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Research on Robust Stochastic Dynamic Economic Dispatch Model Considering the Uncertainty of Wind Power

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ABSTRACT The increasing penetration rates of wind power in power systems bring challenges to the dynamic economic dispatching. This paper proposes a Robust Stochastic Optimization (RSO) model to handle the uncertainty of wind power in dynamic economic dispatch. Based on the event-wise ambiguity set and event-wise recourse adaptation, the RSO model has generality and enables the ambiguity set to be constructed irrelatively to the specific problem. Furthermore, by introducing the detail-variables, the adjustment of event-wise ambiguity set of the RSO model can reduce the conservativeness. To the dynamic economic dispatch problem, simulations studies on the IEEE 118-bus system and IEEE 300-bus system verify that 1) RSO model is flexible and adjustable; 2) RSO model has excellent performance under different penetration rates of wind power; 3) Compared with the results of Robust Optimization (RO) and Stochastic Optimization (SO), RSO model can balance the economy and robustness effectively; 4) The RSO model has better performance in dealing with the small sample volume of wind power data.

INDEX TERMS Optimization methods, power generation dispatch, wind power generation.

NOMENCLATURE

A. SETS

$\mathcal{B}$	Set of bus
$\mathcal{G}$	Set of generation unit
$\mathcal{W}$	Set of wind power unit
$\mathcal{L}$	Set of line
$\mathbb{R}$	Set of all real numbers
$[S]$	Set of Random scenario

B. DETERMINISTIC VARIABLES/ PARAMETERS

T	Total number of scheduling periods
m_i	Reactance of line i
$\mathbf{P}_{\text{line}}/\mathbf{P}^g/\mathbf{P}^L$	Line/Generator/Load power matrix
$\mathbf{B}$	Admittance coefficient matrix
$\mathbf{L}$	Connection matrix of the branch nodes of the system
C_T/C_D	Start-up and shutdown costs of thermal power units
C_G/C_W	Adjusting penalty coefficient of corresponding variables under real-time operation
R_U/R_D	Climbing / Landslide rate of thermal units
$\hat{P}_i^w/P_i^w$	Real wind power / Forecasted wind power output at bus i
$\hat{P}_i^g/P_i^g$	Real / Based-point thermal output at bus i
P^L	Demand of load
$P_i^{\max}/P_i^{\min}$	Upper / Lower limits of generator power of thermal unit i
$-r_i/\bar{r}_i$	Upper/Lower spinning reserve capacity of thermal unit i
$g_{i,t}^{on}/g_{i,t}^{off}$	Unit start-up and shutdown variables, binary, 1 for on, 0 for off
$a/b/c$	Coefficient of generation unit cost function

C. UNCERTAINTY VARIABLES

$\tilde{w}$	A discrete random scenario in $[S]$
$\tilde{v}$	Wind power forecasting error

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I. INTRODUCTION

Developing and utilizing renewable energy is the inevitable trend of developing energy-saving, emission reduction, and low carbon environmental protection. As an important renewable energy, wind power is particularly concerned due to renewability, low-consumption, and non-pollution. With the large-scale grid-connected wind power, the uncertainty of wind power output brings significant challenges to the optimal dispatch of power system [1].

In a power system, many decisions are 'wait-and-see'. The uncertain variables representing 'wait-and-see' decisions depend on the portion of the real data that reveals itself before the moment when the decision is made. As one of the main uncertainties in power system, wind power is one of the main 'wait-and-see' decisions. For reducing the impact of wind power dispatch in the power system, Stochastic Optimization (SO) [2], [3] and Robust Optimization (RO) [4], [5] are reported to tackle the uncertainty of wind power of the optimal dispatch problem in power system.

The characteristic of SO is to generate a large number of discrete samples through probability description for calculation and solution. However, it is difficult to obtain an accurate probability distribution of wind power output in practice [6], [7]. Moreover, its computing burden increases heavily with the numbers of samples.

RO assumes that the output range of wind power is in a given set of uncertainty, which can be polyhedron, ellipsoid, interval, and others. The goal of RO solution is to find the worst possible scenario for the system. In most cases, RO may be over-conservative, but even in the worst case, the solution obtained is still feasible [8]–[10].

In order to compensate for the deficiencies of RO model and SO model, DRO model was proposed in [11] and [12]. At present, DRO has been applied to various fields in the power system optimization problems, such as transmission expansion planning [13], joint unit dispatching [14], and unit commitment[15].

A class of hypothesis model of distribution set assumes that the random variables satisfy a specific distribution form (such as normal distribution) and their statistical moments can vary in a certain range. Therefore, the uncertainty of the distribution can be described by limiting the range of moments. [16] assumes that wind power forecast error obeys independent normal distribution. It gives a polyhedral set describing the uncertainty degree of the mean (first moment) and variance (second moment), and controls the conservativeness of the model by setting conservative coefficients. The method has good computational efficiency, but the rationality of its normal distribution hypothesis is difficult to apply in reality.

Another kind of distribution set gives the first and second moments of random vectors without presupposing their specific probability distribution form [17]. In the distribution set, all joint probability distributions of first and second moments that satisfy the given conditions are all possible distributions to characterize the uncertainty of distribution. In [18], the integral definitions of the first and second moments of wind power prediction errors and their corresponding distribution sets are given, and the dispatching model is transformed into bilinear matrix inequality problem by mathematical deduction. In [19], the data-driven theory is used to construct ambiguity sets and Column-and-Constraint generation algorithm is used to solve them. In [15], the Wasserstein metric is used to construct the ambiguity set, and the dual theory is used to transform and solve the model. Also, the theory of K-means [20] can be used to construct the ambiguity set. These theories and models are often effective for specific problems but lack of generality.

DRO has different decision models and corresponding processing methods for different ambiguity sets, which might cause models and solutions only suitable for specific problems in power system. In order to find a general model expression, the RSO model is proposed in [21]. The author proposed event-wise ambiguity set to construct DRO and proved that event-wise ambiguity set had a good generality. The event-wise ambiguity set can describe discrete distribution in distributed robustness, distribution ambiguity set based on moment information, K-means ambiguity set driven by data, and Wasserstein ambiguity set.

This paper proposes an RSO model to solve the dynamic economic dispatch optimization model considering the uncertainty of wind power in the power system. The main contributions of this paper are as follows:

1) A novel RSO model composed of event-wise ambiguity set and event-wise recourse adaptations is applied to the day-ahead dynamic economic dispatch problem of power system considering the uncertainty of wind power, which can compensate for the conservativeness of RO model and the inefficiency of SO model. The validity of the model is verified by simulation analysis based on IEEE 118-bus system and IEEE 300-bus system.
2) In order to apply the RSO model to solve the dynamic economic dispatch problem with uncertain wind power, we propose to use the Monte Carlo method and scenarios reduction technology to construct affine functions related to wind power. Besides, we slightly adjust the ambiguity set of RSO by introducing detail-variables, which reduced the conservativeness of the adjusted model. For decision-maker, there are more choices. At the end of the simulation section, the application scenarios of different models are presented.
3) Generalized moment information of wind power output is difficult to obtain accurately. Therefore, the DRO model based on Wasserstein ambiguity set is often used to solve the dynamic economic dispatch problem. However, this method is prone to over-conservative when the sample volume of wind power data is small. RSO model makes up for this defect. In addition, the results of RSO model and DRO model are compared based on generalized moment information ambiguity set and Wasserstein ambiguity set. It shows that the difference

$$\mathcal{P}=\left\{\mathbb{P}\in\mathcal{P}_0(\mathbb{R}^D+\mathbb{R}^L):\begin{array}{c}(\boldsymbol{\xi},\tilde{w})\sim\mathbb{P}\\ \mathbb{E}_{\mathbb{P}}\left[\rho\left(\boldsymbol{\xi},\widehat{\boldsymbol{\xi}}\right)\middle|\,\tilde{w}\in[S]_k\right]=\sigma_k\leq\vartheta\in\psi\\ \mathbb{P}\left[\boldsymbol{\xi}\in\mathcal{C}|\,\tilde{w}=w\right]=1,\mathbb{P}\left[\tilde{w}=w\right]=p_w\end{array}\right\}\tag{7}$$

$$\mathcal{P}=\left\{\mathbb{P}\in\mathcal{P}_0(\mathbb{R}^D+\mathbb{R}^L):\begin{array}{c}((\boldsymbol{\xi},\mathbf{u}),\tilde{w})\sim\mathbb{P}\\ \mathbb{E}_{\mathbb{P}}\left[\mathbf{u}|\,\tilde{w}\in[S]_k\right]\leq\vartheta\\ \mathbb{E}_{\mathbb{P}}\left[\boldsymbol{\xi}\right]=\mu\\ \mathbb{P}\left[(\boldsymbol{\xi},\mathbf{u})\in\mathcal{C}|\,\tilde{w}=w\right]=1,\mathbb{P}\left[\tilde{w}=w\right]=p_w\end{array}\right\}\tag{8}$$

between the RSO model and DRO model using two ambiguity sets is less than 1% when the sample volume is 5000. Hence, it is rational that RSO has good generality and can be used to solve the dynamic economic dispatch problem in power system considering the uncertainty of wind power.

The rest of this paper is organized as follows. In section II, the DRO model under different ambiguity sets and RSO models are proposed. In section III, a three-level two-stage dynamic economic dispatch model with the participation of wind power is proposed. Section IV presents the reformulation of the proposed RSO model. In section V, numerical simulation is given to verify the generality and correctness of RSO model. The conclusion is given in section VI.

II. SOLVING DYNAMIC ECONOMIC DISPATCH PROBLEM BASED ON RSO MODEL

The RSO model for solving multi-stage problems introduced in this section can be extended from the DRO model. The DRO model for a multi-stage optimization problem is as follows [22]:

$$\begin{aligned}&\min f_0\left(\mathbf{x},\boldsymbol{\xi}_0\right)+\max_{\mathbb{P}(\boldsymbol{\xi})\in\mathcal{P}}\mathbb{E}_{\mathbb{P}}\left[f\left(\mathbf{x},\boldsymbol{\xi}\right)\right]\\ &s.t.\ \mathbf{h}\left(\mathbf{x},\boldsymbol{\xi}\right)\leq 0,\quad\forall\boldsymbol{\xi}\in U\left(\Gamma\right)\end{aligned}\tag{1}$$

where $\mathbf{x}$ is set of decision variables, $f\left(\mathbf{x},\boldsymbol{\xi}\right)$ and $\mathbf{h}\left(\mathbf{x},\boldsymbol{\xi}\right)$ are functions of $\mathbf{x}$, $\boldsymbol{\xi}$ is set of uncertain parameters, $U\left(\Gamma\right)$ is uncertainty set, Γ adjusts the scale of uncertainty set, $\mathbb{E}_{\mathbb{P}}$ represents the expectations of functions, P is the ambiguity set, $\mathbb{P}\left(\boldsymbol{\xi}\right)$ represents the set of all probability distributions on $\mathbb{R}$. When the distribution $\mathbb{P}\left(\boldsymbol{\xi}\right)$ of $\boldsymbol{\xi}$ is known, this model is SO, when only the uncertainty set $U\left(\Gamma\right)$ of $\boldsymbol{\xi}$ is known, this model is RO. Therefore, DRO has different decision models and corresponding processing methods for different ambiguity set $\mathcal{P}$. Different from DRO model, a general model expression of RSO is as follows:

$$\begin{aligned}&\min f_0\left(\mathbf{x},\boldsymbol{\xi}_0\right)+\max_{\mathbb{P}(\boldsymbol{\xi})\in\mathcal{P}}\mathbb{E}_{\mathbb{P}}\left[f\left(\mathbf{x},\boldsymbol{\xi}\right)\right]\\ &s.t.\ \mathbf{h}\left(\mathbf{x},\boldsymbol{\xi}\right)\leq 0,\quad\forall\boldsymbol{\xi}\in U\left(\Gamma\right)\\ &\qquad f\left(\mathbf{x},\boldsymbol{\xi}\right)\in\mathcal{A}\left(K,\mathcal{I}\right)\end{aligned}\tag{2}$$

The biggest difference between RSO and DRO is the existence of dynamic decision constraints $f\left(\mathbf{x},\boldsymbol{\xi}\right)\in\mathcal{A}\left(\mathbf{x},\boldsymbol{\xi}\right)$, which means that dynamic decision $f\left(\mathbf{x},\boldsymbol{\xi}\right)$ is a different affine function for random variables $\boldsymbol{\xi}$ in each scenario. where $\mathcal{A}$ is event-wise recourse adaptations expressed as follows:

$$\mathcal{A}\left(K,\mathcal{I}\right)=\left\{f:\begin{array}{c}f\left(\mathbf{x},\boldsymbol{\xi}\right)=f_0+\sum\limits_{i\in\mathcal{I}}f_i(\mathbf{x},\boldsymbol{\xi})\\ for\ some\ f_0,f_i(x)\in\mathcal{A}\left(K\right)\end{array}\right\}\tag{3}$$

where $\mathcal{I}$ is the information index set. $\mathcal{A}\left(K\right)$ is an event-wise static adaptation. The formula is as follows:

$$\mathcal{A}\left(K\right)=\left\{f':\begin{array}{c}x(\xi)=x^{\tau}\\ \tau=\mathcal{F}(\xi)\\ for\ some\ x^{\tau}\in\mathbb{R}\end{array}\right\}\tag{4}$$

Correspondingly, a mapping $\mathcal{F}(\xi):[S]\mapsto K$, $[S]$ is the range of random variables, K is a collection in the partition of scenarios and K is mutually exclusive and collectively exhaustive events.

Whether DRO or RSO, the effect of the ambiguity set on the final solution is very important. The ambiguity set of DRO with generalized moment information is as follows [23]:

$$\mathcal{P}=\left\{\mathbb{P}\in\mathcal{P}_0(\mathbb{R}^D+\mathbb{R}^L):\begin{array}{c}(\boldsymbol{\xi},\mathbf{u})\sim\mathbb{P}\\ \mathbb{E}_{\mathbb{P}}\left[\mathbf{A}\boldsymbol{\xi}+\mathbf{B}\mathbf{u}\right]=\mathbf{b}\\ \mathbb{P}\left[(\boldsymbol{\xi},\mathbf{u})\in\mathcal{C}\right]\in\left[\underline{p}_k,\bar{p}^k\right]\end{array}\right\}\tag{5}$$

where $\mathbb{P}$ represents a joint probability distribution of the random vector $\boldsymbol{\xi}\in\mathbb{R}^D$ appearing in the constraint function $f\left(\mathbf{x},\boldsymbol{\xi}\right)$ and some detail variables $\mathbf{u}\in\mathbb{R}^L$, $\underline{p}_k/\bar{p}^k$ represent upper/lower bounds of the probability boundary, and the confidence set $\mathcal{C}$ is defined as:

$$\mathcal{C}=\{(\boldsymbol{\xi},\mathbf{u}):C_i\boldsymbol{\xi}+D_i\mathbf{u}\preceq\mathcal{K}_ic_i\}\tag{6}$$

where $\mathcal{K}_i$ represents a proper cone, $\preceq$ means less than or equal to in cone space. In this model, the highlight is the introduction of detail-variables $\mathbf{u}\in\mathbb{R}^L$, which can transform the model into a solvable form.

Different from (5), the Wasserstein ambiguity set of DRO is as follows (7), as shown at the top of this page, where w conditionings on the realization of a scenario $w\in[S]$, ϑ is an adjustable parameter representing the radius, $\widehat{\boldsymbol{\xi}}$ represents historical empirical data, $\rho(\boldsymbol{\xi},\widehat{\boldsymbol{\xi}})$ represents the Euclidean norm distance between $\boldsymbol{\xi}$ and $\widehat{\boldsymbol{\xi}}$, and $\sum\limits_{w\in[S]}p_w=1$. Because the original RSO model described the problem very vaguely. Inspired by Wasserstein ambiguity set, the detail variables $\mathbf{u}\in\mathbb{R}^L$ is introduced in the optimization of the RSO model in this paper. Hence, the event-wise ambiguity set of RSO model can be adjusted as follows (8), as shown at the top of this page, where μ is the expected value of $\boldsymbol{\xi}$, the confidence sets $\mathcal{C}$ are defined as:

$$\mathcal{C} = \{(\boldsymbol{\xi}, \mathbf{u}) : \mu + \mathbf{u} \geq \boldsymbol{\xi}, \mu + \mathbf{u} \leq \boldsymbol{\xi}\} \tag{9}$$

Note that detail variables $\mathbf{u}$ are auxiliary variables without any specific meaning in the original RSO model, but in the dynamic economic dispatch problem, $\mathbf{u}$ can represent the difference between expected and actual wind power output.

III. DYNAMIC ECONOMIC DISPATCH MODEL WITH WIND POWER

A. OBJECTIVE FUNCTION

In the actual power grid, it is necessary to take into account the minimum sum of day-ahead economic dispatch cost and adjustment cost under real-time operation. Aiming at this problem, the objective function is as follows:

$$\begin{aligned}
\min \; & \sum_{t=1}^{T}\sum_{i\in\mathcal{G}}(C_{Ti}g_i^{on} + C_{Di}g_i^{off}) \\
& + \sum_{t=1}^{T}\sum_{i\in\mathcal{G}}(a_i(\hat{P}_{i,t}^{g})^2 + b_i\hat{P}_{i,t}^{g} + c_i) \\
& + \max_{\mathbb{P}\in\mathcal{P}}\sum_{s\in S}\mathbb{P}_s\Bigg[\min\sum_{i\in\mathcal{G}}(C_{Gi,t}\Delta P_{i,t}^{g}) \\
& + \sum_{k\in\mathcal{W}}C_{Wk}\Delta P_{k,t}^{w}\Bigg]
\end{aligned} \tag{10}$$

The formula (10) is a min-max-min three-level two-stage optimization problem. The first part of the formula (10) expresses the total cost of day-ahead economic dispatch, including the start-up and shutdown costs and operation costs of conventional units. The second part expresses the total cost of adjustment of units and devices in real-time dispatch considering the uncertainties of wind power output.

B. CONSTRAINTS

For the simplicity of the model, the uncertainty of wind power is expressed as the total wind forecasting error. At the time t, the wind power output can be expressed as:

$$\tilde{\nu} = \sum_{k\in\mathcal{W}}(\hat{P}_{k,t}^{w} - P_{k,t}^{w}) \tag{11}$$

The affine policy between wind power output and thermal units is as follows:

$$\begin{aligned}
\hat{P}_{i,t}^{g} &= P_{i,t}^{g} - \delta_{i,t}\tilde{\nu}, \quad i\in\mathcal{G} \\
0 &\leq \delta_{i,t} \leq 1, \quad i\in\mathcal{G} \\
\sum_{i\in\mathcal{G}}\delta_{i,t} &= 1, \quad i\in\mathcal{G}
\end{aligned} \tag{12}$$

where δ is the participation factor of the automatic generation control system in response to the total wind power forecasting errors. Besides, it is necessary to satisfy the reserve capacity constraints of thermal units.

$$-r_{i,t} \leq -\delta_{i,t}\tilde{\nu} \leq \bar{r}_{i,t}, \quad i\in\mathcal{G} \tag{13}$$

$$P_{i,t}^{\min} \leq \hat{P}_{i,t}^{g} \leq P_{i,t}^{\max}, \quad i\in\mathcal{G} \tag{14}$$

The power balance constraints are as follows:

$$\sum_{i}^{\mathcal{G}}P_{i,t}^{g} + \sum_{i}^{\mathcal{W}}P_{i,t}^{w} = \sum_{i}^{\mathcal{B}}P_{i,t}^{L} \tag{15}$$

The climbing constraints of the unit are as follows:

$$P_{i,t}^{g} - P_{i,t-1}^{g} \leq R_{Ui}, \quad i\in\mathcal{G} \tag{16}$$

$$P_{i,t-1}^{g} - P_{i,t}^{g} \leq R_{Di}, \quad i\in\mathcal{G} \tag{17}$$

We choose direct current power flow method to do the calculation, and the constraints are as follows:

$$\mathbf{P}_{\text{line}} = \mathbf{B}_{\text{diag}}\mathbf{L}\mathbf{B}^{-1}\left(\mathbf{P}^{g} - \mathbf{P}^{L}\right) \tag{18}$$

$$-\bar{\mathbf{P}}_{\text{line}} \leq \mathbf{P}_{\text{line}} \leq \bar{\mathbf{P}}_{\text{line}} \tag{19}$$

$$\mathbf{B}_{\text{diag}} = diag(\frac{1}{m_1}, \frac{1}{m_2}, \ldots, \frac{1}{m_i}), \quad i\in\mathcal{L} \tag{20}$$

IV. MODEL REFORMULATION

It is difficult to solve the RSO model mainly because the number of functional variables is over that of the ambiguity set in the worst case. Therefore, it usually needs some relaxation to solve. The dual theory can be used to obtain the transformation form [21]. Although the transformation is not strictly valid, [21] has proved that the difference between the result after transformation and the original result is less than 0.1%. More detailed and relevant discussions can be found in [21]. Hence, the dual theory method is used to relax the model.

To simplify the description, $\mathbf{C}^{\mathrm{T}}\mathbf{g}$ is equivalent to start-stop cost $\sum_{t=1}^{T}\sum_{i\in\mathcal{G}}(C_{Ti}g_{i,t}^{on} + C_{Di}g_{i,t}^{off})$, $f_0\left(\mathbf{y}, \boldsymbol{\xi}_0\right)$ is equivalent to $\sum_{t=1}^{T}\sum_{i\in\mathcal{G}}(a_i(\hat{P}_{i,t}^{g})^2 + b_i\hat{P}_{i,t}^{g} + c_i)$, $L\left(\mathbf{a}^{\mathrm{T}}\mathbf{y}\right)$ is equivalent to operating cost $\sum_{t=1}^{T}\sum_{i\in\mathcal{G}}(a_i(P_{i,t}^{g})^2 + b_iP_{i,t}^{g} + c_i)$, $\mathbf{b}^{\mathrm{T}}\Delta\boldsymbol{\xi}$ is equivalent to $\sum_{k\in\mathcal{W}}^{C_{Wk}\Delta P_{k,t}^{w}}$, $\boldsymbol{\xi}$ represents wind power output.

Hence, the objective function includes operating cost and start-stop cost, which can be described as follows:

$$\begin{aligned}
&\min\ \mathbf{C}^{\mathrm{T}}\mathbf{g} + f_0\left(\mathbf{y}, \boldsymbol{\xi}_0\right) + \max_{\mathbb{P}(\boldsymbol{\xi})\in\mathcal{P}}\mathbb{E}_{\mathbb{P}}\left[L\left(\mathbf{a}^{\mathrm{T}}\Delta\mathbf{y}\right) + {}^{\mathrm{T}}\Delta\boldsymbol{\xi}\right] \\
&\Updownarrow \\
&\min\ \mathbf{C}^{\mathrm{T}}\mathbf{g} + f_0\left(\mathbf{y}, \boldsymbol{\xi}_0\right) + \max_{\mathbb{P}(\boldsymbol{\xi})\in\mathcal{P}}\mathbb{E}_{\mathbb{P}}\left[f\left(\Delta\mathbf{y}, \Delta\boldsymbol{\xi}\right)\right]
\end{aligned} \tag{21}$$

In this model, $\mathbf{y}, \boldsymbol{\xi}_0$ represents $P_{i,t}^{g}, \hat{P}_{k,t}^{w}$, the variables in the first stage are day-ahead robust decision variables which do not change with the actual scenario, and the second stage variables are adjustable variables and uncertain variables.

The event-wise recourse adaptations are as follows

$$\mathcal{A}(K, \mathcal{I}) = \left\{ f : f(\Delta \mathbf{y}, \Delta \boldsymbol{\xi}) = f_0(\Delta \mathbf{y}) + \sum_{i \in \mathcal{I}} f_i(\Delta \mathbf{y}, \Delta \boldsymbol{\xi}) \right\} \tag{22}$$

It means that dynamic decision f is an affine function of the random variable $\Delta\boldsymbol{\xi}$ under each scenario K, and $\mathcal{I}$ is specific scenarios. Each $\mathcal{I}$ is generated by a finite number of K.

According to the duality theory, the following theorem is established. The specific contents and proofs can be seen in Section 4 of [21].

The expectation of the worst case is

$$\sup_{\mathbb{P}(\boldsymbol{\xi}) \in P} \mathbb{E}_{\mathbb{P}} \left[\mathbf{r}^{\top}(\tilde{w}) G_m(\tilde{w}) \tilde{z} + h_m(\tilde{w}) \right] \tag{23}$$

It is equivalent to the optimal value of the following classical robust optimization problem:

$$\begin{aligned}
&\inf \ \gamma \\
&s.t. \ \gamma \geq \alpha^{\top} \mathbf{p} + \sum_{k \in [K]} \beta_k^{\top} \mu_k \quad \forall \mathbf{p} \in \mathcal{P}, \\
&\quad \frac{\mu_k}{\sum_{w \in \mathcal{E}_k} p_w} \in Q_k, \quad k \in [K] \\
&\quad \alpha_w + \sum_{k \in \mathcal{K}_s} \beta_k^{\top} z \geq r^{\top}(w) G_m(w) z + h_m(w) \\
&\quad \forall z \in Z_w, w \in [S] \\
&\quad \gamma \in \mathbb{R}, \quad \alpha \in \mathbb{R}^S, \ \beta_k \in \mathbb{R}^{I_z} \ \forall k \in [K]
\end{aligned} \tag{24}$$

where $k \in [K]$ correspond to different events.

Therefore, the RSO model can be reformulated as follows:

$$\begin{aligned}
&\max_{\mathbb{P}(\boldsymbol{\xi}) \in P} \mathbb{E}_{\mathbb{P}} [f(\Delta \mathbf{y}, \Delta \boldsymbol{\xi})] \\
&= \max_{\mathbb{P}(\boldsymbol{\xi}) \in P} \mathbb{E}_{\mathbb{P}} \left[L\left(\mathbf{a}^{\mathrm{T}} \Delta \mathbf{y}\right) + {}^{\mathrm{T}} \Delta \boldsymbol{\xi} \right] \\
&= \max_{\mathbb{P}(\boldsymbol{\xi}) \in P} \mathbb{E}_{\mathbb{P}} \left[\sum_{t=1}^{T} \sum_{i=1}^{G} (a_i (\Delta P_{i,t}^g)^2 + b_i \Delta P_{i,t}^g + c_i) + {}^{\mathrm{T}} \Delta \boldsymbol{\xi} \right] \\
&= \begin{cases}
\inf \ \gamma \\
s.t. \ \gamma \geq \alpha^{\mathrm{T}} \mathbf{p} + \sum_{k \in [K]} \beta_k^{T} \sigma_k \quad \forall \mathbf{p} \in \mathcal{P}, \\
\quad \frac{\sigma_k}{\sum_{w \in \varepsilon_k} p_w} \in Q_k, \quad k \in [K] \\
\alpha + \sum_{k \in K_s} \beta_k^{\mathrm{T}} \Delta \boldsymbol{\xi} \geq \sum_{t=1}^{T} \sum_{i=1}^{G} (a_i (\Delta P_{i,t}^g)^2 + b_i \Delta P_{i,t}^g + c_i) \\
\quad + \mathbf{b}^{\mathrm{T}} \Delta \boldsymbol{\xi} \quad \forall z \in Z_w, w \in [s] \\
\gamma, \alpha, \beta \in \mathbb{R}
\end{cases}
\end{aligned} \tag{25}$$

where α and β are Lagrange multipliers, ı stands for p_w vector. Besides, the operating cost function is a nonlinear function and is linearized by using the method in [24]. Divide the x interval $\left[x^{\min}, x^{\max}\right]$ into N segments, and the boundary points are $x_1^s, x_2^s, \ldots, x_{N+1}^s$. The state variable l and continuous variable $x$is introduced into each segment of the linear function.

$$\begin{aligned}
\hat{P}_{i,t}^g(x) &\approx \sum_{i}^{N} (k_i^1 x_{i,t} + k_i^2 l_{i,t}) \\
k_i^1 &= \left(\hat{P}_{i,t}^g(x_{i+1}^s) - \hat{P}_{i,t}^g(x_i^s)\right) / \left(x_{i+1}^s - x_i^s\right) \\
k_i^2 &= \hat{P}_{i,t}^g(x_i^s) - k_i^1 x_i^s \\
x_i^s l_{i,t} &\leq x_{i,t} \leq x_{i+1}^s l_{i,t}
\end{aligned} \tag{26}$$

In (26), k_i^1 is the slope and k_i^2 is the intercept.

Now the transformation of the dynamic economic dispatch model is completed, which is mixed-integer linear programming and can be solved using Cplex [25] or Gurobi [26].

V. NUMERICAL SIMULATION

This section presents numerical results of IEEE 118-bus system and IEEE 300-bus system. All models have been implemented using MATLAB R2018b, Gurobi 8.1 and RSOME [27], while the simulation is running on a desktop computer with 3.10 GHz processor and 8 GB memory. In IEEE 118-bus system, wind farms are connected at nodes 20, 21, 30, 32, 65, 86, and 92 with each capacity of 100MW. In IEEE 300-bus system, wind farms are connected at node 18, 20, 56, 57, 58, 81, 119, 156, 189, 215, and 245 with each capacity of 100MW. The penetration rates of wind power are 21.2% in IEEE 118-bus system. The penetration rates of wind power are 19.8% in IEEE 300-bus system. The value of p_w is $\frac{1}{S}$.

Output data of wind farm come from a 100MW wind farm in Nanning, Guangxi, China. For example, for node 18 in IEEE 118-bus system, Figure 1 shows the output curve of wind power on typical summer days, the blue line represents the actual processing curve and the orange line and the grey line represents the upper and lower bounds of the output interval. Based on this output interval and real output, the Monte Carlo method and scenario reduction technology [28] are used to generate scenarios and specific scenarios of wind power output. The output data of other wind farms also use this method.

In order to verify the validity and correctness of the proposed model, the simulation will be analyzed from the following four aspects. 1) The effect of sample volume, different radius parameters and the existence of detail-variables on the results. 2) The comparison of model solution results under different wind power penetration rates. 3) The comparison of calculation results of different models. 4) The comparison of RSO and DRO model under different ambiguity sets.

A. THE EFFECT OF DIFFERENT RADIUS PARAMETERS AND THE EXISTENCE OF DETAIL-VARIABLES ON THE CALCULATION RESULTS OF RSO MODEL

The detail-variables $\mathbf{u}$ is introduced in formula (8). Through the analysis of formula (8), it is seen that the optimal value will be different for different choices of ϑ.

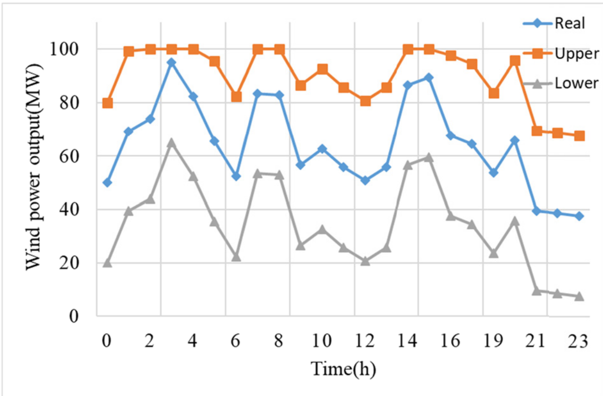

FIGURE 1. Output curve of 18 nodes wind farm in IEEE 118-bus system.

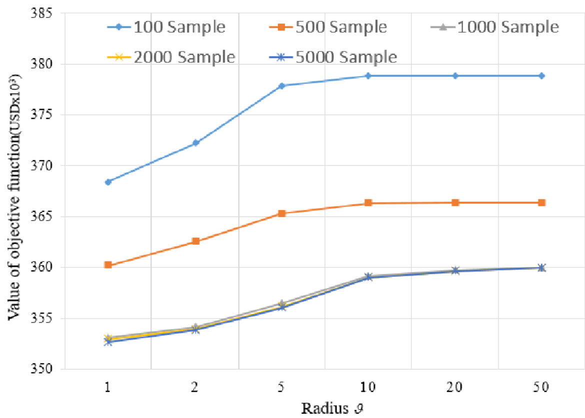

FIGURE 2. The change of optimal value under different radius.

TABLE 1. The growth rate of objective function value without introducing the detail-variables.

Sample volume	ϑ					
	1	2	3	5	10	20
100	1.37%	1.81%	1.94%	3.13%	4.36%	7.34%
500	1.25%	1.62%	1.74%	2.12%	3.76%	4.87%
1000	1.21%	1.53%	1.74%	2.12%	3.55%	4.59%
2000	1.21%	1.53%	1.73%	2.11%	3.52%	4.54%

We run 100, 500, 1000, 2000 and 5000 sample volume generated by Monte Carlo method and compare their performance: (with the participation of detail-variables **u**)

As shown in Figure 2, with the increase of the value of ϑ, the optimal value of RSO model is also increasing. This is because with the increase of ϑ, the uncertainties increase. In order to maintain the stable operation of the power system, more spare capacity of thermal units is needed, which causes an increase in the objective function value. However, when ϑ exceeds 10, the optimal value increases slowly. This is because ϑ represents the degree of migration of historical samples date, while the whole sample is based on the simulation of the upper and lower bounds of the actual data, which means that there is an upper limit of the degree of migration.

To verify the validity of detail-variables **u**, this section also compares the results based on the same sample volume.

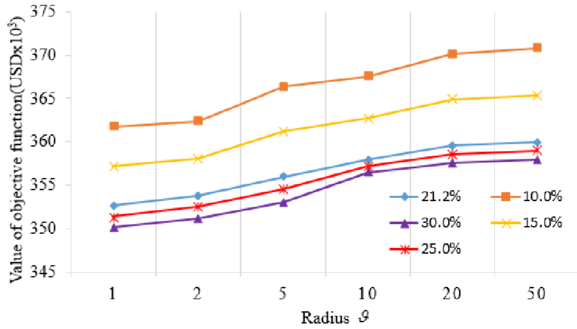

FIGURE 3. Comparison of objective values on IEEE 118-bus system with different wind power penetration rates.

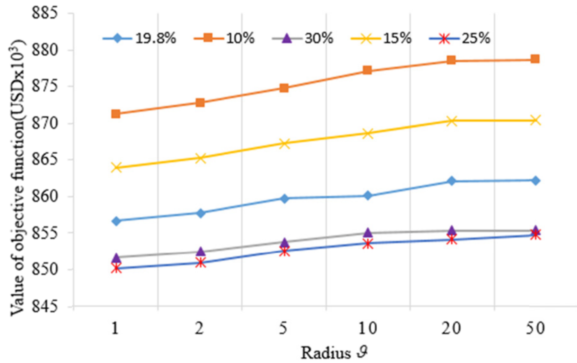

FIGURE 4. Comparison of objective values on IEEE 300-bus system with different wind power penetration rates.

As presented in Table 1, in any sample volume, without introducing detail-variables **u**, the optimal value of the function will rise. As the sample volume increases, the effect of detail-variables **u** on the optimal value decreases. This is because the large sample volume will make up for the lack of detail in ambiguity sets. But when ϑ becomes larger, there is still an offset that cannot be ignored. Therefore, the existence of detail-variables **u** is important to reduce the conservativeness of RSO model.

B. THE IMPACT OF WIND POWER PENETRATION RATES ON THE VALUE OF THE OBJECTIVE FUNCTION IN RSO MODEL

The challenge of wind power uncertainty to the power system is usually directly related to wind power penetration rates. In the simulations, the different scenarios of wind power under different wind power penetration rates have been adjusted according to the capacity of the wind farm in equal proportion. To analyze the performance of RSO model under different penetration rates of wind power, the following comparative analysis is made.

From Figure 3 and 4, it shows that the higher the penetration rates of wind power, the better the economy of the system. The relationship between the penetration rates and the optimal value of the model is not linear growth, because it involves the safe and stable operation of the system.

TABLE 2. Result comparison under different numbers of historical data.

Sample Volume	IEEE 118-bus/USD+e3		
	RSO	SO	(RSO-SO)/SO
100	368.42	345.26	6.7%
500	360.18	345.26	4.3%
1000	353.12	345.26	2.3%
2000	352.96	345.26	2.2%
5000	352.67	345.26	2.1%

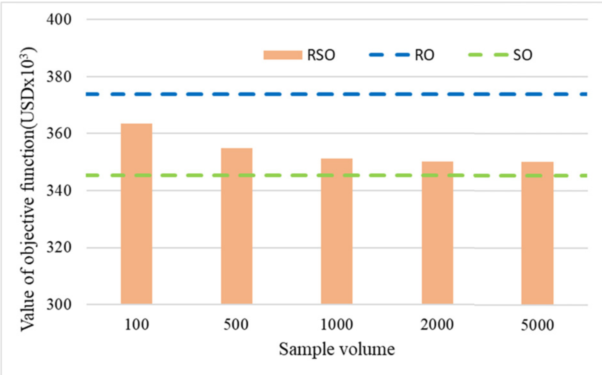

FIGURE 5. Comparison of objective values among RO, RSO, and SO on IEEE 118-bus system with different sample volumes.

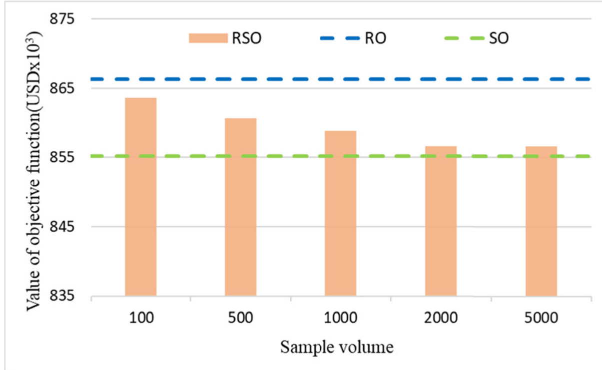

FIGURE 6. Comparison of objective values among RO, RSO, and SO on IEEE 300-bus system with different sample volumes.

TABLE 3. Computational speed comparison of different models.

	IEEE 118-bus system			IEEE 300-bus system		
	RSO	RO	SO	RSO	RO	SO
Run time(s)	3.43	1.98	7.96	6.12	2.11	13.64

C. COMPARISON OF OPERATION COSTS AMONG UNDER DIFFERENT MODELS AND DIFFERENT SAMPLE VOLUME

In order to verify the effect of sample volume on the results of model solving, we made the following comparative analysis under $\vartheta = 1$. SO is based on 5000 sample volume.

Table 2 shows the comparison of optimization results between RSO and SO under different sample volumes. Notably, with the increasing number of samples volume, the results of RSO model are closer to SO, which leads to the affine function in the ambiguity set closer to the reality, thus reducing the conservativeness of the problem. That is, with the increasing number of samples volume, the RSO model is less conservative.

The RO model deals with uncertainties by interval uncertainty set, and the results are the most conservative. SO results are calculated based on a large number of scenarios, and the results are the least conservative. The uncertainty interval of RO is based on the upper and lower bounds as shown in Figure 1. To analyze the relationship between the conservativeness of RSO, RO, and SO models, the value of ϑ is set to 0.1 with the participation of detail-variables $\mathbf{u}$.

Figure 5 and figure 6 show that RSO tends to RO when the sample volume is small. As the sample volume increases, the results of RSO tend to SO from RO. SO is optimized by selecting scenarios and the expectations corresponding to the probability of these scenarios. Compared with SO, RSO also needs to consider the influence of uncertain sets in the model description. Therefore, the economy of RSO is generally very close to that of SO, but better than RO which only describes uncertainty in interval form. In a word, RSO's conservativeness is between RO and SO.

We also make a comparative analysis of the computational speed of different models. The RSO model is based on 5000 sample volume, and the value of ϑ is 1.

It can be seen clearly from Table 3 that although RSO is slower than RO, the computation time of SO is about two times that of RSO.

D. COMPARISON OF OBJECTIVE VALUES BETWEEN RSO MODEL AND DRO MODEL UNDER DIFFERENT AMBIGUITY SETS

To verify the generality and advantage of RSO model in dynamic economic dispatch problem, we compared the RSO model and DRO model under different ambiguity sets. For clarity, the DRO model under generalized moment information ambiguity set is named as GDRO, the DRO model under Wasserstein ambiguity set is named as WDRO. Set the value of ϑ to 0.1 for WDRO and RSO. In order to reduce the randomness of model calculation, the simulation is carried out by calculating 50 times and taking the expected value for comparative analysis. The results are shown in Figure 7 and Figure 8.

As can be seen from Figures 7 and 8, the gap between RSO, GDRO, and WDRO is getting smaller as the sample volume increases. When the sample volume is 5000, the difference between RSO, GDRO, and WDRO is less than 1%. Therefore, it is rational that RSO, GDRO, and WDRO are equivalent in the three-level two-stage dynamic economic dispatch optimization problem.

For the dynamic economic dispatch optimization model considering the uncertainty of wind power in the power system, the application of RO model, SO model, DRO model, and RSO model are as follows:

When the accurate probability distribution of wind power output data can be obtained, the SO model has high accuracy, but it is not conservative. When the decision maker can only

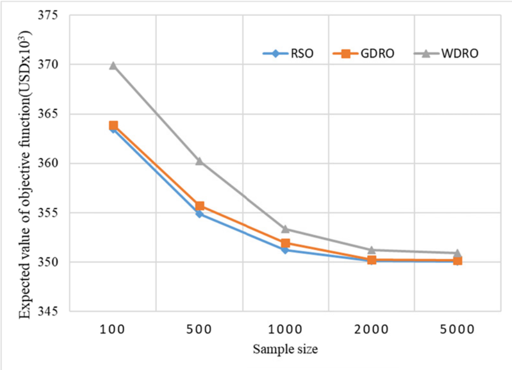

FIGURE 7. Comparison of average values on IEEE 118-bus system with different model.

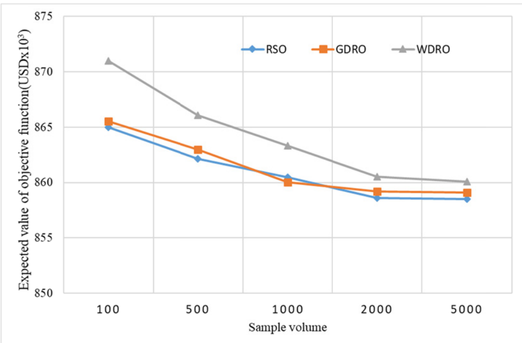

FIGURE 8. Comparison of average values on IEEE 300-bus system with different models.

get the output range of wind power, the selection of RO model can effectively ensure the stable operation of the power system, but it is not economical.

DRO model can effectively balance conservativeness and economy. When the accuracy of the probability distribution of wind power is not clear, WDRO model is a better choice, but when the sample volume of wind power is small, the optimization result is too conservative. When the distribution of wind power output can be accurately obtained, GDRO model is a better choice, however, it is difficult to gain accurate distribution in reality. RSO model combines the advantages of WDRO and GDRO. When the sample volume of wind power is small, and the distribution is not clear, RSO model is a better choice.

VI. CONCLUSION

This paper has proposed a novel framework to solve a three-level two-stage dynamic economic dispatch optimization problem in power system considering the uncertainty of wind power. To solve the original RSO model, this paper has transformed the RSO model into a mixed-integer programming model by using dual theory and piecewise linearization method. Affine function sets have been generated by the Monte Carlo method and scenarios reduction technology. The RSO model has good flexibility, adjustability and can deal with dispatching problems under different penetration rates of wind power. Besides, the introduction of detail variables reduces the conservativeness of RSO model and makes the RSO model available on the extensive range. Compared with WDRO model and GDRO model, RSO model has better performance in small samples of wind power data.

The RSO model has been proposed to solve a dynamic economic dispatch problem based on direct current power flow. Due to the higher accuracy alternating current power flow, our future research is to find an efficient alternating current power flow computing method to apply on the dynamic economic dispatch problem.

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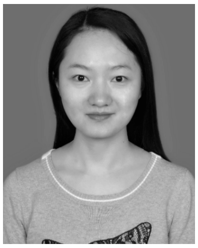

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