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The Local Delay and the Throughput in Cooperative Cognitive Radio Ad Hoc Networks

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ABSTRACT Cognitive radio ad hoc network is a framework which combines cognitive technique and ad hoc network. How to improve the performance of that network has always been a popular research for the past decades. In this paper, we study the local delay and the throughput in cooperative cognitive radio ad hoc networks. In order to forward the packets of primary users, a τ-slotted ALOHA protocol is adopted by secondary users in which a slot is divided into the first τ-slot and the latter $(1-\tau)$-slot. By modeling the location of primary and secondary users as homogeneous Poisson point processes, we give the closed-form expression of the local delay and the throughput of both networks with two access strategies. Then, we optimize the two performance parameters with the intensity of secondary users. Numerical results show the feasibility of the optimal problem about the network performance metrics we proposed and could get an obvious better performance for primary users than that of ALOHA protocol by less sacrificing the performance of secondary users.

INDEX TERMS Cooperative, cognitive radio, ad hoc, local delay, throughput.

I. INTRODUCTION

As is well known, cognitive radio (CR) technology is an efficient and widespread method to improve the spectrum efficiency of licensed frequency [1]. In CR networks, cognitive (secondary) users opportunistically access the spectrum used by licensed (primary) users. In recent years, the Internet of Things (IoT) has grown rapidly which is aimed to connect all kinds of different devices for making our daily life more convenient. In IoT, all nodes can be assumed as mobile transeivers and communicate with each other without infrastructure controlling. CR ad hoc networks [2] can be regarded as those networks in which a secondary ad hoc network underlaid with a cellular network.

Delay and throughput are two important indicators to measure the quality of service (QoS) of wireless networks. In [3], Baccelli et al. first defined the local delay of mobile ad hoc networks with ALOHA medium access control (MAC) protocol. Based on this framework, Martin discussed the closed-form expression of the local delay in different types of nodes mobility and transmission strategies [4], [5]. Furtherly, in [6], Gao et al. analyzed the local delay with slotted-ALOHA based on CR ad hoc networks. They gave the analytical expression of the local delay by modeling the channel occupied by primary nodes as a continuous-time Markov on-off process. In [7], Gao et al. conducted the research of end-to-end delay in CR ad hoc networks with two different traffic models.

In [8], Xie et al. gave the scaling law of transport capacity about nodes density. In [9], Jovicic et al. analyzed the scaling law of transport capacity about the transmission character of channel. In [10], Yin et al. testified that there is a tradeoff between throughput and delay in an overlaid wireless network. Jeon et al. proved in [11] that the throughput of the coexist two networks could achieve the same scaling law in a two-tier network which is same to that of single network. Then, in [12], Gao et al. studied the scaling law in a two-tier network with cooperative transmission. They specified that the throughput and the delay had the same scaling law in the two networks when secondary nodes assisted primary nodes to send packets.

On the other hand, some research had been done about the closed-form of the throughput in CR ad hoc networks. In [13], Weber et al. analyzed the transmission capacity of single-hop

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network based on the theory of stochastic geometry [14]. In [15], Baccelli et al. proposed the spatial density of progress to measure the capacity of multi-hop network. These results were later spread to analyze the network capacity of cognitive radio ad hoc networks (CRAHNs). In [16], the transmission capacity was derived when an ad hoc network coexists with a cellular network. And the transmission capacity could be improved by changing some important parameters such as link diversity gain and link distance etc.. In [17], the throughput of CRAHNs was given to propose a distributed spectrum allocation policy. In [18], the upper bound of broadcast transmission capacity was studied in heterogeneous networks. In [19], transport capacity was redefined for CRAHNs as the product of transmission capacity times hop distance. Aiming to find out how much traffic load by the source can be handled by a network, Wang and Song [20] proposed a novel end-to-end congestion control scheme that considered the unique features in multi-hop CR ad hoc networks. In [21], Demarchou et al. modeled channel traffic with time-space Poisson point processes and provided an analytical framework for the performance of the asynchronous system.

Since no infrastructure is an obvious feature of the CR ad hoc networks, the users have to communicate with those far away from them by forwarding. Hence, cooperative transmission is an important transmission scheme. Until now, little research has been conducted with the performance of the cooperative CR ad hoc networks due to the complexity of it. In this paper, we address the performance measurement and the optimization of cooperative CRAHNs. We propose a τ-slotted ALOHA MAC protocol in order to give the closed-form of the local delay and the throughput of CR ad hoc networks. Finally, numerical results are done to show that the two parameters could measure the performance and be used for the optimization of CR ad hoc networks.

The paper is organized as follows. Section II gives the system model and the τ-slotted ALOHA MAC protocol. Section III analyzes the local delay and the throughput of the cooperative CR ad hoc networks. Section IV discusses the optimization of the two performance parameters. Section V presents the numerical results with some observations of them. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL for the τ-SLOTTED ALOHA PROTOCOL

We consider an infinite planar network. Both primary and secondary users locate in the plane $\mathbb{R}^2$. Primary network is authorized network and secondary network could only access the licensed spectrum while keeping the QoS of primary network. Different from [6] and [7], we assume that SU could forward the packets from certain primary transmitters to its corresponding receivers, and SU adopt underlay accessing strategy which is elaborated in detail in the following.

A. NETWORK MODEL

Since Poisson point process (PPP) can better depict the geometric locations of the users in ad hoc networks and its

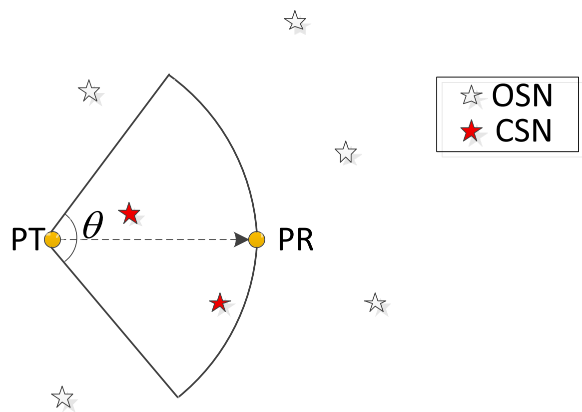

FIGURE 1. Schematic diagram of cooperative area.

tractability [3], [4], we also consider PU and SU are distributed as PPPs. Let Φ_{1T} be a PPP with intensity λ_1 on the plane. Φ_{1T} is a points set which denotes the locations of primary transmitters (PT). Each primary receiver (PR) associates with one designated PT with R_1 distant away. According to the displacement theorem [14], the locations of PR are also distributed as a PPP which is denoted by Φ_{1R}.

Let Φ_2 be a PPP with intensity λ_2 on the plane. Φ_2 denotes the locations of secondary users (SU). All SU are supposed to be able to get the location information of PT, PR and be sure about of their own locations. Therefore, SU are split into two categories. Those SU locate in the cooperative area are defined as cooperative SU (CSU), other SU are defined as ordinary SU (OSU). Cooperative area is a sector for the center of PT with spread angle θ in the direction from PT to PR as illustrated in figure 1. In the following, this sector is denoted as $s_\theta(o, R_1)$. Since SU are assumed to distribute as a homogeneous PPP, CSU and OSU are both distributed as homogeneous PPPs, respectively. Let p_c be the thinning probability [14], the locations of CSU and OSU follow the distribution of PPP Φ_{2C} with intensity $\lambda_2 p_c$ and PPP Φ_{2O} with intensity $\lambda_2(1-p_c)$, respectively.

B. SIR BASED SUCCESSFUL TRANSMISSION

In this model, signal is assumed to undergo path loss and Rayleigh fading only. The function $l(x, y) = \|x-y\|^{-\alpha}$ gives the attenuation from y to x in $\mathbb{R}^2$, which $\alpha > 2$ is the path loss factor. h is the small-scale fading coefficient having exponential distribution with mean of 1. Suppose there is a user located at x that transmits with power ρ and requires SIR β. The user can establish a channel to another user located at y $\in \mathbb{R}^2$ with a given bit rate if and only if

$$\mathrm{SIR}_{xy} = \frac{\rho h \|x-y\|^{-\alpha}}{I_\Phi(y)} \geq \beta, \tag{1}$$

where $\Phi = \{X_i\}$ denotes the locations of concurrent transmitters. I_Φ is the shot-noise process of Φ: $I_\Phi(y) = \sum_{X_i \in \Phi} \rho h \|y - X_i\|^{-\alpha}$.

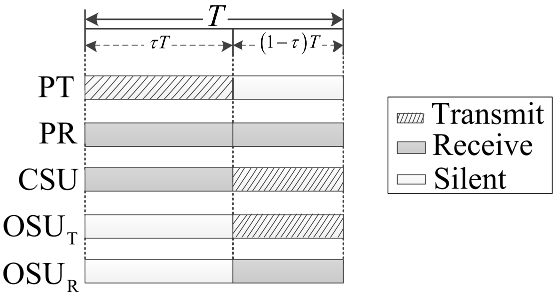

FIGURE 2. Accessing diagram of strategy 1.

Supposing a typical user u locates at the origin ($u = o$), the palm probability [14] of user v can successfully accept the packets from u at time slot n is

$$\begin{aligned} \mathbf{P^o}(n) &= \mathbf{P^o}(\mathrm{SIR}_{uv}(n) \geq \beta) \\ &= \mathbf{P^o}\left(\rho h r^{-\alpha} \geq \beta I_\Phi(y)\right), \end{aligned} \quad (2)$$

where r is the link distance from user u to user v.

In the following analysis, all PT are supposed to transmit their packets with power ρ_1 and successful decoding threshold β_1. And all SU are supposed to transmit their packets with power ρ_2 and successful decoding threshold β_2.

C. τ-SLOTTED ALOHA PROTOCOL

Time is divided into many slots and all users are synchronized to one slot. A slot is further divided into two parts: the first τ-slot ($0 < \tau < 1$) and the latter $(1 - \tau)$-slot. During one slot, PT and SU access the licensed spectrum according to the two strategies discussed in detail in the following.

Strategy 1: Within the first τ-slot, PT transmits their packets while PR and CSU receive the data packets from PT, and OSU keep silent. Within the latter $(1 - \tau)$-slot, PT keep silent, selected CSU (those nearest to PT) forward packets to PR while PR receive data packets, and OSU transmit with a probability p. Combining the former network model, OSU transmitters ($\mathrm{OSU_T}$) and receivers ($\mathrm{OSU_R}$) follow PPP $\Phi_{2\mathrm{O_T}}$ with intensity $\lambda_2 p(1 - p_c)$ and PPP $\Phi_{2\mathrm{O_R}}$ with intensity $\lambda_2(1 - p)(1 - p_c)$, respectively. Denote a time slot by T, the accessing strategy is illustrated in figure 2.

Strategy 2: Within the first τ-slot, PT transmit their packets while PR and CSU receive the data packets from PT. Different from the circumstance in strategy 1, $\mathrm{OSU_T}$ send packets to their nearest $\mathrm{OSU_R}$ within transmission distance R_2 and $\mathrm{OSU_R}$ receive packets. The accessing process is illustrated in figure3.

D. DEFINITIONS OF PERFORMANCE METRICS

In the next section, the performance metrics that we studied mainly includes local delay and network throughput. We investigate the performance metrics of τ-slotted Aloha protocol under strategy 1 and strategy 2, respectively.

1) LOCAL DELAY

The local delay D of ad hoc network is defined in [6] as the average number of time slots needed by a typical transmitter $u = o$ successfully sending packets to its receiver $v = y$, and

$$D = \mathrm{E}\left\{\inf\left\{n \geq 1 : \delta_0(n) = 1\right\}\right\}, \quad (3)$$

where $\delta_0(n) = 1$ is an indicator function that (1) holds in time slot n.

Let $\pi_c = \mathbf{P^o}(n)$, and π_c is a variable which is irrelevant to n and equals to the probability of successful transmission in and time slot since the distribution of nodes is independent from time slot to time slot. Therefore, the local delay is defined as

$$D = \frac{1}{\mathrm{E}\{\pi_c\}} = \frac{1}{\mathrm{E}\{\mathbf{P^o}(\mathrm{SIR}_{uv} \geq \beta)\}}. \quad (4)$$

2) NETWORK THROUGHPUT

Network throughput is defined as the average transmission rate of successful transmitters during one slot. Let T_h be throughput, we have

$$T_h = \frac{r_t \Pr(SIR \geq \beta)}{T}. \quad (5)$$

where r_t denotes transmission rate of a transmitter.

In the following analysis, PT and OSU are assumed to send packets at transmission rate r_{t1} and r_{t2}, respectively.

For clearly stating the following analysis, a list of symbol notations is shown in Table 1.

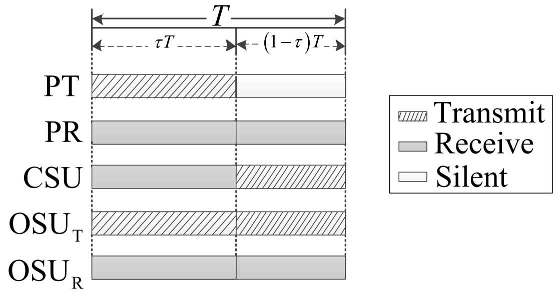

FIGURE 3. Accessing diagram of strategy 2.

TABLE 1. Mainly used symbols.

Symbol	Description
$\Phi_{1\mathrm{T}}$	point sets of primary transmitters (PT)
$\Phi_{1\mathrm{R}}$	point sets of primary receivers (PR)
λ_1	intensity of PT and PR
R_1	transmission distance of PT
ρ_1	transmission power of PT
β_1	Successful decoding threshold of primary network
Φ_2	point sets of secondary users (SU)
$\Phi_{2\mathrm{C}}$	point sets of CSU
$\Phi_{2\mathrm{O}}$	point sets of OSU
λ_2	intensity of SU
$\Phi_{2\mathrm{O_T}}$	point sets of transmitters of ordinary secondary users (OSU)
$\Phi_{2\mathrm{O_R}}$	point sets of receivers of OSU
R_2	transmission distance of SU
ρ_2	transmission power of SU
β_2	Successful decoding threshold of secondary network
p_c	thinning probability of CSU
p	accessing probability of OSU
T	time slot
r_{t1}	transmission rate of PT
r_{t2}	transmission rate of OSU

III. PERFORMANCE ANALYSIS

In this section, local delay and throughput are going to be analyzed on the supposition that all receivers are at the origin. It will not affect the results but greatly simplify the difficulty of studying according to the Slivnyak's theorem [14]. In addition, p_c is an important parameter because of its determination in transmitters sets. Based on the definition, p_c is the probability that SU locate in the cooperative area and

$$\begin{aligned} p_c &= 1 - \mathbf{P}(\text{no SU in the cooperative area}) \\ &= 1 - \mathbf{P}(\Phi_2 (s_\theta (o, R_1) = 0)) \\ &= 1 - \exp\left(-\frac{\theta}{2}\lambda_2 R_1^2\right). \end{aligned} \tag{6}$$

A. THE LOCAL DELAY AND THE NETWORK THROUGHPUT FOR STRATEGY 1

As shown in figure 2, the activity of primary and secondary users in the first τ-slot is different from the activity of them in the latter $(1-\tau)$-slot. Hence the performance analysis is divided into two parts, the first τ-slot and the latter $(1-\tau)$-slot.

1) THE LOCAL DELAY OF PRIMARY NETWORK

According to the definition in (4), local delay is determined by the probability of primary successful transmission. Further, it is determined by the SIR based successful probability of PT. In the first τ-slot, with interference coming from concurrent PT, according to the Lemma 1 in [15], the successful probability of PT transmission to PR is

$$\begin{aligned} P_{11} &= \mathrm{P_r}\left(\mathrm{SIR}_{11} \geq \beta_1\right) \\ &= \mathrm{P_r}\left(\frac{\rho_1 h R_1^{-\alpha}}{\sum_{X_i\in\Phi_{1T}} \rho_1 h \|X_i\|^{-\alpha}} \geq \beta_1\right) \\ &= \exp\left(-\lambda_1 K_\alpha \beta_1^{2/\alpha} R_1^2\right), \end{aligned} \tag{7}$$

where $K_\alpha = \frac{2\pi^2}{\alpha sin(2\pi/\alpha)}$.

In the latter $(1-\tau)$-slot, PT keep silent and CSU forward the packets they received from PT in the first τ-slot. Hence the successful probability improved by CSU comprises two parts: successful probability of CSU receiving in the first τ-slot and successful probability of CSU transmitting in the latter $(1-\tau)$-slot.

Firstly, similar to the results of equation (7), we give the probability that a secondary user is to be a CSU as

$$p_c = 1 - \exp\left(-\frac{\theta}{2}\lambda_2 R_1^2\right). \tag{8}$$

As given in [7], if a random CSU is chosen to receive the packets from PT, the average link distance of PT to a CSU is

$$h_a = \frac{2R_1\left(1-\exp\left(-\frac{\theta}{2}\lambda_2 p_c R_1^2\right)\right)}{3\theta}. \tag{9}$$

Thus the successful probability of a CSU successfully receiving the packets from its relevant PT in the first τ-slot is

$$P_{1c} = \exp\left(-\lambda_1 K_\alpha \beta_2^{2/\alpha} h_a^2\right). \tag{10}$$

In the latter $(1-\tau)$-slot, PT keep silent and CSU forward the packets. With considering the interference from $\mathrm{OSU_T}$ and concurrent CSU, the forwarding successful probability of CSU is

$$\begin{aligned} &P'_{1c} \\ &= \mathrm{P_r}\left(\mathrm{SIR}'_{1c} \geq \beta_1\right) \\ &= \mathrm{P_r}\left(\frac{\rho_2 h l_1^{-\alpha}}{\sum_{X_i\in\Phi_{2C}} \rho_2 h \|X_i\|^{-\alpha} + \sum_{Y_i\in\Phi_{2O_T}} \rho_2 h \|Y_i\|^{-\alpha}} \geq \beta_1\right) \\ &= \exp\left\{-\lambda_2 K_\alpha \beta_1^{2/\alpha} l_1^2 \left[p_c + p(1-p_c)\right]\right\}, \end{aligned} \tag{11}$$

where l_1 is the link distance from CSU to its corresponding PR.

Combining (7), (10) and (11), we take the **Local delay of primary network** as

$$D_{11} = \frac{\tau}{E(P_{11})} + \frac{(1-\tau)}{E\left(P_{1c}P'_{1c}\right)}. \tag{12}$$

2) THE LOCAL DELAY OF SECONDARY NETWORK

During the whole slot, SU deliver data only in the latter $(1-\tau)$-slot. The local delay of secondary network is determined by the successful transmission probability of $\mathrm{OSU_T}$ during the time slot. Denoting the probability by P_{21}, we have

$$\begin{aligned} &P_{21} \\ &= \mathrm{P_r}\left(\mathrm{SIR}_{21} \geq \beta_2\right) \\ &= \mathrm{P_r}\left(\frac{\rho_2 h l_2^{-\alpha}}{\sum_{X_i\in\Phi_{2C}} \rho_2 h \|X_i\|^{-\alpha} + \sum_{Y_i\in\Phi_{2O_T}} \rho_2 h \|Y_i\|^{-\alpha}} \geq \beta_2\right) \\ &= \exp\left\{-\lambda_2 K_\alpha \beta_2^{2/\alpha} l_2^2 \left[p_c + p(1-p_c)\right]\right\}. \end{aligned} \tag{13}$$

And the **local delay of secondary network** is

$$D_{21} = \frac{1}{E(P_{21})}. \tag{14}$$

3) THE THROUGHPUT OF PRIMARY AND SECONDARY NETWORK

Based on the successful probability of PT in the first τ-slot P_{11} and the successful probability of CSU in the latter $(1-\tau)$-slot P_{1c}, it is easy to take the throughput of primary network in the whole slot as

$$T_{h1} = r_{t1}\tau P_{11} + r_{t2}(1-\tau) P_{1c}P'_{1c}. \tag{15}$$

And the throughput of secondary network in the whole slot is

$$T_{h2} = r_{t2}(1-\tau) P_{21}. \tag{16}$$

B. THE LOCAL DELAY AND THE NETWORK THROUGHPUT FOR STRATEGY 2

As shown in figure 3, PT access the spectrum in the first τ-slot and keep silent in the latter $(1-\tau)$-slot, which is same as that of strategy 1. It is noted that $\mathrm{OSU_T}$ and $\mathrm{OSU_R}$ are active in the whole time slot. This determines the interference different from that of strategy 1. The local delay and the network throughput are also analyzed in two categories, the first τ-slot and the latter $(1-\tau)$-slot.

1) THE LOCAL DELAY OF PRIMARY NETWORK

In the first τ-slot, PT and $\mathrm{OSU_T}$ deliver their packets concurrently. The successful probability of PT transmission to PR is

$$\begin{aligned} P_{12} &= \mathrm{P_r}\left(\mathrm{SIR}_{12} \geq \beta_1\right) \\ &= \mathrm{P_r}\left(\frac{\rho_1 h R_1^{-\alpha}}{\sum_{X_i' \in \Phi_{1T}} \rho_1 h \left\|X_i'\right\|^{-\alpha} + \sum_{Y_i' \in \Phi_{2O_T}} \rho_2 h \left\|Y_i'\right\|^{-\alpha}} \geq \beta_1\right) \\ &= \exp\left\{-K_\alpha \beta_1^{2/\alpha} R_1^2 \left[\lambda_1 + \lambda_2 p \left(1-p_c\right)\left(\rho_2/\rho_1\right)^{2/\alpha}\right]\right\}. \end{aligned} \tag{17}$$

Since the CSU are supposed to be able to successfully receive the packets from PT in the first τ-slot. The successful probability of CSU forwarding data is taken as the same to that of strategy 1, i.e., $P_{2c} = P_{1c}$, $P'_{2c} = P'_{1c}$. Thus the local delay of primary network is

$$D_{12} = \frac{\tau}{P_{12}} + \frac{(1-\tau)}{E\left(P_{2c}P'_{2c}\right)}. \tag{18}$$

2) THE LOCAL DELAY OF SECONDARY NETWORK

$\mathrm{OSU_T}$ propagate data during the whole time slot. The successful probability of $\mathrm{OSU_R}$ receiving data from $\mathrm{OSU_T}$ in the first τ-slot is

$$\begin{aligned} P_{22} &= \mathrm{P_r}\left(\mathrm{SIR}_{22} \geq \beta_2\right) \\ &= \mathrm{P_r}\left(\frac{\rho_2 h l_{22}^{-\alpha}}{\sum_{X_i' \in \Phi_{1T}} \rho_1 h \left\|X_i'\right\|^{-\alpha} + \sum_{Y_i' \in \Phi_{2O_T}} \rho_2 h \left\|Y_i'\right\|^{-\alpha}} \geq \beta_2\right) \\ &= \exp\left\{-K_\alpha \beta_2^{2/\alpha} l_{22}^2 \left[\lambda_1 \left(\rho_1/\rho_2\right)^{2/\alpha} + \lambda_2 p \left(1-p_c\right)\right]\right\}, \end{aligned} \tag{19}$$

where l_{22} is the link distance from $\mathrm{OSU_T}$ to $\mathrm{OSU_R}$ in strategy 2.

In the latter $(1-\tau)$-slot, the successful probability of $\mathrm{OSU_T}$ transmission is given as

$$\begin{aligned} P_{2O} &= \mathrm{P_r}\left(\mathrm{SIR}_{2O} \geq \beta_2\right) \\ &= \mathrm{P_r}\left(\frac{\rho_2 h l_{2O}^{-\alpha}}{\sum_{X_i'' \in \Phi_{2C}} \rho_2 h \left\|X_i''\right\|^{-\alpha} + \sum_{Y_i'' \in \Phi_{2O_T}} \rho_2 h \left\|Y_i''\right\|^{-\alpha}} \geq \beta_2\right) \\ &= \exp\left\{-\lambda_2 K_\alpha \beta_2^{2/\alpha} l_{2O}^2 \left[p_c + p\left(1-p_c\right)\right]\right\}, \end{aligned} \tag{20}$$

where l_{2O} is the transmission distance from $\mathrm{OSU_T}$ to its corresponding receiver.

The local delay of secondary network is

$$D_{22} = \frac{\tau}{E\left(P_{22}\right)} + \frac{(1-\tau)}{E\left(P_{2O}\right)}. \tag{21}$$

3) THE THROUGHPUT OF PRIMARY AND SECONDARY NETWORK

It is obvious that the throughput of primary network is given as

$$T'_{h1} = r_{t1}\tau P_{12} + r_{t2}\left(1-\tau\right) P_{2c}P'_{2c}. \tag{22}$$

And the throughput of secondary network is

$$T'_{h2} = r_{t2}\tau P_{22} + r_{t2}\left(1-\tau\right) P_{2o}. \tag{23}$$

IV. PERFORMANCE OPTIMIZATION

Aiming to optimize the performance parameters, we investigate the optimal λ_2 to minimize the local delay or maximize the throughput.

A. BEST λ_2 FOR LOCAL DELAY

1) THE LOCAL DELAY FOR STRATEGY 1

In order to minimize the local delay of primary network, we look for the intensity of SU

$$\lambda_{2min1} = \arg\min_{\lambda_2 \geq \lambda_1}\left\{D_{11}\right\}. \tag{24}$$

In general, the link distance from CSU to its corresponding PR l_1 is a random variable since the uncertainty of a PT choosing a cooperative CSU. Thus λ_{2min} is rewritten as

$$\lambda_{2min1} = \arg\min_{\lambda_2 \geq \lambda_1}\left\{\frac{\tau}{P_{11}} + \frac{(1-\tau)}{P_{1c}E_{l_1}\left(P'_{1c}\right)}\right\}. \tag{25}$$

For computing the expectation $E_{l_1}\left(P'_{1c}\right)$, the probability density distribution of l_1 need to be determined which is related to the selection of CSU. If it is the nearest one to a given PT, the cumulative distribution function (CDF) of l_1 is easy to be got as

$$F_{l_1}(x) = 1 - \exp\left(-\frac{\theta}{2}\lambda_2 l_1^2\right), \quad 0 < l_1 < R_1 \tag{26}$$

$E_{l_1}\left(P'_{1c}\right)$ is an integration shown in the following.

$$E_{l_1}\left(P'_{1c}\right) = \int_0^{R_1} P'_{1c} f_{l_1}(x)\,dx = \frac{\theta\lambda_2\left[1-\exp\left(-\delta_{\lambda_2}R_1^2\right)\right]}{2\delta_{\lambda_2}}, \tag{27}$$

where $\delta_{\lambda_2} = \lambda_2\left\{\frac{\theta}{2} + K_\alpha\beta_1^{2/\alpha}\left[p_c + p\left(1-p_c\right)\right]\right\}$.

Furtherly, taking a derivative with respect to λ_2, we get the equality satisfied by λ_{2min} by setting the result equal to zero as followed,

$$\exp\left(-\delta_{\lambda_{2min}}R_1^2\right)\left(1+\delta_{\lambda_{2min}}R_1^2\right) = 1. \tag{28}$$

If the optimization target is the local delay of secondary network, the optimal intensity of SU becomes

$$\begin{aligned} \lambda'_{2min1} &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{D_{21}\right\} \\ &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{\frac{1}{E_{l_2}\left(P_{21}\right)}\right\}. \end{aligned} \tag{29}$$

As same to the discussion of λ_{2min} in (24), the PDF of l_2 also need to be determined. If the nearest OSU is selected to be the receiver, the PDF of l_2 is easy to be got as

$$F_{l_2}(x) = 1 - \exp\left[-\lambda_2(1-p)(1-p_c)l_2^2\right].0 < l_2 < R_2 \tag{30}$$

So $E_{l_2}(P_{21})$ is an integration as followed,

$$E_{l_2}(P_{21}) = \int_0^{R_2} P_{21} f_{l_2}(x)\,dx = \frac{K_{\lambda_2}\left[1-\exp\left(-\delta'_{\lambda_2}R_2^2\right)\right]}{\delta'_{\lambda_2}}, \tag{31}$$

where $K_{\lambda_2} = \lambda_2(1-p)(1-p_c)R_2^2$, $\delta'_{\lambda_2} = K_{\lambda_2} + \lambda_2 K_\alpha \beta_2^{2/\alpha}[p_c + p(1-p_c)]$.

2) THE LOCAL DELAY FOR STRATEGY 2

Based on the analysis above, it is easy to derive the optimal intensity of SU for the minimum local delay of primary network as followed.

$$\lambda_{2\min 2} = \arg\min_{\lambda_2 \geq \lambda_1}\left\{\frac{\tau}{P_{12}} + \frac{\tau}{E_{l_1}\left(P_{2C}P'_{2C}\right)}\right\}. \tag{32}$$

If the nearest SU is chosen to be the receiver, the optimal λ_2 for the least delay of secondary network is

$$\lambda'_{2\min 2} = \arg\min_{\lambda_2 \geq \lambda_1}\left\{\frac{\tau}{E_{l_{22}}(P_{22})} + \frac{(1-\tau)}{E_{l_{2O}}(P_{2O})}\right\}. \tag{33}$$

If OSU_T always transmit their packets to the nearest OSU_R, the probability density function (PDF) of l_{22} is

$$f_{l_{22}}(x) = 2K_{\lambda_2}xe^{-K_{\lambda_2}x^2}, 0<x \leq R_2$$

where $K_{\lambda_2} = \pi\lambda_2(1-p)(1-p_c)$.

And $E_{l_{22}}(P_{22})$ is easy to be given as

$$\begin{aligned} E_{l_{22}}(P_{22}) &= \int_0^{R_2} f_{l_{22}}(x)\,e^{-K_\lambda x^2}dx \\ &= \frac{K_{\lambda_2}}{K_{\lambda_2}+K_\lambda}\left(1-e^{-(K_{\lambda_2}+K_\lambda)R_2^2}\right), \end{aligned} \tag{34}$$

where $K_\lambda = K_\alpha \beta_2^{2/\alpha}\left[\lambda_1(\rho_1/\rho_2)^{2/\alpha} + \lambda_2 p(1-p_c)\right]$.

B. BEST λ_2 FOR THE THROUGHPUT GIVEN P

For strategy 1, best λ_2 for maximizing the throughput of primary network is

$$\begin{aligned} \lambda_{2\max 1} &= \arg\min_{\lambda_2 \geq \lambda_1}\{T_{h1}\} \\ &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{r_{t1}\tau P_{11} + r_{t2}(1-\tau)P_{1c}E_{l_1}\left(P'_{1c}\right)\right\}. \end{aligned} \tag{35}$$

Thus the optimal τ for maximizing the throughput of secondary network is

$$\begin{aligned} \lambda_{2\max 2} &= \arg\min_{\lambda_2 \geq \lambda_1}\{T_{h2}\} \\ &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{r_{t2}(1-\tau)E_{l_2}(P_{21})\right\}. \end{aligned} \tag{36}$$

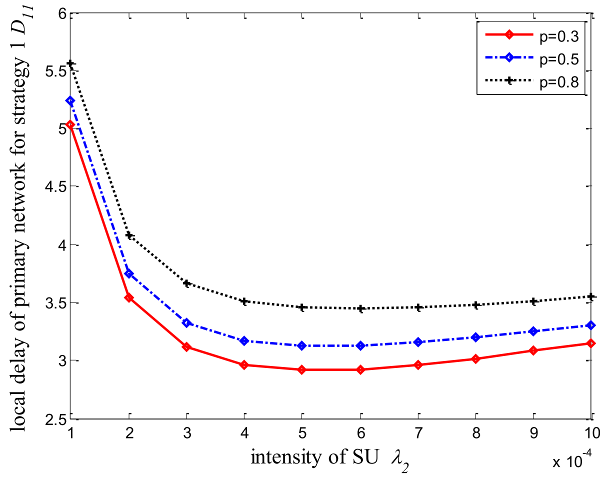

FIGURE 4. **Local delay of primary network for strategy 1 vs. intensity of secondary users.**

For strategy 2, in order to maximize the throughput of primary network, λ_2 should satisfy the expression in the following,

$$\begin{aligned} \lambda'_{2max1} &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{T'_{h1}\right\} \\ &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{r_{t1}\tau P_{12} + r_{t2}(1-\tau)P_{2c}E_{l_1}(P'_{2c})\right\}. \end{aligned} \tag{37}$$

Same to the best τ for the throughput of primary network, the expression of best τ for the throughput of secondary network is given as

$$\begin{aligned} \lambda'_{2max2} &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{T'_{h2}\right\} \\ &= \arg\min_{\lambda_2 \geq \lambda_1}\left\{r_{t2}\tau P_{22} + r_{t2}(1-\tau)P_{2o}\right\}. \end{aligned} \tag{38}$$

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present some numerical results based on the above theoretical analysis. According to the rationality between the parameters and formal experience, unless other specified, the network parameters are set as follows: $\alpha = 4c\beta_1 = 3$dB, β_2=1dB, $\theta = 2\pi/3$, $p = 0.5$,$\tau = 0.4$, $\lambda_1 = 10^{-4}$nodes$/$m^2, $R_1 = 30$m, $R_2 = 15$m, $\rho_1/\rho_2 = 2$.

In figure 4, we show the local delay of primary network for strategy 1 versus the intensity of secondary users λ_2 with different p. It is illustrated that the local delay of the primary network for strategy 1 is convex with the intensity of SU with any transmission probability p. This because that the increasing intensity of SU leads to a lager number of forwarding SU which definitely increases the probability of forwarding by SU in the latter $1-\tau$ slot. However, when λ_2 goes beyond a certain value, the successful probability of forwarding packets will degrade by the serious intra-interference. It also shows in the figure, the local delay is proportional to the transmission probability of OSU. Since the probability reflects the number of transmitting OSU in the latter $1-\tau$ slot which will increase the interference to CSU. And the local delay increases which is caused by the decreasing successful probability of forwarding packets.

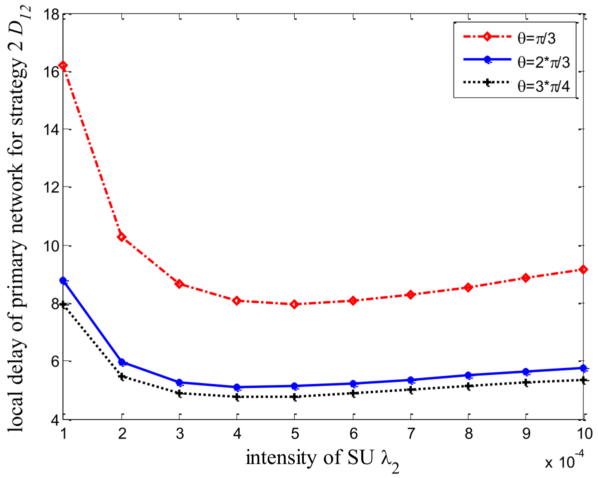

FIGURE 5. Local delay of primary network for strategy 2 vs. intensity of secondary users.

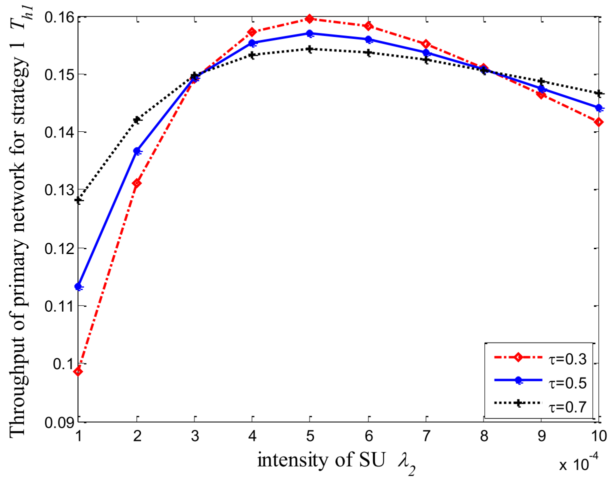

FIGURE 7. Throughput of primary network for strategy 1 vs. intensity of secondary users.

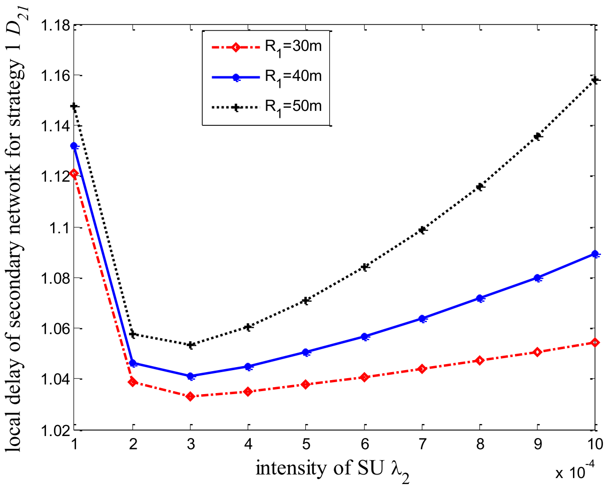

FIGURE 6. Local delay of secondary network for strategy 1 vs. intensity of secondary users.

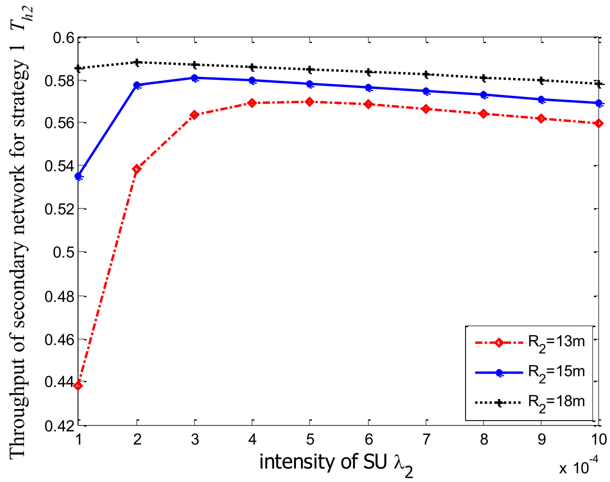

FIGURE 8. Throughput of secondary network for strategy 1 vs. intensity of secondary users.

In figure 5, we present the local delay of primary network for strategy 2 versus the intensity of SU. Same to that of strategy 1, it is convex with the intensity of secondary users. The reason is omitted due to the similarity of D_{11}. In addition, D_{12} is decreasing when the angle becomes larger. This because that there is more CSU could be selected to forward the packets of primary transmitters. Hence, the successful probability of primary network in the latter $(1-\tau)$-slot and a smaller local delay of primary network.

In figure 6, we draw the local delay of secondary network for strategy 1 versus the intensity of secondary users. It is shown that a certain value can be found to minimize the local delay of secondary network. When λ_2 increases, D_{21} will become smaller firstly as more OSU could be found to accept the packets of secondary network. But after a certain λ_2, the concurrent secondary transmitters lead to a decreasing successful probability of SIR transmission in secondary network. Additionally, D_{21} will become larger with the increasing R_1 since that less secondary users are chosen to be ordinary users because of larger coverage radius of primary transmitters.

In figure 7, we illustrate that the throughput of primary network versus the intensity of secondary users with varied τ. It can be concluded that we can find an optimal λ_2 to obtain the maximal T_{h1}. The reason is obvious and quite similar to that of the local delay of primary network. But $T_{h1}$is not monotonically related with τ. When $\tau$is very small, the time length of primary transmission is short which results in the less throughput of primary network. On the other hand, if $\tau$is too large, the time length of primary transmission is too long which leads to shorter time for CSU forwarding the packets of primary transmitters. This certifies that an optimal τexists to maximize the throughput of primary network.

In figure 8, we present the throughput of secondary network for strategy 1 versus the intensity of secondary users with varied R_2. It is shown in the figure that there is a certain λ_2 which can maximize the throughput. At the same time, T_{h2} is monotonous increasing with R_2 within a certain range

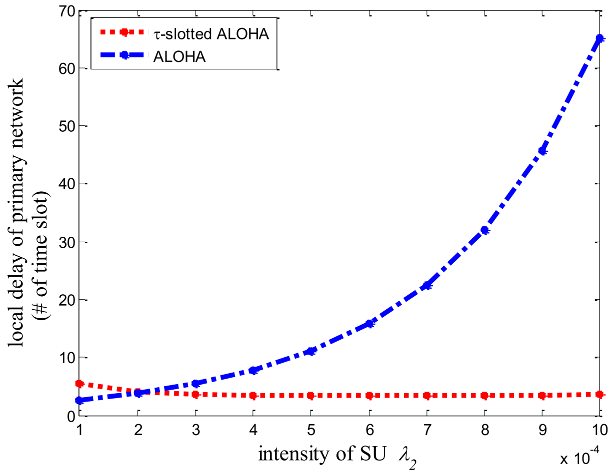

FIGURE 9. Local delay of primary network of τ-slotted ALOHA comparing that of ALOHA.

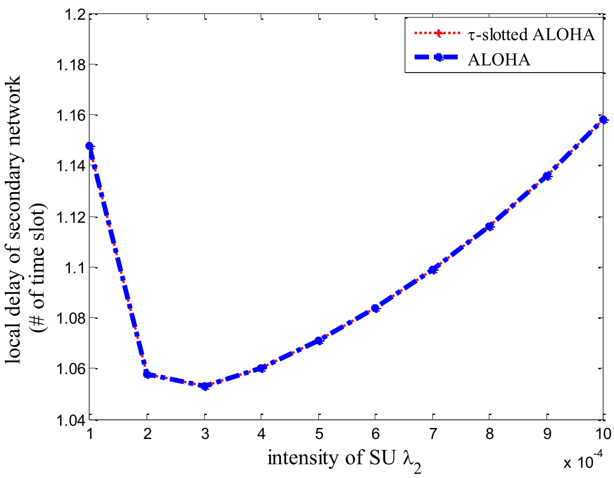

FIGURE 10. Local delay of secondaty network of τ-slotted ALOHA comparing that of ALOHA.

because of the higher successful probability of secondary transmission.

In figure 9 and figure 10, we compare the local delay of τ-slotted ALOHA with that of ALOHA MAC protocol for both primary and secondary networks with strategy 1. It is shown that the local delay of primary network of τ-slotted ALOHA is obvious better than that of ALOHA when the intensity of SU is larger than a certain value. And the local delay of secondary network of τ-slotted ALOHA is a bit worse than that of ALOHA which is hardly seen through numerical results.

VI. CONCLUSION

We conducted a research of the performance of cooperative CR ad hoc networks. Using the PPP to model the primary and secondary users and τ-slotted Aloha protocol, we presented the analytical express of the local delay and the throughput of both primary and secondary network. In the τ-slotted Aloha protocol, one slot is divided into two parts: first τ-slot and the latter $(1-\tau)$-slot. Some SU are assumed to be able to forward the primary packets in the latter $(1-\tau)$-slot after receiving them in the first τ-slot. In addition, we studied the optimization of the metrics we proposed above. Both theoretical and numeral results show that an optimal intensity of SU could be found to minimize the local delay or maximize the throughput of the two overlaid networks. It is illustrated in the numerical results that our τ-slotted Aloha protocol can promote the performance of primary network obviously by less sacrificing the performance of secondary network.

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