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Adaptive Finite-Time Consensus Tracking for Nonstrict Feedback Nonlinear Multi-Agent Systems With Unknown Control Directions

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ABSTRACT This paper investigates the finite-time consensus tracking problem for nonstrict feedback nonlinear multi-agent systems with unknown dynamics and control directions. Firstly, the backstepping scheme based on command filter and fractional power control law is proposed, which can not only solve the problem of computational explosion, but also ensure that the state of each agent can follow the leader's output within a finite time. Then, the error compensation signals are designed to compensate the error caused by filtering. The unknown nonlinear dynamics are approximated by the fuzzy logic system. The state variables problem in the non-strict feedback system is well solved through the scaling of inequalities. By introducing the Nussbaum function, the intermediate input signal and control input signal based on adaptive control law are designed respectively, and the problem of unknown control directions is solved. Simulations are given to show the effectiveness of the presented method.

INDEX TERMS Finite-time control, nonstirct feedback multi-agent systems, unknown control directions, Nussbaum type function.

I. INTRODUCTION

Recently, the consensus control problem of multi-agent systems (MASs) has received many concerns since the extensive engineering applications for multi-unmanned boats, multi-unmanned aircrafts, multi-sensor network and so on [1]–[4]. Consensus problems include MASs with leaderless-following case and leader-following case. The leader-following system can strengthen the communication between individuals, improve the anti-interference ability, save energy and so forth. Therefore, many people are committed to study such problem, such as in [5] a leader-following structure of the consensus algorithm is designed; in [6]–[13], the single-intergrator, double-intergrator and high-order nonlinear dynamics for MASs are considered, respectively.

The nonlinear dynamic in the MASs, especially the higher-order nonlinear characteristics is a challenging situation. As a mature method, the backstepping control can solve this problem well [14], [15]. But in backstepping process, the explosion of complexity problem will occur. Therefore, backstepping technology combined with some advanced control techniques have been given. For example the dynamic surface control (DSC), by using the first-order filter can solve the expansion of the differential term and make the controller design simple. [16] puts forward a DSC approach for MIMO nonlinear systems under immeasurable states; in [17] and [18], the distributed DSC design approach is proposed to design the controllers, and the DCS on the containment control problem with dynamic leaders is further considered. But how to compensate the filtering error is not proposed in them. The command filtered backstepping control is also an effective scheme [19]–[22], in which the error compensation signal is constructed to eliminate errors generated by filtering. So that the higher control quality and desired tracking performance can be obtained. For the unknown nonlinear dynamics, the adaptive neural network (NN) [23], [24] or fuzzy logical system (FLS) [25], [26] is usually introduced to approximate it. However, the closed-loop systems under the above design schemes are all asymptotic stable. Compared with finite-time control, they lack faster convergence and higher tracking performance. So the finite-time control method are proposed to improve the

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convergence rate and achieve the fast stability of MASs. In [27], the authors study the finite-time control for second-order multi-agent systems with antagonistic interactions. In [28], the authors consider the finite-time backstepping control for nonlinear MASs, and the finite-time formation for spacecrafts subject to external disturbances is addressed in [29].

It is worth emphasizing that all the control schemes mentioned above are not aimed at non-strict feedback systems. They are only apply to strict or semi-strict feedback systems. Therefore, there will be some limitations in practical engineering applications. In order to solve the problem of non-strict feedback, some different schemes have been proposed continuously. In [30], [31] they usually use the method of variable separation to deal with the non-strict feedback structure, and in [32] the author studies the trajectory tracking problem for nonstrict-feedback systems. But the variable-separation method has a disadvantage that the stability analysis is complex in general. So other simpler approaches should be further studied for stability analysis. Up to now, to our best knowledge, the problem of finite-time consensus tracking for nonstrict feedback nonlinear MASs is remain unsolved, which need to be further study. Further, in the control systems, the signs of control gains are called control directions that are required to be known *a priori*. But in some practical control systems, such as uncalibrated visual servo system, autopilot design of uncertain ships and unmanned sailboat heading control, their control directions are often unknown. Therefore, the design of control signals becomes difficult in the process of backstepping, and the adaptive problem is also difficult to solve. It is gratifying to note that the Nussbaum gain can solve the unknown control directions (UCDs) well. After this, the authors in [33] present a robust adaptive control approach for uncertain nonlinear systems with completely unknown control coefficients. In [34] and [35], the authors consider the cooperative output regulation problems for nonlinear multi-agent systems with unknown control directions. Although the design of controllers for signal nonlinear systems or nonlinear MASs with UCDs are all very concerned problems, the work for nonstrict feedback nonlinear MASs is little.

This paper will consider the above discussion, study the adaptive finite-time consensus tracking problems for nonstrict feedback nonlinear MASs with UCDs, and design a controller of the MASs under a general directed graph. The main contributions and advantages of this article are summarized in the following two points.

1) Compared to [17]–[19], [21], a fuzzy approximation based finite-time command filtering backstepping technology is design to solve the explosion of complexity problem, and guarantee the closed-loop system converge quickly and ensure better tracking accuracy. The filtering error compensation scheme is designed, which can further improve the control quality.

2) In [15], [19], [21], [28], they study the systems with strict or semi-strict feedback structure, a non-strict feedback nonlinear MAS is investigated in this paper. The state variables problem in the non-strict feedback system is well solved through the scaling of inequalities based on the basis function vector of FLS. Moreover, the UCDs of the MASs are solved by using the Nussbaum type function, which is more adopt to engineering applications.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. GRAPH THEORY

In this paper, the weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to describe the communications about N agents, where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edges and $\mathcal{V} = \{1, 2, \ldots, N\}$ represents the nodes, respectively. $N_i = \{j|(j, i) \in \mathcal{E}\}$ describe the neighbors of node i. If similar to this form of continuous edge sequence $\{(i, n), (n, k), \ldots, (m, j)\}$, we say from node i to note j exists a direct path. $A = [a_{ij}] \in \mathbf{R}^{N\times N}$ is the weighted adjacency matrix where $a_{ii} = 0$ for $\forall i$, and $a_{ij} > 0$ when $(j, i) \in \mathcal{E}$, $a_{ij} = 0$ when $(j, i) \notin \mathcal{E}$. the Laplacian matrix is $L = D - A$, where $D = diag\{d_1, d_2, \ldots, d_N\}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. If there exists a node that has a directed path from it to other nodes, then it is a root node. If a spanning tree is existed in $\mathcal{G}$, then the root node exists.

We use an extended graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ to describe the communications between a leader agent 0 and the N following agents, the adjacency matrix is described as $B = diag\{b_1, b_2, \ldots, b_N\}$, in which $b_i > 0$ means from nodes 0 to i has an edge, otherwise, $b_i = 0$.

B. SYSTEM DESCRIPTION

Under the directed graph $\bar{\mathcal{G}}$, we discuss the communications between N following agents and one leader. And the equations about the ith follower of the MASs are given

$$\begin{aligned} \dot{x}_{i,q} &= f_{i,q}(x_i) + g_{i,q}(\bar{x}_{i,q})x_{i,q+1} \\ \dot{x}_{i,n_i} &= f_{i,n_i}(x_i) + g_{i,n_i}(x_i)K_i u_i \\ y_i &= x_{i,1}, \quad q = 1, 2, \ldots, n_i - 1 \end{aligned} \tag{1}$$

in which x_i is the state vector and $i \in \mathcal{V}$, $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}]^T \in \mathbf{R}^{n_i}$ with $\bar{x}_{i,q} = [x_{i,1}, x_{i,2}, \ldots, x_{i,q}]^T$. $y_i \in \mathbf{R}$ is the output signal and $u_i \in \mathbf{R}$ is the input signal. In the MAS, $f_{i,q}(\cdot)$ is smooth nonlinear function but unknown. $g_{i,q}(\cdot)$ is a smooth bounded and known function, which $o_1 < |g_{i,q}(\cdot)| < o_2$, and o_1, o_2 are known positive constants. The gain K_i is transfer coefficient and the sign of it is unknown. The leader's signal $r(t) \in \mathbf{R}$ is known and the functions $r(t)$, $\dot{r}(t)$ are assumed to be bounded and smooth.

Remark 1: $f_{i,q}(\cdot)$ is the unknown smooth nonlinear function and it need all states of the system (1), so that it is non-strict feedback case. Many practical systems can be described as or transformed into (1). For example, the electromechanical system in [20] and the one-link manipulator system in [31] and so on.

C. SOME LEMMAS AND ASSUMPTIONS

Assumption 1: $\bar{\mathcal{G}}$ contains a spanning tree and the root node is the leader node.

Note that Assumption 1 guarantees that all eigenvalues of the matrix $H = L + B$ have positive real parts [28].

Lemma 1 [28]: If the numbers $\sigma_1 > 0$, $\sigma_2 > 0$, and $\gamma \in (0, 1)$, and the Lyapunov function satisfies $\dot{V} + \sigma_1 V(x) + \sigma_2 V^{\gamma}(x) \leq 0$, then the system is finite-time stable, and the setting time is $T \leq t_0 + [1/(\sigma_1(1-\gamma))] \ln\left[\left(\sigma_1 V^{1-\gamma}(t_0) + \sigma_2\right)/\sigma_2\right]$.

Lemma 2 [35]: If the continuous function $\aleph(\cdot) : \mathbf{R} \to \mathbf{R}$ satisfies

$$\lim_{n\to\infty} \sup \frac{1}{n}\int_0^n \aleph(\nu)d\nu = +\infty$$
$$\lim_{n\to\infty} \inf \frac{1}{n}\int_0^n \aleph(\nu)d\nu = -\infty \tag{2}$$

then it is a Nussbaum type function. $\nu^2\cos(\nu)$ and $e^{\nu^2}\cos(\pi/(2\nu))$ are the common Nussbaum type functions.

Lemma 3 [33]: Let $V(t), \zeta(t)$ defined in the interval $[0, t_f)$ and they are all smooth functions and $\forall t \in [0, t_f)$, $V(t) \geq 0$, and the function $\aleph(\zeta)$ is even smooth Nussbaum-type function. If there is the following inequality:

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t [g(\tau)\aleph(\zeta) + 1]\,\dot{\zeta} e^{c_1\tau}d\tau \tag{3}$$

in which c_0 is a suitable constants, c_1 is positive constant, and the time-varying parameter $g(\tau)$ takes values in the $[l-, l+]$ with $0 \notin I$ and the interval is unknown. Then on $[0, t_f)$, the functions $V(t)$ and $\int_0^t g(\tau)\aleph(\zeta)\dot{\zeta}d\tau$ must be bounded.

Lemma 4 [20]: If the continuous function $f(x)$ is defined on a compact set Ψ. Then the FLS $W^T S(x)$ meets the following inequality

$$\sup_{x\in\psi} |f(x) - W^T S(x)| \leq \psi \tag{4}$$

where the any scalar $\psi > 0$ and $W = [w_1, w_2, \ldots, w_Q]^T$ is the ideal weight vector, $S(x) = [s_1(x), s_2(x), \ldots, s_Q(x)]T/\sum_{i=1}^{Q} s_i(x)$ is the basis function vector, and $s_i(x) = \exp[(-(x-\gamma_i)^T(x-\gamma_i))/(\tau_i^2)]$ is the Gaussian function, respectively. where $\gamma_i = [\gamma_{i,1}, \gamma_{i,2}, \ldots, \gamma_{i,n}]$ for $i = 1, 2, \ldots N$ is the center vector, and the width is τ_i.

III. MAIN RESULTS

Define the following tracking errors for the ith agent:

$$\varepsilon_{i,1} = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + b_i(y_i - r)$$
$$\varepsilon_{i,q} = x_{i,q} - \pi_{i,q}, \quad q = 2, \cdots, n_i \tag{5}$$

where $\pi_{i,q+1}(t) = \varphi_{i,q,1}(t)$ and $\varphi_{i,q,1}(t)$ is defined as follows.

Lemma 5 [28]: The following second-order command filter

$$\dot{\varphi}_{i,q,1} = \chi_{i,q,1}$$
$$\chi_{i,q,1} = -r_{i,q,1}\left|\varphi_{i,q,1} - \alpha_{i,q}\right|^{\frac{1}{2}} \mathrm{sign}\left(\varphi_{i,q,1} - \alpha_{i,q}\right) + \varphi_{i,q,2}$$
$$\dot{\varphi}_{i,q,2} = -r_{i,q,2}\mathrm{sign}\left(\varphi_{i,q,2} - \chi_{i,q,1}\right) \tag{6}$$

can guarantee $\varphi_{i,q,1} = \alpha_{i,q}$ and $\chi_{i,q,1} = \dot{\alpha}_{i,q}$ are satisfied in finite time by choosing proper $r_{i,q,1}$ and $r_{i,q,2}$.

Remark 2: The parameters $r_{i,q,1}$ and $r_{i,q,2}$ of command filter (6) should be chosen large enough, and $r_{i,q,2}$ should be firstly selected. Then by inputting the virtual signal $\alpha_{i,q}$, we can get $\pi_{i,q+1}$ and $\dot{\pi}_{i,q+1}$.

Remark 3: In the process of backstepping, the command filter (6) is used to get all intermediate functions and their derivatives by inputting $\alpha_{i,q}$ of each step, it can also filter the virtual signals precisely and get its derivatives and guarantee the conditions $\varphi_{i,q,1} = \alpha_{i,q}$ and $\chi_{i,q,1} = \dot{\alpha}_{i,q}$ are satisfied in finite time. The application of command filter can well solve the problem of computational explosion in traditional backstepping control design in [14], [15].

Now, through the command filtered backstepping scheme based on fractional power control law, the virtual control signals $\alpha_{i,q}$, α_{i,n_i} are designed as follows:

$$\alpha_{i,1} = \frac{1}{(d_i+b_i)g_{i,1}}\Bigg(-\frac{1}{2}v_{i,1} - k_{i,1}\varepsilon_{i,1} - \frac{v_{i,1}\hat{\theta}_i}{2\phi_{i,1}^2 S_{i,1}^T S_{i,1}} - \varrho_{i,1}v_{i,1}^{\gamma} + \sum_{j=1}^{N} a_{i,j}g_{j,1}x_{j,2} + b_i\dot{r}\Bigg)$$

$$\alpha_{i,2} = \frac{1}{g_{i,2}}\Bigg(-\frac{1}{2}v_{i,2} - k_{i,2}\varepsilon_{i,2} - \frac{v_{i,2}\hat{\theta}_i}{2\phi_{i,2}^2 S_{i,2}^T S_{i,2}} - \varrho_{i,2}v_{i,2}^{\gamma} + \dot{\pi}_{i,2} - (d_i+b_i)g_{i,1}\varepsilon_{i,1}\Bigg)$$

$$\alpha_{i,q} = \frac{1}{g_{i,q}}\Bigg(-\frac{1}{2}v_{i,q} - k_{i,q}\varepsilon_{i,q} - \frac{v_{i,q}\hat{\theta}_i}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} - \varrho_{i,q}v_{i,q}^{\gamma} + \dot{\pi}_{i,q} - g_{i,q-1}\varepsilon_{i,q-1}\Bigg)$$

$$u_i = \alpha_{i,n_i} = \frac{\aleph(\zeta_i)}{g_{i,n_i}}\Bigg(\frac{1}{2}v_{i,n_i} + k_{i,n_i}\varepsilon_{i,n_i} + \frac{v_{i,n_i}\hat{\theta}_i}{2\phi_{i,n_i}^2 S_{i,n_i}^T S_{i,n_i}} + \varrho_{i,n_i}v_{i,n_i}^{\gamma} - \dot{\pi}_{i,n_i} + g_{i,n_i-1}\varepsilon_{i,n_i-1}\Bigg) \tag{7}$$

where $\frac{1}{2} < \gamma = \frac{\gamma_1}{\gamma_2} < 1$, γ_1, γ_2 are positive odd integers, $k_{i,q}$, $\varrho_{i,q}$ are all positive constants, $\aleph(\zeta_i)$ is a smooth Nussbaum-type function and the updating process of ζ_i is designed as

$$\dot{\zeta} = v_{i,n_i}\Bigg(\frac{1}{2}v_{i,n_i} + k_{i,n_i}\varepsilon_{i,n_i} + \frac{v_{i,n_i}\hat{\theta}_i}{2\phi_{i,n_i}^2 S_{i,n_i}^T S_{i,n_i}} + \varrho_{i,n_i}v_{i,n_i}^{\gamma} - \dot{\pi}_{i,n_i} + g_{i,n_i-1}\varepsilon_{i,n_i-1}\Bigg) \tag{8}$$

We further define the compensated tracking error $v_{i,q}$ as

$$v_{i,q} = \varepsilon_{i,q} - \vartheta_{i,q}, \quad q = 1, \ldots, n_i \tag{9}$$

and give the following error compensation signals to eliminate the error $\pi_{i,q+1} - \alpha_{i,q}$ caused by command filter (6)

$$\dot{\vartheta}_{i,1} = -k_{i,1}\vartheta_{i,1} + (d_i+b_i)g_{i,1}\vartheta_{i,2} - \iota_{i,1}\mathrm{sign}(\vartheta_{i,1}) + (d_i+b_i)g_{i,1}(\pi_{i,2} - \alpha_{i,1})$$

$$\begin{aligned}\dot{\vartheta}_{i,2} &= -k_{i,2}\vartheta_{i,2} - (d_i+b_i)g_{i,1}\vartheta_{i,1} + g_{i,2}(\pi_{i,3}-\alpha_{i,2}) \\ &\quad - \iota_{i,2}\text{sign}(\vartheta_{i,2}) + g_{i,2}\vartheta_{i,3} \\ \dot{\vartheta}_{i,q} &= -k_{i,q}\vartheta_{i,q} - g_{i,q-1}\vartheta_{i,q-1} + g_{i,q}(\pi_{i,q+1}-\alpha_{i,q}) \\ &\quad - \iota_{i,q}\text{sign}(\vartheta_{i,q}) + g_{i,q}\vartheta_{i,q} \\ \dot{\vartheta}_{i,n_i} &= -k_{i,n_i}\vartheta_{i,n_i} - \iota_{i,n_i}\text{sign}(\vartheta_{i,n_i}) - g_{i,n_i-1}\vartheta_{i,n_i-1} \end{aligned} \tag{10}$$

Now we establish the following Lyapunov functions with n_i steps to show the designed controller is effective.

step 1: Firstly, choose the following Lyapunov function

$$V_{i,1} = \frac{1}{2}v_{i,1}^2 \tag{11}$$

then $\dot{V}_{i,1}$ is

$$\begin{aligned}\dot{V}_{i,1} &= v_{i,1}(\dot{\varepsilon}_{i,1} - \dot{\vartheta}_{i,1}) \\ &= v_{i,1}\Big(-\dot{\vartheta}_{i,1} + \sum_{j=1}^{N} a_{ij}(\dot{x}_{i,1}-\dot{x}_{j,1}) + b_i(\dot{x}_{i,1}-\dot{r})\Big) \\ &= v_{i,1}\Big(-\dot{\vartheta}_{i,1} + (d_i+b_i)(f_{i,1}(x_i) + g_{i,1}(x_{i,1})x_{i,2}) \\ &\quad - \sum_{j=1}^{N} a_{ij}(f_{j,1}(x_j) + g_{j,1}(x_{j,1})x_{j,2}) - b_i\dot{r}\Big) \\ &= v_{i,1}\Big(-\dot{\vartheta}_{i,1} + (d_i+b_i)f_{i,1}(x_i) - \sum_{j=1}^{N} a_{ij}\big(f_{j,1}(x_j) \\ &\quad + g_{j,1}(x_{j,1})x_{j,2}\big) + (d_i+b_i)g_{i,1}(x_{i,1})\varepsilon_{i,2} \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})(\pi_{i,2}-\alpha_{i,1}) \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})\alpha_{i,1} - b_i\dot{r}\Big) \\ &= v_{i,1}\Big(-\dot{\vartheta}_{i,1} + \bar{f}_{i,1}(x_i,x_j) - \sum_{j=1}^{N} a_{ij}g_{j,1}(x_{j,1})x_{j,2} \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})\varepsilon_{i,2} \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})(\pi_{i,2}-\alpha_{i,1}) \\ &\quad + (d_i+b_i)g_{i,1}(x_{i,1})\alpha_{i,1} - b_i\dot{r}\Big) \end{aligned} \tag{12}$$

where $\bar{f}_{i,1}(x_i,x_j) = (d_i+b_i)f_{i,1}(x_i) - \sum_{j=1}^{N} a_{ij}f_{j,1}(x_j)$ and it is unknown, from Lemma 4 it can be approximated by the FLS, that is

$$\bar{f}_{i,1}(x_i,x_j) = W_{i,1}^T S_i(x_i,x_j) + \delta_{i,1} \tag{13}$$

where x_i, x_j are state vectors of agent i and j, $|\delta_{i,1}| \le \psi_{i,1}$, and $\psi_{i,1}$ is a positive constant. Because $0 \le S_i^T S_i \le 1$ and through inequality scaling yields

$$\begin{aligned} v_{i,1}\bar{f}_{i,1} &\le \frac{v_{i,1}^2\|W_{i,1}\|^2 S_i^T S_i}{2\phi_{i,1}^2} + \frac{\phi_{i,1}^2}{2} + \frac{\psi_{i,1}^2}{2} + \frac{v_{i,1}^2}{2} \\ &\le \frac{v_{i,1}^2\|W_{i,1}\|^2}{2\phi_{i,1}^2 S_{i,1}^T S_{i,1}} + \frac{\phi_{i,1}^2}{2} + \frac{\psi_{i,1}^2}{2} + \frac{v_{i,1}^2}{2} \end{aligned} \tag{14}$$

where $\phi_{i,1} > 0$ is a constant, $S_i = S_i(x_i,x_j)$ and $S_{i,1} = S_{i,1}(x_{i,1},x_{j,1})$. Now, substituting (7), (10) and (14) into (12), we can get

$$\begin{aligned}\dot{V}_{i,1} \le v_{i,1}\Bigg(&\frac{v_{i,1}\left(\|W_{i,1}\|^2 - \hat{\theta}_i\right)}{2\phi_{i,1}^2 S_{i,1}^T S_{i,1}} \\ &- k_{i,1}v_{i,1} + (d_i+b_i)g_{i,1}v_{i,2} - \varrho_{i,1}v_{i,1}^{\gamma} \\ &+ \iota_{i,1}\text{sign}(\vartheta_{i,1})\Bigg) + \frac{\psi_{i,1}^2}{2} + \frac{\phi_{i,1}^2}{2} \end{aligned} \tag{15}$$

step 2: The second Lyapunov function is constructed by

$$V_{i,2} = V_{i,1} + \frac{1}{2}v_{i,2}^2 \tag{16}$$

So we can get

$$\begin{aligned}\dot{V}_{i,2} &= \dot{V}_{i,1} + v_{i,2}\dot{v}_{i,2} = \dot{V}_{i,1} + v_{i,2}(-\dot{\vartheta}_{i,2} + \dot{x}_{i,2} - \dot{\pi}_{i,2}) \\ &= \dot{V}_{i,1} + v_{i,2}\Big(-\dot{\vartheta}_{i,2} + f_{i,2}(x_i) + g_{i,2}(\bar{x}_{i,2})x_{i,3} - \dot{\pi}_{i,2}\Big) \\ &= \dot{V}_{i,1} + v_{i,2}\Big(-\dot{\vartheta}_{i,2} + f_{i,2}(x_i) + g_{i,2}(\bar{x}_{i,2})\varepsilon_{i,3} \\ &\quad + g_{i,2}(\bar{x}_{i,2})\alpha_{i,2} + g_{i,2}(\bar{x}_{i,2})(\pi_{i,3}-\alpha_{i,2}) - \dot{\pi}_{i,2}\Big) \end{aligned} \tag{17}$$

From the FLS, there exists a $W_{i,2}^T S_i(x_i)$ so that

$$f_{i,2}(x_i) = W_{i,2}^T S_i^T(x_i) + \delta_{i,2} \tag{18}$$

where $|\delta_{i,2}| \le \psi_{i,2}$, and $\psi_{i,2} > 0$ is a positive constant. Similarly we use the inequality technique like in (14) and substitute (7), (10) into (17) yields

$$\begin{aligned}\dot{V}_{i,2} \le &\sum_{q=1}^{2}\left[-k_{i,q}v_{i,q}^2 + v_{i,q}\iota_{i,q}\text{sign}(\vartheta_{i,q}) - \varrho_{i,q}v_{i,q}^{\gamma+1}\right] \\ &+ \sum_{q=1}^{2}\left(\frac{1}{2}\phi_{i,q}^2 + \frac{1}{2}\psi_{i,q}^2\right) + g_{i,2}v_{i,2}v_{i,3} \\ &+ \sum_{q=1}^{2}\frac{v_{i,q}^2(\|W_{i,q}\|^2 - \hat{\theta}_i)}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} \end{aligned} \tag{19}$$

where $S_{i,q} = S_{i,q}(\bar{x}_{i,q})$.

step q: The Lyapunov function equation of step q is constructed

$$V_{i,q} = V_{i,q-1} + \frac{1}{2}v_{i,q}^2, \; q = 3, \ldots n_i - 1 \tag{20}$$

Then we get

$$\begin{aligned}\dot{V}_{i,q} &= \dot{V}_{i,q-1} + v_{i,q}\dot{v}_{i,q} = \dot{V}_{i,q-1} + v_{i,q}(-\dot{\vartheta}_{i,q} + \dot{x}_{i,q} - \dot{\pi}_{i,q}) \\ &= \dot{V}_{i,q-1} + v_{i,q}(-\dot{\vartheta}_{i,q} + f_{i,q}(x_i) + g_{i,q}(\bar{x}_{i,q})x_{i,q+1} - \dot{\pi}_{i,q}) \end{aligned} \tag{21}$$

Similarly, the given constant $\psi_{i,q} > 0$, from the FLS, there exists $W_{i,q}^T S_i(x_i)$ so that

$$f_{i,q}(x_i) = W_{i,q}^T S_i^T(x_i) + \delta_{i,q} \tag{22}$$

where $|\delta_{i,q}| \leq \psi_{i,q}$,and $S_{i,q} = S_{i,q}(\bar{x}_{i,q})$ .Through the inequality technique, we can get the following results:

$$v_{i,q}f_{i,q}(x_i) \leq \frac{v_{i,q}^2\|W_{i,q}\|^2}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} + \frac{1}{2}\phi_{i,q}^2 + \frac{1}{2}\psi_{i,q}^2 + \frac{1}{2}v_{i,q}^2 \quad (23)$$

Then combine (7), (10), (23) and (21) we have

$$\begin{aligned}\dot{V}_{i,q} \leq& \sum_{p=1}^{q}\Big[-k_{i,p}v_{i,p}^2 + v_{i,p}\iota_{i,p}\mathrm{sign}(\vartheta_{i,p}) - \varrho_{i,p}v_{i,p}^{\gamma+1}\Big] \\ &+ \sum_{p=1}^{q}\left(\frac{\phi_{i,p}^2}{2} + \frac{\psi_{i,p}^2}{2}\right) \\ &+ \sum_{p=1}^{q}\frac{v_{i,p}^2(\|W_{i,p}\|^2 - \hat{\theta}_i)}{2\phi_{i,p}^2 S_{i,p}^T S_{i,p}} + g_{i,q}v_{i,q}v_{i,q+1} \quad (24)\end{aligned}$$

where $S_{i,p} = S_{i,p}(\bar{x}_{i,p})$.

step n_i: The Lyapunov function equation of step n_i is constructed as

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2}v_{i,n_i}^2 \quad (25)$$

and

$$\begin{aligned}\dot{V}_{i,n_i} &= \dot{V}_{i,n_i-1} + v_{i,n_i}\dot{v}_{i,n_i} \\ &= \dot{V}_{i,n_i-1} + v_{i,n_i}(\dot{x}_{i,n_i} - \dot{\vartheta}_{i,n_i} - \dot{\pi}_{i,n_i}) \\ &= \dot{V}_{i,n_i-1} + v_{i,n_i}(f_{i,n_i}(x_i) + g_{i,n_i}K_iu_i \\ &\quad - \dot{\vartheta}_{i,n_i} - \dot{\pi}_{i,n_i}) + \dot{\zeta}_i - \dot{\zeta}_i \quad (26)\end{aligned}$$

Similarly, there exists a $W_{i,n_i}^T S_i(x_i)$ so that

$$f_{i,n_i}(x_i) = W_{i,n_i}^T S_i^T(x_i) + \delta_{i,n_i} \quad (27)$$

where $|\delta_{i,n_i}| \leq \psi_{i,n_i}$, and ψ_{i,n_i} is a positive constant and where $S_{i,n_i} = S_{i,n_i}(\bar{x}_{i,n_i})$. We can get the following results:

$$v_{i,n_i}f_{i,n_i}(x_i) \leq \frac{v_{i,n_i}^2\|W_{i,n_i}\|^2}{2\phi_{i,n_i}^2 S_{i,n_i}^T S_{i,n_i}} + \frac{1}{2}\phi_{i,n_i}^2 + \frac{1}{2}\psi_{i,n_i}^2 + \frac{1}{2}v_{i,n_i}^2 \quad (28)$$

So substitutes (7), (8), (10) and (28) into (26) we can get the results

$$\begin{aligned}\dot{V}_{i,n_i} \leq& \sum_{q=1}^{n_i}\Big[-k_{i,q}v_{i,q}^2 + v_{i,q}\iota_{i,q}\mathrm{sign}(\vartheta_{i,q}) - \varrho_{i,q}v_{i,q}^{\gamma+1}\Big] \\ &+ \sum_{q=1}^{n_i}\left(\frac{\phi_{i,q}^2}{2} + \frac{\psi_{i,q}^2}{2}\right) + \sum_{q=1}^{n_i}\frac{v_{i,q}^2(\|W_{i,q}\|^2 - \hat{\theta}_i)}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} \\ &+ (K_i\aleph(\zeta_i) + 1)\dot{\zeta}_i \quad (29)\end{aligned}$$

According to Yang's inequality we can get

$$\begin{aligned}\iota_{i,q}v_{i,q}\mathrm{sign}(\vartheta_{i,q}) &\leq \frac{1}{2}\iota_{i,q}\big[\mathrm{sign}(\vartheta_{i,q})\big]^2 + \frac{1}{2}\iota_{i,q}v_{i,q}^2 \\ &\leq \frac{1}{2}\iota_{i,q} + \frac{1}{2}\iota_{i,q}v_{i,q}^2 \quad (30)\end{aligned}$$

Further, substitutes (30) into (29) we can get

$$\begin{aligned}\dot{V}_{i,n_i} \leq& -\sum_{q=1}^{n_i}\left(k_{i,q} - \frac{1}{2}\iota_{1,q}\right)v_{i,q}^2 - \sum_{q=1}^{n_i}\varrho_{i,q}v_{i,q}^{\gamma+1} \\ &+ \sum_{q=1}^{n_i}\left(\frac{1}{2}\psi_{i,q}^2 + \frac{1}{2}\iota_{i,q} + \frac{1}{2}\phi_{i,q}^2\right) \\ &+ \sum_{q=1}^{n_i}\frac{v_{i,q}^2(\|W_{i,q}\|^2 - \hat{\theta}_i)}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} \\ &+ (K_i\aleph(\zeta_i) + 1)\,\dot{\zeta}_i \quad (31)\end{aligned}$$

We denote $\theta_i = \max\{\|W_{i,1}\|^2, \|W_{i,2}\|^2, \ldots, \|W_{i,n_i}\|^2\}$, and the adaptive updating law about $\hat{\theta}_i$ is designed as

$$\dot{\hat{\theta}}_i = -2\lambda_i\mu_i\hat{\theta}_i + \sum_{q=1}^{n_i}\frac{\lambda_i v_{i,q}^2}{2\phi_{i,q}^2 S_{i,q}^T S_{i,q}} \quad (32)$$

where the constants λ_i, μ_i are all positive.

We further define

$$V = \sum_{i=1}^{N}V_{i,n_i} + \sum_{i=1}^{N}\frac{1}{2\lambda_i}\tilde{\theta}_i^2 \quad (33)$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, so we can get

$$\begin{aligned}\dot{V} \leq& -\sum_{i=1}^{N}\sum_{q=1}^{n_i}\left(k_{i,q} - \frac{1}{2}\iota_{i,q}\right)v_{i,q}^2 - \sum_{i=1}^{N}\sum_{q=1}^{n_i}\varrho_{i,q}v_{i,q}^{\gamma+1} \\ &+ \sum_{i=1}^{N}\sum_{q=1}^{n_i}\left(\frac{1}{2}\psi_{i,q}^2 + \frac{1}{2}\iota_{i,q} + \frac{1}{2}\phi_{i,q}^2\right) + \sum_{i=1}^{N}2\mu_i\tilde{\theta}_i\hat{\theta}_i \\ &+ \sum_{i=1}^{N}(K_i\aleph(\zeta_i) + 1)\,\dot{\zeta}_i \quad (34)\end{aligned}$$

where $\mu_i\tilde{\theta}_i\hat{\theta}_i \leq -\big[(\mu_i(2c_i - 1))/2c_i\big]\tilde{\theta}_i^2 + (\mu_ic_i/2)\theta_i^2$, and $c_i > (1/2)$. So we have

$$\begin{aligned}\dot{V} \leq& -\sum_{i=1}^{N}\sum_{q=1}^{n_i}\left(k_{i,q} - \frac{1}{2}\iota_{i,q}\right)v_{i,q}^2 - \sum_{i=1}^{N}\sum_{q=1}^{n_i}\varrho_{i,q}v_{i,q}^{\gamma+1} \\ &+ \sum_{i=1}^{N}\sum_{q=1}^{n_i}\left(\frac{1}{2}\iota_{i,q} + \frac{1}{2}\phi_{i,q}^2 + \frac{1}{2}\psi_{i,q}^2\right) \\ &- \sum_{i=1}^{N}\left(\frac{\varsigma_i}{\lambda_i}\tilde{\theta}_i^2\right)^{\frac{\gamma+1}{2}} + \sum_{i=1}^{N}\left(\frac{\varsigma_i}{\lambda_i}\tilde{\theta}_i^2\right)^{\frac{\gamma+1}{2}} - 2\sum_{i=1}^{N}\frac{\varsigma_i}{\lambda_i}\tilde{\theta}_i^2 \\ &+ \sum_{i=1}^{N}\mu_ic_i\theta_i^2 + \sum_{i=1}^{N}(K_i\aleph(\zeta_i) + 1)\,\dot{\zeta}_i \quad (35)\end{aligned}$$

in which $\varsigma_i = \lambda_i\big[(\mu_i(2c_i - 1))/2c_i\big]$, if $(\varsigma_i/\lambda_i)\tilde{\theta}_i^2 \geq 1$, so that $[(\varsigma_i/\lambda_i)\tilde{\theta}_i^2]^{(\gamma+1)/2} - (\varsigma_i/\lambda_i)\tilde{\theta}_i^2 + \mu_ic_i\theta_i^2 \leq \mu_ic_i\theta_i^2$, if $(\varsigma_i/\lambda_i)\tilde{\theta}_i^2 < 1$, we have $[(\varsigma_i/\lambda_i)\tilde{\theta}_i^2]^{(\gamma+1)/2} - (\varsigma_i/\lambda_i)\tilde{\theta}_i^2 + \mu_ic_i\theta_i^2 < 1 + \mu_ic_i\theta_i^2$. Applying these to push and we can get

$$\dot{V} \leq -\sigma_1V - \sigma_2V^{\frac{\gamma+1}{2}} + \eta + \sum_{i=1}^{N}(K_i\aleph(\zeta_i) + 1)\,\dot{\zeta}_i \quad (36)$$

where $\sigma_1 = \min\{2k_{i,q} - \iota_{i,q}, 2\varsigma_i\}$, $\sigma_2 = \min\{\varrho_{i,q} \cdot 2^{(\gamma+1)/2}, (2\varsigma_i)^{(\gamma+1)/2}\}$ and $\eta = \sum_{i=1}^{N}\sum_{q=1}^{n_i}(\iota_{i,q}/2+\phi_{i,q}^2/2+\psi_{i,q}^2/2) + N + \sum_{i=1}^{N}\mu_i c_i \theta_i^2$. (36) implies that $\dot{V} \le -\sigma_1 V + \eta + \sum_{i=1}^{N}(K_i\aleph(\zeta_i)+1)\dot{\zeta}_i$, and from Lemma 3 we know $\sum_{i=1}^{N}(K_i\aleph(\zeta_i)+1)\dot{\zeta}_i$ is bounded in $[0, t_f)$, and $\eta_{\max} = \max_{t\in[0,t_f]}\sum_{i=1}^{N}(K_i\aleph(\zeta_i)+1)\dot{\zeta}_i$, $\bar{\eta} = \eta + \eta_{\max}$. So the inequality (36) can be rewritten as

$$\dot{V} \le -\sigma_1 V - \sigma_2 V^{\frac{\gamma+1}{2}} + \bar{\eta} \tag{37}$$

Then there has a constant $\upsilon \in (0, 1)$ so that (38) can be written as

$$\dot{V} \le -\upsilon\sigma_1 V - (1-\upsilon)\sigma_1 V - \sigma_2 V^{\frac{\gamma+1}{2}} + \bar{\eta} \tag{38}$$

or

$$\dot{V} \le -\sigma_1 V - \upsilon\sigma_2 V^{\frac{\gamma+1}{2}} - (1-\upsilon)\sigma_2 V^{\frac{\gamma+1}{2}} + \bar{\eta} \tag{39}$$

We can get $\dot{V} \le -\upsilon\sigma_1 V - \sigma_2 V^{(\gamma+1)/2}$ when $V > [\bar{\eta}/(1-\upsilon)\sigma_1]$. According to Lemma 1 the decrease of V will put $v_{i,q}, \tilde{\theta}_i$ into the region

$$(v_{i,q}, \tilde{\theta}_i) \in \left\{V \le \frac{\bar{\eta}}{(1-\upsilon)\sigma_1}\right\} \tag{40}$$

in finite time $T_{1,1} \le [1/(\upsilon\sigma_1(1-(\gamma+1)/2))]\ln[(\upsilon\sigma_1 V^{1-(\gamma+1)/2}(0)+\sigma_2)/\sigma_2]$. From (39) we have $\dot{V} \le -\sigma_1 V - \upsilon\sigma_2 V^{(\gamma+1)/2}$ if $V^{(\gamma+1)/2} > [\bar{\eta}/(1-v)\sigma_2]$, similarly, we know that $v_{i,q}, \tilde{\theta}_i$ will be driven into the region

$$(v_{i,q}, \tilde{\theta}_i) \in \left\{V \le \left[\frac{\bar{\eta}}{(1-v)\sigma_2}\right]^{\frac{2}{\gamma+1}}\right\} \tag{41}$$

in finite time $T_{1,2} \le [1/\sigma_1(1-(\gamma+1)/2)]\ln[(\sigma_1 V^{1-(\gamma+1)/2}(0)+\upsilon\sigma_2)/\upsilon\sigma_2]$. So that we know $v_{i,q}$ will arrive $|v_{i,q}| \le \min\{\sqrt{2[\bar{\eta}/((1-\upsilon)\sigma_1)]}, \sqrt{2[\bar{\eta}/((1-v)\sigma_2)]^{2/(\gamma+1)}}\}$ in finite time $T_1 = \max\{T_{1,1}, T_{1,2}\}$. In order to prove that ε_i is bounded, we need also to prove that ϑ_i is bounded in finite time since $\varepsilon_i = v_i + \vartheta_i$. Now we choose

$$\bar{V} = \frac{1}{2}\sum_{i=1}^{N}\sum_{q=1}^{n_i}\vartheta_{i,q}^2 \tag{42}$$

Then, one have

$$\begin{aligned}\dot{\bar{V}} = \sum_{i=1}^{N}\Big(&(d_i+b_i)g_{i,1}\vartheta_{i,1}(\pi_{i,2}-\alpha_{i,1}) - k_{i,1}\vartheta^2{}_{i,1}\\ &+(d_i+b_i)g_{i,1}\vartheta_{i,1}\vartheta_{i,2} - \vartheta_{i,1}\iota_{i,1}\mathrm{sign}(\vartheta_{i,1})\\ &-k_{i,2}\vartheta^2{}_{i,2} + g_{i,2}\vartheta_{i,2}(\pi_{i,3}-\alpha_{i,2}) + g_{i,2}\vartheta_{i,2}\vartheta_{i,3}\\ &-(d_i+b_i)g_{i,1}\vartheta_{i,1}\vartheta_{i,2} - \vartheta_{i,2}\iota_{i,2}\mathrm{sign}(\vartheta_{i,2})\\ &+\ldots+g_{i,q}\vartheta_{i,q}(\pi_{i,q+1}-\alpha_{i,q})+g_{i,q}\vartheta_{i,q}\vartheta_{i,q+1}-k_{i,q}\vartheta^2{}_{i,q}\\ &-g_{i,q-1}\vartheta_{i,q-1}\vartheta_{i,q} - \vartheta_{i,q}\iota_{i,q}\mathrm{sign}(\vartheta_{i,q})\\ &+\ldots-k_{i,n_i}\vartheta^2{}_{i,n_i} - g_{i,n_i-1}\vartheta_{i,n_i-1}\vartheta_{i,n_i}\\ &-\vartheta_{i,n_i}\iota_{i,n_i}\mathrm{sign}(\vartheta_{i,n_i})\Big)\end{aligned}$$

$$\begin{aligned}&= -\sum_{i=1}^{N}\sum_{q=1}^{n_i}k_{i,q}\vartheta_{i,q}^2 - \sum_{i=1}^{N}\sum_{q=1}^{n_i}\vartheta_{i,q}\iota_{i,q}\mathrm{sign}(\vartheta_{i,q})\\ &\quad+\sum_{i=1}^{N}\sum_{q=2}^{n_i-1}g_{i,q}\vartheta_{i,q}(\pi_{i,q+1}-\alpha_{i,q})\\ &\quad+\sum_{i=1}^{N}(d_i+b_i)g_{i,1}\vartheta_{i,1}(\pi_{i,2}-\alpha_{i,1})\end{aligned} \tag{43}$$

Form lemma 5, we have $|\pi_{i,q+1}-\alpha_{i,q}| = 0$ is satisfied in finite time T_2, so we rewrite (43) as

$$\begin{aligned}\dot{\bar{V}} &\le -\sum_{i=1}^{N}\sum_{q=1}^{n_i}k_{i,q}\vartheta_{i,q}^2 - \sum_{i=1}^{N}\sum_{q=1}^{n_i}\iota_{i,q}|\vartheta_{i,q}|\\ &\le -k_0\bar{V} - l_0\bar{V}^{\frac{1}{2}}\end{aligned} \tag{44}$$

where $\iota_0 = \sqrt{2}\min\{\iota_{i,q}\}$, $k_0 = 2\min\{k_{i,q}\}$, then we have $\lim_{t\to T_3}\vartheta_{i,q} = 0$ for $T_3 \le T_2 + [1/(k_0(1-1/2))]\ln[(k_0\bar{V}^{1-1/2}(T_2)+\iota_0)/\iota_0]$.

Finally, we will get for t $\ge T_4 = \max\{T_1, T_3\}$, $|\varepsilon_{i,1}| \le \min\{\sqrt{2\bar{\eta}/[(1-\upsilon)\sigma_1]}, \sqrt{2[\bar{\eta}/((1-v)\sigma_2)]^{2/(\gamma+1)}}\}$. Denote $\Upsilon_1 = [\varepsilon_{1,1}, \varepsilon_{2,1}, \ldots, \varepsilon_{N,1}]^T$ and $\Phi_1 = [y_1-r, y_2-r, \ldots, y_N-r]^T$, $\Phi_1 = (H\otimes I_{N\times N})^{-1}\Upsilon_1$. Then we can get $|y_i - r| \le [\sqrt{N}\min\{\sqrt{2[\bar{\eta}((1-\upsilon)\sigma_1)]}, \sqrt{2\bar{\eta}/[(1-\upsilon)\sigma_2]^{2/(\gamma+1)}}\}/\rho_{min}(H)]$, where the minimum singular value of H is represented by $\rho_{min}(H)$.

Theorem 1: Consider the MASs (1) satisfies Assumption 1, and $r(t)$ represents the leader signal. Using the finite time command filters (6), design the virtual control functions (7) and the error compensation signals (10), then the control law is chosen $u_i = \alpha_{i,n_i}$ and combine with the adaptive law (33) can guarantee that the consensus tracking error of the closed-loop system can converge to a sufficiently small neighborhood of the origin in finite time.

Remark 4: We can see that the steady-state error is determined by the parameters σ_1 and σ_2, which are defined in (36). And the control parameters $2k_{i,q} > \varrho_{i,q}$ should be satisfied. so that if we choose larger control parameters $k_{i,q}, \varrho_{i,q}$ and smaller $\iota_{i,q}, \gamma$ we will get bigger σ_1 and σ_2. Then we will obtain the smaller steady-state error. But the small $\iota_{i,q}$ will affect the convergence rate of the error compensation signals.

IV. NUMERICAL RESULTS

In this part, we will through a specific simulation example and comparison of different control schemes to illustrate the effectiveness of the current method. As in Fig.1, consider the system has three followers and one leader, and the followers models are chosen by

$$\begin{aligned}&f_{1,1} = \cos(x_{1,1}x_{1,2}), \quad g_{1,1} = 1,\\ &f_{1,2} = x_{1,1}x_{1,2}, \quad g_{1,2} = 1\\ &f_{2,1} = \sin(0.5x_{2,1}x_{2,2}), \quad g_{2,1} = 1,\\ &f_{2,2} = x_{2,1}x_{2,2}e^{-0.3x_{2,2}}, \quad g_{2,2} = 1\\ &f_{3,1} = \cos(-0.5x_{3,1}x_{3,2}), \quad g_{3,1} = 1,\\ &f_{3,2} = x_{3,1}x_{3,2}, \quad g_{3,2} = 1.\end{aligned} \tag{45}$$

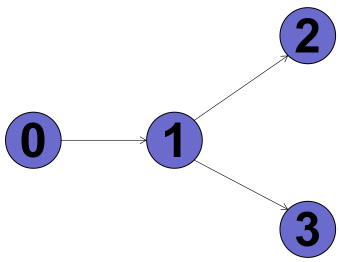

FIGURE 1. The leader and three followers.

in which $x_1(0) = [1.5, -0.5]^T$ is the initial state of agent 1, the agent 2 is $x_2(0) = [-0.7, 0.3]^T$, and agent 3 is $x_3(0) = [1.4, -0.4]^T$. The Laplacian matrix is $L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, the leader adjacency matrix $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and the output of the leader is given by $r(t) = \sin(0.5t)$. Control parameters of the system are set as $k_{i,q} = 10$, $\iota_{i,q} = 8$, $\varrho_{i,q} = 20$, $\gamma = (3/5)$, $r_{i,q,1} = 450$, $\lambda_i = 1$, $\mu_i = 1$, $r_{i,q,2} = 2000$, $\phi_{i,q} = 1$. The unknown control gain $K_i = -1$ and the Nussbaum type function is chosen as $\zeta^2 cos(\zeta)$. In the FLS, the number of the fuzzy rules is 10, and the basis function whose centers are distributed in $[-3, 3] \times \cdots \times [-3, 3]$, and the width $\tau = 4$. The responses of $x_{i,1}, i = 1, 2, 3$ and r are shown

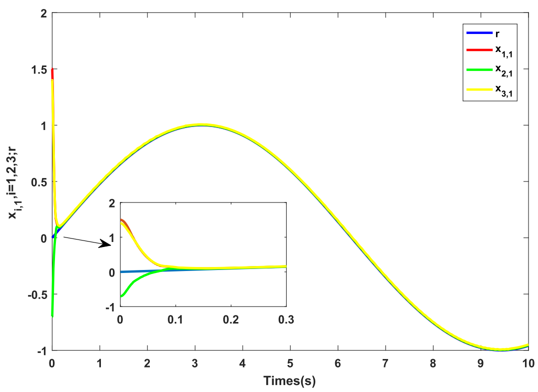

FIGURE 2. The responses of $x_{i,1}, i = 1, 2, 3$, and r.

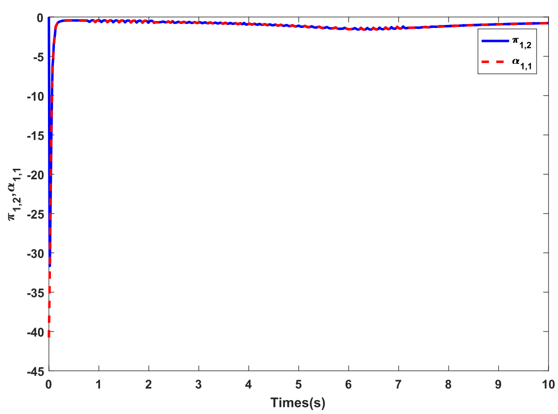

FIGURE 3. The responses of $\pi_{1,2}$ and $\alpha_{1,1}$.

in Fig.2, we can see that the closed-loop system has realized consensus tracking within a finite time although the control directions is unknown and the system is a non-strict feedback. Fig.3-Fig.5 reflect the virtual control signal $\alpha_{i,1}$ and the intermediate virtual signal $\pi_{i,2}$ designed in this paper. The overall tracking error OTE $= \|[\varepsilon_{1,1}, \varepsilon_{2,1}, \varepsilon_{3,1}]^T\|$ is used to compare the consensus tracking performances with the command filtered backstepping subject to different control parameters. Fig. 6 shows the different OTEs under the distributed finite-time command filtered backstepping control scheme and distributed command filtered backstepping control scheme, respectively. It is obvious that the better tracking

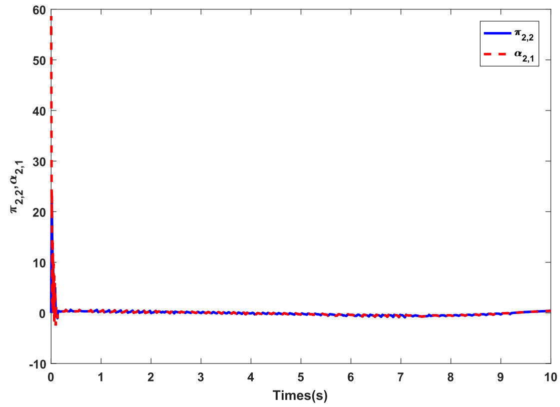

FIGURE 4. The responses of $\pi_{2,2}$ and $\alpha_{2,1}$.

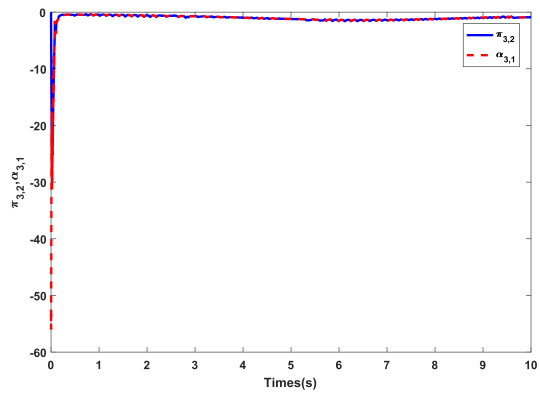

FIGURE 5. The responses of $\pi_{3,2}$ and $\alpha_{3,1}$.

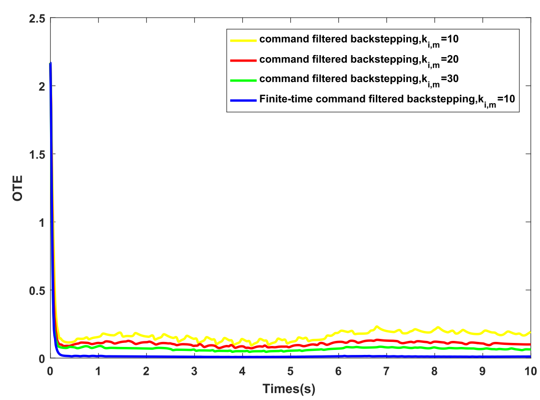

FIGURE 6. The OTEs of different control approaches.

performance and faster convergence rate are achieved under our proposed algorithm.

V. CONCLUSION

In this paper, the consensus tracking problem for nonstrict feedback MASs with nonlinear dynamics and unknown control directions has been solved by the presented method. In the designed method, the command filtered backstepping scheme based on fractional power control law is proposed, in which the error compensation signals, intermediate input signals and control input signals combined with adaptive control law are all constructed, the filtering errors are eliminated and the states of each agent can fast track the leader's output in finite time. In addition, the state variables problem in the nonstrict feedback system and unknown control directions are well solved by introducing Nussbaum function and through the scaling of inequalities. Further, if the state of each agent cannot be completely measured, the state observer will be studied in the future. And the switching topology can also be considered by using the presented method.

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