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Projection Modification Based Robust Adaptive Backstepping Control for Multipurpose Quadcopter UAV

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ABSTRACT In this paper, the robust adaptive control scheme based on backstepping technique is presented that improves the trajectory tracking performance of the quadrotor unmanned aerial vehicles (UAVs), specially tasked for supply, rescue and combat missions. The proposed control scheme is designed to estimate all the system parameters that may posses uncertainties and effectively rejects the completely unknown time varying external disturbances. The adaptive laws, derived through Lyapunov stability theorem are robustified by merging with derivative-integral (DI) term, resulting in rapid and accurate adaptation. In addition, to avoid parametric drift phenomenon, we introduce the projection modification (PM) in the designed DI-adaptive laws that ensures the closed-loop system signals bounded. The trajectory tracking and parameter estimation performance of the UAV in the presence of external disturbances, the payload pick up/drop off effect on altitude and recoil effect on attitude is analyzed by means of numerical simulations. The results validate strict robustness with extended applicability of proposed control scheme.

INDEX TERMS Adaptive control, backstepping, mass estimation, projection modification, parameter estimation, quadrotor.

I. INTRODUCTION

In recent years, quadcopter UAVs have gained enormous popularity due to its simple mechanical structure and capability of conducting various task-based unmanned flights [1], [2]. Compared with fixed-wing aircraft, the quadcopter has the ability to hover, vertically take-off and land, and can be operated in indoor environment, which particularly is the reason that it has replaced piloted/manned or fixed-wing aircraft UAVs in various applications, such as military operations [3], surveillance [4], traffic control [5], agricultural [6], tourism, photography, rescue, delivery [7]–[10] tasks and various other applications [11]–[19]. However, holding such abilities with the simple structure, quadcopter UAV can be particularly very sensitive to the wind gusts or environmental changes termed as external disturbances, mass variations, sensor and actuator faults [20]–[22]. These events can cause the quadcopter UAV to fly unstably and hence, end-up crashing itself.

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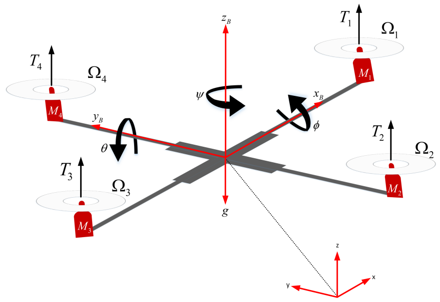

FIGURE 1. Quadcopter configuration with forces.

Therefore, designing an appropriate controller for the stable and smooth flight operations is a very interesting and significantly challenging objective. Before designing the automatic controller for the quadcopter, following points must be kept in the mind. (i) The quadcopter is severely underactuated system, i.e., with six degrees of freedom (6-DOF), it has only 4 independent control inputs, shown in Fig. 1.

Therefore, its translational and rotational motions are coupled, thus exhibiting highly non-linear dynamics and typically, is open-loop unstable. (ii) The quadcopter UAV has very low friction force to stop its motion. Hence, controller design has to be efficient and robust enough to keep stable at certain position. Additionally, momentum of the quadcopter is dependent on the rotors that makes it more reactive to variable external disturbances, such as wind. To address such problems, several control methods have been proposed and implemented, such as proportional-integral-derivative (PID) control and linear quadratic regulator (LQR) control [23]–[25]. However, these methods are often vulnerable to external disturbances, thus have limited performance. On other hand, nonlinear controls such as sliding mode control (SMC) and backstepping control can be very efficient, as they form the overall control law based on system parameters. However, performance may be compromised, if there exist modeling errors or uncertainties in the system parameters. Although SMC is known for its robustness, the approach always suffers from its chattering effect incurred due to discontinuous control that could cause mechanical damage to the aerial vehicle. Therefore, researchers usually combine various control techniques to eliminate certain drawbacks to attain better performance. In [26], second-order SMC based on PID sliding surface is presented, in which the switching term is combined with adaptive technique to estimate the upper bound of disturbance. Approach presented in [27], deals with the mass varying disturbance of the quadcopter UAV during flight, by mean of model reference adaptive control (MRAC) based LQR control. In [28], weakly modeled system is considered, additionally the modeling errors are allowed to depend on system states and presents robust adaptive backstepping control to adjust the modeling errors. The robust integral of the signum of the error (RISE) approach is implemented for disturbance rejection and adaptive control based immersion and invariance (I&I) methodology is proposed for trajectory tracking of quadcopter UAV in [29]. These methods are capable of solving the specified problems. However, these methods require complete knowledge of system's parameters, intensive computation and are highly complicated.

A. MAIN CONTRIBUTIONS

This study focuses on designing the adaptive control law for altitude and attitude tracking of uncertain quadcopter under the influence of time varying external disturbances. In addition, the quadcopter UAV is considered to experience the disturbances caused by; (i) payload drop or pick up. Usually appears when UAV is involved in agricultural, delivery, rescue or supply missions. (ii) recoil energy of gun, occurs in military or guarding missions. We introduce a DI-PM based robust adaptive backstepping controller, which guarantees good tracking performance with fast convergence rate. The adaptive laws are derived from the Lyapunov stability theorem that are improvised by merging with the respective channel-error based DI term, resulting fast and accurate adaptation with extended robustness. Furthermore, a convex set is defined that contains all the unknown system parameters and we propose the projection based modification in adaptive laws: (i) if, the estimated value is on the boundary of the convex set and update law is pointing outside, the update law is modified to keep estimate inside the defined convex set. (ii) else, the update law remains the same. Hence, ensuring all the system and control signals are bounded. We briefly compare our approach with the existing techniques as follows: (i) Most of the existing schemes rely on the fixed or known models [23], [24], [26], which can be inaccurate due to uncertainties present in the models. Whereas in our approach, all the system parameters are estimated, eventually eliminating the uncertainties present in the modeled system and therefore, exhibiting more robust and stable tracking performance. (ii) The parameters and mass adaptation is faster compared with the fuzzy adaptive backstepping control suggested in [25], [27], due to proposed robust modifications in adaptive laws. (iii) Our method is applicable to the systems ranging from partially known to fully known, by just adjusting the boundary of convex set. Additionally, it is robust, effective and easy to implement comparative to other approaches [22]–[24].

II. QUADCOPTER MODELLING

The mathematical model of quadcopter UAV under external disturbances is given as:

$$\begin{cases} \ddot{x} = \dfrac{U_1}{m}(S_\theta C_\phi C_\psi + S_\phi S_\psi) \\ \ddot{y} = \dfrac{U_1}{m}(S_\theta C_\phi S_\psi + S_\phi C_\psi) \\ \ddot{z} = \dfrac{U_1}{m}(C_\theta C_\phi) - g \\ \ddot{\phi} = \dfrac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} - I_m \dfrac{\dot{\theta}}{I_x}\Omega_\Phi + \dfrac{U_2}{I_x} + \varrho_\phi - Q\chi_\phi(t) \\ \ddot{\theta} = \dfrac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} + I_m \dfrac{\dot{\phi}}{I_y}\Omega_\Phi + \dfrac{U_3}{I_y} + \varrho_\theta - Q\chi_\theta(t) \\ \ddot{\psi} = \dfrac{(I_x - I_y)}{I_z}\dot{\phi}\dot{\theta} + \dfrac{U_4}{I_z} + \varrho_\psi \end{cases} \tag{1}$$

$\varrho_{(.)} \in \mathbb{R}$ denotes the external disturbances affecting the dynamics of the quadcopter. $Q\chi_{(.)}(t) \in \mathbb{R}^+$ is the recoil effect, where $Q = \frac{h_g m_b}{I_g m_g}$ is the constant and $\chi_{(.)}(t)$ is angular acceleration due to recoil energy by firing of a weapon. The gravitational constant is $g = 9.8m/s^2$. $U_i \in \mathbb{R}^+, i \in [1, 2, 3, 4]$ are the control inputs to quadcopter system, can be described in details as:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} lk & lk & lk & lk \\ 0 & -lk & 0 & lk \\ lk & 0 & -lk & 0 \\ -b & b & -b & b \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{2}$$

The remaining parameters from (1) and (2) are indexed in Table 1.

Lets consider $X \in \mathbb{R}^{12}$ be the set of state variables in state space, which can be given as: $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$. Then (1) can be written in state space form as:

$$\dot{X} = \begin{bmatrix} x_2 \\ \frac{U_1}{m}(S_{x_9}C_{x_7}C_{x_{11}} + S_{x_7}S_{x_{11}}) \\ x_4 \\ \frac{U_1}{m}(S_{x_9}C_{x_7}S_{x_{11}} + S_{x_7}C_{x_{11}}) \\ x_6 \\ \frac{U_1}{m}(C_{x_9}C_{x_7}) - g \\ x_8 \\ a_1x_{10}x_{12} - a_2\Omega_\Phi x_{10} + \varrho_\phi - Q\chi_\phi + b_1U_2 \\ x_{10} \\ a_3x_8x_{12} + a_4\Omega_\Phi x_8 + \varrho_\theta - Q\chi_\theta + b_2U_3 \\ x_{12} \\ a_5x_8x_{10} + \varrho_\psi + b_3U_4 \end{bmatrix} \tag{3}$$

where $a_1 = \frac{(I_y - I_z)}{I_x}$, $a_2 = \frac{I_m}{I_x}$, $a_3 = \frac{I_z - I_x}{I_y}$, $a_4 = \frac{I_m}{I_y}$, $a_5 = \frac{I_x - I_y}{I_z}$, $b_1 = \frac{1}{I_x}$, $b_2 = \frac{1}{I_y}$, $b_3 = \frac{1}{I_z}$, $Q = \frac{h_g m_b}{m_g}$

Remark-1. The parameters $b_1, b_2, b_3 > 0$ and control input $U_1 \neq 0$.

Remark-2. $C_{x_7}, C_{x_9} > 0$, as system is bounded with $-\frac{\pi}{2} < \phi < \frac{\pi}{2}, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

Definition 1: A subset $\xi \subset \mathbb{R}^n$ is convex if, for all $u, v \in \mathbb{R}$, implies:

$$\varepsilon u + (1-\varepsilon)v \leq \varepsilon u + (1-\varepsilon)v; \quad 0 \leq \varepsilon \leq 1 \tag{4}$$

Definition 2: A function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex on $\mathbb{R}^n$ if $f(\varepsilon u + (1-\varepsilon)v) \leq \varepsilon f(u) + (1-\varepsilon)f(v)$.

Lemma 1: Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be the convex function. Then for any constant $\Pi > 0$, the subset $\xi_\Pi = \{\Lambda \in \mathbb{R}^n \mid f(\Lambda) \leq \Pi\}$ is convex set.

Proof: Let $\Lambda_1, \Lambda_2 \in \xi_\Pi$, which implies, $f(\Lambda_1) \leqslant \Pi$ and $f(\Lambda_2) \leqslant \Pi$, then:

$$\begin{aligned} &f(\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2) \leq \varepsilon f(\Lambda_1) + (1-\varepsilon)f(\Lambda_2) \\ &f(\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2) \leq \varepsilon\Pi + (1-\varepsilon)\Pi \\ &f(\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2) \leq \varepsilon\Pi + \Pi - \varepsilon\Pi \\ &f(\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2) \leq \Pi \\ &f(\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2) \leq \xi_\Pi \end{aligned} \tag{5}$$

Hence, $\varepsilon\Lambda_1 + (1-\varepsilon)\Lambda_2 \in \xi_\Pi$, which is a convex set.

Lemma 2: Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be differentiable convex function. For $\Pi > 0$, consider the subset as, $\xi_\Pi = \{\Lambda \in \mathbb{R}^n \mid f(\Lambda) \leq \Pi\} \subset \mathbb{R}^n$. Consider an interior point $\Lambda_a \in \xi_\Pi$ such that $f(\Lambda_a) < \Pi$. Consider another point $\Lambda_b \in \xi_\Pi$, which is on the boundary of the convex set, such that $f(\Lambda_b) = \Pi$, then:

$$(\Lambda_a - \Lambda_b)^T \; \nabla f(\Lambda_b) \leq 0; \quad \nabla f(\Lambda_b) = \begin{bmatrix} \frac{\partial f(\Lambda_b)}{\partial \Lambda_1} \\ \frac{\partial f(\Lambda_b)}{\partial \Lambda_2} \\ \vdots \\ \frac{\partial f(\Lambda_b)}{\partial \Lambda_n} \end{bmatrix} \tag{6}$$

Proof: Since the function f is convex

$$\begin{aligned} f(\varepsilon\Lambda_a + (1-\varepsilon)\Lambda_b) &\leq \varepsilon f(\Lambda_a) + (1-\varepsilon)f(\Lambda_b) \\ f(\Lambda_b + \varepsilon(\Lambda_a - \Lambda_b)) &\leq f(\Lambda_b) + \varepsilon(f(\Lambda_a) - f(\Lambda_b)) \\ f(\Lambda_b + \varepsilon(\Lambda_a - \Lambda_b)) - f(\Lambda_b) &\leq \varepsilon(f(\Lambda_a) - f(\Lambda_b)) \\ \frac{f(\Lambda_b + \varepsilon(\Lambda_a - \Lambda_b)) - f(\Lambda_b)}{\varepsilon} &\leq (f(\Lambda_a) - f(\Lambda_b)) \end{aligned} \tag{7}$$

As $f(\Lambda_a) < \Pi$ and $f(\Lambda_b) = \Pi, f(\Lambda_a) - f(\Lambda_b) < 0$

$$\frac{f(\Lambda_b + \varepsilon(\Lambda_a - \Lambda_b)) - f(\Lambda_b)}{\varepsilon} \leq 0 \tag{8}$$

Taking limit $\lim_{\varepsilon \to 0}$ on both sides, we get:

$$\begin{aligned} \lim_{\varepsilon \to 0} \frac{f(\Lambda_b + \varepsilon(\Lambda_a - \Lambda_b)) - f(\Lambda_b)}{\varepsilon} &\leq 0 \\ \nabla f(\Lambda_b)(\Lambda_a - \Lambda_b) &\leq 0 \\ (\Lambda_a - \Lambda_b)^T \; \nabla f(\Lambda_b) &\leq 0 \end{aligned} \tag{9}$$

Lemma 3: Suppose $(\Lambda_a - \Lambda_b)^T \; \nabla f(\Lambda_b) > 0$, i.e., Λ_b is breaking out of the bounded convex set, then the projection of $\dot{\Lambda}_b$ on the tangential plane can be given as:

$$\tan(\Lambda_b, \dot{\Lambda}_b) = \dot{\Lambda}_b - \delta\nabla f(\Lambda_b) \tag{10}$$

where $\delta\nabla f(\Lambda_b)$ is a magnitude of the projection.

$$\begin{cases} \tan(\Lambda_b, \dot{\Lambda}_b)^T = [\dot{\Lambda}_b - \delta\nabla f(\Lambda_b)]^T \\ \tan(\Lambda_b, \dot{\Lambda}_b)^T \nabla f(\Lambda_b) = [\dot{\Lambda}_b - \delta\nabla f(\Lambda_b)]^T \nabla f(\Lambda_b) \\ \dot{\Lambda}_b^T \nabla f(\Lambda_b) = \delta\nabla f(\Lambda_b)^T \nabla f(\Lambda_b) \\ \delta = \dfrac{\dot{\Lambda}_b^T \nabla f(\Lambda_b)}{\nabla f(\Lambda_b)^T \nabla f(\Lambda_b)} \\ \delta = \dfrac{\dot{\Lambda}_b^T \nabla f(\Lambda_b)}{\parallel \nabla f(\Lambda_b) \parallel^2} \end{cases} \tag{11}$$

To divert back $\dot{\Lambda}_b$ into the convex set, we get:

$$Tan(\Lambda_b, \dot{\Lambda}_b) = \dot{\Lambda}_b - \frac{\dot{\Lambda}_b^T \nabla f(\Lambda_b)}{\parallel \nabla f(\Lambda_b) \parallel^2} \nabla f(\Lambda_b) \tag{12}$$

III. CONTROL DESIGN

This section presents the complete control system, consisting of an inner loop i.e., attitude controller and outer loop i.e., position controller for the quadcopter system, shown in Fig. 2.

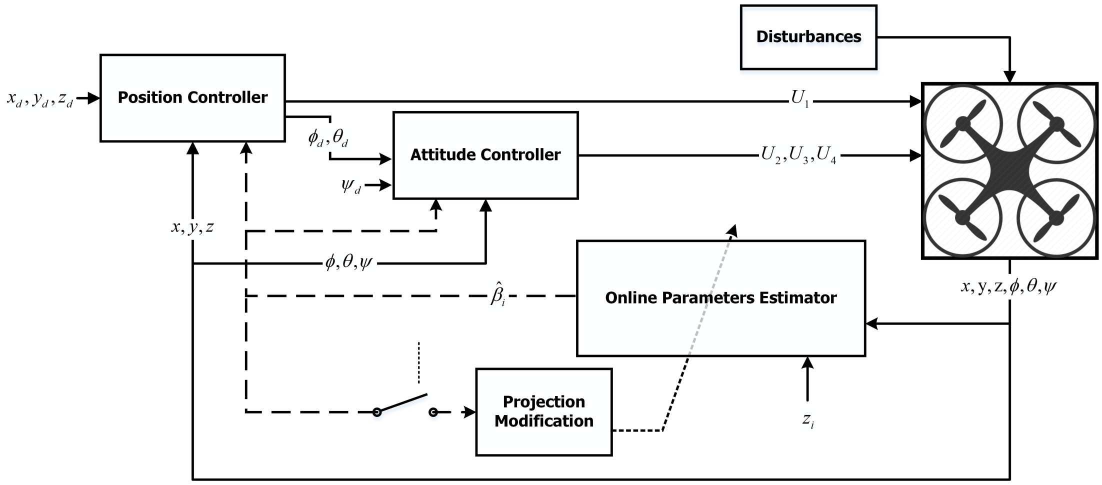

FIGURE 2. Block diagram of proposed control system.

A. ATTITUDE CONTROLLER

Let x_{7_d}, x_{9_d} and x_{11_d} are the desired set values of the roll, pitch and yaw angles, respectively. The roll system can be given as:

$$\begin{cases} \dot{x}_7 = x_8 \\ \dot{x}_8 = a_1 x_{10} x_{12} - a_2 \Omega_\Phi x_{10} \\ \qquad + \varrho_\phi - Q\chi_\phi + b_1 U_2 \end{cases} \tag{13}$$

where a_1, a_2 and b_1 are the system parameters with unknown values.

The controller is designed by following the backstepping approach. To design the overall controller and adaptive laws, the coordinates are modified as:

$$z_p = x_q - x_{q_d}; \quad z_{p+1} = x_{q+1} - \alpha_r \tag{14}$$

where α_r is the virtual control, $p, q \in [1, 2, \dots, 11]$, and $r \in [1, 2, \dots, 6]$.

Step 1: The derivative of z_1 from (14) can be written as

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{x}_{7_d} \tag{15}$$

Choosing the Lyapnov function candidate $V_{\phi_1}(z_1) = \frac{1}{2} z_1^2 > 0$. Based on derivative of $V_{\phi_1}(z_1)$, the virtual control α_1 can be obtained as:

$$\alpha_1 = -c_{\phi_1} z_1 + \dot{x}_{7_d} \tag{16}$$

where $c_{\phi_1} > 0$ is design parameter, then derivative of $V_{\phi_1}(z_1)$ is

$$\dot{V}_{\phi_1}(z_1) = -c_{\phi_1} z_1^2 + z_1 z_2 \tag{17}$$

If z_2 converges to zero, then $\dot{V}_{\phi_1}(z_1) \leq 0$.

Step 2: The derivative of z_2 is

$$\begin{aligned} \dot{z}_2 &= \dot{x}_8 - \dot{\alpha}_1 \\ \dot{z}_2 &= a_1 x_{10} x_{12} - a_2 \Omega_\Phi x_{10} + \varrho_\phi - Q\chi_\phi \\ &\quad + b_1 U_2 - (-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d}) \\ \frac{\dot{z}_2}{b_1} &= \frac{a_1}{b_1} x_{10} x_{12} - \frac{a_2}{b_1} \Omega_\Phi x_{10} + \frac{1}{b_1} \varrho_\phi - \frac{1}{b_1} Q\chi_\phi \\ &\quad + U_2 - \frac{1}{b_1}(-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d}) \\ \dot{z}_2 \beta_{\phi_1} &= \beta_{\phi_2} x_{10} x_{12} - \beta_{\phi_3} \Omega_\Phi x_{10} + \beta_{\phi_1} \varrho_\phi - \beta_{\phi_1} Q\chi_\phi \\ &\quad + U_2 - \beta_{\phi_1}(-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d}) \end{aligned} \tag{18}$$

where $\beta_{\phi_1} = \frac{1}{b_1}$, $\beta_{\phi_2} = \frac{a_1}{b_1}$, $\beta_{\phi_3} = \frac{a_2}{b_1}$. To obtain overall control law U_2, following augmented Lyapnov function candidate is considered:

$$V_{\phi_2}(z_1, z_2) = V_\phi(z_1) + \frac{|\beta_{\phi_1}| z_2^2}{2} + \frac{\tilde{\beta}_{\phi_1}^2}{2\gamma_{\phi_1}} + \frac{\tilde{\beta}_{\phi_2}^2}{2\gamma_{\phi_2}} + \frac{\tilde{\beta}_{\phi_3}^2}{2\gamma_{\phi_3}} \tag{19}$$

where $\tilde{\beta}_{\phi_1} = \beta_{\phi_1} - \hat{\beta}_{\phi_1}$, $\tilde{\beta}_{\phi_2} = \beta_{\phi_2} - \hat{\beta}_{\phi_2}$, $\tilde{\beta}_{\phi_3} = \beta_{\phi_3} - \hat{\beta}_{\phi_3}$. Taking the time derivative of (19), we get

$$\begin{aligned} \dot{V}_{\phi_2} &= -c_{\phi_1} z_1^2 + z_1 z_2 + |\beta_{\phi_1}| z_2 \dot{z}_2 + \frac{\tilde{\beta}_{\phi_1}}{\gamma_{\phi_1}} \dot{\tilde{\beta}}_{\phi_1} + \frac{\tilde{\beta}_{\phi_2}}{\gamma_{\phi_2}} \dot{\tilde{\beta}}_{\phi_2} \\ &\quad + \frac{\tilde{\beta}_{\phi_3}}{\gamma_{\phi_3}} \dot{\tilde{\beta}}_{\phi_3} \\ &= -c_{\phi_1} z_1^2 + z_2(z_1 + |\beta_{\phi_1}| \dot{z}_2) - \frac{\tilde{\beta}_{\phi_1}}{\gamma_{\phi_1}} \dot{\hat{\beta}}_{\phi_1} + \frac{\tilde{\beta}_{\phi_2}}{\gamma_{\phi_2}} \dot{\hat{\beta}}_{\phi_2} \\ &\quad + \frac{\tilde{\beta}_{\phi_3}}{\gamma_{\phi_3}} \dot{\hat{\beta}}_{\phi_3} \\ &= -c_{\phi_1} z_1^2 + z_2[sgn(\beta_{\phi_1}) z_1 + \beta_{\phi_2} x_{10} x_{12} - \beta_{\phi_3} \Omega_\Phi x_{10} \\ &\quad + \beta_{\phi_1} \varrho_\phi - \beta_{\phi_1} Q\chi_\phi + U_2 - \beta_{\phi_1}(-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d})] \\ &\quad - sgn(\beta_{\phi_1}) \frac{\tilde{\beta}_{\phi_1}}{\gamma_{\phi_1}} \dot{\hat{\beta}}_{\phi_1} - sgn(\beta_{\phi_1}) \frac{\tilde{\beta}_{\phi_2}}{\gamma_{\phi_2}} \dot{\hat{\beta}}_{\phi_2} \\ &\quad - sgn(\beta_{\phi_1}) \frac{\tilde{\beta}_{\phi_3}}{\gamma_{\phi_3}} \dot{\hat{\beta}}_{\phi_3} \end{aligned} \tag{20}$$

The adaptive control law U_2 is chosen as:

$$\begin{aligned} U_2 &= -sgn(\beta_{\phi_1}) z_1 - \hat{\beta}_{\phi_2} x_{10} x_{12} + \hat{\beta}_{\phi_3} \Omega_\Phi x_{10} - \hat{\beta}_{\phi_1} \varrho_\phi \\ &\quad + \hat{\beta}_{\phi_1} Q\chi_\phi + \hat{\beta}_{\phi_1}(-c_{\phi_1} - \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d}) - c_{\phi_2} z_2 \end{aligned} \tag{21}$$

where $c_{\phi_2} > 0$ is design parameter and $\hat{\beta}_{\phi_i}$ are estimates of $\beta_{\phi_i}(i = 1, 2, 3)$, we obtained

$$\dot{V}_{\phi_2} = -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + z_2[\tilde{\beta}_{\phi_2}x_{10}x_{12} - \tilde{\beta}_{\phi_3}\Omega_\Phi x_{10} + \tilde{\beta}_{\phi_1}\varrho_\phi - \tilde{\beta}_{\phi_1}Q\chi_\phi - \tilde{\beta}_{\phi_1}(-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d})] - sgn(\beta_{\phi_1})\frac{\tilde{\beta}_{\phi_1}}{\gamma_{\phi_1}}\dot{\hat{\beta}}_{\phi_1} - sgn(\beta_{\phi_1})\frac{\tilde{\beta}_{\phi_2}}{\gamma_{\phi_2}}\dot{\hat{\beta}}_{\phi_2} - sgn(\beta_{\phi_1})\frac{\tilde{\beta}_{\phi_3}}{\gamma_{\phi_3}}\dot{\hat{\beta}}_{\phi_3} \quad (22)$$

The update laws are chosen as

$$\begin{cases} \dot{\hat{\beta}}_{\phi_1} = \gamma_{\phi_1}sgn(\beta_{\phi_1})z_2[\hat{\varrho}_\phi - Q\hat{\chi}_\phi - (-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d})] \\ \dot{\hat{\beta}}_{\phi_2} = \gamma_{\phi_2}sgn(\beta_{\phi_1})z_2x_{10}x_{12} \\ \dot{\hat{\beta}}_{\phi_3} = -\gamma_{\phi_3}sgn(\beta_{\phi_1})z_2\Omega_\Phi x_{10} \end{cases} \quad (23)$$

Substituting U_2 from (21) and update laws from above equation, $\dot{V}_{\phi_2}$ is expressed as:

$$\dot{V}_{\phi_2} = -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + z_2[\tilde{\beta}_{\phi_1}\tilde{\varrho}_\phi - \tilde{\beta}_{\phi_1}Q\tilde{\chi}_\phi] \quad (24)$$

Step 3: Considering Lyapnov function candidate $V_{\phi_3}(z_1, z_2, \varrho, \chi)$ as

$$V_{\phi_3}(z_1, z_2, \varrho, \chi) = V_{\phi_2}(z_1, z_2) + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\varrho}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\chi}_\phi^2 \quad (25)$$

where $\tilde{\varrho}_\phi = \varrho_\phi - \hat{\varrho}_\phi$, $\tilde{\chi}_\phi = \chi_\phi - \hat{\chi}_\phi$ and both are bounded by unknown constant. The derivative of Lyapunov function $\dot{V}_{\phi_3}(z_1, z_2, \varrho, \chi)$ is given as

$$\begin{aligned}\dot{V}_{\phi_3} &= -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + z_2[\tilde{\beta}_{\phi_1}\tilde{\varrho}_\phi - \tilde{\beta}_{\phi_1}Q\tilde{\chi}_\phi] + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\varrho}_\phi\dot{\tilde{\varrho}}_\phi + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\tilde{\chi}_\phi\dot{\tilde{\chi}}_\phi \\ &= -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + z_2[\tilde{\beta}_{\phi_1}\tilde{\varrho}_\phi - \tilde{\beta}_{\phi_1}Q\tilde{\chi}_\phi] + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\varrho}_\phi\dot{\varrho}_\phi - 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\varrho}_\phi\dot{\hat{\varrho}}_\phi + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\tilde{\chi}_\phi\dot{\chi}_\phi - 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\tilde{\chi}_\phi\dot{\hat{\chi}}_\phi\end{aligned}$$

Then, we choose following update laws

$$\begin{cases} \dot{\hat{\varrho}}_\phi = \frac{\gamma_{\varrho_\phi}}{2}sgn(\tilde{\beta}_{\phi_1})z_2 \\ \dot{\hat{\chi}}_\phi = \frac{\gamma_{\chi_\phi}}{2}sgn(\tilde{\beta}_{\phi_1})Qz_2 \end{cases} \quad (26)$$

which yield

$$\begin{aligned}\dot{V}_{\phi_3} &= -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\tilde{\varrho}_\phi\dot{\varrho}_\phi + 2\frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\tilde{\chi}_\phi\dot{\chi}_\phi \\ &\leq -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + \tilde{\varrho}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\dot{\varrho}_\phi^2 + \tilde{\chi}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\dot{\chi}_\phi^2 \\ &\leq -c_{\phi_1}z_1^2 - c_{\phi_2}z_2^2 + \tilde{\varrho}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\overline{\varrho}_\phi^2 + \tilde{\chi}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\overline{\chi}_\phi^2 \\ &\leq -\Xi_\phi\left\|Z_\phi\right\|^2 + \delta_\phi\end{aligned}$$

where $\overline{\varrho}_\phi$ and $\overline{\chi}_\phi$ are the upper bounds of ϱ_ϕ and χ_ϕ, respectively. $\delta_\phi = \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\varrho_\phi}}\overline{\varrho}_\phi^2 + \frac{|\tilde{\beta}_{\phi_1}|}{\gamma_{\chi_\phi}}\overline{\chi}_\phi^2$, $\Xi_\phi = \begin{bmatrix} c_{\phi_1} & c_{\phi_2} & -1 & -1\end{bmatrix}$ and $Z_\phi = \begin{bmatrix} z_1 & z_2 & \tilde{\varrho}_\phi & \tilde{\chi}_\phi\end{bmatrix}^T$.

Choosing $\Xi_{\phi_{min}} > \delta_\phi$, satisfies

$$\dot{V}_{\phi_3}(z_1, z_2, \varrho, \chi) \leq -\Xi_{\phi_{min}}\left\|Z_\phi\right\|^2 + \delta_\phi \quad (27)$$

Since (27) satisfies that the system is globally and asymptotically stable.

Updating (21) as:

$$U_2 = -sgn(\beta_{\phi_1})z_1 - \hat{\beta}_{\phi_2}x_{10}x_{12} + \hat{\beta}_{\phi_3}\Omega_\Phi x_{10} - \hat{\beta}_{\phi_1}\hat{\varrho}_\phi + \hat{\beta}_{\phi_1}Q\hat{\chi}_\phi + \hat{\beta}_{\phi_1}(-c_{\phi_1} - \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d}) - c_{\phi_2}z_2 \quad (28)$$

We propose DI term in each update law given in (23), aiming to enhance adaptation rate and robustness

$$\begin{cases} \dot{\hat{\beta}}_{\phi_1} = \gamma_{\phi_1}sgn(\beta_{\phi_1})z_2[\{\hat{\varrho}_\phi - Q\hat{\chi}_\phi - (-c_{\phi_1} + \ddot{x}_{7_d})(x_8 - \dot{x}_{7_d})\} - sgn(\tilde{\beta}_{\phi_1})\{kd_{\phi_1}(|\ddot{x}_8 - \dddot{x}_{7_d}|) + ki_{\phi_1}(|\dot{x}_7 - \dot{x}_{7_d}|)\}] \\ \dot{\hat{\beta}}_{\phi_2} = \gamma_{\phi_2}sgn(\beta_{\phi_1})z_2[x_{10}x_{12} - sgn(\tilde{\beta}_{\phi_2})\{kd_{\phi_2}(|\ddot{x}_8 - \dddot{x}_{7_d}|) + ki_{\phi_2}(|\dot{x}_7 - \dot{x}_{7_d}|)\}] \\ \dot{\hat{\beta}}_{\phi_3} = -\gamma_{\phi_3}sgn(\beta_{\phi_1})z_2[\Omega_\Phi x_{10} - sgn(\tilde{\beta}_{\phi_3})\{kd_{\phi_3}(|\ddot{x}_8 - \dddot{x}_{7_d}|) + ki_{\phi_3}(|\dot{x}_7 - \dot{x}_{7_d}|)\}] \end{cases} \quad (29)$$

with this modification

$$\begin{aligned}\dot{V}_\phi \leq& -\Xi_{min}\left\|Z_\phi\right\|^2 - |\tilde{\beta}_{\phi_1}|[kd_{\phi_1}(|\ddot{x}_8 - \dddot{x}_{7_d}|) + ki_{\phi_1}(|\dot{x}_7 - \dot{x}_{7_d}|)] - |\tilde{\beta}_{\phi_2}|[kd_{\phi_2}(|\ddot{x}_8 - \dddot{x}_{7_d}|) \\ &+ ki_{\phi_2}(|\dot{x}_7 - \dot{x}_{7_d}|)] - |\tilde{\beta}_{\phi_3}|[kd_{\phi_3}(|\ddot{x}_8 - \dddot{x}_{7_d}|) + ki_{\phi_3}(|\dot{x}_7 - \dot{x}_{7_d}|)] + \delta_\phi \leq \dot{V}_{\phi_3}(z_1, z_2, \varrho, \chi)\end{aligned} \quad (30)$$

Comparatively (30) depicts faster convergence then (27), still we need to prove the boundedness of the closed-loop system. Rewriting U_2 as

$$U_2 = -sgn(\beta_{\phi_1})z_1 - c_{\phi_2}z_2 + \eta_\phi^{1\times 3}\hat{\beta}_\phi^{3\times 1} \quad (31)$$

where $\eta_\phi^{3\times 1} = \begin{bmatrix} -\hat{\varrho}_\phi + Q\hat{\chi}_\phi + \dot{\alpha}_1 & -x_{10}x_{12} & \Omega_\Phi x_{10}\end{bmatrix}$, $\hat{\beta}_\phi^{3\times 1} = \begin{bmatrix}\hat{\beta}_{\phi_1} \\ \hat{\beta}_{\phi_2} \\ \hat{\beta}_{\phi_3}\end{bmatrix}$

By assuming $\|\beta_\phi\| \leq \beta_{\phi M}$; $\beta_{\phi_M} \in \mathbb{R}^{3\times 1}$, we define a convex function $f(\hat{\beta}_\phi) = \|\hat{\beta}_\phi\|^2 - \beta_{\phi M}^2$ and convex set $\xi_{\beta_{\phi M}} = \{\hat{\beta}_\phi \mid f(\hat{\beta}_\phi) \leq \beta_{\phi M}\}$.

By recalling *lemma 3*, we propose projection modification in adaptive laws as:

$$\dot{\hat{\beta}}_{\phi_{proj}} = \begin{cases} \dot{\hat{\beta}}_\phi & \dot{\hat{\beta}}_\phi^T\nabla f(\hat{\beta}_\phi) \leq 0 \\ \dot{\hat{\beta}}_\phi - \frac{\nabla f(\hat{\beta}_\phi)\nabla f(\hat{\beta}_\phi)^T}{\|\nabla f(\hat{\beta}_\phi)\|^2}\dot{\hat{\beta}}_\phi^T & \dot{\hat{\beta}}_\phi^T\nabla f(\hat{\beta}_\phi) > 0 \end{cases} \quad (32)$$

Lets see how this modification affects the stability analysis.

Case 1: When $\hat{\beta}_\phi^T\nabla f(\hat{\beta}_\phi) \leq 0$, the adaptive laws are not modified, hence yields (30).

Case 2: When $\hat{\beta}_{\phi}^{T}\nabla f(\hat{\beta}_{\phi}) > 0$, rewriting (19) as:

$$V_{\phi_2}^{*} = V_{\phi}(z_1) + \frac{|\beta_{\phi_1}|z_2^2}{2} + \frac{1}{2}\tilde{\beta}_{\phi}^{T}\Gamma^{-1}\tilde{\beta}_{\phi} + \frac{|\tilde{\beta}_{\phi_1}|}{2\gamma_{\varrho_\phi}}\tilde{\varrho}_{\phi}^2 + \frac{|\tilde{\beta}_{\phi_1}|}{2\gamma_{\varrho_\phi}}\tilde{\chi}_{\phi}^2;\ \Gamma^{-1} = diag\{\frac{1}{\gamma_{\phi_1}}, \frac{1}{\gamma_{\phi_2}}, \frac{1}{\gamma_{\phi_3}}\} > 0 \quad (33)$$

Derivative of $V_{\phi_2}^{*}(z_1, z_2)$ with U_2 from (31) is:

$$\begin{aligned} V_{\phi_2}^{*} &\leq -\Xi_{\phi_{min}}\left\|Z_{\phi}\right\|^2 + \delta_{\phi} + z_2\eta_{\phi}\tilde{\beta}_{\phi} \\ &\quad - sgn(\beta_{\phi_1})\tilde{\beta}_{\phi}\Gamma^{-1}\dot{\hat{\beta}}_{\phi_{proj}} \\ &\leq -\Xi_{\phi_{min}}\left\|Z_{\phi}\right\|^2 + \delta_{\phi} + z_2\eta_{\phi}\tilde{\beta}_{\phi} \\ &\quad - sgn(\beta_{\phi_1})\tilde{\beta}_{\phi}\Gamma^{-1}(\dot{\hat{\beta}}_{\phi} - \frac{\hat{\beta}_{\phi}\hat{\beta}_{\phi}^{T}}{\|\hat{\beta}_{\phi}\|^2}\dot{\hat{\beta}}_{\phi}^{T}) \\ &\leq -\Xi_{\phi_{min}}\left\|Z_{\phi}\right\|^2 + \delta_{\phi} + z_2\eta_{\phi}\tilde{\beta}_{\phi} \\ &\quad - sgn(\beta_{\phi_1})\tilde{\beta}_{\phi}\Gamma^{-1}\dot{\hat{\beta}}_{\phi} + sgn(\beta_{\phi_1})\frac{\tilde{\beta}_{\phi}}{\gamma_{\phi}}\Gamma^{-1}\frac{\hat{\beta}_{\phi}\hat{\beta}_{\phi}^{T}}{\|\hat{\beta}_{\phi}\|^2}\dot{\hat{\beta}}_{\phi}^{T}) \end{aligned}$$

with adaptive laws given in (29), we get

$$\dot{V}_{\phi_2}^{*} \leq \dot{V}_{\phi} + \tilde{\beta}_{\phi}\dot{\hat{\beta}}_{\phi}^{T}\frac{\hat{\beta}_{\phi}\hat{\beta}_{\phi}^{T}}{\|\hat{\beta}_{\phi}\|^2} \quad (34)$$

By *Lemma 2*, we assure that $\tilde{\beta}_{\phi}\dot{\hat{\beta}}_{\phi}^{T}\frac{\hat{\beta}_{\phi}\hat{\beta}_{\phi}^{T}}{\|\hat{\beta}_{\phi}\|^2} \leq 0$, which guarantees the boundedness of all parameters and the system is globally and asymptotically stable.

Similar procedure can be followed to obtain following adaptive laws and controller design for pitch and yaw motion.

Pitch Controller:

$$\begin{cases} z_3 = x_9 - x_{9_d},\ z_4 = x_{10} - \alpha_2 \\ \beta_{\theta_1} = \dfrac{1}{b_2},\ \beta_{\theta_2} = \dfrac{a3}{b_2},\ \beta_{\theta_3} = \dfrac{a4}{b_2} \\ \alpha_2 = -c_{\theta_1}z_3 + \dot{x}_{9_d},\ \dot{\alpha}_2 = (-c_{\theta_1} - \ddot{x}_{9_d})(x_{10} - \dot{x}_{9_d}) \\ \dot{\hat{\beta}}_{\theta_1} = \gamma_{\theta_1}sgn(\beta_{\theta_1})z_4[\{\hat{\varrho}_{\theta} - Q\hat{\chi}_{\theta} - \dot{\alpha}_2)\} \\ \qquad - sgn(\tilde{\beta}_{\theta_1})\{kd_{\theta_1}(|\ddot{x}_{10} - \dddot{x}_{9_d}|) + ki_{\theta_1}(|\dot{x}_9 - \dot{x}_{9_d}|)\}] \\ \dot{\hat{\beta}}_{\theta_2} = \gamma_{\theta_2}sgn(\beta_{\theta_1})z_4[x_8x_{12} \\ \qquad - sgn(\tilde{\beta}_{\theta_2})\{kd_{\theta_2}(|\ddot{x}_{10} - \dddot{x}_{9_d}|) + ki_{\theta_2}(|\dot{x}_9 - \dot{x}_{9_d}|)\}] \\ \dot{\hat{\beta}}_{\theta_3} = -\gamma_{\theta_3}sgn(\beta_{\theta_1})z_4[\Omega_{\Phi}x_8 \\ \qquad - sgn(\tilde{\beta}_{\phi_3})\{kd_{\theta_3}(|\ddot{x}_{10} - \dddot{x}_{9_d}|) + ki_{\theta_3}(|\dot{x}_9 - \dot{x}_{9_d}|)\}] \\ \dot{\hat{\varrho}}_{\theta} = \dfrac{\gamma_{\varrho_\theta}}{2}sgn(\tilde{\beta}_{\theta_1})z_4,\ \ \dot{\hat{\chi}}_{\theta} = \dfrac{\gamma_{\chi_\theta}}{2}sgn(\tilde{\beta}_{\theta_1})Qz_4 \\ U_3 = -sgn(\beta_{\theta_1})z_3 - \hat{\beta}_{\phi_2}x_8x_{12} + \hat{\beta}_{\theta_3}\Omega_{\Phi}x_8 - \hat{\beta}_{\theta_1}\hat{\varrho}_{\theta} \\ \qquad + \hat{\beta}_{\theta_1}Q\hat{\chi}_{\theta} + \hat{\beta}_{\theta_1}(-c_{\theta_1} - \ddot{x}_{9_d})(x_{10} - \dot{x}_{9_d}) - c_{\theta_2}z_4 \end{cases}$$

Yaw Controller:

$$\begin{cases} z_5 = x_{11} - x_{11_d},\ z_6 = x_{12} - \alpha_3 \\ \beta_{\psi_1} = \dfrac{1}{b_3},\ \beta_{\psi_2} = \dfrac{a5}{b_3},\ \alpha_3 = -c_{\psi_1}z_5 + \dot{x}_{11_d}, \\ \dot{\alpha}_3 = (-c_{\psi_1} - \ddot{x}_{11_d})(x_{12} - \dot{x}_{11_d}) \\ \dot{\hat{\beta}}_{\psi_1} = \gamma_{\psi_1}sgn(\beta_{\psi_1})z_6[\{\hat{\varrho}_{\psi} - \dot{\alpha}_3\} - sgn(\tilde{\beta}_{\psi_1}) \\ \qquad \{kd_{\psi_1}(|\ddot{x}_{12} - \dddot{x}_{11_d}|) + ki_{\psi_1}(|\dot{x}_{11} - \dot{x}_{11_d}|)\}] \\ \dot{\hat{\beta}}_{\psi_2} = \gamma_{\psi_2}sgn(\beta_{\psi_1})z_6[x_8x_{10} - sgn(\tilde{\beta}_{\psi_2}) \\ \qquad \{kd_{\psi_2}(|\ddot{x}_{12} - \dddot{x}_{11_d}|) + ki_{\psi_2}(|\dot{x}_{11} - \dot{x}_{11_d}|)\}] \\ \dot{\hat{\varrho}}_{\psi} = \dfrac{\gamma_{\varrho_\psi}}{2}sgn(\tilde{\beta}_{\psi_1})z_6 \\ U_4 = -sgn(\beta_{\psi_1})z_5 - \hat{\beta}_{\psi_2}x_8x_{10} - \hat{\beta}_{\psi_1}\hat{\varrho}_{\psi} \\ \qquad + \hat{\beta}_{\psi_1}(-c_{\psi_1} - \ddot{x}_{11_d})(x_{12} - \dot{x}_{11_d}) - c_{\psi_2}z_6 \end{cases}$$

Strictly, $c_{\theta_1}, c_{\theta_2}, c_{\psi_1}, c_{\psi_2} > 0$

B. ALTITUDE CONTROLLER

Let x_{5d} be the desired height, then error systems and its derivative are considered as: $z_7 = x_5 - x_{5_d}$, $z_8 = x_6 - \alpha_4$, $\dot{z_7} = z_8 + \alpha_4 - \dot{x_{5_d}}$, and choosing Lyapunov function candidate $V_{z_1}(z_7) = \frac{1}{2}z_7^2$ with virtual control, $\alpha_4 = -c_{z_1}z_7 + \dot{x_{5_d}}$, $c_{z_1} > 0$, we get:

$$\dot{V}_{z_1}(z_7) = -c_{z_1}z_7^2 + z_7z_8 \quad (35)$$

derivative of z_8 can be written as:

$$\dot{z}_8 = \left(\frac{U_o + U_p}{m_0 + m_p}\right)\Delta_z - g - \dot{\alpha}_4$$

$$\frac{m_0}{\Delta_z}\dot{z}_8 + \frac{m_p}{\Delta_z}\dot{z}_8 = U_o + U_p - \frac{m_0}{\Delta_z}(g + \dot{\alpha}_4) - \frac{m_p}{\Delta_z}(g + \dot{\alpha}_4) \quad (36)$$

where $\Delta_z = C_{x_9}C_{x_7}$, m_o and m_p represents the masses of quadrotor and payload, respectively, and the control effort required is $U_1 = U_o + U_p$, then we split the system as

$$\begin{aligned} \frac{m_0}{\Delta_z}\dot{z}_8 &= U_o - \frac{m_0}{\Delta_z}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d})) \\ \beta_{z_1}\dot{z}_8 &= U_o - \beta_{z_1}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d})) \end{aligned} \quad (37)$$

For obtaining U_o, consider following Lyapunov function candidate

$$\begin{aligned} V_{z_2}(z_7, z_8) &= V_{z_1}(z_7) + \frac{|\beta_{z_1}|}{2}z_8^2 \\ \dot{V}_{z_2} &= -c_{z_1}z_7^2 + z_8(z_7 + |\beta_{z_1}|\dot{z}_8) \\ &= -c_{z_1}z_7^2 + z_8(sign(\beta_{z_1})z_7 + \beta_{z_1}\dot{z}_8) \\ &= -c_{z_1}z_7^2 + z_8[sign(\beta_{z_1})z_7 + U_o \\ &\quad - \beta_{z_1}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d}))] \\ \dot{V}_{z_2} &= -c_{z_1}z_7^2 - c_{z_2}z_8^2 \leq 0 \end{aligned} \quad (38)$$

we obtain

$$U_o = -sign(\beta_{z_1})z_7 + \beta_{z_1}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d})) - c_{z_2}z_8 \quad (39)$$

where $c_{z_2} > 0$.

$$\begin{aligned} \frac{m_p}{\Delta_z}\dot{z}_8 &= U_p - \frac{m_p}{\Delta_z}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d})) \\ \beta_{z_2}\dot{z}_8 &= U_p - \beta_{z_2}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d}))) \end{aligned} \quad (40)$$

The Lyapunov function $V_{z_3}(z_7, z_8)$ is given as:

$$
\begin{aligned}
V_{z_3}(z_7, z_8) &= V_{z_1}(z_7) + \frac{|\beta_{z_2}|}{2} z_8^2 + \frac{1}{2\gamma_{z_1}} \tilde{\beta}_{z_2}^2 \\
\dot{V}_{z_3} &= -c_{z_1} z_7^2 + z_8(z_7 + |\beta_{z_2}| \dot{z}_8) + \frac{1}{\gamma_{z_1}} \tilde{\beta}_{z_2} \dot{\tilde{\beta}}_{z_2} \\
&= -c_{z_1} z_7^2 + z_8(sign(\beta_{z_2}) z_7 + \beta_{z_2} \dot{z}_8) \\
&\quad + \frac{sign(\beta_{z_2})}{\gamma_{z_1}} \tilde{\beta}_{z_2} \dot{\tilde{\beta}}_{z_2} \\
&= -c_{z_1} z_7^2 + z_8[sign(\beta_{z_2}) z_7 + U_p \\
&\quad - \beta_{z_2}(g + (-c_{z_1} + \ddot{x}_{5_d})(x_6 - \dot{x}_{5_d}))] \\
&\quad - \frac{sign(\beta_{z_2})}{\gamma_{z_1}} \tilde{\beta}_{z_2} \dot{\hat{\beta}}_{z_2}
\end{aligned} \tag{41}
$$

By choosing U_p and $\dot{\hat{\beta}}_{z_2}$ as:

$$
\begin{cases}
U_p = -sign(\beta_{z_2}) z_7 + \hat{\beta}_{z_2}(g + (-c_{z_1} + \ddot{x}_{5_d}) \\
\qquad (x_6 - \dot{x}_{5_d}) - c_{z_2} z_8 \\
\dot{\hat{\beta}}_{z_2} = -\gamma_{z_1} sign(\beta_{z_2}) z_8[(g + (-c_{z_1} + \ddot{x}_{5_d}) \\
\qquad (x_6 - \dot{x}_{5_d}) - sgn(\tilde{\beta}_{z_2})\{kd_{z_1}(|\ddot{x}_6 - \dddot{x}_{5_d}|) \\
\qquad + ki_{z_1}(|\dot{x}_5 - \dot{x}_{5_d}|)\}]
\end{cases} \tag{42}
$$

we get

$$
\begin{aligned}
\dot{V}_{z_3} = -c_{z_1} z_7^2 - c_{z_2} z_8^2 - |\tilde{\beta}_{z_2}|[kd_{z_2}(|\ddot{x}_6 - \dddot{x}_{5_d}|) \\
+ ki_{z_2}(|\dot{x}_5 - \dot{x}_{5_d}|)] + \tilde{\beta}_{z_2} \dot{\tilde{\beta}}_{z_1} \leq 0
\end{aligned} \tag{43}
$$

For tracking x and y positions, the desired commands x_{1_d}, x_{3_d} can easily be translated into respective angular reference inputs by:

$$
\begin{cases}
x_{9_d} = \arcsin[-sign(\beta) z_9 + \beta(\ddot{x}_{1_d} - c_{x_1}) \\
\qquad (x_2 - \dot{x}_{1_d}) - c_{x_2} z_{10}] \\
x_{11_d} = \arcsin[-sign(\beta) z_{11} + \beta(\ddot{x}_{3_d} - c_{y_1}) \\
\qquad (x_4 - \dot{x}_{3_d}) - c_{y_2} z_{12}]
\end{cases} \tag{44}
$$

where $c_{x_1}, c_{x_2}, c_{y_1}, c_{y_2} > 0$, $\beta = \frac{m}{U_1}$.

IV. SIMULATION STUDY

In this section, the numerical simulation is carried out to prove and validate the performance of the proposed PM based robust adaptive backstepping control (PMABC). The simulation is set with following assumptions: (i) the recoil effect is generated with respect to "*.32 S&W-Long*" bullet weighing 98 grains (6 grams), produces recoil velocity of 1.88976 m/s with the standard gun; (ii) the effect of time varying external disturbance is characterized by function given in (45) affecting roll, pitch and yaw movements, all the time during the flight; (iii) the controlled mechanism is assumed for payload pick up or drop off and firing the gun. The remaining parameters are presented in Tables 1 and 2.

$$
\varrho_\phi(t) = \varrho_\theta(t) = \varrho_\psi(t) = \sin(\frac{8\pi}{25} t) \tag{45}
$$

TABLE 1. System parameters.

Parameter	Value (unit)	Description
I_x,I_y I_z	0.0085136, 0.0085136 0.0085136 (kgm^2)	Inertia in respective axes
m	1.5 (kg)	Mass of quadcopter
k	1.4865×10^{-7} (Ns^2)	Thrust coefficient
b	1.27513×10^{-7} (Nms^2)	Drag coefficient
I_m	8.5×10^{-4} (kgm^2)	Moment of inertia of motor
h_g	0.12192 (m)	Perpendicular distance of the center of mass of the weapon below its barrel axis
m_b	0.00635029 (kg)	Mass of the bullet
m_g	0.907185 (kg)	Mass of the gun

TABLE 2. Controller design parameters.

Parameter	X	Y	Z	ϕ	θ	ψ
c_1, c_2	0.48, 5	0.48,5	4.9,2	20,4.776	20,5.98	20,2.8
$\gamma_1, \gamma_2, \gamma_3$	-	-	1.1,-,-	0.28,4.98, 1.03	1.98,4.82, 1.03	12.98,0.28, -
$\gamma_\varrho, \gamma_\chi$	-	-	-	10,1.32	1.2,1.32	10,-
kd_1, ki_1	-	-	4.8,1.1	0.045,0.1	0.21, 0.00001	0.2,0.1
kd_2, ki_2	-	-	-	0.45,0.01	0.41,0.001	0.99,0.5
kd_3, ki_3	-	-	-	0.35,0.01	0.35,0.01	-

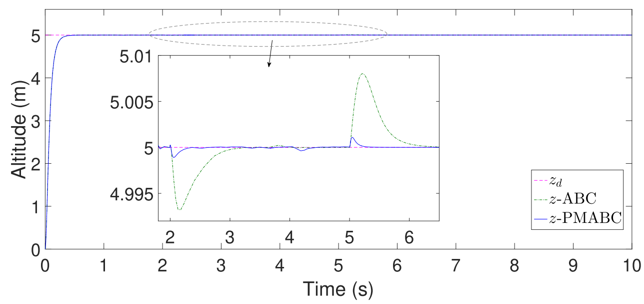

FIGURE 3. Comparison of altitude tracking performance.

To analyze the performance of proposed controller and effect of abrupt change in mass on quadcopter, the payload weighing $1kg$ is set to being picked up and dropped off by quadcopter at $t = 2s$ and $t = 5s$, respectively. Fig. 3, shows comparison of the altitude tracking performances, one is obtained with proposed altitude controller (PMABC) with DI-PM based mass estimator design and other is obtained with adaptive backstepping controller (ABC) with simple adaptive mass estimator. It can be seen that PMABC essentially rejects the disturbing force caused by abrupt mass change and exhibits steady and robust performance. While, the simple adaptive backstepping controller designed with $\hat{m}$ without DI-PM, reflects small deviations in the trajectory tracking, for the short period of time. Nevertheless, the tracking performance under both approaches is very acceptable.

The Fig. 4, validates the effect of robust DI-PM in adaptive law for mass estimation. It clearly depicts the superiority over the simple adaptive mass estimator. The robust adaptive law, estimates the mass change, rapidly. Hence, the amount of effort U_p respective to the mass change, is being added with the controller, in order to prevent any disturbance in the altitude motion. Whereas, the non-robust adaptive mass estimator, is slower than the proposed one, causing the altitude controller to respond slowly against the mass change, hence altitude, slightly deviates from steady position, already shown in Fig. 3.

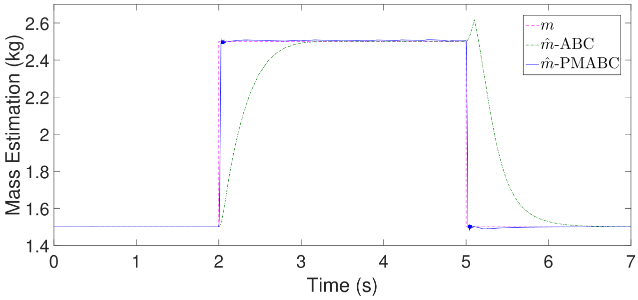

FIGURE 4. **Comparison of mass estimator performance.**

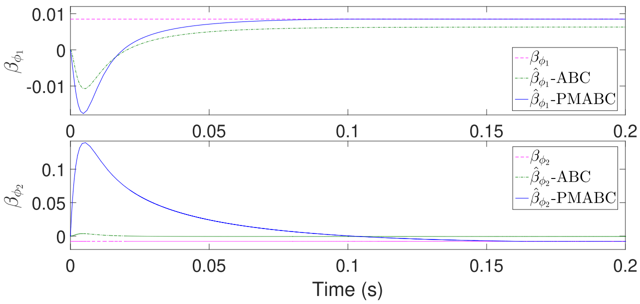

FIGURE 5. **Comparison of proposed and simple adaptive estimator for roll parameters.**

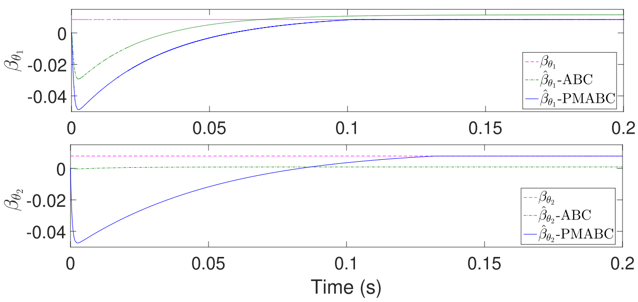

FIGURE 6. **Comparison of proposed and simple adaptive estimator for pitch parameters.**

The attitude tracking performance is presented in Figs. 5-7, it can be seen that during the flight time, all system parameters are being estimated. The robust adaptive laws exhibit remarkable performance even in the presence of external disturbances, as compared with simple adaptive law. The estimation process is very smooth with a higher convergence rate, approximately $< 0.1sec$. In addition, the robust adaptive controller is able to trace the unknown external disturbances, in short time after the parameters are being estimated (Fig. 8), which are effectively rectified from the system. Due to the fact that the proposed controller is based

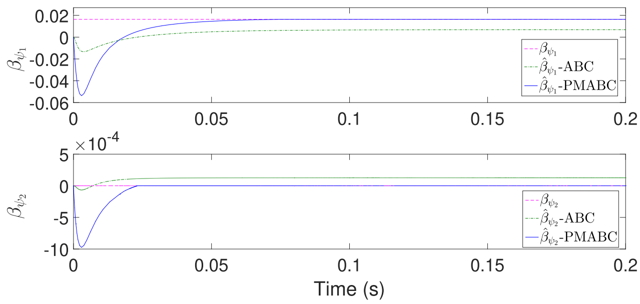

FIGURE 7. **Comparison of proposed and simple adaptive estimator for yaw parameters.**

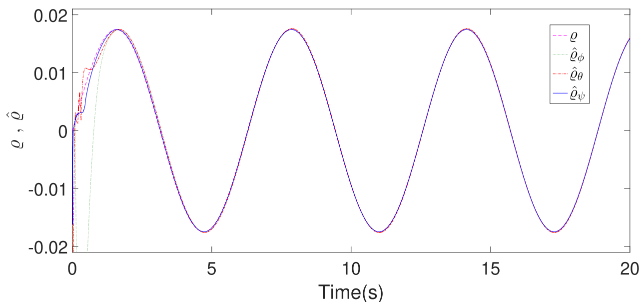

FIGURE 8. **Disturbance estimation in attitude system by proposed method.**

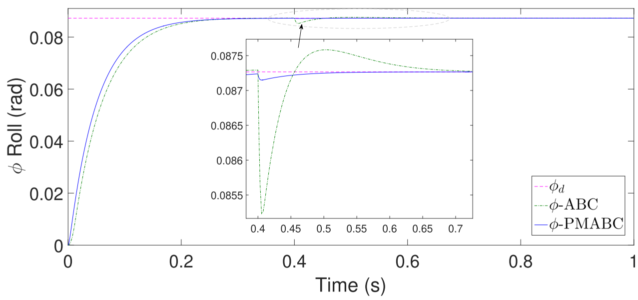

FIGURE 9. **Comparison of roll tracking performance.**

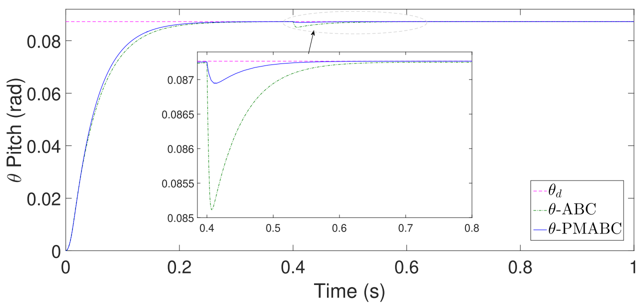

FIGURE 10. **Comparison of pitch tracking performance.**

on estimated parameters, hence producing good asymptotic attitude tracking performance shown in Figs. 9-11. The impact of recoil energy on the system at $t = 0.4s$ is tested, which is successfully damped by the proposed controller.

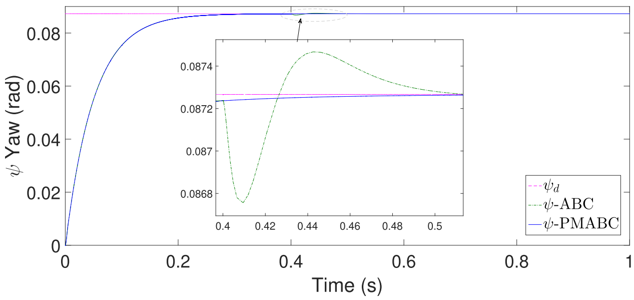

FIGURE 11. **Comparison of yaw tracking performance.**

V. CONCLUSION

This study proposed the projection modification based robust adaptive backstepping controller, which effectively and accurately estimates all the parameters of the uncertain quadcopter system. Based on estimated parameters, the overall controller is designed to improve the altitude and attitude tracking performance of the UAV. In this study, it has been shown that DI-PM based adaptive laws fortifying the backstepping controller against the external disturbances and uncertainties present in system parameters. The proposed algorithm will be capable to perform adequately for the systems ranging from partially known to fully known, by just defining the bound for the system parameters. The results validate that the DI-PM enhances the robustness of the adaptive laws, where the estimation of system parameters evolve inside the bounded set, resulting fast and accurate adaptation. Future work will focus on utilizing the proposed method on UAVs with actuator or sensor faults, without compromising the tracking performance. The numerical simulations presented the effectiveness of the proposed control system. and validating it experimentally, is also planned for future study.

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