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Fractional Order Adaptive Sliding Mode Control System of Micro Gyroscope

FANG CHEN AND JUNTAO FEI, (Senior Member, IEEE)
College of IoT Engineering, Hohai University, Changzhou 213022, China
Jiangsu Key Laboratory of Power Transmission and Distribution Equipment Technology, Hohai University, Changzhou 213022, China

Corresponding author: Juntao Fei (jtfei@hhu.edu.cn)

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ABSTRACT In this paper, an adaptive sliding mode control method based on fractional order method is proposed for the estimation of unknown parameters in micro gyroscope system. Firstly, compared with the traditional integer order sliding surface, the fractional order sliding surface in this paper increases the order terms which can adjust the fractional order, and improve the control performance and precision. Additionally, the adaptive laws are derived to tune the control parameters online based on the Lyapunov stability theorem to deal with the uncertainty of the system. Moreover, simulation results show that the proposed control system's convergence speed, output signal tracking error and parameter fitting are better than the integral sliding mode control system, proving that the proposed control method is practicable and effective.

INDEX TERMS Fractional order, adaptive control, sliding mode control, micro gyroscope.

I. INTRODUCTION

The principle of gyroscope is mainly the law of conservation of angular momentum, which is a device with sensing, maintaining direction stability and angular motion detection. Compared with traditional gyroscopes, micro gyroscopes have a lot of advantages, which make them widely used in aviation, aerospace, navigation, automotive safety, environmental monitoring and other fields, especially in areas where the size and weight requirements are extremely strict, micro gyroscopes have extremely significant advantages.

More and more control methods are applied to the closed-loop control system of the micro gyroscopes to improve its performance and measurement accuracy. A novel silicon micro gyroscope temperature prediction and control system in a narrow space was proposed in [1]. Sliding mode control has the advantages of fast response [2], strong robustness, simple physical implementation, and independent design of object parameters and disturbances. Due to these practical and robust advantages, the sliding mode control suits various fields, such as piezoelectric actuators [3], grid-connected inverter [4], small-scaled autonomous aerial vehicles [5] and randomly varying actuator faults [6]. Moreover, sliding mode control has been incorporated into adaptive controller to control the MEMS gyroscopes, to regulate the amplitude of the drive axis vibration of micro gyroscope [7]–[10]. Fei and Zhou [11] proposed a robust adaptive control strategy using a fuzzy compensator for a MEMS triaxial gyroscope. The adaptive control method provides online estimation of the key parameters and the proper control strategy for the system [12]. In the nonlinear systems, unknown nonlinearities can be approximated by intelligent methods such as fuzzy systems and neural networks [13]–[16]. Intelligent control methods have been investigated for dynamic systems [17]–[19]. Adaptive control with intelligent control methods are also widely used in other systems, i.e. active power filter system [20]–[23], and chaotic systems [24]. dynamical system [25]

To suppress the steady-state error, introducing an integer order integral term into the sliding surface is a good approach, which has been studied sufficiently. However, the fractional order integral provides the controller design with more degree of freedom, so it is more flexible. In this regard, we can improve sliding mode by integrating the fractional order integral in the sliding surface and constructing an adaptive algorithm to reduce loss of precision. In recent decades, the fractional order integral calculus has been successfully utilized for model analyzes and controller designs [26]–[32].

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The combination of fractional order integral and adaptive control is also applied to the industrial applications, such as linear motor [28], [29], lighting control [30], active suspension actuator [31], and robot manipulators [32].

Motivated by the above literatures, a fractional order sliding mode control method for micro gyroscopes based on adaptive control algorithm is proposed in this paper. The main characteristics of this control method are as follows:

(1) In the presence of the model uncertainties and external disturbance, the proposed adaptive control algorithm uses Lyapunov stability theory to design the angular velocity, cross stiffness and damping coefficients of the system, identifies these unknown parameters of the micro-gyroscope system online, and meanwhile reduces the uncertainty of the system.

(2) By introducing fractional order integral into the sliding surface to soften the controller design by adding adjustable order, obtain smaller steady-state error than the integer order controller, and meanwhile the control law is more flexible. The combination of adaptive control and fractional order can also track the trajectory of the system well, obtain high precision and fast-tracking performance.

This paper is organized as follows. In the second part, the dynamic model of micro gyroscopes is introduced. The third part introduces the adaptive fractional sliding mode control method, and proves the stability of sliding mode motion and the reachability of the sliding mode surface base on the Lyapunov stability theory. The proposed method is simulated and compared with the integer order adaptive sliding mode control in the fourth part. The fifth part draws the conclusion.

II. DYNAMICS OF MICRO GYROSCOPE

The driving mode and sensing mode of micro gyroscope can be considered as a spring-mass-damping second-order system. The basic dynamic model of micro gyroscope is shown in Figure 1.

The rotational coordinates system of the micro gyroscope model is established. The x-axis is the direction of micro gyroscope driving vibration, the y-axis is the direction of micro gyroscope sensing vibration, the z-axis is the direction of input angular velocity. The basic dynamic equation of the micro gyroscope will be written as follows:

$$m\ddot{x} + d_x\dot{x} + k_x x = u_x + 2m\Omega_z\dot{y} + m\Omega_z^2 x + m\dot{\Omega}_z y \quad (1)$$
$$m\ddot{y} + d_y\dot{y} + k_y y = u_y - 2m\Omega_z\dot{x} - m\dot{\Omega}_z x + m\Omega_z^2 y \quad (2)$$

where m is the mass of the mass block, d_x, k_x, u_x and d_y, k_y, u_y denote the damping coefficients stiffness coefficients and control inputs, in x-axis and y-axis, respectively, Ω_z is the angular velocity on the z-axis.

Considering the influence of the structural error, the basic dynamic equation can be obtained as:

$$m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + k_{xx}x + k_{xy}y = u_x + 2m\Omega_z\dot{y} \quad (3)$$
$$m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + k_{xy}x + k_{yy}y = u_y - 2m\Omega_z\dot{x} \quad (4)$$

where d_{xx}, d_{yy} and k_{xx}, k_{yy} denote the x-axis and y-axis' damping coefficients, and stiffness coefficients, respectively,

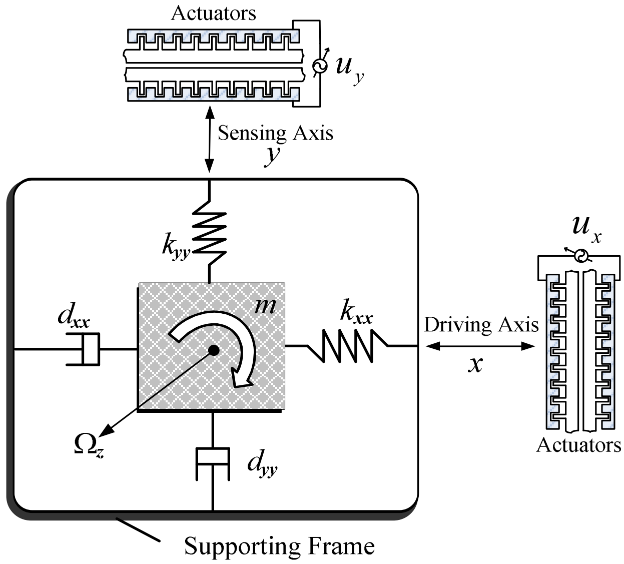


FIGURE 1. Dynamic model of micro gyroscope.

d_{xy} and k_{xy} are the coupling stiffness coefficients and coupling damping coefficient.

In order to reduce the complexity of controller design, the dynamics model is dimensionless. Dividing both sides of equations (3)-(4) by mass blocks m of micro gyroscope, reference length q_0 and natural resonance frequency ω_0, and getting the dimensionless model as:

$$\frac{\ddot{q}}{q_0} + \frac{D}{m\omega_0}\frac{\dot{q}}{q_0} + \frac{K}{m\omega_0^2}\frac{q}{q_0} = \frac{u}{m\omega_0^2 q_0} - 2\frac{\Omega}{\omega_0}\frac{\dot{q}}{q_0} \quad (5)$$

where $q = \begin{bmatrix} x \\ y \end{bmatrix}$, $\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$, $\ddot{q} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix}$, $D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}$, $K = \begin{bmatrix} \omega_{xx} & \omega_{xy} \\ \omega_{xy} & \omega_{yy} \end{bmatrix}$, $u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}$, $\Omega = \begin{bmatrix} 0 & -\Omega_z \\ \Omega_z & 0 \end{bmatrix}$.

The form in which equations (5) are rewritten as vectors is as follows:

$$\ddot{q} + D\dot{q} + Kq = u - 2\Omega\dot{q} \quad (6)$$

III. ADAPTIVE FRACTIONAL SLIDING MODE CONTROLLER DESIGN

Fig. 2 is a block diagram of adaptive fractional order sliding mode control system.

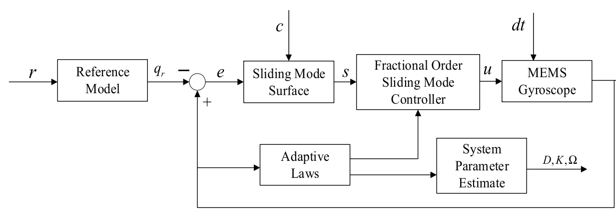


FIGURE 2. Block diagram of adaptive fractional order sliding mode control system.

Considering the parameters uncertainty and external disturbance in the micro gyroscope system, the Eq. (6) can be expressed as:

$$\ddot{q} + (D + 2\Omega + \Delta D)\dot{q} + (K + \Delta K)\, q = u + d \tag{7}$$

where ΔD and ΔK are the uncertainties of unknown parameters $D + 2\Omega$ and K, d is the external disturbance.

Then f is defined as lumped parameter uncertainty and external disturbance, represented as:

$$f = d - \Delta D\dot{q} - \Delta K q \tag{8}$$

The Eq. (7) can be modified as:

$$\ddot{q} + (D + 2\Omega)\, \dot{q} + Kq = u + f \tag{9}$$

The lumped parameter uncertainty and external disturbance f is assumed to satisfy:

$$\|f\| \le \rho \tag{10}$$

where ρ is a given positive constant.

The fractional sliding mode surface is defined as follows:

$$s = ce + \dot{e} + \lambda e D^{\alpha-1} \tag{11}$$

in which α is a fractional order used to obtain more accurate performance; e denotes the tracking error defined as $e = q - q_r$ where q_r is the actual position; $\dot{e}$ denotes the differential of the tracking error defined as $\dot{e} = \dot{q} - \dot{q}_r$; c and λ are given positive constants.

The derivative of the Eq. (11) is as follows:

$$\dot{s} = c\dot{e} + \ddot{e} + \lambda e D^{\alpha} \tag{12}$$

The Eq. (12) can be modified as:

$$\dot{s} = -(D + 2\Omega)\, \dot{q} - Kq + u + f + c\dot{e} + \lambda e D^{\alpha} - \ddot{q}_r \tag{13}$$

According to the hitting condition of the sliding mode control $\dot{s} = 0$, the following formula can be obtained:

$$-(D + 2\Omega)\, \dot{q} - Kq + u + f - \ddot{q}_r + c\dot{e} + \lambda e D^{\alpha} = 0 \tag{14}$$

The equivalent control law can be obtained by Eq. (14):

$$u_{eq} = (D + 2\Omega)\, \dot{q} + Kq + \ddot{q}_r - c\dot{e} - \lambda D^{\alpha} e \tag{15}$$

The design of switch control law is as follows:

$$u_{sw} = -\rho \frac{s}{\|s\|} \tag{16}$$

Using the method of combination of equivalent sliding mode control and switching control, the control law u of fractional order sliding mode control is expressed as follows:

$$u = (D + 2\Omega)\, \dot{q} + Kq + \ddot{q}_r - c\dot{e} - \lambda D^{\alpha} e - \rho \frac{s}{\|s\|} \tag{17}$$

According to the general idea of adaptive control, the unknown real values of D, K and Ω are replaced by the estimated values of $\hat{D}$, $\hat{K}$ and $\hat{\Omega}$, and the adaptive algorithm of $\hat{D}$, $\hat{K}$ and $\hat{\Omega}$ are designed to update the estimated values of the system in real time to ensure the stability of the system.

Then Eq. (15) and Eq. (17) can be rewritten as:

$$u'_{eq} = -c\dot{e} - \lambda D^{\alpha} e + \left(\hat{D} + 2\hat{\Omega}\right) \dot{q} + \hat{K} q + \ddot{q}_r \tag{18}$$

$$\begin{aligned} u' &= u_{eq} + u_{sw} \\ &= -c\dot{e} - \lambda D^{\alpha} e + \left(\hat{D} + 2\hat{\Omega}\right) \dot{q} + \hat{K} q + \ddot{q}_r - \rho \frac{s}{\|s\|} \end{aligned} \tag{19}$$

Estimation error $\tilde{D}$, $\tilde{K}$ and $\tilde{\Omega}$ are defined as follows:

$$\begin{cases} \tilde{D} = \hat{D} - D \\ \tilde{K} = \hat{K} - K \\ \tilde{\Omega} = \hat{\Omega} - \Omega \end{cases} \tag{20}$$

Substituting Eq. (19) – (20) into Eq. (8) generates:

$$\begin{aligned} &\ddot{q} - \ddot{q}_r + c\dot{e} + \lambda D^{\alpha} e \\ &\quad = -(D + 2\Omega)\, \dot{q} + \left(\hat{D} + 2\hat{\Omega}\right) \dot{q} \\ &\qquad - Kq + \hat{K} q - \Delta K q - \Delta D\dot{q} - \rho(t) \frac{s}{\|s\|} + d \end{aligned} \tag{21}$$

Substitute Eq. (12) into Eq. (21), it is revealed that:

$$\begin{aligned} \dot{s} = &-(D + 2\Omega)\, \dot{q} + \left(\hat{D} + 2\hat{\Omega}\right) \dot{q} \\ &- Kq + \hat{K} q - \Delta K q - \Delta D\dot{q} - \rho \frac{s}{\|s\|} + d \end{aligned} \tag{22}$$

Then combined with Eq. (8), (20) and (22), the following expression is obtained:

$$\dot{s} = -\left(\tilde{D} + 2\tilde{\Omega}\right) \dot{q} + \tilde{K} q + f - \rho \frac{s}{\|s\|} \tag{23}$$

Select the Lyapunov candidate function as:

$$\begin{aligned} V = \frac{1}{2} s^T s + \frac{1}{2} tr\left\{\tilde{D} M^{-1} \tilde{D}^T\right\} + \frac{1}{2} tr\left\{\tilde{K} N^{-1} \tilde{K}^T\right\} \\ + \frac{1}{2} tr\left\{\tilde{\Omega} P^{-1} \tilde{\Omega}^T\right\} \end{aligned} \tag{24}$$

where $M = M^T > 0$, $N = N^T > 0$, $P = P^T > 0$, M, N and P are positive definite symmetric matrices. $tr\{\cdot\}$ represents the tracing operation of the matrix.

The first derivative of Eq. (24) can be obtained as follows:

$$\dot{V} = s^T \dot{s} + tr\left\{\tilde{D} M^{-1} \dot{\tilde{D}}^T\right\} + tr\left\{\tilde{K} N^{-1} \dot{\tilde{K}}^T\right\} + tr\left\{\tilde{\Omega} P^{-1} \dot{\tilde{\Omega}}^T\right\} \tag{25}$$

Substitute Eq. (23) into Eq. (25), it is revealed that:

$$\begin{aligned} \dot{V} &= s^T \left(\tilde{D}\dot{q} + 2\tilde{\Omega}\dot{q} + \tilde{K} q + f - \rho(t) \frac{s}{\|s\|}\right) \\ &\quad + tr\left\{\tilde{D} M^{-1} \dot{\tilde{D}}^T\right\} + tr\left\{\tilde{K} N^{-1} \dot{\tilde{K}}^T\right\} + tr\left\{\tilde{\Omega} M^{-1} \dot{\tilde{\Omega}}^T\right\} \end{aligned} \tag{26}$$

Since D, K and Ω are symmetric matrices, then there are $D = D^T$, $K = K^T$ and $\Omega = -\Omega^T$, and there is $s^T \tilde{D} \dot{q} = \dot{q}^T \tilde{D} s$ for the matrix D, namely:

$$s^T \tilde{D} \dot{q} = \frac{1}{2} \left(s^T \tilde{D} \dot{q} + \dot{q}^T \tilde{D} s\right) \tag{27}$$

Similarly, it can get the equations for the matrix K and Ω:

$$s^T \tilde{K} \dot{q} = \frac{1}{2} \left(s^T \tilde{K} q + q^T \tilde{K} s\right) \tag{28}$$

TABLE 1. Root mean square error of different orders.

Order α	Root mean square error of x-axis tracking error	Root mean square error of y-axis tracking error
0.1	0.0011	0.0016
0.3	0.0011	0.0015
0.5	0.0010	0.0015
0.8	0.00098	0.0014
0.99	0.00096	0.0014

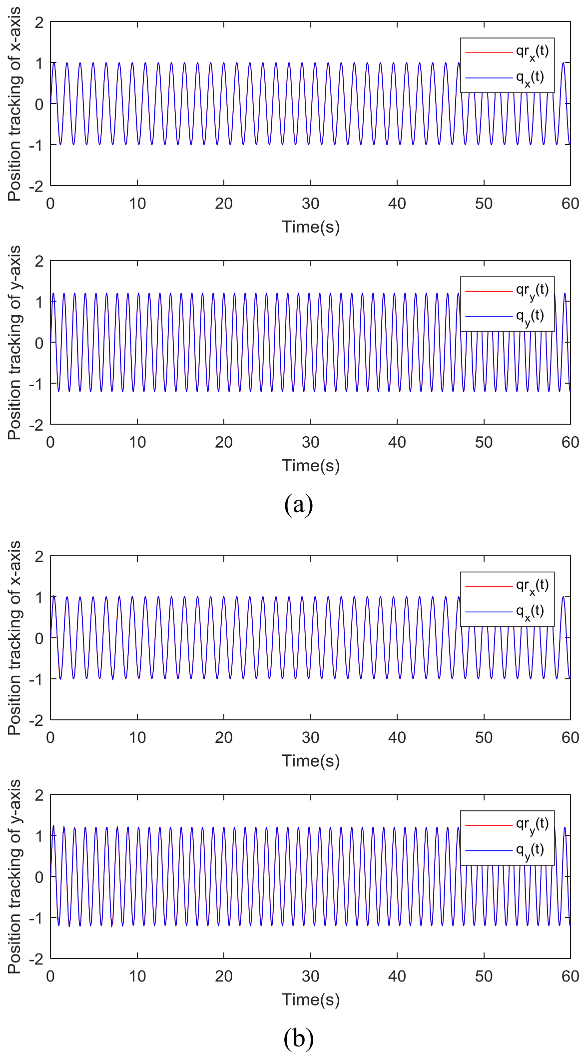


FIGURE 3. Track trajectory of x-axis and y-axis of micro gyroscope. (a) Adaptive fractional order sliding mode control. (b) Adaptive integral order sliding mode control.

$$s^T\tilde{\Omega}\dot{q} = \frac{1}{2}\left(s^T\tilde{\Omega}\dot{q} - \dot{q}^T\tilde{\Omega}s\right) \tag{29}$$

Substitute Eq. (27) – (29) into Eq. (26), it can be represented as:

$$\dot{V} = s^T\left(f - \rho(t)\frac{s}{\|s\|}\right) + \frac{1}{2}\left(s^T\tilde{D}\dot{q} + \dot{q}^T\tilde{D}s\right)$$

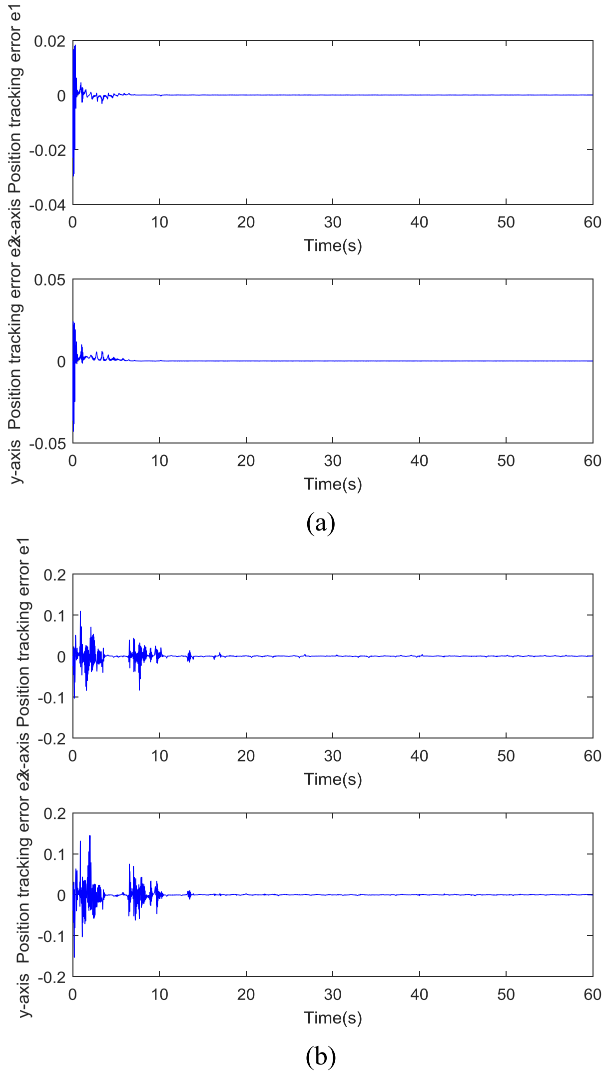


FIGURE 4. Tracking error of x-axis and y-axis of micro gyroscope. (a) adaptive fractional order sliding mode control. (b) adaptive integral order sliding mode control.

$$\begin{aligned} &+ tr\left\{\tilde{D}M^{-1}\dot{\tilde{D}}^T\right\} + \frac{1}{2}\left(s^T\tilde{K}q + q^T\tilde{K}s\right) \\ &+ tr\left\{\tilde{K}N^{-1}\dot{\tilde{K}}^T\right\} \\ &+ \frac{1}{2}\left(2s^T\tilde{\Omega}\dot{q} - 2\dot{q}^T\tilde{\Omega}s\right) + tr\left(\tilde{\Omega}P^{-1}\dot{\tilde{\Omega}}^T\right) \end{aligned} \tag{30}$$

Because of $\dot{\hat{D}} = \dot{\tilde{D}}$, the Eq. (30) is simplified as follows:

$$\begin{aligned} \dot{V} = s^T&\left(f - \rho\frac{s}{\|s\|}\right) + tr\left\{\tilde{D}\left[M^{-1}\dot{\hat{D}}^T + \frac{1}{2}\left(s^T\dot{q} + \dot{q}^T s\right)\right]\right\} \\ &+ tr\left\{\tilde{K}\left[N^{-1}\dot{\hat{K}}^T\right] + \frac{1}{2}\left(s^T q + q^T s\right)\right\} \\ &+ tr\left(\tilde{\Omega}\left[P^{-1}\dot{\hat{\Omega}}^T\right] + \frac{1}{2}\left(2\dot{q}s^T - 2s\dot{q}^T\right)\right) \end{aligned} \tag{31}$$

In order to guarantee $\dot{V} \leq 0$, let

$$tr\left\{\tilde{D}\left[M^{-1}\dot{\hat{D}}^T + \frac{1}{2}\left(s^T\dot{q} + \dot{q}^T s\right)\right]\right\} = 0,$$

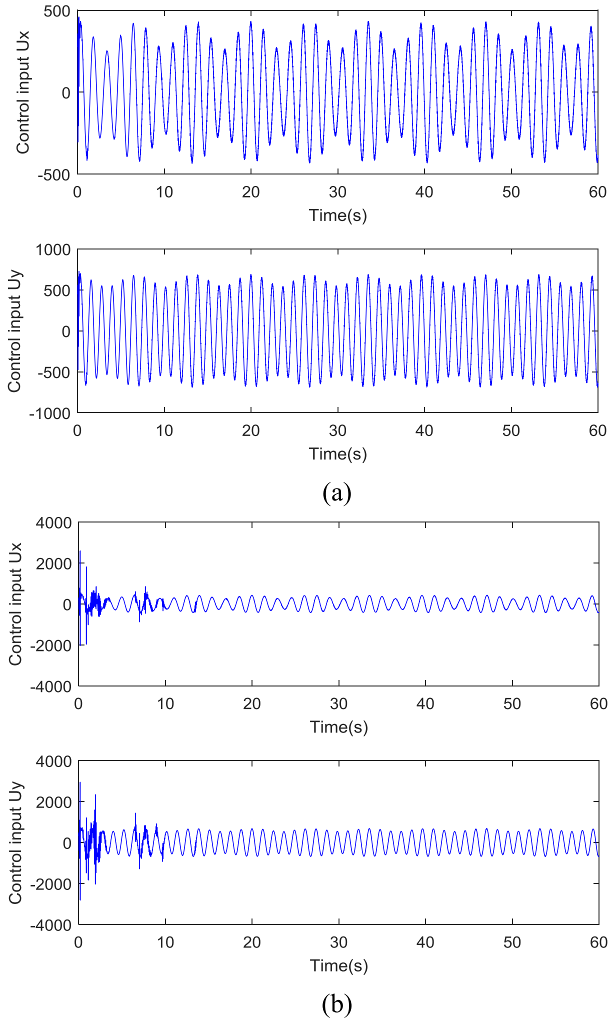


FIGURE 5. Control input of X and Y axes of micro gyroscope. (a) adaptive fractional order sliding mode control. (b) adaptive integral order sliding mode control.

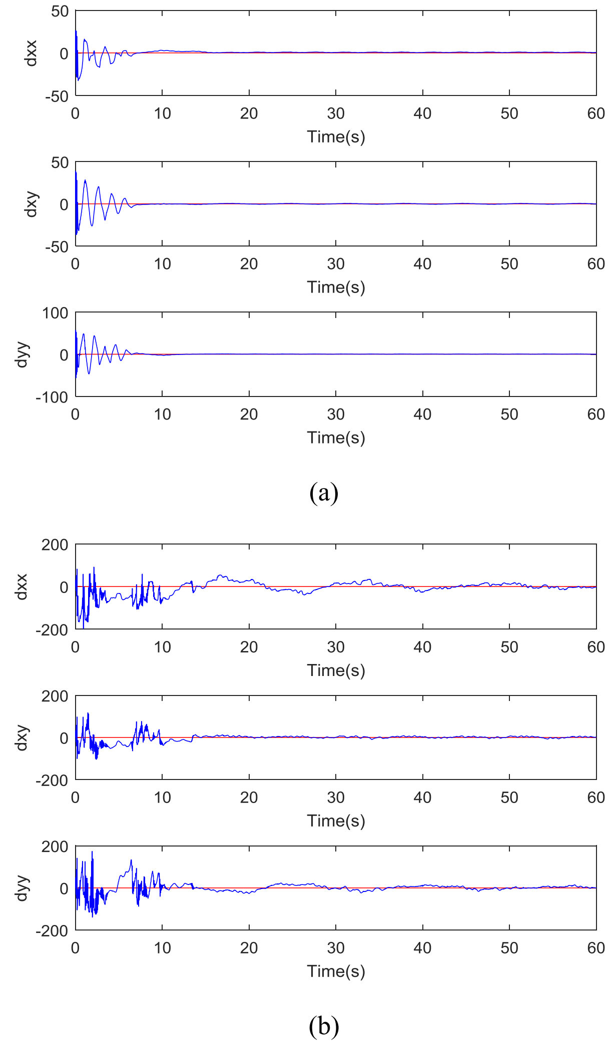


FIGURE 6. Adaptive identification of d_{xx}, d_{xy}, d_{yy}. (a) adaptive fractional order sliding mode control. (b) adaptive integral order sliding mode control.

$$tr\left\{\tilde{K}\left[N^{-1}\dot{\hat{K}}^{T}\right]+\frac{1}{2}\left(s^{T}q+q^{T}s\right)\right\}=0,$$

and

$$tr\left(\tilde{\Omega}\left[P^{-1}\dot{\hat{\Omega}}^{T}\right]+\frac{1}{2}\left(2\dot{q}s^{T}-2s\dot{q}^{T}\right)\right)=0.$$

The adaptive laws of $\hat{D}$, $\hat{K}$ and $\hat{\Omega}$ are designed as follows:

$$\begin{cases}\dot{\hat{D}}^{T}=-\frac{1}{2}M\left(\dot{q}s^{T}+s\dot{q}^{T}\right)\\ \dot{\hat{K}}^{T}=-\frac{1}{2}N\left(qs^{T}+sq^{T}\right)\\ \dot{\hat{\Omega}}^{T}=-P\left(\dot{q}s^{T}-s\dot{q}^{T}\right)\end{cases}\tag{32}$$

In order to analyze system stability, Eq. (31) is rewritten as follows:

$$\begin{aligned}\dot{V}&=s^{T}\left(f-\rho\frac{s}{\|s\|}\right)=s^{T}f-s^{T}\rho\frac{s}{\|s\|}\\&\leq\left\|s^{T}\right\|\|f\|-\rho\|s\|=\|s\|\left(\|f\|-\rho\right)\end{aligned}\tag{33}$$

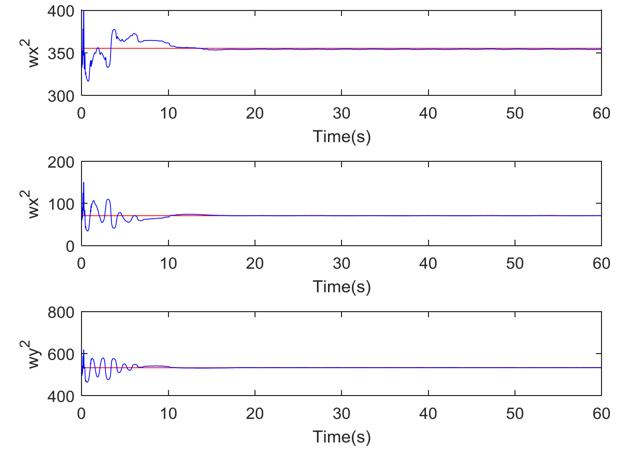
Since $\|f\|\leq\rho$, Eq. (33) can be expressed as:

$$\dot{V}\leq\|s\|\left(\|f\|-\rho\right)\leq 0\tag{34}$$

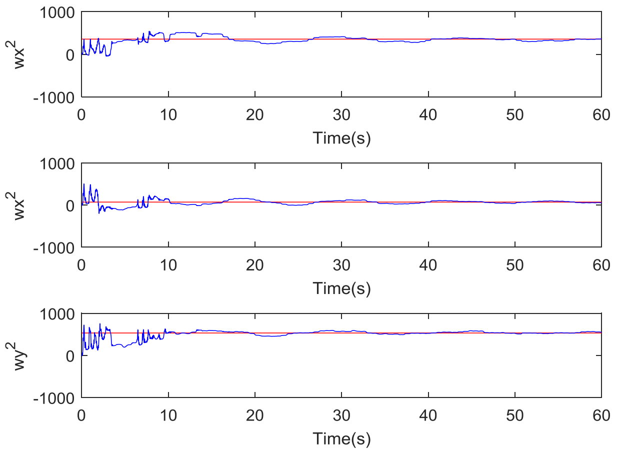
According to Eq. (34), $\dot{V}$ is semi-negative definite, that is, the system tracking trajectory can reach the designed fractional sliding mode surface in finite time. Integrating $\dot{V}$ with respect to time, we have $\int_{0}^{t}\|s\|\left(\|f\|-\rho(t)\right)dt\leq V(t)-V(0)$. Since $V(t)$ and $V(0)$ are bounded, and $V(t)$ is non-increasing, then $\int_{0}^{t}\|s\|\left(\|f\|-\rho(t)\right)dt$ is also bounded. According to the Barbalat lemma and inference, we can get $\lim_{t\to\infty}s(t)=0$, in other words, the tracking error and fractional sliding mode surface converge asymptotically to zero, the system is asymptotic stability.

IV. SIMULATION STUDY

The adaptive fractional order sliding mode control method is simulated with MATLAB/Simulink, and the sliding surface

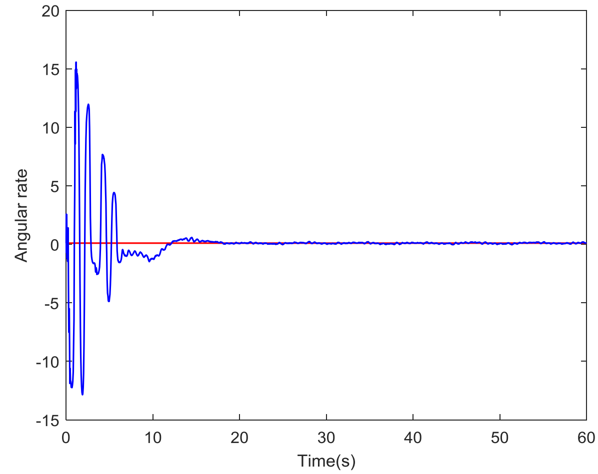


(a)

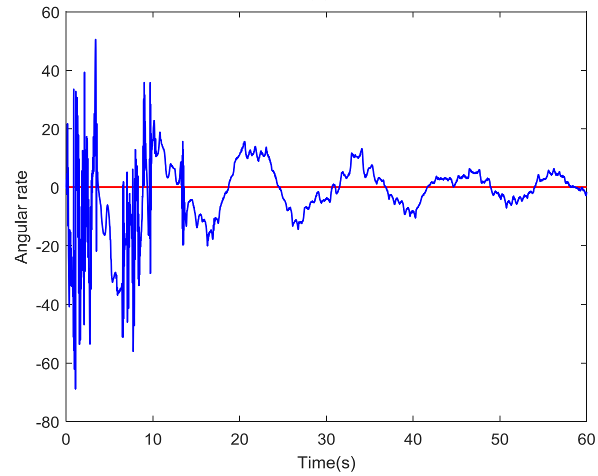


(b)

FIGURE 7. Adaptive identification of ω_x^2, ω_{xy}, ω_y^2. (a) adaptive fractional order sliding mode control. (b) adaptive integral order sliding mode control.



(a)



(b)

FIGURE 8. Adaptive identification of Ω_z. (a) adaptive fractional order sliding mode control. (b) adaptive integral order sliding mode control.

is designed as an integer sliding mode surface, which is compared with the proposed method.

The parameters of the micro gyroscope are selected as follows:

$m = 1.8 \times 10^{-7} kg$, $d_{xx} = 1.8 \times 10^{-6} N \cdot s/m$, $q_0 = 1\mu m$, $d_{xy} = 3.6 \times 10^{-7} N \cdot s/m$, $d_{yy} = 1.8 \times 10^{-6} N \cdot s/m$, $\omega_0 = 1kHz$,

$k_{xy} = 12.779N/m$, $k_{yy} = 95.92N/m$, $\Omega_z = 100rad/s$.

The dimensionless parameters of the micro gyroscope calculated from the above data are as follows:

$d_{xx} = 0.01$, $d_{xy} = 0.002$, $d_{yy} = 0.01$, $\omega_x^2 = 355.3$, $\omega_{xy} = 70.99$, $\omega_y^2 = 532.9$, $\Omega_z = 0.1$.

In the experiments, the initial conditions for setting the system are: $q_1(0) = 0$, $\dot{q}_1(0) = 1$, $q_2(0) = 0$, $\dot{q}_2(0) = 1$; the expected trajectory of the micro gyroscope's two axes is set as: $q_{r1} = \sin(4.17t)$, $q_{r2} = 1.2\sin(5.11t)$; the fractional order sliding mode surface parameters are set as: $c = 85$, λ=1, $\alpha = 0.99$; the adaptive fixed gain is set as: $M = diag(200, 200)$, $N = diag(950, 950)$, $P = diag(120, 120)$; the estimated initial values of the three-parameter matrix is set as: $\hat{D}(0) = 0.95 * D$, $\hat{K}(0) = 0.95 * K$, $\hat{\Omega}(0) = 0.95 * \Omega$; the upper bound of the system uncertainty and external disturbance is set as: $\rho = 10$; the random signal $d = [0.5 * randn(1, 1); 0.5 * randn(1, 1)]$ is considered as lumped uncertainty.

In order to determine the order of fractional order, let $\alpha = 0.1, 0.3, 0.5, 0.8, 0.99$, and calculate the root mean square error of different orders respectively, $RMSE = sqrt\left(sum\left((q_i - q_{ri})^2\right)/n\right)$, where $i = 1, 2$, and n is the observation times. Table 1 shows the root mean square error of different orders. By comparison, the fractional order $\alpha = 0.99$ is selected in the simulation experiment.

Set the simulation time to 60s, and the simulation results are shown in Fig. 3 – Fig. 8. Fig. 3(a) is the tracking trajectory of x-axis and y-axis obtained by adaptive fractional order sliding mode control for micro gyroscope. Fig. 3(b) is the tracking trajectory obtained by adaptive integer order sliding mode control. Fig. 4 are the tracking errors in the x-axis and y-axis using two different controllers, showing that the tracking error of adaptive fractional order sliding mode control system can converge to zero in a limited time well. Fig. 5(a) and (b) show the control input of the x-axis and y-axis of the two kinds of controller, and the control effect of the adaptive fractional order sliding mode control law is better. Fig. 6 – Fig.8 are the adaptive identification of unknown parameters of micro gyroscope system. It can be seen that unknown parameters can converge to their truth values in both cases, but the convergence effect of the adaptive fractional order sliding mode control law is much better. The root means square error of the two-axis tracking error of adaptive integral order sliding mode control is 0.0094 and 0.0132, respectively. Compared with the root mean square error and simulation results of the adaptive fractional order sliding mode control, the trajectory tracking of the adaptive sliding mode control method based on fractional order is smaller than that based on the adaptive integral order sliding mode control method.

V. CONCLUSION

In this paper, an adaptive sliding mode control method based on fractional order is proposed to estimate the unknown parameters of micro gyroscope system. Compared with the integral order sliding mode surface, the fractional order sliding mode surface increases the order term related to the fractional order, and moreover improves the control performance and precision. Fractional order integral is more flexible than the integer order one, which provides the controller design with another degree of freedom, i.e., the order. In addition, the parameters of the micro gyroscope can be adaptively updated based on the Lyapunov analysis and the stability of the closed-loop system can be guaranteed with the proposed control strategy. Consequently, the fractional order and the adaptive algorithms are utilized to guarantee the high-precision and fast-response performance with robustness against uncertainties. The simulation results demonstrate that the adaptive fractional sliding mode controller has the benefits of higher tracking precision and properly faster response than the conventional adaptive integral order sliding mode controller.

REFERENCES

[1] D. Xia, L. Kong, Y. Hu, and P. Ni, "Silicon microgyroscope temperature prediction and control system based on BP neural network and fuzzy-PID control method," *Meas. Sci. Technol.*, vol. 26, no. 2, Jan. 2015, Art. no. 025101.

[2] J. Song, Y. Niu, and Y. Y. Zou, "Finite-time stabilization via sliding mode control," *IEEE Trans. Autom. Control*, vol. 62, no. 3, pp. 1478–1483, Mar. 2017.

[3] Q. Xu, "Continuous integral terminal third-order sliding mode motion control for piezoelectric nanopositioning system," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 4, pp. 1828–1838, Aug. 2017.

[4] Y. Zhu and J. Fei, "Disturbance observer based fuzzy sliding mode control of PV grid connected inverter," *IEEE Access*, vol. 6, pp. 21202–21211, 2018.

[5] M. Choi, B. Shirinzadeh, and R. Porter, "System identification-based sliding mode control for small-scaled autonomous aerial vehicles with unknown aerodynamics derivatives," *IEEE/ASME Trans. Mechatronics*, vol. 21, no. 6, pp. 12944–12952, Dec. 2016.

[6] J. Song, Y. Niu, and Y. Zou, "A parameter-dependent sliding mode approach for finite-time bounded control of uncertain stochastic systems with randomly varying actuator faults and its application to a parallel active suspension system," *IEEE Trans. Ind. Electron.*, vol. 65, no. 10, pp. 8124–8132, Oct. 2018.

[7] J. Fei and Z. Feng, "Adaptive fuzzy super-twisting sliding mode control for microgyroscope," *Complexity*, vol. 2019, Feb. 2019, Art. no. 6942642. doi: 10.1155/2019/6942642.

[8] Y. Fang, J. Fei, and Y. Yang, "Adaptive backstepping design of a microgyroscope," *Micromachines*, vol. 9, no. 7, p. 338, Jul. 2018. doi: 10.3390/mi9070338.

[9] R. P. Leland, "Adaptive control of a MEMS gyroscope using Lyapunov methods," *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 2, pp. 278–283, Mar. 2006.

[10] N.-C. Tsai and C.-Y. Sue, "Integrated model reference adaptive control and time-varying angular rate estimation for micro-machined gyroscopes," *Int. J. Control*, vol. 83, no. 2, pp. 246–256, 2010.

[11] J. Fei and J. Zhou, "Robust adaptive control of MEMS triaxial gyroscope using fuzzy compensator," *IEEE Trans. Syst., Man, Cybern. B. Cybern.*, vol. 42, no. 6, pp. 1599–1607, Dec. 2012.

[12] D. Xia, Y. Hu, and P. Ni, "A digitalized gyroscope system based on a modified adaptive control method," *Sensors*, vol. 16, no. 3, p. 321, Mar. 2016.

[13] J. Fei and C. Lu, "Adaptive sliding mode control of dynamic systems using double loop recurrent neural network structure," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1275–1286, Apr. 2018.

[14] Y. Chu, J. Fei, and S. Hou, "Adaptive global sliding-mode control for dynamic systems using double hidden layer recurrent neural network structure," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published. doi: 10.1109/TNNLS.2019.2919676.

[15] Y. Liu, M. Gong, S. Tong, C. L. P. Chen, and D.-J. Li, "Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 2607–2617, Oct. 2018. doi: 10.1109/TFUZZ.2018.2798577.

[16] Y. Li and S. Tong, "Adaptive fuzzy control with prescribed performance for block-triangular-structured nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1153–1163, Jun. 2018. doi: 10.1109/TFUZZ.2017.2710950.

[17] H. Wang, P. Liu, and B. Niu, "Robust fuzzy adaptive tracking control for nonaffine stochastic nonlinear switching systems," *IEEE Trans. Cybern.*, vol. 48, no. 8, pp. 2462–2471, Aug. 2018.

[18] H.-N. Wu and S. Feng, "Mixed fuzzy/boundary control design for nonlinear coupled systems of ODE and boundary-disturbed uncertain beam," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3379–3390, Dec. 2018. doi: 10.1109/TFUZZ.2018.2826475.

[19] J. Fei and H. Ding, "Adaptive sliding mode control of dynamic system using RBF neural network," *Nonlinear Dyn.*, vol. 70, no. 2, pp. 1563–1573, Oct. 2012.

[20] J. Fei and T. Wang, "Adaptive fuzzy-neural-network based on RBFNN control for active power filter," *Int. J. Mach. Learn. Cybern.*, vol. 10, no. 5, pp. 1139–1150, May 2019.

[21] S. Hou, J. Fei, and C. Chen, "Finite-time adaptive fuzzy-neural-network control of active power filter," *IEEE Trans Power Electron.*, vol. 34, no. 10, pp. 10298–10313, Oct. 2019.

[22] Y. Chu and J. Fei, "Dynamic global PID sliding mode control using RBF neural compensator for three-phase active power filter," *Trans. Inst. Meas. and Control*. vol. 40, no. 12, pp. 3549–3559, 2018.

[23] J. Fei and Y. Chu, "Double hidden layer output feedback neural adaptive global sliding mode control of active power filter," *IEEE Trans. Power Electron.*, to be published. doi: 10.1109/TPEL.2019.2925154.

[24] M. Roopaei, B. Sahraei, and T.-C. Lin, "Adaptive sliding mode control in a novel class of chaotic systems," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 15, no. 12, pp. 4158–4170. Dec. 2010.

[25] M. Hua, D. Zheng, F. Deng, J. Fei, P. Cheng, and X. Dai, "H∞ filtering for nonhomogeneous Markovian jump repeated scalar nonlinear systems with multiplicative noises and partially mode-dependent characterization," *IEEE Trans. Syst., Man, Cybern.*, to be published. doi: 10.1109/TSMC.2019.2919146.

[26] J. Fei and X. Liang, "Adaptive backstepping fuzzy neural network fractional-order control of microgyroscope using a nonsingular terminal sliding mode controller," *Complexity*, vol. 2018, Sep. 2019, Art. no. 5246074. doi: 10.1155/2018/5246074.

[27] Y. Fang, J. Fei, and D. Cao, "Adaptive fuzzy-neural fractional-order current control of active power filter with finite-time sliding controller," *Int. J. Fuzzy Syst.*, vol. 21, no. 5, pp. 1533–1543, Jul. 2019. doi: 10.1007/s40815-019-00648-4.
[28] G. Sun, Z. Ma, and J. Yu, "Discrete-time fractional order terminal sliding mode tracking control for linear motor," *IEEE Trans. Ind. Electron.*, vol. 65, no. 4, pp. 3386–3394, Apr. 2018.
[29] S.-Y. Chen, H.-H. Chiang, and T.-S. Liu, and C.-H. Chang, "Precision motion control of permanent magnet linear synchronous motors using adaptive fuzzy fractional-order sliding-mode control," *IEEE/ASME Trans. Mechatronics*, vol. 24, no. 1, pp. 741–752, Apr. 2019.
[30] C. Yin, B. Stark, Y. Q. Chen, S.-M. Zhong, and E. Lau, "Fractional-order adaptive minimum energy cognitive lighting control strategy for the hybrid lighting system," *Energy Buildings*, vol. 87, pp. 176–184, Jan. 2015.
[31] X. Dong, D. Zhao, and B. Yang, "Fractional-order control of active suspension actuator based on parallel adaptive clonal selection algorithm," *J. Mech. Sci. Technol.*, vol. 30, no. 6, pp. 2769–2781, Jun. 2016.
[32] D. Nojavanzadeh and M. Badamchizadeh, "Adaptive fractional-order nonsingular fast terminal sliding mode control for robot manipulators," *IET Control Theory Appl.*, vol. 10, no. 13, pp. 1565–1572, 2016.

FANG CHEN received the B.S. degree in electrical engineering from Henan University, China, in 2018. She is currently pursuing the M.S. degree in electrical engineering with Hohai University, China. Her research interests include adaptive control, power electronics, nonlinear control, and neural networks.

JUNTAO FEI (M'03–SM'14) received the B.S. degree in electrical engineering from the Hefei University of Technology, China, in 1991, the M.S. degree in electrical engineering from the University of Science and Technology of China, in 1998, and the M.S. and Ph.D. degrees in mechanical engineering from The University of Akron, OH, USA, in 2003 and 2007, respectively. He was a Visiting Scholar with the University of Virginia, VA, USA, from 2002 to 2003. He was a Postdoctoral Research Fellow and an Assistant Professor with the University of Louisiana, LA, USA, from 2007 to 2009. He is currently a Professor with Hohai University, China. His research interests include adaptive control, nonlinear control, intelligent control, dynamics and control of MEMS, and smart materials and structures.