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Spreading Dynamics of a CPFB Group Booking Preferential Information Model on Scale-Free Networks

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ABSTRACT With the development of the Internet, online shopping has become a popular way of life in contemporary society. In recent years, online group booking has become an important strategy for e-commerce to promote commodities. In order to study the impact of the products' discount rate, customer's repurchase intention and the heterogeneity of the networks on the spreading of group booking preferential information, we present a novel *CPFB* (customer-participant-forwarder-beneficiary) group booking preferential information spreading model based on scale-free networks. The dynamic propagation of preferential information in group booking online is analyzed in detail by the mean-field theory. The basic reproductive number R_0 and the two equilibriums are obtained. Then, locally asymptotical stability of equilibriums is discussed based on the Routh–Hurwitz criterion. The global asymptotic stability of the group booking free equilibrium, the permanence of preferential information spreading and the global attractivity of the group booking equilibrium is proved in detail. Theoretical results show that the basic reproductive number R_0 depends mainly on the discount rate and the topology of the underlying networks. And the size of the repurchase intention rate η can promote the spreading of preferential information about online group booking, but do not affect the size of the basic reproductive number R_0. Finally, numerical simulations confirm the analytical results of the theories.

INDEX TERMS *CPFB* model, scale-free networks, stability, permanence, global attractivity.

I. INTRODUCTION

With the development of socialized e-commerce, the spreading of online shopping preferential information about commodities is a very common phenomenon [1]–[3]. Since online shopping has many unique advantages over physical stores, more and more consumers like to shop online [4], [5]. Nowadays, the e-commerce website platforms Taobao and 360Buy often launch group shopping activities. Some group buying Apps are designed for people who like to join the group booking, such as Pinduoduo [6], [7]. Online group buying behavior has the dual value creation concept of marketing [8]. In order to find out the factors that affect the behavior of group buying launched by online merchants, it is necessary to study the spread of the group buying phenomenon and effectively propose promotional strategies of group buying [9].Therefore, it is very important to study the dissemination of network information.

As we all know, group buying is based on dynamic pricing mechanism [10], [11]. In online group buying, a large number of consumers gather together to buy the same commodities and then they can get the price discount [12], [13]. However, there is an e-commerce mode where the discount prices are determined by the price-quantity function or price-quantity table defined by the seller in online group buying [14]. This model can be called online group booking in China. It works as follows: if it is in a predetermined time, buyers succeed to form a group of including a few people, then everyone in this group receives the product or the service at the same discounted price [15], [16]. This e-commerce model has been very successful in China, and the online group booking is very popular [15]. Because the dissemination of information

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can be regarded as some kinds of communication behavior occurring on the network, we can analyze this phenomenon of information propagation from the perspective of propagation dynamics [17], [18].

Regarding the dynamic process of information dissemination, a consumer is represented by a node in the network, the edges are the connected relationship between consumers, and the individual transmits information to neighbors through their connections [19], [20]. Since the study of network information dissemination is a broad research topic, many scholars have studied online shopping behavior [3], [4], [21], [22]. But most of them are only qualitative analysis of how psychological, institutional and social factors affect consumer's willingness to purchase or repurchase [21], [22]. For instance, Xiao [23] conducted a comprehensive study on the motivation of online shopping behavior of online group-buying (OGB) customers from the perspective of user orientation. Che *et al.* [24] studied the factors affecting consumer's willingness to revisit from the perspective of transaction cost economics (TCE). These researches are only analysis of the phenomenon. They do not consider the impact of the individual's repurchase intention of group booking, the discount rate of commodity and the heterogeneity of the network on the dissemination of preferential information from the direction of propagation dynamics.

Since the introduction of scale-free networks in 1999, it has been found that scale-free features are an important aspect of social networks [25]. Based on the mean-field theory [25], [26], many scientists are dedicated to the study of complex networks, driven by the pioneering work of revealing the small world effects of various real networks [27] and scale-free features [25]. Therefore, the mean-field theory plays a crucial role in the study of complex networks. For the study of complex network propagation dynamics, many classical researches are in the direction of disease transmission [28]–[30]. However, the dissemination of information on social networks is similar to the nature of disease transmission. For instance, Wang *et al.* [31] changed the SEIR (susceptible-exposed-infective-removed) model of infectious disease to the SEIR-based model of information that can reflect the model of information dissemination in Social Networking Services (SNS).

Therefore, we can find that mathematical modeling is an important method to study the dissemination of network information [32]–[34]. For instance, Rui *et al.* [35] introduced a potential spreader set in the model, solving the problem of repeated calculation effectively. Cao *et al.* [36] established a general model system by considering the degree of forgetting in the process of knowledge dissemination in complex networks, which proved that the level of forgetfulness depends mainly on the number in a crowd who possess knowledge. In recent years, time delay has often been considered in studying system stability [37]–[40]. But in addition to studying the effects of time delay on stability, it is also valuable to study the topology of the underlying network [41], [42]. For instance, Zhao *et al.* [42] found that the topology of the network was not considered because of the limitations of complex network research, which limited the wider application of the model. Wan *et al.* [43] studied that preferential degree and the heterogeneity of underlying networks can affect the spread of preferential information in e-commerce networks. Therefore, in order to study the impact of the heterogeneity of the networks, the products' discount rate and the repurchase intention on the online dissemination of preferential information, we propose a novel *CPFB* model based on scale-free networks.

The rest of this paper is organized as follows. In Second 2, we present a *CPFB* model of group booking preferential information dissemination on scale-free networks. In Second 3, the basic reproductive number and two equilibriums are obtained at first. Then, locally asymptotical stability of equilibriums is discussed based on the Routh–Hurwitz criterion. The globally asymptotic stability of the group booking free equilibrium, the global attractivity of the group booking equilibrium and the permanence of preferential information spreading of group booking is analyzed in detail. In Second 4, we present the results of the numerical simulation. Finally, we conclude the paper in Section 5.

II. MODEL FORMULATION

Considering the impact of the products' discount rate, the repurchase intention and the heterogeneity of the networks on the online dissemination of preferential information, we consider a novel *CPFB* model on scale-free networks. It is assumed that each individual is abstracted as a node.

In the dynamic spreading process of group booking preferential information, each node has four states: Customer (C) refers to the individual who has not participated in the group buying activity but may have the intention to buy commodities. Participant (P) refers to the individual who participates in online group booking activities. Forwarder (F) refers to the individual who participates in online shopping activity and forwards shopping preferential information. Beneficiary (B) is the individual who succeeds in group booking and obtains benefit.

Considering the heterogeneous structure of the networks, let $C_k(t)$, $P_k(t)$, $F_k(t)$ and $B_k(t)$ be the relative densities of nodes with degree k at time t in the whole network. And $N_k(t) = C_k(t)+P_k(t)+F_k(t)+B_k(t)$ means the density of whole population with degree k. In the process of spreading the preferential information, if a customer contacts with a participant or a forwarder, then he or she will become a participant with a probability of α_1 or α_2. The participant joining the online activity of group buying will convert into the forwarder with forwarding probability δ. When the number of participants meets the requirement of the online group booking activities, and then the participant will get the benefits and convert into the beneficiary with probability ε. During joining the online group booking activity, the forwarder who offers preferential information of online group booking may convert to the beneficiary of the successful purchase with

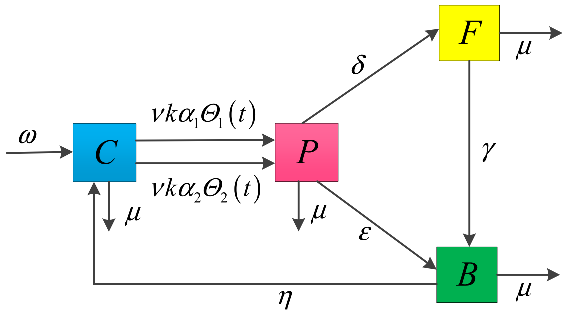


FIGURE 1. The transmission sketch of the *CPFB* model.

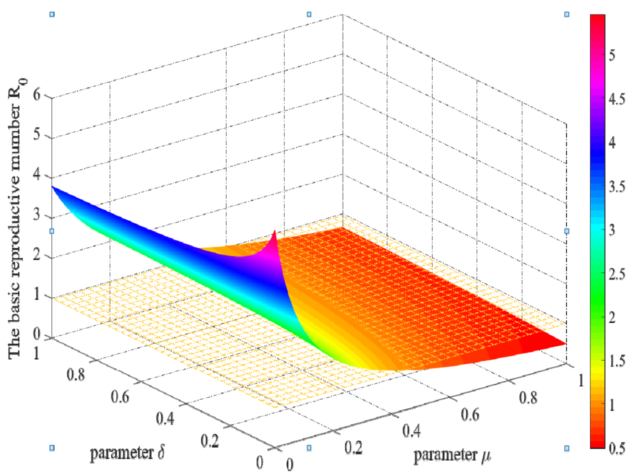


FIGURE 2. The size of the basic reproductive number with the diffident value of μ, δ.

probability γ. Since the purchased products are very good and cheap, the beneficiary may become the customer again with probability η named the repurchase intention rate. Assuming that the new registered users all become into customers and the growth rate is ω. The number of exit rate of each individual state is μ. And the discount rate is ν. The flow diagram of transmission is described in Figure 1.

According to the above model and the mean-field theory, the *CPFB* model can be written as

$$\begin{cases} \dfrac{dC_k(t)}{dt} = \omega + \eta B_k(t) - \nu k(\alpha_1\Theta_1(t) + \alpha_2\Theta_2(t))\, C_k(t) \\ \qquad - \mu C_k(t), \\ \dfrac{dP_k(t)}{dt} = \upsilon k(\alpha_1\Theta_1(t) + \alpha_2\Theta_2(t))\, C_k(t) - \varepsilon P_k(t) \\ \qquad - \delta P_k(t) - \mu P_k(t), \\ \dfrac{dF_k(t)}{dt} = \delta P_k(t) - \gamma F_k(t) - \mu F_k(t), \\ \dfrac{dB_k(t)}{dt} = \varepsilon P_k(t) + \gamma F_k(t) - \eta B_k(t) - \mu B_k(t), \end{cases} \tag{1}$$

where $\Theta_1(t)$ is the probability that a link is connected to a participant at time t.

$$\Theta_1(t) = \frac{1}{\langle k\rangle}\sum_k kP(k)P_k(t). \tag{2}$$

And $\Theta_2(t)$ is the probability that a link is connected to a forwarder at time t.

$$\Theta_2(t) = \frac{1}{\langle k\rangle}\sum_k kP(k)F_k(t). \tag{3}$$

Here, $\langle k\rangle$ represents the average degree of nodes, i.e., $\langle k\rangle = \sum_k kP(k)$, and $P(k)$ represents the degree distribution. Let $P(t) = \sum_k P(k)P_k(t)$, which denotes the density of all individual of participants and $F(t) = \sum_k P(k)F_k(t)$, which denotes the density of all individuals of forwarders. Therefore, if we make $\rho(t) = (\alpha_1\Theta_1(t) + \alpha_2\Theta_2(t))$, the system (1) can be simplified as:

$$\begin{cases} \dfrac{dC_k(t)}{dt} = \omega + \eta B_k(t) - (\nu k\rho + \nu\beta + \mu)\, C_k(t), \\ \dfrac{dP_k(t)}{dt} = \upsilon k\rho(t)\, C_k(t) - (\varepsilon + \delta + \mu)\, P_k(t), \\ \dfrac{dF_k(t)}{dt} = \delta P_k(t) - (\gamma + \mu)\, F_k(t), \\ \dfrac{dB_k(t)}{dt} = \varepsilon P_k(t) + \gamma F_k(t) - (\eta + \mu)\, B_k(t). \end{cases} \tag{4}$$

From the system (4), we can know

$$\frac{dN_k(t)}{dt} = \frac{dC_k(t)}{dt} + \frac{dP_k(t)}{dt} + \frac{dF_k(t)}{dt} + \frac{dB_k(t)}{dt} = \omega - \mu N_k(t) \tag{5}$$

Then, we can get that $N_k(t) = \omega/\mu + N_k(0)\,e^{-\mu t}$, where $N_k(0)$ represents the initial density of whole population with degree k. Therefore, $\lim_{t\to\infty}\sup N_k(t) = \omega/\mu$, then $N_k(t) = C_k(t)+P_k(t)+F_k(t)+B_k(t) \le \omega/\mu$ for all $t \ge 0$. We imply that the region $\Omega = \{(C_k(t), P_k(t), F_k(t), B_k(t)) \in R_+^{4n} : 0 \le C_k(t) + P_k(t) + F_k(t) + B_k(t) \le \omega/\mu, k = 1, 2, \cdots, n\}$ is the positively invariant set for the system (4). Thus, each solution of the system (4), with initial conditions and the limit sets are contained in Ω for all $t \ge 0$. For simplicity, assuming that $\omega = \mu$. According to the normalization condition, the initial conditions satisfy:

$$C_k(t)+P_k(t)+F_k(t)+B_k(t) = 1, \quad k = 1, 2, \cdots n. \tag{6}$$

$$0 \le C_k(0), P_k(0), F_k(0), B_k(0) \le 1. \tag{7}$$

III. STABILITY ANALYSIS OF MODEL

In this section, we will derive the expression of the basic reproductive number R_0, which will pave the way for the next stability analysis. And we present an analytical solution of *CPFB* for describing the dynamic of preferential information spreading process.

Theorem 1: Define the basic reproductive number

$$R_0 = \frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{\nu[\alpha_1(\gamma+\mu) + \delta\alpha_2]}{(\varepsilon+\delta+\mu)(\gamma+\mu)}.$$

When $R_0 < 1$, there is a group booking free equilibrium $E_0(1, 0, 0, 0)$ in system (1). When $R_0 > 1$, there is a group booking equilibrium $E_+(C_k^*, P_k^*, F_k^*, B_k^*)$ in system (1).

Proof: To get the group booking equilibrium $E_+(C_k^*, P_k^*, F_k^*, B_k^*)$, we need to make the right side of system (4) equal to zero, it should satisfy

$$\begin{cases} \omega + \eta B_k^*(t) - (\upsilon k\rho + \mu) C_k^*(t) = 0, \\ \upsilon k\rho C_k^*(t) - (\varepsilon + \delta + \mu) P_k^*(t) = 0, \\ \delta P_k^*(t) - (\gamma + \mu) F_k^*(t) = 0, \\ \varepsilon P_k^*(t) + \gamma F_k^*(t) - (\eta + \mu) B_k^*(t) = 0. \end{cases} \tag{8}$$

Then, we can get

$$\begin{cases} C_k^*(t) = \dfrac{(\gamma+\mu)(\delta+\varepsilon+\mu)}{\upsilon k\delta\rho} F_k^*(t), \\ P_k^*(t) = \dfrac{\gamma+\mu}{\delta} F_k^*(t), \\ B_k^*(t) = \dfrac{\varepsilon(\gamma+\mu)+\gamma\delta}{\delta(\eta+\mu)} F_k^*(t). \end{cases} \tag{9}$$

And then, according to the normalization condition $C_k(t) + P_k(t) + F_k(t) + B_k(t) = 1$, we can get

$$\begin{cases} C_k^*(t) = \dfrac{(\gamma+\mu)(\varepsilon+\delta+\mu)(\eta+\mu)}{H_k}, \\ P_k^*(t) = \dfrac{\upsilon k\rho(\gamma+\mu)(\eta+\mu)}{H_k}, \\ F_k^*(t) = \dfrac{\upsilon\delta k\rho(\eta+\mu)}{H_k}, \\ B_k^*(t) = \dfrac{\upsilon k\rho[\varepsilon(\gamma+\mu)+\gamma\delta]}{H_k}, \end{cases} \tag{10}$$

where

$$H_k = (\gamma+\mu)(\eta+\mu)(\varepsilon+\delta+\mu) + \upsilon k\rho(\gamma+\mu)(\eta+\mu) + \upsilon\delta k\rho(\eta+\mu) + \upsilon k\rho[\varepsilon(\gamma+\mu)+\gamma\delta].$$

Because of $\rho(t) = \frac{\sum_k kP(k)}{\langle k\rangle}\left(\alpha_1 P_k^*(t) + \alpha_2 F_k^*(t)\right)$, and from the system (10), we can easily find

$$\begin{aligned} \rho(\infty) &= \alpha_1\Theta_1(\infty) + \alpha_2\Theta_2(\infty) \\ &= \frac{\sum_k kP(k)}{\langle k\rangle}\left(\alpha_1\frac{(\gamma+\mu)}{\delta} + \alpha_2\right)F_k^* \\ &= f(\rho(\infty)). \end{aligned} \tag{11}$$

Clearly, $\rho(\infty) = 0$ is a solution of Eq. (11). To ensure the equation has a nontrivial solution, the following condition must be satisfied

$$\frac{d}{d\rho(\infty)}(f(\rho(\infty)))\big|_{\rho(\infty)=0} > 1 \text{ and } f(1) \le 1. \tag{12}$$

We can obtain the basic reproductive number

$$R_0 = \frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{\upsilon[\alpha_1(\gamma+\mu)+\delta\alpha_2]}{(\gamma+\mu)(\varepsilon+\delta+\mu)}, \tag{13}$$

where $\langle k^2\rangle = \sum_k k^2 P(k)$. Then we can easily find that $E_0(1, 0, 0, 0)$ is always a group booking free equilibrium of the system (4).

Next, we can also find that $0 < C_k^*(t) < 1$, $0 < P_k^*(t) < 1$, $0 < F_k^*(t) < 1$, $0 < B_k^*(t) < 1$. Hence, the group booking equilibrium $E_+(C_k^*, P_k^*, F_k^*, B_k^*)$ is easy to be defined. Therefore, if $R_0 > 1$, only one positive equilibrium $E_+(C_k^*, P_k^*, F_k^*, B_k^*)$ of system (4) exists. Hence, the information spreading will die out if $R_0 < 1$, and it will break out if $R_0 > 1$. The proof is completed.

A. THE GLOBAL ASYMPTOTIC STABILITY OF THE EQUILIBRIUM

Theorem 2: If $R_0 < 1$, the group booking free equilibrium E_0 of the system (4) is locally asymptotically stable, and it is unstable if $R_0 > 1$.

Proof: Since the initial conditions for the system (4) satisfies $C_k(t) + P_k(t) + F_k(t) + B_k(t) = 1$, we can get $C_k(t) = 1 - P_k(t) - F_k(t) - B_k(t)$. The system (4) can be equivalent to the following model:

$$\begin{cases} \dfrac{dP_k(t)}{dt} = \upsilon k\rho(1 - P_k(t) - F_k(t) - B_k(t)) \\ \qquad - (\varepsilon+\delta+\mu)P_k(t), \\ \dfrac{dF_k(t)}{dt} = \delta P_k(t) - (\gamma+\mu)F_k(t), \\ \dfrac{dB_k(t)}{dt} = \varepsilon P_k(t) + \gamma F_k(t) - (\eta+\mu)B_k(t). \end{cases} \tag{14}$$

The system (14) can be written as

$$\frac{dx}{dt} = u(x) - m(x),$$

and

$$\begin{aligned} x &= (P_k, F_k, B_k)^T, \\ u(x) &= \begin{pmatrix} \upsilon k\rho(1 - P_k - F_k - B_k) \\ 0 \\ 0 \end{pmatrix}, \\ m(x) &= \begin{pmatrix} (\varepsilon+\delta+\mu)P_k \\ (\gamma+\mu)F_k - \delta P_k \\ (\eta+\mu)B_k - \varepsilon P_k - \gamma F_k \end{pmatrix}, \end{aligned}$$

Because the Jacobian matrices of $u(x)$ and $m(x)$ at E_0 are as following:

$$\begin{aligned} U &= du(E_0) = \begin{pmatrix} U_{11} & U_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ M &= dm(E_0) = \begin{pmatrix} M_{11} & 0 & 0 \\ M_{21} & M_{22} & 0 \\ M_{31} & M_{32} & M_{33} \end{pmatrix}, \end{aligned}$$

and

$$M_{11} = -(\varepsilon+\delta+\mu)I, \quad M_{21} = \delta I, \; M_{22} = -(\gamma+\mu)I, \\ M_{31} = \varepsilon I, \quad M_{32} = \gamma I, \; M_{33} = -(\eta+\mu)I,$$

where I represents the identity matrix, and having

$$U_{11} = \frac{\upsilon\alpha_1}{\langle k\rangle}\begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix}[1P(1), 2P(2), \cdots, nP(n)],$$

$$U_{12} = \frac{\upsilon\alpha_2}{\langle k\rangle}\begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix}[1P(1), 2P(2), \cdots, nP(n)],$$

TABLE 1. The Routh-Hurwitz table.

λ^3	a_3	a_1
λ^2	a_2	a_0
λ^1	b_1	0
λ^0	a_0	

Next, Jacobian matrix at the group booking information free equilibrium E_0 is

$$J(E_0) = \begin{bmatrix} -(\varepsilon+\delta+\mu)+U_{11} & U_{12} & 0 \\ \delta & -(\gamma+\mu) & 0 \\ \varepsilon & \gamma & -(\eta+\mu) \end{bmatrix},$$

the eigenfunction of $J(E_0)$ is

$$|\lambda I - J(E_0)| = \begin{vmatrix} \lambda+(\varepsilon+\delta+\mu)-U_{11} & -U_{12} & 0 \\ -\delta & \lambda+(\gamma+\mu) & 0 \\ -\varepsilon & -\gamma & \lambda+(\eta+\mu) \end{vmatrix},$$

where the matrix I is identity matrix and λ is eigenvalue.

Hence, the eigenfunction of $J(E_0)$ is equal to

$$f(\lambda) = a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0. \tag{15}$$

we can obtain

$$\begin{aligned} a_3 &= 1, \\ a_2 &= 3\mu+\gamma+\varepsilon+\delta+\eta-U_{11}, \\ a_1 &= (\varepsilon+\delta+\mu)(2\mu+\gamma+\eta)+(\eta+\mu)(\mu+\gamma) \\ &\quad -[U_{11}(2\mu+\gamma+\eta)+\delta U_{12}], \\ a_0 &= (\eta+\mu)[(\gamma+\mu)(\varepsilon+\delta+\mu)-(\gamma+\mu)U_{11}-\delta U_{12}], \\ b_1 &= \left\{ \begin{array}{l} W[(\varepsilon+\delta+\mu)(2\mu+\gamma+\eta)+(\eta+\mu)(\mu+\gamma)] \\ +(\eta+\mu)[(\gamma+\mu)U_{11}+\delta U_{12}]-O \end{array} \right\} \Big/ W, \end{aligned}$$

where

$$\begin{aligned} W &= (3\mu+\gamma+\varepsilon+\delta+\eta-U_{11}), \\ O &= (\eta+\mu)(\gamma+\mu)(\varepsilon+\delta+\mu) \\ &\quad +(3\mu+\gamma+\varepsilon+\delta+\eta-U_{11})[(\gamma+\mu)U_{11}+\delta U_{12}], \end{aligned}$$

and

$$R_0 = \frac{\langle k^2\rangle}{\langle k\rangle}\cdot\frac{\nu[\alpha_1(\gamma+\mu)+\delta\alpha_2]}{(\gamma+\mu)(\varepsilon+\delta+\mu)} = \frac{(\gamma+\mu)U_{11}+\delta U_{12}}{(\gamma+\mu)(\varepsilon+\delta+\mu)}.$$

Based on the Routh-Hurwitz criterion, the Routh-Hurwitz table of the system (15) is written in the form as shown in Table 1.

And $b_1 = \frac{a_2a_1-a_3a_0}{a_2}$.

According to theorem in [44], the necessary condition for the system to be stable is that all the elements of the first column of Routh's array must have positive values. It means that the necessary condition for the model to be stable is that $a_3 > 0$, $a_2 > 0$, $b_1 > 0$, $a_0 > 0$.

Hence, if $R_0 < 1$, we can easily gain $a_0 > 0$, which also implies that $(\varepsilon+\delta+\mu) > U_{11}$. Similarly, if it is $R_0 < 1$, we can also find $(\varepsilon+\delta+\mu) > U_{11} + \delta/(\gamma+\mu)U_{12}$, which is equivalent to $(\varepsilon+\delta+\mu)(2\mu+\gamma+\eta) > (2\mu+\gamma+\eta)U_{11} + \delta\, U_{12}$. Therefore, we can find $a_1 > 0$. By adjusting the parameters, we can make $a_2a_1 > a_3a_0$ to mean. In the word, if $R_0 < 1$, we can verify $a_3 > 0$, $a_2 > 0$, $b_1 > 0$, $a_0 > 0$. Therefore, based on the existing works on stability analysis, we use the Routh-Hurwitz theorems to prove locally asymptotically stable for $R_0 < 1$. And if $R_0 > 1$, we find $a_0 < 0$. Then it implies the group booking free equilibrium E_0 is unstable.

Theorem 3: If $R_0 < 1$, the group booking free equilibrium E_0 is globally asymptotically stable; If there is $R_0 > 1$, the system (4) is permanent, i.e., there exists a $\xi > 0$, such that $\lim_{t\to\infty}\inf\{P_k(t), F_k(t), B_k(t)\}_{k=1}^n \geq \xi$, where $(P_k(t), F_k(t), B_k(t))$ is any solution of the system (4), satisfying $P_k(0) > 0$, $F_k(0) > 0$.

Proof: According to this paper, it is denoted that $P_i = iP(i)/\langle k\rangle$ and $n = k_{\max}$. Next, the Jacobian matrix at the group booking free equilibrium E_0 can be obtained from the equations (14).

$$L = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix},$$

where

$$A_{11} = \begin{pmatrix} -(\varepsilon+\delta+\mu)+\nu\alpha_1P_n & \nu\alpha_2P_n & 0 \\ \delta & -(\gamma+\mu) & 0 \\ \varepsilon & \gamma & -(\eta+\mu) \end{pmatrix},$$

$$A_{1n} = \begin{pmatrix} \nu\alpha_1P_n & \nu\alpha_2P_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_{n1} = \begin{pmatrix} \nu n\alpha_1P_1 & \nu n\alpha_2P_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$A_{nn} = \begin{pmatrix} -(\varepsilon+\delta+\mu)+\nu n\alpha_1P_n & n\nu\alpha_2P_n & 0 \\ \delta & -(\gamma+\mu) & 0 \\ \varepsilon & \gamma & -(\eta+\mu) \end{pmatrix},$$

Solving the characteristic equation directly

$$(\lambda+\eta+\mu)^n(\lambda+\varepsilon+\delta+\mu)^{n-1} \times (\lambda+\gamma+\mu+\beta)^{n-1}(\lambda^2+p\lambda+q) = 0,$$

where $p = 2\mu+\varepsilon+\delta+\gamma-\nu\alpha_1\sum_{k=1}^n kP(k)$, and

$$q = (\gamma+\mu)(\varepsilon+\delta+\mu) - [\nu\alpha_1(\gamma+\mu)+\nu\delta\alpha_2]\sum_{k=1}^n kP(k).$$

It is easy to see that $R_0 < 1$. And we can easily find

$$(\varepsilon+\delta+\mu)(\gamma+\mu) < \nu[\alpha_1(\gamma+\mu)+\delta\alpha_2]\sum_{k=1}^n kP(k),$$

where

$$(\varepsilon+\delta+\mu) < \nu\alpha_1\sum_{k=1}^n kP(k) \text{ and } (\mu+\gamma) > 0.$$

Hence, if $R_0 < 1$, we can obtain $q < 0$. However, because of $q < 0$. we can obtain $2\mu+\gamma+\varepsilon+\delta < \nu\alpha_1\sum_{k=1}^n kP(k)$, which means that $p < 0$. In a word, when $R_0 < 1$, all real eigenvalues of the matrix L are negative. When $R_0 > 1$, the matrix L has and only has a unique positive eigenvalue. According to Perron-Frobenius theorem, this indicates that when $R_0 > 1$. Then, the maximum value of the real part of all eigenvalues of the matrix L is positive. Finally, a theorem of Lajmanovich and York [45] yields the results of this theorem. The proof is completed.

B. THE GLOBAL ATTRACTIVITY OF THE EQUILIBRIUM

Lemma 1 [46]: If $a > 0$, $b > 0$ and $\frac{dx(t)}{dt} \geq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $\lim_{t\to\infty} \inf x(t) \geq \frac{b}{a}$; if $a > 0$, $b > 0$ and $\frac{dx(t)}{dt} \leq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $\lim_{t\to\infty} \sup x(t) \leq \frac{b}{a}$.

Next, a novel monotone iterative algorithm in [47] is used to discuss the global attractivity of the equilibrium point of cluster information.

Theorem 4: If we suppose that $(P_k(t), F_k(t), B_k(t))$ is a solution of the system (14). It should satisfy $P_k(0) > 0$, $F_k(0) > 0$, $B_k(0) > 0$. Therefore, if $R_0 > 1$, then $\lim_{t\to\infty} (P_k(t), F_k(t), B_k(t)) = \left(P_k^*(t), F_k^*(t), B_k^*(t)\right)$. However, $\left(P_k^*(t), F_k^*(t), B_k^*(t)\right)$ is the group booking equilibrium of the system (14) for $k = 1, 2, \cdots, n$.

Proof: Assuming that k is any positive integer in $\{1, 2, \cdots, n\}$ as follow. The theorem tells us that there is a normal number $0 < \varphi < 1/3$, and a big enough constant of $T > 0$. Such that $P_k(t) \geq \varphi$, $F_k(t) \geq \varphi$ for all $t > T$, which is true. So, $\rho(t) > \varphi(\alpha_1 + \alpha_2)$ for $t > T$. Though the first equation in the system (14) with these shows

$$\frac{dP_k(t)}{dt} \leq \upsilon k(\alpha_1 + \alpha_2)(1 - P_k(t)) - (\varepsilon + \delta + \mu) P_k(t), \quad t > T.$$

According to the comparison theorem of differential equation theory, for any given normal number

$$0 < \varphi_1 < \frac{\varepsilon + \mu + \delta}{2[\nu k(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)]}.$$

There exists a $t_1 > T$, such that $P_k(t) \leq X_k^{(1)} - \varphi_1$ for $t > t_1$, there is

$$X_k^{(1)} = \frac{\nu k(\alpha_1 + \alpha_2)}{\nu k(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)} + 2\varphi_1 < 1.$$

According to the second equation in system (14) with there shows

$$\frac{dF_k(t)}{dt} \leq \delta(1 - F_k(t)) - (\gamma + \mu)F_k(t), \quad t > t_1.$$

Therefore, for any given normal constant

$$0 < \varphi_1 < \min\left\{\frac{1}{2}, \varphi_1, \frac{\gamma + \mu}{2[\gamma + \mu + \delta]}\right\},$$

there exists a $t_2 > t_1$, such that $F_k(t) \leq Y_k^{(1)} - \varphi_2$, there is

$$Y_k^{(1)} = \frac{\delta}{\gamma + \mu + \delta} + 2\varphi_2 < 1.$$

On the other hand, we can get

$$\frac{dB_k(t)}{dt} \leq (\varepsilon + \gamma)(1 - B_k(t)) - (\mu + \eta) B_k(t), \quad t > t_2.$$

Therefore, as follows any given constant

$$0 < \varphi_3 < \min\left\{\frac{1}{3}, \varphi_2, \frac{\eta + \mu}{2[\varepsilon + \gamma + \eta + \mu]}\right\},$$

there exists a $t_3 > t_2$, such that $B_k(t) \leq Z_k^{(1)} - \varphi_3$ for $t > t_3$, where

$$Z_k^{(1)} = \frac{\varepsilon + \gamma}{\varepsilon + \gamma + \eta + \mu} + 2\varphi_3 < 1.$$

On the other hand, if we replace $P_k(t) \geq \varphi$, $F_k(t) \geq \varphi$ and $\rho(t) > \varphi(\alpha_1 + \alpha_2)$ into the first equation of the system (14), we can get

$$\begin{aligned}\frac{dP_k(t)}{dt} &\geq \nu k\varphi(\alpha_1 + \alpha_2)(1 - P_k(t) - F_k(t) - B_k(t)) \\ &\quad - (\varepsilon + \mu + \delta) P_k(t) \\ &= \nu k\varphi(\alpha_1 + \alpha_2)(1 - F_k(t) - B_k(t)) \\ &\quad - [\nu k\varphi(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)] P_k(t) \\ &\geq \nu k\varphi(\alpha_1 + \alpha_2)\left(1 - Y_k^{(1)} - Z_k^{(1)}\right) \\ &\quad - [\nu k\varphi(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)] P_k(t), \quad t > T.\end{aligned}$$

Then, as follows any given constant

$$0 < \varphi_4 < \min\left\{\frac{1}{4}, \varphi_3, \frac{\nu k\varphi(\alpha_1 + \alpha_2)\left(1 - Y_k^{(1)} - Z_k^{(1)}\right)}{2[\nu k\varphi(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)]}\right\},$$

there exists a $t_4 > t_3$, So, there is $P_k(t) \leq x_k^{(1)} + \varphi_4$ for $t > t_4$, then

$$x_k^{(1)} = \frac{\nu k\varphi(\alpha_1 + \alpha_2)\left(1 - Y_k^{(1)} - Z_k^{(1)}\right)}{\nu k\varphi(\alpha_1 + \alpha_2) + (\varepsilon + \mu + \delta)} - 2\varphi_4 > 0.$$

Hence, as the second equation of the system (14), where

$$\frac{dF_k(t)}{dt} \geq \delta x_k^{(1)} - (\gamma + \mu) F_k(t), \quad t > t_4.$$

Therefore, for any given normal number

$$0 < \varphi_5 < \min\left\{\frac{1}{5}, \varphi_4, \frac{\delta x_k^{(1)}}{2[\gamma + \mu]}\right\},$$

there exists a $t_5 > t_4$, So, there is $F_k(t) \leq y_k^{(1)} + \varphi_5$ for $t > t_5$, where

$$y_k^{(1)} = \frac{\delta x_k^{(1)}}{\gamma + \mu} - 2\varphi_5 > 0.$$

Next, we have

$$\frac{dB_k(t)}{dt} \geq (\varepsilon + \gamma) x_k^{(1)} - (\eta + \mu) B_k(t), \quad t > t_5.$$

Hence, for any given a constant

$$0 < \varphi_6 < \min\left\{\frac{1}{6}, \varphi_5, \frac{(\varepsilon + \gamma) x_k^{(1)}}{2(\eta + \mu)}\right\},$$

there exists a $t_6 > t_5$, So, there is $B_k(t) \le z_k^{(1)} + \varphi_6$ for $t > t_6$, where

$$z_k^{(1)} = \frac{(\varepsilon+\gamma)\, x_k^{(1)}}{\eta+\mu} - 2\varphi_6 > 0$$

Since φ is a very small constant, it has that $0 < x_k^{(1)} < X_k^{(1)} < 1$, $0 < y_k^{(1)} < Y_k^{(1)} < 1$, $0 < z_k^{(1)} < Z_k^{(1)} < 1$. Next

$$q^{(j)} = \sum_{i=1}^{n} G_i\left(\alpha_1 x_i^j + \alpha_2 y_i^{(j)}\right),$$
$$Q^{(j)} = \sum_{i=1}^{n} G_i\left(\alpha_1 X_i^j + \alpha_2 Y_i^{(j)}\right), \quad j = 1, 2, \cdots.$$

Similarly, through the first equation in the system (14). where

$$\begin{aligned}\frac{dP_k(t)}{dt} &\le \nu k Q^{(1)}\left(1 - P_k(t) - y_k^{(1)} - z_k^{(1)}\right) \\ &\quad - (\varepsilon+\mu+\delta)\, P_k(t) \\ &= \nu k Q^{(1)}\left(1 - y_k^{(1)} - z_k^{(1)}\right) \\ &\quad - \left[\nu k Q^{(1)} + (\varepsilon+\mu+\delta)\right] P_k(t), \quad \mathrm{t} > \mathrm{t}\end{aligned}$$

Hence, for any given constant $0 < \varphi_7 < \min\left\{1/7, \varphi_7\right\}$, there exists a $t_7 > t_6$, such that

$$\begin{aligned}P_k(t) &\le X_k^{(2)} \\ &\le \min\left\{X_k^{(1)} - \varphi_1, \frac{\nu k Q^{(1)}\left(1 - y_k^{(1)} - z_k^{(1)}\right)}{\nu k Q^{(1)} + \varepsilon+\mu+\delta} + \varphi_7\right\}, \\ &\qquad t > t_7.\end{aligned}$$

Therefore, by the second equation in the system (14),

$$\frac{dF_k(t)}{dt} \le \delta X_k^{(2)} - (\gamma+\mu)\, F_k(t), \quad t > t_7.$$

So, for any given constant $0 < \varphi_8 < \min\left\{1/8, \varphi_7\right\}$, there exists a $t_8 > t_7$, such that

$$F_k(t) \le Y_k^{(2)} \le \min\left\{Y_k^{(1)} - \varphi_2, \frac{\delta X_k^{(2)}}{\gamma+\mu} + \varphi_8\right\}, \quad t > t_8.$$

Next, by the third equation in the system (14), having

$$\frac{dB_k(t)}{dt} \le (\varepsilon+\gamma)\, X_k^{(2)} - (\mu+\eta) B_k(t), \quad t > t_8.$$

Hence, for any given constant $0 < \varphi_9 < \min\left\{1/9, \varphi_8\right\}$, there exists a $t_9 > t_8$, such that

$$B_k(t) \le Z_k^{(2)} \le \min\left\{Z_k^{(1)} - \varphi_3, \frac{(\gamma+\varepsilon)\, X_k^{(2)}}{\eta+\mu} + \varphi_9\right\}, \quad t > t_9.$$

Then, turning back to the system (14), we can get

$$\begin{aligned}\frac{dP_k(t)}{dt} &\ge \nu k q^{(1)}\left(1 - P_k(t) - Y_k^{(2)} - Z_k^{(2)}\right) \\ &\quad - (\varepsilon+\mu+\delta)\, P_k(t), \quad t > t_9.\end{aligned}$$

Hence, for any given constant

$$0 < \varphi_{10} < \min\left\{\frac{1}{10}, \varphi_9, \frac{\nu k q^{(1)}\left(1 - Y_k^{(2)} - Z_k^{(2)}\right)}{2\left[\nu k q^{(1)} + (\varepsilon+\mu+\delta)\right]}\right\},$$

there exists a $t_{10} > t_9$, So, there is $P_k(t) \le x_k^{(1)} + \varphi_{10}$ for $t > t_{10}$, where

$$x_k^{(2)} = \max\left\{x_k^{(1)} + \varphi_4, \frac{\nu k q^{(1)}\left(1 - Y_k^{(2)} - Z_k^{(2)}\right)}{\nu k q^{(1)} + \varepsilon+\mu+\delta} - 2\varphi_{10}\right\}.$$

Thus,

$$\frac{dF_k(t)}{dt} \ge \delta x_k^{(2)} - (\gamma+\mu) F_k(t), \quad t > t_{10}.$$

So, for any given constant

$$0 < \varphi_{11} < \min\left\{\frac{1}{11}, \varphi_{10}, \frac{\delta x_k^{(2)}}{2(\gamma+\mu)}\right\},$$

there exists a $t_{11} > t_{10}$, So, there is $F_k(t) \le y_k^{(1)} + \varphi_{11}$ for $t > t_{11}$, where

$$y_k^{(2)} = \max\left\{y_k^{(1)} + \varphi_5, \frac{(\nu\beta+\delta)\, x_k^{(2)}}{\gamma+\mu} - 2\varphi_{11}\right\}.$$

Next,

$$\frac{dB_k(t)}{dt} \ge (\varepsilon+\gamma)\, x_k^{(2)} - (\mu+\eta) B_k(t), \quad t > t_{11}.$$

Hence, for any given constant

$$0 < \varphi_{12} < \min\left\{\frac{1}{12}, \varphi_{11}, \frac{(\varepsilon+\gamma)\, x_k^{(2)}}{2(\eta+\mu)}\right\},$$

there exists a $t_{12} > t_{11}$, So, there is $B_k(t) \le z_k^{(1)} + \varphi_{12}$ for $t > t_{12}$, where

$$z_k^{(2)} = \max\left\{z_k^{(1)} + \varphi_6, \frac{(\gamma+\varepsilon)\, x_k^{(2)}}{\eta+\mu} - 2\varphi_{12}\right\}.$$

However, we can carry out step $h\,(h = 3, 4, \cdots)$ of the calculation and obtain six sequence: $\left\{X_k^{(h)}\right\}$, $\left\{Y_k^{(h)}\right\}$, $\left\{Z_k^{(h)}\right\}$, $\left\{x_k^{(h)}\right\}$, $\left\{y_k^{(h)}\right\}$ and $\left\{z_k^{(h)}\right\}$. Because the first three sequences are monotone increasing and the last three sequences are strictly monotone decreasing, there exists a large positive integer M, when there is $h \ge M$, we can get

$$x_k^{(h)} = \frac{\nu k q^{(h-1)}\left(1 - Y_k^{(h-1)} - Z_k^{(h-1)}\right)}{\nu k q^{(h-1)} + \varepsilon+\mu+\delta} - 2\varphi_{6h-2},$$
$$y_k^{(h)} = \frac{\delta x_k^{(h)}}{\gamma+\mu} - 2\varphi_{6h-1},\ z_k^{(h)} = \frac{(\varepsilon+\gamma)\, x_k^{(h)}}{\eta+\mu} - 2\varphi_{6h},$$
$$Z_k^{(h)} = \frac{(\varepsilon+\gamma)\, X_k^{(h)}}{\eta+\mu} + \varphi_{6h-3},$$

$$X_k^{(h)} = \frac{vkQ^{(h-1)}\left(1 - y_k^{(h-1)} - z_k^{(h-1)}\right)}{vkQ^{(r-1)} + \varepsilon + \mu + \delta} + \varphi_{6h-5},$$
$$Y_k^{(h)} = \frac{\delta X_k^{(h)}}{\gamma + \mu} + \varphi_{6h-4}.$$

So, we can get that

$$\begin{cases} x_k^{(h)} \le P_k(t) \le X_k^{(h)} \\ y_k^{(h)} \le F_k(t) \le Y_k^{(h)}, \\ z_k^{(h)} \le B_k(t) \le Z_k^{(h)} \end{cases} \quad t > t_{6h}. \tag{16}$$

Since the sequence limit in system (14) exists, supposing that $\lim_{t\to\infty} \Omega^{(h)} = \Omega_k$, where $\Omega_k^{(h)} \in \left\{X_k^{(h)}, Y_k^{(h)}, Z_k^{(h)}, x_k^{(h)}, y_k^{(h)}, z_k^{(h)}, Q_k^{(h)}, q_k^{(h)}\right\}$ and $\Omega_k \in \{X_k, Y_k, Z_k, x_k, y_k, z_k, Q_k, q_k\}$. If there is $0 < \varphi_h < \frac{1}{h}$, with $h \to \infty$ and $\varphi_h \to 0$, we can calculate the equation of the system (14), such that

$$\begin{cases} X_k = \dfrac{vkQ(1 - y_k - z_k)}{vkQ + \varepsilon + \mu + \delta}, Y_k = \dfrac{\delta X_k}{\gamma + \mu}, Z_k = \dfrac{(\varepsilon + \gamma) X_k}{\eta + \mu}, \\ x_k = \dfrac{vkq(1 - Y_k - Z_k)}{vkq + \varepsilon + \delta + \mu}, y_k = \dfrac{\delta x_k}{\gamma + \mu}, z_k = \dfrac{(\varepsilon + \gamma) x_k}{\eta + \mu}, \end{cases} \tag{17}$$

where

$$q = \sum_{i=1}^{n} P_i(\alpha_1 x_i + \alpha_2 y_i), \quad Q = \sum_{i=1}^{n} P_i(\alpha_1 X_i + \alpha_2 Y_i).$$

So,

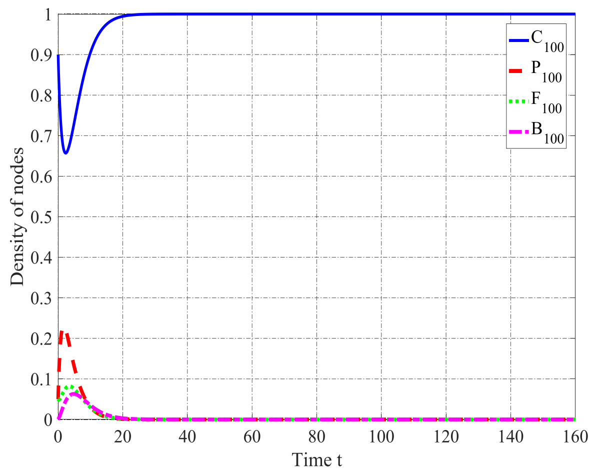
$$\begin{cases} Y_k = \dfrac{vkQ(\gamma+\mu)(\eta+\mu)}{H_k} \begin{Bmatrix} (vkq+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -vkq[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix}, \\ y_k = \dfrac{vkq(\gamma+\mu)(\eta+\mu)}{H_k} \begin{Bmatrix} (vkQ+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -vkQ[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix}, \end{cases} \tag{18}$$

where

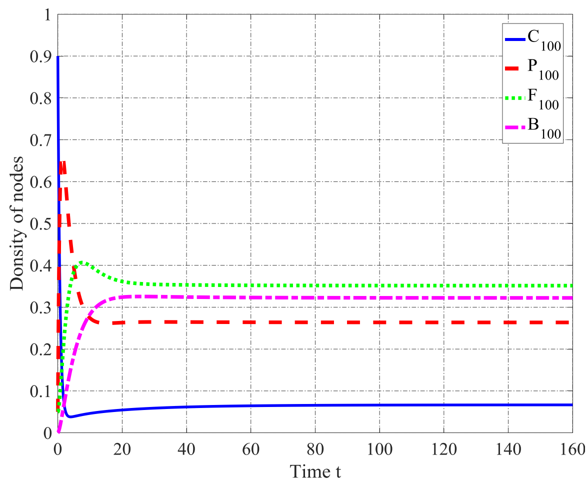
$$H_k = (vkq+\varepsilon+\delta+\mu)(vkQ+\varepsilon+\delta+\mu)(\gamma+\mu)^2(\eta+\mu)^2 - v^2k^2qQ[\delta(\eta+\mu) + (r+\mu)(\varepsilon+\gamma)]^2.$$

Replacing (17) and (18) into Q and q, respectively, we can

$$\begin{aligned} 1 = {} & [\alpha_1\delta(\eta+\mu)] \\ & \times \sum_{i=1}^{n} \frac{ivG_i}{H_i} \begin{Bmatrix} (viQ+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -viQ[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix} \\ & + \alpha_2(\varepsilon+\gamma)(\gamma+\mu) \\ & \times \sum_{i=1}^{n} \frac{ivG_i}{H_i} \begin{Bmatrix} (viQ+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -viQ[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix}, \\ 1 = {} & [\alpha_1\delta(\eta+\mu)] \\ & \times \sum_{i=1}^{n} \frac{ivG_i}{H_i} \begin{Bmatrix} (viq+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -viq[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix} \\ & + \alpha_2(\varepsilon+\gamma)(\gamma+\mu) \\ & \times \sum_{i=1}^{n} \frac{ivG_i}{H_i} \begin{Bmatrix} (viq+\varepsilon+\delta+\mu)(\gamma+\mu)(\eta+\mu) \\ -viq[\delta(\eta+\mu)+(\gamma+\mu)(\varepsilon+\gamma)] \end{Bmatrix}. \end{aligned}$$



(a) $R_0 = 0.5056 < 1$



(b) $R_0 = 7.2800 > 1$

FIGURE 3. The time series and orbits of four states with $R_0 < 1$, $R_0 > 1$ and initial values $C(0) = 0.9$, $P(0) = 0.05$, $F(0) = 0.05$, $B(0) = 0$.

Hence, we can get

$$(q-Q)\{\delta(\alpha_1-1)(\eta+\mu)+\alpha_2(\varepsilon+\gamma)(\gamma+\mu)\}\sum_{i=1}^{n}\frac{iv^2G_i}{H_i} \equiv 0.$$

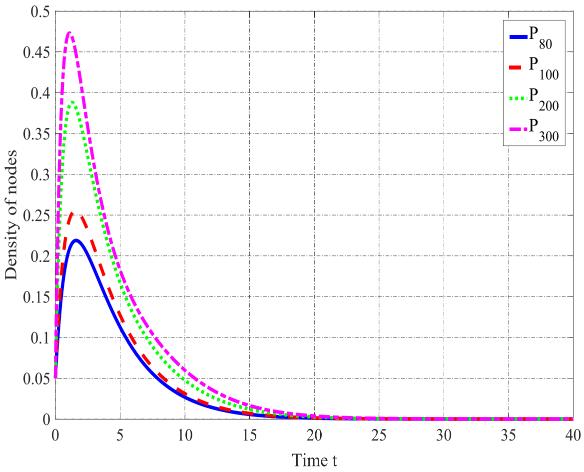
It means that $Q = q$, So,

$$\sum_{i=1}^{n} G_i[\alpha_1(X_i - x_i) + \alpha_2(Y_i + y_i)] = 0,$$

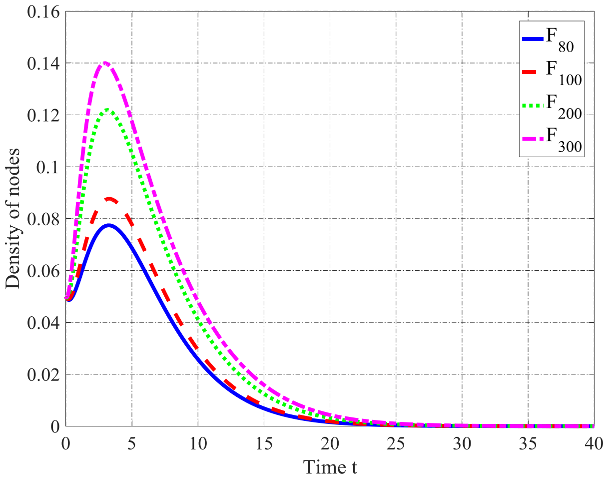
while is equivalent to $X_i = x_i$ and $Y_i = y_i$ for $1 \le i \le n$. Next, according to the equation of the system (17) and (18), we can find that

$$\lim_{t\to\infty} P_k(t) = X_k = x_k, \quad \lim_{t\to\infty} F_k(t) = Y_k = x_k,$$
$$\lim_{t\to\infty} B_k(t) = Z_k = z_k.$$

Finally, by substituting $Q = q$ into equation (18) to get $X_k = P_k^{\infty}$, $Z_k = B_k^{\infty}$, $Y_k = F_k^{\infty}$, from which the conclusion can be obtained. This evidence is complete, and we know that there always exists the group booking equilibrium when $R_0 > 1$.

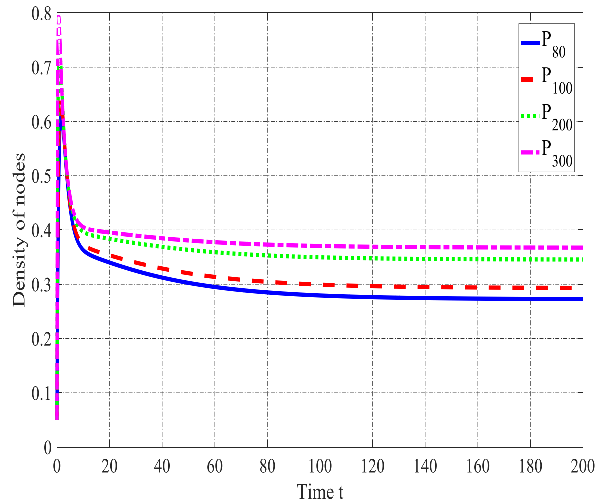


(a) Individuals of participant $P(t)$

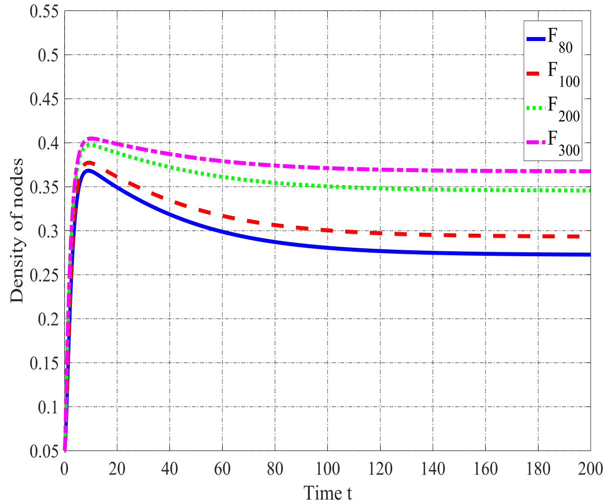


(b) Individuals of forwarder $F(t)$

FIGURE 4. The time series and orbits of the participants or forwarders with $R_0 < 1$ and $k = 80, 100, 200, 300$.



(a) Individuals of participant $P(t)$



(b) Individuals of forwarder $F(t)$

FIGURE 5. The time series and orbits of the participants or forwarders with $R_0 > 1$ and $k = 80, 100, 200, 300$.

IV. NUMERICAL SIMULATIONS

A. DISCUSSION OF PARAMETER SETTING

First of all, we know that some model parameters have an effect on the basic reproductive number, next, we can get

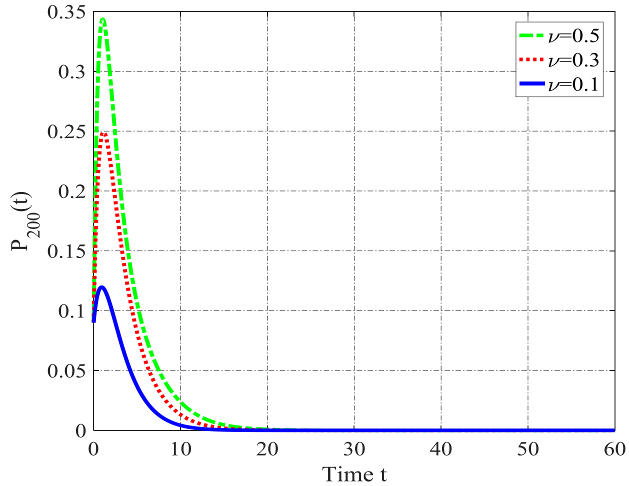
$$\begin{aligned}
\frac{\partial R_0}{\partial \alpha_1} &= \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\nu}{(\varepsilon + \delta + \mu)} > 0, \\
\frac{\partial R_0}{\partial \alpha_2} &= \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\nu \delta}{(\varepsilon + \delta + \mu)(\gamma + \mu)} > 0, \\
\frac{\partial R_0}{\partial \nu} &= \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\alpha_1 (\gamma + \mu) + \delta \alpha_2}{(\varepsilon + \delta + \mu)(\gamma + \mu)^2} > 0, \\
\frac{\partial R_0}{\partial \varepsilon} &= -\frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\nu [\alpha_1 (\gamma + \mu) + \delta \alpha_2]}{(\varepsilon + \delta + \mu)^2 (\gamma + \mu)} < 0, \\
\frac{\partial R_0}{\partial \gamma} &= -\frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\delta \alpha_2}{(\varepsilon + \delta + \mu)(\gamma + \mu)^2} < 0, \\
\frac{\partial R_0}{\partial \delta} &= \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\alpha_2 (\varepsilon + \delta + \mu) - \nu [\alpha_1 (\varepsilon + \delta) + \delta \alpha_2]}{(\varepsilon + \delta + \mu)^2 (\gamma + \mu)},
\end{aligned}$$

$$\frac{\partial R_0}{\partial \mu} = \frac{\langle k^2 \rangle}{\langle k \rangle} \cdot \frac{\nu \{\alpha_1 (\gamma + \mu)(\varepsilon + \delta + \mu) - D\}}{(\varepsilon + \delta + \mu)^2 (\gamma + \mu)^2},$$

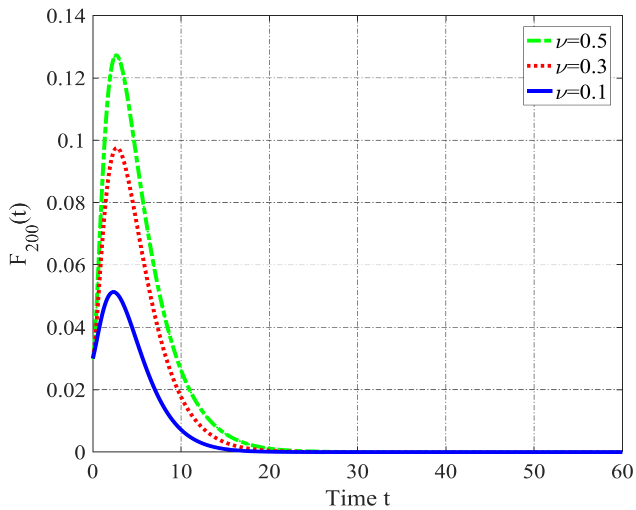
where

$$D = [\alpha_1 (\gamma + \mu) + \delta \alpha_2] (2\mu + \varepsilon + \delta + \gamma)$$

Hence, we can find that increasing the discount rate ν, the participating rate α_1 and the forwarding rate α_2, respectively, will promote the preferential propagation of online group booking. The change of the repurchase intention rate η has no effect on the value of the basic reproductive number R_0. And the discount rate ν can enhance the spreading of the group booking preferential information. However, reducing the probability γ, ε, respectively, will decrease the preferential propagation of online group booking. For forwarding probability δ and exit rate μ, it is necessary to consider the values other different parameters. What's more, the preferential propagation of online group booking is promoted, which

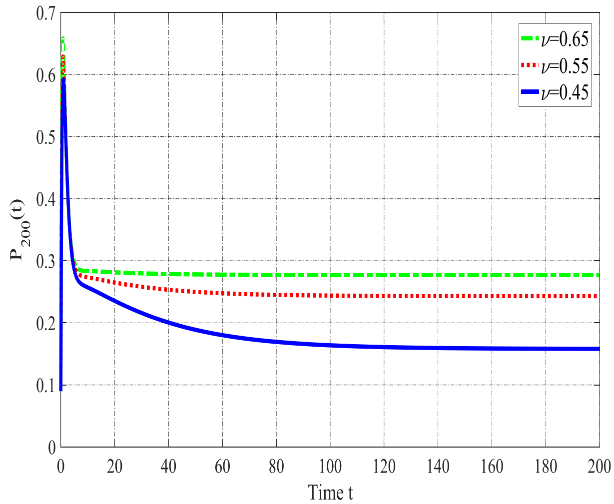


(a) Individuals of participant $P(t)$

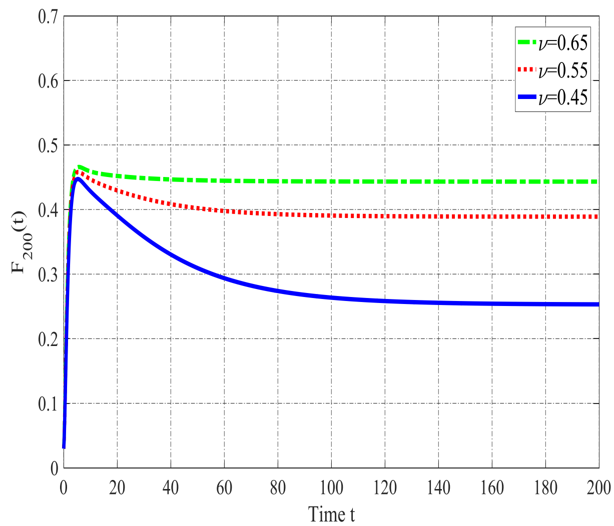


(b) Individuals of forwarder $F(t)$

FIGURE 6. Prevalence $P_{200}(t)$, $F_{200}(t)$ versus t corresponding to different ν with $R_0 < 1$ and initial values $P(0) = 0.09$, $F(0) = 0.03$.



(a) Individuals of participant $P(t)$



(b) Individuals of forwarder $F(t)$

FIGURE 7. Prevalence $P_{200}(t)$, $F_{200}(t)$ versus t corresponding to different ν with $R_0 > 1$ and initial values $P(0) = 0.09$, $F(0) = 0.03$.

is helping the e-commerce company forecasts in the future situation of sale and formulating a plan of beneficial sale.

In Figure 2, the size of the basic reproductive number R_0 is changed with the diffident value of μ, δ. If the parameters are chosen as $\eta = 0.1$, $\nu = 0.4$, $\alpha_1 = 0.3$, $\alpha_2 = 0.6$, $\varepsilon = 0.1$, $\gamma = 0.3$. We can clearly see that if larger forwarding probability δ or small exit rate μ can lead to larger value of the basic reproductive number R_0, which means actively forwarding the information and preventing the exit can increase the spread of the group booking preferential information.

B. THE GLOBAL ATTRACTIVITY OF THE EQUILIBRIUM

In this part, we conduct theoretical simulation on the numerical value. The *CPFB* model in this paper is in scale-free networks with degree distribution of $P(k) = \omega k^{-3}$, and the parameter is $\sum_{k=0}^{n} \omega k = 1$, $n = 1000$.

In Figure 3(a), if the parameters are chosen as $\mu = 0.3$, $\eta = 0.1$, $\nu = 0.2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.6$, $\delta = 0.2$, $\varepsilon = 0.1$, $\gamma = 0.15$, the basic reproductive number $R_0 = 0.5056 < 1$. In figure 3(b), if the parameters are chosen as $\mu = 0.1$, $\eta = 0.1$, $\nu = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\delta = 0.1$, $\varepsilon = 0.05$, $\gamma = 0.1$, the basic reproductive number $R_0 = 7.2800 > 1$. We can clearly see that there is almost no transmission of the online group booking preferential information when $R_0 < 1$, which means that online group booking preferential information will eventually disappear. It also suggests that the group booking free equilibrium E_0 is globally asymptotically stable when $R_0 < 1$. We can see that group booking is permanent on the scale-free networks when $R_0 > 1$, the group booking preferential information will exist.

In Figure 4(a) and (b), the parameters are chosen as $\mu = 0.1$, $\eta = 0.1$, $\nu = 0.2$, $\alpha_1 = 0.2$, $\alpha_2 = 0.4$, $\delta = 0.2$, $\varepsilon = 0.3$,

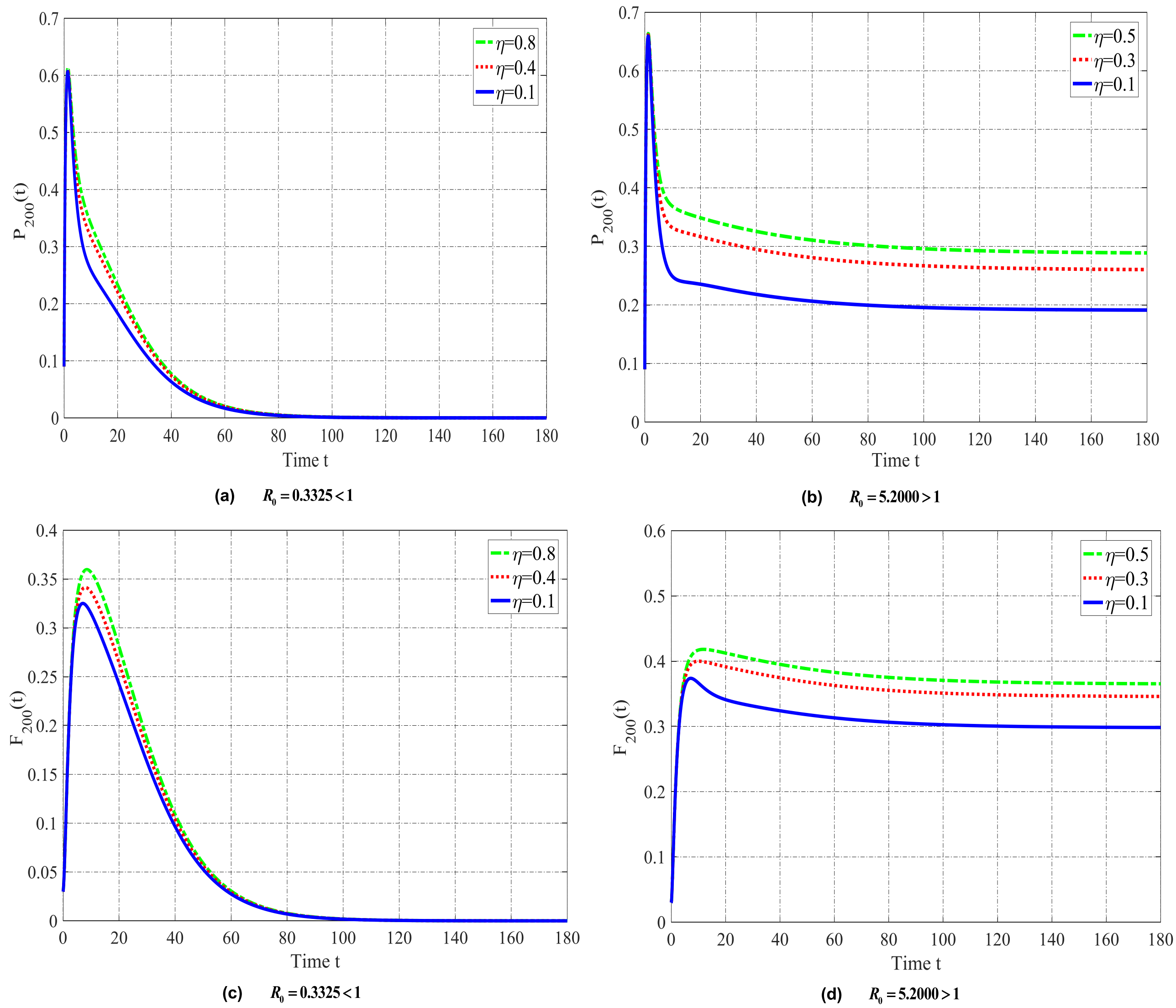


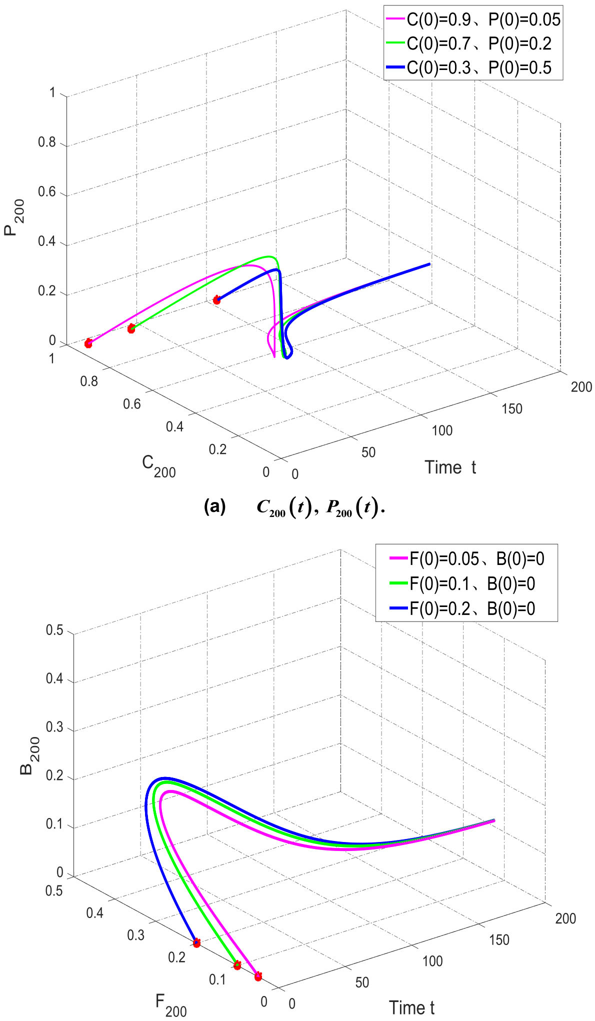
FIGURE 8. **Prevalence $P_{200}(t)$, $F_{200}(t)$ versus t corresponding to different η.**

$\gamma = 0.35$, the basic reproductive number $R_0 = 0.5730 < 1$. The Figure 4 describe the time series of the participants $P(t)$ or forwarders $F(t)$ with different degree k. Apparently, we can see that when $R_0 < 1$, $P(t)$ and $F(t)$ both grow to a positive constant with the increasing of degree k. It can be seen that the larger degree k can lead to the faster disappear spreading preferential information in online group purchase.

In Figure 5(a) and (b), the parameters are chosen as $\mu = 0.1$, $\eta = 0.3$, $\nu = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\delta = 0.2$, $\varepsilon = 0.05$, $\gamma = 0.1$, the basic reproductive number $R_0 = 5.2000 > 1$. The Figure 5 describe the time series of the participants $P(t)$ and forwarders $F(t)$ with different degree of k. Apparently, we can see that $P(t)$ and $F(t)$ both grow to a positive constant with the increasing of degree k when $R_0 > 1$. We can also see that the larger degree k can increase the convergence value, which means the larger k can lead to the more people spreading preferential information in online group booking.

In Figure 6(a) and (b), the parameters are chosen as $\mu = 0.1$, $\eta = 0.1$, $\alpha_1 = 0.2$, $\alpha_2 = 0.4$, $\delta = 0.2$, $\varepsilon = 0.4$, $\gamma = 0.45$, the basic reproductive number $R_0 < 1$. As shown in Figure 6, the smaller the discount rate ν is, the fewer people will spread the preferential information in the online group booking. Apparently, we can find that the smaller ν can accelerate the disappearance of the preferential information.

In Figure 7(a) and (b), the parameters are chosen as $\mu = 0.1$, $\eta = 0.3$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\delta = 0.4$, $\varepsilon = 0.1$, $\gamma = 0.15$, the basic reproductive number $R_0 > 1$. As shown in Figure 7, the larger the discount rate ν is, the faster people will spread the preferential information in the online group booking. Apparently, we can find that the larger ν can lead to greater convergence value at the group booking equilibrium.



(a) $C_{200}(t)$, $P_{200}(t)$.

(b) $F_{200}(t)$, $B_{200}(t)$

FIGURE 9. Prevalence $C_{200}(t)$, $P_{200}(t)$, $F_{200}(t)$, $B_{200}(t)$ corresponding to different initial values with $R_0 > 1$.

In Figure 8(a) and (c), the parameters are chosen as $\mu = 0.1$, $\nu = 0.2$, $\alpha_1 = 0.3$, $\alpha_2 = 0.6$, $\delta = 0.2$, $\varepsilon = 0.1$, $\gamma = 0.1$. The Figure 8 describe the time series of the participants $P(t)$ and forwarders $F(t)$ with different rate η of the repurchase intention rate. Under the condition that the basic reproductive number $R_0 = 0.3325 < 1$, it can be seen that a smaller the rate η of repurchase intention can accelerate the disappearance of the group booking preferential information. In Figure 8(b) and (d), if the parameters are chosen as $\mu = 0.1$, $\nu = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\delta = 0.2$, $\varepsilon = 0.05$, $\gamma = 0.1$. Under the condition that the basic reproductive number $R_0 = 5.2000 > 1$, it can be seen that when η is larger, the convergence value is greater at the group booking equilibrium. Simultaneously, we can easily find that the size of the repurchase intention rate η does not affect the basic reproductive number R_0.

In Figure 9(a) and (b), the parameters are chosen as $\mu = 0.1$, $\eta = 0.2$, $\nu = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.6$, $\delta = 0.2$, $\varepsilon = 0.1$, $\gamma = 0.15$, the basic reproductive number $R_0 > 1$. As time t changes, we can find that the density of $C_{200}(t)$, $P_{200}(t)$, $F_{200}(t)$ and $B_{200}(t)$ is no longer changes. And we can also find that no matter how the number of individuals in initial network, the population of individuals will eventually stabilize to a fixed value as long as other parameters in our model remains unchanged.

V. CONCLUSION

In this paper, in order to study the impact of the products' discount rate, the repurchase intention and the heterogeneity of the networks on the online dissemination of preferential information, we have proposed a novel *CPFB* model based on scale-free networks. Through the topology of networks and mean-field theory, we have analyzed the dynamic propagation of preferential information in group booking. The theoretical results of this paper were verified and proved to be consistent with the reality. We also determined that the diffusion dynamics of this model depend on the basic reproductive number R_0. If $R_0 < 1$, the group booking free equilibrium is globally stability, which means the spreading of group booking preferential information will eventually disappear regardless of the initial values of the participants and the forwarders. If $R_0 > 1$, the preferential information spreading is permanent and the group booking equilibrium is globally asymptotically stable, which means the spreading of preferential information about online group booking will persist and converge to a positive level. Furthermore, we investigated the impact of the parameter ν of the discount rate about online products, which can control the spreading of group booking preferential information. We found that the repurchase intention rate η can promote the spreading of preferential information about online group booking. The size of the repurchase intention rate η does not affect the basic reproductive number R_0. The study has a vital guiding significance in studying the spreading dynamics of the group booking preferential information in the heterogeneous networks and controlling strategies of group booking.

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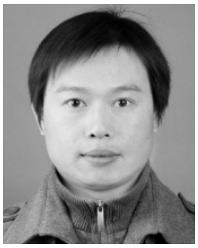

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