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Comparative Analysis of Two- and Three-Coil WPT Systems Based on Transmission Efficiency

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ABSTRACT It is obvious that the conventional two-coil wireless power transfer (WPT) system has a short power transmission distance, which is determined by system parameters such as the loaded Q of the resonators and the coupling coefficient. There have been many attempts to improve the power transfer efficiency (PTE) or transmission distance. A typical approach is to use multiple coils like a three- or four-coil WPT system. In this paper, we propose a new method to obtain the PTE of a multi-coil WPT system based on the scattering parameter and impedance parameter. Also, we compare the two- and three-coil WPT systems in terms of their transmission efficiency rather than their system energy efficiency. For high transmission efficiency, we determined that the three-coil WPT system should have a symmetric structure and the coupling coefficient between the transmitting and receiving coils has to be as zero as possible. Additionally, we found that $1/\sqrt{3}$ of the conventional critical coupling is the boundary coupling coefficient at which the two- and three-coil WPT systems have the same transmission efficiency. We successfully verified these theoretical analyses by implementing two- and three-coil WPT systems at the operating frequency of 6.78 MHz and measuring their transmission efficiency and spectra.

INDEX TERMS Power transfer efficiency, three-coil WPT system, transmission efficiency, two-coil WPT system, wireless charging, wireless power transfer.

I. INTRODUCTION

The conventional wireless power transfer (WPT) system is commonly modeled as two magnetically coupled resonators, in which each resonator consists of an inductor, a capacitor, and a parasitic resistor, and is connected to a source or load in series. Owing to the use of two coils, it is often called a two-coil WPT system that has maximum output power and power transfer efficiency (PTE)–it is referred to as transmission efficiency or system energy efficiency depending on the definition of the input power [1]– in the critical coupled state. The transmission distance corresponding to the critical coupling is considerably shorter than might be expected. Additionally, the two-coil WPT system is very sensitive to alignment between the coils. Because of these problems, multiple coils are used in a WPT system instead of two coils.

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In 2007, an MIT group introduced a four-coil WPT system, which transferred the power to a load 2.4-m away [2]. It was generally believed that use of multiple coils could extend the power transmission distance. However, soon many researchers revealed that the source and load coils of the four-coil WPT system play the role of a matching network and the four-coil WPT system acts as a two-coil WPT system [3]–[5]. The main reason the transmission distance is increased in the four-coil WPT system is that resonators with high Q-factor are implemented using large coil dimensions and a high frequency band.

Afterward, there still have been attempts to increase the transmission distance using multiple coils. Those can be categorized into multiple transmitting coils structure and relay coils structure. The former structure adopts multiple transmitting coils instead a single transmitting coil to achieve a high PTE over long distances [6]–[9]. Because it is usually hard to fabricate small coils with high Q factor, multiple coils share the burden of power transmission. A more important characteristic of the multiple transmitting coils structure is that power can be transmitted highly efficiently regardless of the alignment state by adjusting the magnitude and phase of the power supplied to the transmitting coils. This is sometimes called the magnetic beamforming. However, there is a disadvantage that the source part becomes very complicated. The latter structure is a method of arranging additional coils on the power transmission path for long distance using the concept of a relay in a communication system [5], [10]–[18]. These additional coils are called relay or intermediate coils. However, there is also a disadvantage that spaces for the additional coils are required between transmitting and receiving coils. Among WPT systems with multiple coils, three-coil WPT systems have been mainly studied. Especially, there have been many attempt to apply a three-coil WPT system to implant devices, since high transmission efficiency can be achieved by changing system parameters of the intermediate coil as a matching network of a transmitting or receiving coil [19]–[20]. On the other hand, Kiani *et al.* [5] derived transmission and system energy efficiency of a three-coil WPT system in closed-form and demonstrated that the three-coil WPT system can concurrently achieve high transmission and system energy efficiency through an optimization process. Zhang *et al.* [10] revealed that there is an optimal position of a relay coil to maximize the output power. Kim *et al.* [11] analyzed a symmetric three-coil WPT system using the temporal coupled mode theory and showed that the optimum position of a relay coil is the center between the transmitting and receiving coils. Recently, from the perspective of system energy efficiency, comparative analysis with two- and three-coil WPT systems has been conducted in several studies [12]–[15]. Their results showed that the three-coil WPT system can achieve higher system energy efficiency and less sensitivity to load change in comparison to its two-coil counterpart. In [13]–[15], slightly different conditions under which the three-coil WPT system has higher system energy efficiency than the two-coil system were also presented.

In low-frequency (LF) band such as 110–205 kHz, since the source (or generator) usually has the source impedance of a negligible value, the system energy efficiency is commonly used to evaluate the performance of a WPT system. By contrast, the source of high-frequency (HF) band such as 6.78 MHz generally has the source impedance of 50 ohms. If the system energy efficiency is used as the PTE in HF band, the source impedance acts as a loss factor [1]. This is why the transmission efficiency rather than the system energy efficiency is preferred for the PTE of a WPT system in HF band. Therefore, in HF band, it is highly required to assess the two- and three-coil WPT system from the perspective of transmission efficiency instead of system energy efficiency.

On the other hand, the transmission efficiency, which is proportional to the output power, is simply expressed as the absolute value squared of the transmission coefficient, $|S_{21}|^2$, under the condition that the source and load impedances are perfectly matched to the input and output impedance of the two-port network, respectively [20], [21]. Therefore, the transmission efficiency using the transmission coefficient can usually apply to a static WPT system, which has constant system parameters such as constant source and load impedances and constant coupling coefficient. There also have been various attempts to express the transmission efficiency of multi-coil WPT system such as three- and four-coil WPT systems as well as two-coil WPT system in closed form by using the definition of the transducer power gains [4], [22]–[24]. The transmission efficiency of the two-coil WPT system is simply expressed in closed form, but to derive that of WPT system with three or more coils in closed form is very cumbersome. Therefore, in order to express the PTE of multi-coil WPT systems as simply as possible, only the coupling between adjacent coils has been considered. In other words, most previous researches assume that couplings among coils except adjacent coils would be zero [21]–[24]. In this case, it is only suitable for the relay type and it is difficult to identify the precise power transfer mechanism. Therefore, in HF band, it is highly required to present the general transmission efficiency of a multi-coil WPT system in closed-form.

Compared with the previous literature, the main contribution of this paper is the expression of the transmission efficiency of a multi-coil WPT system, in which all couplings among coils are considered, as well as a useful guideline on which of two- and three-coil WPT systems would be more suitable to use in a specific transmission environment. First, we consider a multi-coil WPT system as series RLC resonators and express the relationship among the resonators as an impedance matrix. Without any assumption such as zero coupling between faraway coils, we derive the transmission efficiency of a multi-coil WPT system in closed-form from the definition of the generalized scattering parameters. Second, we find the conditions of coils for the three-coil WPT system to have higher transmission efficiency. Next, through a comparison of the transmission efficiencies of the two-coil and three-coil WPT systems, we derive the boundary coupling coefficient determining of which two- and three-coil WPT systems have much higher transmission efficiency. Then measurements are presented to verify these theoretical analyses.

The remainder of the paper is organized as follow. Section II derives the general expression of the transmission efficiency of multi-coil WPT systems based on definition of the generalized scattering parameter and discloses the transmission efficiency characteristics of the three-coil WPT systems by comparing with those of the two-coil WPT systems. Then, Section III carries out measurements and simulations to verify the characteristics of the three-coil WPT system. Finally, some conclusions are drawn in Section IV.

II. THEORETICAL ANALYSIS

In this section, we derive the transmission efficiency of a multi-coil WPT system using elements of the inverse of an

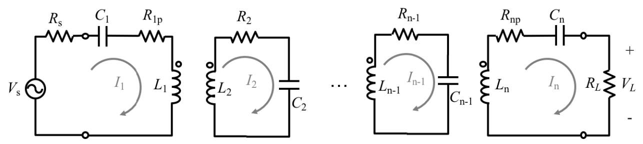

FIGURE 1. Equivalent circuit model of a multi-coil WPT system using multiple series RLC resonators that are mutually coupled each other.

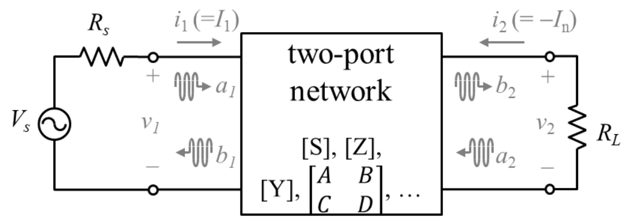

FIGURE 2. Equivalent circuit model of a multi-coil WPT system using a general two-port network parameter.

impedance matrix, and disclose the characteristics of the multi-coil WPT system by comparing transmission efficiencies of the WPT systems. From this, therefore, the term of the PTE refers to the transmission efficiency.

A. PTE OF MULTI-COIL WPT SYSTEM

Figure 1 shows the simplified circuit model of a multi-coil WPT system with n coils. Each resonator includes a capacitor C, a coil L, and a parasitic resistor R_p. All coils are coupled with each other, and the coupling between the coils is modeled by introducing a mutual inductance M_{ij}, which is defined as the ratio of voltage induced in inductor j to the rate of change of current in inductor i, and reciprocity implies that $M_{ij} = M_{ji}$. R_s and R_L are the source and load resistances, respectively. V_s is the transmit voltage of the source. Applying Kirchhoff's current laws, we obtain (1), as shown at the bottom of this page, or, in matrix form,

$$\mathbf{ZI} = \mathbf{V}, \tag{2}$$

where $\mathbf{Z}$ is the n by n impedance matrix, and R_i is the total resistance of the ith resonator. For example, $R_1 = R_s + R_{1p}$, $R_2 = R_{2p}$, and $R_n = R_{np} + R_L$.

The n-coil WPT system shown in Fig. 1 can be equivalently modeled as a combination of a two-port network, a generator, and a load, as shown in Fig. 2. The PTE of the n-coil WPT system is given by

$$\eta = |S_{21}|^2 = \left| \frac{b_2}{a_1} \bigg|_{a_2=0} \right|^2, \tag{3}$$

where S_{21} is the transmission coefficient from port 1 to port 2, and it is defined as the ratio of the outgoing power wave b_2 to the incoming power wave a_1, when there is no incoming power wave at port 2, $a_2 = 0$. The incoming power wave a_1 and the outgoing power wave b_2 can be expressed as [25]

$$a_1 = \frac{v_1 + R_s i_1}{2\sqrt{R_s}}, \quad b_2 = \frac{v_2 - R_L i_2}{2\sqrt{R_L}}. \tag{4}$$

Substituting the total voltages of port 1 and 2, $v_1 = V_s - R_s i_1$ and $v_2 = -R_L i_2$ into (4) gives

$$a_1 = \frac{V_s}{2\sqrt{R_s}}, \quad b_2 = -i_2\sqrt{R_L}. \tag{5}$$

Therefore, the PTE of (3) becomes

$$\eta = |S_{21}|^2 = 4R_sR_L \left| \frac{I_n}{V_s} \right|^2, \tag{6}$$

where I_n/V_s can be found from the nth row and the first column element of the inverse of the impedance matrix, $\mathbf{Z}^{-1}$. This results in the PTE of the n-coil WPT system in closed-form:

$$\eta = 4R_sR_L \left| \left[\mathbf{Z}^{-1} \right]_{n1} \right|^2 = 4R_sR_L \left| \frac{\tilde{\mathbf{Z}}_{n1}}{\Delta} \right|^2, \tag{7}$$

where $\tilde{\mathbf{Z}}$ is the adjugate matrix of $\mathbf{Z}$, and Δ is the determinant of $\mathbf{Z}$.

This method does not require the complete inverse of the impedance matrix but needs only the nth row and the first column element of the inverse. Therefore, although a multi-coil WPT system has a high-order impedance matrix because of the number of the coils, the determinant and nth row and the first column element can be easily obtained by calculating cofactors of the impedance matrix from properties of a square matrix which is called Cramer's rule [26]. Consequently, the PTE can be obtained without complete inversion of the matrix.

$$\begin{bmatrix} R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1} & -j\omega M_{12} & \cdots & -j\omega M_{1n} \\ -j\omega M_{21} & R_2 + j\omega L_2 + \dfrac{1}{j\omega C_2} & \cdots & -j\omega M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -j\omega M_{n1} & -j\omega M_{n2} & \cdots & R_n + j\omega L_n + \dfrac{1}{j\omega C_n} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{1}$$

B. PTE OF TWO-COIL AND THREE-COIL WPT SYSTEMS

For the two-coil WPT system, the impedance matrix is given by

$$\mathbf{Z} = \begin{bmatrix} R_1 + j\omega L_1 + \dfrac{1}{j\omega C_1} & -j\omega M_{12} \\ -j\omega M_{21} & R_2 + j\omega L_2 + \dfrac{1}{j\omega C_2} \end{bmatrix}. \quad (8)$$

If each resonator resonates at the operating frequency, the impedance matrix can be simply rewritten by

$$\mathbf{Z} = \begin{bmatrix} R_1 & -j\omega M_{12} \\ -j\omega M_{21} & R_2 \end{bmatrix}. \quad (9)$$

By inversing (9), the PTE of the two-coil WPT system is expressed as

$$\eta_{2\text{-coil}} = |S_{21}|^2 = 4R_sR_L \left| \frac{j\omega M_{12}}{R_1R_2 + \omega^2 M_{12}^2} \right|^2. \quad (10)$$

The relationship between the mutual inductance and coupling coefficient is

$$M_{12} = k_{12}\sqrt{L_1L_2}. \quad (11)$$

The loaded Q of the primary and secondary resonators having the series RLC structure are, respectively,

$$Q_1 = \frac{\omega_0 L}{R_{1p} + R_s} = \frac{\omega_0 L}{R_1}, \; Q_2 = \frac{\omega_0 L_2}{R_{2p} + R_L} = \frac{\omega_0 L_2}{R_2}. \quad (12)$$

Substituting (11) and (12) into (10), the PTE can be obtained by

$$\eta_{2\text{-coil}} = \frac{4R_sR_L}{R_1R_2} \frac{k_{12}^2 Q_1 Q_2}{\left(1 + k_{12}^2 Q_1 Q_2\right)^2}. \quad (13)$$

This expression is identical to the PTE of [22], which is derived from the transducer power gain of the two-coil WPT system. Let $A = k_{12}^2 Q_1 Q_2$, then the PTE of the two-coil WPT system can be written as

$$\eta_{2\text{-coil}} = \frac{4R_sR_L}{R_1R_2} \frac{A}{(1 + A)^2}. \quad (14)$$

Through differentiating (14) by A, it can be seen that the PTE is maximized when $A = 1$, that is, at the critical coupling, $k_{critical} = 1/\sqrt{Q_1Q_2}$. In addition, the term A is frequently referred to as the figure-of-merit (FoM) of a two-coil WPT system and has positive value, $A \geq 0$.

Similarly, by obtaining the cofactors of the impedance matrix of the three-coil WPT system, the PTE of the system is expressed as

$$\eta_{3\text{-coil}} = |S_{21}|^2 = 4R_sR_L \left| \frac{\omega^4 M_{12}^2 M_{23}^2 + \omega^2 M_{13}^2 R_2^2}{\Delta} \right|^2, \quad (15)$$

where Δ is the determinant of the 3 by 3 impedance matrix and is given by

$$\Delta = \left(R_1R_2R_3 + \omega^2 M_{23}^2 R_1 + \omega^2 M_{12}^2 R_3 + \omega^2 M_{13}^2 R_2\right)^2 + 4\omega^6 M_{12}^2 M_{23}^2 M_{13}^2. \quad (16)$$

Using the coupling coefficient k and loaded Q, the PTE of (15) can be rewritten as (17), as shown at the bottom of this page.

If we let $A = k_{12}^2 Q_1 Q_2$, $B = k_{23}^2 Q_2 Q_3$, and $C = k_{13}^2 Q_1 Q_3$, then we get a simple expression of the PTE as

$$\eta_{3\text{-coil}} = \frac{4R_sR_L}{R_1R_3} \frac{AB + C}{(1 + A + B + C)^2 + 4ABC}, \quad (18)$$

where the terms A, B, and C denote the FoMs between the primary and secondary resonators, between the secondary and tertiary resonators, and between the primary and tertiary resonators, respectively.

Similarly, for a four-coil WPT system, the PTE is (19), shown at the bottom of this page, in which A, B, C, D, E, and F represent FoMs between resonators and are shown in Fig. 3.

C. CHARACTERISTICS OF PTE OF THREE-COIL WPT SYSTEMS

If the PTE of (18) is a maximum, then $\partial\eta/\partial A = \partial\eta/\partial B = 0$. The first-order partial derivatives are the following:

$$(1+A+B+C)\left(B^2+B+BC-AB-2C\right)-4BC^2=0, \quad (20a)$$

$$(1+A+B+C)\left(A^2+A+AC-AB-2C\right)-4AC^2=0. \quad (20b)$$

These imply that $A = B$, and the symmetric three-coil WPT system has a higher PTE than the asymmetric system. This

$$\eta_{3\text{-coil}} = \frac{4R_sR_L}{R_1R_3} \times \frac{k_{12}^2 k_{23}^2 Q_1 Q_2^2 Q_3 + k_{13}^2 Q_1 Q_3}{\left(1 + k_{13}^2 Q_1 Q_3 + k_{12}^2 Q_1 Q_2 + k_{23}^2 Q_2 Q_3\right)^2 + 4k_{12}^2 k_{23}^2 k_{13}^2 Q_1^2 Q_2^2 Q_3^2}. \quad (17)$$

$$\eta_{4\text{coil}} = \frac{4R_sR_L}{R_1R_4} \times \frac{\left(\sqrt{AE}+\sqrt{CD}\right)^2+\left(\sqrt{F}+\sqrt{F}B-\sqrt{ABD}-\sqrt{BCE}\right)^2}{\left[1+A+B+C+D+E+F-2\left(\sqrt{ACDE}+\sqrt{ABDF}+\sqrt{BCEF}\right)+AD+BF\right]^2+4\left(\sqrt{BDE}+\sqrt{ABC}+\sqrt{AEF}+\sqrt{CDF}\right)^2}. \quad (19)$$

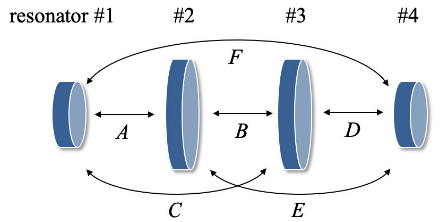

FIGURE 3. **Definition of FoMs between four resonators.**

result is identical to the experimental results of [10]. Here, the interesting point is that the symmetric three-coil WPT system does not mean that $k_{12} = k_{23}$ and $Q_1 = Q_3$. Even when the primary and termitary resonators are not the same, the symmetric structure can be obtained by adjusting the coupling coefficients k_{12} and k_{13}. That is, only the condition of $k_{12}^2 Q_1 Q_2 = k_{23}^2 Q_2 Q_3$ (or $A = B$) should be satisfied.

If the three-coil WPT system has a symmetric structure ($A = B$), then the PTE can be written as

$$\eta_{3\text{-coil}} = \frac{4R_s R_L}{R_1 R_3} \frac{A^2 + C}{(1 + 2A + C)^2 + 4A^2 C}. \tag{21}$$

When $A = B = 0$, equation (21) becomes the PTE of a two-coil WPT system like (14). Comparing the PTE of a two-coil WPT system with that of a three-coil WPT, (21), can gives us a guideline on applying either a three-coil or two-coil WPT systems. To find out the condition for use of the three-coil WPT system rather than the two-coil WPT system, let the PTE of the three-coil WPT system be higher than that of the two-coil WPT system as follows:

$$\begin{aligned} \eta_{2\text{-coil}} &= \frac{4R_s R_L}{R_1 R_3} \frac{C}{(1 + C)^2} \\ &< \eta_{3\text{-coil}} = \frac{4R_s R_L}{R_1 R_3} \frac{A^2 + C}{(1 + 2A + C)^2 + 4A^2 C}. \end{aligned} \tag{22}$$

Solving for C gives

$$0 < C < \frac{A}{4 + 3A}. \tag{23}$$

Because C is the FoM between the primary (or transmitting) and tertiary (or receiving) resonators, it is related to the transmission distance. That is, a C of 0 means an infinite transmission distance, and a large C value means a short transmission distance. Equation (23) tells us that the three-coil WPT system has a higher PTE than the two-coil WPT system for long transmission distance, and the two-coil WPT system is suitable for a short transmission distance, as we would expect. The term of A determines the boundary at which the two systems have the same PTE. As A increases, the three-coil WPT system can be used at shorter transmission distances. If $A \gg 1$, (23) reduces to

$$0 < C < \frac{1}{3} \text{ or } 0 < k_{13} < \frac{1}{\sqrt{3Q_1 Q_3}}. \tag{24}$$

If the coupling coefficient between the transmitting and receiving resonators is less than $k_{critical}/\sqrt{3}$, a three-coil WPT system is recommended. If not, a two-coil WPT system is recommended. Therefore, $k_{critical}/\sqrt{3}$ can be used as a reference point for determining whether a two-coil or three-coil WPT system should be used.

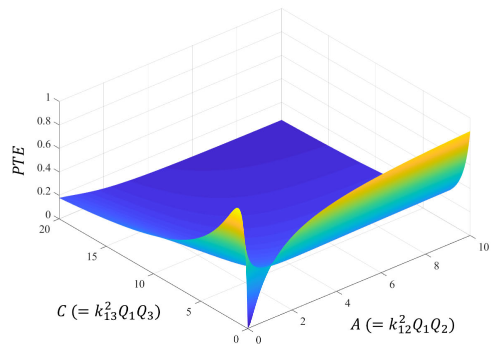

FIGURE 4. **Calculated PTE of a symmetric three-coil WPT system as functions of A and C.**

On the other hand, from the condition that the PTE differentiated by C is 0, the value of C maximizing the PTE can be obtained by

$$\frac{\partial \eta_{3-coil}}{\partial C} = 0 \rightarrow C = -A^2 + \sqrt{(1 - A)(A + 1)^2 (3A + 1)}. \tag{25}$$

For $0 \leq A \leq 0.945$, C of (25) is positive, and then the PTE of the three-coil WPT system has a local maximum. On the other hand, since C satisfying (25) is imaginary for $A > 0.945$, the PTE monotonically increases or decreases with respect to C.

The PTE of a symmetric three-coil WPT system is numerically calculated by using (18) under the assumption that $R_s/R_1 = R_L/R_3 = 1$. These assumptions are ideal, but they are usually valid due to the relatively small parasitic resistance of the coils in comparison to source or load resistance. The simulated results as functions of FoMs (A and C) among resonators are shown in Fig. 4. It can be seen that the three-coil WPT system has meaningful PTE when $A \approx 0$ or $C \approx 0$. The case of $A = 0$ means the two-coil WPT system because of the symmetric structure. When $C \approx 0$, the three-coil WPT system has high PTE despite very week coupling between the primary and tertiary resonators. This is because there is little power directly transferred from the primary to tertiary resonators and most of the power is transferred to the tertiary resonator through the secondary resonator. If $A \approx 0$ and $C \approx 0$, the coupling between all of the coils is very weak and it results in very low PTE.

Figure 5(a) and 5(b) represents the PTE of the symmetric three-coil WPT system as functions of C and A, respectively. As shown in Fig. 5(a), when $A = 0$, the symmetric three-coil WPT system becomes a two-coil WPT system that has maximum PTE at critical coupling, $C = 1$. As A increases, the peak and the overall levels of the PTE decrease, but the PTE near $C = 0$ increases. When A is 0.945, the peak is at $C = 0$. If A is larger than 0.945, the PTE decreases

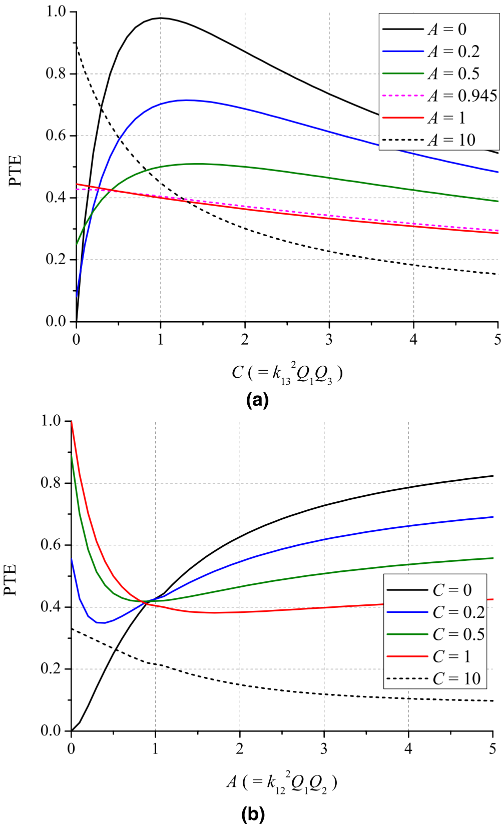

FIGURE 5. Calculated PTE of a symmetric three-coil WPT system (a) as a functions of *A* for several *C* values and (b) as a function of *C* for several *A* values.

monotonically with C. If A is extremely large at 10, the PTE increases to 90% when $C = 0$. This means that the addition of a coil does not help to increase the PTE over most of the range except near $C = 0$. In Fig. 5(b), it is obvious that the larger A is, the higher the PTE is when $C = 0$. If C is smaller than 1, A of 0.945 can achieve a PTE of about 40%. That is, when C is close to 0 and A is large, a higher PTE can be achieved. In most previous research on a three-coil WPT system, it has been simply assumed that the coupling coefficient between transmitting and receiving coils can be ignored, $k_{13} = 0$. However, this simple assumption is valid for relay or domino coil structure, in which the secondary coil is usually located in the middle of transmitting and receiving coils, not for multiple transmitting or intermediate coils, in which the secondary coil is usually located near the transmitting coil. On the other hand, if C is larger than 1, the PTE does not increase even if A increases. That is, a three-coil WPT system is not suitable when C is large. To summarize, A must be greater than 0.945, and C must be as close to 0 as possible.

In a three-coil WPT system, if the secondary coil is not coupled with the tertiary coil, $k_{23} = 0$ or $B = 0$, then the PTE of (18) becomes

$$\eta_{\text{3-coil}} = \frac{4R_sR_L}{R_1R_3}\frac{C}{(1+A+C)^2}, \tag{26}$$

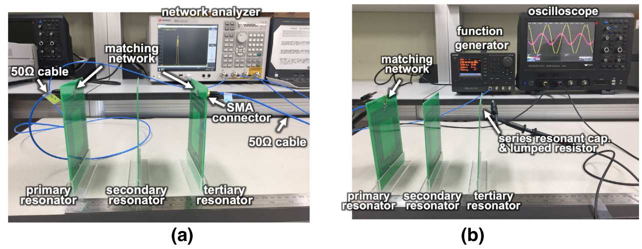

FIGURE 6. Experiment setup used to measure the PTEs with respect to (a) distance between the primary and tertiary resonators and (b) load resistance.

where the term A is included in the denominator, and then it reduces the PTE as a loss. For the conventional three-coil WPT system in which all resonators resonate at the operating frequency, if the secondary coil is coupled with only the primary coil, even though three coils are used, the WPT system always has a lower PTE than a two-coil WPT system.

In the conventional four-coil WPT system, the source and load coils are coupled with only the primary and secondary coils, respectively, and are not coupled with the others. Therefore, $C = E = F = 0$, as shown in Fig. 3. Thus, the PTE of (19) can be written as

$$\eta_{\text{4-coil}} = \frac{4R_sR_L}{R_1R_4}\frac{B}{(1+A+D+B)^2}. \tag{27}$$

This PTE form is also similar to the PTE of a two-coil WPT system, but the FoMs between the source (or load) and primary (or secondary) coils, A or D, act as loss. Therefore, the PTE of a four-coil WPT system is slightly lower than that of a two-coil WPT system. This is identical to the results of [4]. According to these analyses, the PTE is reduced when there is a coil coupled with only a single coil in a multi-coil WPT system.

III. MEASUREMENT SETUP AND MEASURED RESULTS

A. MEASUREMENT SETUP

Figure 6 shows the measurement setups that were employed to compare the two- and three-coil WPT systems. For the fixed source and load impedances, a network analyzer was used to measure the PTE as a function of distance between the transmitting and receiving coils, as shown in Fig. 6(a). Since the network analyzer had a port impedance of 50 ohms, matching networks were adopted to transform the port impedance into the designated source and load impedances, and the PTE of the WPT systems could be obtained from (3) by measuring the S-parameter. Meanwhile, when the effect of the load impedance variation was measured, we used the function generator as a power source and an oscilloscope to measure the output power, as shown in Fig. 6(b). We actually changed the chip resistance manually without the matching network to emulate the change in load impedance, and the tertiary resonant capacitor was connected to the coil in series like the receiving resonator of Fig 1. The received power was obtained by measuring the voltage across the known resistor with an oscilloscope. Because the maximum available power

TABLE 1. Circuit parameters of experimental prototype.

Symbol	Notes	Value
f_0	operating frequency	6.78 MHz
L	coil inductance	1.129 μH
R_p	coil's parasitic resistance	0.189 Ω
R_s	source resistance	1 Ω
R_L	load resistance	1 Ω

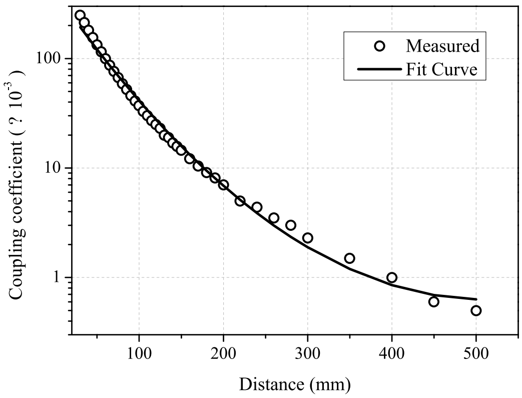

FIGURE 7. Measured coupling coefficients and the fit curve with respect to the distance between coils.

TABLE 2. Parameters of fit curve.

Equation	$\log_{10} y = a + bx + cx^2$	
Parameters	Value	Error
a	2.60548	0.02694
b	-0.01099	2.933×10^{-4}
c	1.07×10^{-5}	5.98×10^{-7}
Adjusted R-squared	0.99331	

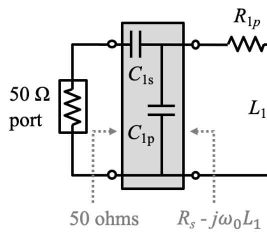

FIGURE 8. Transmitting resonator with an arbitrary source impedance R_s and resonant capacitor, matched into 50 ohms using an L-section matching network.

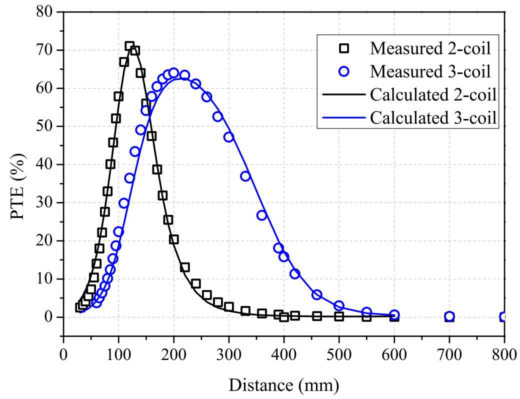

FIGURE 9. Measured and calculated PTE of two-coil and three-coil WPT systems as a function of distance between transmitting and receiving resonators.

of a general function generator is 250 mW, the PTEs can be obtained by dividing the measured received power by 250 mW.

In the measurement, we used identical coils printed on an FR-4 substrate that had three turns and a single layer. The outer dimension of the coils was 95.7 mm × 105.7 mm, and the line width and gap between lines were 5.7 mm and 2.4 mm, respectively. The system parameters including those of the coils are summarized in Table 1.

First, we measured the coupling coefficient between the coils using the network analyzer of Keysight's E5071C. The measured values are plot in the Fig. 7. To compare the simulated and measured PTEs, the measured log-scaled coupling coefficients are fitted with a second-order polynomial function. The parameters of the fit curve are summarized in Table 2. The adjusted R-squared is very close to 1.0, which means that the model has a good fit. However, the fit curve has a global minimum at approximately 600 mm. Because it is abnormal that the coupling coefficient increases as distance increases, the fit curve is available within the range from 30 mm to 600 mm.

The source and load impedances were implemented through the matching network, which consisted of series and parallel capacitors, as shown in Fig. 8. The impedance seen at port and the coil are 50 ohms and R_s - $j\omega_0 L_1$, respectively. Therefore, it seems that the primary coil is connected to the source with R_s and the resonant capacitor with a reactance of -$\omega_0 L_1$. That is, even though the matching network has a series-parallel compensation structure, the primary resonator equivalently has a series capacitor. In our case, C_{1s} and C_{1p} were 420 pF (270 pF // 150 pF) and 69.8 pF (68 pF // 1.8 pF), respectively, for the source and load impedances of 1.0-ohm, which capacitance values can be calculated by using the analytic solutions of the L-section matching network [25]. Meanwhile, the resonant capacitor in the secondary resonant was implemented by the parallel connection of 170 pF and 18 pF, since the secondary resonant does not need any matching network or port.

B. MEASURED RESULTS

Figure 9 shows the measured and calculated PTEs of the two-coil and three-coil WPT systems in relation to the distance between the transmitting and receiving coils, where two systems have the constant system parameters of Table 1. The measured results show good agreement with the calculated ones, which were obtained from (14) and (18). The slight difference between them is presumably due to imperfect curve fitting. First of all, the PTE peak of the two-coil WPT system

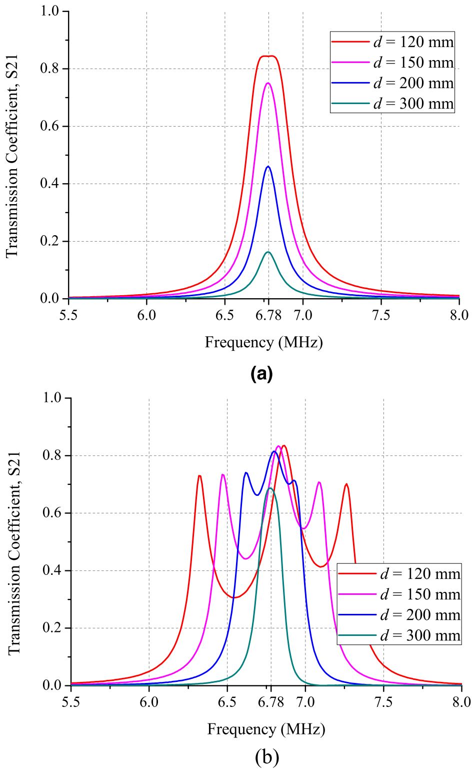

FIGURE 10. **Measured spectrum of transmission coefficient for several in the (a) two-coil and (b) three-coil WPT system.**

occurs at the critical coupling coefficient, $k_{critical} = 0.0247$, of which the corresponding distance is about 120 mm.

The two-coil WPT system has a higher PTE than the three-coil system at distances shorter than 150 mm. If the distance is greater than 150 mm, the three-coil system has a higher PTE. Putting the system parameters into (24) gives the asymptotic boundary between the two-coil and three-coil WPT systems. In this case, $1/\sqrt{3Q_1Q_3}$ is 0.0143, and its corresponding distance is approximately 150 mm, as seen in Fig. 7. The three-coil WPT system has a greater PTE than the two-coil system when the distance is greater than 150 mm.

The transmission coefficients, S_{21}, of the two-coil and three-coil WPT systems were measured by using the network analyzer for several transmission distances, and the results are presented in Fig. 10. It is well known that the two-coil WPT system has two peaks on its spectrum in the over-coupled state, and it has one peak at the operating frequency, 6.78 MHz, in the critical and under-coupled states. Since critical coupling occurs at about 120 mm, the peak value decreases as the distance increases, as seen in Fig. 10(a). On the other hand, it is known that the PTE of the three-coil WPT system has three peaks on its spectrum [19]. As seen in Fig. 10(b), when the primary and tertiary resonators are close to each other, the three peaks appear. As the distance between coils increases, the middle maximum point moves to the operating frequency, and the three peaks converge to the middle maximum points. The interesting point is that although the PTE of the three-coil WPT system has

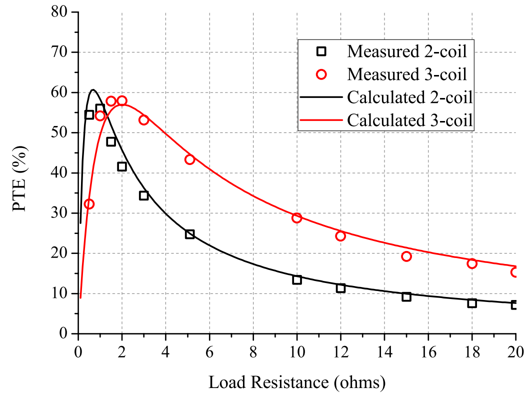

FIGURE 11. **Measured and calculated PTEs of the two-coil and three-coil WPT system with respect to the load resistance.**

a maximum at the operating frequency of 6.78 MHz and the distance of about 200 mm, as seen in Fig. 9, there are still three peak frequencies on the spectrum, as seen in Fig. 10(b). That is, the phenomenon called frequency splitting still appears when the maximum PTE of the three-coil WPT system is achieved. In addition, the middle maximum frequency is not far from the operating frequency when the distance between transmitting and receiving coils is farther than the maximum PTE. Therefore, we can expect that tuning the operating frequency into the middle maximum frequency could result in a high PTE even if the distance is short.

To investigate the effect of load resistance variation on the PTE, we calculated and measured the PTE with respect to the load resistance at the distance of 150 mm, at which the two systems with the source and load resistance of 1 ohm have the same PTE. The source resistance was fixed to 1 ohm, while the load resistance was varied from 0.1 ohm to 20 ohms. As seen in Fig. 11, it is difficult to say which system is absolutely better than the other. However, the two-coil WPT system has a peak with load resistances lower than 1 ohm, while the three-coil WPT system has a peak with load resistances higher than 1 ohm. Therefore, if the load resistance tends to increase from the original resistance, then the three-coil WPT system is favorable. In the opposite case, the two-coil WPT system is more advantageous.

IV. CONCLUSION

Based on the definition of scattering parameters, we derived the general expression for the PTE of a multi-coil WPT system. The derived expression was verified by measuring the PTE of a symmetric three-coil WPT system, and its expression has a potential to be expanded to a WPT system with an arbitrary number of coils. Through analysis of the PTE of the three-coil WPT system, we showed that the system should have a symmetric structure for a high PTE. In addition, by comparing the PTEs of two-coil and three-coil WPT systems, we identified the boundary at which the PTEs of the two system are the same. This boundary can be used as a measure to determine the number of coils–two or three coils–for wireless power transmission. On the other hand, the effect of load variation on the PTE was also investigated at the boundary. The results showed that both of the systems are not robust to load variation. However, if the load resistance tends to increase, the three-coil WPT system is advantageous; if not, the two-coil WPT system is recommended. Finally, we noticed that the three-coil WPT system has three split frequencies at the peak PTE unlike the two-coil WPT system. It is well-known that the two-coil WPT system has two split frequencies that are far away from the operating frequency as the coupling coefficient increases beyond the critical coupling coefficient. However, in the three-coil WPT system, the middle of the three split frequencies does not deviate far from the operating frequency. If the operating frequency can be fine-tuned within a narrow bandwidth, the three-coil WPT system would achieve a high PTE even over a short transmission distance.

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