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Using WCS-FDTD Method to Study the Plasma Frequency Selective Surface

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ABSTRACT The plasma is very useful to design the dynamically tunable Frequency Selective Surface (FSS) because of its tunable property. But it is difficult to simulate the plasma FSS by using the conventional finite-difference time-domain (FDTD) method, because the FSS often has fine structure which confines the time step size. To solve this problem, the weakly conditionally stable finite-difference time-domain (WCS-FDTD) method is developed in this paper. The WCS-FDTD method uses auxiliary difference equation to represent plasma's dispersive characteristic, and removes the confinement of FSS's fine structure on time step size by using hybrid implicit-explicit (HIE) technique. So, compared with the FDTD method, the presented method can save a large number of computational time, which are validated by numerical examples. Also, by using the proposed method, the tunability of plasma FSS are validated and analyzed numerically.

INDEX TERMS Finite difference time domain, plasma, frequency selective surface.

I. INTRODUCTION

Frequency selective surface (FSS) is widely used in electromagnetic devices [1]–[3], such as antenna, filter, radar system, etc. Conventional FSS is made of metallic material, and often is untunable once the structure is designed. Plasma has excellent tunable property by just tuning the plasma density and collision frequency, which makes it have great potential to design dynamically tunable FSS. The plasma FSS can filter out any undesirable electromagnetic wave at any frequency and can also transmit waves at other frequency. So, it becomes more and more important because of its great adaptability. However, plasma FSS often has very thin layer, so it's difficult to analyze it by applying conventional finite difference time domain (FDTD) method [4], [5]. The FDTD method needs very long computational time when it is used to simulate plasma, because its time step size is confined by the fine spatial cell size in the thin layer.

The weakly conditionally stable (WCS)-FDTD method has been developed recently [6]–[9]. In this method, the size of time step size is only determined by one spatial cell size, and has no relation with other two cell sizes in the computational domain. So this method is widely used in the simulations of some fine structures, such as the graphene based absorber [8], small holes [9], etc. Compared with conventional FDTD scheme, the computational time of the WCS-FDTD method is reduced greatly, which has been validated by extensive numerical examples.

In this paper, the WCS-FDTD method is developed to simulate plasma FSS. It calculates the dispersive characteristics of plasma by using auxiliary difference equation, and uses the HIE technique to the spatial derivative operators in the fine direction. Based on these two techniques, the presented WCS-FDTD method not only can simulate the plasma FSS accurately, but also has much less computational time than conventional FDTD scheme, which are well validated through numerical examples. By using this method, the tunable properties of plasma FSS are discussed numerically, and the effects of plasma density and collision frequency on the electromagnetic performance are analyzed. The results show that by controlling the plasma density and collision frequency, the plasma FSS exhibits tunable resonant frequency and transmitted magnitude obviously.

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II. FORMULATION

In a lossy medium, wave propagating obeys the Maxwell's equations as follows

$$\nabla \times \vec{H} = j\omega\varepsilon_r\varepsilon_0\vec{E} \tag{1}$$
$$\nabla \times \vec{E} = -j\omega\mu_r\mu_0\vec{H} \tag{2}$$

where ε_r is the relative permittivity, $\mu_r\mu_r$ is the relative permeability. For plasma medium, it has

$$\varepsilon_r = 1 + \frac{\omega_p^2}{j\omega\,(j\omega + \nu_c)} \tag{3}$$
$$\mu_r = 1 \tag{4}$$

where, ν_c is the collision frequency and ω_p is the plasma frequency which can be expressed as

$$\omega_p^2 = \frac{e^2 N}{m\varepsilon_0} \tag{5}$$

where m and e represent the mass of electron and charge, respectively.

Equation (1) can be written as

$$\nabla \times \vec{H} = j\omega\varepsilon_0\vec{E} + \vec{J}_p \tag{6}$$

where, $\vec{J}_p$ is the polarized current introduced by plasma, and has the form as follows

$$\vec{J}_p = \frac{\varepsilon_0\omega_p^2}{j\omega + \nu_c}\vec{E} \tag{7}$$

By substituting $j\omega$ by $\partial/\partial t$, and replacing $\partial/\partial t$ by centered second-order finite differences, it has

$$\vec{J}_p^{n+1/2} = \frac{2 - \nu_c\Delta t}{2 + \nu_c\Delta t}\vec{J}_p^{n-1/2} + \frac{2\varepsilon_0\omega_p^2\Delta t}{2 + \nu_c\Delta t}\vec{E}^n \tag{8}$$

where, n and Δt are the index and size of time step, respectively.

Equation (8) is the auxiliary difference equation which represents the dispersive characteristic of plasma.

Suppose the plasma FSS has fine structures in x and z directions. So, the HIE difference scheme should be used to the spatial derivative operators $\partial/\partial x$ and $\partial/\partial z$ in the WCS-FDTD method. In such a case, the basic formulas of the WCS-FDTD method for analyzing the plasma are presented as follows,

$$\varepsilon_0\frac{(E_x^{n+1/2} - E_x^n)}{\Delta t} = -\frac{\partial(H_y^n + H_y^{n+1/2})}{2\partial z} + \frac{\partial H_z^n}{\partial y} - J_{px}^n \tag{9-1}$$

$$\varepsilon_0\frac{(E_y^{n+1/2} - E_y^n)}{\Delta t} = -\frac{\partial(H_z^n + H_z^{n+1/2})}{2\partial x} - J_{py}^n \tag{9-2}$$

$$E_z^{n+1/2} = E_z^n \tag{9-3}$$

$$H_x^{n+1/2} = H_x^n \tag{9-4}$$

$$\mu_0\frac{(H_y^{n+1/2} - H_y^n)}{\Delta t} = -\frac{\partial(E_x^n + E_x^{n+1/2})}{2\partial z} \tag{9-5}$$

$$\mu_0\frac{(H_z^{n+1/2} - H_z^n)}{\Delta t} = \frac{\partial E_x^{n+1/2}}{\partial y} - \frac{\partial(E_y^n + E_y^{n+1/2})}{2\partial x} \tag{9-6}$$

$$E_x^{n+1} = E_x^{n+1/2} \tag{10-1}$$

$$\varepsilon_0\frac{(E_y^{n+1} - E_y^{n+1/2})}{\Delta t} = \frac{\partial(H_x^{n+1/2} + H_x^{n+1})}{2\partial z} \tag{10-2}$$

$$\varepsilon_0\frac{(E_z^{n+1} - E_z^{n+1/2})}{\Delta t} = -\frac{\partial H_x^{n+1/2}}{\partial y} + \frac{\partial(H_y^{n+1/2} + H_y^{n+1})}{2\partial x} - J_{pz}^n \tag{10-3}$$

$$\mu_0\frac{(H_x^{n+1} - H_x^{n+1/2})}{\Delta t} = -\frac{\partial E_z^{n+1}}{\partial y} + \frac{\partial(E_y^{n+1/2} + E_y^{n+1})}{2\partial z} \tag{10-4}$$

$$\mu_0\frac{(H_y^{n+1} - H_y^{n+1/2})}{\Delta t} = \frac{\partial(E_z^{n+1/2} + E_z^{n+1})}{2\partial x} \tag{10-5}$$

$$H_z^{n+1} = H_z^{n+1/2} \tag{10-6}$$

Equations (9-1) $\sim$ (9-6) are the first updating from time step n to n+1/2 and equations (10-1) $\sim$ (10-6) are the second updating from n+1/2 to n+1. By replacing the spatial derivative operators in above equations with the centered second-order finite differences, and computing J_{px}^{n+1}, J_{py}^{n+1} and J_{pz}^{n+1} by using (8), the final equations for the WCS-FDTD method to solve plasma can be obtained.

III. NUMERICAL ANALYSIS

A. VALIDATION OF ACCURACY AND EFFICIENCY

To demonstrate the numerical performance of the proposed WCS-FDTD method in the simulation of plasma, we proceed to calculate the transmission of a plane wave incidence on a plasma FSS in free space, and compare the numerical results with those of the conventional FDTD method.

The geometry of the FSS is shown in Figure 1. Each plasma element which is actually closed gaseous plasma contained in a tube is set to be L=15 cm in length and W=1 mm in width and height h=1 mm, and arranged periodically in x and y directions. The thickness of plasma is 1mm. The period of each element is taken to be P1=17 cm and P2=9.1cm. The parameters of plasma are $\nu_c = 0$ Hz, $\omega_p = 2\pi F_P$ and F_P =10 GHz.

A z-directed plane wave polarized in y direction normally impinges on the plasma FSS. The time dependence of the excitation function is

$$g(t) = \exp\left[-4\pi\,(t - t_0)^2 \big/ \tau_0^2\right] \tag{9}$$

where, $\tau_0 = t_0 = 2 \times 10^{-9}$s. So the highest frequency of the plane wave is $2/\tau_0$ =1 GHz, and the minimum wavelength is 30 cm.

We use the proposed WCS-FDTD method and conventional FDTD method to calculate the transmitted field of the FSS. To model the infinite plane, the periodic boundary is used in x and y directions, and convolutional perfectly matched layer (CPML) with ten layer [10] is applied in z direction to truncate the outgoing wave. Fine cells with size $\Delta x_{min} = \Delta z_{min} = 0.25$ mm are used to discretize the plasma. In y direction, uniform meshes with

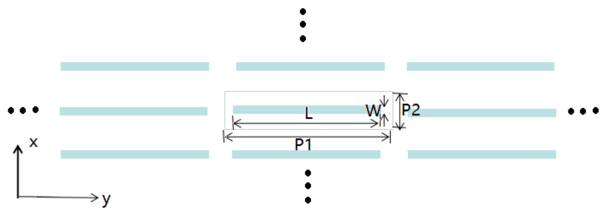

FIGURE 1. Geometry of plasma FSS.

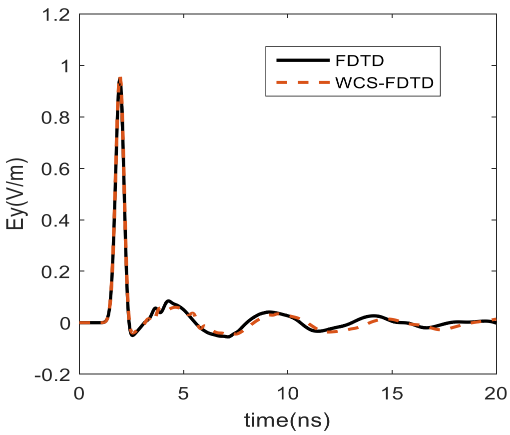

FIGURE 2. The transmitted field of plasma FSS calculated by using FDTD and WCS-FDTD methods.

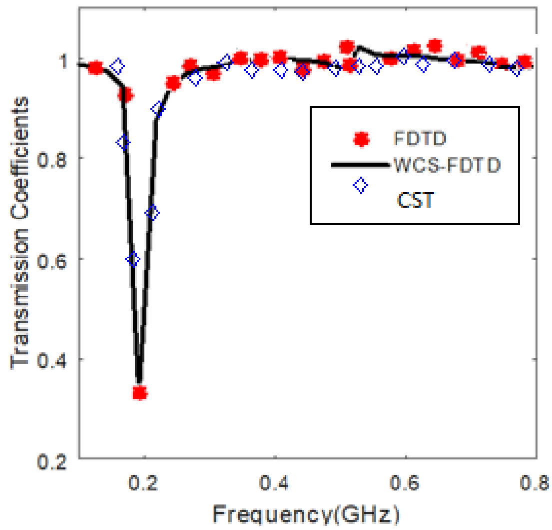

FIGURE 3. The transmission coefficients of plasma FSS calculated by using F DTD and WCS-FDTD methods.

TABLE 1. The computational times of WCS-FDTD method and FDTD method. in the simulation of plasma FSS.

Method	Δt (ps)	Time(s)
WCS-FDTD	30	49
FD-TD	0.58	1920

size $\Delta y = 1$ cm, corresponding 1/15 wavelength, are applied. Thus, the size of time step in the FDTD method is $\Delta t \leq 1/c\sqrt{(1/\Delta x_{min})^2 + (1/\Delta y)^2 + (1/\Delta z_{min})^2} =$ 0.58 ps [5], while, in the WCS-FDTD method, it can increase to $\Delta t \leq \Delta y/c = 35.33$ ps, which is about 60 times as that of FDTD method.

The simulated transmitted field and transmission coefficients by using WCS-FDTD method and FDTD method are plotted in Figures 2 and 3. To improve the accuracy of the simulation, the time step size of the WCS-FDTD method is 30 ps. It can be seen from these two figures that the results of the WCS-FDTD method and FDTD method agree very well with each other, no matter in time domain or in frequency domain. Besides, to validate the computational accuracy of the proposed method, the transmission coefficients simulated by using CST Microwave Software are also plotted in Fig.3. These results demonstrate that the WCS-FDTD method has high computational accuracy. However, to complete the simulation, the computational times of these two methods are different greatly. The CPU times for the FDTD method and WCS-FDTD method are 1920 seconds and 49 seconds, respectively, as shown in Table 1. Obviously, due to much larger time step size, the computational efficiency of WCS-FDTD scheme is considerably improved from that of FDTD method. Besides, from Figure 3, we can see that the FSS has obvious band-stop characteristic at resonant frequency 0.2 GHz. At this frequency, the transmission coefficient is below to 0.3.

The resonant frequency of the FSS is determined by the structure of the plasma. We simulate the FSS with different plasma's width and length by using WCS-FDTD method and compare the transmission coefficients. The plasma frequency Fp is 30 GHz and the collision frequency is 0 GHz. The variations of the transmission coefficients with respect to plasma's width and length are presented in Figures 4 and 5. In Figure 4, the width of the plasma varies from 1 mm to 8 mm, and the length of the plasma L is 15 cm. In Figure 5, the length of the plasma varies from 10 cm to 20 cm, and the width of the plasma is set to be 1mm. It can be seen from these two figures that as the width or length of the plasma increases, the resonant frequency of the FSS increases obviously, namely, we can realize a low-pass filter by increasing the width or length of the plasma.

B. TUNABILITY ANALYSIS OF FSS

It's well known that the relative permittivity of plasma varies with its collision frequency ν_c and plasma frequency F_p, as shown in (3). So, by changing the collision frequency and/or plasma frequency of plasma, the transmission coefficient of plasma FSS will varies. We demonstrate this performance numerically by using WCS-FDTD method.

We take different plasma frequencies F_p to analyze transmission coefficients of the FSS by using WCS-FDTD method. The structures of the FSS are set to be L=15 cm in length and W=1 mm in width and height h=1 mm. The thickness of plasma is 1mm. The period of each element is taken to be P1=17 cm and P2=9.1cm.The results are

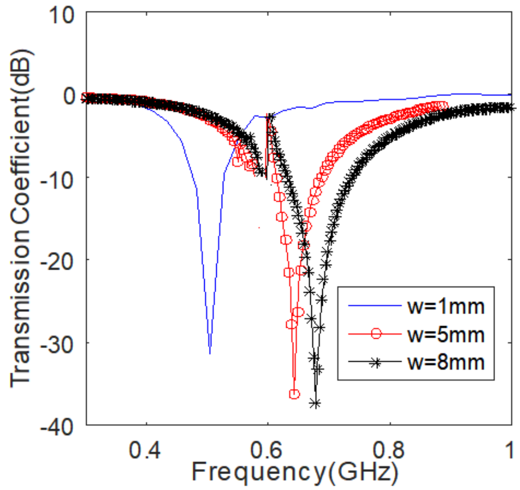

FIGURE 4. The variation of transmission coefficients with respect to width of plasma.

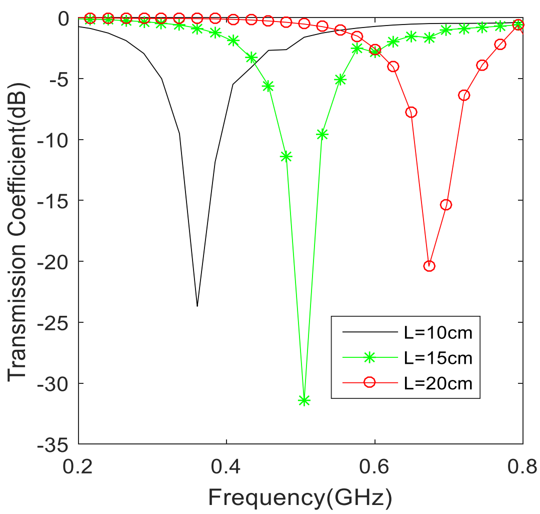

FIGURE 5. The variation of transmission coefficients with respect to length of plasma.

plotted in Figure 6, here, the collision of the electron of the plasma is neglected, namely, plasma collision frequency v_c is assumed to be 0 GHz. It can be seen from Fig.6 that, by increasing the plasma frequencies from 10 GHz to 30 GHz the resonant frequency of the FSS shifts from 0.2 GHz to 0.5 GHz. From the simulation, we can conclude that if the plasma frequency continues to increase, the resonant frequency increases prospectively.

The collision frequency v_c of plasma also has effects on the transmission of FSS. Here, we consider the collision frequency varies from 0 GHz to 5 GHz, and the plasma frequency is 30 GHz unchanged. The variation of transmission coefficients with respect to v_c is plotted in Figure 7. It can be seen from this figure that as the collision frequency increases, the transmission coefficients also increase, from −30 dB to −2 dB, which means the transmitted field strengthens and the band-stop effect attenuates. But the resonant frequency remains the same.

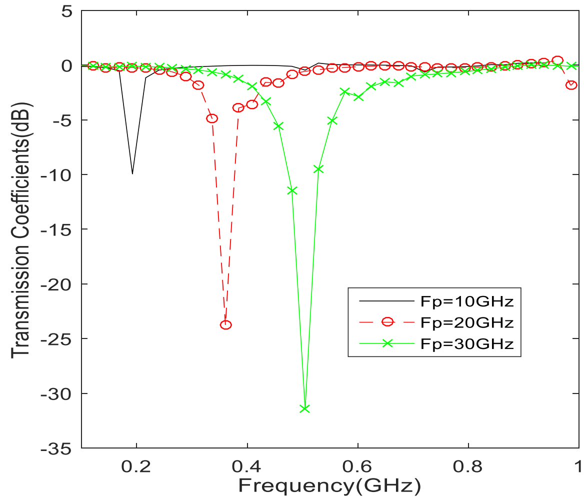

FIGURE 6. The variation of transmission coefficients with respect to plasma frequency.

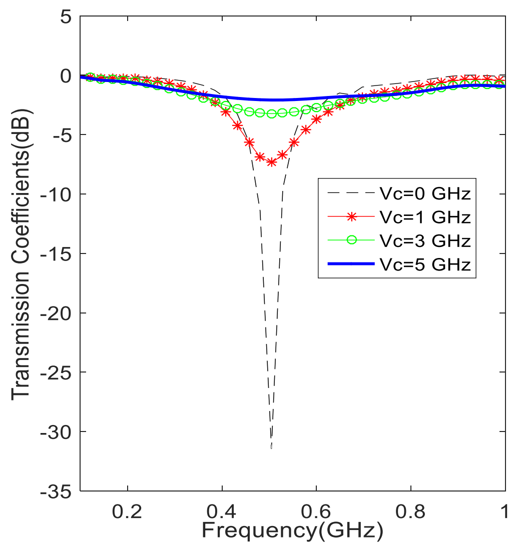

FIGURE 7. The variation of transmission coefficients with respect to collision frequency.

From the analysis above, we can conclude that by tuning the collision frequency and plasma frequency, it can realize dynamically tunable FSS. The collision frequency of plasma can influence the transmitted magnitude of the plasma FSS, and the plasma frequency determines the resonant frequency. Thus, by just changing the plasma density and collision frequency, without changing the structure, the plasma FSS can filter out any undesirable electromagnetic wave at desirable frequency and desirable magnitude.

IV. CONCLUSION

The WCS-FDTD method is developed to solve the plasma FSS in this paper. The dispersive characteristic of the plasma is incorporated into this method and the HIE difference scheme is applied. So, the proposed method has much higher computational efficiency compared with the conventional

FDTD method, which are well validated by numerical examples. The simulated results show that the resonant frequency and transmitted magnitude of the plasma FSS are tunable, just by controlling the collision frequency and plasma frequency. These conclusions are valuable for the design of plasma-based electromagnetic devices.

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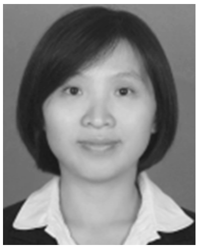

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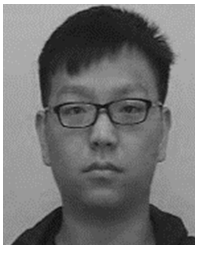

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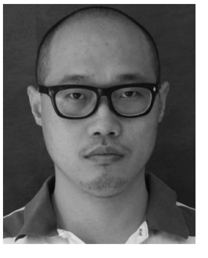
XUMIN YU, photograph and biography not available at the time of publication.



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