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A Novel Reliability Analysis Method for Fuzzy Multi-State Systems Considering Correlation

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ABSTRACT Fuzzy multi-state systems (FMSSs) exist widely in practical engineering. It is usually difficult to evaluate the reliability of FMSSs because the reliability data is usually fuzzy due to the inaccuracy or imperfection of information, and there is often correlation between the main components and others which constitute the FMSSs. Although many research works with respect to the independent failure of components have been carried out, the master-slave relationship between the main components and others of the FMSSs is often ignored, thus unrealistic results are often obtained with this treatment. Based on fuzzy universal generating function (FUGF), an effective reliability analysis method of FMSSs considering the correlation and fuzziness is proposed in this paper. In the novel method, the fuzziness of reliability data and the master-slave relationship between the main and other components are taken into consideration, and the performance levels and corresponding probabilities of the non-main components are considered as conditional probability distributions. A case study with respect to the reliability analysis of hydraulic system is presented to illustrate the application of the proposed method.

INDEX TERMS Multi-state system (MSS), fuzzy universal generating function (FUGF), composition operator, three-leg robot.

I. INTRODUCTION

Due to abrasion, fatigue, deformation and so on, the degradation of the performance level is inevitable in a mechanical system. As systems are getting more and more complex, the simplest binary-state systems turn into multi-state systems (MSSs) associated with the performance degradation. The conventional MSS models regard the observed reliability data of all components as crisp values, but some unrealistic results are often obtained with this treatment because of the inaccuracy or fluctuation of information, thus fuzzy multi-state systems (FMSSs) are proposed to overcome the deficiencies of conventional MSS models [1], [2].

The idea of MSS which reflects the polymorphism of systems or components was first touched in Hirsch *et al.* [3]. Subsequently, the MSS theory is applied in coherent systems [4], [5]. In the past few decades, many techniques and methods are introduced and extended, which have made significant progress on the theoretical side. However, it is usually assumed that the observed reliability data of all components in MSS are precise real numbers for reliability evaluation, risk assessment, and maintenance policy making [6]–[12]. For better capture the actual components or system behaviors, FMSS models which take the performance level and the corresponding probabilities as fuzzy values are developed. The reliability analysis for FMSS is usually difficult and how to accurately evaluate the reliability of a FMSS under uncertainties is an issue needs to be solved and has drawn a lot of attention. Huang *et al.* [13] proposed a Bayesian reliability analysis method to determine the membership function of parameter estimation and reliability function for fuzzy lifetime data. Ding and Lisnianski [1] and Ding *et al.* [2] proposed fuzzy universal generating function (FUGF) to analyze and evaluate the reliability of FMSS, with the consideration of incomplete and imprecise data and reducing computational complexity. The definition and concepts of FMSS and reliability of series-parallel systems were further elaborated by Lisnianski *et al.* [9]. Liu and Huang [14] proposed a

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modified reliability evaluation method for the uncertainty in state transition probability. Liu *et al.* [15] discussed the reliability evaluation by fuzzy Bayesian method and preventive maintenance policy making under fuzzy environments. Ren and Kong [16] designed a fuzzy expert system and applied it to fault tree model for reliability analysis of MSS. Based on qualitative data processing, Purba [17] and Purba *et al.* [18] developed a fuzzy reliability algorithm to obtain the failure probabilities of basic event of fault trees. Li *et al.* [19] investigated a random fuzzy extension based on the UGF method to consider aleatory and epistemic uncertainties, and applied the fuzzy extension to assess the reliability of MSS. Mousavi *et al.* [20] evaluated the MSS availability by FUGF to find the optimal redundancy.

According to our literature review, although a lot of research works with respect to the independent failure of multiple components are carried out, the master-slave relationship between the main components and others are often ignored in reliability analysis. Therefore, a novel reliability assessment method for FMSSs which can take the fuzziness of reliability data caused by the inaccuracy and insufficiency of information, and the master-slave relationship between the main and other components into account is proposed in this paper.

The rest of this paper is organized as follows. In Section II and Section III the preliminaries of multi-state system and universal generating function are reviewed. A novel fuzzy reliability analysis method is proposed in Section IV. Section V uses a case study with respect to the reliability analysis of hydraulic system to demonstrate all developments. Finally, Section VI summarizes and concludes.

II. MULTI-STATE SYSTEM

In aerospace, communications, power, nuclear industry, and many other fields, a system and its components can show multiple performance levels (states), which is called multi-state system (MSS). MSS is usually used to set up models for complex behaviors, such as performance degradation, imperfect coverage (IPC), and maintenance activities. The traditional binary system comprises of two distinct states: a perfect functioning state and a complete failure state, which can be regarded as the simplest case of a multi-state system. In practice, there are many different situations in which a system should be considered a MSS: [9]

1) A system consisting of different binary-state units that have a cumulative effect on the entire system performance, and the most typical example of such a situation is k-out-of-n systems.

2) Due to component performance degradation (fatigue, partial failure) or external environment change, which leads to the lead to the degradation of the entire MSS performance (or state).

Suppose a multi-state system consists of n components and component $j, 1 \leq j \leq n$ has k_j different states with corresponding performance levels which can be represented by a set

$$\mathbf{g}_j = \left\{g_{j1}, g_{j2}, \cdots, g_{jk_j}\right\} \tag{1}$$

where g_{ji} is the performance level of component j in the state $i, i \in \left\{1, 2, \cdots, k_j\right\}$.

For any time instant $t \geq 0$, the performance level $G_j(t)$ of component j is a random variable and takes its value from $\mathbf{g}_j$: $G_j(t) \in \mathbf{g}_j$. Suppose the MSS operation duration is T, and for the time interval $[0, T]$, the performance level $G_j(t)$ is a stochastic process. The probability of component j associated with different states at any time instant t can be expressed as

$$\mathbf{p}_j(t) = \left\{p_{j1}(t), p_{j2}(t), \cdots, p_{jk_j}(t)\right\} \tag{2}$$

where $p_{ji}(t) = \Pr\left\{G_j(t) = g_{ji}\right\}, i \in \left\{1, 2, \cdots, k_j\right\}$ and the mapping $g_{ji} \rightarrow p_{ji}$ is usually called the probability mass function (p.m.f.).

The component states compose the complete group of mutually exclusive events and yields

$$\sum_{i=1}^{k_j} p_{ji}(t) = 1, 0 \leq t \leq T \tag{3}$$

Suppose a multi-state system, composed of n independent components, and the performance levels of the system are completely determined by the performance levels of its components. Given a certain time instant, the state of the system component determines the state of the system. Suppose that the whole system has K different states and g_i is the performance level of the whole system at state $i, i \in \{1, 2, \cdots, K\}$.

The maximum of the possible states for the MSS is

$$K = \prod_{j=1}^{n} k_j \tag{4}$$

The performance level of the multi-state system can be expressed as a random variable and takes values from the set $\mathbf{V} = \{g_1, g_2, \cdots, g_K\}$. Let $\mathbf{R}^n = \left\{g_{11}, g_{12}, \cdots, g_{1k_1}\right\} \times \left\{g_{21}, g_{22}, \cdots, g_{2k_2}\right\} \times \cdots \times \left\{g_{n1}, g_{n2}, \cdots, g_{nk_n}\right\}$ be the space of all possible combinations of performance levels for all n components. The function $f(G_1(t), G_2(t), \cdots, G_n(t))$: $\mathbf{R}^n \rightarrow \mathbf{V}$, which maps the space of performance levels of components into the space of system's performance levels, is defined as the system structure function. The probability distribution (PD) of performance levels for each component at any time instant $t \geq 0$ can be expressed as

$$\begin{gathered}\mathbf{g}_j = \left\{g_{j1}, g_{j2}, \cdots, g_{jk_j}\right\}, \\ \mathbf{p}_j(t) = \left\{p_{j1}(t), p_{j2}(t), \cdots, p_{jk_j}(t)\right\}, \\ (j = 1, 2, \cdots, n)\end{gathered}$$

and the system structure function

$$G_s(t) = f(G_1(t), G_2(t), \cdots, G_n(t))$$

Without regard to different combinations corresponding to the same performance levels, the probability of the system performance staying in state i at time instant t yields

$$p_i(t) = \prod_{j=1}^{n} p_{ji_j}(t), i = 1, 2, \cdots, K; \; 1 \le i_j \le k_j \tag{5}$$

where $p_i(t)$ represents the probability of the whole system performance staying in state i at time instant t, $p_{ji_j}(t)$ is the corresponding probability of component j.

III. FUZZY UNIVERSAL GENERATING FUNCTION

Considering independent random variables $X_1, X_2, \cdots, X_n$ and an arbitrary function $f(X_1, X_2, \cdots, X_n)$ with respect to $X_1, X_2, \cdots, X_n$. Suppose that the PD of each variable X_i can be expressed as

$$\begin{aligned} \mathbf{x}_i &= \left\{x_{i1}, x_{i2}, \cdots, x_{ik_i}\right\}, \\ \mathbf{p}_i &= \left\{p_{i1}, p_{i2}, \cdots, p_{ik_i}\right\}, i = 1, 2, \cdots, n \end{aligned} \tag{6}$$

where k_i is the number of possible values for variable X_i.

The z-transformation of X_i can be defined as

$$u_i(z) = \sum_{j_i=1}^{k_i} p_{ij_i} z^{x_{ij_i}}, i = 1, 2, \cdots, n \tag{7}$$

To obtain the z-transformation of function $f(X_1, X_2, \cdots, X_n)$, a composition operator $\otimes_f$ is defined as

$$\begin{aligned} U_s(z) &= \otimes_f\left(u_1(z), u_2(z), \cdots, u_n(z)\right) \\ &= \otimes_f\left(\sum_{j_1=1}^{k_1} p_{1j_1} z^{x_{1j_1}}, \sum_{j_2=1}^{k_2} p_{2j_2} z^{x_{2j_2}}, \cdots, \sum_{j_n=1}^{k_n} p_{nj_n} z^{x_{nj_n}}\right) \\ &= \sum_{j_1=1}^{k_1}\sum_{j_2=1}^{k_2}\cdots\sum_{j_n=1}^{k_n}\left[\prod_{i=1}^{n} p_{ij_i} z^{f(x_{1j_1}, x_{2j_2}, \cdots, x_{nj_n})}\right] \end{aligned} \tag{8}$$

where $U_s(z)$ represents the z-transformation of function $f(X_1, X_2, \cdots, X_n)$.

The algorithm based on z-transformation and composition operator is called the universal generating function (UGF) method. In the UGF method, the z-transformation is also known as u-function. It should be noted that the u-function has the form of ordinary polynomials, but the operational rule defined by the composition operator is different from multiplication rule of polynomials.

For convenience, we take a system whose performance level $G = f(X_1, X_2) = X_1 X_2$ is determined by two random variables X_1, X_2 for an example. Suppose the PDs of X_1 and X_2 are

$$\begin{aligned} \mathbf{x}_1 &= \{x_{11}, x_{12}\} = \{4, 9\}, \\ \mathbf{p}_1 &= \{p_{11}, p_{12}\} = \{0.2, 0.8\} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \mathbf{x}_2 &= \{x_{21}, x_{22}, x_{23}\} = \{0, 1, 2\}; \\ \mathbf{p}_2 &= \{p_{21}, p_{22}, p_{23}\} = \{0.2, 0.2, 0.6\} \end{aligned} \tag{10}$$

According to Eq. (7), the z-transformation of X_1 and X_2 can be gained

$$u_1(z) = p_{11} z^{x_{11}} + p_{12} z^{x_{12}} = 0.2z^4 + 0.8z^9 \tag{11}$$

$$\begin{aligned} u_2(z) &= p_{21} z^{x_{21}} + p_{22} z^{x_{22}} + p_{23} z^{x_{23}} \\ &= 0.2z^0 + 0.2z^1 + 0.6z^2 \end{aligned} \tag{12}$$

Applying the composition operator to Eqs. (11)-(12), the z-transformation of the system yields

$$\begin{aligned} U_s(z) &= \otimes_f\left(u_1(z), u_2(z)\right) \\ &= \otimes_f\left(0.2z^4 + 0.8z^9, 0.2z^0 + 0.2z^1 + 0.6z^2\right) \\ &= 0.04z^0 + 0.04z^4 + 0.12z^8 + 0.16z^0 + 0.16z^9 + 0.48z^{18} \\ &= 0.2z^0 + 0.04z^4 + 0.12z^8 + 0.16z^9 + 0.48z^{18} \end{aligned} \tag{13}$$

According to Eq. (4), the maximum of the possible states is $K = 2 \times 3 = 6$. In Eq. (13), the number of states is reduced from 6 to 5 because there exists different combinations corresponding to the same state.

Through expanding the method of UGF, FUGF method is proposed to consider the fuzziness of data. The difference is that FUGF considers the performance levels and/or the corresponding probabilities as fuzzy values, but both of them are deemed to crisp values in UGF.

The z-transformation of a system in FUGF can be expressed as

$$\tilde{U}_s(z) = \tilde{\Omega}_\Theta\left(\tilde{u}_1(z), \tilde{u}_2(z), \cdots, \tilde{u}_n(z)\right) = \sum_{i=1}^{M} \tilde{p}_i z^{\tilde{g}_i} \tag{14}$$

where $\tilde{U}_s(z)$ is the z-transformation of the performance level for MSS, $\tilde{\Omega}_\Theta$ is the fuzzy composition operator, M is the number of system states, and $\tilde{g}_i$ and $\tilde{p}_i$ denote the fuzzy performance levels and the corresponding probabilities, respectively.

IV. THE PROPOSED FUZZY RELIABILITY ANALYSIS METHOD FOR FMSS

A. RELIABILITY MODELING AND ANALYSIS OF FMSS CONSIDERING CORRELATION

The conventional MSS models regard the PDs of all components as crisp values and the components which constitute the system are independent from each other, and this cannot always be satisfied. In order to be more reasonable, the performance levels and the corresponding probabilities of a component can be measured as fuzzy values in practical engineering such as the stress is about 500MPa or the probability that the mean stress equals to 500MPa is about 0.8. That is, this kind of MSS should be regard as a FMSS. In addition to this, the components may be correlated, and for FMSS, there is often an component whose performance level can affect the performance levels of the others, that is, the performance level of any other components in FMSS is affected by the main component, such as the electric current of trunk current has impact on that of branch current in electrical system. Due to the master-slave relationship between the main components and others are often ignored in reliability analysis method for FMSSs, a novel fuzzy reliability analysis method consider the correlation is proposed in this paper.

TABLE 1. The relationship between component 1 and the others.

Component 1	Component 2	…	Component n
$\tilde{g}_1^1$, $\tilde{p}_1^1$	$\tilde{\mathbf{g}}_{2\|1}=\left\{\tilde{g}_{2\|1}^1,\tilde{g}_{2\|1}^2,\cdots,\tilde{g}_{2\|1}^{m_{21}}\right\}$, $\tilde{\mathbf{p}}_{2\|1}=\left\{\tilde{p}_{2\|1}^1,\tilde{p}_{2\|1}^2,\cdots,\tilde{p}_{2\|1}^{m_{21}}\right\}$	…	$\tilde{\mathbf{g}}_{n\|1}=\left\{\tilde{g}_{n\|1}^1,\tilde{g}_{n\|1}^2,\cdots,\tilde{g}_{n\|1}^{m_{n1}}\right\}$, $\tilde{\mathbf{p}}_{n\|1}=\left\{\tilde{p}_{n\|1}^1,\tilde{p}_{n\|1}^2,\cdots,\tilde{p}_{n\|1}^{m_{n1}}\right\}$
$\tilde{g}_1^2$, $\tilde{p}_1^2$	$\tilde{\mathbf{g}}_{2\|2}=\left\{\tilde{g}_{2\|2}^1,\tilde{g}_{2\|2}^2,\cdots,\tilde{g}_{2\|2}^{m_{22}}\right\}$, $\tilde{\mathbf{p}}_{2\|2}=\left\{\tilde{p}_{2\|2}^1,\tilde{p}_{2\|2}^2,\cdots,\tilde{p}_{2\|2}^{m_{22}}\right\}$	…	$\tilde{\mathbf{g}}_{n\|2}=\left\{\tilde{g}_{n\|2}^1,\tilde{g}_{n\|2}^2,\cdots,\tilde{g}_{n\|2}^{m_{n2}}\right\}$, $\tilde{\mathbf{p}}_{n\|2}=\left\{\tilde{p}_{n\|2}^1,\tilde{p}_{n\|2}^2,\cdots,\tilde{p}_{n\|2}^{m_{n2}}\right\}$
…	…	…	…
$\tilde{g}_1^{k_1}$, $\tilde{p}_1^{k_1}$	$\tilde{\mathbf{g}}_{2\|k_1}=\left\{\tilde{g}_{2\|k_1}^1,\tilde{g}_{2\|k_1}^2,\cdots,\tilde{g}_{2\|k_1}^{m_{2k_1}}\right\}$, $\tilde{\mathbf{p}}_{2\|k_1}=\left\{\tilde{p}_{2\|k_1}^1,\tilde{p}_{2\|k_1}^2,\cdots,\tilde{p}_{2\|k_1}^{m_{2k_1}}\right\}$	…	$\tilde{\mathbf{g}}_{n\|k_1}=\left\{\tilde{g}_{n\|k_1}^1,\tilde{g}_{n\|k_1}^2,\cdots,\tilde{g}_{n\|k_1}^{m_{nk_1}}\right\}$, $\tilde{\mathbf{p}}_{n\|k_1}=\left\{\tilde{p}_{n\|k_1}^1,\tilde{p}_{n\|k_1}^2,\cdots,\tilde{p}_{n\|k_1}^{m_{nk_1}}\right\}$

Suppose a FMSS consists of n components, the component 1 has effect on the performance levels of the other $n-1$ independent components. In this case, we define the component numbered 1 as the main component. The PD of the main component is represented by fuzzy sets $\tilde{\mathbf{g}}_1=\left\{\tilde{g}_1^1,\tilde{g}_1^2,\cdots,\tilde{g}_1^{k_1}\right\}$ and $\tilde{\mathbf{p}}_1=\left\{\tilde{p}_1^1,\tilde{p}_1^2,\cdots,\tilde{p}_1^{k_1}\right\}$. The conditional probability distributions (CPDs) of the other $n-1$ components are represented by $\tilde{\mathbf{g}}_{j|i}=\left\{\tilde{g}_{j|i}^1,\tilde{g}_{j|i}^2,\cdots,\tilde{g}_{j|i}^{m_{ji}}\right\}$ and $\tilde{\mathbf{p}}_{j|i}=\left\{\tilde{p}_{j|i}^1,\tilde{p}_{j|i}^2,\cdots,\tilde{p}_{j|i}^{m_{ji}}\right\}$, where $i=1,2,\cdots,k_1$, $j=2,3,\cdots,n$, and m_{ji} is the number of possible fuzzy states for component j at conditional state i. The relationship between the main component and the others can be listed as Table 1. According to Table 1, the possible fuzzy performance levels of component j can be redefined as

$$\tilde{\mathbf{g}}_j=\bigcup_{i=1}^{k_1}\tilde{\mathbf{g}}_{j|i}=\left\{\tilde{g}_j^1,\tilde{g}_j^2,\cdots,\tilde{g}_j^{n_j}\right\},j=2,3,\cdots,n \tag{15}$$

where n_j is the number of all the possible fuzzy values for component j, and $n_j\le\sum_{h=1}^{k_1}m_{jh}$. Meanwhile, the corresponding conditional probability can be redefined as follows.

$$\begin{aligned}\tilde{\mathbf{p}}_{j|i}&=\left\{\tilde{p}_{j|i}^1,\tilde{p}_{j|i}^2,\cdots,\tilde{p}_{j|i}^{n_j}\right\},\\ &\quad i=1,2,\cdots,k_1\quad j=2,3,\cdots,n\end{aligned} \tag{16}$$

where

$$\tilde{p}_{j|i}^h=\begin{cases}0,\ \tilde{g}_j^h\notin\tilde{\mathbf{g}}_{j|i}\\ \tilde{p}_{j|i}^h,\ \tilde{g}_j^h\in\tilde{\mathbf{g}}_{j|i},\end{cases}$$
$$(i=1,2,\cdots,k_1;\ j=2,3,\cdots,n;\ h=1,2,\cdots,n_j) \tag{17}$$

The CPD of component j can be listed as

$$\begin{aligned}\tilde{\mathbf{g}}_j&=\left\{\tilde{g}_j^1,\tilde{g}_j^2,\cdots,\tilde{g}_j^{n_j}\right\}\\ \tilde{\mathbf{p}}_{j|1}^h&=\left\{\tilde{p}_{j|1}^1,\tilde{p}_{j|1}^2,\cdots,\tilde{p}_{j|1}^{n_j}\right\}\\ \tilde{\mathbf{p}}_{j|2}^h&=\left\{\tilde{p}_{j|2}^1,\tilde{p}_{j|2}^2,\cdots,\tilde{p}_{j|2}^{n_j}\right\}\\ &\cdots\\ \tilde{\mathbf{p}}_{j|k_1}^h&=\left\{\tilde{p}_{j|k_1}^1,\tilde{p}_{j|k_1}^2,\cdots,\tilde{p}_{j|k_1}^{n_j}\right\}\end{aligned} \tag{18}$$

The states of component j are ordered by the following way:

$$h<k\Leftrightarrow g_j^{h,\max}>g_j^{k,\max}$$

where $g_j^{h,\max}=\max\left\{g_j^h\,\middle|\,g_j^h\in G_j^h\right\}$ and $g_j^{k,\max}=\max\left\{g_j^k\,\middle|\,g_j^k\in G_j^k\right\}$, G_j^h and G_j^k are collections of objects denoted generically by g_j^h and g_j^k.

According to Eq. (14), the z-transformation of the performance levels for component j yields

$$\tilde{u}_j(z)=\sum_{h=1}^{n_j}\tilde{\mathbf{p}}_j^h z^{\tilde{g}_j^h},j=2,3,\cdots,n \tag{19}$$

where $\tilde{\mathbf{p}}_j^h$ is a vector and satisfies

$$\tilde{\mathbf{p}}_j^h=\left(\tilde{p}_{j|1}^h,\tilde{p}_{j|2}^h,\cdots,\tilde{p}_{j|k_1}^h\right),h=1,2,\cdots,n_j \tag{20}$$

The PD of performance levels for the entire FMSS can be calculated as

$$\begin{aligned}\tilde{U}_s(z)&=\tilde{\Omega}_\Theta\left(\tilde{u}_1(z),\tilde{u}_2(z),\cdots,\tilde{u}_n(z)\right)\\ &=\tilde{\Omega}_\Theta\left(\sum_{h=1}^{k_1}\tilde{p}_1^h z^{\tilde{g}_1^h},\sum_{h=1}^{n_2}\tilde{\mathbf{p}}_2^h z^{\tilde{g}_2^h},\cdots,\sum_{h=1}^{n_n}\tilde{\mathbf{p}}_n^h z^{\tilde{g}_n^h}\right)\\ &=\sum_{i=1}^{M}\tilde{p}_i z^{\tilde{g}_i}\end{aligned} \tag{21}$$

where Θ represents the structure function of the FMSS, $\tilde{\Omega}_\Theta$ is the composition operator, $\tilde{U}_s(z)$ is the z-transformation of performance level for the entire FMSS, M is the number of the possible fuzzy performance levels, $\tilde{g}_i$ and $\tilde{p}_i$ are the performance level and corresponding probability, respectively.

In the conventional MSS model, if the system performance level g_i is no less than the demand d, it is considered performance adequacy for the state i definitely; whereas, performance deficiency if the system performance level g_i is smaller than the system demand d. In these situations, the boundary between success and failure is distinct. However, in FMSS model, the boundary is ambiguous. For example, it is assumed that the performance level $\tilde{g}_i$ for state i can be denoted as triangular fuzzy number $\left(a^L,a^M,a^U\right)$ and the system demand is a constant d. From Figure 1, the system is in a definitely successful state if $a^L\ge d$ and a definitely failure state if $a^U<d$. However, for $a^L<d\le a^U$, the state i

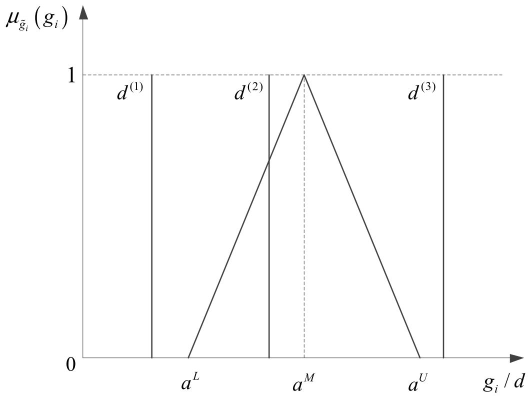

FIGURE 1. The relationship between the fuzzy performance level and the crisp demand.

can be seen as a fuzzy state, and the system may function normally or abnormally.

Sometimes, it is more reasonable to treat the system demand as a fuzzy value $\tilde{d}$. Then, the difference between the system performance and demand is defined as the system state adequacy index, which can be represented by a fuzzy set.

$$\tilde{\phi}_i = \left\{ \phi_i, \mu_{\tilde{\phi}_i}(\phi_i) \left| \phi_i = g_i - d, g_i \in G_i, d \in D \right. \right\} \tag{22}$$

where $\mu_{\tilde{\phi}_i}(\phi_i) = \sup_{\phi_i = g_i - d} \min \left\{ \mu_{\tilde{g}_i}, \mu_{\tilde{d}} \right\}$, G_i and D are collections of objects denoted generically by g_i and d, respectively. The system performance level is adequate if $\phi_i \geq 0$ and deficient if $\phi_i < 0$ for the state i.

Szmidt and Kacprzyk [21] defined the scalar cardinality of a fuzzy set $\tilde{\phi}_i : \Phi_i \to [0, 1]$ as the sum of the membership of the fuzzy set.

$$\left| \tilde{\phi}_i \right| = \sum_{\phi_i \in \Phi_i} \mu_{\tilde{\phi}_i}(\phi_i) \tag{23}$$

where $\left| \tilde{\phi}_i \right|$ is the cardinality or so-called the sigma-count of $\tilde{\phi}_i$, Φ_i is the collection of objects denoted generically by ϕ_i.

Denoting Φ_i^+ as a subset of Φ_i, which satisfies

$$\Phi_i^+ = \{ \phi_i \geq 0 \, | \phi_i \in \Phi_i \} \tag{24}$$

and

$$\tilde{\phi}_i^+ = \left\{ \phi_i^+, \mu_{\tilde{\phi}_i^+}\left(\phi_i^+\right) \left| \mu_{\tilde{\phi}_i^+}\left(\phi_i^+\right) = \mu_{\tilde{\phi}_i}\left(\phi_i^+\right), \phi_i^+ \in \Phi_i^+ \right. \right\} \tag{25}$$

According to Eq. (23), the cardinality of the fuzzy set $\tilde{\phi}_i^+$ can be calculated as

$$\left| \tilde{\phi}_i^+ \right| = \sum_{\phi_i^+ \in \Phi_i^+} \mu_{\tilde{\phi}_i^+}\left(\phi_i^+\right) \tag{26}$$

The relative cardinality of the fuzzy set $\tilde{\phi}_i^+$ is defined as

$$\left| \tilde{\phi}_i^+ \right|_r = \frac{\left| \tilde{\phi}_i^+ \right|}{\left| \tilde{\phi}_i \right|} \tag{27}$$

where $\left| \tilde{\phi}_i^+ \right|_r$ is the relative cardinality and it satisfies: a) $\left| \tilde{\phi}_i^+ \right|_r = 0$, if the performance level is deficient for the state i definitely; b) $\left| \tilde{\phi}_i^+ \right|_r = 1$, if the performance level is adequate for the state i definitely.

Define a composition operator $\tilde{\Omega}_{sys}$, then the fuzzy reliability of FMSS under state i can be evaluated as

$$\begin{aligned} \tilde{R} &= \tilde{\Omega}_{sys}\left(\tilde{U}_s(z), \tilde{d} \right) \\ &= \tilde{\Omega}_{sys}\left(\sum_{i=1}^{M} \tilde{p}_i z^{\tilde{g}_i}, \tilde{d} \right) \\ &= \tilde{\Omega}_{sys}\left\{ \cdots, \left\{ p_i \cdot \left| \tilde{\phi}_i^+ \right|_r, \mu_{\tilde{p}_i}(p_i) \left| p_i \in P_i \right. \right\}, \cdots \right\} \\ &= \left\{ R, \mu_{\tilde{R}}(R) \left| R = \sum_{i=1}^{M} p_i \cdot \left| \tilde{\phi}_i^+ \right|_r, p_i \in P_i \right. \right\} \end{aligned} \tag{28}$$

where $\mu_{\tilde{R}}(R) = \sup_{R = \sum_{i=1}^{M} p_i \cdot \left| \tilde{\phi}_i^+ \right|_r} \min \left\{ \mu_{\tilde{p}_1}, \mu_{\tilde{p}_2}, \cdots, \mu_{\tilde{p}_M} \right\}$, and $\tilde{R}$ is the fuzzy reliability of the FMSS.

From the above, the performance level and the corresponding probability are regarded as fuzzy value to consider the fuzziness of reliability data in the proposed method. Meanwhile, the main component can be recognized by combining theoretical analysis with pragmatic verification, and based on the relationship between the main and other components in MSSs, the CPDs of the latter can be obtained and the correlation is introduced into reliability analysis. Then, the probability distribution of performance level of the whole FMSS can be obtained based on FUGF, and with the consideration of fuzzy performance demand under state i, the cardinality of fuzzy set fuzzy sets $\tilde{\phi}_i$ and $\tilde{\phi}_i^+$ and the relative cardinality, that is, $\left| \tilde{\phi}_i \right|$, $\left| \tilde{\phi}_i^+ \right|$ and $\left| \tilde{\phi}_i^+ \right|_r$ can be calculated by adopting Eqs. (23), (26) and (27). Finally, all of the states can be considered by defining fuzzy composition operator and the reliability of FMSS can be obtained through Eq. (28). This is the novel reliability analysis method which can consider the master-slave relationship between the main component and others. The generalized flowchart of the proposed method can be drawn as Figure 2.

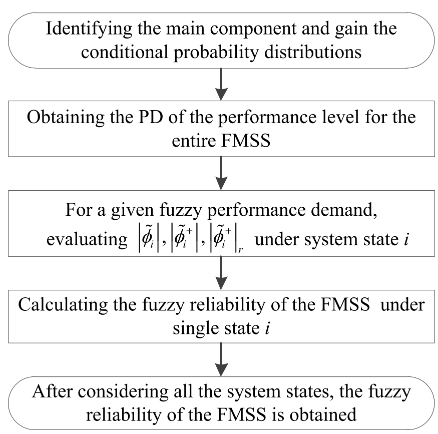

FIGURE 2. The flowchart of the proposed method.

According to the flowchart shown in Figure 2, the reliability analysis considering the correlation between the main and the other components in FMSSs can be implemented by defining fuzzy composition operator. And then, the key point is turned into the definition and calculation of fuzzy composition operators. Currently, a lot of complex mechanical systems with different structures are developed to meet the requirement of engineering practice. Different types of systems corresponding to different fuzzy composition operators. The definition and calculation under different situations will be illustrated hereinafter.

B. FUZZY COMPOSITION OPERATOR FOR DIFFERENT SUBSYSTEMS.

1) FUZZY COMPOSITION OPERATOR FOR SERIES SUBSYSTEMS

For a series subsystem, the performance level of the entire system is the minimum of the subsystems performances. According to the resolution theorem of the fuzzy mathematics, the system performance $\tilde{g}_i$ can be evaluated with α-cut as

$$\begin{aligned}\tilde{g}_i &= \tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}, \cdots, \tilde{g}_{ji_j}, \cdots, \tilde{g}_{ni_n}\right)\\ &= \bigcup_{\alpha} \alpha \cdot \tilde{g}_i^{\alpha}\\ &= \bigcup_{\alpha} \alpha \cdot \left[a_i^{\alpha}, c_i^{\alpha}\right] \end{aligned} \tag{29}$$

where $\tilde{\Omega}_s$ is the series composition operator, $\tilde{g}_i^{\alpha}$ is the α-cut of the fuzzy set $\tilde{g}_i$ and $\tilde{g}_i^{\alpha} = \left\{g_i \left| \mu_{\tilde{g}_i}(g_i) \geq \alpha \right.\right\}$.

Suppose a system consists of two components and the performance levels can be represented by $\left(a_{1i_1}, b_{1i_1}, c_{1i_1}\right)$ and $\left(a_{2i_2}, b_{2i_2}, c_{2i_2}\right)$, respectively. The α-cut of the fuzzy set $\tilde{g}_i$ is expressed as an interval $\left(a_i^{\alpha}, c_i^{\alpha}\right)$ and $\tilde{g}_i$ can be represented by solid lines as shown in Figure 3.

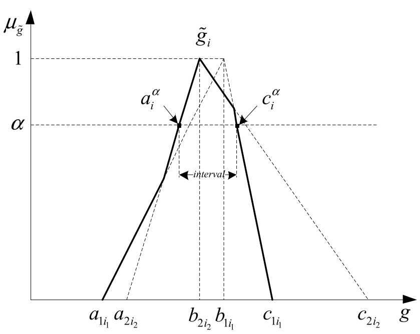

FIGURE 3. α**-cut of the fuzzy set** $\tilde{g}_i$**.**

Eq. (29) is rewritten as

$$\begin{aligned}\tilde{g}_i &= \tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)\\ &= \bigcup_{\alpha} \alpha \cdot \left[a_i^{\alpha}, c_i^{\alpha}\right]\\ &= \bigcup_{\alpha} \alpha \cdot \left[\min\left(a_{1i_1}^{\alpha}, a_{2i_2}^{\alpha}\right), \min\left(c_{1i_1}^{\alpha}, c_{2i_2}^{\alpha}\right)\right] \end{aligned} \tag{30}$$

There are eight possible results for Eq. (30) corresponding to different numerical value relationships.

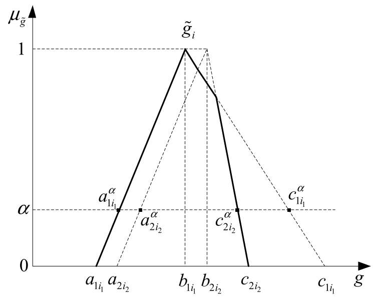

FIGURE 4. **The membership function of** $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ **in Case 2.**

Case 1: $a_{1i_1} \leq a_{2i_2}, b_{1i_1} \leq b_{2i_2}$ and $c_{1i_1} \leq c_{2i_2}$. In this case, $\tilde{g}_{1i_1}$ is less than or equal to $\tilde{g}_{2i_2}$ definitely, $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ can be represented by a triplet $\left(a_{1i_1}, b_{1i_1}, c_{1i_1}\right)$.

Case 2: $a_{1i_1} \leq a_{2i_2}, b_{1i_1} \leq b_{2i_2}$ and $c_{1i_1} \geq c_{2i_2}$. As is shown in Figure 4, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)}(g) = \begin{cases} 0, & g < a_{1i_1} \\ \dfrac{g - a_{1i_1}}{b_{1i_1} - a_{1i_1}}, & a_{1i_1} \leq g \leq b_{1i_1} \\ \dfrac{c_{1i_1} - g}{c_{1i_1} - b_{1i_1}}, & b_{1i_1} \leq g \leq \dfrac{c_{2i_2} b_{1i_1} - c_{1i_1} b_{2i_2}}{b_{1i_1} - c_{1i_1} - b_{2i_2} + c_{2i_2}} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & \dfrac{c_{2i_2} b_{1i_1} - c_{1i_1} b_{2i_2}}{b_{1i_1} - c_{1i_1} - b_{2i_2} + c_{2i_2}} \leq g \leq c_{2i_2} \\ 0, & g > c_{2i_2} \end{cases} \tag{31}$$

Case 3: $a_{1i_1} \leq a_{2i_2}, b_{1i_1} \geq b_{2i_2}$ and $c_{1i_1} \leq c_{2i_2}$. As is shown in Figure 3, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)}(g) = \begin{cases} 0, & g < a_{1i_1} \\ \dfrac{g - a_{1i_1}}{b_{1i_1} - a_{1i_1}}, & a_{1i_1} \leq g \leq \dfrac{a_{1i_1} b_{2i_2} - a_{2i_2} b_{1i_1}}{b_{2i_2} - b_{1i_1} - a_{2i_2} + a_{1i_1}} \\ \dfrac{g - a_{2i_2}}{b_{2i_2} - a_{2i_2}}, & \dfrac{a_{1i_1} b_{2i_2} - a_{2i_2} b_{1i_1}}{b_{2i_2} - b_{1i_1} - a_{2i_2} + a_{1i_1}} \leq g \leq b_{2i_2} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & b_{2i_2} \leq g \leq \dfrac{c_{2i_2} b_{1i_1} - c_{1i_1} b_{2i_2}}{b_{1i_1} - c_{1i_1} - b_{2i_2} + c_{2i_2}} \\ \dfrac{c_{1i_1} - g}{c_{1i_1} - b_{1i_1}}, & \dfrac{c_{2i_2} b_{1i_1} - c_{1i_1} b_{2i_2}}{b_{1i_1} - c_{1i_1} - b_{2i_2} + c_{2i_2}} \leq g \leq c_{1i_1} \\ 0, & g > c_{1i_1} \end{cases} \tag{32}$$

Case 4: $a_{1i_1} \leq a_{2i_2}, b_{1i_1} \geq b_{2i_2}$ and $c_{1i_1} \geq c_{2i_2}$. As is shown in Figure 5, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)}(g) = \begin{cases} 0, & g < a_{1i_1} \\ \dfrac{g - a_{1i_1}}{b_{1i_1} - a_{1i_1}}, & a_{1i_1} \leq g \leq \dfrac{a_{1i_1} b_{2i_2} - a_{2i_2} b_{1i_1}}{b_{2i_2} - b_{1i_1} - a_{2i_2} + a_{1i_1}} \\ \dfrac{g - a_{2i_2}}{b_{2i_2} - a_{2i_2}}, & \dfrac{a_{1i_1} b_{2i_2} - a_{2i_2} b_{1i_1}}{b_{2i_2} - b_{1i_1} - a_{2i_2} + a_{1i_1}} \leq g \leq b_{2i_2} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & b_{2i_2} \leq g \leq c_{2i_2} \\ 0, & g > c_{2i_2} \end{cases} \tag{33}$$

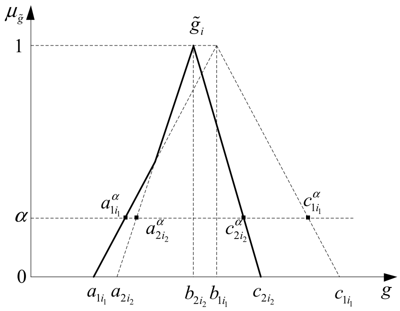

FIGURE 5. The membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ in Case 4.

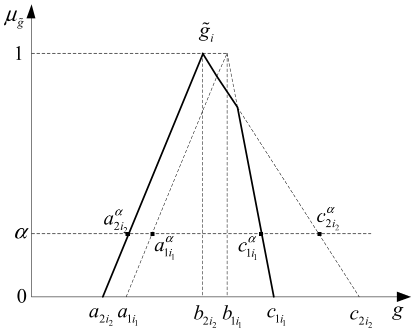

FIGURE 6. The membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ in Case 6.

Case 5: $a_{1i_1} \geq a_{2i_2}, b_{1i_1} \geq b_{2i_2}$ and $c_{1i_1} \geq c_{2i_2}$. In this case, $\tilde{g}_{1i_1}$ is more than or equal to $\tilde{g}_{2i_2}$ definitely, $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ can be represented by a triplet $\left(a_{2i_2}, b_{2i_2}, c_{2i_2}\right)$.

Case 6: $a_{1i_1} \geq a_{2i_2}, b_{1i_1} \geq b_{2i_2}$ and $c_{1i_1} \leq c_{2i_2}$. As is shown in Figure 6, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s(\tilde{g}_{1i_1},\tilde{g}_{2i_2})}(g) = \begin{cases} 0, & g < a_{2i_2} \\ \dfrac{g - a_{2i_2}}{b_{2i_2} - a_{2i_2}}, & a_{2i_2} \leq g \leq b_{2i_2} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & b_{2i_2} \leq g \leq \dfrac{c_{1i_1} b_{2i_2} - c_{2i_2} b_{1i_1}}{b_{2i_2} - c_{2i_2} - b_{1i_1} + c_{1i_1}} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & \dfrac{c_{1i_1} b_{2i_2} - c_{2i_2} b_{1i_1}}{b_{2i_2} - c_{2i_2} - b_{1i_1} + c_{1i_1}} \leq g \leq c_{1i_1} \\ 0, & g > c_{1i_1} \end{cases} \tag{34}$$

Case 7: $a_{1i_1} \geq a_{2i_2}, b_{1i_1} \leq b_{2i_2}$ and $c_{1i_1} \geq c_{2i_2}$. As is shown in Figure 7, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s(\tilde{g}_{1i_1},\tilde{g}_{2i_2})}(g) = \begin{cases} 0, & g < a_{2i_2} \\ \dfrac{g - a_{2i_2}}{b_{2i_2} - a_{2i_2}}, & a_{2i_2} \leq g \leq \dfrac{a_{2i_2} b_{1i_1} - a_{1i_1} b_{2i_2}}{b_{1i_1} - b_{2i_2} - a_{1i_1} + a_{2i_2}} \\ \dfrac{g - a_{1i_1}}{b_{1i_1} - a_{1i_1}}, & \dfrac{a_{2i_2} b_{1i_1} - a_{1i_1} b_{2i_2}}{b_{1i_1} - b_{2i_2} - a_{1i_1} + a_{2i_2}} \leq g \leq b_{1i_1} \\ \dfrac{c_{1i_1} - g}{c_{1i_1} - b_{1i_1}}, & b_{1i_1} \leq g \leq \dfrac{c_{1i_1} b_{2i_2} - c_{2i_2} b_{1i_1}}{b_{2i_2} - c_{2i_2} - b_{1i_1} + c_{1i_1}} \\ \dfrac{c_{2i_2} - g}{c_{2i_2} - b_{2i_2}}, & \dfrac{c_{1i_1} b_{2i_2} - c_{2i_2} b_{1i_1}}{b_{2i_2} - c_{2i_2} - b_{1i_1} + c_{1i_1}} \leq g \leq c_{2i_2} \\ 0, & g > c_{2i_2} \end{cases} \tag{35}$$

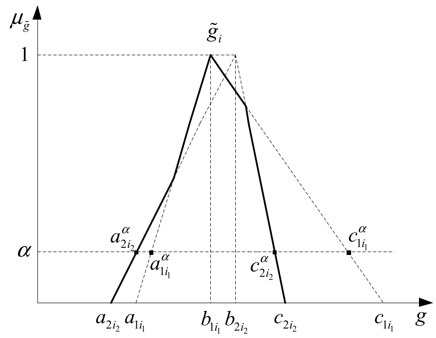

FIGURE 7. The membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ in Case 7.

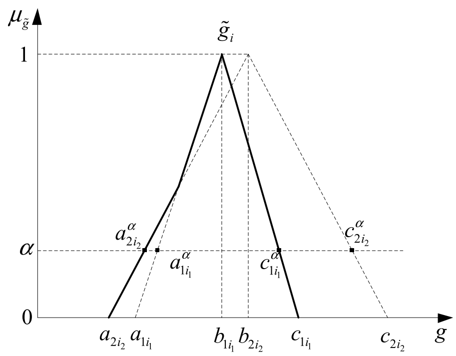

FIGURE 8. The membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ in Case 8.

Case 8: $a_{1i_1} \geq a_{2i_2}, b_{1i_1} \leq b_{2i_2}$ and $c_{1i_1} \leq c_{2i_2}$. As is shown in Figure 8, the membership function of $\tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right)$ is

$$\mu_{\tilde{\Omega}_s(\tilde{g}_{1i_1},\tilde{g}_{2i_2})}(g) = \begin{cases} 0, & g < a_{2i_2} \\ \dfrac{g - a_{2i_2}}{b_{2i_2} - a_{2i_2}}, & a_{2i_2} \leq g \leq \dfrac{a_{2i_2} b_{1i_1} - a_{1i_1} b_{2i_2}}{b_{1i_1} - b_{2i_2} - a_{1i_1} + a_{2i_2}} \\ \dfrac{g - a_{1i_1}}{b_{1i_1} - a_{1i_1}}, & \dfrac{a_{2i_2} b_{1i_1} - a_{1i_1} b_{2i_2}}{b_{1i_1} - b_{2i_2} - a_{1i_1} + a_{2i_2}} \leq g \leq b_{1i_1} \\ \dfrac{c_{1i_1} - g}{c_{1i_1} - b_{1i_1}}, & b_{1i_1} \leq g \leq c_{1i_1} \\ 0, & g > c_{1i_1} \end{cases} \tag{36}$$

2) FUZZY COMPOSITION OPERATOR FOR PARALLEL SUBSYSTEMS

For a parallel subsystem, the performance level of the system is the sum of the performances of all the subsystems. The fuzzy arithmetic operations of the triangular fuzzy numbers are applied to obtain the performance level of the system.

$$\begin{aligned} \tilde{g}_i &= \tilde{\Omega}_p\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}, \cdots, \tilde{g}_{ji_j}, \cdots, \tilde{g}_{ni_n}\right) \\ &= \left(\sum_{j=1}^{n} a_{ji_j}, \sum_{j=1}^{n} b_{ji_j}, \sum_{j=1}^{n} c_{ji_j}\right) \end{aligned} \tag{37}$$

where $\tilde{\Omega}_p$ is the parallel composition operator, triplet $\left(a_{ji_j}, b_{ji_j}, c_{ji_j}\right)$ represents the performance levels for component j.

The corresponding probability $\tilde{p}_i$ can be expressed as

$$\tilde{p}_i = \left(\prod_{j=1}^{n} a^p_{ji_j}, \prod_{j=1}^{n} b^p_{ji_j}, \prod_{j=1}^{n} c^p_{ji_j} \right) \tag{38}$$

where the triplet $\left(a^p_{ji_j}, b^p_{ji_j}, c^p_{ji_j}\right)$ represents the probability that the performance levels equals to $\left(a_{ji_j}, b_{ji_j}, c_{ji_j}\right)$ for component j.

3) FUZZY COMPOSITION OPERATOR FOR SERIES-PARALLEL HYBRID SUBSYSTEMS

If a subsystem is a series-parallel hybrid system, the system can usually be divided into different subsystems. When the number of series subsystems or parallel subsystems is more than two, a three-step procedure can be adopt.

Step 1: The FUGF of two components in the series or parallel subsystem is calculated

Step 2: Simplifying the original two components with a new single component through the FUGF obtained in the first step and substituting them with the new one.

Step 3: After the replacement, if the components in the series or parallel subsystem is more than two, go to Step 1, repeat the procedure until there is only one component in the series or parallel subsystem.

Through the above steps, a series-parallel hybrid subsystem can be transformed to a system with a single component. But the computational burden is costly when there are a large number of series components in the system. To simplify the calculation, an approximation can be made to series subsystems.

$$\begin{aligned} \tilde{g}_i &= \tilde{\Omega}_s\left(\tilde{g}_{1i_1}, \tilde{g}_{2i_2}\right) \\ &= \bigcup_\alpha \alpha \cdot \left[a^\alpha_i, c^\alpha_i\right] \\ &= \bigcup_\alpha \alpha \cdot \left[\min\left(a^\alpha_{1i_1}, a^\alpha_{2i_2}\right), \min\left(c^\alpha_{1i_1}, c^\alpha_{2i_2}\right)\right] \\ &= \left(\min\left(a_{1i_1}, a_{2i_2}\right), \min\left(b_{1i_1}, b_{2i_2}\right), \min\left(c_{1i_1}, c_{2i_2}\right)\right) \end{aligned} \tag{39}$$

Therefore, the fuzzy composition operator for series, parallel and series-parallel hybrid subsystem can be defined and calculated, and combining with the proposed method aforementioned and Eqs. (23), (26)-(28), the master-slave relationship between the main and other components of FMSSs can be considered in reliability analysis.

V. NUMERICAL EXAMPLES

In this section, the reliability analysis for the hydraulic system of a three-leg robot is carried out to demonstrate the availability of the proposed method. As shown in Figure 9, the hydraulic system of the three-leg robot contains several hydraulic subsystems, and each subsystem is composed of basic circuits and auxiliary circuits. In the course of movement, the pressures of hydraulic cylinders 1 and 2 are identified by hydraulic cylinder 3, and the hydraulic cylinders 1 and 2 have three performance levels corresponding to different states of motion. The hydraulic cylinder 3 is the main component and has two different performance levels, the maximal pressure (State 1), and the minimal pressure (State 2), corresponding to different functional requirements. The PD and CPDs of pressure levels for the three hydraulic subsystems are listed in Table 2. The requirement is that the total output pressure of hydraulic loop is not lower than 45MPa and the reliability is 0.9.

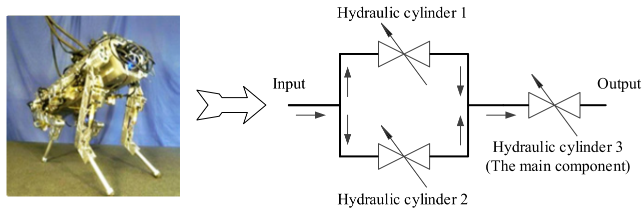

FIGURE 9. The hydraulic system of a three-leg robot and the hydraulic loop.

The probability distribution for the pressures of hydraulic cylinder 3 is given by

$$\begin{aligned} \tilde{\mathbf{g}}_3 &= \left\{\tilde{g}^1_3, \tilde{g}^2_3\right\} = \{60, 0\}; \\ \tilde{\mathbf{p}}_3 &= \left\{\tilde{p}^1_3, \tilde{p}^2_3\right\} = \{(0.65, 0.70, 0.90), (0.20, 0.30, 0.55)\}. \end{aligned} \tag{40}$$

where $\tilde{g}^1_3, \tilde{g}^2_3$ are the pressure levels of hydraulic cylinder 3 under two different states, and $\tilde{p}^1_3, \tilde{p}^2_3$ are the corresponding probabilities.

When the hydraulic cylinder 3 is in State 1 or State 2, the CPDs of the pressure levels of hydraulic cylinder 1 can be expressed by Eqs. (41)-(42).

$$\begin{aligned} \tilde{\mathbf{g}}_{1|1} &= \left\{\tilde{g}^1_{1|1}, \tilde{g}^2_{1|1}, \tilde{g}^3_{1|1}\right\} = \{20, (10, 15, 16), 0\}; \\ \tilde{\mathbf{p}}_{1|1} &= \left\{\tilde{p}^1_{1|1}, \tilde{p}^2_{1|1}, \tilde{p}^3_{1|1}\right\} \\ &= \{(0.50, 0.60, 0.65), (0.15, 0.20, 0.25), \\ &\quad (0.18, 0.20, 0.24)\}. \end{aligned} \tag{41}$$

$$\begin{aligned} \tilde{\mathbf{g}}_{1|2} &= \left\{\tilde{g}^1_{1|2}, \tilde{g}^2_{1|2}, \tilde{g}^3_{1|2}\right\} = \{20, (10, 12, 13), 0\}; \\ \tilde{\mathbf{p}}_{1|2} &= \left\{\tilde{p}^1_{1|2}, \tilde{p}^2_{1|2}, \tilde{p}^3_{1|2}\right\} \\ &= \{(0.60, 0.70, 0.75), (0.15, 0.20, 0.30), \\ &\quad (0.05, 0.10, 0.25)\}. \end{aligned} \tag{42}$$

where $\tilde{g}^1_{1|1} \sim \tilde{g}^3_{1|1}$ and $\tilde{g}^1_{1|2} \sim \tilde{g}^3_{1|2}$ denote the pressure levels of hydraulic cylinder 1 under two different states, and $\tilde{p}^1_{1|1} \sim \tilde{p}^3_{1|1}$ and $\tilde{p}^1_{1|2} \sim \tilde{p}^3_{1|2}$ are the corresponding probabilities.

Accordingly, the CPDs of the pressure levels of hydraulic cylinder 2 can be expressed by Eqs. (43)-(44).

$$\begin{aligned} \tilde{\mathbf{g}}_{2|1} &= \left\{\tilde{g}^1_{2|1}, \tilde{g}^2_{2|1}, \tilde{g}^3_{2|1}\right\} = \{40, (26, 30, 32), 0\}; \\ \tilde{\mathbf{p}}_{2|1} &= \left\{\tilde{p}^1_{2|1}, \tilde{p}^2_{2|1}, \tilde{p}^3_{2|1}\right\} \\ &= \{(0.60, 0.70, 0.85), (0.08, 0.10, 0.15), \\ &\quad (0.16, 0.20, 0.35)\}. \end{aligned} \tag{43}$$

TABLE 2. The PD and CPDs of the pressure levels.

Component 3	Component 1	Component 2
$\tilde{g}_3^1 = 60$, $\tilde{p}_3^1 = (0.65, 0.70, 0.90)$	$\tilde{\mathbf{g}}_{1\|1} = \{20, (10, 15, 16), 0\}$ $\tilde{\mathbf{p}}_{1\|1} = \{(0.50, 0.60, 0.65), (0.15, 0.20, 0.25), (0.18, 0.20, 0.24)\}$	$\tilde{\mathbf{g}}_{2\|1} = \{40, (26, 30, 32), 0\}$ $\tilde{\mathbf{p}}_{2\|1} = \{(0.60, 0.70, 0.85), (0.08, 0.10, 0.15), (0.16, 0.20, 0.35)\}$
$\tilde{g}_3^2 = 0$, $\tilde{p}_3^2 = (0.20, 0.30, 0.55)$	$\tilde{\mathbf{g}}_{1\|2} = \{20, (10, 12, 13), 0\}$ $\tilde{\mathbf{p}}_{1\|2} = \{(0.60, 0.70, 0.75), (0.15, 0.20, 0.30), (0.05, 0.10, 0.25)\}$	$\tilde{\mathbf{g}}_{2\|2} = \{40, (26, 30, 32), 0\}$ $\tilde{\mathbf{p}}_{2\|2} = \{(0.50, 0.70, 0.80), (0.05, 0.10, 0.12), (0.10, 0.20, 0.25)\}$

$$\begin{aligned}
\tilde{\mathbf{g}}_{2|2} &= \left\{\tilde{g}_{2|2}^1, \tilde{g}_{2|2}^2, \tilde{g}_{2|2}^3\right\} = \{40, (26, 30, 32), 0\};\\
\tilde{\mathbf{p}}_{2|2} &= \left\{\tilde{p}_{2|2}^1, \tilde{p}_{2|2}^2, \tilde{p}_{2|2}^3\right\}\\
&= \{(0.50, 0.70, 0.80), (0.05, 0.10, 0.12),\\
&\qquad (0.10, 0.20, 0.25)\}. \qquad (44)
\end{aligned}$$

where $\tilde{g}_{2|1}^1 \sim \tilde{g}_{2|1}^3$ and $\tilde{g}_{2|2}^1 \sim \tilde{g}_{2|2}^3$ denote the pressure levels of hydraulic cylinder 2 under two different states, and $\tilde{p}_{2|1}^1 \sim \tilde{p}_{2|1}^3$ and $\tilde{p}_{2|2}^1 \sim \tilde{p}_{2|2}^3$ are the corresponding probabilities.

According to Eqs. (18)-(19) and (41)-(42), the CPD of the pressure level of hydraulic cylinder 1 is

$$\begin{aligned}
\tilde{\mathbf{g}}_1 &= \left\{\tilde{g}_1^1, \tilde{g}_1^2, \tilde{g}_1^3, \tilde{g}_1^4\right\} = \{20, (10, 15, 16), (10, 12, 13), 0\};\\
\tilde{\mathbf{p}}'_{1|1} &= \left\{\tilde{p}_{1|1}^1, \tilde{p}_{1|1}^2, \tilde{p}_{1|1}^3, \tilde{p}_{1|1}^4\right\}\\
&= \{(0.50, 0.60, 0.65), (0.15, 0.20, 0.25), 0,\\
&\qquad (0.18, 0.20, 0.24)\};\\
\tilde{\mathbf{p}}'_{1|2} &= \left\{\tilde{p}_{1|2}^1, \tilde{p}_{1|2}^2, \tilde{p}_{1|2}^3, \tilde{p}_{1|2}^4\right\}\\
&= \{(0.60, 0.70, 0.75), 0, (0.15, 0.20, 0.30),\\
&\qquad (0.05, 0.10, 0.25)\}. \qquad (45)
\end{aligned}$$

where $\tilde{g}_1^1 \sim \tilde{g}_1^4$ are the all possible pressure levels of hydraulic cylinder 1, $\tilde{p}_{1|1}^1 \sim \tilde{p}_{1|1}^4$ and $\tilde{p}_{1|2}^1 \sim \tilde{p}_{1|2}^4$ denote the corresponding probabilities under States 1 and 2.

According to Eqs. (18), (20) and (45), Eq. (46) can be obtained.

$$\begin{aligned}
\tilde{\mathbf{p}}_1^1 &= \left\{\tilde{p}_{1|1}^1, \tilde{p}_{1|2}^1\right\} = \{(0.50, 0.60, 0.65), (0.60, 0.70, 0.75)\};\\
\tilde{\mathbf{p}}_1^2 &= \left\{\tilde{p}_{1|1}^2, \tilde{p}_{1|2}^2\right\} = \{(0.15, 0.20, 0.25), 0\};\\
\tilde{\mathbf{p}}_1^3 &= \left\{\tilde{p}_{1|1}^3, \tilde{p}_{1|2}^3\right\} = \{0, (0.15, 0.20, 0.30)\};\\
\tilde{\mathbf{p}}_1^4 &= \left\{\tilde{p}_{1|1}^4, \tilde{p}_{1|2}^4\right\} = \{(0.18, 0.20, 0.24), (0.05, 0.10, 0.25)\}. \qquad (46)
\end{aligned}$$

where $\tilde{p}_{1|1}^h, \tilde{p}_{1|2}^h$, $(h = 1, 2, 3, 4)$ denote the probability when the pressure level of hydraulic cylinder 1 is equal to $\tilde{g}_1^h$ under States 1 and 2, respectively.

Similarly, the CPD, and the probability when the pressure level of hydraulic cylinder 2 is equal to $\tilde{g}_2^h$ under States 1 and 2 can be obtained.

$$\begin{aligned}
\tilde{\mathbf{g}}_2 &= \left\{\tilde{g}_2^1, \tilde{g}_2^2, \tilde{g}_2^3\right\} = \{40, (26, 30, 32), 0\};\\
\tilde{\mathbf{p}}'_{2|1} &= \left\{\tilde{p}_{2|1}^1, \tilde{p}_{2|1}^2, \tilde{p}_{2|1}^3\right\}\\
&= \{(0.60, 0.70, 0.85), (0.08, 0.10, 0.15),\\
&\qquad (0.16, 0.20, 0.35)\};\\
\tilde{\mathbf{p}}'_{2|2} &= \left\{\tilde{p}_{2|2}^1, \tilde{p}_{2|2}^2, \tilde{p}_{2|2}^3\right\}\\
&= \{(0.50, 0.70, 0.80), (0.05, 0.10, 0.12),\\
&\qquad (0.10, 0.20, 0.25)\}. \qquad (47)\\
\tilde{\mathbf{p}}_2^1 &= \left\{\tilde{p}_{2|1}^1, \tilde{p}_{2|2}^1\right\} = \{(0.60, 0.70, 0.85),\\
&\qquad (0.50, 0.70, 0.80)\};\\
\tilde{\mathbf{p}}_2^2 &= \left\{\tilde{p}_{2|1}^2, \tilde{p}_{2|2}^2\right\} = \{(0.08, 0.10, 0.15),\\
&\qquad (0.05, 0.10, 0.12)\};\\
\tilde{\mathbf{p}}_2^3 &= \left\{\tilde{p}_{2|1}^3, \tilde{p}_{2|2}^3\right\} = \{(0.16, 0.20, 0.35),\\
&\qquad (0.10, 0.20, 0.25)\}. \qquad (48)
\end{aligned}$$

where $\tilde{g}_2^1 \sim \tilde{g}_2^3$ are the all possible pressure levels of hydraulic cylinder 2, $\tilde{p}_{2|1}^1 \sim \tilde{p}_{2|1}^3$ and $\tilde{p}_{2|2}^1 \sim \tilde{p}_{2|2}^3$ denote the corresponding probabilities, and $\tilde{p}_{2|1}^h, \tilde{p}_{2|2}^h$, $(h = 1, 2, 3)$ are the probability when the pressure level of hydraulic cylinder 2 is equal to $\tilde{g}_2^h$ under States 1 and 2, respectively.

According to Eq. (19), the z-transformation of the fuzzy performance levels for the three hydraulic subsystems can be defined as

$$\begin{aligned}
\tilde{u}_3(z) &= \tilde{p}_3^1 \cdot z^{\tilde{g}_3^1} + \tilde{p}_3^2 \cdot z^{\tilde{g}_3^2}\\
&= (0.65, 0.70, 0.90)\, z^{60} + (0.20, 0.30, 0.55)\, z^0 \qquad (49)\\
\tilde{u}_1(z) &= \sum_{h_1=1}^{n_1} \tilde{\mathbf{p}}_1^{h_1} z^{\tilde{g}_1^{h_1}}\\
&= \{(0.50, 0.60, 0.65), (0.60, 0.70, 0.75)\}\, z^{20}\\
&\quad + \{(0.15, 0.20, 0.25), 0\}\, z^{(10,15,16)}\\
&\quad + \{0, (0.15, 0.20, 0.30)\}\, z^{(10,12,13)}\\
&\quad + \{(0.18, 0.20, 0.24), (0.05, 0.10, 0.25)\}\, z^0 \qquad (50)\\
\tilde{u}_2(z) &= \sum_{h_2=1}^{n_2} \tilde{\mathbf{p}}_2^{h_2} z^{\tilde{g}_2^{h_2}}\\
&= \{(0.60, 0.70, 0.85), (0.50, 0.70, 0.80)\}\, z^{40}\\
&\quad + \{(0.08, 0.10, 0.15), (0.05, 0.10, 0.12)\}\, z^{(26,30,32)}\\
&\quad + \{(0.16, 0.20, 0.35), (0.10, 0.20, 0.25)\}\, z^0 \qquad (51)
\end{aligned}$$

From Figure 9, the hydraulic system can be regarded as a series-parallel hybrid system, where hydraulic cylinders 1 and 2 connect in parallel, and then connect in series with hydraulic cylinder 3. The z-transformation of the pressure levels for the parallel subsystem and the entire hydraulic system can be expressed as

$$\begin{aligned}\tilde{U}_p(z) &= \tilde{\Omega}_p(\tilde{u}_1(z), \tilde{u}_2(z)) \\ &= \{(0.30, 0.42, 0.5525), (0.30, 0.49, 0.60)\} z^{60} \\ &\quad + \{(0.04, 0.06, 0.0975), \\ &\qquad (0.03, 0.07, 0.09)\} z^{(46,50,52)} \\ &\quad + \{(0.08, 0.12, 0.2275), (0.06, 0.14, 0.1875)\} z^{20} \\ &\quad + \{(0.09, 0.14, 0.2125), 0\} z^{(50,55,56)} \\ &\quad + \{(0.012, 0.02, 0.0375), 0\} z^{(36,45,48)} \\ &\quad + \{(0.024, 0.04, 0.0875), 0\} z^{(10,15,16)} \\ &\quad + \{0, (0.075, 0.14, 0.24)\} z^{(50,52,53)} \\ &\quad + \{0, (0.0075, 0.02, 0.036)\} z^{(36,42,45)} \\ &\quad + \{0, (0.015, 0.04, 0.075)\} z^{(10,12,13)} \\ &\quad + \{(0.108, 0.14, 0.204), (0.025, 0.07, 0.2)\} z^{40} \\ &\quad + \{(0.0144, 0.02, 0.036), \\ &\qquad (0.0025, 0.01, 0.03)\} z^{(26,30,32)} \\ &\quad + \{(0.0288, 0.04, 0.084), (0.005, 0.02, 0.0625)\} z^{0}\end{aligned} \tag{52}$$

$$\begin{aligned}\tilde{U}_s(z) &= \tilde{\Omega}_s\left(\tilde{\Omega}_p(\tilde{u}_1(z), \tilde{u}_2(z)), \tilde{u}_3(z)\right) \\ &= (0.195, 0.294, 0.49725) z^{\min\{60,60\}} \\ &\quad + (0.026, 0.042, 0.08775) z^{\min\{60,(46,50,52)\}} \\ &\quad + (0.052, 0.084, 0.20475) z^{\min\{60,20\}} \\ &\quad + (0.0585, 0.098, 0.19125) z^{\min\{60,(50,55,56)\}} \\ &\quad + (0.0078, 0.014, 0.03375) z^{\min\{60,(36,45,48)\}} \\ &\quad + (0.0156, 0.028, 0.07875) z^{\min\{60,(10,15,16)\}} \\ &\quad + (0.0702, 0.098, 0.1836) z^{\min\{60,40\}} \\ &\quad + (0.00936, 0.014, 0.0324) z^{\min\{60,(26,30,32)\}} \\ &\quad + (0.01872, 0.028, 0.0756) z^{\min\{60,0\}} \\ &\quad + (0.06, 0.147, 0.33) z^{\min\{0,0\}} \\ &\quad + (0.006, 0.021, 0.0495) z^{\min\{0,0\}} \\ &\quad + (0.012, 0.042, 0.103125) z^{\min\{0,0\}} \\ &\quad + (0.015, 0.042, 0.132) z^{\min\{0,0\}} \\ &\quad + (0.0015, 0.006, 0.0198) z^{\min\{0,0\}} \\ &\quad + (0.003, 0.012, 0.04125) z^{\min\{0,0\}} \\ &\quad + (0.005, 0.021, 0.11) z^{\min\{0,0\}} \\ &\quad + (0.0005, 0.003, 0.0165) z^{\min\{0,0\}} \\ &\quad + (0.001, 0.006, 0.034375) z^{\min\{0,0\}} \\ &= (0.195, 0.294, 0.49725) z^{60} \\ &\quad + (0.0585, 0.098, 0.19125) z^{(50,55,56)} \\ &\quad + (0.026, 0.042, 0.08775) z^{(46,50,52)} \\ &\quad + (0.0078, 0.014, 0.03375) z^{(36,45,48)} \\ &\quad + (0.0702, 0.098, 0.1836) z^{40} \\ &\quad + (0.00936, 0.014, 0.0324) z^{(26,30,32)} \\ &\quad + (0.052, 0.084, 0.20475) z^{20} \\ &\quad + (0.0156, 0.028, 0.07875) z^{(10,15,16)} \\ &\quad + (0.12272, 0.328, 0.91215) z^{0}\end{aligned} \tag{53}$$

From Eq. (53), the hydraulic system has nine different states. Suppose the system demand is a fuzzy value and can be represented by a triangle fuzzy number $\tilde{d} = (43, 45, 46)$. The relative cardinality of fuzzy set $\tilde{\phi}_i^+, i = 1, 2, \cdots, 9$ can be evaluated as follows.

a) For states $i = 1, 2, 3$, the performance level $\tilde{g}_i > \tilde{d}$ definitely, $\left|\tilde{\phi}_i^+\right|_r = 1$;

b) For state $i = 5, 6, 7, 8, 9$, the performance level $\tilde{g}_i < \tilde{d}$ definitely, $\left|\tilde{\phi}_i^+\right|_r = 0$;

c) For state $i = 4$, $\tilde{\phi}_i = (36, 45, 48) + (-46, -45, -43) = (-10, 0, 5)$, $\left|\tilde{\phi}_i\right| = 1/2 \times (5 + 10) \times 1 = 7.5$, $\left|\tilde{\phi}_i^+\right| = 1/2 \times (5 - 0) \times 1 = 2.5$, $\left|\tilde{\phi}_i^+\right|_r = \left|\tilde{\phi}_i^+\right| / \left|\tilde{\phi}_i\right| = 0.3333$.

According to Eq. (28), the fuzzy reliability of the FMSS is evaluated as

$$\begin{aligned}\tilde{R} &= \Omega_{sys}\left(\tilde{U}_s(z), \tilde{d}\right) = \sum_{i=1}^{9} \tilde{p}_i z^{\tilde{g}_i} \\ &= (0.195, 0.294, 0.49725) + (0.0585, 0.098, 0.19125) \\ &\quad + (0.026, 0.042, 0.08775) \\ &\quad + (0.0078, 0.014, 0.03375) \times 0.3333 + 0 + 0 + 0 + 0 + 0 \\ &= (0.2821, 0.4387, 0.7875)\end{aligned} \tag{54}$$

Due to the requirement that the total output pressure via hydraulic loop is not lower than 45MPa and the reliability is 0.9, it is clear that the FMSS cannot meet the required performance after evaluation.

VI. CONCLUSION

In this paper, the definitions of multi-state systems and universal generating functions are reviewed firstly, and then a novel reliability analysis method considering the correlation between components is proposed based on fuzzy universal generating functions. The proposed method takes the master-slave relationship between the main and other components into consideration, which is an extension of the existing fuzzy universal generating functions-based methods. A generalized flow chart for the calculation of the fuzzy reliability is provided in this paper. In addition, different composition operators are also introduced to calculate the fuzzy output performance distribution and evaluate its fuzzy reliability. The reliability analysis for the hydraulic system of a three-leg robot is used to illustrate the proposed method and demonstrate the feasibility. In this paper, the correlation between the main component and the others is supposed to be unidirectional. Moreover, the performance levels and corresponding probabilities of the system is regarded as random variables. Therefore, the stochastic process performance levels and probabilities and the more complicated correlation between different components or subsystems will be studied in our future research works.

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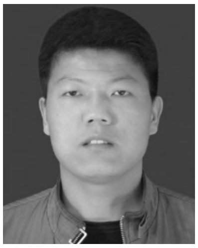

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