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New Improved Normal Parameter Reduction Method for Fuzzy Soft Set

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ABSTRACT There are many redundant parameters in the actual decision-making process based on fuzzy soft sets. Kong et al. [Z. Kong, J. Ai, L. Wang, P. Li, L. Ma, and F. Lu, "New Normal Parameter Reduction Method in Fuzzy Soft Set Theory," in IEEE Access, vol. 7, pp. 2986-2998, 2019] described the parameter reduction method with regard to four special dispensable sets based on score decision criteria. However, this method is complicated and has high computational complexity. This paper firstly gives some new calculation methods of comparison matrix, and then proposes a new improved normal parameter reduction method. The comparison results on two real-life datasets validate that the proposed algorithm reduces the complexity of the algorithm and greatly simplifies the process of parameter reduction compared with the algorithm of Kong et al.

INDEX TERMS Fuzzy soft sets, score decision criteria, dispensable sets, parameter reduction, decision making.

I. INTRODUCTION

Soft set theory [21], [23] as a new mathematical tool for dealing with uncertainty was introduced by Molodtsov in 1999, which overcomes the shortcomings of parameterization of traditional mathematical tools such as probability theory, fuzzy sets [2], rough sets [3], intuitionistic fuzzy sets [4], fuzzy rough set [24], and vague sets [5]. The development of mathematical tools for dealing with inaccuracy and fuzziness is promoted, and the combination of soft set models and other mathematical models is impelled to form many extended models of soft sets, such as fuzzy soft set, intuitionistic fuzzy soft sets [6], interval-valued fuzzy soft set [7] and trapezoidal interval type-2 fuzzy soft sets [8] et al. Consequently, the soft set theory is more mature and widely applied in the fields of economy, biology, medicine, and computer science.

It is known to all that fuzzy soft set is one of the most successful extended models of soft set. Maji *et al.* [10] put forward the fuzzy soft set theory by combining fuzzy set and soft set for the first time. This theory has been further generalized in [15]. Fuzzy soft set theory as a major subfield of soft set theory has been widely applied to the field of decision making [19], [20] and the combined forecasting [9]. Maji and Roy [11] outlined a method of object recognition from an imprecise multi-observer data to deal with decision making based on fuzzy soft sets, which has been improved in the paper of [12]. An adjustable decision making approach for the fuzzy soft set was introduced by means of level soft set in [16]. Basu et al. employed the concept of mean potentiality approach to obtain a balanced decision making solution [17]. For the incomplete fuzzy soft set, the article of [18] gave the object-parameter approach to predict unknown data. However, there are many redundant parameters in the actual decision-making process based on fuzzy soft sets. The parameter reduction set is the smallest subset of parameters. When redundant parameters are reduced, the smallest parameter set keeps the same decision-making or description ability as the original parameter set. As a result, parameter reduction is extraordinarily important for decision making problem of fuzzy soft sets. Kong *et al.* [14] proposed the normal parameter reduction of fuzzy soft set theory. Ma and Qin [13]further discussed the simpler and more convenient parameter reduction algorithm named by distance-based parameter reduction. In the paper of [1], parameter reduction problem is studied from a new angle of the score decision criteria and then the new method of normal parameter reduction in fuzzy soft set theory is illustrated. The method is called S-normal parameter reduction. However, the reduction methods have some deficiencies by Kong et al. proposed in the paper [1]. It is plain to see that the S-normal parameter reduction method

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TABLE 1. The tabular representation of fuzzy soft set (F, E).

U	e_1	e_2	e_3	e_4	e_5	e_6
p_1	0.89	0.81	0.87	0.89	0.84	0.87
p_2	0.98	0.9	0.78	1.0	0.86	1.0
p_3	0.55	0.55	0.75	0.75	0.55	0.75
p_4	0.96	0.9	0.96	0.97	0.9	0.96
p_5	0.8	0.73	0.8	0.8	0.67	0.73

has a large amount of calculation and is not easily implemented. Accordingly, in order to remedy this shortcoming, this paper proposes a new improved normal parameter reduction method. The comparison results on two real-life datasets and computational complexity validate that the proposed algorithm reduces the complexity of the algorithm and greatly simplifies the process of parameter reduction compared with the algorithm of Kong et al.

The rest of this paper is structured below. Section II reviews the basic concepts of soft set theory and fuzzy soft set theory. Section III discusses the parameter reduction method for fuzzy soft sets proposed in the paper of [1]. In Section IV, a new improved normal parameter reduction method is proposed. Section V compares the proposed algorithm with the algorithm of paper [1] based on two datasets and then illustrates our comparison of computational complexity. Finally, section VI gives the conclusion of our study.

II. THEORETICAL BACKGROUND

In this section, we recall some underlying concepts about soft set and fuzzy soft set theory.

Definition 1: (See [21]). Let U be a non-empty initial universe of objects, E be a set of parameters in relation to objects in U, P(U) be the power set of U, and A be a subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

Definition 2: (See [22]). Let U be an initial universe of objects, E be a set of parameters in relation to objects in U, $\xi(U)$ be the set of all fuzzy subsets of U. A pair($\tilde{F}$,E)is called a fuzzy soft set over $\xi(U)$, where $\tilde{F}$ is a mapping given by

$$\tilde{F} : E \to \xi(U).$$

From the above definition, we realize that the soft set is a special subset of the fuzzy soft set. In order to understand the fuzzy soft set easily, we give the representation of the data table by example, where each element value range is between 0 and 1.

Example 1: Suppose that there is a student who wants to buy a computer. Let $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a set of five computers considered. E is used to describe the characteristics of the computer. $E = \{e_1, e_2, e_3, e_4, e_5, e_6\} = \{$ cost-effective, battery life, screen, appearance, cooling, speed$\}$. The fuzzy soft set (F,E) describes the characteristics of the computer that the students intend to purchase, which is clarified by the tabular form in Table 1.

III. RELATED WORK

In Section III,we briefly discuss the S-normal parameter reduction method with regard of dispensable parameters for score decision criteria in fuzzy soft set, which was proposed by Kong in [1] . The specific algorithm is given in Figure 1.

Algorithm: S-Normal parameter reduction method(see[1])
Step 1: Input the fuzzy-soft-sets (F, E) with parameter set and object set;
Step 2: For any subset T ⊂ E, calculate comparison matrix W_{E-T} and check the matrix W_E-W_{E-T}, if it is symmetric, then T put into dispensable set R.
Step 3:Check whether R is the maximal dispensable subset of E, if satisfying, then E−R is the s-normal parameter reduction of R.

FIGURE 1. S-Normal parameter reduction method in [1].

Example 2: Supposing that U={p1, p2, p3, p4, p5} be the set of five objects and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ be the set of factors that decision maker considered, the fuzzy soft set (F, E) is illustrated by a tabular representation in Table 2.

In the S-normal parameter reduction method, firstly, calculating the comparison table and score table of the fuzzy soft set (F, E),which are shown in Table 3, we can get the score of each scheme is $\{-2, -6, 4, 4, 0\}$,and the priority of the these schemes is $p_3 = p_4 > p_5 > p_1 > p_2$, so the comparison

TABLE 2. The tabular representation of fuzzy soft set (F, E).

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
p_1	0.2	0.7	0.5	0.3	0.2	0.4	0.9	0.1
p_2	0.6	0.65	0.6	0.2	0.5	0.2	0.3	0.1
p_3	0.7	0.5	0.2	0.8	0.2	0.9	0.9	0.1
p_4	0.75	0.3	0.8	0.5	0.4	0.3	0.8	0.1
p_5	0.9	0.2	0.9	0.4	0.7	0.1	0.1	0.1

TABLE 3. The comparison and score table of fuzzy soft set (F, E).

	p_1	p_2	p_3	p_4	p_5	row-sum (r_i)	column-sum (t_i)	Score (s_i)
p_1	8	5	5	4	4	26	28	-2
p_2	4	8	4	3	4	23	29	-6
p_3	6	5	8	5	5	29	25	4
p_4	5	6	4	8	5	28	24	4
p_5	5	5	4	4	8	26	26	0

matrix W_E as

$$W_E = \begin{bmatrix} 8 & 5 & 5 & 4 & 4 \\ 4 & 8 & 4 & 3 & 4 \\ 6 & 5 & 8 & 5 & 5 \\ 5 & 6 & 4 & 8 & 5 \\ 5 & 5 & 4 & 4 & 8 \end{bmatrix}$$

From Table 2, we can found that two special parameters in fuzzy soft set. The parameter set $T = \{e_1, e_2\}$ is arranged in ascending order and descending order respectively, and the parameter set $T' = \{e_5, e_7\}$ is arranged in semi-ascending order and semi-descending order respectively. If the parameter set $T = \{e_1, e_2\}$ is deleted and then W_{E-T} and $W_E - W_{E-T}$ are calculated, we can get the following equation:

$$W_{E-T} = \begin{bmatrix} 6 & 4 & 4 & 3 & 3 \\ 3 & 6 & 3 & 2 & 4 \\ 5 & 4 & 6 & 5 & 5 \\ 4 & 5 & 4 & 6 & 5 \\ 4 & 5 & 4 & 4 & 6 \end{bmatrix} W_E - W_{E-T} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}$$

From the above results, we can see that $W_E - W_{E-T}$ is a symmetric matrix. Therefore, the parameter set is put into the dispensable parameters R. Similarly, the parameter set $T' = \{e_5, e_7\}$ is put into the dispensable parameters R. Due to the parameter set of each object in e_8 is equal, it is concluded that the S-normal parameter reduction of fuzzy soft sets is $E - R = \{e_3, e_4, e_6\}$. However, this method involves a great amount of computation and is not easily to implement. In order to solve these problems, we propose an improved parameter reduction method for score decision criteria of fuzzy soft set below.

IV. A NEW IMPROVED PARAMETER REDUCTION METHOD FOR SCORE DECISION CRITERIA OF FUZZY SOFT SET (I-S-NORMAL METHOD)

In this section, we propose an improved parameter reduction method for score decision criteria of fuzzy soft set, as shown in Figure 2.

Let $U = \{p_1, p_2, \cdots, p_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, (F, E) be a fuzzy soft set with tabular representation. p_{ij} belongs to a unit entity between 0 and 1 in the fuzzy soft set (F, E). We employ $\mu_{S(e_j)}(p_i)$ and $\mu_{S(e_j)}(p_j)$ to represent the membership value respectively.

The comparison table is computed by comparing the values of each object, that is, this table indicates the priority of an

Algorithm: I-S-Normal parameter reduction method

Step 1: Input the fuzzy-soft-sets (F, E), the parameter set E and object set U.

Step 2: For any subset $T' \subset E$, calculate the comparison matrix $W_{T'}$ of the subset T', if it is symmetric, then parameter subset T' is dispensable set.

Step 3: we get $E-T'$ is the parameter reduction of fuzzy soft sets (F, E).

FIGURE 2. Proposed algorithm.

object over others. We will describe in detail the specific steps when constructing comparison tables.

Step 1: According to formula (1) and (2) to calculate the upper triangle (or lower triangle) of the comparison table and mark the number of equal membership values of between two object values.

$$C_{ij} = Count\{\mu_{S(e_i)}(p_j) > \mu_{S(e_j)}(p_i)\} \quad (1)$$

$$C' = Count\{\mu_{S(e_i)}(p_j) = \mu_{S(e_j)}(p_i)\} \quad (2)$$

where C_{ij} is equal the number of parameters whose membership value of P_i exceeds to the membership value of P_j in a set of the comparison table. C' is the number of parameters whose membership values are equivalent for the comparison objects.

Step 2: Combining the upper triangle (or lower triangle) data set mentioned above with formula 3, another lower triangle (or upper triangle) comparison table is calculated.

$$C_{ji} = N - C_{ij} - C' = N - \left(C_{ij} + C'\right) \quad (3)$$

In this formula, N denotes the number of parameters in a fuzzy soft set.

Step 3: Get a complete comparison table, and then get the comparison matrix.

In order to explain our proposed I-S-normal parameter reduction method, the parameter reduction process of the fuzzy soft set by the proposed method in Example 2 will be described below.

In Example 2, we found that a set of special parameters $T'' = \{e_1, e_2, e_5, e_7, e_8\}$. In order to analyze whether the parameter set is dispensable set, firstly, let's calculate the upper triangle comparison table (or lower triangle) of the fuzzy soft subset (F, T'') by formulas (1) and (2) and record the equal count C'(in brackets) between comparative objects, as shown in Table 4. Since the object itself is not very meaningful to compare, $C_{ii} = 0$.

Then, since the fuzzy soft set (F, T'') contains five parameters, N value is 5 in the formula $C_{ji} = N - \left(C_{ij} + C'\right)$. Let's take C_{12} as an example, the calculated results discover that $C_{ij} = 2$ and $C' = 1$, when calculating the corresponding value of C_{21} in the lower triangle, we get $C_{21} = N - (C_{12} + C') = 5 - (2 + 1) = 2$, the corresponding lower triangle comparison table as shown in Table 5.

Finally, we get a complete triangle comparison table and calculate the score (s_i) of each scheme, as shown in Table 6.

TABLE 4. The comparison table of the upper triangle of (F, T').

	p_1	p_2	p_3	p_4	p_5
p_1	0	2 (1)	1 (3)	2(1)	2 (1)
p_2		0	2 (1)	2 (1)	2 (1)
p_3			0	2 (1)	2(1)
p_4				0	2 (1)
p_5					0

TABLE 5. The comparison table of lower triangle of (F, T'').

	p_1	p_2	p_3	p_4	p_5
p_1					
p_2	2				
p_3	1	2			
p_4	2	2	2		
p_5	2	2	2	2	

The comparison matrix $M_{T''}$ of fuzzy soft sets (F, T'') is described as

$$M_{T''} = \begin{bmatrix} 0 & 2 & 1 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 1 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix}$$

From the above results, the score of each scheme is 0. According to our improved comparison table calculation method, the calculation score tables S_i and S_i' before and after reduction are calculated in Table 7, we can see that the corresponding scores of each scheme are equal. Therefore, the parameter set T'' is dispensable parameters.

Theorem 1: Let (F, E) be a fuzzy soft set on U, $U = \{p_1, p_2, \cdots, p_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$. For any $P_i \in U$, score S_i and priority ranking PR_i are obtained by S-normal parameter reduction while score S_i' and Priority ranking PR_i' are obtained by I-S-normal parameter reduction. Then we have $S_i = S_i'$ and $PR_i = PR_i'$.

Proof: Suppose that the fuzzy soft set (F, E), and the choice values $F(e_m) = \{p_1/f_{11}, \ldots, p_i/f_{n-1,m-1}, p_n/f_{n,m}\}$, For $\sum_{j=1}^{n} C_{ij} = \sum_{g=1}^{m} R(p_i)(e_g) = \{f_{n,j}, \ldots, f_{n-s,j}, \ldots, f_{n,j}\}$, and $\sum_{j=1}^{n} C_{ji} = \sum_{g=1}^{m} T(p_i)(e_g) = \{f_{i,q}, \ldots, f_{i,m-q}, \ldots, f_{i,m}\}$,

TABLE 6. The comparison and score table of fuzzy soft set (F, T'').

	p_1	p_2	p_3	p_4	p_5	row-sum (r_i)	column-sum (t_i)	Score (s_i)
p_1	0	2	1	2	2	7	7	0
p_2	2	0	2	2	2	8	8	0
p_3	1	2	0	2	2	7	7	0
p_4	2	2	2	0	2	8	8	0
p_5	2	2	2	2	0	8	8	0

TABLE 7. Comparison of scores before and after reduction.

U	p_1	p_2	p_3	p_4	p_5
S_i	-2	-6	4	4	0
S_i'	-2	-6	4	4	0

We obtain

$$\begin{aligned} S_i &= r_i - t_i = \sum\nolimits_{j=1}^{n} C_{ij} - \sum\nolimits_{j=1}^{n} C_{ji} \\ &= \sum\nolimits_{g=1}^{m} R(p_i)(e_g) - \sum\nolimits_{g=1}^{m} T(p_i)(e_g) \\ &= \sum\nolimits_{g=1}^{m} (R(p_i)(e_g) - T(p_i)(e_g)) \\ &= \sum\nolimits_{g=1}^{m} S(p_i)(e_g) \\ &= S_i' \end{aligned}$$

If there is a special fuzzy soft subsets (F, T') in the fuzzy soft set, these subsets have $S_{T'} = r_{T'} - t_{T'} = 0$. Then, T' can be reduced. The detailed scoring computational process of each scheme for each special fuzzy soft subset is as follows:

$$\begin{aligned} S_1 &= r_1 - t_1 \\ &= \sum\nolimits_{j=1}^{n} C_{1j} - \sum\nolimits_{j=1}^{n} C_{j1} \\ &= \sum\nolimits_{g=1}^{m} R(p_1)(e_g) - \sum\nolimits_{g=1}^{m} T(p_1)(e_g) \\ &= 0; \end{aligned}$$

$$\begin{aligned} S_2 &= r_2 - t_2 \\ &= \sum\nolimits_{j=1}^{n} C_{2j} - \sum\nolimits_{j=1}^{n} C_{j2} \\ &= \sum\nolimits_{g=1}^{m} R(p_2)(e_g) - \sum\nolimits_{g=1}^{m} T(p_2)(e_g) \\ &= 0; \end{aligned}$$

$$\vdots$$

$$\begin{aligned} S_i &= r_i - t_i \\ &= \sum\nolimits_{j=1}^{n} C_{ij} - \sum\nolimits_{j=1}^{n} C_{ji} \\ &= \sum\nolimits_{g=1}^{m} R(p_i)(e_g) - \sum\nolimits_{g=1}^{m} T(p_i)(e_g) \\ &= 0_{\circ} \end{aligned}$$

Next, suppose the comparison table data to represent the $n \times n$ comparison matrix M_E of the element are M_{ij}^{E}, which M_{ij}^{E} (i = j) are equal to 0 and described below.

$$M_E = \begin{bmatrix} 0 & M_{1,2}^{E} & \dots & M_{1,n}^{E} \\ M_{2,1}^{E} & 0 & \dots & M_{2,n}^{E} \\ \vdots & \vdots & 0 & \vdots \\ M_{n,1}^{E} & M_{n,2}^{E} & \dots & M_{n,n}^{E} \end{bmatrix}_{n\times n}.$$

Consequently, we will continue to use the formula $S_i = r_i - t_i$ to calculate the score of the comparison matrix as mentioned above. Here, we have

$$S_i^E = r_i^E - t_i^E = \sum_{j=1}^{n} M_{ij}^{E} - \sum_{j=1}^{n} M_{ji}^{E}.$$

Assume that the comparison matrix $M_{T'}$ is symmetrical, the comparison matrix $M_{T'}$ consisting of special fuzzy soft subsets (F, T') is described as

$$M_{T'} = \begin{bmatrix} 0 & M_{1,2}^{T'} & \dots & M_{1,n}^{T'} \\ M_{2,1}^{T'} & 0 & \dots & M_{2,n}^{T'} \\ \vdots & \vdots & 0 & \vdots \\ M_{n,1}^{T'} & M_{n,2}^{T'} & \dots & M_{n,n}^{T'} \end{bmatrix}_{n\times n}$$

Based on that, if the above assumptions are valid, then

$$r_1^{T'} = \sum_{j=1}^{n} (M_{1,j}^{T'}) = M_{1,1}^{T'} + M_{1,2}^{T'} + \dots + M_{1,n}^{T'} t_1^{T'}$$

$$= \sum_{j=1}^{n} (M_{j,1}^{T'}) = M_{1,1}^{T'} + M_{2,1}^{T'} + \dots + M_{n,1}^{T'}$$

$$S_1^{T'} = r_1^{T'} - t_1^{T'} = \sum_{j=1}^{n}(M_{1,j}^{T'}) - \sum_{j=1}^{n}(M_{j,1}^{T'}) = \sum_{j=1}^{n}(M_{1,j}^{T'} - M_{j,1}^{T'})$$

$$= M_{1,1}^{T'} + M_{1,2}^{T'} + \ldots + M_{1,n}^{T'} - \left(M_{1,1}^{T'} + M_{2,1}^{T'} + \ldots + M_{n,1}^{T'}\right)$$

$$r_2^{T'} = \sum_{j=1}^{n}(M_{2,j}^{T'}) = M_{2,1}^{T'} + M_{2,2}^{T'} + \ldots + M_{2,n}^{T'}$$

$$t_2^{T'} = \sum_{j=1}^{n}(M_{j,2}^{T'}) = M_{1,2}^{T'} + M_{2,2}^{T'} + \ldots + M_{n,2}^{T'}$$

$$S_2^{T'} = r_2^{T'} - t_2^{T'} = \sum_{j=1}^{n}(M_{2,j}^{T'}) - \sum_{j=1}^{n}(M_{j,2}^{T'}) = \sum_{j=1}^{n}(M_{2,j}^{T'} - M_{j,2}^{T'})$$

$$= M_{2,1}^{T'} + M_{2,2}^{T'} + \ldots + M_{2,n}^{T'} - \left(M_{1,2}^{T'} + M_{2,2}^{T'} + \ldots + M_{n,2}^{T'}\right)$$

$$r_{i1}^{T'} = \sum_{j=1}^{n}(M_{ij}^{T'}) = M_{i,1}^{T'} + M_{i,2}^{T'} + \ldots + M_{i,n}^{T'}$$

$$\vdots$$

$$t_{i1}^{T'} = \sum_{j=1}^{n}(M_{ji}^{T'}) = M_{1,i}^{T'} + M_{2,i}^{T'} + \ldots + M_{n,i}^{T'}$$

$$S_i^{T'} = r_i^{T'} - t_i^{T'} = \sum_{j=1}^{n}(M_{i,j}^{T'}) - \sum_{j=1}^{n}(M_{j,i}^{T'}) = \sum_{j=1}^{n}(M_{i,j}^{T'} - M_{j,i}^{T'})$$

$$= M_{i,1}^{T'} + M_{i,2}^{T'} + \ldots + M_{i,n}^{T'} - \left(M_{1,i}^{T'} + M_{2,i}^{T'} + \ldots + M_{n,i}^{T'}\right)$$

Since it is a symmetric matrix, then we have $S_1^{T'} = 0$, $S_2^{T'} = 0, \ldots, S_i^{T'} = 0$. Namely, $S_1^{T'} = S_2^{T'} = \ldots = S_i^{T'} = 0$.

So the score value before and after delete dispensable parameters is unchanged. Briefly, the decision order is unchanged. That is, $PR_i = PR_i'$. This completes the proof. □

V. THE COMPARISON RESULT

We first analyze and compare the computational complexity of the two algorithms. Then, two real-life cases are given to further verify the superiority of I-S-normal algorithm.

A. THE COMPARISON OF COMPUTATIONAL COMPLEXITY

We analyze the computational complexity of the two algorithms by calculating their basic operations. The basic operations may vary with the diversity of the algorithm implementation, so we only need to discuss the predominant number of access entry. Assuming a fuzzy soft set (F, E) with initial universe U, the number of parameters column of the fuzzy soft set is denoted as m, the special parameters subset is represented by T. Suppose that we only consider a set of special parameter subsets and the number of parameters columns is counted as m'', the analysis of the two algorithms is as follow:

1) S-NORMAL ALGORITHM

By calculating the comparison matrix of the initial fuzzy soft set, we get the number of element accesses n^2m. If we found a set of special parameter subsets T, then the number of parameter columns of the fuzzy soft set (F, E-T) is equal to $m' = m - m''$, and $m' < m$, we get the number of element accesses is n^2m' by calculating the comparison matrix of the fuzzy soft set $(F, E - T)$. Then, we calculate the difference between the two comparison matrix of which the number of element accesses is n^2. Therefore, the computational complexity of S-normal algorithm is $n^2m + n^2m' + n^2 (m' < m)$. Taking big O notation, the computational complexity of this algorithm is $o(n^2m)$.

2) I-S-NORMAL ALGORITHM

In our algorithm, we only need to compute the number of accessing elements of comparison matrix of the fuzzy soft subset (F, T). Assume that there is a set of M column parameters in a special subset T, we will get the complexity of our improved algorithm is $[1+2+\ldots+(n-2)+(n-1)]m''$, which simplifies to $\frac{1}{2}n^2m'' - \frac{3}{2}nm'' + m''$. From the above results, it is easy to find that if there are enough parameter sets in the fuzzy soft set (F, E) and only one or several of parameter subsets satisfying the reduction condition, then $m'' << m$. Taking big O notation, the computational complexity of our improved algorithm is $o(n^2m'')$. From the above analysis, we can see the improved algorithm reduces the computational complexity compared with the S-normal algorithm. Hence our algorithm is superior to the S-normal algorithm.

B. COMPARISON ON OBTAINING PARAMETER REDUCTION

Here, two real-life cases are given to further verify the superiority of I-S-normal algorithm.

Case 1 Evaluation System of Recruitment for Company: HR department of Sunshine Company which major business involves the software design released information about recruitment for one position of one programmer. After that, they received six resumes from six applicants. HR department designs ten parameters to evaluate six applicants during the interviews in order to find the most competitive candidate. We use fuzzy soft set to describe the evaluation system. Assume that $U = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ is defined as the set including six applicants. $E = \{e_1, e_2, e_3, \ldots e_9, e_{10}\}$ is defined as one parameter set including ten evaluation system parameters, where e_i stand for "Training experience", "computer major", "working experience", "young", "highly educated", "marital status" and "health", "calmness", "self-motivation", "Flexibility", respectively. The fuzzy soft set is illustrated by a tabular representation in Table 8.

S-normal algorithm

Step 1: Input the fuzzy soft set for the company job description.

TABLE 8. Case 1:Evaluation system of recruitment for company dataset.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
p_1	0.62	0.6	0.7	0.3	0.3	0.7	0.8	0.7	0.1	0.6
p_2	0.58	0.57	0.5	0.63	0.6	0.4	0.4	0.3	0.2	0.6
p_3	0.4	0.4	0.3	0.7	0.68	0.3	0.5	0.8	0.8	0.4
p_4	0.59	0.9	0.2	0.5	0.5	0.57	0.3	0.6	0.8	0.3
p_5	0.5	0.66	0.55	0.66	0.8	0.2	0.6	0.4	0.5	0.2
p_6	0.2	0.8	0.36	0.8	0.6	0.4	0.9	0.5	0.6	0.5

Step 2: It is found that there exists a special parameter subset $T = \{e_1, e_4\}$ and $T = \{e_5, e_6\}$ in the fuzzy soft set. According to the score decision criterion to calculate the comparison matrix W_E of the fuzzy soft set with the comparison matrix W_{E-T} and $W_E - W_{E-T}$ of the two special parameter subsets are as follows. From the above calculation results, we can conclude that the comparison matrix $W_E - W_{E-T}$ are symmetrical, That is, the parameters $\{e_1, e_4, e_5, e_6\}$ are dispensable in the fuzzy soft set for the company job description.

The comparison matrix of parameter subset $T = \{e_1, e_4\}$:

$$W_E = \begin{bmatrix} 10 & 7 & 6 & 6 & 6 & 5 \\ 4 & 10 & 5 & 5 & 3 & 5 \\ 4 & 5 & 10 & 7 & 5 & 4 \\ 4 & 5 & 4 & 10 & 6 & 5 \\ 4 & 7 & 5 & 4 & 10 & 3 \\ 5 & 7 & 6 & 5 & 7 & 10 \end{bmatrix}$$

$$W_{E-T} = \begin{bmatrix} 8 & 6 & 5 & 5 & 5 & 4 \\ 3 & 8 & 4 & 4 & 2 & 4 \\ 3 & 4 & 8 & 6 & 4 & 3 \\ 3 & 4 & 3 & 8 & 5 & 4 \\ 3 & 6 & 4 & 3 & 8 & 2 \\ 4 & 6 & 5 & 4 & 6 & 8 \end{bmatrix} \quad W_E - W_{E-T} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

The comparison matrix of parameter subset $T = \{e_5, e_6\}$:

$$W_E = \begin{bmatrix} 10 & 7 & 6 & 6 & 6 & 5 \\ 4 & 10 & 5 & 5 & 3 & 5 \\ 4 & 5 & 10 & 7 & 5 & 4 \\ 4 & 5 & 4 & 10 & 6 & 5 \\ 4 & 7 & 5 & 4 & 10 & 3 \\ 5 & 7 & 6 & 5 & 7 & 10 \end{bmatrix}$$

$$W_{E-T} = \begin{bmatrix} 8 & 6 & 5 & 5 & 5 & 4 \\ 3 & 8 & 4 & 4 & 2 & 3 \\ 3 & 4 & 8 & 6 & 4 & 3 \\ 3 & 4 & 3 & 8 & 5 & 4 \\ 3 & 6 & 4 & 3 & 8 & 2 \\ 4 & 5 & 5 & 4 & 6 & 8 \end{bmatrix}$$

$$W_E - W_{E-T} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 & 1 & 2 \end{bmatrix}$$

Step 3: Finally, we get parameter reduction is $\{e_2, e_3, e_7, e_8, e_9, e_{10}\}$.

I-S-normal algorithm

Step 1: Input the fuzzy soft set for the company job description.

Step 2: We find two parameter subsets $T = \{e_1, e_4\}$ and $T = \{e_5, e_6\}$ satisfying special conditions, we use our improved comparison matrix method to compute the comparison matrix composed of all special parameter subsets $\{e_1, e_4, e_5, e_6\}$.The result is shown below.

$$M_{e_{1,4,5,6}} = \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 1 & 2 & 2 & 2 & 0 \end{bmatrix}$$

From above results, we can see that the comparison matrix $M_{e_{1,4,5,6}}$ is symmetrical. Therefore, $\{e_1, e_4, e_5, e_6\}$ are deleted.

Step 3: The parameter reduction is $\{e_2, e_3, e_7, e_8, e_9, e_{10}\}$.

TABLE 9. The comparison result for case1.

Comparison	S-normal algorithm	I-S-normal algorithm	Improvement %
parameter reduction	$\{e_2,e_3,e_7,e_8,e_9,e_{10}\}$	$\{e_2,e_3,e_7,e_8,e_9,e_{10}\}$	The same
The number of comparison matrix	6	1	5/6=83.3%
The number of element accesses	684	40	644/684=94.2%
The involved key steps for comparison matrix	3	1	2/3=66.7%
The number of parameters involved in the calculation of comparison matrix	10	4	6/10=60%

According to the analysis of the above real-life application, we find that:

(1) We can obtain the same parameter reduction result $\{e_2, e_3, e_7, e_8, e_9, e_{10}\}$, which means two methods are equivalent.

(2) We should get six comparison matrices by S-normal algorithm while one comparison matrix is needed by our proposed method.

(3) Based on S-normal algorithm, we should calculate the comparison matrix W_E of the fuzzy soft set with the comparison matrix W_{E-T} and $W_E - W_{E-T}$ and check the matrix W_E-W_{E-T}, if it is symmetric, then T put into dispensable set R. That is, this method needs three steps for comparison matrix; By our I-S-normal algorithm, we only calculate the comparison matrix $W_{T'}$ of the subset T', if it is symmetric, then parameter subset T' is dispensable set, that is, our proposed method only need one step for comparison matrix;

(4) The number of element accesses is 684 by S-normal algorithm while The number of element accesses is only 40 by I-S-normal algorithm;

(5) The number of parameters involved in the calculation of comparison matrix is 10, that is, all of parameters should be involved by S-Normal algorithm; our method only involve 4 parameters to calculate the comparison matrix.

It is obvious that our method is easier to implement and involve less computation than S-normal algorithm on this case, that is, the S-normal algorithm is redundant and cumbersome in the process of reduction. Table 9 expresses in detail the comparison results by two methods for this real-life application of Recruitment for Company.

Case 2 Evaluation of Scientific Research Achievements: Academic papers are the scientific research achievements of researchers, and academic evaluation is mainly the evaluation of academic research results. A researcher wants to read an excellent paper of scientific journals in the field of "data mining". There are currently five related papers and seven assessment criteria are provided in order to get the most valuable paper to this reader. We choose fuzzy soft set to depict this evaluation system. These data are from Baidu Scholar. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5\}$ ={"Design and Application of Teaching Quality Monitoring and Evaluation System Based on Data Mining", "Study on Data Mining for Combat Simulation", "Research on Intrusion Detection of Data Mining Model Based on Improved Apriori Algorithm", "Construction of Cloud Service Platform for Network Big Data Mining", "Overview of Data Mining"}, which are measured according to seven attributes, such as "downloads", "cited frequency", "number of results", "H-index", "number of cited", "reading volume" and "impact factor". All of datasets are normalized and preprocessed to get the tables as shown in Table 10, where each entry of data set is between 0 and 1.

S-normal algorithm

Step 1: Input the fuzzy soft set (F, E) for scientific research achievements.

Step 2: Finding a set of special parameter subset $T = \{e_1, e_6\}$ for fuzzy soft set in Table 9, then calculating the comparison matrix W_E with the comparison matrix W_{E-T} and $W_E - W_{E-T}$ of the fuzzy soft set according to S-normal method, the comparison matrix results are described as follows:

$$W_E = \begin{bmatrix} 7 & 1 & 3 & 4 & 4 \\ 6 & 7 & 6 & 6 & 5 \\ 4 & 2 & 7 & 4 & 4 \\ 4 & 1 & 3 & 7 & 4 \\ 3 & 2 & 3 & 3 & 7 \end{bmatrix} W_{E-T} = \begin{bmatrix} 5 & 0 & 2 & 3 & 3 \\ 5 & 5 & 5 & 5 & 4 \\ 3 & 1 & 5 & 3 & 3 \\ 3 & 0 & 2 & 5 & 3 \\ 2 & 1 & 2 & 2 & 5 \end{bmatrix}$$

$$W_E - W_{E-T} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

From the above conclusions, we can see that the comparison matrix $W_E - W_{E-T}$ is symmetrical.

Step 3: Finally, we get the new parameter reduction of E is $\{e_2, e_3, e_4, e_5, e_7\}$.

TABLE 10. Case 2: scientific research datasets.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7
h_1	0.05	0.09	0.12	0.16	0.05	0.95	0.44
h_2	0.17	0.95	0.95	0.95	0.23	0.75	0.95
h_3	0.22	0.40	0.11	0.61	0.23	0.55	0.05
h_4	0.72	0.43	0.08	0.5	0.05	0.15	0.18
h_5	0.95	0.05	0.05	0.05	0.95	0.05	0.63

TABLE 11. The comparison result for case 2.

Comparison	S-normal algorithm	I-S-normal algorithm	Improvement %
parameter reduction	$\{e_2,e_3,e_4,e_5,e_7\}$	$\{e_2,e_3,e_4,e_5,e_7\}$	The same
The number of comparison matrix	3	1	2/3=66.7%
The number of element accesses	325	12	313/325=96.3%
The involved key steps for comparison matrix	3	1	2/3=66.7%
The number of parameters involved in the calculation of comparison matrix	7	2	5/7=71.4%

I-S-normal algorithm

Step 1: Input the fuzzy soft set for scientific research achievements.

Step 2: Using our improved comparison matrix method to calculate the comparison matrix composed of a set of special parameter subsets $T = \{e_1, e_6\}$ found for the fuzzy soft set in table 9. The result is shown below.

$$W_T = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

From the above calculation results, we can see that the comparison matrix W_T is symmetrical. So $T = \{e_1, e_6\}$ can be reduced

Step 3: We get the parameter reduction of E is $T = \{e_2, e_3, e_4, e_5, e_7\}$.

By means of making the contrast of two methods, we can discover that:

(1) Both of two methods can get the same parameter reduction as $\{e_2, e_3, e_4, e_5, e_7\}$.

(2) Three comparison matrices need to be computed by S-normal algorithm while one comparison matrix is needed by our proposed method.

(3) The number of element accesses is up to 325 by S-normal algorithm in this case; however the number of element accesses is down to 12 by I-S-normal algorithm.

(4) In this evaluation system of scientific research achievements, using the S-Normal algorithm the whole parameters should be involved for the calculation of comparison matrix. But applying the proposed method, only two parameters are involved.

The comparison results between S-normal algorithm and I-S-normal algorithm on the application of scientific research achievement evaluation are shown in Table 11.

Based on the above two real-life applications, we can draw the conclusion that:

(1) The two methods can get the same parameter reduction results;

(2) The improvement average of the number of comparison matrices is up to 75% by the proposed method.

(3) The improvement average of the number of element accesses is 95.25% by I-S-normal algorithm.

(4) The improvement average of the number of involved parameters for comparison matrix is 65.7% on two real-life application cases.

Hence our proposed algorithm is easier to implement and has less computation compared with the S-Normal algorithm.

VI. CONCLUSION

This paper analyzes the S-normal parameter reduction method put forward by Kong et al., and then finds that the S-normal parameter reduction method has a large amount of calculation and high redundancy in the calculation process, in order to overcome this shortcoming, the new I-S-normal reduction algorithm is proposed. Compared with S-normal algorithm, I-S-normal algorithm has less computational complexity and is easier to implement. Comparison results on two real-life application cases between two methods verify superiority of the proposed algorithm. Accordingly, our improved algorithm is an effective and efficient parameter reduction method.

REFERENCES

[1] Z. Kong, J. Ai, L. Wang, P. Li, L. Ma, and F. Lu, "New normal parameter reduction method in fuzzy soft set theory," *IEEE Access*, vol. 7, pp. 2986–2998, 2018.
[2] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, pp. 338–353, Jun. 1965.
[3] Z. Pawlak, "Rough sets," *Int. J. Comput. Inf. Sci.*, vol. 11, no. 5, pp. 341–356, Oct. 1982.
[4] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, pp. 87–96, Aug. 1986.
[5] W. L. Gau and D. J. Buehrer, "Vague sets," *IEEE Trans. Syst., Man, Cybern.*, vol. 23, no. 2, pp. 610–614, Apr. 2013.
[6] M. Agarwal, K. K. Biswas, and M. Hanmandlu, "Generalized intuitionistic fuzzy soft sets with applications in decision-making," *Appl. Soft Comput.*, vol. 13, pp. 3552–3566, Aug. 2013.
[7] X. Ma, H. Qin, N. Sulaiman, T. Herawan, and J. Abawajy, "The parameter reduction of the interval-valued fuzzy soft sets and its related algorithms," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 57–71, Feb. 2014.
[8] A. M. Khalil and N. Hassan, "A note on 'a novel approach to multi attribute group decision making based on trapezoidal interval type-2 fuzzy soft sets,"' *Appl. Math. Model.*, vol. 41, pp. 684–690, Jan. 2017.
[9] Z. Xiao, K. Gong, and Y. Zou, "A combined forecasting approach based on fuzzy soft sets," *J. Comput. Appl. Math.*, vol. 228, no. 1, pp. 326–333, 2009.
[10] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.
[11] P. K. Maji and A. R. Roy, "A fuzzy soft set theoretic approach to decision making problems," *J. Comput. Appl. Math.*, vol. 203, pp. 412–418, Jun. 2007.
[12] Z. Kong, L. Gao, and L. Wang, "Comment on 'a fuzzy soft set theoretic approach to decision making problems,"' *J. Comput. Appl. Math.*, vol. 223, pp. 540–542, Jan. 2009.
[13] X. Ma and H. Qin, "A distance-based parameter reduction algorithm of fuzzy soft sets," *IEEE Access*, vol. 6, pp. 10530–10539, 2018.
[14] Z. Kong, L. Gao, L. Wang, and S. Li, "The normal parameter reduction of soft sets and its algorithm," *Comput. Math. Appl.*, vol. 56, no. 12, pp. 3029–3037, 2008.
[15] P. Majumdar and S. K. Samanta, "Generalized fuzzy soft sets," *Comput. Math. Appl.*, vol. 59, no. 4, pp. 1425–1432, 2010.
[16] F. Feng, Y. B. Jun, X. Liu, and L. Li, "An adjustable approach to fuzzy soft set based decision making," *J. Comput. Appl. Math.*, vol. 234, pp. 10–20, May 2010.
[17] T. M. Basu, N. K. Mahapatra, and S. K. Mondal, "A balanced solution of a fuzzy soft set based decision making problem in medical science," *Appl. Soft Comput.*, vol. 12, pp. 3260–3275, Oct. 2012.
[18] T. Deng and X. Wang, "An object-parameter approach to predicting unknown data in incomplete fuzzy soft sets," *Appl. Math. Model.*, vol. 37, pp. 4139–4146, Mar. 2013.
[19] H. Tang, "A novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence," *Appl. Soft Comput.*, vol. 31, pp. 317–325, Jun. 2015.
[20] J. C. R. Alcantud, "A novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set," *Inf. Fusion*, vol. 29, pp. 142–148, May 2016.
[21] D. Molodtsov, "Soft set theory—First results," *Comput. Math. Appl.*, vol. 37, nos. 4–5, pp. 19–31, 1999.
[22] P. K. Maji, A. R. Roy, and R. Biswas, "On intuitionistic fuzzy soft sets," *J. Fuzzy Math.*, vol. 12, no. 3, pp. 669–683, 2004.
[23] J. Zhan and J. C. R. Alcantud, "A survey of parameter reduction of soft sets and corresponding algorithms," *Artif. Intell. Rev.*, vol. 52, pp. 1839–1872, Oct. 2019.
[24] L. Zhang, J. Zhan, Z. Xu, and J. C. R. Alcantud, "Covering-based general multigranulation intuitionistic fuzzy rough sets and corresponding applications to multi-attribute group decision-making," *Inf. Sci.*, vol. 494, pp. 114–140, Aug. 2019.

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