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Compressive Sparse Data Gathering With Low-Rank and Total Variation in Wireless Sensor Networks

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ABSTRACT Wireless Sensor Networks (WSNs) have been deeply studied by many researchers and been widely used in many fields. Since a large amount of energy for WSNs is used for sensing and transmitting, researchers come up with many methods to reduce the number of sensed and transmitted data packets. Compressive Data Gathering (CDG) is a well-known method to gather WSNs data, but it does not realize sparse sensing as it needs to sense all data and compress them. The efficiency of Low-rank and TV regularizations for recovering WSNs data has been demonstrated, however, they are not combined to enable utilization of data correlation throughout the network. To recover the data accurately and to reduce the energy consumption in WSNs, we propose a Compressive Sparse Data Gathering (CSDG) scheme including a Compressive Sparse Sampling (CSS) method and a data recovery algorithm based on low-rank and Total Variation (TV) regularizations fully exploiting the sparsity and low-rank characteristics of WSNs data. The alternating direction method of multipliers and the steepest descent method are used to solve the problem. Simulations show that the CSDG method outperforms the state-of-the-art methods in terms of the recovery accuracy. Moreover, with fairly low sparse sampling ratio and high compression ratio, CSDG method can still recover the original signal with little error. As the number of sensed data and transmitted data is reduced greatly with sparse sampling and compression, the energy consumption of WSNs is lessen and the lifetime is prolonged.

INDEX TERMS Wireless sensor networks, data gathering, optimization methods, total variation, low-rank.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) are wireless networks composed of a large number of stationary or mobile nodes. Broadly speaking, the nodes can be divided into sensor nodes and sink nodes. The sensor nodes are self-organized and in the multi-hop manner to collaboratively sense, acquire, process and transmit the perceived information in the area that the network coverages. Ultimately, the sink receives the information and apply the information in practice [1]. WSNs have been deeply studied by many researchers and been widely used in agriculture and environmental protection fields to monitor environmental parameters [2], [3].

The sensor nodes are usually powered by a battery and they need to monitor the environment for a long time, so that the control of energy consumption is of great importance for WSNs. Since a large amount of energy for WSNs is used for sensing and transmitting [4], researchers come up with many methods to reduce the number of sensed and transmitted data packets. Making full use of the general characteristic of WSNs data is a main research orientation and has been proved to be effective.

Many methods consider the sparsity feature of WSNs data. Compressive Data Gathering (CDG) [5] is the most well-known method to apply Compressive Sensing (CS) theory [6] to WSNs because the representation coefficients of data are usually sparse under some specific bases. However, the recovery accuracy of CDG fails to promise high level. Recently, a Sequential Compressed Sensing with Progressive Signal Reconstruction method (Seq-Prog-CS) [7] has been proposed, demonstrating that the recovering accuracy can be guaranteed from a sequence of periodically

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delivered CS measurements. Later, the method utilizing both the low-rank and temporal sparsity feature was proposed, named as Compressive Data Gathering with Low-Rank constraints (CDGLR) [8]. CDGLR method introduced the historic data, efficiently improving the recovery accuracy. Discovering the gradient sparsity of WSNs data, The Compressive Multi-timeslots Data Gathering method with Total Variation Regularization (CMDGTV) [9] is proposed, using the Total Variation (TV) regularization. With CMDGTV method, people can recover WSNs data with high accuracy without need to look for a suitable sparse representation basis.

CDG has shown its effectiveness to reduce energy consumption and to recover data accurately in WSNs, however, the number of samples is not actually reduced because sensor nodes need to sample all data and then compress them when using CDG method in most related works [5], [8]–[10]. Later according to the research progress of the Matrix Completion (MC) theory [11], researchers exploited the low-rank characteristic of sensor data and applied the MC theory to WSNs [4]. In MC, the low-rank data matrix can be recovered accurately from entries sampled uniformly at random [11], which means only a portion of data are needed to be sensed and transmitted to the sink, realizing a real sense of sparse sensing.

Since the application of MC theory in WSNs, many algorithms for solving the MC problem are adapted to the WSNs. In the beginning, people used the alternative least squares (ALS) method to solve the data recovery problem exploiting the spatial correlation of WSNs data, which is called the Efficient Data Collection Approach (EDCA) [4]. Later the Spatio-Temporal Compressive Data Collection (STCDG) [12] was proposed adding the short-term stability features to improve the accuracy. These methods assume the rank of the data is fixed and known, which is unlikely to hold in the practical system. He et al. combined the matrix completion with sparsity constraints, proposing the Data Recovery method with joint Matrix Completion and Sparsity Constraints (DRMCSC) [13], and used the alternating minimization to solve the problem.

Although the data recovery methods based on MC theory reduce the number of sensed data of sensor nodes, it does not mean they reduce the number of transmitted data packets compared with the methods based on CS theory. Because these methods did not show higher recovery accuracy with the same sampling ratio. Moreover, many methods based on MC theory reduce energy consumption with nodes in the sleep mode, which has a strict requirement for the communication limits of sensor nodes. Actually, the data gathering method based on MC theory is not opposite to the method based on the CS theory. But they can not be combined easily and require a higher requirement for the corresponding sampling method, transmitting method and recovery algorithm.

TV regularization has been used for recovering WSNs data effectively [9], however, the method only utilizes information from local neighborhoods, neglecting useful information from remote data. In this paper, we propose a Compressive Sparse Data Gathering (CSDG) scheme to recover the data accurately and to reduce the energy consumption in a WSN, which monitors slowly time-changing environment parameters for a long time such as the temperature, the humidity and the light. The scheme includes a Compressive Sparse Sampling (CSS) method and a data recovery algorithm based on low-rank and TV regularizations (LRTV). The main contributions of this paper are summarized as follows:

- We propose a compressive sparse data gathering scheme for WSNs including a compressive sparse sampling method and a data recovery algorithm based on low-rank and TV regularizations to recover the data without loss of generality accurately and to reduce the energy consumption.
- We design a compressive sparse sampling method fully exploiting the sparsity and time-relevant characteristics of WSNs data. In this method, only part of data are sensed and then compressed and transmitted to the sink. Sparse sampling and compression reduce the number of sensed data and transmitted data greatly so that the energy consumption of WSNs is lessen. It is notable that this sampling method is generally applicable and can be used with other recovery algorithms easily.
- We construct the model with low-rank and TV regularizations that integrates both local and global information for recovering signal from the samples. This is achieved by, in addition to TV, low-rank regularization that enables utilization of information throughout the network. The alternating direction method of multipliers and the steepest descent method are used to solve the problem.
- With a real-world data set, we evaluate the proposed compressive sparse data gathering method and the state-of-the-art data gathering methods. The simulation results show that the proposed method outperforms other methods in terms of the reconstruction accuracy. Moreover, the CSDG method with lower sparse sampling ratios still performs well.

The rest of this paper is organized as follows. Section II presents the basic CDG scheme. The proposed CSDG method is described in Section III. It includes the compressive sparse sampling method, the problem formulation and the algorithm. Section IV presents the experiments results of the proposed methods compared with the state-of-the-art methods in the term of reconstruction accuracy and the proposed method with different sparse sampling ratios. Last but not least, Section VI concludes the paper and looks to research direction in the future.

II. OVERVIEW OF CDG

CDG method presents the first complete design to apply compressed sensing theory to sensor data gathering for large-scale wireless sensor networks. Instead of traditional transmitting each sensed value, sensor nodes transmits a sum of products of sensed values and measurements vectors in CDG method

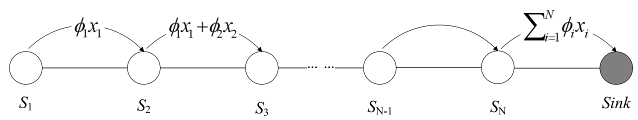

FIGURE 1. **Compressive data gathering in a multi-hop route.**

so that the global scale communication cost is reduced and load is balanced.

Fig. 1 shows the basic idea of CDG method, where x_i denotes the value sensed by the node i and ϕ_i denotes the i_{th} column vector of measurement matrix Φ. As Fig. 1 shown, the sink finally gets the sums of $\phi_i x_i$, so the CDG process can be easily formulated as

$$\mathbf{b} = \Phi \mathbf{x}, \tag{1}$$

where $\mathbf{b} \in \mathbb{R}^M$ is the measurements the sink gets, $\mathbf{x} \in \mathbb{R}^N$ is the sensed value vector and $\Phi \in \mathbb{R}^{M \times N}$ is the measurement matrix, which is usually a random Gaussian matrix.

After the sink obtains the measurements, it recovers $\mathbf{x}$ from the measurements $\mathbf{b}$. According to the CS theory, this can be formulated as a l_1 minimization problem if $\mathbf{x}$ is sparse under a basis, which is as follows:

$$\min_{\mathbf{s}} \|\mathbf{s}\|_{l_1} \quad \text{s.t.} \quad \mathbf{b} = \Phi \mathbf{x}, \mathbf{x} = \Psi \mathbf{s}, \tag{2}$$

where Ψ is the sparse basis and $\mathbf{s}$ is the sparse representation coefficient. As Ψ is known, the original data $\mathbf{x}$ can be obtained easily after $\mathbf{s}$ is gotten. The solution of Eq. (2) is deeply studied and can be obtained fast and accurately by many algorithms [14], [15].

However, in CDG, each sensor node needs to transmit M data packets in each round of data gathering. That is, the total number of data transmissions for a network of N sensor nodes is $M \times N$, which still incurs high communication cost. To address this problem, hybrid CDG (H-CDG) approaches are proposed in [16]–[18]. In the hybrid methods, the nodes close to the leaf nodes transmit the original data without using the CS technique, while the nodes close to the sink transmit data to sink using the CS method. In [19], a sparsest random scheduling scheme is proposed for CDG (SRS-CDG) in WSNs to further reduce the transmission cost by treating each sensor reading as one CS measurement, where the measurement matrix is a sparsest one. It is notable that other CDG related methods can also be used in the proposed scheme. In the later section, the basic CDG method is used for better understanding.

III. THE PROPOSED METHOD

We consider a slowly time-changing WSN system with a sink and N sensor nodes, where N sensor nodes transmit their data to the sink. The data are sent to the sink periodically and a cycle includes C time slot t. The sensor node i senses data at time slot t so that it senses C data values during a cycle. The sensed original data without loss of generality can form the matrix $X_F \in \mathbb{R}^{N \times C}$, where the row and column number corresponds to the node ID and time slot number, respectively.

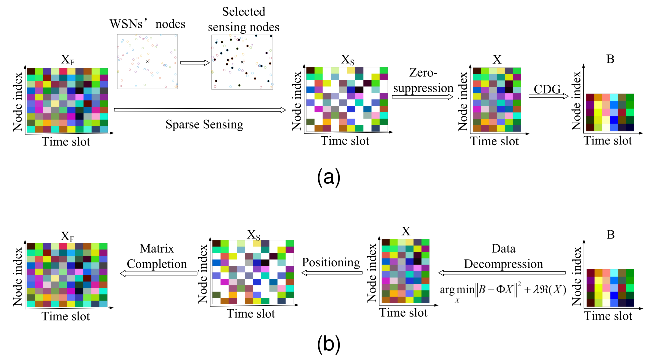

FIGURE 2. **The proposed CSDG method (a) Compressive sparse sampling. (b) Data recovery.**

Each time slot is with same time interval and is usually several to dozens of seconds.

To recover X_F accurately and to reduce energy consumption in WSNs, we propose a compressive sparse data gathering method. With the CSDG method, only a portion of data in X_F are sensed forming the sensing data with 0 values X_S. And then the 0 values are deleted according to the highly temporal-correlated characteristic of WSNs data. Finally the matrix X without 0 are compressed and transmitted to the sink using the CDG method. After receiving the measurements B, the sink recovers X with the proposed optimization algorithm at first, and then recovers the original data. The scheme of the proposed method is shown in Fig. 2.

In this section, we will describe the proposed CSDG method from three aspects, the proposed Compressive Sparse Sampling (CSS) method, the scheme of data recovery process and the proposed optimization method in detail.

A. COMPRESSIVE SPARSE SAMPLING (CSS) METHOD

A cycle of the periodic CSS method is explained here to show the proposed CSS method as all cycles are the same. In the CSS method, The data transmitting process is only carried out once in a cycle and each cycle includes a decision period, C time slots and a transmitting period. The sparse sensing ratio and the compression ratio are defined as p_s and p_c, respectively.

The CSS method can be simply divided into two parts, the sparse sampling and the compressive data gathering. The sparse sampling is completed during the decision period and the following C time slots, and during the transmitting period, the compressive data gathering is completed.

Because the data of WSNs are highly temporal-correlated, only part of the data along the dimension of time are sensed to realize the sparse sensing. The number of sensed data of each sensor node in a cycle is $T = [C \times p_s]$, where $[S]$ means S rounds to integer number.

In the decision period of the cycle, each sensor node i randomly selects T integer numbers in the set $\{1, 2, \ldots, C\}$, forming the subset defined as I_i. The set I_i decides at which time slot the sensor node i senses data in the cycle. In the cycle, the sensor node i only senses data at the selected time slots while is in the sleep mode during the other time slots to reduce the energy consumption. According to this, a sparse sensing matrix $Q \in \mathbb{R}^{N\times C}$ can be constructed as follows:

$$Q_{ij} = \begin{cases} 1 & j_i \in I_i \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where j_i means the column number of i^{th} row. In practice, Q_i, the i^{th} row vector of Q is saved in the i^{th} sensor node and will be covered in the following cycle.

The sparse sensing process can be simply expressed as

$$X_S = Q \circ X_F, \tag{4}$$

where $\circ$ represents the Hadamard product of two matrices. Namely, $(X_S)_{ij} = Q_{ij} \times (X_F)_{ij}$. Because the data of WSNs is highly temporal-correlated and numbers of zero values of all rows in X_S are the same, zero values can be deleted and the smaller-size data matrix without zeros values X is obtained, where $X \in \mathbb{R}^{N\times T}$. The operator $\mathcal{C}(\cdot)$ is used to delete the zero values and compress the size, which is defined as follows:

$$X = \mathcal{C}(X_S), \tag{5}$$

where $X \in \mathbb{R}^{N\times T}$, $X_S \in \mathbb{R}^{N\times C}$ and $T = [C \times p_s]$.

When entering the transmitting period, the CDG method is used to compress the data X and to transmit the measurements B. Besides the measurements B, the Q_i is also needed to be transmitted to the sink by sensor node i. As there is only 1 and 0 value in Q_i, Q_i can be easily transmitted with a few of bits accompanying the measurements. It is notable that CSS method can be completed with but not limited to the basic CDG method, other related CDG method can also be used here to improve the transmitting efficiency.

Because the WSN is slowly time-changing and each time slot is short, the characteristic of data can be remained to a great extent with CSS method. The compressed matrix X can represent the original full data X_F to a certain degree so that X_F can also be recovered accurately if X can be reconstructed accurately. Under the acceptable reconstruction accuracy, the CSS method can reduce the number of samples and the transmitting data, which certainly reduces the energy consumption.

B. DATA RECOVERY SCHEME

To recover the original sensing data, $X_F \in \mathbb{R}^{N\times C}$, without loss of generality, the recovery process can be divided into two steps. As Fig. 2b shown, $X \in \mathbb{R}^{N\times T}$ is recovered from measurements B at first. Later, X is filled according to the low rank characteristic of WSNs data and the X_F is obtained. The corresponding formulations are described as follows.

1) RECOVERING X

According to the CSS method, X is compressed and transmitted to the sink using the CDG method. Each time slot, this process is formulated as Eq. (2). After a cycle, this process can be easily formulated with a same measurement matrix Φ

$$B = \Phi X, \tag{6}$$

where B is the measurements obtained after a cycle and $B \in \mathbb{R}^{M\times T}$. It is notable that the sensing matrix Φ is independent on time changing so that the occupied memory of Φ in sensor nodes is small [9]. To recover X from the measurements, it usually minimizes the following cost function:

$$\hat{X} = \arg\min_X \|B - \Phi X\|^2. \tag{7}$$

This is a data fidelity term for penalizing the difference between the obtained measurements B and the measurements from the original data X using a measurement matrix Φ. Since this is an ill-posed inverse problem, regularization terms are added to stabilize the solution. The cost function is thus rewritten as:

$$\hat{X} = \arg\min_X \|B - \Phi X\|^2 + \lambda\Re(X), \tag{8}$$

where $\Re(X)$ is the regularization term often defined based on prior knowledge. The parameter λ is used to balance the contributions of the fidelity term and the regularization term. Eq. (8) can be solved by many methods. In this paper, a method with Low-rank and TV regularizations is proposed and explained later.

2) RECOVERING X_F

As Q is transmitted to the sink, the positions of elements of X in X_S are known. After recovering X, X_S can be easily obtained with Q. Because of the low-rank characteristic of WSNs data, the recovery of X_F from X_S can be simply seen as a matrix completion problem, which is formulated as

$$\min_X \{\text{rank}(X_F), \quad \text{subject to } X_S = Q \circ X_F\}, \tag{9}$$

where rank(X_F) is the rank of X_F. However, this problem is NP-hard [20]. Thus, people turn to solve its convex approximation [21] which reads as

$$\min_X \{\|X_F\|_*, \quad \text{subject to } X_S = Q \circ X_F\}, \tag{10}$$

where $\|X_F\|_*$ is the nuclear norm defined as the sum of all singular values of X_F.

Eq. (10) can be solved by many methods [21]–[23]. In this paper, a Tikhonov regularization [24] is added and a Fast Singular Value Thresholding (FSVT) algorithm is used, which is shown in [25] in detail. It is noteworthy that here this method will update all the values including those in X recovered by CDG, which is different from previous ones [21]–[23], [25].

C. OPTIMIZATION METHOD FOR RECOVERING X

The effectiveness of TV regularization and low-rank property for recovering WSNs data has been demonstrated by various existing works theoretically and numerically. In this paper, we consider combining these two tools and the optimization method with TV+low-rank regularization is as follows:

$$\hat{X} = \arg\min_X \left\{\|B - \Phi X\|^2 + \lambda_{TV}\|X\|_{\text{TV}} + \lambda_{LR}\|X\|_{\text{LR}}\right\}, \tag{11}$$

where λ_{TV} and λ_{LR} are the parameters for controlling the contributions of TV term and low-rank term, respectively. Their specified definitions are shown as follows.

1) TOTAL-VARIATION REGULARIZATION

Although nonconvex TV models and algorithms have also been developed [26], [27], this paper just considers the convex methodology. The TV norm of X, $\|X\|_{\mathrm{TV}}$ can either be the anisotropic TV norm, 1-norm or the isotropic TV norm, 2-norm. Here, the anisotropic TV norm is used defined as follows [28]:

$$\|X\|_{\mathrm{TV1}} = \|\mathbf{D}X\|_1 = \sum_{\mathrm{i,j}}([\mathbf{D}_n X]_{\mathrm{i,j}} + [\mathbf{D}_t X]_{\mathrm{i,j}}), \quad (12)$$

where operator $\mathbf{D}$ is a collection of two sub operators $\mathbf{D} = [\mathbf{D}_n^T \quad \mathbf{D}_t^T]^T$, $\mathbf{D}_n$ and $\mathbf{D}_t$ are the first-order forward finite-difference operators along the vertical and horizontal directions of the matrix X, respectively. Here, $[X]_{i,j}$ denotes the $(i,j)^{\mathrm{th}}$ element of the matrix X. The definitions of each individual sub operators with periodic boundary conditions are

$$\mathbf{D}_n X = [X(n+1,t) - X(n,t) \;\; X(1,t) - X(N,t)], \quad (13)$$

$$\mathbf{D}_t X = [X(n,t+1) - X(n,t) \;\; X(n,1) - X(n,T)], \quad (14)$$

thus, $D_n X \in \mathbb{R}^{N\times T}$ and $D_t X \in \mathbb{R}^{N\times T}$. In conclusion, $\|\mathbf{D}X\|_1$ is the sum of all the elements of two matrices. It is obvious that the definition of $\|\mathbf{D}X\|_1$ does not agree with traditional 1-norm of a matrix, but here it is used for numerical algorithms later.

2) LOW-RANK REGULARIZATION

Previous studies have shown that data collected from WSNs are highly spatio-temporal correlated [29], which brings the low-rank characteristic of the matrix X. As described above, the nuclear norm can be used to to show the low-rank characteristic, so the low-rank regularization term can be $\|X\|_*$.

3) JOINT TV AND LOW-RANK REGULARIZATION TERMS

According to the description above, the regularization terms in the Eq. (11) can be specified and the Eq. (11) can be written as

$$\hat{X} = \arg\min_X \Big\{\|B - \Phi X\|^2 + \lambda_{TV}\|\mathbf{D}X\|_1 + \lambda_{LR}\|X\|_*\Big\}. \quad (15)$$

To solve the optimization problem, we employ variable splitting techniques and rewrite Eq. (15) as follows:

$$\{\hat{X}, \hat{U}, \hat{W}\} = \arg\min_{X,U,W} \frac{1}{2}\|B - \Phi X\|^2 + \lambda_{TV}\|U\|_1 + \lambda_{LR}\|W\|_*, \quad \text{subject to } U = \mathbf{D}X,\; W = X. \quad (16)$$

The augmented Lagrangian function of (16) is as follows by introducing two Lagrangian multipliers Z and Y

$$\begin{aligned}\{\hat{X}, \hat{U}, \hat{W}, \hat{Z}, \hat{Y}\} &= \arg\min_{X,U,W,Z,Y} \Big\{\frac{1}{2}\|B - \Phi X\|^2 \\ &+ \lambda_{TV}\|U\|_1 + < Z, U - \mathbf{D}X > + \frac{\mu}{2}\|U - \mathbf{D}X\|_2^2 \\ &+ \lambda_{LR}\|W\|_* + < Y, W - X > + \frac{\beta}{2}\|W - X\|_2^2\Big\}, \quad (17)\end{aligned}$$

The idea of augmented Lagrangian method is to find a critical point of Eq. (17), which is also the solution of original problem Eq. (15). To this end, the alternating direction method of multipliers (ADMM) [30] is used to solve the following subproblems iteratively:

$$\begin{aligned}X_{k+1} &= \arg\min_X \Big\{\frac{1}{2}\|B - \Phi X\|^2 \\ &+ < Z_k, U_k - \mathbf{D}X > + \frac{\mu}{2}\|U_k - \mathbf{D}X\|_2^2 \\ &+ < Y_k, W_k - X > + \frac{\beta}{2}\|W_k - X\|_2^2\Big\}, \quad (18)\\
U_{k+1} &= \arg\min_U \Big\{\lambda_{TV}\|U\|_1 + < Z_k, U - \mathbf{D}X_{k+1} > \\ &+ \frac{\mu}{2}\|U - \mathbf{D}X_{k+1}\|_2^2\Big\}, \quad (19)\\
W_{k+1} &= \arg\min_W \Big\{\lambda_{LR}\|W\|_* + < Y_k, W - X_{k+1} > \\ &+ \frac{\beta}{2}\|W - X_{k+1}\|_2^2\Big\}, \quad (20)\\
Z_{k+1} &= Z_k - \mu(U_{k+1} - \mathbf{D}X_{k+1}), \quad (21)\\
Y_{k+1} &= Y_k - \beta(U_{k+1} - X_{k+1}). \quad (22)\end{aligned}$$

For the X-sub problem, Eq. (18), clearly it is a minimization of a quadratic function and can be solved by letting the gradient of the function be 0. Its gradient is

$$\begin{aligned}d_k(X) = \Phi^T(\Phi X - B) - \mathbf{D}^T Z + \mu\mathbf{D}^T(\mathbf{D}X - U) \\ -Y^T + \beta(X - W) \quad (23)\end{aligned}$$

resulting the solution

$$(\Phi^T\Phi + \mu\mathbf{D}^T\mathbf{D} + \beta)X = \Phi^T B + \mathbf{D}^T Z + \mu\mathbf{D}^T U + Y^T + \beta. \quad (24)$$

Fast Fourier Transform (FFT) method can be used to solve Eq. (24). Here, the one-step steepest descent method is used to solve Eq. (18) with BB-like method [31] to choose the step size, which is derived by the BB method proposed by Barzilai and Borwein [32]. To solve the U-subproblem, Eq. (19) can be written as

$$\begin{aligned}U_{k+1} = \arg\min_U \Big\{\lambda_{TV}\|U\|_1 + \frac{\mu}{2}\|U - (\mathbf{D}X_{k+1} - \frac{1}{\mu}Z_k)\|_F^2 \\ - \frac{1}{2\mu}\|Z_k\|_F^2\Big\} \quad (25)\end{aligned}$$

Because $\mathbf{D}X$ generates two matrices $\mathbf{D}_n X$ and $\mathbf{D}_t X$, the corresponding solutions U_n and U_t of Eq. (25) through a shrinkage formula are as follows:

$$\mathbf{U}_t = (|\mathbf{D}_t X_{k+1} - \frac{1}{\mu} Z_t| - \frac{\lambda_{TV}}{\mu})_+, \tag{26}$$

$$\mathbf{U}_n = (|\mathbf{D}_n X_{k+1} - \frac{1}{\mu} Z_t| - \frac{\lambda_{TV}}{\mu})_+, \tag{27}$$

where t_+ is the positive part of t, namely, $t_+ = \max(0, t)$. In words, this simply applies a soft-thresholding rule [33]. To solve the W subproblem, Eq. (20) can be written as

$$W_{k+1} = \arg\min_W \{\lambda_{LR} \|W\|_* + \frac{\beta}{2} \|W - (X_{k+1} - \frac{1}{\beta} Y_k)\|_F^2 - \frac{1}{2\beta} \|Y_k\|_F^2. \tag{28}$$

It is easily seen that Eq. (28) can be solved by a singular value shrinkage operator [21]:

$$W_{k+1} = \mathbf{S}_{\frac{\lambda_{LR}}{\beta}} (X_{k+1} - \frac{1}{\beta} Y_k). \tag{29}$$

The shrinkage operator $\mathbf{S}_\tau(X)$ means to apply a soft-thresholding rule to the singular values of X. Consider the singular value decomposition of the matrix $X \in \mathbb{R}^{N \times T}$ of rank r, then the $\mathbf{S}_\tau(X)$ can be defined as follows:

$$\begin{aligned} X &= U \Sigma V^*, \qquad \Sigma = \mathrm{diag}(\{\sigma_i\}_{1 \le i \le r}), \\ \mathbf{S}_\tau(X) &:= U \mathbf{S}_\tau(\Sigma) V^*, \quad \mathbf{S}_\tau(\Sigma) = \mathrm{diag}(\{(\sigma_i - \tau)_+\}), \end{aligned} \tag{30}$$

where $U \in \mathbb{R}^{N \times r}$ and $V \in \mathbb{R}^{T \times r}$ with orthonormal columns and σ_i and τ are positive. In practical implementation, the algorithm was initialized with null matrices Z_0 and Y_0. Then $X = \Phi^{-1} B$ with Z_0 and Y_0 is used to obtain U_0 and W_0. The algorithm is terminated when $\|X_{k+1} - X_k\|_F$ is smaller than a predefined tolerance parameter, or k exceeds a maximum number of iterations.

IV. SIMULATION

A real-world data set generated recently is used to evaluate the performance of data recovery methods accurately. [34]. The data-set is the sensing mote data from the Data Sensing Lab [34], named as the Strata New York 2012 held at the New York Hilton Midtown generated on October, 2012 in New York, NY. The brief map indicating the position of each sensor mote is shown in Figure 3. The hexagons with numbers inside in Fig. 3 represent the sensor nodes with their number ID.

From the sensor nodes of the Data Sensing Lab, the sensed temperature and humidity data are selected. There are 40 sensors and 1724 time slots forming the matrix $X \in \mathbb{R}^{40 \times 1724}$. However, with the disturbance when sensing and transmitting, there are some missing data in the two matrices. To obtain the complete data for evaluating the performance of the methods, the data during the time slots from 71 to 110 are selected. The four nodes do not sense any data during the 40 time slots, so the complete data $X \in \mathbb{R}^{36 \times 40}$ are obtained when the zero values are deleted.

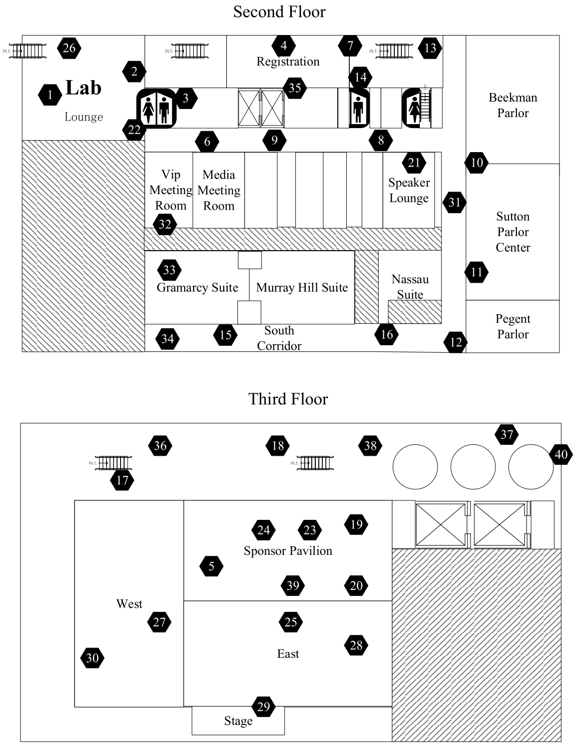

FIGURE 3. The positions of sensors from data sensing lab.

The Normalized Mean Absolute Error (NMAE) is used to measure the accuracy of the reconstructed data which is defined as:

$$\mathrm{NMAE} = \frac{\sum |\hat{X}_{ij} - X_{ij}|}{|X_{ij}|}, \tag{31}$$

where $\hat{X}$ is the recovered data. The NMAE is used in many previous data gathering works for WSNs [8], [12], [13], [25] and is suitable to evaluate the performance of data collection methods. Experiments over different compression ratios p_c were carried out and p_c is set as $p_c = 1, 2, \ldots 10$. The compression ratios determine the size of the column of the measurements, where $M = \lfloor \frac{N}{p_c} \rfloor$. The sensing matrix Φ is a normally distributed random matrix. Each experiment is tested 200 times to calculate the average values.

The proposed CSDG method is compared with CDG [5], Seq-Prog-CS [7], CDGLR [8] and CMDGTV [9] methods over different compression ratios. For the CSDG, CMDGTV and CDGLR method, the size of the collected data varies with different time slots T, forming the original data matrix $X_{N \times T}$. The sparse sampling ratio p_s is set as 1 so that the proposed LRTV method is evaluated.

Fig.4 shows the recovery accuracy of the humidity data from the Data Sensing Lab. As can be seen in Fig. 4, the CSDG method with different T yields noticeable better recovery accuracy than other methods. With increasing compression ratios, the NMAE values of all the methods increase and are not stable, but the advantage of CSDG method expands. It is notable that the recovery accuracy

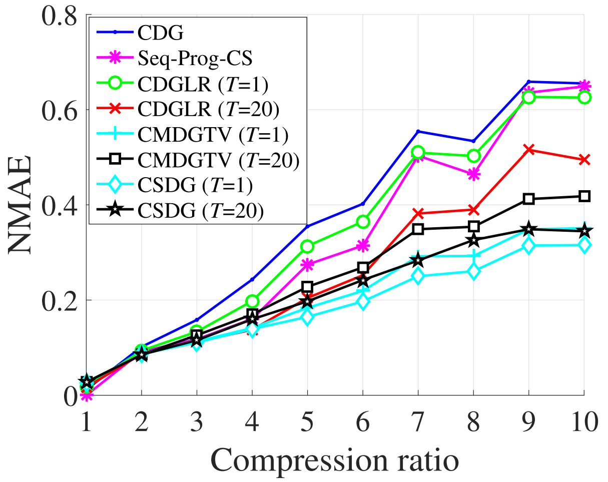

FIGURE 4. NMAE over different compression ratios for humidity data.

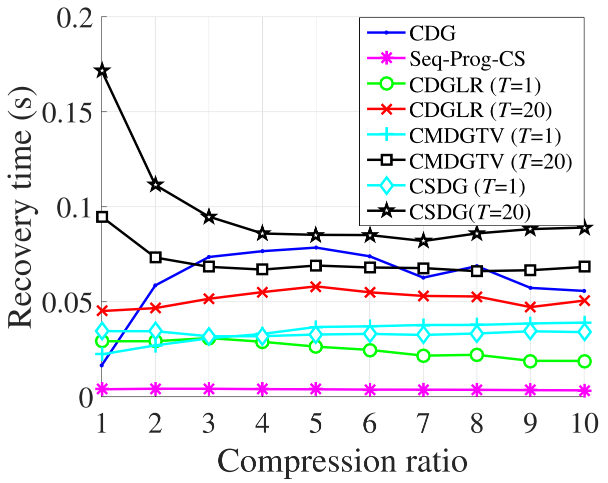

FIGURE 5. Recovery time over different compression ratios for humidity data.

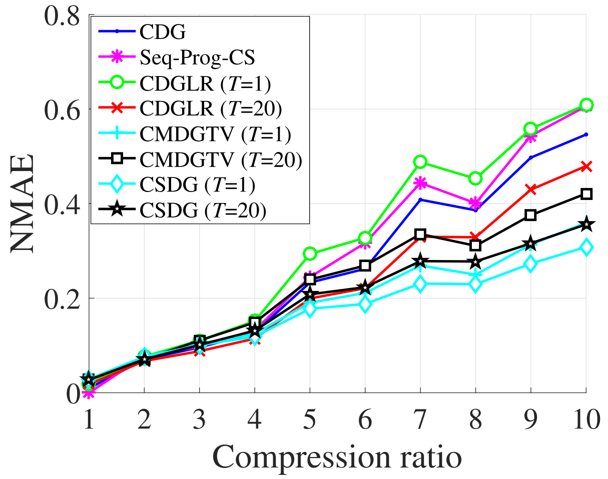

FIGURE 6. NMAE over different compression ratios for temperature data.

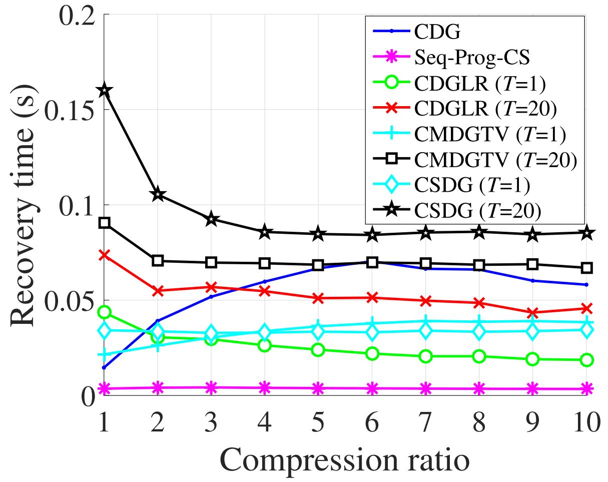

FIGURE 7. Recovery time over different compression ratios for temperature data.

of the CSDG method decreases as T increases, in our opinion, this is because values of different time slots are similar and there are no missing values so that the values of term $\mathbf{D}_t X$ are very close. It becomes more difficult to recover X with the total variation regularization, which has been clarified in paper [9]. In paper [9], it also shows that the performance of recovery method with TV regularization with $T = 20$ is better than that with $T = 1$ for a real-world data set. The data set includes the humidity and temperature information from Intel Berkeley Research Lab, which has many missing values so that the TV regularization makes sense. Although the recovery accuracy does not improve with T increasing, the sparse sampling method can be used with large T so that the number of transmitted data will decrease, which greatly reduce the overall energy consumption in WSNs.

As the proposed method adds a step to fill the sparse sensing data, the recovery time needs to be considered. The corresponding recovery time of methods for humidity data is shown in Fig. 5. The time of the CSDG method is the most when $T = 20$ while is close to the CMDGTV method when $T = 1$. It is easily explainable that the two algorithms of the proposed scheme, the ADMM algorithm and the SVT algorithm contribute to the time when $T = 20$. However. it is only when the compression ratio is 1 or 2, the recovery time of CSDG is especially large compared with that of other methods. Low compression ratio can not reduce the energy consumption and is not the main focus. In overall, the recovery time is acceptable with high recovery accuracy.

The recovery accuracy and recovery time for the temperature data from the Data Sensing Lab are shown in Fig. 6 and Fig. 7, separately. They show the similar performance to those for the humidity data. The CSDG method with $T = 1$ still shows the best recovery accuracy while the relative performance among other methods is a little different. CDG performs better than Seq-Prog-CS and CDGLR when $T = 1$. CDGLR method enhances greatly when $T = 20$.

To evaluate the recovery accuracy of CSDG when the sparse sampling method is used, the experiments with different sparse sampling ratios p_s are carried out and p_s is set as $p_s = \{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}\}$. The larger p_s means that more data

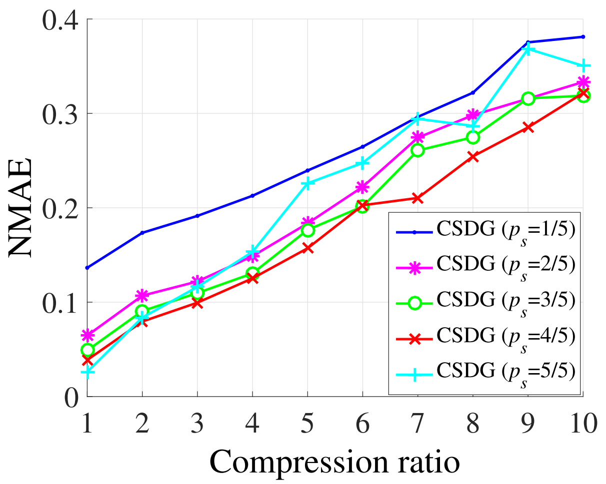

FIGURE 8. NMAE over different compression ratios with different sparse sampling ratios for humidity data.

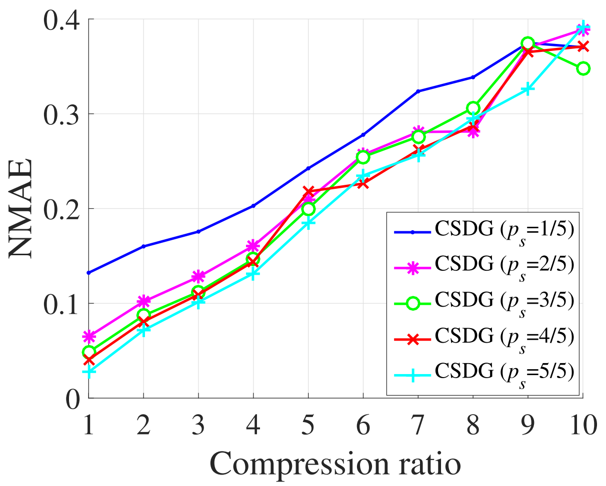

FIGURE 9. NMAE over different compression ratios with different sparse sampling ratios for temperature data.

are sensed. $p_s = 1$ means that all data are sensed and only the LRTV algorithm is used to recovery the original data. The results for humidity data and the temperature data are shown in Fig. 8 and Fig. 9 respectively.

As seen in Fig. 8, the recovery accuracy improves when p_s increases in general except when $p_s = 1$. The CSDG ($p_s = 1$) becomes unstable with increasing compression ratios and its recovery error even exceeds that of CSDG ($p_s = \frac{2}{5}$) over most compression ratios. Except for CSDG ($p_s = 1$), The CSDG ($p_s = \frac{1}{5}$) method shows a gap of recovery accuracy with CSDG ($p_s = \{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$). However, the recovery accuracy of CSDG method with different p_s values are similar when $p_s = \{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}\}$. In other words, CSDG method with relative low sparse sampling ratio can still maintain better recovery accuracy, which reduce the number of sensing data and transmitted data greatly.

Fig. 9 shows a similar result to Fig. 8. It verifies that the CSDG method performs well with different data sets. In Fig. 9, the recovery results of CSDG method with different sparse sampling ratios are very close to each other when $p_s = \{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}\}$. When compression ratio is larger than 4, then results are unstable and cannot tell which sparse sampling ratio is the best.

In summary, the proposed CSDG outperforms CDG, Seq-Prog-CS, CDGLR and CMDGTV in terms of reconstruction accuracy. Moreover, with fairly low sparse sampling ratio and high compression ratio, CSDG can still recover original signal with little error so that the number of sensed data and transmitted data can be reduced greatly.

V. CONCLUSION

In this paper, a new data gathering scheme is proposed called the Compressive Sparse Data Gathering method, which fully exploits the sparsity and low-rank characteristics of WSNs data. In this method, sensor nodes only sense part of data, and then compress data and transmit data to the sink based on CS. A recovery algorithm is developed based on low-rank and TV regularizations. The ADMM method and the steepest descent method is used to solve the problem. Simulations show that the CSDG method outperforms the state-of-the-art methods in terms of the recovery accuracy. Moreover, with fairly low sparse sampling ratio and high compression ratio, CSDG method can still recover the original signal with little error. As the number of sensed data and transmitted data is reduced greatly with sparse sampling and compression, the energy consumption of WSNs is lessen and the lifetime is prolonged.

In the future, as different kinds of WSNs data are relevant, the tensor completion with TV regularization will be considered to be applied for Heterogeneous WSNs to further reduce the sampling ratio and compression ratio.

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