

Received September 16, 2019, accepted October 9, 2019, date of publication October 14, 2019, date of current version October 30, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2947158

# Design of Closed-Loop Parameters With High Dynamic Performance for Micro-Grating Accelerometer

HUI LI<sup>ID</sup>, (Member, IEEE), KEKE DENG<sup>ID</sup>, SHAN GAO, AND LISHUANG FENG

Key Laboratory of Precision Opto-mechatronics Technology, Ministry of Education, Beihang University, Beijing 100191, China

Key Laboratory of Micro-nano Measurement-Manipulation and Physics, Ministry of Education, Beihang University, Beijing 100191, China

Corresponding author: Hui Li (lihui@buaa.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61875006, and in part by the Defense Industrial Technology Development Program under Grant JCKY201601C006.

**ABSTRACT** A novel closed-loop parameter design scheme with timing sequence control method is proposed to obtain fast tracking ability of micro-grating accelerometer. Firstly, we investigate a timing sequence control method to achieve the minimum time-delay of signal processing only dependent on the response capability of sensing element, which is the same as half modulation period in the designed closed-loop accelerometer system. Considering optical sensing principle and time-delay of signal processing, we establish a dynamic equation for closed-loop micro-grating accelerometer. Then, we analyze the design principle of closed-loop parameters on the condition of system stability, which describes the relationship between time-delay and control parameters to guarantee fast tracking performance of micro-grating accelerometer system. And, through proposed parameter design principle, the closed-loop parameters of micro-grating accelerometer can be obtained with the gains of forward channel and feedback channel in closed-loop detection system. The conducted experiment results show that micro-grating accelerometer can achieve the nonlinearity within 0.28%, the fast step response time of 0.8ms and  $-3$ dB bandwidth up to 525Hz, which validate the effectiveness of our parameter design scheme and timing sequence control method.

**INDEX TERMS** Dynamic performance, micro-grating accelerometer, signal processing, time-delay.

## I. INTRODUCTION

As a key component in inertial measurement unit, accelerometers are widely applied in the field of the attitude control and inertial navigation [1]–[3]. Especially, micro-grating accelerometer has been a dominant development tendency due to its advantages of high integration, excellent theoretical precision and immunity to electro-magnetic interference [4]–[6]. Furthermore, to promote the practical application of micro-grating accelerometer, signal detection method is of vital importance in this process.

Recently, a lot of effort has been paid to the signal detection for micro-grating accelerometer. A reference detection circuit was designed to reduce the laser relative intensity noise and improve the sensitivity of micro-grating accelerometer [7]. Garcia et al. put forward electromagnetic actuation to realize the closed-loop feedback for optimizing the

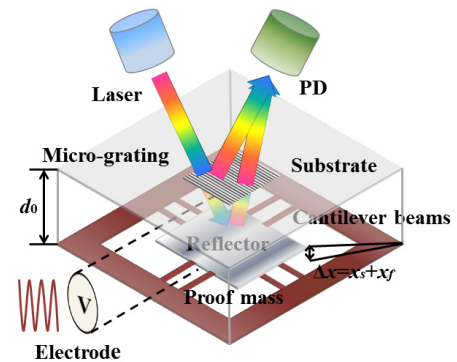
stability of grating interferometric seismometer [8]. Aimed at extracting the optical signal with acceleration information, Zhao et al. investigated intensity modulation method through piezoelectric translator actuation to improve the precision of optical accelerometer [9]. A differential detection method was introduced so as to reduce the common-mode noise and enhance the sensitivity of the micro-grating accelerometer [10]. Lu et al. focused on the optical path mechanism and improved the contrast ratio of interference signal by modifying the design of symmetrical structure [11]. The design of electromagnetic actuation utilized on micro-grating accelerometer was presented to optimize the linear relation between the electrostatic feedback and input acceleration [12]. In addition, a dual modulation method was proposed combining intensity modulation of the laser and phase modulation on electrodes, which reveals a decrease background noise in the micro-grating accelerometer system [13]. In order to eliminate the light power fluctuation and various noises, a PDH modulation technology was adopted to

The associate editor coordinating the review of this manuscript and approving it for publication was Prakasam Periasamy<sup>ID</sup>.

improve the linearity and detection accuracy of micro-grating accelerometer [14]. Although above schemes make contributions to the linearity and detection accuracy, the dynamic performance is also essential for the wide application of micro-grating accelerometer. The methods of applying automatic gain control [15] and PI control [16] were separately proposed to improve the dynamic performance of mechanical accelerometer, but they are not aimed at and suitable for micro-grating accelerometer which is based on optical sensing principle. Hall et al. analyzed the influence of positive or negative feedback gain on the micro-grating accelerometer's bandwidth [17]. Qin et al. demonstrated linear response of fiber bragg grating accelerometer in a frequency range [18]. A white light interferometry demodulation algorithm was developed for optical accelerometer, which obtains a reliable sensitivity within the frequency bandwidth [19]. Recently, related work has been devoted to the dynamic response of optical accelerometer with high sensitivity. However, there are still some challenging problems about the improvement of dynamic performance which confine the application of optical accelerometer in the practical engineering field.

The closed-loop error of micro-grating accelerometer is a weak signal containing numerous noises [20], which adds difficulty in designing the signal processing method to extract it precisely. Thus, researchers apply the modulation and demodulation method based on correlation detection principle to improve the SNR of closed-loop error and further optimize the detection accuracy of accelerometer [9], [13], [14], [19]. Unavoidably, the modulation and demodulation method brings time-delay in signal processing, and the minimum value of modulation and demodulation period is limited by response capability of optical sensing element. Thus, one of our motivations is to resolve the contradiction between improving the dynamic performance and promoting detection precision of micro-grating accelerometer. Especially, the influence mechanism of closed-loop parameters on the dynamic performance of micro-grating accelerometer hasn't been studied yet. Furthermore, the time-delay existed in the micro-grating accelerometer system will make the dynamic performance worse and even lead the closed-loop system to an unstable state. Thus, how to find out the design principle of closed-loop parameters for system stability and minimize the time-delay in signal processing is very important to improve the dynamic performance of micro-grating accelerometer.

In this work, a novel closed-loop parameter design scheme with timing sequence control method is proposed to obtain fast tracking ability of micro-grating accelerometer. Firstly, we introduce the optical sensing principle of micro-grating accelerometer and design a closed-loop detection scheme for it. Secondly, we propose the timing sequence control method to minimize the time-delay in entire signal processing, which is only dependent on the response capability of sensing element. Thirdly, we theoretically analyze the dynamic mathematic model of closed-loop accelerometer system considering unavoidable time-delay to obtain the design principle of closed-loop parameters for system stability. Then, the design



**FIGURE 1.** The sensing principle of closed-loop micro-grating accelerometer.

of controller parameters can be determined through proposed parameter design principle, which guarantees fast tracking performance of micro-grating accelerometer system. Finally, the experimental results are given to demonstrate the effectiveness of proposed closed-loop parameter design scheme with timing sequence control method.

## II. PROBLEM DESCRIPTION

The optical mechanism of closed-loop micro-grating accelerometer is schematically described in Fig. 1. The architecture consists of proof mass, cantilever beams, micro-grating and reflector. When the light emitted from laser propagates to the transparent substrate, a portion of light is reflected off the micro-grating and the other portion travels through it. The latter portion continues transmitting and arrives at the reflector, which is attached to the proof mass. Then the latter portion will be reflected and pass through the micro-grating gap again. Finally, the light directly reflected by the micro-grating meets with the light that travels the distance between micro-grating and reflector twice, and they form the interference and diffraction field [21]. Input of acceleration can result in the movement of proof mass. Because we consider that the proof mass has reached the equilibrium state, the sensing mechanism of micro-grating accelerometer can be treated as linear process. Moreover, the cantilever beams and micro-grating made of metal enable the feedback signal to counterbalance the displacement change of proof mass and simultaneously implement the modulation process. Displacement change of proof mass brings about phase difference between two portions of interference light. Consequently, the interference light intensity varies, which is detected by photodetector(PD).

According to the optical sensing mechanism, we design a signal processing and closed-loop control scheme shown in Fig. 2. The interference intensity reflecting acceleration information is a weak signal with various noises. To extract the weak closed-loop error signal, sinusoidal modulating signal is imposed on the cantilever beams. Based on the scalar diffraction theory [22], we can obtain the detected signal by

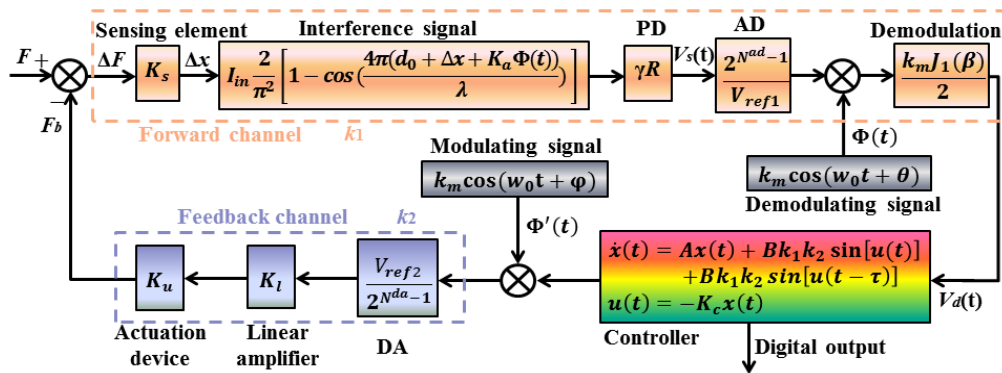


FIGURE 2. The block diagram of closed-loop detection scheme of micro-grating accelerometer.

PD as

$$V_s(t) = \frac{k_0}{2} \left\{ 1 - \cos \left[ \frac{4\pi (d_0 + \Delta x_s(t) + \Delta x_f(t) + K_a \Phi(t))}{\lambda} \right] \right\} \quad (1)$$

where  $d_0$ ,  $\Delta x_s(t)$ ,  $\Delta x_f(t)$ ,  $K_a$  and  $\lambda$  are initial distance between the micro-grating and reflector, the displacement induced by input acceleration, the displacement generated from feedback voltage, the conversion coefficient of displacement and actuation voltage and the wavelength of laser, respectively. And  $k_0$  includes sensing mechanism's gain  $K_s$ , interference process gain  $8I_{in}/\pi\lambda$ , the PD convertor's gain  $\gamma \cdot R$  and AD convertor's gain  $2^{N_{ad}-1}/V_{ref1}$ , where  $I_{in}$ ,  $\gamma$  and  $R$ ,  $N_{ad}$  and  $V_{ref1}$  are output power of laser, conversion coefficient and transimpedance of PD, conversion bit and reference voltage of AD converter, respectively.  $\Phi(t) = k_m \cos(\omega_0 t + \varphi)$  is the modulating signal, where  $k_m$ ,  $\omega_0$  and  $\varphi$  is modulating signal's amplitude, frequency and the phase delay, respectively. Here  $\Delta x(t) = \Delta x_f(t) + \Delta x_s(t)$  is defined as the closed-loop error of micro-grating accelerometer. The expansion of Equation (1) includes  $n\omega_0$  ( $n=0, 1, 2, 3 \dots$ ) frequency components.

Modulating signal moves the closed-loop error signal into high frequency carrier wave for improving the signal to noise ratio (SNR) of closed-loop error signal. However, the modulation and demodulation method induces time-delay in signal processing which diminishes the fast tracking ability of micro-grating accelerometer. In this work, we aim to resolve the contradiction between improving the dynamic performance and promoting detection precision of micro-grating accelerometer. Therefore, we take the unavoidable time-delay into consideration and analyze the design principle of closed-loop parameters to obtain the high dynamic performance of micro-grating accelerometer.

### III. DESIGN AND ANALYSIS OF CLOSED-LOOP DETECTION SYSTEM

On the basis of correlation detection principle, the detected closed-loop error signal is multiplied by  $\Phi'(t) = k_m \cos(\omega_0 t + \theta)$ , where  $\Phi'(t)$  has the same frequency and amplitude as  $\Phi(t)$

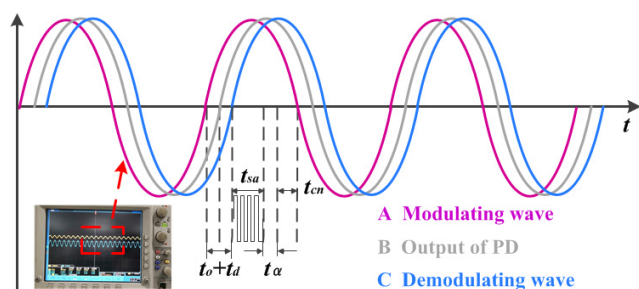


FIGURE 3. The time relation of modulating wave, out of PD and demodulating wave according to the timing sequence control method.

and its phase bias  $\theta$  can be adjusted to counterbalance  $\varphi$ . To obtain closed-loop error as fast as possible, the designed low pass filter (LPF) performs integral in half modulation period and removes  $m\omega_0$  ( $m=2, 3, 4 \dots$ ) frequency components, which is illustrated as

$$V_T(t) = \frac{k_0 k_m}{2T} \int_0^{T/2} J_1(\beta) \sin \left( \frac{4\pi (d_0 + \Delta x(t))}{\lambda} \right) dt + \frac{k_0 k_m}{2T} \int_0^{T/2} \cos(\omega_0 t + \theta) dt - \frac{k_0 k_m}{2T} \int_0^{T/2} \cos \left( \frac{4\pi (d_0 + \Delta x(t))}{\lambda} \right) \times [J_0(\beta) \cos(\omega_0 t + \theta) - J_2(\beta) \cos(\omega_0 t)] dt \quad (2)$$

where  $\beta = 4\pi k_m K_a / \lambda$  and  $T = 2\pi / \omega_0$ . For our system, we design  $4\pi d_0 / \lambda = 2n\pi$ . Because the frequency of closed-loop error signal is far less than modulation frequency, we can suppose that the closed-loop error signal in half modulation period is unchanged.

Then, we will introduce how to guarantee the minimum time delay in closed-loop detection system of micro-grating accelerometer, as can be seen in Fig. 3. Because the response capability of sensing element also limits the modulation frequency, and the minimum time-delay of signal processing in

theory is a half modulation period. The main idea of our timing sequence method is taking the system's unavoidable time-delay into first consideration. In the optical sensing element, the time-delay of the optical path stays constant denoted as  $t_o$ . And, AD convertor provides data with a delay of twelve clocks, which can be represented as  $t_d$ . Next, the LPF contains sample realized through the frequency divider and filter implemented by several registers, whose sample time is  $t_{sa}$  and time-delay of filter is  $t_\alpha$ . Functionally, the LPF is designed to make adjacent half period of positive and negative data add together. In addition, the resonant frequency of optical structure is the response capability of sensing element, which determines the minimum value of the modulation period. And the minimum value of closed-loop control period is equal to half modulation period. Taking unavoidably time delay  $t_o$ ,  $t_d$ ,  $t_{sa}$  and  $t_\alpha$  into consideration, it is important to make sure that the closed-loop feedback signal is generated in current control period to realize the minimum value of time-delay in signal processing so that the system time-delay is minimized. Thus, we should control the processing time of LPF to guarantee that there is time  $t_{cn}$  left for the controller to operate before the next half modulation period. Considering the unavoidably time delay  $t_o$ ,  $t_d$ ,  $t_{sa}$ ,  $t_{cn}$  and the closed-loop period, we obtain the maximum value of LPF time-delay  $t_\alpha$ . Furthermore, the order and parameters of LPF can be calculated to precisely confine the processing time of LPF  $t_\alpha$ .

This principle of timing sequence control method can guide the design of modulation and demodulation as well as the time distribution in signal processing. Therefore, the whole demodulation and control process can finish within a half of modulation period  $\tau$ , which guarantees that the minimum time-delay in signal processing. Dependent on the response capability of sensing element, the minimum time-delay  $\tau$  is determined as 100us, which is also the half modulation period.

After that, in one modulation period, adjacent half period of positive and negative data are added together, consequently fundamental frequency components are eliminated. Then, the demodulation result of half modulation period is further deduced as

$$V_d(t) = k_1 \left[ \sin\left(\frac{4\pi \Delta x(t)}{\lambda}\right) + \sin\left(\frac{4\pi \Delta x(t - \tau)}{\lambda}\right) \right] \quad (3)$$

where  $k_1 = k_0 k_m J_1(\beta)/2$  is the gain of forward channel, and  $\tau$  represents the minimum value of time-delay in theory, which is dependent on the response capability of sensing element and determines the modulation period.

In order to analyze the relation between closed-loop parameters and time-delay on the condition of system stability, we deduce the closed-loop system model in continuous domain to improve the dynamic performance of micro-grating accelerometer. Based on the demodulation result in Equation (3), we build a dynamic model considering closed-loop error with time-delay of half modulation period to analyze the design of system parameters. The dynamic equation

of closed-loop micro-grating accelerometer is given as

$$\dot{x}(t) = Ax(t) - Bk_1 k_2 \sin[K_c x(t)] - Bk_1 k_2 \sin[K_c x(t - \tau)] \quad (4)$$

where  $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ ,  $K_c = [k_{c1} \ k_{c2}]$ , and  $k_{c1} > 0, k_{c2} > 0$ .  $x(t) \in R^2$  denotes the state variable with the same order as second-order controller. As is shown in Fig. 2, the feedback channel gain  $k_2$  includes DA convertor's gain  $V_{ref2}/2^{N_{da}-1}$ , linear amplifier's gain  $K_l$  and actuation device's gain  $K_u$ , where  $V_{ref2}$  and  $N_{da}$  are reference voltage and conversion bit of DA converter, respectively.

Our ambition is to figure out the design principle of closed-loop parameter and further realize the fast tracking ability of micro-grating accelerometer. Since the feedback signal  $\Delta x_f(t)$  is generally designed to counterbalance the  $\Delta x_s(t)$  induced by input acceleration, we have  $\Delta x(t) \approx 0$ . Thus, the closed-loop accelerometer system can work around the optimal point, which yields to the result of  $\sin K_c x(t) \approx K_c x(t)$ . Furthermore, the dynamic equation of closed-loop micro-grating accelerometer is considered as linear. In order to ensure the tracking performance and fast dynamic response, we present the design method of feedback gain matrix  $K_c$  to guarantee that micro-grating accelerometer satisfies the fast stability.

**Theorem 1.** Consider the dynamic equation of closed-loop micro-grating accelerometer with time delay. The system (4) solves the stability problem, if  $\tau < \frac{2}{w'} \arctan \frac{k_{c2}}{k_{c1}} w'$ , where  $w' = (2 k_{c1} k_1 k_2 / (1 - 2 k_{c2}^2 k_1 k_2 / k_{c1}))^{-1/2}$ , and  $k_{c1}, k_{c2} > 0$  are the gains of feedback gain matrix  $K_c$ , respectively.

*Proof:* using the Laplace transform for Equation (4), we deduce that

$$[j\omega I - A + Bk_1 k_2 K_c + Bk_1 k_2 K_c e^{-\tau j\omega}] u = 0 \quad (5)$$

where  $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ . We expand Equation (5), and obtain

$$j\omega u_1 = u_2 \quad (6)$$

Due to  $u^* u = 1$ , we also have

$$u_1^* u_1 = \frac{1}{w^2 + 1} \quad (7)$$

$$u_2^* u_2 = \frac{w^2}{w^2 + 1} \quad (8)$$

Then Equation (5) is multiplied by  $u^*$ , and the equation is still satisfied

$$u^* [j\omega I - A + Bk_1 k_2 K_c + Bk_1 k_2 K_c e^{-\tau j\omega}] u = 0$$

where  $u^* = [u_1^* \ u_2^*]$ . It can be expanded as

$$j\omega u_1^* u_1 + (1 + e^{-\tau j\omega}) k_{c1} k_1 k_2 u_2^* u_1 - u_1^* u_2 + j\omega u_2^* u_2 + u_2^* u_2 k_{c2} k_1 k_2 (1 + e^{-\tau j\omega}) = 0 \quad (9)$$

Applying Equations (6)-(8) to Equation (9), then

$$(1 + e^{-\tau j\omega}) k_{c1} k_1 k_2 - w^2 + k_{c2} k_1 k_2 j\omega (1 + e^{-\tau j\omega}) = 0 \quad (10)$$



To satisfy Equation (10), the real and imaginary part should equal to 0, which is

$$k_{c1} k_1 k_2 + k_{c1} k_1 k_2 \cos w\tau - w^2 + k_{c2} k_1 k_2 w \sin w\tau = 0 \quad (11)$$

$$-k_{c1} k_1 k_2 \sin w\tau + k_{c2} k_1 k_2 w + k_{c2} k_1 k_2 w \cos w\tau = 0 \quad (12)$$

From Equation (12),  $\tau$  depends on  $w$  as

$$\tau = \frac{2}{w} \arctan \frac{k_{c2} w}{k_{c1}} \quad (13)$$

Similarly, from Equation (11), we have

$$(2k_{c1} + \frac{2k_{c2}^2 w^2}{k_{c1}}) k_1 k_2 \cos^2 \frac{w\tau}{2} = w^2$$

Since  $|\cos w\tau/2| \leq 1$ , we obtain  $w^2 - 2k_{c1} k_1 k_2 - 2k_{c2}^2 w^2 k_1 k_2 / k_{c1} \leq 0$ . Thus,  $k_{c1}$  and  $k_{c2}$  satisfy

$$\frac{2k_{c2}^2 k_1 k_2}{k_{c1}} < 1 \quad (14)$$

We obtain the upper bound of  $w$  as

$$w \leq \frac{2k_{c1} k_1 k_2}{1 - 2k_{c1} k_1 k_2 / k_{c1}} \quad (15)$$

For Equation (13), we find that  $d\tau/dw < 0$  in the case of  $w > 0$ . In consideration of Equation (15), we have  $\tau \geq 2\arctan(k_{c2} w' / k_{c1}) / w'$  where  $w' = (2k_{c1} k_1 k_2 / (1 - 2k_{c2}^2 k_1 k_2 / k_{c1}))^{-1/2}$ .

According to the above analysis, we can draw a conclusion. When Equation (14) is satisfied, if

$$0 \leq \tau < \frac{2}{w'} \arctan \frac{k_{c2} w'}{k_{c1}} \quad (16)$$

There isn't any imaginary root and the real part of the root is less than 0 in the system. Therefore, the closed-loop micro-grating accelerometer system is stable. By Theorem 1, we can see that the micro-grating accelerometer with time-delay satisfies the stability condition. This completes the proof.

Theorem 1 provides a sufficient condition for the design of control matrix  $K_c$  of the closed-loop micro-grating accelerometer system with time-delay. Particularly, we obtain the relation between closed-loop parameters and time-delay on the condition of second-order control system stability in Equation (16). Although there is time-delay in signal processing of micro-grating accelerometer, the eigenvalues of second-order controller can guarantee negative real parts to further improve the dynamic performance of micro-grating accelerometer. And, the time-delay of closed-loop detection system is minimized by proposed timing sequence control method and determined by response capability of sensing element. Furthermore, Equation (16) obtains the allowable values of closed-loop parameters of second-order controller to guarantee the system stability of micro-grating accelerometer with time-delay. Thus, Theorem 1 provides the design principle of second-order control parameters to ensure fast tracking ability of the closed-loop micro-grating accelerometer system.

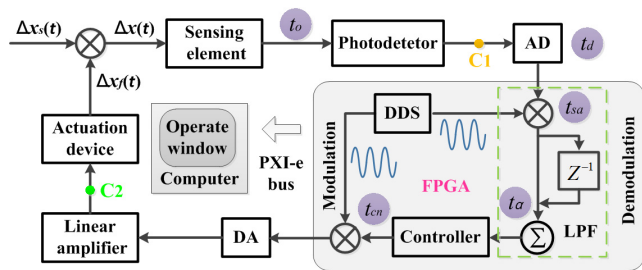


FIGURE 4. The designed hardware scheme of closed-loop micro-grating accelerometer.

In this work, we propose the closed-loop parameter design scheme with timing sequence control method to obtain the fast tracking ability of micro-grating accelerometer. The timing sequence control method can minimize time-delay in the closed-loop detection system, which only dependent on the response capability of sensing element of micro-grating accelerometer. Particularly, with our proposed closed-loop parameter design principle the micro-grating accelerometer can obtain high dynamic performance. And, we can see that improving the optical structure of micro-grating accelerometer can optimize the response capability of sensing element. Therefore, our work can be extended with higher frequency and magnitude levels on condition that the optical structure of micro-grating accelerometer is optimized.

#### IV. SYSTEM IMPLEMENTATION DESCRIPTION

The output light intensity signal of sensing element containing the closed-loop error information is received by PD. However, it is still a weak signal accompanied by strong noises such as light source noise and mechanical thermal noise. Therefore, it is difficult to extract the closed-loop error signal with high SNR. To solve this problem, we design the hardware circuit including modulation, the LPF filter and demodulation according to the design of closed-loop micro-grating accelerometer in Section III. The block diagram of the designed hardware is illustrated in Fig. 4. And, the generation of modulating signal, the processing of demodulation and controller are implemented in the digital signal processing unit by NI5781R FPGA board card. In the designed hardware, the modulated closed-loop error signal is collected by AD convertor, then is demodulated by multiplying the same frequency sinusoidal signal, finally the closed-loop error signal is recovered by the LPF. At the output port of FPGA, the feedback signal and sinusoidal modulation signal are added together and then sent to DA convertor. Thus, the designed hardware can realize modulation, the LPF filter, demodulation and feedback control of the closed-loop error signal for micro-grating accelerometer.

The FPGA is connected to host computer by PXI-e bus. Therefore, designed platform has the advantage of monitoring the nano-second state variables and generating frequency-adjustable signal to test the dynamic performance of accelerometer system. In digital signal processing unit, a 100MHz oscillator is utilized to generate system clock and

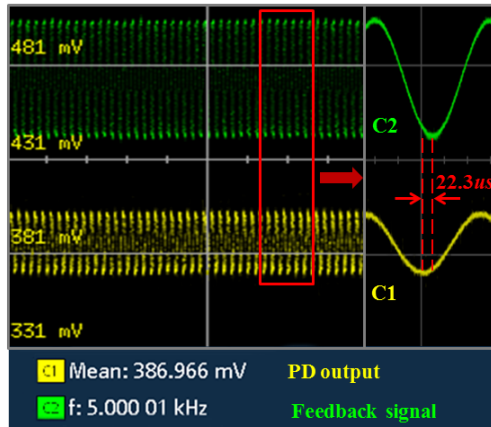


FIGURE 5. The key signals of closed-loop micro-grating accelerometer.

reference clock. Additionally, the modulating and demodulating sinusoidal signal are generated by direct digital synthesizers (DDS) in FPGA.

The implementation of timing sequence control method for closed-loop micro-grating accelerometer is shown in Fig. 5. And, the time-delay between PD output and feedback signal is 22.3 us as shown in Fig. 5, which contains the time delay of AD conversion, filter of demodulation, controller operation and actuation device of feedback signal. It achieves the minimum time-delay of signal detection by the proposed timing sequence control method. Considering the response time of sensing element which is also the time delay of optical path, we can obtain the total time delay of closed-loop micro-grating accelerometer system. According to the modulating frequency and dynamic response characteristic of sensing element, the optical path time-delay  $t_o$  is 66.7 us. Therefore, the total time delay of closed-loop micro-grating accelerometer system is less than 100us. Thus, we can see that the timing sequence control method can ensure that the whole close-loop control process is only limited by response capability of optical sensing element.

The timing sequence control method guarantees the minimum time delay in closed-loop detection system of micro-grating accelerometer, which is in accordance with the response capability of sensing element. Next, our ambition is figuring out the closed-loop parameter values in second-order controller to optimize the dynamic performance of micro-grating accelerometer by Theorem 1. Table 1 shows the parameter values of micro-grating accelerometer based on the closed-loop detection scheme of Fig. 2. Then, we obtain the gains of forward channel and feedback channel  $k_1 = 1.65 \times 10^7$  and  $k_2 = 8.97 \times 10^{-9}$  in our closed-loop micro-grating accelerometer system. Based on the minimum time-delay  $\tau = 100$  us, the discretized closed-loop feedback matrix  $Kc = [2.38 \ 1.19]$  is obtained by the closed-loop parameter design principle in Equation (16), which guarantees the fast tracking ability of closed-loop micro-grating accelerometer system. Therefore, the software design of closed-loop micro-grating accelerometer can be implemented according

TABLE 1. The parameter values of micro-grating accelerometer system.

Parameter	Value	Parameter	Value
$I_{in}$	0.93 mW	T	200us
$\lambda$	850nm	$K_u$	$1.47 \times 10^5 N/V$
$\gamma$	0.92A/W	$K_f$	20
R	100k $\Omega$	$N_{sat}$	14-bit

to the theoretical analysis of loop gains and timing sequence control method.

Different from traditional method, the timing sequence control method shortens the time-delay of LPF in demodulation process, which ensures enough time left for the controller to operate, as is illustrated in Fig. 3. The LPF is designed to make adjacent half period of positive and negative data add together, and the closed-loop period of controller is equal to half modulation period. In this case, the feedback signal generates during current control period. Consequently, the system time-delay is minimized, which is equal to the response capability of sensing element  $\tau$ . In addition,  $\tau$  represents the minimum value of time-delay in signal processing theoretically and determines the modulation period. Thus, the feedback signal is transmitted to DA convertor before the next half modulation period. Then, we can obtain the dynamic equation of closed-loop micro-grating accelerometer, as shown in Equation (4).

However, without timing sequence control, the LPF of the demodulation process in traditional method can't finish until next modulation period. Therefore, the feedback signal is generated in next half modulation period. And, Equation (4) is rewritten as

$$\dot{x}(t) = Ax(t) - Bk_1k_2 \sin[K_c x(t - \tau)] - Bk_1k_2 \sin[K_c x(t - 2\tau)] \quad (17)$$

We can see that the time-delay of signal processing with traditional method is too large, so that it is difficult to design high-order controller. Then, the commonly one-order integrator is used to guarantee the system stability in traditional signal processing method, which restricts the dynamic performance of micro-grating accelerometer with time-delay. However, compared with one-order system, the higher order controller can theoretically obtain faster tracking ability for closed-loop detection system with time-delay [23], [24].

In our work, we propose a timing sequence control method to confine the LPF time-delay in the demodulation process, which achieves the minimum time-delay of signal processing. Considering this time-delay, we further deduce the design principle of closed-loop parameter in second-order controller to realize the fast tracking ability of micro-grating accelerometer.

## V. EXPERIMENTS AND RESULTS

In this section, some experiments have been conducted to verify the correctness and effectiveness of the designed signal processing method to optimize the micro-grating accelerometer's high dynamic performance. The experimental prototype

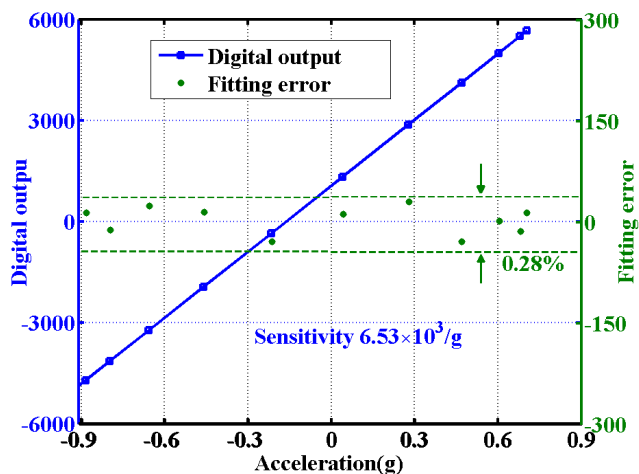


FIGURE 6. The sensitivity measurement of closed-loop micro-grating accelerometer system with proposed signal processing scheme.

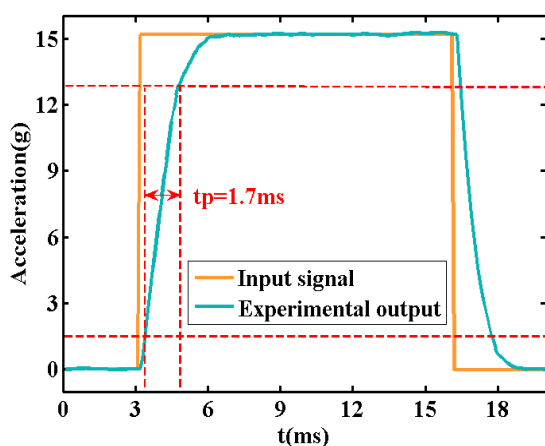


FIGURE 7. The step response of micro-grating accelerometer with traditional signal processing scheme.

is built up based on optical sensing element of Fig. 1 and signal processing scheme of Fig. 2.

Firstly, we carry out the experiment to obtain the sensitivity of closed-loop micro-grating accelerometer system. By turntable, the accelerometer is revolved under different angles to measure its sensitivity and relative measurement error. Illustrated in Fig. 6, the feedback voltage is fitted as a linear function of acceleration from  $-0.9g$  to  $0.9g$ . And we obtain the sensitivity of  $6.53 \times 10^3/g$  and the nonlinearity within 0.28%. Compared to the previous measurement nonlinearity results of 0.35% with the same optical components [14], our proposed closed-loop parameter design method with timing sequence control obtains better nonlinearity when optimizing the dynamic performance of micro-grating accelerometer.

Furthermore, we perform the contrast experiments about step response between our proposed scheme and traditional signal processing scheme to demonstrate the effectiveness of closed-loop parameter design scheme with timing sequence control method. A step signal with amplitude of 15g is imposed on the sensing element. Fig. 7 illustrates that the

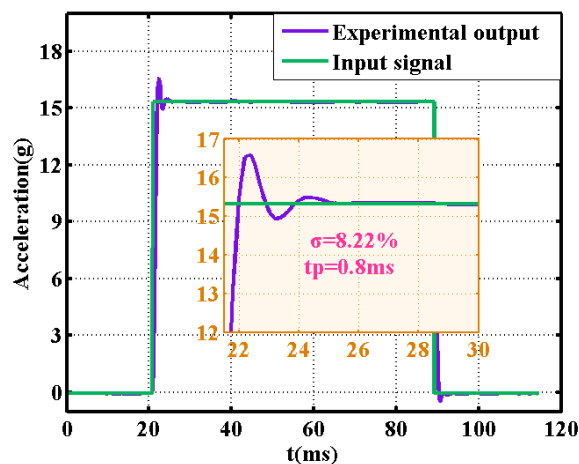


FIGURE 8. The step response of micro-grating accelerometer with proposed signal processing scheme.

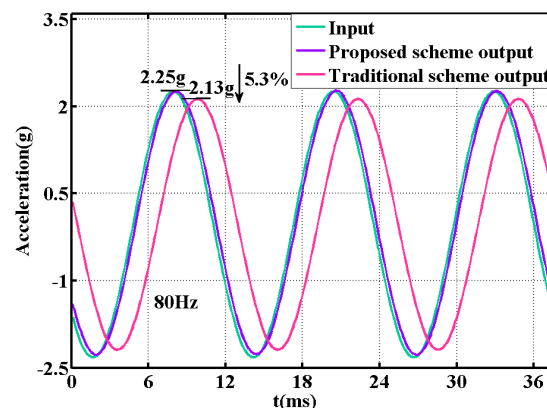
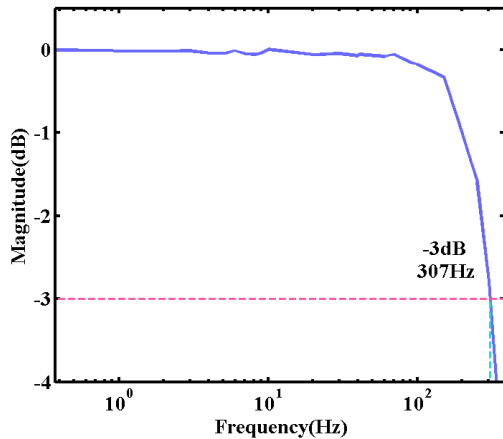


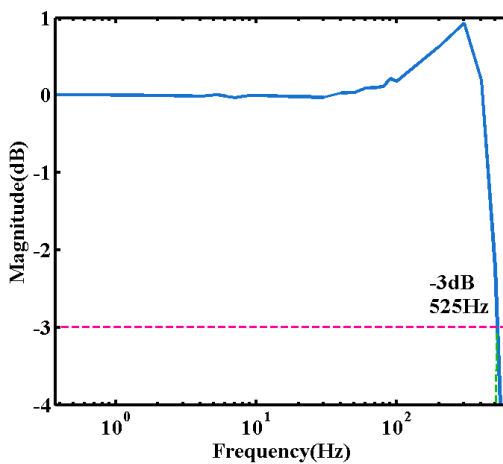
FIGURE 9. The comparison result of sine response experiment between proposed signal processing scheme and traditional scheme.

closed-loop micro-grating accelerometer system with traditional scheme is overdamped and has a rise time of 1.7ms. The step response experiment of traditional method also verifies one-order integrator is difficult to improve the dynamic performance of micro-grating accelerometer system due to the large time-delay in the signal processing. In contrast, the result shown in Fig. 8 reveals that the micro-grating accelerometer system with proposed control algorithm and timing sequence control method has an overshoot of 8.22% and its rise time is 0.8ms. The step experimental results demonstrate that our proposed timing sequence control method can minimize the time-delay of signal processing, and the design principle of second-order control parameters described in Theorem 1 can guarantee the fast tracking ability of micro-grating accelerometer in the practical application.

Finally, we conduct the comparison experiment of sine response of micro-grating accelerometer with our proposed signal processing scheme and traditional scheme, separately. In the experiment, we simultaneously record the feedback signal and input sine signal of micro-grating accelerometer while applying sine signal on the sensing element.



**FIGURE 10.** The frequency response result of the micro-grating accelerometer with traditional signal processing scheme.



**FIGURE 11.** The frequency response result of the micro-grating accelerometer with proposed signal processing scheme.

Fig. 9 reveals that the output of our proposed method is almost in agreement with 80Hz sine signal input. However, the output amplitude of traditional method is decreased by 5.2% compared with 80Hz sine signal input. Furthermore, we measure the closed-loop system bandwidth of micro-grating accelerometer with traditional scheme and our proposed signal processing scheme. As is shown in Fig. 10,  $-3$ dB bandwidth of the micro-grating accelerometer with traditional scheme is 307Hz. However, with our proposed scheme,  $-3$ dB bandwidth of the micro-grating accelerometer system is improved and reaches 525Hz, which is illustrated in Fig. 11.

Compared with traditional method, the rise time of micro-grating accelerometer is shortened from 1.7ms to 0.8ms, and the bandwidth of micro-grating accelerometer is improved from 307Hz to 525Hz, which verify the effectiveness of our proposed closed-loop parameter design principle and timing sequence control method. The contrast experiment results show that our proposed method has the superiority in improving fast tracking ability and bandwidth of micro-grating accelerometer in the practical application. To the best

of our knowledge, it has the best dynamic performance of closed-loop micro-grating accelerometer with this kind of high resolution sensing structure, which can meet the practical engineering demands. Compared with related works [18], [19], our proposed closed-loop signal processing method greatly enhances the system's dynamic performance, which makes the micro-grating accelerometer stand out in novel optical-sensing accelerometer for the engineering application in the future.

## VI. CONCLUSION

In engineering practice, the response capability of sensing element limits the modulation period, which causes time-delay of signal processing in micro-grating accelerometer. Considering optical sensing principle and time-delay of signal processing, we design a novel closed-loop parameter design scheme with timing sequence control method for system stability and further excellent dynamic performance of micro-grating accelerometer. After achieving the minimum time-delay of signal processing by timing sequence control method, we build a closed-loop dynamic equation to analyze the design principle of closed-loop parameters considering unavoidable time-delay. Then, the design of controller parameters can be determined through proposed parameter design principle, which guarantees fast tracking ability of closed-loop micro-grating accelerometer system. The experimental results demonstrate the effectiveness of our closed-loop parameter design scheme and timing sequence control method, which show that both the nonlinearity and dynamic performance of micro-grating accelerometer have been greatly improved. Our research result is of great significance in promoting the practical application of micro-grating accelerometer.

## REFERENCES

- [1] N. Barbour and G. Schmidt, "Inertial sensor technology trends," *IEEE Sensors J.*, vol. 1, no. 4, pp. 332–339, Dec. 2001.
- [2] N. M. Barbour, "Inertial navigation sensors," Charles Stark Draper Lab., MIT, Cambridge, MA, USA, Tech. Rep. RTO-EN-SET-116, 2010.
- [3] X. Liu, S. Wang, X. Guo, W. Yang, and G. Xu, "A method for gravitational apparent acceleration identification and accelerometer bias estimation," *IEEE Access*, vol. 7, pp. 38115–38122, Mar. 2019.
- [4] B. E. N. Keeler, D. W. Carr, J. P. Sullivan, T. A. Friedmann, and J. R. Wendt, "Experimental demonstration of a laterally deformable optical nanoelectromechanical system grating transducer," *Opt. Lett.*, vol. 29, no. 11, pp. 1182–1184, Nov. 2004.
- [5] A. G. Krause, M. Winger, T. D. Blasius, Q. Lin, and O. Painter, "A high-resolution microchip optomechanical accelerometer," *Nature Photon.*, vol. 6, pp. 768–772, Oct. 2012.
- [6] A. Sheikhalah, K. Abedi, and K. Jafari, "A proposal for an optical MEMS accelerometer relied on wavelength modulation with one dimensional photonic crystal," *J. Lightw. Technol.*, vol. 34, no. 22, pp. 5244–5249, Nov. 15, 2016.
- [7] R. H. Olsson, B. E. N. Keeler, D. A. Czaplowski, and D. W. Carr, "Circuit techniques for reducing low frequency noise in optical MEMS position and inertial sensors," in *Proc. IEEE Int. Symp. Circuits Syst.*, New Orleans, LA, USA, May 2007, pp. 2391–2394.
- [8] C. T. Garcia, G. Onaran, B. Avenson, B. A. Yocom, and N. A. Hall, "Microseismometers via advanced meso-scale fabrication," Silicon Audio, LLC, Tech. Rep. DE-FG02-08ER85106, Sep. 2010, pp. 280–288.
- [9] S. S. Zhao, J. Zhang, C. Hou, J. Bai, and G. Yang, "Optical accelerometer based on grating interferometer with phase modulation technique," *Appl. Opt.*, vol. 51, no. 29, pp. 7005–7010, Oct. 2012.



[10] X. Wang, L. Feng, B. Yao, and X. Ren, "Sensitivity improvement of micro-grating accelerometer based on differential detection method," *Appl. Opt.*, vol. 52, no. 18, pp. 4091–4096, Jun. 2013.

[11] Q. Lu, C. Wang, J. Bai, K. Wang, S. Lou, X. Jiao, D. Han, G. Yang, D. Liu, and Y. Yang, "Minimizing cross-axis sensitivity in grating-based optomechanical accelerometers," *Opt. Express*, vol. 24, no. 8, pp. 9094–9111, Apr. 2016.

[12] Y. Wang, L. Feng, and X. Wang, "Design and experimental demonstration of electromagnetic actuation utilized on micrograting accelerometers," *Appl. Opt.*, vol. 55, no. 19, pp. 5063–5068, Jul. 2016.

[13] T. Zhang, H. Liu, L. Feng, X. Wang, and Y. Zhang, "Noise suppression of a micro-grating accelerometer based on the dual modulation method," *Appl. Opt.*, vol. 56, no. 36, pp. 10003–10008, Dec. 2017.

[14] H. Li, S. Li, K. Deng, S. Gao, and L. Feng, "Analysis and design of closed-loop detection technique for micro-grating accelerometer," *J. Lightw. Technol.*, vol. 36, no. 24, pp. 5738–5745, Dec. 15, 2018.

[15] S. Sung, C. J. Kim, J. Park, Y. J. Lee, and J. G. Park, "Oscillation amplitude-controlled resonant accelerometer design using a reference tracking automatic gain control," *Int. J. Control, Autom. Syst.*, vol. 7, no. 2, pp. 203–210, Apr. 2009.

[16] W. Zhou, H. Yu, J. Zeng, B. Peng, Z. Zeng, X. He, and Y. Liu, "Improving the dynamic performance of capacitive micro-accelerometer through electrical damping," *Microsyst. Technol.*, vol. 22, no. 12, pp. 2961–2969, Dec. 2016.

[17] N. A. Hall, M. Okandan, R. Littrell, D. K. Serkland, G. A. Keeler, K. Peterson, B. Bicen, C. T. Garcia, and F. L. Degertekin, "Micromachined accelerometers with optical interferometric read-out and integrated electrostatic actuation," *J. Microelectromech. Syst.*, vol. 17, no. 1, pp. 37–44, Feb. 2008.

[18] Q. P. Liu, X. G. Qiao, J. L. Zhao, Z. A. Jia, H. Gao, and M. Shao, "Novel fiber Bragg grating accelerometer based on diaphragm," *IEEE Sensors J.*, vol. 12, no. 10, pp. 3000–3004, Oct. 2012.

[19] Z. Zhao, Z. Yu, K. Chen, and Q. Yu, "A fiber-optic Fabry–Perot accelerometer based on high-speed white light interferometry demodulation," *J. Lightw. Technol.*, vol. 36, no. 9, pp. 1562–1567, May 1, 2018.

[20] E. B. Cooper, E. R. Post, S. Griffith, J. Levitan, S. R. Manalis, M. A. Schmidt, and C. F. Quate, "High-resolution micromachined interferometric accelerometer," *Appl. Phys. Lett.*, vol. 76, pp. 3316–3318, Apr. 2000.

[21] L. M. Smith and C. C. Dobson, "Absolute displacement measurements using modulation of the spectrum of white light in a Michelson interferometer," *Appl. Opt.*, vol. 28, no. 16, pp. 3339–3342, Aug. 1989.

[22] N. A. Hall and F. L. Degertekin, "Integrated optical interferometric detection method for micromachined capacitive acoustic transducers," *Appl. Phys. Lett.*, vol. 80, no. 20, pp. 3859–3861, May 2002.

[23] A. Nedić and A. Ozdaglar, "Convergence rate for consensus with delays," *J. Global Optim.*, vol. 47, no. 3, pp. 437–456, Jul. 2010.

[24] P. Lin and Y. Jia, "Consensus of a class of second-order multi-agent systems with time-delay and jointly-connected topologies," *IEEE Trans. Autom. Control*, vol. 55, no. 3, pp. 778–784, Mar. 2010.



**HUI LI** was born in Liaocheng, Shandong, China, in 1981. She received the Ph.D. degree from the School of Instrumentation and Optoelectronic Engineering, Beihang University, China, in 2009, where she is currently an Associate Professor. She has authored or coauthored more than 40 articles in refereed journals and conference proceedings. Her current research interests include optical sensors, integrated optics, signal processing, time-delay systems, and robust control.



**KEKE DENG** was born in Luoyang, Henan, China, in 1995. She received the B.S. degree in measurement and control technology and instrumentation from the Beijing University of Chemical Technology. She is currently pursuing the M.S. degree of optical engineering with the School of Instrumentation and Optoelectronic Engineering, Beihang University. Her research interests include optical sensors and closed-loop control algorithms.



**SHAN GAO** was born in Baoding, Hebei, China, in 1992. He received the B.S. degree from the School of Instrumentation and Optoelectronic Engineering, Beihang University, where he is currently pursuing the Ph.D. degree. His research interest includes optical accelerometers.



**LISHUANG FENG** was born in Hebei, China, in 1968. She received the Ph.D. degree from the Saint Petersburg Institute of Fine Mechanics and Optics, Russia, in 1996. From 1997 to 2001, she was an Associate Professor with the School of Photoelectronic Information and Communication Engineering, Beijing Information Science and Technology University. She joined the School of Instrumentation Science and Opto-Electronics Engineering, Beihang University, in 2001, where she is currently a Professor. She has authored or coauthored more than 100 articles in refereed journals and conference proceedings. Her research interests include integrated optics and MOEMS systems, advanced optical sensors, and optoelectronics devices.

...