IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received September 22, 2019, accepted October 3, 2019, date of publication October 10, 2019, date of current version October 23, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2946387

A Robust Fundamental Matrix Estimation Method Based on Epipolar Geometric Error Criterion

KUN YAN[1], RUJIN ZHAO[1], ENHAI LIU[1], AND YUEBO MA[1,2]
[1]Institute of Optics and Electronics of Chinese Academy of Sciences, Chengdu 610209, China
[2]University of Chinese Academy of Sciences, Beijing 100149, China

Corresponding author: Kun Yan (yankunioe@163.com)

This work was supported by the Institute of Optics and Electronics, Chinese Academy of Sciences, in part by the National Natural Science Foundation under Grant 61501429, and in part by the Project of Young Scholars in Western China, Chinese Academy of Sciences, under Grant Y929181C21.

ABSTRACT In this paper, a robust fundamental matrix estimation method based on epipolar geometric error criterion is proposed. First, the method removes outliers into the computation of the fundamental matrix instead of taking it as an independent processing step. The potential error corresponding points are eliminated by iteration to achieve the stable estimation of the fundamental matrix. Then, the epipolar geometry error criterion is used to identify outliers and the estimation results of the fundamental matrix are obtained during each iteration. The iterative process can converge quickly. Even if a large number of matched outliers are present, the calculated values will soon become stable. Experiments have been carried out for synthetic and real image pairs, which show that the proposed method performs very well in terms of robustness to noises and outliers. Additionally it has a low computational cost and is convenient for use in practical applications.

INDEX TERMS Computer vision, fundamental matrix, epipolar geometry, robustness.

I. INTRODUCTION

Two perspective projective images of a single rigid scene are related by the epipolar geometry [1], which can be described by a matrix called fundamental matrix (F-matrix). All the geometric information contained in the two views is completely captured by it. The fundamental matrix is independent of the scene structure, and can be computed from correspondences of imaged scene points alone, without requiring knowledge of the cameras' internal parameters or relative pose [2]. Estimating the fundamental matrix is a basic step for a wide variety of vision-based applications [3], [4], such as 3D reconstruction [5], camera-pair calibration [5], [6], object matching & tracking [7] object recognition [8], *etc*. The estimated F-matrix is useful for recovering the relative motion of camera, guiding correspondences establishment in stereo matching, and so on. Therefore, developing a method for fundamental matrix estimation with high efficiency and robustness is of great significance.

Due to the great importance of the F-matrix, it has received much attention in both photogrammetry and computer vision communities. An application of scene reconstruction using epipolar geometry was first published by H.C.Longuet-Higgins [9]. Over the years, numerous algorithms for F-matrix estimation have been developed. In [10], [11], different types of estimation are described and compared in detail. They can roughly be divided into linear methods, iterative methods, and robust methods. Linear methods are mainly based on least-squares minimization and eigen values, which are sensitive to noise. In order to improve the estimation accuracy in noisy situation with point localization error, iterative methods are proposed by minimizing the error cost function. Fathy *et al.* [12] summarized and compared the accuracy and efficiency of the different error criteria for computing the fundamental matrix. Subsequently, an iterative algorithm based on the least absolute deviation is put forward [13], further improves the estimation accuracy. While in practical applications, due to the drawback of feature extraction and matching methods, the correspondences data are corrupted not only by noise but also by gross outliers (false matches). In which case, the estimation accuracy of linear and iterative methods would be seriously reduced.

Robust methods are based on computing a more accurate geometry relation and removing false matches. An early example of a robust algorithm is the random sample consensus paradigm (RANSAC) of Fischler and Bolles [14]. As small a subset of the data as is feasible to estimate the parameters is used (e.g, seven correspondences for fundamental

The associate editor coordinating the review of this manuscript and approving it for publication was Vincenzo Piuri.

matrix estimation), and this process is repeated enough times, trying to find the largest consensus on an estimated F-matrix. Later, some researchers have presented different methods for improvement, such as LMedSeig, MLESAC and MAPSAC, SDO, etc [11], [15]–[18]. The key idea of these algorithms is to remove outliers independently based on the hypothesis testing strategy. As they require random sampling test for times, and all the datasets are involved to distinguish inliers or outliers for each testing process. Thus, they are computationally expensive with a low processing speed. It is noteworthy that in [19], two nonparametric robust methods are proposed to compute the fundamental matrix by modifying the ORSA method and using an empirical reference distribution of the image features. Besides, in literature [20], the authors first establish initial correspondences by feature description matches, and then estimate the fundamental matrix and homography using L2E-LSC and get the refined correspondences. However, the time complexities of their method needs to be improved.

So far different methods have been presented [21]–[31], while the problem of F-matrix estimation has not been well solved. On the whole, linear methods don't perform quite well if the points are badly located in the image; iterative methods can deal with some localization error of points, but they are inefficient in the presence of outliers; robust methods can cope with discrepancies in the localization of points and false matches at the same time, but with an extra computational complexity. Therefore, robust computation is still a subject for wide research focusing mainly on proposing new estimators to improve the accuracy of the fundamental matrix and on reducing computation expenses.

In this paper, we propose a novel F-matrix estimation method that integrates an outlier-rejection mechanism and does not need to resort to an independent and separate strategy. Like in RANSAC, our approach also iterates to remove outliers. Where at each step, the criterion that results from the geometric error (geometric distance between image point and corresponding epipolar line) is used to reject them. This process has a fast converge, even in situations where many correspondences outliers exist. As we will demonstrate in the experimental section, this results in speed-ups of computation time compared to the methods with hypothesis testing strategy, while yielding similar or comparable accuracy.

The remainder of this paper is organized as follows. The basics of epipolar geometry are revisited in Section 2. The linear formation for fundamental matrix estimation and the details of the proposed algorithm are presented in Section 3. In Section 4, both simulative and real data are used to validate the proposed method, and the paper is concluded in Section 5.

II. EPIPOLAR GEOMETRY AND FUNDAMENTAL MATRIX

The epipolar geometry of scene is illustrated in Figure 1, where a 3D point $\mathbf{P}_i$ is projected onto image point $\mathbf{x}_i$ and $\mathbf{x}'_i$ in the two different image planes, respectively. Where O, O' represent the camera centers of the two cameras.

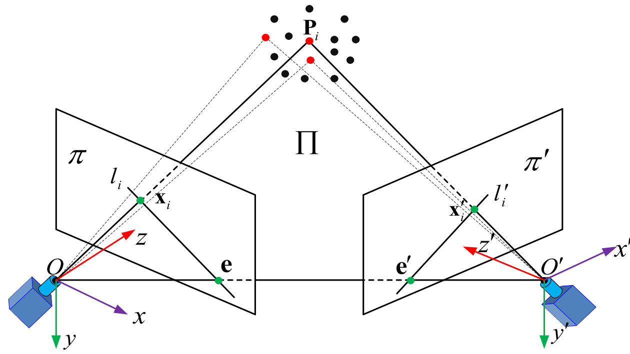


FIGURE 1. Epipolar geometry.

The line linking the two optical centers is called baseline, which intersects with corresponding image plane $\pi'(\pi)$ at $\mathbf{e}'(\mathbf{e})$, called epipole. The plane $\prod$ that passes through both the camera centers and the real world 3D point $\mathbf{P}_i$ is called the epipolar plane. And the line through the epipole and an image point is the epipolar line, which is also the intersection of epipolar plane with image plane.

From these two images, we can get the homogenous coordinates of a pair of corresponding points, $\mathbf{x}_i$ and $\mathbf{x}'_i$, corresponding to the same 3D point. The relation of the point correspondence can be expressed by the fundamental matrix $\mathbf{F}$, as shown in the following equation,

$$\mathbf{x}'^T_i \mathbf{F} \mathbf{x}_i = 0 \tag{1}$$

This is known as the epipolar constraint, whose geometrical meaning is that, point pairs $\mathbf{x}'_i$ and $\mathbf{x}_i$ must lie on its corresponding epipolar lines, denoted by $l'_i = \mathbf{F}\mathbf{x}'_i$, and $l_i = \mathbf{F}^T\mathbf{x}'_i$.

Actually, the fundamental matrix contains the intrinsic parameters of both cameras and the rigid transformation of one camera related to the other. Eq. (1) characterizes the fundamental matrix only in terms of correspondences, which provides a cue to estimate the fundamental matrix without knowing the camera matrices.

III. PROPOSED ROBUST METHOD FOR F-MATRIX ESTIMATION

With the inspiration of ref. [32], in which a robust solution to the Perspective-n-Point (PnP) problem is presented by integrating the outlier removal procedure. We apply the idea to estimate the fundamental matrix, the details of the proposed algorithm is described in this section. Firstly, the linear formulation of the F-matrix estimation is revisited. Then we combine the robust outlier rejection scheme with the linear formulation. With several iterations, the outliers are discarded progressively based on the criterion of geometric error, and the F-matrix is solved by estimating the null space of the linear system. Although we adopt the idea of ref. [32], the method of this paper is different. We have made corresponding improvements. In ref. [32], the author did not calculate the fundamental matrix separately. In addition, the selection of error criteria is different. We choose polar geometric errors, while algebraic errors are used in ref. [32].

A. LINEAR FORMULATION FOR F-MATRIX ESTIMATION

Given two images of the same scene, a point $\mathbf{x}_i$ in the first image and the corresponding point $\mathbf{x}'_i$ in the second satisfy the epipolar constraint equation Eq. (1). Assume that the homogenous coordinates of a pair of imaged points corresponding to the same 3D point are $\mathbf{x}_i = (u_i, v_i, 1)^T$ and $\mathbf{x}'_i = (u'_i, v'_i, 1)^T$, respectively. And the Eq. (1) can be rewritten as,

$$(u'_i, v'_i, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u_i \\ v_i \\ 1 \end{pmatrix} = 0 \tag{2}$$

Then we formulate the following equation which can be deduced from Eq. (2),

$$\mathbf{U}_i\mathbf{f} = [(u_i, v_i, 1) \otimes (u'_i, v'_i, 1)]\mathbf{f} = \left[\mathbf{x}_i^T \otimes \mathbf{x'}_i^T\right]\mathbf{f} = \mathbf{0} \tag{3}$$

where $\otimes$ denotes Kronecker product, with vector $\mathbf{f}$ as,

$$\mathbf{f} = (F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33})^T \tag{4}$$

Finally, concatenating these equations for n correspondences $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ $(i = 1, \cdots, n)$ can be expressed as a linear system,

$$\mathbf{Mf} = \mathbf{0} \tag{5}$$

where $\mathbf{M}$ is a $n \times 9$ matrix, generated by the n image correspondences. It provides a cue to estimate the fundamental matrix based on solving a linear system in terms of correspondences.

B. INTEGRATE OUTLIER REMOVAL WITHIN LINEAR F-MATRIX ESTIMATIONS

The solution of Eq. (5) can be expressed as a linear combinations of the null eigenvectors of $\mathbf{M}$. If the correspondences are noiseless, then the rank of null-space of $\mathbf{M}$ should be exactly one and we can solve for F-matrix up to a scale factor. However, in the presence of noise and outliers, $\mathbf{M}$ has no eigenvalue that is strictly zero, though some of which are very close to zero. Following the suggestion of ref. [32], we assume the rank of the null-space of $\mathbf{M}$ to be always one. Then a robust method is proposed to compute this null space while removing outliers and gross error correspondences.

In order to remove the outliers, we denote matrix $\mathbf{L}$ to be a noise-free version of $\mathbf{M}$, and solving $\mathbf{L}$ is converted into the following minimization,

$$\begin{aligned} &\arg\min_{\mathbf{L},\mathbf{W}} \|\mathbf{W}(\mathbf{M} - \mathbf{L})\|^2 \\ &\textit{subject to } rank(\mathbf{L}) = rank(\mathbf{M}) - 1 \end{aligned} \tag{6}$$

where $\mathbf{W} = diag(w_1, w_2, \cdots, w_{n-1}, w_n)$ is a $n \times n$ diagonal matrix. If the i-th data-pair is considered as inliers, then $w_i = 1$. And $w_i = 0$ indicates an outlier of the i-th correspondence.

Here we aim to compute the null eigenvector of $\mathbf{M}$, i.e, the F-matrix vector $\mathbf{f}$ as defined in Eq. (4). By distinguishing the outliers from data correspondences, we have the matrix $\mathbf{L}$ with $\mathbf{Lf} = \mathbf{0}$. Then, we multiply the both sides of Eq. (6) by $\mathbf{f}$, the constrained minimization of Eq. (6) is turned into the following minimization with vector $\mathbf{f}$ and matrix $\mathbf{W}$,

$$\arg\min_{\mathbf{f},\mathbf{W}} \|\mathbf{WMf}\|^2 \tag{7}$$

To solve this optimization, we compute $\mathbf{f}$ and $\mathbf{W}$ iteratively by integrating outlier removal, as detailed in Algorithm 1. Initially, all correspondences are assumed to be inliers, and thus $\mathbf{W}$ is initialized to the $n \times n$ identity matrix. We then compute the singular value decomposition of $\mathbf{M}^T\mathbf{WM}$, and take $\mathbf{f}$ to be the eigenvector associated to the smallest singular value. In each iteration, the criterion that results from the geometric error is used to distinguish the inliers and outliers. That is, all the entries of the matrix $\mathbf{W}$ are updated according the function as follows,

$$w_i = \begin{cases} 1, & \textit{if } \varepsilon_i \le \max(\varepsilon_{\max}, \delta_{\max}) \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where the geometric error (i.e, geometric distance between image point and corresponding epipolar line) associated to

TABLE 1. Algorithm 1.

Algorithm 1 *Robust estimation for the fundamental matrix* $\mathbf{F}$

Input: $\mathbf{M}$: $n \times 9$ matrix generated by correspondence; $\delta_{\max}$: maximal geometric error (geometric distance between image point and corresponding epipolar line).
Output: the fundamental matrix $\mathbf{F}$.
Initialize: $\mathbf{W} = \mathbf{I}_n$, $\xi = \text{Inf}$.

1: loop
2: $[\mathbf{U},\mathbf{S},\mathbf{V}] = svd(\mathbf{M}^T\mathbf{WM})$; where $\mathbf{V} = [\mathbf{v}_1, \quad, \mathbf{v}_9]$; $\mathbf{S} = diag(s_1, \quad, s_9)$.
3: $\mathbf{f} \leftarrow \mathbf{v}_k$ **; where** $k : s_k = \min(s_1, \quad, s_9)$ **.**
4: Compute the fundamental matrix $\mathbf{F}$ corresponding to vector $\mathbf{f}$ using Eq. (4).
5: Compute $\{\varepsilon_i\}(i = 1, \quad, n)$, the geometric distance between image points and corresponding epipolar lines using Eq. (9).
6: $\varepsilon_{\max} = Q_{25\%}(\varepsilon_1, \quad, \varepsilon_n)$.
7: if $\varepsilon_{\max} > \xi$ **then**
    return F
  else
    $\xi = \varepsilon_{\max}$
end if
8: Update matrix $\mathbf{W}$ by computing $\{w_i\}(i = 1, \quad, n)$ using Eq. (8).
9: end loop

the i-th correspondence is computed as,

$$\varepsilon_i = \left(\frac{1}{\sqrt{(\mathbf{F}\mathbf{x}'_i)_1^2 + (\mathbf{F}\mathbf{x}'_i)_2^2}} + \frac{1}{\sqrt{(\mathbf{F}^T\mathbf{x}_i)_1^2 + (\mathbf{F}^T\mathbf{x}_i)_2^2}}\right) \left\|\mathbf{x}_i^T \mathbf{F}\mathbf{x}'_i\right\| \tag{9}$$

And $\varepsilon_{\max} = Q_{25\%}(\varepsilon_1, \cdots, \varepsilon_n)$, the lowest 25% quartile of geometric error for all the correspondences, is used to remove the outliers with large geometric errors. Note that in an outlier-free case, the inliers correspondences would be wrongly considered to be outliers and rejected only by the criterion $\varepsilon_{\max}$. For this, a geometric error threshold $\delta_{\max}$ is also introduced, avoiding this situation and achieving faster convergence at the same time. While the threshold $\delta_{\max}$ is related with the location accuracy of data correspondence, we set $\delta_{\max} = 0.18$ in our experiments. With several iterations, it would converge as the outliers are removed; the robust estimation for the fundamental matrix $\mathbf{F}$ is obtained.

C. DATA NORMALIZATION

A normalization process is needed before applying the algorithm for fundamental matrix estimation to reduce its susceptibility to noise. According to the suggestion of ref. [10], the raw data is normalized by using Hartley's method [33]. The normalization is done by translating the centroid of the measured image points to the origin and scaling them to a average distance of $\sqrt{2}$ from the centroid (origin). In which method, image coordinates are transformed by multiplying them by $\mathbf{T}$ in one image and $\mathbf{T}'$ in the second image, where $\mathbf{T}$ and $\mathbf{T}'$ are scaling and translation normalizations. The fundamental matrix approximation $\hat{\mathbf{F}}$ is estimated from the normalized data then. And it is de-normalized to obtain the final estimation of fundamental matrix $\mathbf{F} = \mathbf{T}'^T\hat{\mathbf{F}}\mathbf{T}$ that corresponds to the original image correspondences.

IV. EXPERIMENTAL RESULTS

We adopt the two testing datasets from the Affine Covariant Regions Datasets and USAC Datasets.The two Datasets were released by the Oxford University.We compared the proposed method with other state-of-art robust methods(M-estimator [10], LmeSeig [11], RANSAC [14], FNS [26], SDO [18], L2E-LSC [20]) using simulated data that contained different levels of Gaussian noise and false matches, as well as real images with different scenarios. In which process, the average epipolar distance of inliers were used as the evaluation performance criteria to test the accuracy [7]. All the algorithms are implemented with Matlab R2017a and performed on a PC with Intel(R) Core(TM) i5-4300 M CPU @ 2.6 GHz.

A. EXPERIMENTS WITH SYNTHETIC DATA

In our simulation experiment, we generate 125 pairs of correspondences points that distributed in the synthetic images. We perturb the synthetic data by Gaussian noise and outliers to simulate the feature location error and false matching. The proposed method is compared with other six robust methods: M-estimator, LmedSeig, RANSAC, FNS, SDO and L2E-LSC. The implementations of the compared methods are provided in reference [11]. In which, the parameters of the compared methods used for the experiments are detailed. Their accuracy and robustness are analyzed with the synthetic data varying the Gaussian noise and the percentage of outliers. The mean and standard deviation of the distance between points and epipolar lines are computed on the set of inlier matches, which are used as measurement of the accuracy of different F-matrix estimation methods. For each group of data, we perform 100 independent trials, and the results given below are the average.

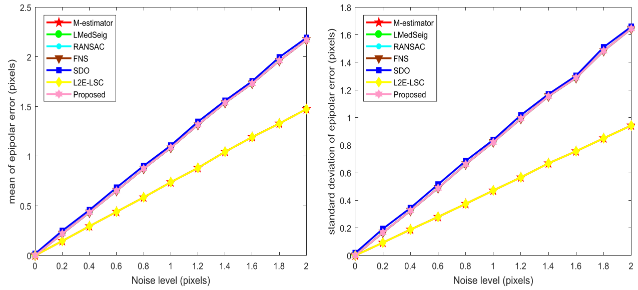


FIGURE 2. Comparison of the performance for different F-matrix estimation methods by varying the standard deviation of Gaussian noise. (a) the mean of the distance between correspondences points and epipolar lines; (b) the standard deviation of the distance between correspondences points and epipolar lines.

In the first simulation experiment, the ratio of outliers is fixed to 0, and the Gaussian noise with a mean of 0 and a standard deviation of 0 to 2 pixels is added to the data set of the simulation points for testing.The results obtained by different F-matrix estimation methods are illustrated in Figure 2, in which the mean and standard deviation of the distance between correspondences points and epipolar lines are used for performance (or accuracy) comparison. As we can see, the accuracy of all the estimation methods decreases linearly with the increasing noise level. The M-estimator method and L2E-LSC method achieve better results in the conditions of different noise levels. The performance of the proposed method is comparable to that of LMedSeig, RANSAC, FNS and SDO methods. For the proposed estimation method with Gaussian noise of 1 pixel, the mean of the distance between correspondences points and epipolar lines is around 1.1 pixels and the standard deviation is less than 1 pixel.

In the second simulation experiment, we do not add noise to the data set and only change the proportion of outliers (0∼45%) to verify the robustness of the estimation method. The results obtained by different F-matrix estimation methods are shown in Figure 3, and the performance (or accuracy) is also compared using the mean and standard deviation of the distance between the correspondences points and epipolar lines. As we can see, the accuracy of all estimation methods decreases approximately linearly with the increase of the proportion of outliers. The LMedSeig method achieves better results in the conditions of different ratio of outliers. Besides, the two methods of RANSAC and FNS have achieved relatively poor results in different proportion of outliers.

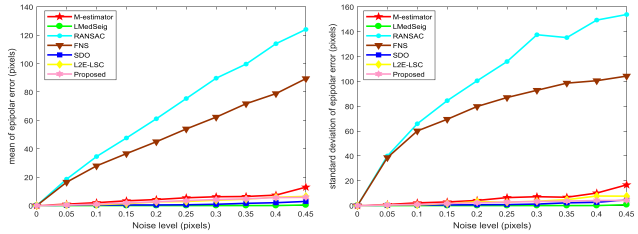


FIGURE 3. **Comparison of the performance for different F-matrix estimation methods by varying the proportion of outliers. (a) the mean of the distance between correspondences points and epipolar lines; (b) the standard deviation of the distance between correspondences points and epipolar lines.**

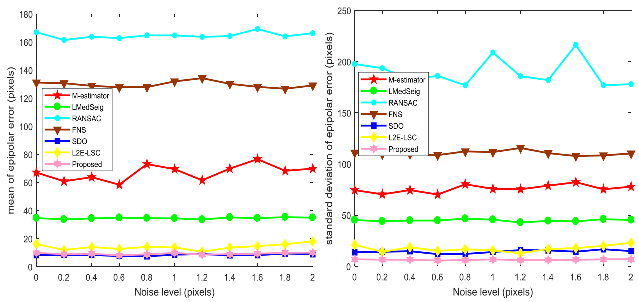


FIGURE 4. **Comparison of the performance for different F-matrix estimation methods by fixing a certain proportion of outliers and varying the standard deviation of Gaussian noise. (a) the mean of the distance between correspondences points and epipolar lines; (b) the standard deviation of the distance between correspondences points and epipolar lines.**

However, the performance of the proposed method is comparable to that of M-estimator, SDO and L2E-LSC methods.

In the third simulation experiment, the proportion of fixed outliers is 60%, and Gaussian noise with mean value of 0 and standard deviation of 0 to 2 pixels is added to verify the robustness of the estimation method in the presence of outliers and noises in the data set. The results obtained by different F-matrix estimation methods are shown in Fig.4, and the performance (or accuracy) is also compared using the mean and standard deviation of the distance between the correspondences points and epipolar lines. As we can see, the accuracy of all estimation methods is approximately unchanged with the increase of noise level. This shows that the influence of outliers is greater than that of low noise when the ratio of outliers is 60%. Moreover, under different noise levels, the proposed method achieves good results.

Next, the estimation results of synthetic data in the presence of different percentages of outliers by different methods are summarized in Table 2. From which we can see that, all the robust methods could partly detect and remove potential outliers. M-estimator reduces the effect of outliers weighting the residual of each correspondence, which starts from a linear initial guess and is limited in presence of a large amount of outliers. RANSAC and LMedSeig are both based on hypothesis testing strategy. RANSAC estimates the F-matrix that maximizes the number of inliers, while LMedSeig choose that minimize the median distance between the points and epipolar lines. That LMedSeig is more restrictive

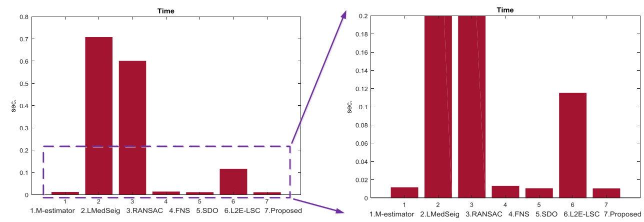


FIGURE 5. **The mean computational time using different methods with all the synthetic data, (b) is a partially enlarged view of the dashed box in (a).**

than RANSAC and the results show that LMedSeig achieve better accuracy. FNS is based on the non-linear gradient criterion and represents the basic data method, but its estimation accuracy is poor. The proposed method also has good robustness to outliers, whose accuracy is close to that obtained by L2E-LSC and SDO with data in presence of different percentages of outliers. Moreover, when the outliers exceed 50%, the proposed method has the smallest polar geometric error compared with other methods.

We also do the comparison about the computational complexity of different methods. The mean computational time with all the synthetic data is illustrated in Fig.5. It's observed that the proposed approach is faster than other four methods. This is because that our method integrates the outlier-rejection mechanism in estimating F-matrix and does not need to resort to an independent strategy that requires extra processing time.

B. EXPERIMENTS WITH REAL DATA

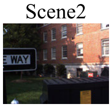
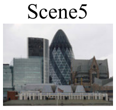
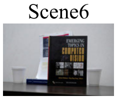
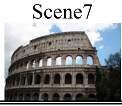
Real images in different scenarios (provided by ref. [10]) were selected and intensive experiments were carried out in order to further validate the proposed method. Here the points correspondence between images have been solved based on the method proposed by Zhang *et al.* [7]. In which, a Harris corner detector is applied to get a list of interesting points, followed by a pixel-based correlation method for points matching between two images. Figure 6 shows four sets of image pairs in different scenarios, the matching results (labeled by white '+'). And the epipolar geometry recovered by the proposed method is also given. In which, 20 pairs of matching inliers (labeled by red 'o') and the corresponding epipolar lines are illustrated. As we can see, F-matrix is accurately estimated by using the proposed method. With each matching point being on its corresponding epipolar line exactly, the epipolar geometry is properly modeled.

For each F-matrix estimation method, the mean and standard deviation of the distance between points and epipolar lines are shown in Table 3. In addition, Table 3 also counts the runtime of different methods for estimating the Fundamental Matrix. From which we can see that the estimation accuracy achieved by the proposed method is comparable with that of LMedSeig, SDO and L2E-LSC; and better than M-estimator, RANSAC and FNS method as a whole. Moreover, compared with the other six methods, the proposed method has the shortest running time. In order to

TABLE 2. Comparison of the performance for different F-matrix estimation methods by adding different percentages of outliers (the standard deviation of the added Gaussian noise is fixed to 1 pixel). Every cell shows the mean and standard deviation of the distance between correspondences points and epipolar lines.

Methods / Data	M-estimator	LMedSeig	RANSAC	FNS	SDO	L2E-LSC	Proposed
outliers 0%	0.809 0.548	1.328 1.088	4.444 16.908	4.184 11.939	1.343 1.203	1.182 0.989	1.179 0.940
outliers 10%	0.826 0.556	1.275 1.234	19.917 48.212	14.568 34.176	1.345 1.246	1.268 1.096	1.133 0.925
outliers 20%	1.065 0.692	1.455 1.874	33.370 61.149	24.939 45.599	1.456 3.286	1.336 0.957	1.253 0.952
outliers 30%	1.402 1.034	1.589 1.783	42.526 73.082	32.919 52.033	1.916 4.391	1.819 1.690	1.673 1.420
outliers 40%	1.423 1.133	3.937 5.624	66.380 101.714	42.762 58.177	1.887 3.678	1.964 1.476	1.552 1.341
outliers 50%	2.026 2.393	5.942 9.634	74.977 88.976	75.616 58.914	2.453 5.349	2.017 2.155	1.755 1.558
outliers 55%	5.771 16.596	11.267 19.984	87.407 102.461	73.718 57.238	2.223 3.658	1.913 1.633	1.586 1.401
outliers 60%	15.082 19.356	16.978 24.709	75.946 83.508	70.362 57.329	2.574 5.938	2.162 2.367	1.405 1.299

TABLE 3. Comparison of performance for the proposed method and other six methods by testing with real images. Each unit shows the average and standard deviation of the distance between the corresponding point and the exterior line, and running time.

Methods / Testing image	M-estimator	LmedSeig	RANSAC	FNS	SDO	L2E-LSC	Proposed
Scene1 	580.241 779.791 0.1371	11.424 18.577 5.8378	146.244 246.307 4.9713	158.41 235.41 0.0905	8.169 14.748 0.1310	6.236 10.239 0.0893	5.968 9.759 0.0810
Scene2 	91.864 101.064 0.2066	2.896 5.879 9.5933	93.967 164.404 7.8694	120.29 100.95 0.1651	2.232 3.439 0.1891	1.415 2.610 0.1792	1.212 2.405 0.1091
Scene3 	512.432 837.22 0.2589	0.629 1.068 12.0024	34.509 124.090 10.0753	29.928 135.09 0.2456	2.815 4.094 0.2535	1.404 2.518 0.2358	1.317 2.493 0.0953
Scene4 	162.901 469.817 0.2614	1.273 3.006 12.4944	100.739 146.594 10.4822	59.088 93.298 0.2382	1.873 2.062 0.3373	1.016 1.997 0.2373	0.976 1.766 0.1371
Scene5 	476.694 1161.497 0.1300	10.914 16.507 4.7394	102.417 117.948 3.3912	128.45 118.17 0.0893	6.638 6.571 0.1638	3.927 4.954 0.0844	3.633 3.554 0.0640
Scene6 	6.395 4.893 0.0797	0.209 0.266 3.4024	15.839 42.159 2.9245	28.778 41.932 0.0508	1.004 1.132 0.0874	0.481 0.834 0.0518	0.308 0.733 0.0378
Scene7 	148.093 304.365 0.0520	6.455 9.946 2.3066	53.018 76.110 2.0061	50.355 63.195 0.0359	3.838 3.909 0.0514	2.027 2.364 0.0410	1.819 2.102 0.0315

compare the robustness of the methods intuitively, Figure 7-Figure 8 give the results with Scene1 and Scene2. Of which, Figure 7(a) and Figure 8(a) show the initial correspondences; Figure 7(b)-(h) and Figure 8(b)-(h) show the inliers obtained by M-estimator, LMedSeig, RANSAC, FNS, SDO, L2E-LSC and the proposed method, respectively. We can see

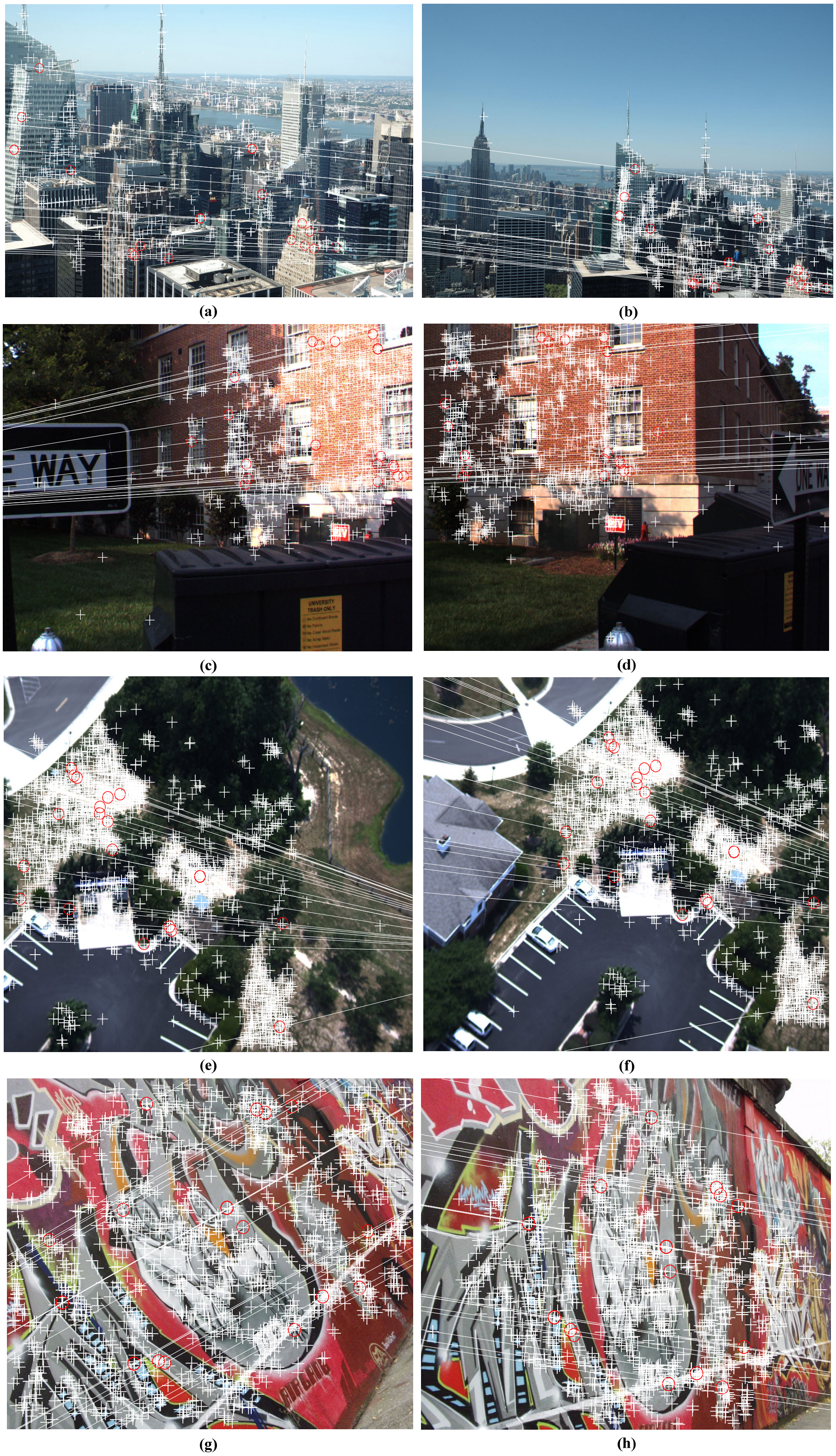


FIGURE 6. Four sets of image pairs in different scenarios: the matching results and epipolar geometry recovered by the proposed method. (a) Scene1, the first view; (b) Scene1, the second view; (c) Scene2, the first view; (d) Scene2, the second view; (e) Scene3, the first view; (f) Scene3, the second view; (g) Scene4, the first view; (h) Scene4, the second view.

that the poor results obtained by M-estimator, some good matchings are removed in Figure 7 (b) while many outliers are still kept in Figure 8(b); RANSAC and FNS do not perform well neither, as many outliers are not detected; LMedSeig, SDO and L2E-LSC have good robustness and they can remove most of the outliers; the proposed method

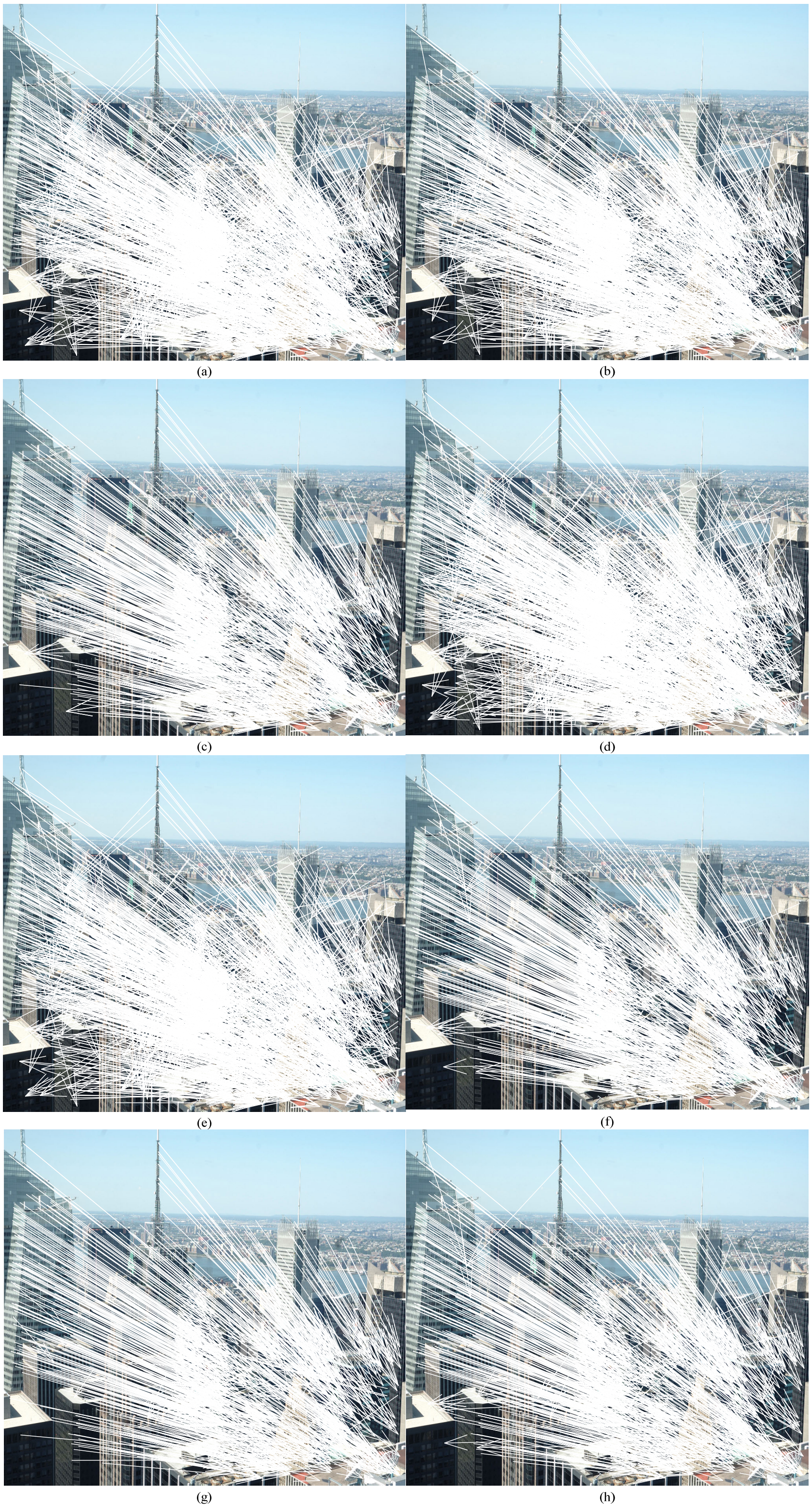


FIGURE 7. Robustness comparison of seven different methods with Scene1. (a) Initial correspondences; (b) Inliers obtained by M-estimator; (c) Inliers obtained by LMedSeig; (d) Inliers obtained by RANSAC; (e) Inliers obtained by FNS; (f) Inliers obtained by SDO; (g) Inliers obtained by L2E-LSC;(h) Inliers obtained by the proposed method.

obtains comparatively the same results with LMedSeig, SDO and L2E-LSC and there is few or no false matching in the obtained inliers.

Similarly, we compared the computation time for F-matrix estimation by M-estimator, LMedSeig, RANSAC, FNS, SDO, L2E-LSC and the proposed method with real images,

FIGURE 8. Robustness comparison of seven different methods with Scene2. (a) Initial correspondences; (b) Inliers obtained by M-estimator; (c) Inliers obtained by LMedSeig; (d) Inliers obtained by RANSAC; (e) Inliers obtained by FNS; (f) Inliers obtained by SDO; (g)Inliers obtained by L2E-LSC; (h) Inliers obtained by the proposed method.

which are shown in Figure 9. It can be seen that the proposed approach is faster than other six methods. It has good real-time performance, the average time for computing F-matrix is about 30 milliseconds.

The experiments with synthetic data and real images demonstrate that the proposed method has fairly good adaptability to noise and false matching. Comparing with other robust methods, good accuracy in fundamental matrix

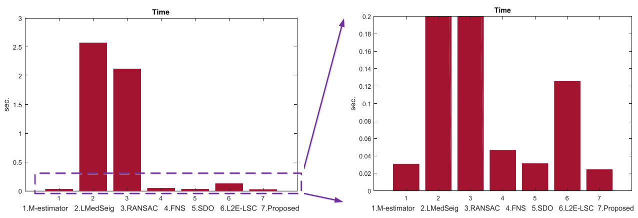


FIGURE 9. The Mean computational time using different methods with all the test images, (b) is a partially enlarged view of the dashed box in (a).

estimation is obtained by our approach. Moreover, it has obvious advantage in terms of computational efficiency.

V. CONCLUSION

A very fast and robust method to estimate the fundamental matrix from image pairs is proposed in this paper. Instead of using the hypothesis testing strategy for outlier removal, we integrate the outlier rejection scheme within the F-matrix estimation pipeline. The validity of the method is demonstrated both in simulation and experimentally. The results show that the new approach achieves a good performance by comparing to several other robust methods. Despite of its robustness to outliers and noises, the computational efficiency is particularly high, which is very appropriate for practical application.

ACKNOWLEDGMENT

The authors greatly appreciate X. Armangué for providing the testing images and the implementations of the M-estimator, LMedSeig, RANSAC, FNS methods. The authors also appreciate the anonymous reviewers whose comments led to substantial improvements in this manuscript.

REFERENCES

[1] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*. Cambridge, U.K.: Cambridge Univ. Press, 2000.
[2] Z. Zhang, "Determining the epipolar geometry and its uncertainty: A review," *Int. J. Comput. Vis.*, vol. 27, no. 2, pp. 161–195, 1998.
[3] C. Steger, "Estimating the fundamental matrix under pure translation and radial distortion," *ISPRS J. Photogramm. Remote Sens.*, vol. 74, pp. 202–217, Nov. 2012.
[4] G. M. Jog, H. Fathi, and I. Brilakis, "Automated computation of the fundamental matrix for vision based construction site applications," *Adv. Eng. Inform.*, vol. 25, pp. 725–735, Oct. 2011.
[5] Q.-T. Luong and O. D. Faugeras, "Camera calibration, scene motion, and structure recovery from point correspondences and fundamental matrices," *Int. J. Comput. Vis.*, vol. 22, no. 3, pp. 261–289, 1995.
[6] Y. Hong, G. Ren, and E. Liu, "Non-iterative method for camera calibration," *Opt. Express*, vol. 23, no. 18, pp. 23992–24003, 2007.
[7] Z. Zhang, R. Deriche, O. Faugeras, and Q.-T. Luong, "A robust technique for matching two uncalibrated images through the recovery of the unknown epipolar geometry," *Artif. Intell.*, vol. 78, nos. 1–2, pp. 87–119, 1995.
[8] Y. Gao, J. Ma, and A. L. Yuille, "Semi-supervised sparse representation based classification for face recognition with insufficient labeled samples," *IEEE Trans. Image Process.*, vol. 26, no. 5, pp. 2545–2560, May 2017.
[9] H. C. Longuet-Higgins, "A computer algorithm for reconstructing a scene from two projections," *Nature*, vol. 293, no. 5828, pp. 133–135, 1981.
[10] P. H. S. Torr and D. W. Murray, "The development and comparison of robust methods for estimating the fundamental matrix," *Int. J. Comput. Vis.*, vol. 24, no. 3, pp. 271–300, 1997.
[11] X. Armangué and J. Salvi, "Overall view regarding fundamental matrix estimation," *Image Vis. Comput.*, vol. 21, no. 2, pp. 205–220, 2003.
[12] M. E. Fathy, A. S. Hussein, and M. F. Tolba, "Fundamental matrix estimation: A study of error criteria," *Pattern Recognit. Lett.*, vol. 32, pp. 383–391, Jan. 2011.
[13] M. Yang, Y. Liu, and Z. You, "Estimating the fundamental matrix based on least absolute deviation," *Neurocomputing*, vol. 74, pp. 3638–3645, Oct. 2011.
[14] M. A. Fischler and R. C. Bolles, *Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography*. San Mateo, CA, USA: Morgan Kaufmann, 1987.
[15] P. H. S. Torr and A. Zisserman, "MLESAC: A new robust estimator with application to estimating image geometry," *Comput. Vis. Image Understand.*, vol. 78, no. 1, pp. 138–156, 2000.
[16] P. H. S. Torr, "Bayesian model estimation and selection for epipolar geometry and generic manifold fitting," *Int. J. Comput. Vis.*, vol. 50, no. 1, pp. 35–61, 2002.
[17] G. Chesi, A. Garulli, A. Vicino, and R. Cipolla, "Estimating the fundamental matrix via constrained least-squares: A convex approach," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 24, no. 3, pp. 397–401, Mar. 2002.
[18] C.-B. Xiao, D.-Z. Feng, and M.-D. Yuan, "Soft decision optimization method for robust fundamental matrix Estimation," *Mach. Vis. Appl.*, vol. 30, pp. 657–669, Jun. 2019.
[19] F. Espuny, P. Monasse, and L. Moisan, "A new a contrario approach for the robust determination of the fundamental matrix," in *Proc. Image Video Technol.-PSIVT Workshops*, 2013, pp. 181–192.
[20] L. Peng, Y. Zhang, H. Zhou, and T. Lu, "A robust method for estimating image geometry with local structure constraint," *IEEE Access*, vol. 6, pp. 20734–20747, 2018.
[21] Z. L. Sun, K. M. Lam, and Q. W. Gao, "Depth estimation of face images using the nonlinear least-squares model," *IEEE Trans. Image Process.*, vol. 22, no. 1, pp. 17–30, Jan. 2013.
[22] P. Chen, "Why not use the Levenberg–Marquardt method for fundamental matrix estimation?" *IET Comput. Vis.*, vol. 4, pp. 286–294, Dec. 2011.
[23] Y. Zheng, S. Sugimoto, and M. Okutomi, "A practical rank-constrained eight-point algorithm for fundamental matrix estimation," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, vol. 9, Jun. 2013, pp. 1546–1553.
[24] F. Bugarin, A. Bartoli, D. Henrion, J.-B. Lasserre, J.-J. Orteu, and T. Sentenac, "Rank-constrained fundamental matrix estimation by polynomial global optimization versus the eight-point algorithm," *J. Math. Imag. Vis.*, vol. 53, no. 1, pp. 42–60, 2014.
[25] F. Zhou, C. Zhong, and Q. Zheng, "Method for fundamental matrix estimation combined with feature lines," *Neurocomputing*, vol. 160, pp. 300–307, Jul. 2015.
[26] W. Chojnacki, M. J. Brooks, A. van den Hengel, and D. Gawley, "FNS, CFNS and HEIV: A unifying approach," *J. Math. Imag. Vis.*, vol. 23, pp. 175–183, Sep. 2005.
[27] Z.-L. Sun, K.-M. Lam, and Q.-W. Gao, "An effective missing-data estimation approach for small-size image sequences," *IEEE Comput. Intell. Mag.*, vol. 10, no. 3, pp. 10–18, Aug. 2015.
[28] Y. Liu, Z.-L. Sun, X. Chen, and L. Shang, "A BRMF-based model for missing-data estimation of image sequence," *Neurocomputing*, vol. 228, pp. 65–70, Mar. 2017.
[29] Y. Liu, Z.-L. Sun, Y.-P. Wang, and L. Shang, "An eigen decomposition based rank parameter selection approach for the NRSFM algorithm," *Neurocomputing*, vol. 198, pp. 109–113, Jul. 2016.
[30] L. Wang, Z. Liu, and Z. Zhang, "Efficient image features selection and weighting for fundamental matrix estimation," *IET Comput. Vis.*, vol. 10, pp. 67–78, Feb. 2016.
[31] Y. Cheng, J. A. Lopez, O. Camps, and M. Sznaier, "A convex optimization approach to robust fundamental matrix estimation," in *Proc. CVPR*, 2015, pp. 2170–2178.
[32] L. Ferraz, X. Binefa, and F. Moreno-Noguer, "Very fast solution to the PnP problem with algebraic outlier rejection," in *Proc. IEEE Conf. Comput. Vis. Pattern Recognit.*, Jun. 2014, pp. 501–508.
[33] R. I. Hartley, "In defense of the eight-point algorithm," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 19, no. 6, pp. 580–593, Jun. 1997.

KUN YAN received the B.S. degree from Sichuan Normal University, Chengdu, China, in 2013, and the Ph.D. degree from the Institute of Optics and Electronics of Chinese Academy of Sciences, Chengdu, in 2018, where he is currently a Researcher.

He was supported by the National Natural Science Foundation of China and the Project of Young Scholars in Western China, Chinese Academy of Sciences. His research interests include binocular vision, computer vision, pose estimation, photogrammetry, and image processing.

RUJIN ZHAO received the B.S. and M.S. degrees from the Southwest University of Science and Technology, Mianyang, China, and the Ph.D. degree from the Institute of Optics and Electronics of Chinese Academy of Sciences, Chengdu, China, where he is currently a Researcher.

He has published relevant articles and is a member and peer reviewer of several journals. He was supported by the National Natural Science Foundation of China and the Youth Innovation Promotion Association CAS. His current research interests include visual measurement, SLAM, and space robotic detection.

ENHAI LIU is a Researcher and a Doctoral Tutor. He is currently the Deputy Director of the Institute of Optoelectronic Technology of Chinese Academy of Sciences. He is a member of the Academic and Academic Degree Committee and is engaged in research and engineering research on photoelectric precision measurement and automatic control technology. He undertaken and completed a number of manned spaceflight and lunar exploration in related fields. The research (development) of the engineering, 863, and 973 engineering projects received four awards for provincial and ministerial level scientific and technological progress. His research interests include space optoelectronic precision measurement, photodetector application and photodetection technology, signal and information processing, optoelectronic measurement system error theory analysis, and system integration technology research.

Mr. Liu is a member of the Chinese Optical Society, Optical Engineering Society, and Space Optical Engineering Society.

YUEBO MA received the B.S. degree in mechanical engineering from the Jincheng College of Sichuan University, Chengdu, China, in 2014, and the M.S. degree in automobile engineering from Xi Hua University, Chengdu, in 2017. He is currently pursuing the Ph.D. degree with the Institute of Optics and Electronics of Chinese Academy of Sciences, Chengdu.

His current research interests include object detection, object tracking, measurement, computer vision, and deep learning.