{16}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{4}{\theta}\left\|S\bar{L}\Psi_y\right\|^2\right] \\
&\leq \frac{\theta}{4}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{8}{\theta}\left\|S\bar{L}z\right\|^2+\frac{8}{\theta}\left\|S\bar{L}\Psi_y\right\|^2 \\
&\leq \frac{\theta}{4}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{8}{\theta}\left\|S\bar{L}\right\|^2\|z\|^2 \\
&\quad +\frac{8}{\theta}\left\|S\bar{L}\right\|^2\left\|\Psi_y\right\|^2 \qquad (25)
\end{aligned}$$

$$\begin{aligned}
&\zeta^TK\left(I_N+\eta^{p-1}\right)\zeta \\
&\leq \frac{\theta}{8}\left\|K\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{2}{\theta}\|\zeta\|^2 \\
&\leq \frac{\theta}{8}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2+\frac{2}{\theta}\|\zeta\|^2 \qquad (26)
\end{aligned}$$

$$\begin{aligned}
&\alpha_k\zeta^T\left(\eta+\frac{\eta^p}{p}\right)S\left(I_N+\eta^{p-1}\right)\zeta \\
&= \alpha_k\zeta^T\eta\left(I_N+\frac{\eta^{p-1}}{p}\right)S\left(I_N+\eta^{p-1}\right)\zeta \\
&\leq \alpha_k\zeta^T\eta\left(I_N+\eta^{p-1}\right)S\left(I_N+\eta^{p-1}\right)\zeta \\
&= \alpha_k\zeta^T\eta S\left(I_N+\eta^{p-1}\right)^2\zeta
\end{aligned}$$

$$
\begin{aligned}
&\le \alpha_k \lambda_{\max}(S)\,\zeta^T \eta \left(I_N + \eta^{p-1}\right)^2 \zeta \\
&\le \frac{\theta}{8}\zeta^T \eta^2 \left(I_N + \eta^{p-1}\right)^2 \zeta + \beta(\theta)\zeta^T\zeta \\
&= \frac{\theta}{8}\zeta^T \left(I_N + \eta^{p-1}\right)\eta^2 \left(I_N + \eta^{p-1}\right)\zeta + \beta(\theta)\zeta^T\zeta \\
&\le \frac{\theta}{8}\zeta^T \left(I_N + \eta^{p-1}\right)(K+\eta^2)\left(I_N + \eta^{p-1}\right)\zeta + \beta(\theta)\zeta^T\zeta \\
&= \frac{\theta}{8}\left\|(K+\eta)\left(I_N + \eta^{p-1}\right)\zeta\right\|^2 + \beta(\theta)\left\|\zeta\right\|^2 \qquad (27)
\end{aligned}
$$

where $\theta$ is a positive design parameter and $\beta : R^+ \to R^+$ is a function that depends on the design parameter.

According to (11) and lemma 4, it can be concluded that below inequality holds:

$$
\left\|\Psi_y\right\|^2 \le \alpha_y \sum_{i=1}^{N}\left(e_i^2 + e_i^{2p}\right) \le \beta_y \sum_{i=1}^{N}\left(\zeta_i^2 + \zeta_i^{2p}\right), \qquad (28)
$$

where $\beta_y$ is a positive real constant.

For (24), using (25), (26), (27), (28) and letting $\theta = |b_1|\,\theta_0$, it can be obtained that

$$
\begin{aligned}
&\dot{V}_{\zeta,\tilde{\omega}} \\
&\le -\frac{\theta}{2}\left\|(K+\eta)\left(I_N + \eta^{p-1}\right)\zeta\right\|^2 + \frac{8}{\theta}\left\|S\bar{L}\right\|^2\|z\|^2 \\
&\quad -\left(k^* - \beta(\theta) - \frac{8}{\theta}\left\|S\bar{L}\right\|^2\beta_y - \frac{2}{\theta}\right)\sum_{i=1}^{N}\left(\zeta_i^2 + \zeta_i^{2p}\right). \qquad (29)
\end{aligned}
$$

Finally, consider the following Lyapunov function candidate:

$$
V = V_{\zeta,\tilde{\omega}} + \frac{8}{\theta}\left\|S\bar{L}\right\|^2 V_z.
$$

Differentiating $V$ with respect to $t$ along the trajectory of (21) and (29), we have

$$
\begin{aligned}
\dot{V} &\le -\frac{\theta}{2}\left\|(K+\eta)\left(I_N + \eta^{p-1}\right)\zeta\right\|^2 - \frac{8}{\theta}\left\|S\bar{L}\right\|^2\|z\|^2 \\
&\quad -\left(k^* - \beta(\theta) - \frac{8}{\theta}\left\|S\bar{L}\right\|^2\beta_y - \frac{2}{\theta} - \frac{8}{\theta}\left\|S\bar{L}\right\|^2\beta_\psi\|P\|^2\right) \\
&\quad \cdot \sum_{i=1}^{N}\left(\zeta_i^2 + \zeta_i^{2p}\right). \qquad (30)
\end{aligned}
$$

Choosing

$$
k^* = \beta(\theta) + \frac{8}{\theta}\left\|S\bar{L}\right\|^2\beta_y + \frac{2}{\theta} + \frac{8}{\theta}\left\|S\bar{L}\right\|^2\beta_\psi\|P\|^2
$$

results in

$$
\dot{V} \le -\frac{\theta}{2}\left\|(K+\eta)\left(I_N+\eta^{p-1}\right)\zeta\right\|^2 - \frac{8}{\theta}\left\|S\bar{L}\right\|^2\|z\|^2. \qquad (31)
$$

Clearly, $V$ is a monotone non-increasing function.

Integrating both sides of (31), we have

$$
\begin{aligned}
V(t) &\le V(0) - \int_0^t \frac{\theta}{2}\left\|(K+\eta)\left(I_N + \eta^{p-1}\right)\zeta\right\|^2 d\tau \\
&\quad - \int_0^t \frac{8}{\theta}\left\|S\bar{L}\right\|^2\|z\|^2 d\tau.
\end{aligned}
$$

Therefore, we can conclude that $V$ is bounded and $k_i - k^* \in L_2 \cap L_\infty$, $\zeta_i \in L_2 \cap L_\infty$, $z_i \in L_2 \cap L_\infty$, $\tilde{\omega}_i \in L_\infty$ for $i = 1, 2, \ldots, N$. The boundedness of $\hat{\omega}_i$ can be obtained from the boundedness of $\omega_i$. Since $k^*$ is a constant, $k_i \in L_\infty$. The boundedness of $u_i$ in (19) follows from $k_i \in L_\infty$, $\zeta_i \in L_\infty$ and $\hat{\omega}_i \in L_\infty$. By further analysis and induction, we can conclude that all the variables in the subsystems are bounded. According to $\zeta_i \in L_2 \cap L_\infty$ and from (14), we can obtain $e_i \in L_2 \cap L_\infty$. Since the derivative of $e_i$ is bounded, using Barbalat Lemma, we conclude that $e_i$ for $i = 1, 2, \ldots, N$ converge to zero as $t$ tends to $\infty$, i.e., $\lim_{t\to\infty} e(t) = 0$, which completes the proof. ◁

## V. ILLUSTRATIVE EXAMPLE

In this section, a numerical example is used to demonstrate the effectiveness of proposed control protocol design and disturbance observer. Suppose that the network-connected nonlinear system consists of 6 subsystems, which includes one leader (labeled by 0) and five followers (labeled from 1 to 5). The dynamic of the subsystem 0 is represented by a second-order nonlinear system as

$$
\dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} \beta\left(y_i - \frac{1}{3}y_i^3\right) \\ -y_i/\beta \end{bmatrix},
$$

for $i = 0$, with $y_i = x_{i,1}$.

Similarly, the dynamics of the subsystem 1-5 are described by

$$
\dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} \beta\left(y_i - \frac{1}{3}y_i^3\right) \\ -y_i/\beta \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}(u_i + \omega_i),
$$

for $i = 1, 2, \ldots, 5$, with $y_i = x_{i,1}$, where $\beta$, $b_1$ and $b_2$ are unknown positive real parameters, $\omega_i$ is unknown disturbance that contains slowly-time-varying functions, $b_1 > 0$ satisfies Assumption 1, so we set $b_1 = b_2 = 1$. For the disturbance, we set it as

$$
\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \end{bmatrix} = 0.1 \times \begin{bmatrix} \sin t \\ \sin 2t \\ \sin 3t \\ \sin 4t \\ \sin 5t \end{bmatrix}.
$$

For the parameter $\beta$,

$$
\beta = \begin{cases} 0.5 & for 0 \le t \le 20, \\ 2 & for t > 20, \end{cases}
$$

which makes the trajectory of the leader has two different limit cycles. It is worth noting that the system satisfies the condition of van der Pol oscillator when $u_i = 0$, and its trajectories are bounded. And a van der Pol oscillator describes the dynamics of a RLC circuit with a nonlinear resistor. Therefore, the Assumption 3 is satisfied with $p = 3$.

The weighted adjacency matrix of the graph corresponding to the subsystems is selected as

$$A=\begin{bmatrix}0&0&0&0&0&0\\1&0&1&1&0&0\\0&1&0&0&1&0\\1&1&0&0&1&1\\0&0&1&1&0&0\\0&0&0&1&0&0\end{bmatrix},$$

from $L=D-A$ and $L=\begin{bmatrix}0&0_{1\times N}\\L_{oc}&\bar{L}\end{bmatrix}$, we can obtain the Laplacian matrix and the $Q$ matrix

$$L=\begin{bmatrix}0&0&0&0&0&0\\-1&3&-1&-1&0&0\\0&-1&2&0&-1&0\\-1&-1&0&4&-1&-1\\0&0&-1&-1&2&0\\0&0&0&-1&0&1\end{bmatrix},$$

$$\bar{L}=\begin{bmatrix}3&-1&-1&0&0\\-1&2&0&-1&0\\-1&0&4&-1&-1\\0&-1&-1&2&0\\0&0&-1&0&1\end{bmatrix}.$$

Note that only two subsystems (subsystem 1 and 3) have access to the leader for information. From the communication topology, it can be seen that the Assumption 2 is satisfied.

The disturbance observer and control protocol are designed as, for $i=1,2,\ldots,5$,

$$\begin{aligned}\dot{\hat{\omega}}_i&=sign\left(b_1\right)s_i\left(k_i+\eta_i\right)\left(\zeta_i+\zeta_i^{2p-1}\right),\\u_i&=-sign\left(b_1\right)\left(k_i+\eta_i\right)\left(\zeta_i+\zeta_i^{2p-1}\right)-\hat{\omega}_i,\ \eta_i=\zeta_i^2,\\\dot{k}_i&=\alpha_k\left(\zeta_i^2+\zeta_i^{2p}\right)\end{aligned}$$

where $s_1=s_2=s_3=s_4=s_5=1$, $\alpha_k=5$.

The simulation study has been carried out for the subsystems with the initial states of the leader and the followers are chosen randomly for $i=0,1,\ldots,5$:

$$\begin{bmatrix}x_{0,1}\\x_{0,2}\end{bmatrix}=\begin{bmatrix}0\\0.5\end{bmatrix},$$
$$\begin{bmatrix}x_{1,1}\\x_{1,2}\end{bmatrix}=\begin{bmatrix}x_{2,1}\\x_{2,2}\end{bmatrix}=\begin{bmatrix}x_{3,1}\\x_{3,2}\end{bmatrix}=\begin{bmatrix}x_{4,1}\\x_{4,2}\end{bmatrix}=\begin{bmatrix}x_{5,1}\\x_{5,2}\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix},$$

and the initial conditions of the controller is $k_i\left(0\right)=k_0=1$.

The state trajectories of all subsystems using disturbance observer (17) and control protocol (19) are shown in Fig.1 and Fig.2. In the simulation, the red curve represents the leader's state trajectory, and the other color curves represent the followers' state trajectory. From Figs. 1 and 2, it can be seen that all the outputs and the states of the followers converge to the trajectories of the leader asymptotically. Figs. 3, 4 and 5 show the inputs $u_i$, dynamic gains $k_i$ and disturbance estimations $\hat{\omega}_i$ of the subsystems, respectively. Since the change in the value of $\mu$, from the graph it can be seen that the trajectories after 20 seconds is different. Then the excellent tracking performance of the disturbance observer and control protocol are verified.

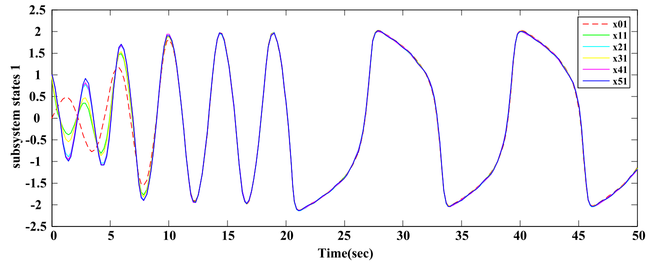


**FIGURE 1.** The subsystem states $x_{i,1}$, i.e., the outputs.

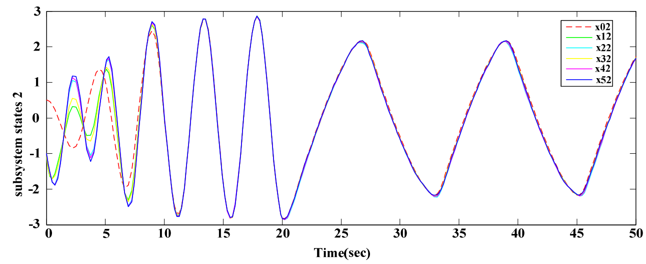


**FIGURE 2.** The subsystem states $x_{i,2}$.

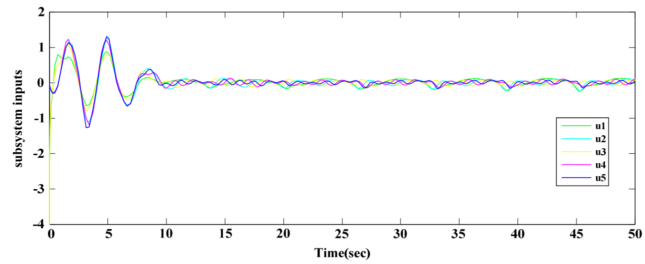


**FIGURE 3.** The subsystem inputs.

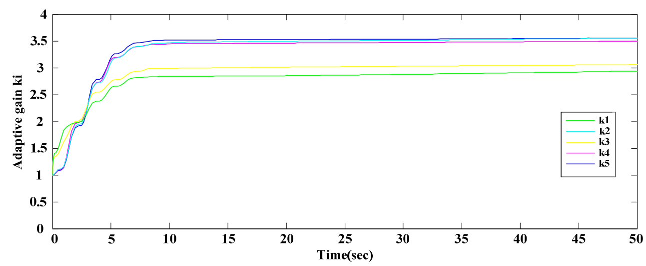


**FIGURE 4.** The controller gains $k_i$.

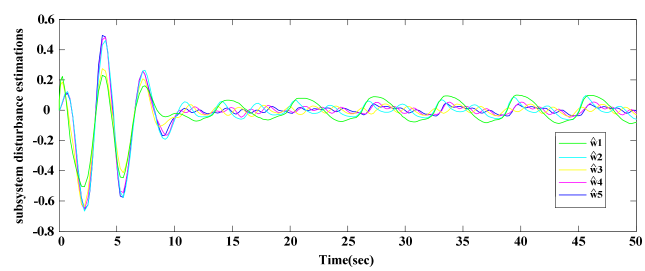


**FIGURE 5.** The subsystem disturbance estimations $\hat{\omega}_i$.

## VI. CONCLUSION

In this paper, consensus control of unknown nonlinear multi-agent systems with unknown disturbance under undirected topologies is investigated. By regarding the leader as a reference model and using relative output information, we proposed a new distributed adaptive control protocol and a disturbance observer of a network-connected unknown nonlinear systems. The method that we have proposed doesn't need any global information, but only the relative output information. Also, unlike the previous disturbance suppression methods, only the relative output information is used for disturbance suppression, and as a result only the part of disturbance that affects the common trajectory will be suppressed. Based on the scheme, all variables of the systems are bounded and the output regulation errors of the subsystems converge to zero asymptotically. The numerical simulation results demonstrate the effectiveness of the proposed control method. Future research will further consider the distributed adaptive consensus control of heterogeneous unknown nonlinear multi-agent systems under directed graphs. In addition, we will further relax the condition of the disturbance and the disturbance we will choose is not matched, which would have more practical application value.

## REFERENCES

[1] A. Dorri, S. S. Kanhere, and R. Jurdak, "Multi-agent systems: A survey," *IEEE Access*, vol. 6, pp. 28573–28593, 2018.
[2] S. Boudoudouh and M. Maâroufi, "Real-time battery state of charge estimation in smart grid application by Multi Agent System," *Int. J. Hydrogen Energy*, vol. 42, no. 30, pp. 19487–19495, Jul. 2017.
[3] R. Yang and L. Wang, "Development of multi-agent system for building energy and comfort management based on occupant behaviors," *Energy Build.*, vol. 56, pp. 1–7, Jan. 2013.
[4] T. Labeodan, K. Aduda, G. Boxem, and W. Zeiler, "On the application of multi-agent systems in buildings for improved building operations, performance and smart grid interaction—A survey," *Renew. Sust. Energ. Rev.*, vol. 50, pp. 1405–1414, Oct. 2015.
[5] D. K. Sahu and P. Kawadkar, "Multiple agents based scheduling and monitoring in cloud system," *Int. J. Data Sci. Technol.*, vol. 1, no. 1, pp. 1–5, Jan. 2016.
[6] W. He, Z. Xu, W. Du, G. Chen, N. Kubota, and F. Qian, "Synchronization control in multiplex networks of nonlinear multi-agent systems," *Chaos, Interdiscipl. J. Nonlinear Sci.*, vol. 27, no. 12, Dec. 2017, Art. no. 123104.
[7] C. He, Z. Feng, and Z. Ren, "A flocking algorithm for multi-agent systems with connectivity preservation under hybrid metric-topological interactions," *PLoS ONE*, vol. 13, no. 2, Feb. 2018, Art. no. e0192987.
[8] L. Cui, Y. Li, R. Yang, and X. Zhang, "Asymptotical cooperative tracking control for unknown high-order multi-agent systems via distributed adaptive critic design," *IEEE Access*, vol. 6, pp. 24650–24659, 2018.
[9] J. Shamma, *Cooperative Control of Distributed Multi-Agent Systems*. New York, NY, USA: Wiley, 2008.
[10] C. E. Ren, Z. Shi, and T. Du, "Distributed observer-based leader-following consensus control for second-order stochastic multi-agent systems," *IEEE Access*, vol. 6, pp. 20077–20084, 2018.
[11] G. Wang, C. Wang, L. Li, and Z. Zhang, "Designing distributed consensus protocols for second-order nonlinear multi-agents with unknown control directions under directed graphs," *J. Franklin Inst.*, vol. 354, pp. 571–592, Jan. 2017.
[12] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.
[13] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.
[14] W. Ren, "Multi-vehicle consensus with a time-varying reference state," *Syst. Control Lett.*, vol. 56, nos. 7–8, pp. 474–483, Jul. 2007.
[15] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 57, no. 1, pp. 213–224, Jan. 2010.
[16] S. Liu, L. Xie, and F. L. Lewis, "Synchronization of multi-agent systems with delayed control input information from neighbors," *Automatica*, vol. 47, pp. 2152–2164, Oct. 2011.
[17] Y. Su and J. Huang, "Stability of a class of linear switching systems with applications to two consensus problems," *IEEE Trans. Autom. Control*, vol. 57, no. 6, pp. 1420–1430, Jun. 2012.
[18] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*. London, U.K.: Springer, 2009.
[19] Z. Ding, *Nonlinear and Adaptive Control Systems*. London, U.K.: Imperial College, 2013.
[20] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York, NY, USA: Wiley, 1995.
[21] R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive and Robust*. Upper Saddle River, NJ, USA: Prentice-Hall, 1996.
[22] G. Wang, L. Li, and C. Wang, "Fully distributed low-complexity control for nonlinear strict-feedback multiagent systems with unknown dead-zone inputs," *IEEE Trans. Syst., Man, Cybern. Syst.*, to be published.
[23] J. Huang, Y. Song, W. Wang, C. Wen, and G. Li, "Fully distributed adaptive consensus control of a class of high-order nonlinear systems with a directed topology and unknown control directions," *IEEE Trans. Cybern.*, vol. 48, no. 8, pp. 2349–2356, Aug. 2018.
[24] J. Huang, C. Wen, W. Wang, and Y.-D. Song, "Adaptive finite-time consensus control of a group of uncertain nonlinear mechanical systems," *Automatica*, vol. 51, pp. 292–301, Jan. 2014.
[25] H. F. Grip, T. Yang, A. Saberi, and A. A. Stoorvogel, "Output synchronization for heterogeneous networks of non-introspective agents," *Automatica*, vol. 48, no. 10, pp. 2444–2453, Oct. 2012.
[26] L. Guo and X.-Y. Wen, "Hierarchical anti-disturbance adaptive control for non-linear systems with composite disturbances and applications to missile systems," *Trans. Inst. Meas. Control*, vol. 33, no. 8, pp. 942–956, Apr. 2011.
[27] Z. Ding, "Global stabilization and disturbance suppression of a class of nonlinear systems with uncertain internal model," *Automatica*, vol. 39, no. 3, pp. 471–479, Mar. 2003.
[28] L. Guo and W.-H. Chen, "Disturbance attenuation and rejection for systems with nonlinearity via DOBC approach," *Int. J. Robust Nonlinear Control*, vol. 15, no. 3, pp. 109–125, Dec. 2005.
[29] Z. Ding, "Asymptotic rejection of general periodic disturbances in output-feedback nonlinear systems," *IEEE Trans. Autom. Control*, vol. 51, no. 2, pp. 303–308, Feb. 2006.
[30] Z. Ding, "Output regulation of uncertain nonlinear systems with nonlinear exosystems," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 498–503, Mar. 2006.
[31] Y. Wei, G.-P. Liu, J. Hu, and F. Meng, "Disturbance attenuation and rejection for nonlinear uncertain switched systems subject to input saturation," *IEEE Access*, vol. 7, pp. 58475–58483, 2018.
[32] A. Isidori, *Nonlinear Control Systems*, 3rd ed. Berlin, Germany: Springer-Verlag, 1995.
[33] S. Li, J. Yang, W.-H. Chen, and X. Chen, *Disturbance Observer-Based Control: Methods and Applications*. Boca Raton, FL, USA: CRC Press, 2014.
[34] J. Huang, Y.-D. Song, W. Wang, C. Wen, and G. Li, "Smooth control design for adaptive leader-following consensus control of a class of high-order nonlinear systems with time-varying reference," *Automatica*, vol. 83, pp. 361–367, Sep. 2017.
[35] C.-E. Ren, T. Du, G. Li, and Z. Shi, "Disturbance observer-based consensus control for multiple robotic manipulators," *IEEE Access*, vol. 6, pp. 51348–51354, 2018.
[36] Y. Liu and Y. Jia, "Adaptive leader-following consensus control of multi-agent systems using model reference adaptive control approach," *IET Control Theory Appl.*, vol. 6, no. 13, pp. 2002–2008, Sep. 2012.
[37] Y. Liu and Y. Jia, "Consensus problem of high-order multi-agent systems with external disturbances: An $H_\infty$ analysis approach," *Int. J. Robust Nonlinear Control*, vol. 20, pp. 1579–1593, Aug. 2010.
[38] H. Yang, Z. Zhang, and S. Zhang, "Consensus of second-order multi-agent systems with exogenous disturbances," *Int. J. Robust Nonlinear Control*, vol. 21, pp. 945–956, Aug. 2011.
[39] Z. Ding, "Consensus disturbance rejection with disturbance observers," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5829–5837, Sep. 2015.

[40] C. Xu, Y. Zheng, H. Su, and H.-B. Zeng, "Containment for linear multi-agent systems with exogenous disturbances," *Neurocomputing*, vol. 160, pp. 206–212, Jul. 2015.
[41] X. Zhang and X. Liu, "Containment of linear multi-agent systems with disturbances generated by heterogeneous nonlinear exosystems," *Neurocomputing*, vol. 315, pp. 283–291, Nov. 2018.
[42] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, Jun. 2012.
[43] Z. Ding, "Consensus output regulation of a class of heterogeneous nonlinear systems," *IEEE Trans. Autom. Control*, vol. 58, no. 10, pp. 2648–2653, Oct. 2013.
[44] Z. Ding and Z. Li, "Distributed adaptive consensus control of nonlinear output-feedback systems on directed graphs," *Automatica*, vol. 72, pp. 46–52, Oct. 2016.
[45] Z. Li, G. Wen, Z. Duan, and W. Ren, "Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs," *IEEE Trans. Autom. Control*, vol. 60, no. 4, pp. 1152–1157, Apr. 2015.
[46] G. Tao, *Adaptive Control Design and Analysis*. Piscataway. NJ, USA: Wiley, 2003.

**QIANGDE WANG** received the B.S. degree in mathematics and the M.S. degree in operational research and cybernetics from Qufu Normal University, Qufu, China, in 1994 and 1997, respectively, and the Ph.D. degree in control theory and control engineering from Northeastern University, Shenyang, China, in 2005. His research interests include nonlinear control, adaptive control, and stochastic nonlinear system control.

**SHAONING LIU** received the B.Eng. degree in electrical engineering and automation from the University of Jinan, Jinan, China, in June 2015. She is currently pursuing the master's degree in mathematics with the College of Engineering, Qufu Normal University, Qufu, China. Her research interests include adaptive control, output feedback control, and nonlinear multiagent systems control.

**CHUNLING WEI** received the B.S. degree in mathematics and the M.S. degree in operational research and cybernetics from Qufu Normal University, Qufu, China, in 1995 and 1998, respectively, and the Ph.D. degree in control theory and control engineering from Southeast University, Nanjing, China, in 2007. Her research interests include nonlinear control, adaptive control, and stochastic nonlinear system control.