

# Efficient Phase II Monitoring Methods for Linear Profiles Under the Random Effect Model

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**ABSTRACT** A profile is a functional relationship between two or more variables used to monitor the process performance and its quality. Sometimes, the aforementioned relationship is linear or nonlinear depending upon the situation. A monitoring method based on the linear profiles is known as linear profiling which is commonly used due to its simplicity and efficacy. Linear profiling methods have been studied by many researchers with a fixed effect model. However random effect model provides a more suitable interpretation as compared to the fixed effect model under different real-time monitoring methods. Therefore in this article, we are intended to propose a linear profiling EWMA method ( $EWMA_{x[R]}-3$  chart) and  $MEWMA_{x[R]}$  chart based on the random effect model using different ranked set sampling techniques such as ranked set sampling (RSS), extreme RSS (ERSS), median RSS (MRSS), double RSS (DRSS), double ERSS (DERSS) and double MRSS (DMRSS). The ranked set sampling (RSS) schemes are not only cost-effective method but also an efficient mechanism as compared to simple random sampling. A designed simulation study used Average Run Length (ARL) as an evaluation measure to witness the detection ability of newly offered  $EWMA_{x[R]}-3$  chart,  $MEWMA_{x[R]}$  chart and existing  $EWMA_{x[SRS]}-3$  chart. The extensive simulation showed that the proposed  $EWMA_{x[R]}-3$  chart and  $MEWMA_{x[R]}$  chart have superiority to detect faults in the process compared to a competitive counterpart. The results are further justified with real data application related to a combined cycle power plant.

**INDEX TERMS** Average run length, double RSS, EWMA, intercept, linear profiles.

## I. INTRODUCTION

Product quality and cost are the most significant consumer preferences in this modern era. This is the reason why most of the production companies engaged in enhancing product quality with the minimum cost of production, in competing with other companies. To achieve this goal, a product should be free from all defects, and these features of a product can be made through effective monitoring of the process. These processes occur with certain variations such as non-assignable and assignable causes of variation. The first one is the natural or integral part of the process that cannot be eliminated from the process. However, the second one damages the process unnaturally and needs intensive care through different statistical tools to deal with it. Statistical tools under the

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umbrella of Statistical Process Control (SPC) are used for the enhancement of process quality by minimizing the assignable cause of variations Mahmood and Xie [1]. A control chart introduced by Shewhart [2] is a primary tool of the SPC toolkit which provides a graphical outlook of the process quality. The control chart designed with the upper and lower specification limits to decide whether the process is under the In-Control (IC) or Out-Of-Control (OOC) state.

In recent times, the profile monitoring has drawn significant consideration from the researchers in which the quality characteristics of interest are defined by a regression model. Such as the density of a wood board is dependent upon the depth of board that is fixed. This functional relationship can be represented by a profile model. This linear profiling approach also terms as monitoring method of the process through regression. Commonly, Phase II methods are applied to distinguish shifts in process parameters defined under the

linear profiles model through different performance measures to run the process smoothly. Initially, Kang and Albin [3] proposed methods based on combined EWMA and R chart in which error term was examined through the EWMA control chart whereas the dispersion of error terms was observed through the R chart, and Hotelling's  $T^2$  charts were designed to monitor the intercept and slope of the linear profiles model. Reference [3] control charting structures are not able to properly advise the OOC parameters of linear profiles. To resolve this problem, Kim *et al.* [4] recommended a methodology established on the transformed simple linear profile model. The proposed EWMA-3 structure consists of separate control limits and test statistics for the error variance as well as for regression parameters. The findings of the EWMA-3 structure were more illustratable as compared to ordinary EWMA chart for linear profiles monitoring. Noorossana *et al.* [5] proposed a Multivariate CUSUM (MCUSUM/R) chart for the profiles monitoring. Zou *et al.* [6] and Mahmoud *et al.* [7] suggested control chart structures based on change-point techniques and Gupta *et al.* [8] made a comparison of the methods used by [4] and Croarkin and Varner [9]. Automated control charts of periodic residuals of simple linear profiles were introduced by Zou *et al.* [10]. The linear profiles model with autocorrelation problem was argued by Jensen *et al.* [11]. Zhang *et al.* [12] recommended a likelihood ratio-based charting structure for simple linear profiles and Saghaei *et al.* [13] proposed a technique based on CUSUM control chart. Mahmoud *et al.* [14] highlighted a linear profiling study when the sample size is at most two in the process, and a method based on the likelihood ratio test for change point models was introduced by Yeh and Zerehsaz [15]. The idea of considering the parametric uncertainty in fixed effect model was well addressed by Abbas *et al.* [16] using Bayesian approach. The parametric uncertainty in profile model is further investigated through Double EWMA and CUSUM charts in Phase II (cf. [17], [18]). Mahmoud *et al.* [19] proposed several alternative methods based on the EWMA structure which are comparatively efficient than the existing EWMA-3 structure. Recently, Saeed *et al.* [20] provided a charting structure based on the progressive linear profile statistics which is effective to monitor small-to-moderate shifts in the parameters of simple linear profiles.

The aforementioned literature and references therein have studied profiling defined by a fixed effect model. These kinds of literature assumed specified values for explanatory variables. However, there are certain situations where this assumption may not hold. It is quite common in regression model applications where the level of the predictors in a process cannot be controlled and are variable in nature. These profiles models are usually referred as the random effect models Greene [21]. For example Atmosphere Pressure ( $Y$ ) is strongly affected by Wind Speed ( $X$ ). The functional relationship among these quality characteristics can be better represented by a profile model Abbasi *et al.* [22]. As the wind speed may vary at different sampling intervals so modeling through fixed effect model is not an appropriate

choice and may produce misleading results. The suitable option for such situation and situations similar to it is random effect models. Sampson [23] study cases of random effect regression models in simple as well as in multivariate situations. Noorossana *et al.* [24] considered a case study of tape thickness that is dependent upon four random locations. This functional relationship is efficiently modeled through random effect model. Abbas *et al.* [25] explored the case in Bayesian perspective for the efficient monitoring of random effect models. There is lots literature available in which researchers used random effect model to represent the profile model for the monitoring of process parameters (cf. [26]–[28]).

The aforementioned studies have used Simple Random Sampling (SRS) strategy to draw random observations from a normal distribution for the case of fixed and random effect models. As time proceeds and development occurred in sciences, new sampling schemes were introduced and effectively used in the different dimension of social and natural sciences. These new sampling schemes not only enhance the literature but also improve the efficiency of experimental results. The notion of Ranked Set Sampling (RSS) was first familiarized by McIntyre [29] to estimate the grazing land and crops and modifications of RSS were provided by Takahasi and Wakimoto [30]. Likewise, other fields of sciences, the RSS schemes and its modified versions are extensively and effectively used in SPC literature (cf. [31]–[34]). The literature suggested above and references therein efficiently used RSS techniques to make proposed control charts more proficient and more reliable. In this study, we have investigated randomness in the independent variable using different RSS techniques. This study investigated the properties of random effect model after the modification of control limits coefficients, choice of sampling distribution for dependent and independent variables using RSS. Three separate EWMA control charts are designed under RSS techniques to monitor the process parameters of profiles model defined by a random effect model.

The rest of the article outlined follows: Section 2 provides the estimation process of the linear profiles model under RSS schemes. Section 3 presents the proposed and competing for control charting structures. Section 4 demonstrates the detailed simulation setup and comparative analysis is reported in section 5. The real data application is presented in section 6, while conclusions and recommendations are made in section 7.

## II. SIMPLE LINEAR PROFILES USING RSS STRATEGIES

In this section, we will describe the structure of the ranked set strategies used in the stated proposal. Further, the derivation of understudy profile model is discussed using the RSS strategies.

### A. RANKED SET SAMPLING SCHEMES

The RSS scheme was first proposed by [29], and reforms on RSS were provided by [30]. The structure of RSS schemes work such as  $n^2$  elements are selected, and  $n$  sets of  $n$  random

samples are constructed. The selected units are ranked in each individual set with the help of auxiliary variable, personal inspection or judgment without any actual measurement. The smallest ranked element is selected from the preliminary set, the second least ranked element from the subsequent set, and then the third set used to pick the third least ranked element. This procedure keeps going on until the highest ranked element is obtained from the last set. In this way,  $n$  elements are selected from  $n$  sets and if the above-mentioned procedure is repeated  $l$  times than  $n^*l$  elements are obtained, and the sampling scheme is named as RSS. Practically these strategies come up with tangible benefits as Dell and Clutter [35] illustrated that mean estimator of RSS is more efficient compared to SRS even if there are errors in the ranking. Further Stokes [36] proved that the estimated variance of RSS is more competent as compared to variance of SRS.

Median Ranked Set Sampling (MRSS) is an extended form of RSS introduced by Muttlak [37]. In MRSS  $n^2$  samples are selected from the target population at random and arranged into  $n$  sets just like RSS, every set having  $n$  units. All the observations in each set are organized on visual scrutiny or with the help of the auxiliary variable. For the case of odd, we choose  $\left(\frac{n+1}{2}\right)$ th ranked element from every set or select the median unit from each set. For the even sample size, we choose  $\left(\frac{n}{2}\right)$ th ranked elements from the first  $\frac{n}{2}$  sets and  $\left(\frac{n+1}{2}\right)$ th ranked elements from the last  $\frac{n}{2}$  sets. In result,  $n$  units are obtained through the aforementioned procedure, and if the above-mentioned procedure is recurrent  $l$  times than  $n^*l$  units are obtained from MRSS.

Extreme Ranked Set Sampling (ERSS) is another type of RSS proposed by Samawi et al. [38]. For the case of ERSS,  $n^2$  random units are arranged into  $n$  sets, where all sets consist of  $n$  samples. All  $n$  observations are arranged in each set on visual judgment. For the case of odd set size, choose the smallest ranked elements from first  $\left(\frac{n-1}{2}\right)$  sets; the largest ranked elements from last the  $\left(\frac{n-1}{2}\right)$  sets and the median ranked element from  $\left(\frac{n+1}{2}\right)$  set. For the case of even set size, we choose the least ranked element from first half sets, and the largest ranked element from last half sets to obtain the sample of  $n$  observations. The aforementioned procedure is repeated  $l$  times to get  $n^*l$  units from ERSS.

Al-Saleh and Al-Kadiri [39] proposed another sampling technique similar to RSS, where  $n^3$  sampling units are selected and divided into  $n$  sets each set consisting of  $n^2$  sampling units known as Double Ranked Set Sampling (DRSS). These sampling units are arranged in each set with respect to the auxiliary variable. The  $n^2$  sampling units are selected from the total of  $n^3$  sampling units using RSS technique. Again  $n$  sets are formed from these  $n^2$  units, and  $n$  units are chosen. The process may be repeated  $l$  times to obtain  $n^*l$  observations from DRSS. The similar extension of MRSS and ERSS can also be obtained and named by Double Median Ranked Set Sampling (DMRSS), and Double Extreme Ranked Set

Sampling (DERSS) respectively. For a detailed understanding, see [33] and [40].

**B. ESTIMATION OF LINEAR PROFILES MODEL UNDER RSS SCHEMES**

This section describes the basic framework of the linear profiles model under RSS schemes. The least-square method is applied to obtain the estimates of intercept, slope and errors variance, which are further used to design control charting structures under RSS. The simple linear profiles model for the  $j^{th}$  order and  $k^{th}$  cycle using RSS techniques can be defined as:

$$Y_{[i]jk} = \beta_{[0]} + \beta_{[1]}X_{[i]jk} + \varepsilon_{[i]jk}, \quad i = 1, 2, 3, \dots, n, \\ k = 1, 2, 3, \dots, m \quad (1)$$

where  $Y_{[i]jk}$  denotes the  $i^{th}$  ordered observation from the  $k^{th}$  cycle and  $j^{th}$  profile using different ranked set sampling techniques denoted by  $[R]$  such as RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS. The errors follow a standard normal distribution, and the explanatory variable follows a normal distribution with mean  $\mu_{x[R]}$  and variance  $\sigma_{x[R]}^2$ . The model provided in Equation (1) can be transformed to obtain the independent estimators for intercept, slope and errors variance as follows:

$$Y_{[i]jk} = A_{[0]} + A_{[1]}X_{[i]jk}^* + \varepsilon_{[i]jk}, \quad i = 1, 2, 3, \dots, n, \\ k = 1, 2, 3, \dots, m \quad (2)$$

where  $A_{[0]} = \beta_{[0]} + \beta_{[1]}\mu_{x[R]}$ ,  $A_{[1]} = \beta_{[1]}$  and  $X_{[i]jk}^* = X_{[i]jk} - \bar{X}_{[R]}$ . The average of the predictor can be obtained by  $\bar{X}_{[R]} = \frac{\sum_{i=1}^n \sum_{k=1}^m X_{[i]jk}}{n*m}$ . This transformed model is further used for the simulation study and comparative analysis. For the case of perfect ranked set sampling, the estimated slope coefficient for the  $j^{th}$  profile and the  $k$ th cycle is obtained by  $\hat{A}_{[1][R]} = \frac{S_{xy[R]}}{S_{xx[R]}}$ , where  $S_{xx[R]} = \sum_{i=1}^n \sum_{k=1}^m (X_{[i]jk} - \bar{X}_{[R]})^2$  and  $S_{xy[R]} = \sum_{i=1}^n \sum_{k=1}^m (X_{[i]jk} - \bar{X}_{[R]})Y_{[i]jk}$ . The estimated intercept can be obtained by  $\hat{A}_{[0][R]} = \hat{\beta}_{[0][R]} + \hat{\beta}_{[1][R]}\mu_{x[R]}$  and the variances of slope and intercept parameters are obtained by,  $Var[\hat{A}_{[1][R]}] = \left[ \frac{\sigma_{(0)}^2}{S_{xx[R]}} \right]$ , and  $var[\hat{A}_{[0][R]}] = \left[ \frac{\sigma_{(0)}^2 + \hat{A}_{[1][R]}^2 \sigma_{x[R]}^2}{n*m} \right]$ . Further, the estimated slope is standardized to control the variability factor due to the random  $X$ , which leads to fix the control limits of the slope parameter. Then the standardized slope coefficient is defined as  $ST(\hat{A}_{[1][R]}) = \frac{(\hat{A}_{[1][R]} - A_1)}{\sqrt{\frac{\sigma_{(0)}^2}{S_{xx[R]}}}}$ , and the resultant standardized slope follows a standard normal distribution. The mean square error defined as  $MSE_{[R]} = \frac{\sum_{i=1}^n \sum_{k=1}^m e_{[i]jk}^2}{n*m-2}$  is the unbiased estimator of error variance in which  $e_{[i]jk}$  is the  $i^{th}$  ordered error term from the  $k^{th}$  cycle and  $j^{th}$  profile for  $[R]$  technique and defined as  $e_{[i]jk} = Y_{[i]jk} - \hat{Y}_{[i]jk}$ . Moreover, the variance of  $MSE_{[R]}$  is  $Var[MSE_{[R]}] = \frac{2\sigma_{(0)}^2}{n-2}$ .

### III. LINEAR PROFILE MONITORING METHODS

In this section, control charting structures of competing and proposed linear profile methods are presented by considering the random effect of the predictor variable.

#### A. EWMA-3 CONTROL CHART UNDER SRS

Reference [24] proposed three independent EWMA statistics for the intercept, slope and error variance monitoring separately for the case of random X. In their study, random samples are drawn from a normal distribution using the SRS scheme. The EWMA statistics are defined as follows:

$$\begin{aligned}
 EWMA_{I[SRS]}(j) &= \theta \hat{A}_{0j} + (1 - \theta) EWMA_{I[SRS]}(j - 1) \\
 EWMA_{S[SRS]}(j) &= \theta ST(\hat{A}_{0j}) + (1 - \theta) EWMA_{S[SRS]}(j - 1) \\
 EWMA_{E[SRS]}(j) &= \max \left\{ \begin{array}{l} \theta \ln(MSE_{j[SRS]}) + (1 - \theta) \\ EWMA_{E[SRS]}(j - 1), \ln(\sigma_0^2) \end{array} \right\} \\
 &\quad , j = 1, 2, \dots \quad (3)
 \end{aligned}$$

where,

$$\begin{aligned}
 EWMA_{I[SRS]}(0) &= \beta_0 + \beta_1 \mu_x, EWMA_{S[SRS]}(0) = 0, \\
 EWMA_{E[SRS]}(0) &= \ln(\sigma_0^2).
 \end{aligned}$$

The smoothing constant  $\theta$  must lie between 0 and 1 (i.e.,  $0 \leq \theta \leq 1$ ). The corresponding control limits of the EWMA statistics for intercept, slope and errors variance are given as follows:

$$\begin{aligned}
 LCL &= \beta_0 + \beta_1 \mu_x - L_{I[SRS]} \sqrt{\frac{\theta}{2 - \theta} \frac{\sigma_0^2 + \beta_1^2 \sigma_x^2}{n}}, \\
 UCL &= \beta_0 + \beta_1 \mu_x + L_{I[SRS]} \sqrt{\frac{\theta}{2 - \theta} \frac{\sigma_0^2 + \beta_1^2 \sigma_x^2}{n}} \\
 LCL &= -L_{S[SRS]} \sqrt{\frac{\theta}{2 - \theta}}, UCL = +L_{S[SRS]} \sqrt{\frac{\theta}{2 - \theta}} \\
 UCL &= \ln(\sigma_0^2) + L_{E[SRS]} \sqrt{\frac{\theta}{2 - \theta} \text{Var}\{\ln(MSE_j)\}} \quad (4)
 \end{aligned}$$

where,

$$\text{Var}\{\ln(MSE_j)\} = \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{4}{3(n-2)^3} - \frac{16}{15(n-2)^5}$$

(cf. [41]).

#### B. EWMA-3 CONTROL CHART UNDER RSS SCHEMES

In the case of the random effect model, the EWMA-3 chart under different ranked set schemes is designed for the intercept, slope and errors variance monitoring. This study has considered the case of perfect ranked set schemes in which main and auxiliary variables are perfectly correlated with  $\rho = 1$ . The memory type structure considered here is represented by the notation  $EWMA_{X[R]} - 3$  control chart. Mathematically it can be proved that the variance of the intercept transforms from  $\frac{\sigma_{[0]}^2}{n * m}$  to  $\frac{\sigma_{(0)}^2 + \hat{A}_{[1][R]}^2 \sigma_{x[R]}^2}{n * m}$  after breaking the condition of the fixed random variable. Then the  $EWMA_{I[j][R]}$  statistic for  $j^{th}$  profile while monitoring of intercept and its

corresponding LCL and UCL using different [R] techniques are:

$$\begin{aligned}
 EWMA_{I[j][R]} &= \theta \hat{A}_{[0][R]} + (1 - \theta) EWMA_{I[j-1][R]} \\
 &\quad j = 1, 2, 3 \dots \quad (5) \\
 LCL_{I[R]} &= \beta_{[0]} + \beta_{[1]} \mu_{x[R]} \\
 &\quad - L_{I[R]} \sqrt{\frac{\theta}{2 - \theta} \frac{\sigma_{(0)}^2 + \hat{A}_{[1][R]}^2 \sigma_{x[R]}^2}{n * m}}, \\
 UCL_{I[R]} &= \beta_{[0]} + \beta_{[1]} \mu_{x[R]} \\
 &\quad + L_{I[R]} \sqrt{\frac{\theta}{2 - \theta} \frac{\sigma_{(0)}^2 + \hat{A}_{[1][R]}^2 \sigma_{x[R]}^2}{n * m}} \quad (6)
 \end{aligned}$$

where,  $EWMA_{I[R]}(0) = \beta_{[0]} + \beta_{[1]} \mu_{x[R]}$ . The process is considered under in-control state until the  $EWMA_{I[j][R]}$  falls within  $LCL_{I[R]}$  and  $UCL_{I[R]}$ .

Now for the monitoring of slope coefficients, it can be seen that the  $EWMA_{S[j][R]}$  statistic depends over explanatory variable X. Under the scenario of random X the  $EWMA_{S[j][R]}$  will lead to variable control limit as it will depend on a new sample, each time the process is repeated and the values of  $EWMA_{S[j][R]}$  will not match. To overcome this problem, standardized slope estimators are used. The  $EWMA_{S[j][R]}$  statistic for  $j^{th}$  profile and its corresponding LCL and UCL for the monitoring of slope are defined as:

$$\begin{aligned}
 EWMA_{S[j][R]} &= \theta ST(\hat{A}_{[1][R]}) + (1 - \theta) EWMA_{S[j-1][R]} \quad (7) \\
 LCL_{S[R]} &= -L_{S[R]} \sqrt{\frac{\theta}{2 - \theta}}, \quad \text{and } UCL_{S[R]} \\
 &= +L_{S[R]} \sqrt{\frac{\theta}{2 - \theta}} \quad (8)
 \end{aligned}$$

where,  $EWMA_{S[R]}(0) = 0$ . The process defined under stable condition till the  $EWMA_{S[j][R]}$  falls within  $LCL_{S[R]}$  and  $UCL_{S[R]}$ .

While discussing the randomness of the explanatory variable of the profile model, it has no effect on  $EWMA_{E[j][R]}$  statistic, so the control limits and test statistic wouldn't be changed for errors variance monitoring defined for the  $j^{th}$  profile as:

$$\begin{aligned}
 EWMA_{E[j][R]} &= \max \left\{ \begin{array}{l} \theta \ln(MSE_{[j][R]}) + \\ (1 - \theta) EWMA_{E[j-1][R]} \ln(\sigma_{(0)}^2) \end{array} \right\}, \\
 &\quad j = 1, 2, 3 \dots \quad (9) \\
 UCL_{E[R]} &= \ln(\sigma_{(0)}^2) + L_{E[R]} \sqrt{\frac{\theta}{2 - \theta} \text{Var}\{\ln(MSE_j)\}} \quad (10)
 \end{aligned}$$

where,  $EWMA_{E[R]}(0) = \ln(\sigma_{(0)}^2)$ . The under study process assumed to be in-control as long as the  $EWMA_{E[j][R]}$  is less than  $UCL_{E[R]}$ .

#### C. MEWMA<sub>X[R]</sub>-3 CONTROL CHART UNDER RSS SCHEMES

In this subsection, we have considered a general case of profiles monitoring and assumed an in-control general profiles

model when explanatory variables are not fixed and defined as:

$$Y_j = X_j C + \epsilon_j, \tag{11}$$

where  $C = (C^{(1)}, C^{(2)}, C^{(3)}, \dots, C^{(k)})$  is the vector of regression coefficients of  $k$ -dimension. The represented error terms  $\epsilon_j$  are identically and independently distributed with a multivariate vector of standard normal distribution. In this model  $X_j$  is provided in the form of  $(1, X_j^*)$  when  $X_j^*$  is orthogonal to 1 on 1 in the situation of  $n_j$ -variates. We have further assumed that  $X_j$  is the multivariate normally distributed vector of  $n_j$ -variates with mean vector  $\mu_X$  and variance-covariance matrix of  $\sigma_X^2$ .

Zou et al. [42] assumed a general profiles model with fixed  $X_j$ 's and designed a MEWMA<sub>[SRS]</sub> chart to monitor simultaneously, the  $m + 1$  regression parameters that includes  $m$  coefficients and the errors variances under SRS. We have further extended MEWMA scheme of [42] when the explanatory variables are random using different imperfect and perfect ranked set schemes (i.e., RSS, MRSS, ERSS) defined with a notation  $[R]$ . Now from the model in Equation (11), we defined a transformation under ranked set schemes as:

$$\omega_j ([R]) = \left( \hat{C}_{[R]j-[R]} \right) / \sigma_{[R]} \tag{12}$$

and

$$\omega_j (\sigma_{[R]}) = \Phi^{-1} \left\{ G \left( \frac{(n-m) \hat{\sigma}_{[R]j}^2}{\sigma_{[R]}^2}; n - m \right) \right\}, \tag{13}$$

where  $\hat{C}_{[R]j} = (X_j^T X_j)^{-1} X_j Y_j$ ,

$\hat{\sigma}_{[R]j}^2 = (Y_j - X_j \hat{C}_{[R]j})^T (Y_j - X_j \hat{C}_{[R]j})$ ,  $\Phi^{-1}(\cdot)$  is defined as the inverse of a multivariate standard normal cumulative function, while  $G(\cdot; \nu)$  is the represented chi-square distribution with pre-specified  $\nu$  degrees of freedom. The standardization of regression coefficients in Equation (12) and the transformation of the process standard deviations in Equation (13) allows accommodating the effect of random explanatory variables and sampling size ( $n$ ) selection. Now define a statistic  $\omega_{[R]j} = \left( \omega_j^T ([R]), \omega_j (\sigma_{[R]}) \right)^T$ , that combines the estimated regression coefficients and standard errors and form a  $(m+1)$  variate of random vector. When the process is in-control the statistic  $\omega_j$  multivariate normally distributed that has zero mean vector and variance-covariance matrix

$$H = \begin{pmatrix} X_j^T X_j & 0 \\ 0 & 1 \end{pmatrix}.$$

Then we can define a MEWMA<sub>x[R]</sub> statistic under ranked set schemes  $[R]$  as:

$$Z_{[R]j} = \lambda \omega_{[R]j} + (1 - \lambda) Z_{[R]j-(j-1)}, \quad j = 1, 2, \dots \tag{14}$$

where,  $Z_{[R]0}$  represents the preliminary vector of  $(m+1)$ -dimension. The value of smoothing constant  $\lambda$  can be anywhere between 0 and 1. The upper control limit for the

MEWMA<sub>x[R]</sub> chart for an out-of-control situation can be defined as:

$$W_{[R]j} = Z_{[R]j}^T H^{-1} Z_{[R]j} > L_{[R]} \frac{\lambda}{2 - \lambda}, \tag{15}$$

The value of the control limit coefficient  $L_{[R]}$  is adjusted to obtain a pre-specified in-control average run length ( $ARL_0$ ).

#### IV. SIMULATION SCHEME AND PERFORMANCE MEASURE

This section discusses the simulative work, and comparative analysis of the control charting structures based on simple and ranked set sampling schemes. The original model in Equation 1 takes the intercept and slope as 3 and 2, respectively, while the transformed model presented in Equation 2 takes intercept as 13 and slope remain the same. The i.i.d. errors are generated from a standard normal distribution, and the explanatory variable  $X \sim N(\text{mean} = 5, \text{variance} = 5/3)$ . At first,  $n^2$  bivariate random numbers are generated for errors and explanatory variables through different RSS techniques when  $n=4$ . The main variables are ranked with respect to auxiliary variables, and four values are selected one in each cycle. The values of the response variable are generated after using Equation 2. Subsequently, parameters have been estimated. In the next phase, all the test statistics are computed on the subject of each control chart with  $\theta = 0.2$  as smoothing constant. The shifts denoted by  $\varphi$  are incorporated in the process parameters (i.e., intercept, slope and error variance) in terms of  $\sigma$  units as:  $A_0 \text{ to } A_0 + (\varphi_I * \sigma)$ ,  $A_1 \text{ to } A_1 + (\varphi_S * \sigma)$ , and  $\sigma \text{ to } \varphi_E * \sigma$ . The magnitude of shifts in intercept is taken as 0.2-2 with a jump of 0.2, and for the slope, 0.025-0.25 are taken with a jump of 0.025. In the error variance, shifts are taken as 1.2-3 by a 0.2 shift difference.

The average run length (ARL) is the performance measure used in this study for the evaluation of charts where ARL is defined as the mean points falling inside the specification limits before a point falls outside of specification limits (cf. [18], [22], [43]). Further, simulation study having 10,000 iterations is carried out to draw the findings of the stated proposal. To obtain the overall in-control ARL ( $ARL_0$ ) 200, the control limit coefficients for each chart are adjusted on the  $ARL_0 = 600$ , while for the overall  $ARL_0 = 370$ , individual control limit coefficients are set on the  $ARL_0 = 1110$  (cf. Table 1).

#### V. COMPARATIVE ANALYSIS

This subsection explains the simulation results and finding of existing and competing for profile monitoring methods. The shifts are initiated in the procedure to check the detection ability of the control charts. These shifts are the illustration of the change in profile parameters in any manufacturing process. We have computed OOC ARL's ( $ARL_1$ ) for the simple and different ranked set sampling techniques to evaluate the performance of EWMA<sub>x[SRS]</sub>-3 and EWMA<sub>x[R]</sub>-3 (i.e., EWMA<sub>x[RSS]</sub>-3, EWMA<sub>x[ERSS]</sub>-3, EWMA<sub>x[MRSS]</sub>-3, EWMA<sub>x[DRSS]</sub>-3, EWMA<sub>x[DERSS]</sub>-3 and EWMA<sub>x[DMRSS]</sub>-3

**TABLE 1.** Coefficients of control limits under SRS and RSS schemes.

Control Charts	ARL <sub>0</sub> =200			ARL <sub>0</sub> =370		
	L <sub>I</sub>	L <sub>S</sub>	L <sub>E</sub>	L <sub>I</sub>	L <sub>S</sub>	L <sub>E</sub>
EWMA <sub>x [SRS]</sub> -3	2.97	3.0109	1.3723	3.24	3.2209	1.428
EWMA <sub>x [RSS]</sub> -3	1.984	5.9	0.95	2.15	6.03	1.068
EWMA <sub>x [MRSS]</sub> -3	1.84	2.7	0.521	1.944	2.813	0.587
EWMA <sub>x [ERSS]</sub> -3	2.094	7.616	1.222	2.24	7.71	1.31
EWMA <sub>x [DRSS]</sub> -3	1.324	9.04	0.42	1.44	9.18	0.477
EWMA <sub>x [DERSS]</sub> -3	1.65	11.95	0.73	1.78	12.0532	0.793
EWMA <sub>x [DMRSS]</sub> -3	1.096	3.495	0.19	1.161	3.59	0.233

**TABLE 2.** Performance comparison of EWMA<sub>x|[SRS]</sub>-3 and EWMA<sub>x|[R]</sub>-3 charts for intercept shift at RL<sub>0</sub>=200.

$\phi_I$	SRS	RSS	ERSS	MRSS	DRSS	DERSS	DMRSS
0	203	198.3	200	200.3	197.5	203.2	199.3
0.2	153.8	129.6	127	120	77.6	107.8	60.5
0.4	89.5	52	54.7	45.5	23.6	37.4	16.6
0.6	49	23.8	26	20.8	10.9	16.8	8
0.8	29	13.7	15	11.9	6.8	9.8	5.2
1	19	9.1	10	8	4.9	6.8	3.8
1.2	13.4	6.8	7	6.1	3.8	5.2	3
1.4	10.2	5.4	5.9	4.8	3.2	4.2	2.6
1.6	8.2	4.5	4.5	4	2.7	3.5	2.2
1.8	6.8	3.8	4.1	3.5	2.4	3.1	2
2	5.8	3.3	3.6	3.1	2.2	2.7	1.9

**TABLE 3.** Performance comparison of EWMA<sub>x|[SRS]</sub>-3 and EWMA<sub>x|[R]</sub>-3 charts for slope shift at ARL<sub>0</sub>=200.

$\phi_S$	SRS	RSS	ERSS	MRSS	DRSS	DERSS	DMRSS
0	203	198.3	200	200.3	197.5	203.2	199.3
0.025	166.1	140.6	128.2	140.9	92.7	92.6	92
0.05	117.3	77.4	70.8	76.1	39.3	40.6	35.6
0.075	77.8	43.8	41.2	41.7	21	23.1	17.2
0.1	51.6	27	26	25.2	13.3	15.5	10.5
0.125	35.5	18.4	18.3	17	9.4	11.6	7.4
0.15	26	13.6	13.7	12.3	7.3	9.2	5.6
0.175	19.6	10	10.8	9.5	5.8	7.6	4.6
0.2	15.5	8	8.9	7.7	4.9	6.4	3.8
0.225	12.6	7.2	7.6	6.5	4.2	5.5	3.3
0.25	10.6	6.2	6.2	5.6	3.7	4.8	2.9

**TABLE 4.** Performance comparison of EWMA<sub>x|[SRS]</sub>-3 and EWMA<sub>x|[R]</sub>-3 charts for error variance shift at ARL<sub>0</sub>=200.

$\phi_E$	SRS	RSS	ERSS	MRSS	DRSS	DERSS	DMRSS
1	203	198.3	200	200.3	197.5	203.2	199.3
1.2	40.5	29.8	23.8	39.6	16.5	13.2	26.2
1.4	15	11.5	10.1	14.2	8.4	7.2	10.8
1.6	8.2	6.9	6.4	7.5	5.7	5.1	6.4
1.8	5.6	4.9	4.8	4.9	4.4	3.9	4.5
2	4.3	3.9	3.9	3.7	3.5	3.2	3.5
2.2	3.6	3	3.3	2.9	2.9	2.7	2.8
2.4	3.1	2.8	2.9	2.5	2.5	2.4	2.4
2.6	2.7	2.4	2.6	2.2	2.2	2.1	2.1
2.8	2.5	2.2	2.4	1.9	2	1.9	1.9
3	2.2	2	2.2	1.8	1.8	1.8	1.7

under RSS, ERSS, MRSS, DRSS, DERSS, DMRSS schemes) charts. For comparison purposes, the focus remains

on the small shifts because the proposed and competing for control charts are efficient for the small shifts in a process.

**TABLE 5.** Performance comparison of EWMA  $x_{[SRS]}-3$  for the joint shift in intercept and slope at  $ARL_0=200$ .

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	105.4	59	34	22	15	11.2	9	7.2	6.1	5.3
0.05	70	40.1	25	17	12.3	9.6	7.7	6.5	5.6	5
0.075	46	28.5	19	13.6	10.4	8.3	7	6	5.1	4.5
0.1	32	21.3	15	11.3	9	7.3	6.2	5.4	4.7	4.2
0.125	23.7	16.6	12.3	9.7	8	6.6	5.6	5	4.4	4
0.15	18.2	13.3	10.4	8.4	7	6	5.2	4.6	4.1	3.8
0.175	14.4	11.1	9	7.4	6.3	5.4	4.8	4.3	4	3.5
0.2	12	9.6	7.9	6.6	5.7	5	4.5	4	3.7	3.4
0.225	10	8.3	7	6	5.2	4.6	4.2	3.8	3.5	3.2
0.25	8.8	7.3	6.3	5.5	4.8	4.3	4	3.6	3.3	3.1

**TABLE 6.** Performance comparison of EWMA  $x_{[R]}-3$  for the joint shift in intercept and slope at  $ARL_0=200$ .

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	66.5	29.9	16.3	10.4	7.5	5.8	4.8	4	3.5	3.1
0.05	37.1	19.5	12	8.4	6.4	5.1	4.3	3.7	3.3	2.9
0.075	23.3	14	9.4	7	5.5	4.6	4	3.4	3	2.8
0.1	16.2	10.7	7.7	6	5	4.1	3.6	3.2	2.9	2.6
0.125	12.1	8.5	6.5	5.2	4.4	3.8	3.3	3	2.7	2.5
0.15	9.5	7.1	5.7	4.7	4	3.5	3.1	2.8	2.6	2.4
0.175	7.8	6.1	5	4.2	3.6	3.2	2.9	2.6	2.4	2.3
0.2	6.6	5.4	4.5	3.8	3.4	3	2.7	2.5	2.3	2.2
0.225	5.8	4.8	4.1	3.5	3.2	2.8	2.6	2.4	2.2	2.1
0.25	5.1	4.3	3.7	3.3	2.9	2.7	2.5	2.3	2.2	2

**TABLE 7.** Performance comparison for combine shift in intercept and slope at  $ARL_0 = 200$  using ERSS.

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	66	32.3	17.9	11.4	8.2	6.4	5.1	4.3	3.8	3.3
0.05	38.1	21.1	13.1	9.1	6.9	5.5	4.6	4	3.5	3.1
0.075	24.2	15.1	10.3	7.6	6	4.9	4.2	3.6	3.2	2.9
0.1	16.9	11.5	8.4	6.5	5.3	4.4	3.8	3.4	3	2.8
0.125	12.7	9.2	7.1	5.7	4.7	4	3.5	3.2	2.9	2.6
0.15	10.1	7.7	6.1	5	4.3	3.7	3.3	3	2.7	2.5
0.175	8.4	6.6	5.4	4.5	3.9	3.4	3.1	2.8	2.6	2.4
0.2	7.1	5.8	4.8	4.1	3.6	3.2	2.9	2.7	2.5	2.3
0.225	6.2	5.1	4.3	3.8	3.4	3	2.8	2.5	2.4	2.2
0.25	5.4	4.6	4	3.5	3.1	2.8	2.6	2.4	2.3	2.1

**TABLE 8.** Performance comparison for combine shift in intercept and slope at  $ARL_0=200$  using MRSS.

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	60.5	26.1	14.3	9.1	6.6	5.2	4.3	3.7	3.2	2.9
0.05	33.4	17.2	10.5	7.4	5.7	4.6	3.9	3.4	3	2.7
0.075	20.9	12.2	8.3	6.2	5	4.1	3.5	3.1	2.8	2.5
0.1	14.5	9.4	6.8	5.4	4.4	3.7	3.3	2.9	2.6	2.4
0.125	10.7	7.6	5.8	4.7	4	3.4	3	2.7	2.5	2.3
0.15	8.5	6.4	5.1	4.2	3.6	3.2	2.8	2.6	2.4	2.2
0.175	7	5.5	4.5	3.8	3.3	2.9	2.7	2.4	2.3	2.1
0.2	6	4.8	4.1	3.5	3.1	2.8	2.5	2.3	2.2	2
0.225	5.2	4.3	3.7	3.2	2.9	2.6	2.4	2.2	2.1	2
0.25	4.6	3.9	3.4	3	2.7	2.5	2.3	2.1	2	1.9

Thus, a control chart with smaller ARL on the small shifts in intercept, slope and error variance will be considered as the most efficient control chart. The ARL results are reported in Tables 2–4 for the monitoring of intercept, slope and errors

variance under SRS and RSS schemes. The impact of ARL performance on the proposed and competing charts with accounting the shifts in intercept, slope and errors variance are described in the following lines.

**TABLE 9.** Performance comparison for combine shift in intercept and slope at  $ARL_0=200$  using DRSS.

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	32.6	13.7	7.9	5.5	4.2	3.4	2.9	2.5	2.2	2.1
0.05	17.6	9.4	6.2	4.6	3.6	3.1	2.6	2.3	2.1	2
0.075	11.3	7.1	5.1	4	3.2	2.8	2.4	2.2	2	1.9
0.1	8.2	5.7	4.3	3.5	2.9	2.6	2.3	2.1	1.9	1.8
0.125	6.4	4.7	3.7	3.1	2.7	2.4	2.1	2	1.8	1.7
0.15	5.2	4.1	3.3	2.8	2.5	2.2	2	1.9	1.8	1.6
0.175	4.4	3.6	3	2.6	2.3	2.1	1.9	1.8	1.7	1.6
0.2	3.9	3.2	2.7	2.4	2.2	2	1.9	1.7	1.6	1.5
0.225	3.4	2.9	2.5	2.3	2.1	1.9	1.8	1.7	1.5	1.4
0.25	3.1	2.7	2.4	2.1	2	1.8	1.7	1.6	1.5	1.3

**TABLE 10.** Performance comparison for combine shift in intercept and slope at  $ARL_0=200$  using DERSS.

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	45	20.7	11.6	7.7	5.7	4.5	3.7	3.2	2.8	2.5
0.05	24.1	13.7	8.7	6.3	4.9	4	3.4	3	2.7	2.4
0.075	15.2	9.9	6.9	5.2	4.2	3.6	3.1	2.7	2.5	2.3
0.1	11.2	7.8	5.8	4.6	3.8	3.2	2.9	2.6	2.3	2.2
0.125	8.6	6.3	5	4	3.4	3	2.7	2.4	2.2	2.1
0.15	6.9	5.4	4.3	3.6	3.2	2.8	2.5	2.3	2.1	2
0.175	5.9	4.7	3.8	3.3	2.9	2.6	2.4	2.2	2	1.9
0.2	5	4.1	3.5	3.1	2.7	2.4	2.2	2.1	2	1.8
0.225	4.4	3.7	3.2	2.8	2.5	2.3	2.1	2	1.9	1.8
0.25	3.9	3.4	3	2.6	2.4	2.2	2.1	1.9	1.8	1.7

**TABLE 11.** Performance comparison for combine shift in intercept and slope at  $ARL_0=200$  using DMRSS.

$\varphi_S \backslash \varphi_I$	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
0.025	24.5	10.4	6	4.2	3.3	2.7	2.4	2.1	1.9	1.8
0.05	13.4	7.2	4.8	3.6	2.9	2.5	2.2	2	1.8	1.7
0.075	8.7	5.4	3.9	3.1	2.6	2.3	2	1.8	1.7	1.6
0.1	6.3	4.4	3.4	2.8	2.4	2.1	1.9	1.8	1.6	1.5
0.125	4.9	3.7	3	2.5	2.2	2	1.8	1.7	1.6	1.4
0.15	4	3.2	2.7	2.3	2.1	1.9	1.7	1.6	1.4	1.3
0.175	3.5	2.9	2.5	2.2	2	1.8	1.7	1.5	1.3	1.2
0.2	3.1	2.6	2.3	2	1.8	1.7	1.6	1.4	1.3	1.2
0.225	2.7	2.4	2.1	1.9	1.8	1.6	1.5	1.3	1.2	1.1
0.25	2.5	2.2	2	1.8	1.7	1.5	1.4	1.2	1.1	1

**A. SHIFTS IN INTERCEPT PARAMETER**

The intercept term in the profile model is the conditional mean of the response variable when  $X=0$ . Taking into account a shift in intercept is vital, as a shift in intercept means changing the origin point of the regression line. Now, considering the case of shifts in intercept, it is seen that at the first shift  $\varphi_I = 0.2$  when  $ARL_0 = 200$ , the  $ARL_1$  values for  $EWMA_{x[SRS]}-3$ ,  $EWMA_{x[RSS]}-3$ ,  $EWMA_{x[ERSS]}-3$ ,  $EWMA_{x[MRSS]}-3$ ,  $EWMA_{x[DRSS]}-3$ ,  $EWMA_{x[DERSS]}-3$  and  $EWMA_{x[DMRSS]}-3$  are 153.8, 129.6, 127, 120, 77.6, 107.8 and 60.5, respectively. The resultant ARL values indicate that the proposed  $EWMA_{x[RSS]}-3$  charts based on [R] schemes outperform the existing chart of  $EWMA_{x[SRS]}-3$  that is based on SRS. Among these ranked set schemes, the  $EWMA_{x[DMRSS]}-3$  chart constructed by using DMRSS scheme showed the best performance as compared to others

(cf. Table 2). It continues with the best performance at different shift values, but the magnitude of difference in ARL is small at larger shift values.

**B. THE SHIFT IN SLOPE PARAMETER**

The slope parameter is vital as it explains the rate of change in response with respect to a unit change in the independent variable. This mean introducing a shift in slope will change the original rate of change in response that ends up with a false decision. For the case of slope shifts, results are provided in Table 3. At fixed  $ARL_0 = 200$ , the  $ARL_1$  values at shift 0.025 for the  $EWMA_{x[SRS]}-3$ ,  $EWMA_{x[RSS]}-3$ ,  $EWMA_{x[ERSS]}-3$ ,  $EWMA_{x[MRSS]}-3$ ,  $EWMA_{x[DRSS]}-3$ ,  $EWMA_{x[DERSS]}-3$  and  $EWMA_{x[DMRSS]}-3$  are 166.1, 140.6, 128.2, 140.9, 92.7, 92.6 and 92, respectively. Similarly, the proposed  $EWMA_{x[R]}-3$  charts based



**TABLE 12.** Performance comparison of  $MEWMA_{x[SRS]}$  and  $MEWMA_{x[R]}$  charts for intercept shift using imperfect ranked set schemes.

$\phi_I$	SRS		RSS				MRSS				ERSS					
	0.0	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1
0	199.4	203.0	199.4	198.5	199.8	198.9	204.9	202.8	203.8	198.9	198.8	199.7	199.6	201.3	200.3	201.0
0.2	157.1	187.1	176.8	174.6	160.3	143.0	179.5	172.1	161.0	150.0	142.6	188.1	182.1	180.1	170.9	148.6
0.4	93.2	156.3	133.9	135.8	110.3	85.4	142.3	127.9	107.6	89.1	83.0	158.1	164.8	157.0	131.3	91.0
0.6	51.8	107.1	90.7	96.6	65.5	49.1	100.1	83.4	62.1	49.6	48.7	130.4	132.4	123.0	89.2	50.9
0.8	30.8	76.1	61.7	73.1	38.6	28.7	61.8	49.6	36.7	29.5	28.1	102.1	100.0	83.1	55.1	30.1
1	19.8	50.7	40.1	50.7	24.6	19.0	39.6	30.0	22.0	19.5	19.0	70.1	74.1	61.7	36.3	20.5
1.2	15.0	35.6	27.8	30.3	17.4	14.5	23.7	19.4	15.5	13.6	14.4	51.2	54.5	43.6	24.0	16.1
1.4	11.4	24.8	19.6	26.7	13.2	11.1	16.4	14.1	11.8	10.6	11.0	37.6	40.9	30.7	18.7	12.0
1.6	9.1	18.7	14.2	19.7	10.5	8.8	12.3	10.6	9.2	8.4	9.1	27.1	29.5	23.4	14.1	9.4
1.8	7.5	14.1	11.4	14.5	8.5	7.4	10.0	8.5	7.3	7.0	7.3	21.2	23.3	18.0	11.4	8.0
2	6.4	12.0	9.5	12.4	7.3	6.4	8.2	7.2	6.2	6.0	6.2	17.3	18.6	14.6	9.5	6.8

**TABLE 13.** Performance comparison of  $MEWMA_{x[SRS]}$  and  $MEWMA_{x[R]}$  charts for slope shift using imperfect ranked set schemes.

$\phi_S$	SRS		RSS				MRSS				ERSS					
	0.0	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1
0	199.4	200.4	198.2	199.4	203.0	198.5	204.9	202.8	203.8	198.8	198.9	201.0	200.3	201.3	199.7	199.6
0.025	165.6	150.3	139.2	131.7	132.0	118.3	175.7	168.7	161.8	154.3	151.8	144.7	141.9	129.0	120.3	107.0
0.05	118.2	100.9	93.2	88.7	86.1	76.0	149.2	136.0	119.5	113.0	111.8	97.7	90.6	84.6	75.1	66.6
0.075	77.4	66.9	60.4	57.9	57.7	50.3	110.4	97.2	82.1	73.4	70.5	66.4	62.7	56.5	48.4	43.7
0.1	51.1	45.0	41.5	40.3	41.5	36.9	82.5	70.5	54.7	47.5	45.1	44.1	43.0	38.4	35.2	30.9
0.125	35.4	31.2	29.4	30.4	30.0	27.8	58.7	49.7	37.5	34.8	32.1	31.0	30.7	29.1	25.5	23.6
0.15	25.6	22.7	22.3	22.7	22.3	21.2	41.8	33.6	26.3	23.3	22.5	23.1	23.6	22.5	20.8	18.5
0.175	19.4	17.6	17.3	18.3	18.2	17.2	30.9	25.0	20.0	18.4	17.7	17.8	18.6	18.1	16.7	15.3
0.2	15.0	14.0	13.7	14.6	14.7	14.5	22.9	19.0	15.9	14.5	14.1	14.0	14.7	15.1	14.4	13.0
0.225	12.2	11.6	11.4	12.1	12.6	11.6	18.2	14.8	12.7	12.0	11.6	11.3	12.9	12.8	12.2	11.5
0.25	10.4	9.7	9.7	10.6	10.8	10.3	14.4	12.1	10.3	10.0	9.4	9.9	10.7	11.0	10.6	10.3

**TABLE 14.** Performance comparison of  $MEWMA_{x[SRS]}$  and  $MEWMA_{x[R]}$  charts for errors variance shift using imperfect ranked set schemes.

$\phi_E$	SRS		RSS				MRSS				ERSS					
	0	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1	0.25	0.5	0.75	0.9	1
1	199.4	200.4	198.2	199.4	203.0	198.5	200.4	198.2	199.4	203.0	204.9	198.0	201.0	201.3	199.7	199.6
1.2	33.2	38.3	39.2	39.7	38.7	35.5	38.3	39.2	39.7	38.7	36.5	36.0	35.7	29.6	27.6	22.7
1.4	12.1	14.1	14.6	16.5	15.8	14.9	14.1	14.6	16.5	15.8	14.9	14.3	13.8	12.0	11.0	10.1
1.6	7.0	8.0	8.8	9.6	9.9	9.6	8.0	8.8	9.6	9.9	9.6	8.5	7.6	7.3	7.0	6.4
1.8	4.9	5.6	6.0	6.9	7.1	7.1	5.6	6.0	6.9	7.1	7.1	6.2	5.5	5.3	5.1	4.5
2	3.8	4.3	4.6	5.4	5.6	5.6	4.3	4.6	5.4	5.6	5.6	5.2	4.1	4.4	3.9	3.5
2.2	3.2	3.9	3.9	4.1	4.6	4.7	3.9	3.9	4.1	4.6	4.7	3.8	3.8	3.9	3.5	3.1
2.4	2.9	2.9	3.2	3.6	4.0	4.3	2.9	3.2	3.6	4.0	4.3	3.4	3.0	3.3	3.2	2.7
2.6	2.3	2.7	3.0	3.2	3.4	3.5	2.7	3.0	3.2	3.4	3.5	3.2	2.6	3.0	2.8	2.4
2.8	1.6	2.1	2.2	2.3	3.0	3.3	2.1	2.2	2.3	3.0	3.3	2.5	2.3	2.1	1.9	1.7
3	1.5	2.0	1.9	2.0	2.0	2.0	2.0	1.9	2.0	2.0	2.0	2.3	2.1	1.9	1.8	1.7

on perfect ranked set schemes have outperformed the  $EWMA_{x[SRS]}-3$  chart. The significance in performance decreases with the increase of shift values. Among the proposed  $EWMA_{x[R]}-3$  charts the chart based on MDRSS schemes has shown superiority over others.

**C. ERROR VARIANCE SHIFTS**

The basic assumption of error terms in the regression model the normality with mean zero and fixed variance. This is the scenario of process under IC state, and this assumption is severally affected after shifts in error variance. This means a change in error variance will change the parameter of the

regression model. For the case of shifts in error variance, results of the linear profile monitoring methods are reported in Table 4. In the presence of a  $1.2\sigma$  shift in the errors, it is seen that the  $EWMA_{x[DERSS]}-3$  chart showed the smallest  $ARL_1$ 's around 13.2 at the fixed  $ARL_0 = 200$ . In conclusion, it is observed that all the proposed  $EWMA_{x[R]}-3$  charts outperform the  $EWMA_{x[SRS]}-3$  chart.

**D. JOINT SHIFTS IN INTERCEPT AND SLOPE**

The efficiency of control charts using RSS techniques for the case of joint shifts in the intercept and slope have also been reviewed at  $ARL_0 = 200$  (cf. Tables 5–11).

TABLE 15. Estimates of the simple linear profile parameters with control charting constant.

Scheme		Intercept	Slope	Error
SRS	Mean	454.015	-2.194	2.939
	Standard deviation	3.149	0.486	0.982
	Charting Constant	3.520	3.650	2.520
DMRSS	Mean	453.423	-2.358	2.893
	Standard deviation	3.101	0.781	1.004
	Charting Constant	3.300	3.450	2.525
DERSS	Mean	454.985	-2.163	2.967
	Standard deviation	2.444	0.299	0.971
	Charting Constant	3.000	3.200	2.530

TABLE 16. Number of OOC profiles with respect to profile indices.

Parameter	Scheme	Profiles				
		1-100	101-125	126-150	151-175	176-200
Intercept	SRS	1	5	0	14	25
	DMRSS	0	18	1	12	25
	DERSS	1	19	1	22	25
Slope	SRS	0	0	0	0	23
	DMRSS	0	0	1	0	21
	DERSS	0	0	15	25	24
Error variance	SRS	0	0	0	0	4
	DMRSS	0	0	0	0	9
	DERSS	1	0	0	0	22

Study of joint shifts is important as the change in the origin of the regression model will mislead in terms of rate of change. The combine shifts are introduced for all possible combinations of intercept and slope shifts using RSS sampling techniques. The purpose is to observe detection ability of the EWMA<sub>x</sub>[SRS]-3, EWMA<sub>x</sub>[RSS]-3, EWMA<sub>x</sub>[ERSS]-3, EWMA<sub>x</sub>[MRSS]-3, EWMA<sub>x</sub>[DRSS]-3, EWMA<sub>x</sub>[DERSS]-3 and EWMA<sub>x</sub>[DMRSS]-3 charts by taking joint shifts into account. The findings are described in the following lines:

The ARL<sub>1</sub> is observed as 105.4, 66.5, 66, 60.5, 32.6, 45, and 24.5 for the EWMA<sub>x</sub>[SRS]-3, EWMA<sub>x</sub>[RSS]-3, EWMA<sub>x</sub>[ERSS]-3, EWMA<sub>x</sub>[MRSS]-3, EWMA<sub>x</sub>[DRSS]-3, EWMA<sub>x</sub>[DERSS]-3 and EWMA<sub>x</sub>[DMRSS]-3 charts respectively, when slope is shifted around 0.025 and intercept is shifted almost 0.2. This indicates a significant improvement in the performance of control charts at small shifts with the use of perfect ranked set sampling schemes, particularly using MDRSS strategy. For the EWMA<sub>x</sub>[SRS]-3 chart, it is seen that the ARL<sub>1</sub> = 5.3 for the pair of shifts (0.025, 2) while ARL<sub>1</sub> is reported as 8.8 for the shifted pairs (0.25, 0.2). The EWMA<sub>x</sub>[RSS]-3 chart provides ARL<sub>1</sub> = 3.1 and 5.1 at (0.025, 2) and (0.25, 0.2) pairs of shifts in the slope and intercepts, respectively. At (0.025, 2) and (0.25, 0.2) shifted pairs, results showed that the EWMA<sub>x</sub>[ERSS]-3 chart provides ARL<sub>1</sub> = 3.3 and 5.4, while the EWMA<sub>x</sub>[MRSS]-3 chart have ARL<sub>1</sub> equals to 2.9 and 4.6. These findings provide the evidence that the linear profile monitoring method under MRSS scheme is slightly better than the ERSS, RSS and SRS schemes. The EWMA<sub>x</sub>[DRSS]-3 and EWMA<sub>x</sub>[DERSS]-3 charts offers ARL<sub>1</sub> = 2.1 and 3.1, and ARL<sub>1</sub> = 2.5 and

3.9 respectively, at shifted pairs (0.025, 2) and (0.25, 0.2). The EWMA<sub>x</sub>[DMRSS]-3 chart offer the best performance among all with ARL<sub>1</sub> = 1.8 and 2.5 at (0.025, 2) and (0.25, 0.2) pairs of shifts in slope and intercept respectively. Overall significant improvement in the detection ability is seen with the proposed EWMA<sub>x</sub>[R]-3 charts because ARL<sub>1</sub> is reduced from 105.4 to 24.5 at the shifted pair (0.025, 0.2) (cf. Tables 5 and 11).

E. OVERALL

The ARL curves for the EWMA<sub>x</sub>[SRS]-3, EWMA<sub>x</sub>[R]-3 charts are presented in Figures 1-6. Under the scenario of shifts in intercept and slope, the EWMA<sub>x</sub>[DMRSS]-3 have lower ARL curve as compared to all other charts, which is the evidence of its superiority. The pattern of the ARL curves also indicates that when there are shifts in errors variance, the EWMA<sub>x</sub>[DERSS]-3 chart is on the lower side. This shows a better performance of EWMA<sub>x</sub>[DERSS]-3 chart compared to all competing charts. The amount of difference is high at a smaller shift while this difference is low at the large shift values among existing and competing charts.

F. EVALUATION MEWMA<sub>x</sub>[R] CHARTS

This subsection describes the performance of newly designed MEWMA<sub>x</sub>[R] charts using the ranked set schemes of RSS, MRSS, and ERSS at different settings of correlation coefficients among main and auxiliary variable. The MEWMA chart under SRS represented by MEWMA<sub>x</sub>[SRS] chart, while under RSS, MRSS and ERSS are represented by MEWMA<sub>x</sub>[RSS], MEWMA<sub>x</sub>[MRSS], and MEWMA<sub>x</sub>[ERSS] charts respectively. The value of smoothing is chosen as

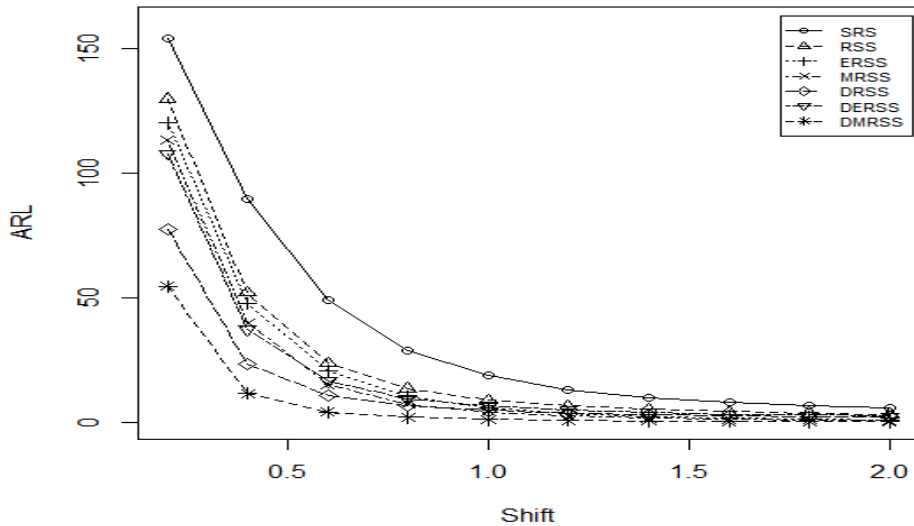


FIGURE 1. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for intercept shifts at  $ARL_0=200$ .

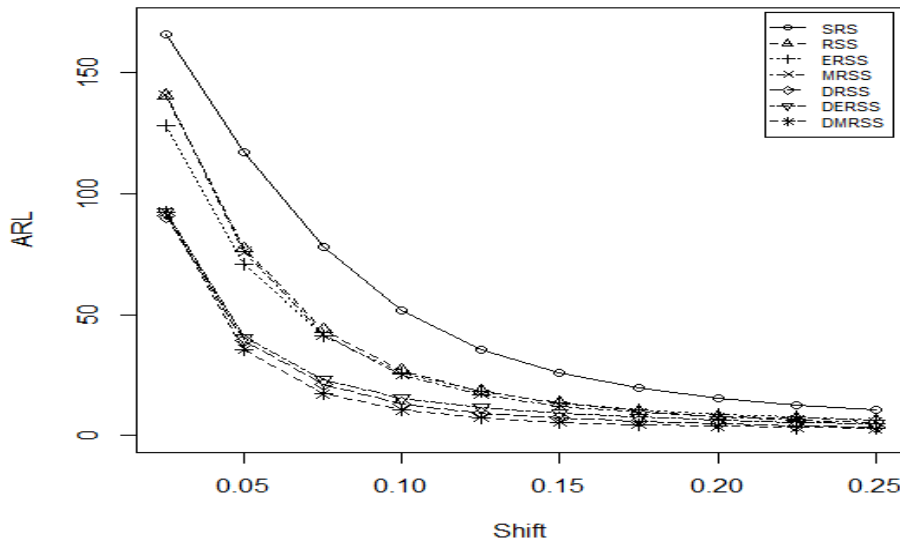


FIGURE 2. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for slope shifts at  $ARL_0=200$ .

0.2, and for imperfect ranked set schemes we have selected  $\rho = 0.25, 0.5, 0.75,$  and  $0.9$  while  $\rho = 1$  for perfect ranked set schemes. The control limit coefficients are adjusted to obtain fixed  $ARL_0$  of 200 for SRS and ranked set sampling at different values of  $\rho$ . The designed MEWMA charts under ranked set schemes can be used to monitor the general linear profiles while for a comparative perspective, let us assume  $m=2$  in Equation (1). For a comparative perspective, the in-control parameter values are considered for the model in Equation (1) as:  $C_0 = 3, C_1 = 2, \sigma_0^2 = 1$ . The explanatory variables are set to follow bivariate normal distribution with  $\mu_{X1} = \mu_{X2} = 5$  and  $\sigma_{X1}^2 = \sigma_{X2}^2 = 5/3$ . The error terms are also set to follow bivariate normal distribution with  $\mu_1 = \mu_2 = 0$  and  $\sigma_1^2 = \sigma_2^2 = 1$ . The designed and competing MEWMA control charts are evaluated for 10,000 simulations. The ARL values are computed for proposed and competing MEWMA charts and reported

in Tables 12–14 taking into account shifts in intercepts, slope and errors variance.

The values of run length measure show that  $MEWMA_{x[R]}$  charts outperformed the  $MEWMA_{x[SRS]}$  charts under perfect ranked set sampling. For the monitoring of the intercept parameter, the  $MEWMA_{x[MRSS]}$  chart showed better performance compared to all other charts in this study when  $\rho = 1$ . When shifts are introduced in slope coefficients the  $MEWMA_{x[ERSS]}$  chart outperformed other charts under perfect and imperfect ranked set schemes and same is the case for error variance monitoring. The simulation results reveal that  $MEWMA_{x[ERSS]}$  charts are best for slope and errors variance monitoring while  $MEWMA_{x[MRSS]}$  charts have advantage over other charts for intercept shifts. Overall, the  $MEWMA_{x[R]}$  charts are proposed for perfect ranked set schemes for the monitoring of general linear profiles (cf. Tables 12–14).

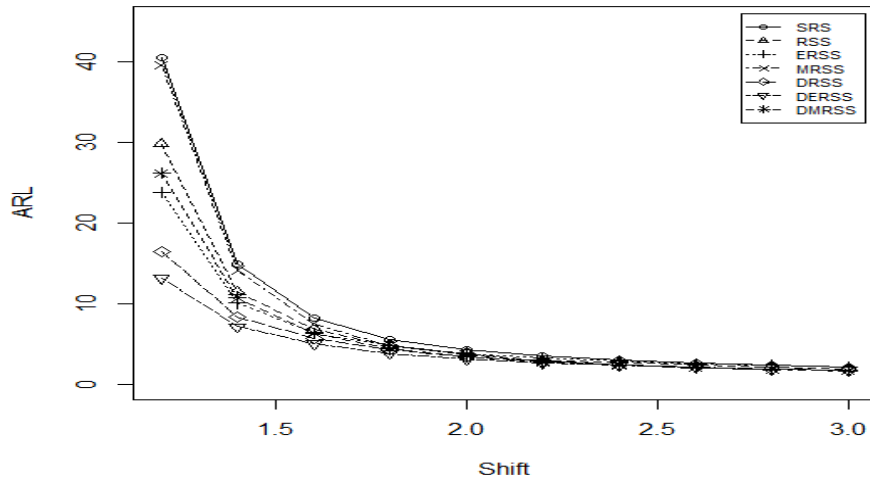


FIGURE 3. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for error variance shifts at  $ARL_0=200$ .

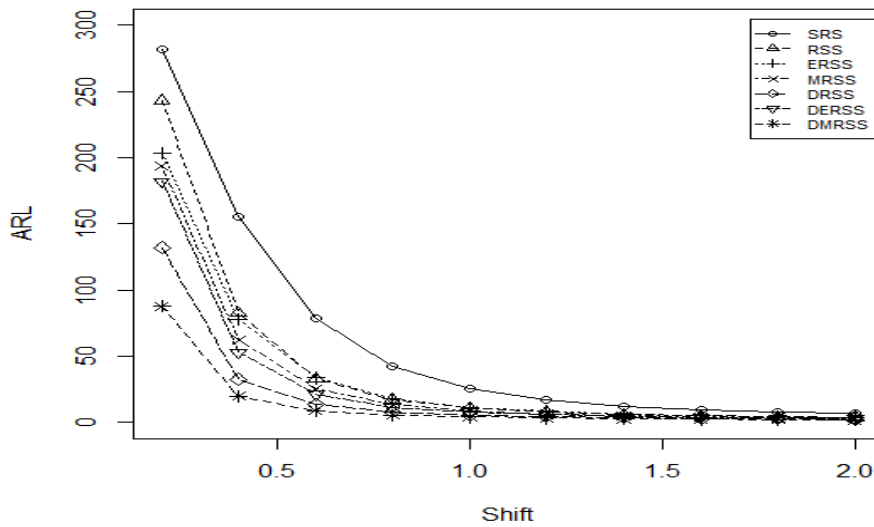


FIGURE 4. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for error intercept shifts at  $ARL_0=370$ .

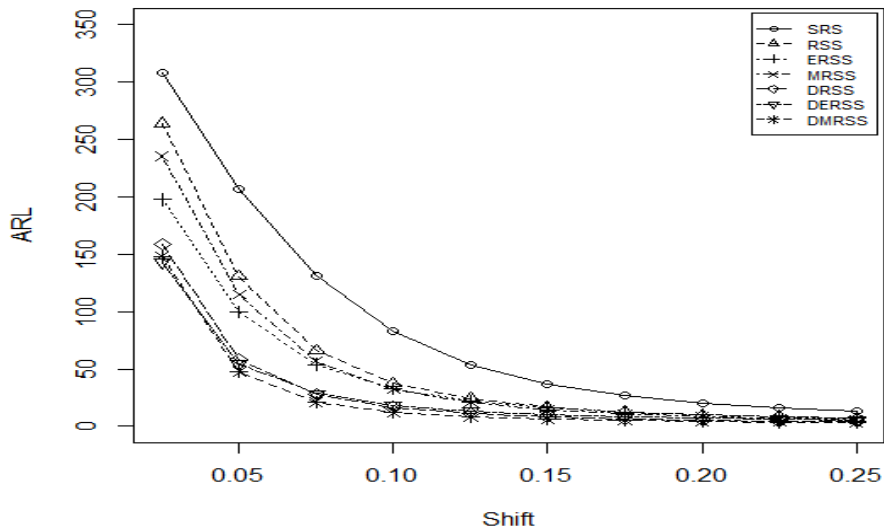


FIGURE 5. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for slope shifts at  $ARL_0=370$ .

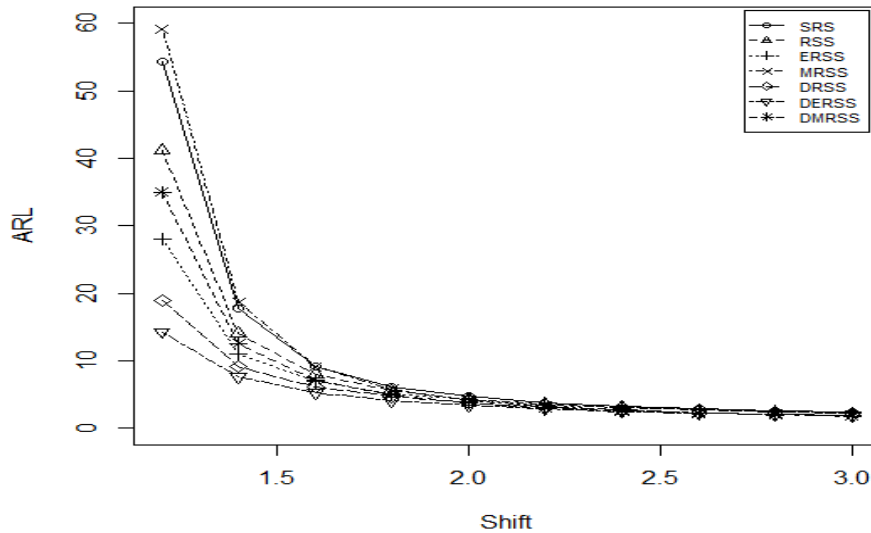


FIGURE 6. ARL comparison of  $EWMA_{x[SRS]}-3$  and  $EWMA_{x[R]}-3$  charts for error variance shifts at  $ARL_0=370$ .

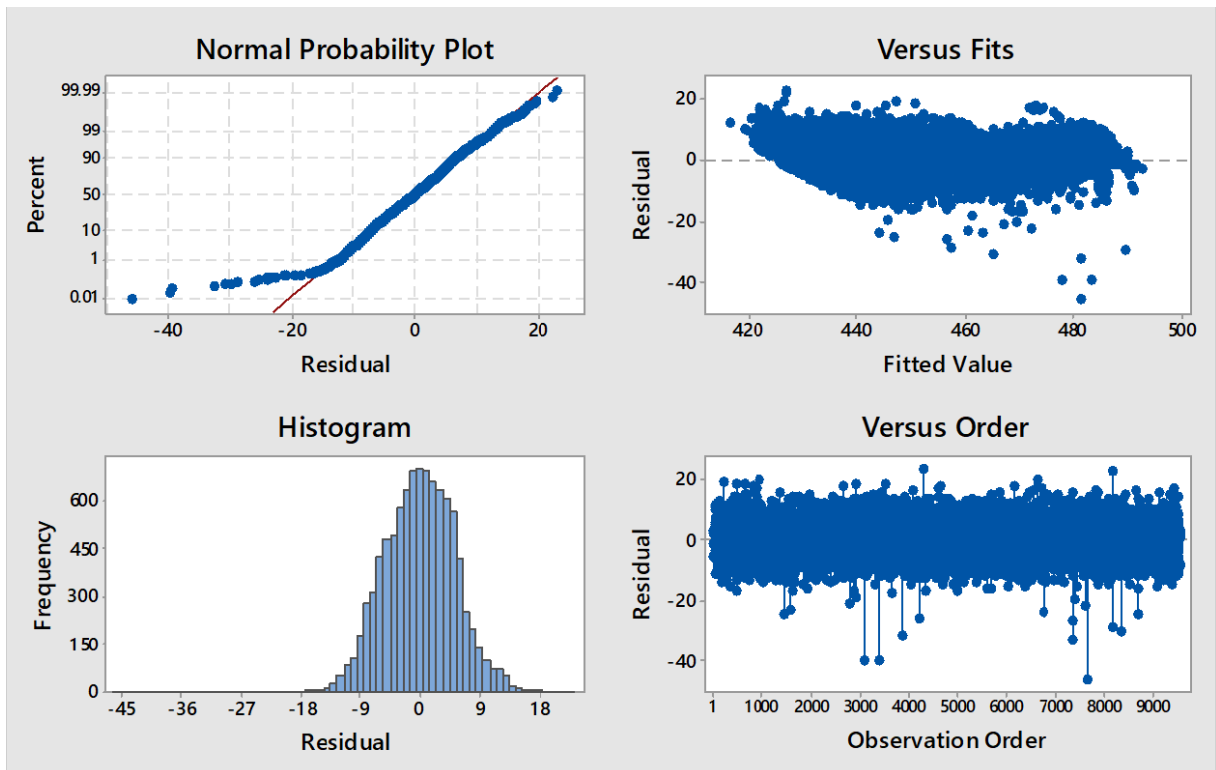


FIGURE 7. Residual plots for model 11.

### VI. A REAL DATA APPLICATION

To highlight the importance of the stated proposal, we have applied proposed charts on the dataset related to combined cycle power (CCP) plant. A CCP plant given in Figure 7 comprises steam turbines, gas turbines, and heat recovery steam generators. In the mechanism of a CCP plant, gas generators are used to generate electrical power, and waste heat of the exhaust gases are further utilized by steam generators to produce electricity.

In this study, we are using a dataset reported by Tüfekci [44]. A CCP plant with electricity generating capacity 480 MW was designed with  $1 \times 160$  MW ABB steam turbine,  $2 \times 160$  MW ABB 13E2 gas turbines, and  $2 \times$  dual heat recovery steam generators. In a CCP plant, the main load is dependent on the gas turbine which is sensitive to the ambient conditions such as atmospheric pressure (AP), ambient temperature (AT), Vacuum (V) and relative humidity (RH). Reference [44] used ambient conditions as explanatory vari-

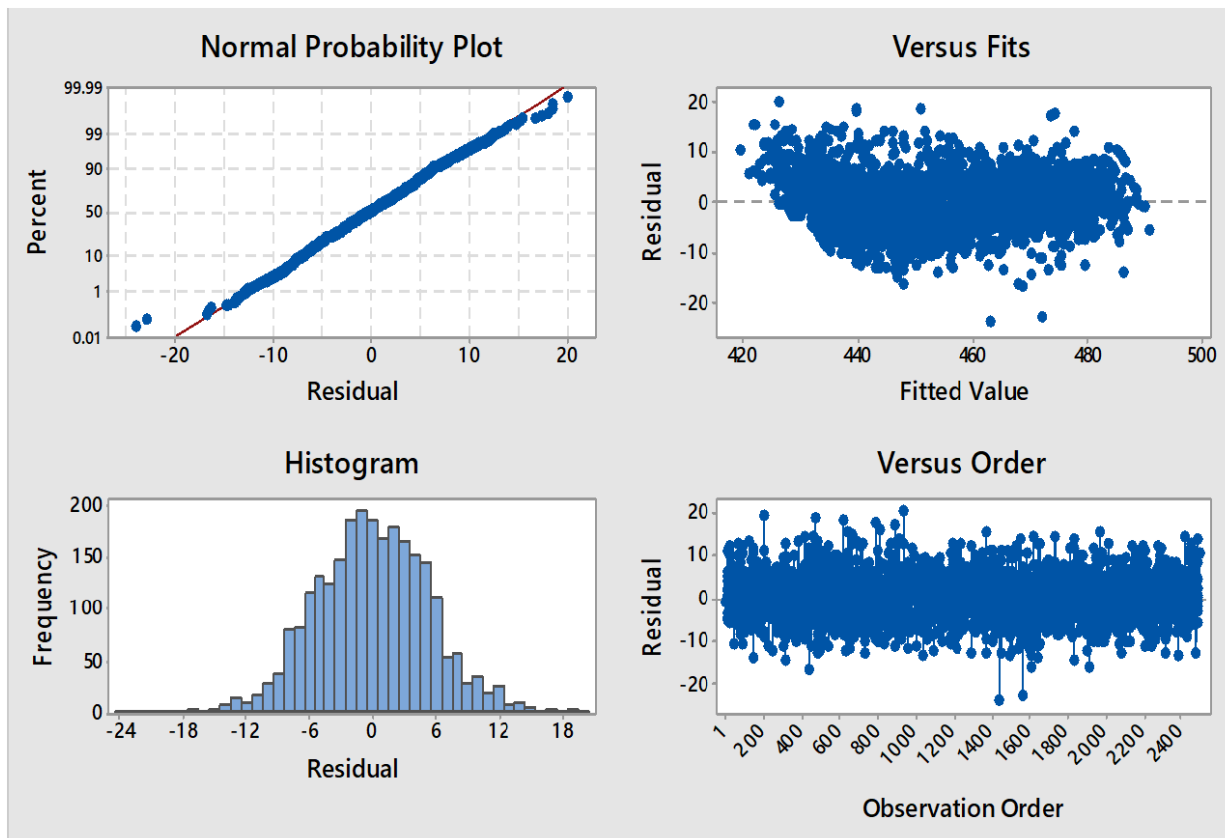


FIGURE 8. Residual plots for model 12.

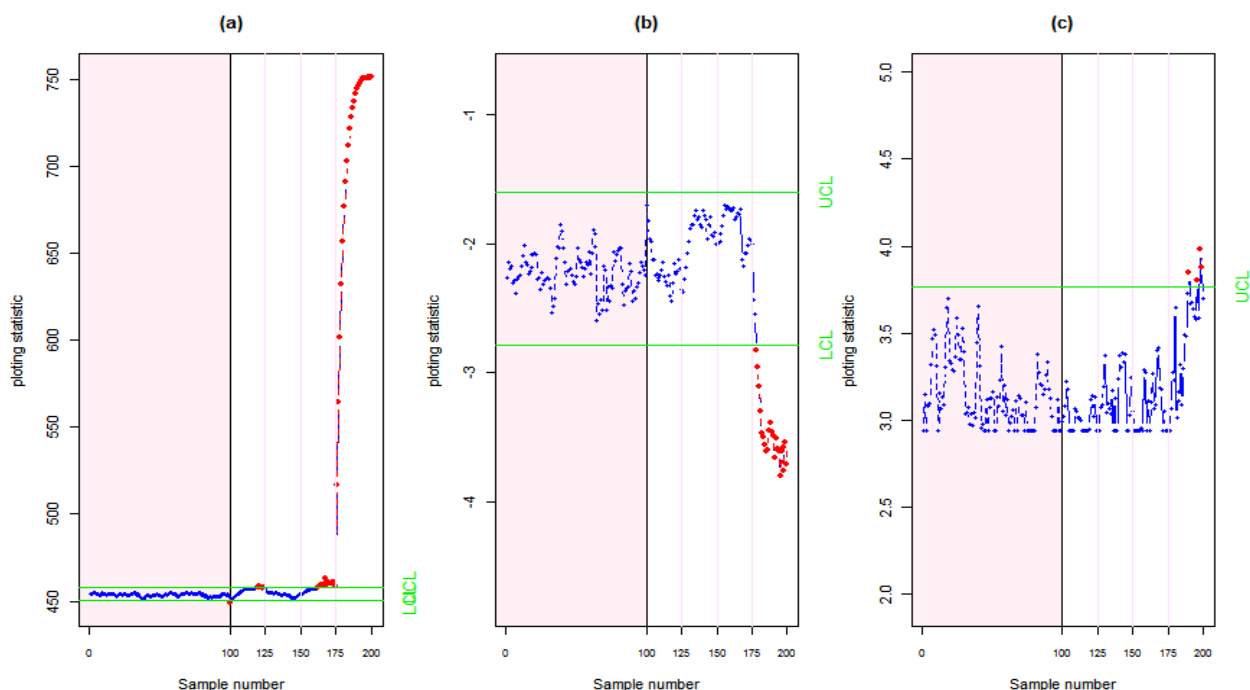
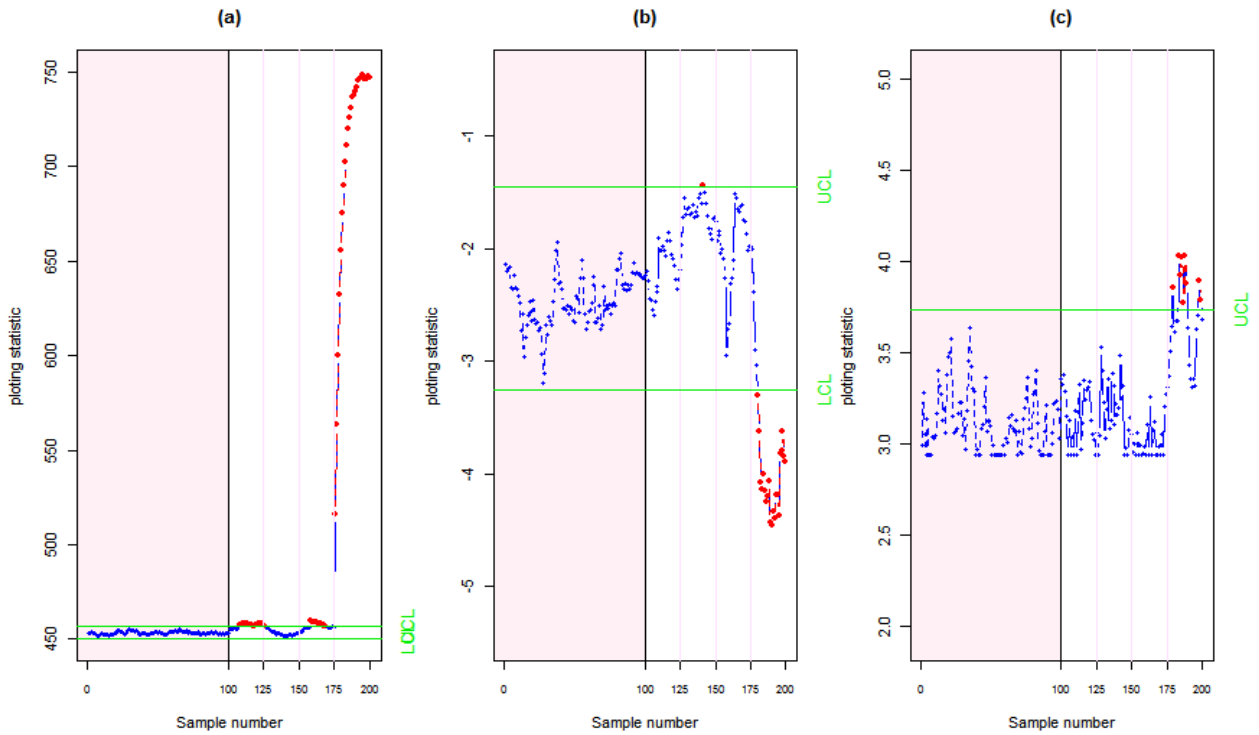


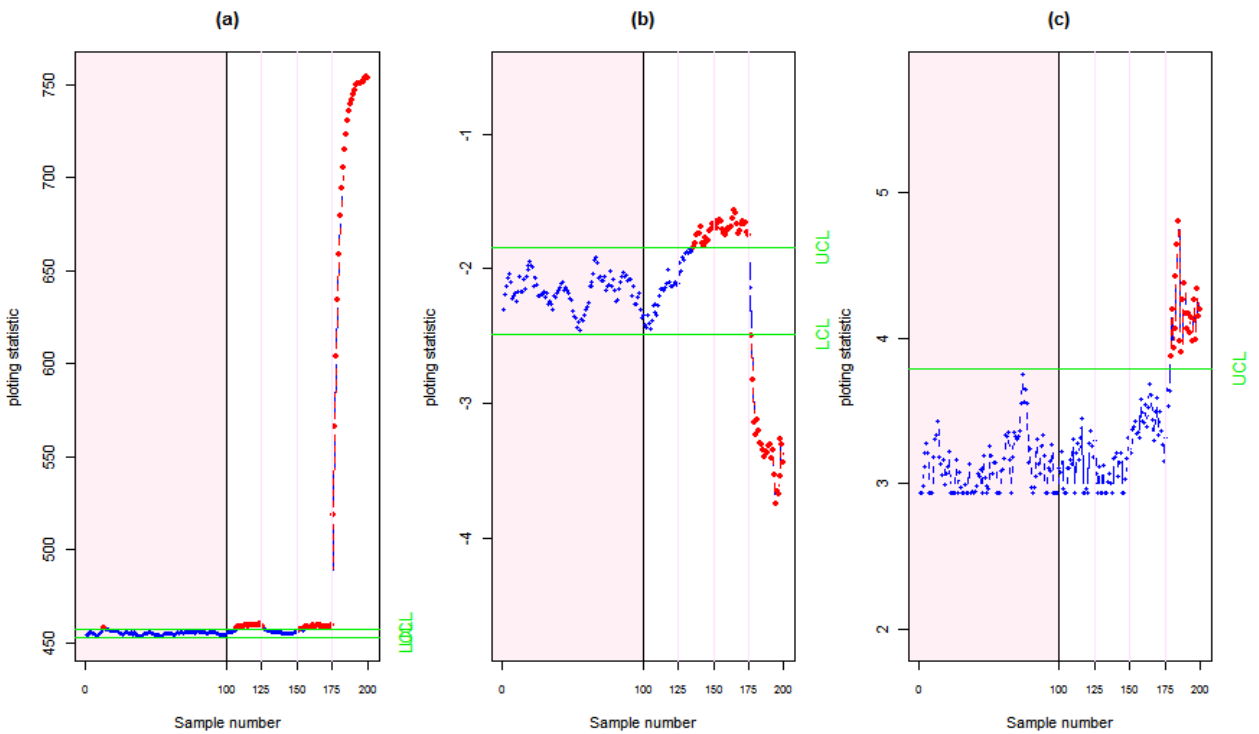
FIGURE 9.  $EWMA_{x[SRS]}-3$  control chart under SRS scheme for shift in (a) intercept parameter; (b) slope parameter; and (c) error variance parameter.

ables and full load electrical power output (PE) as a dependent variable. On the bases of 9568 data points, a possible subset

regression is used to find the significant explanatory variables for the dependent variable PE.



**FIGURE 10.** EWMA<sub>x[DMRSS]</sub>-3 control chart under DMRSS scheme for shift in (a) intercept parameter; (b) slope parameter; and (c) Error variance parameter.



**FIGURE 11.** EWMA<sub>x[DERSS]</sub>-3 control chart under DERSS scheme for shift in (a) intercept parameter; (b) slope parameter; and (c) Error variance parameter.

Although, all ambient conditions play a role in the PE, AT is the most influential factor and most widely studied

about gas turbines. Reference [44] reported that there exists  $-0.95$  correlation between PE and the AT. Further,

a regression model was given as,

$$\widehat{PE} = 497.03 - 2.1713AT \quad (16)$$

which can be interpreted as: if AT increase a unit ( $^{\circ}C$ ), then PE reduced to 2.1713 MW. This model has  $R^2 = 89.89$  but might have the problem of mild normality (cf. Figure 7). Hence, we used only the first 2500 data points and obtained the following model,

$$\widehat{PE} = 497.362 - 2.1860AT \quad (17)$$

which can be interpreted as: if AT increase a unit ( $^{\circ}C$ ), then PE reduced to 2.1860 MW. This model has  $R^2 = 90.06$ , PP-plot is plotted in Figure 8 and Anderson-Darling test with the summary numbers given as AD = 0.736 and P-value = 0.055. From this analysis, we conclude that there is no strong evidence against normality (cf. Figure 8). Therefore, we are using model 12 as an in-control model. The implementation of EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts under SRS, DMRSS and DERSS, respectively on the real data set is described as follows:

*Step 1:* For the analysis, estimates of the simple linear profile parameters are obtained using five random data points ( $n = 5$ ) drawn by SRS, DMRSS and DERSS schemes. Further, these estimates are computed for a large number of time ( $10^6$  iterations) using extensive Monte Carlo simulations. Hence, the average and standard deviation of the estimates with respect to sampling schemes are obtained and reported in Table 15.

*Step 2:* For the analysis, we have fixed the overall  $ARL_0 = 200$  to obtain the charting constants of EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts. These constants are computed by extensive Monte Carlo simulations ( $10^6$  iterations). The resulting control limits are given in Table 15.

*Step 3:* Once, we have established the control limits, we chose 100 profiles as IC profiles (pink shaded in Figures 10-12) for EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts.

Further, the following profiles are used to introduce several amounts of shifts in terms of  $\sigma_{PE} = 16.58$ .

- For the shift in intercept, we have added  $0.25\sigma_{PE}$  in the values of PE, and the resulting 25 profiles with index 101 to 125 for EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts are portrayed in Figures 10-12.
- For the shifts in slope, we have multiplied  $0.075\sigma_{PE}$  with the values of AT, and the resulting 25 profiles with index 126 to 150 for EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts are portrayed in Figures 9-11.
- For the joint shifts in intercept and slope, we have added  $0.25\sigma_{PE}$  in the values of PE and multiplied  $0.075\sigma_{PE}$  with the values of AT, and the resulting 25 profiles with index 151 to 175 for EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts are portrayed in Figures 9-11.

- For the detection of shifts in the variance of disturbance term, we multiply 25 sets of PE with  $0.1\sigma_{PE}$  and the resulting 25 profiles with index 176 to 200 for EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts are portrayed in Figures 9-11.

For EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts the number of OOC profiles with their respective indices are given in Table 16. When there is a shift in intercept parameter, the results revealed that EWMA<sub>x[SRS]-3</sub> chart offers 5 OOC signals while EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> charts alarms 18 and 19 OOC signals, respectively. In the presence of shifts in slope parameter, EWMA<sub>x[SRS]-3</sub> chart signaled no OOC points while EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> signaled 1 and 15 OOC signals, respectively. Further, when the joint shift is introduced in intercept and slope, EWMA<sub>x[SRS]-3</sub> chart signaled 14 OOC points while EWMA<sub>x[DMRSS]-3</sub> and EWMA<sub>x[DERSS]-3</sub> signaled 12 and 55 OOC signals, respectively. Moreover, a similar pattern is also observed when shifts are introduced in error variance. Hence, the EWMA<sub>x[DERSS]-3</sub> has a better detection ability relative to EWMA<sub>x[SRS]-3</sub>, EWMA<sub>x[DMRSS]-3</sub> charts. The findings of the real case study also showed the evidence that the EWMA<sub>x[DMRSS]-3</sub> chart performed well while considering shifts in intercept and slope and EWMA<sub>x[DERSS]-3</sub> has a better detection ability in case of shifts in error variance.

## VII. CONCLUSIONS AND RECOMMENDATIONS

This study presented a new linear profiles monitoring method for random effect model with the application of various ranked set schemes like RSS, MRSS, ERSS, DRSS, DMRSS, and DERSS. The structure is designed on the bases of three independent EWMA<sub>x[R]-3</sub> charts to monitor the process parameters of random effect model such as intercept, slope and errors variance. A comprehensive analysis based on simulation and real data application is carried for the comparison of proposed EWMA<sub>x[R]-3</sub> charts and existing EWMA<sub>x[SRS]-3</sub> chart. The simulation study and practical application provide evidence that the recommended charts have higher detection ability as compared to the existing chart while monitoring shifts in the intercept and slope. For the case of errors variance monitoring, the EWMA<sub>x[DERSS]-3</sub> charts come up with the best detection ability. The significance in performance is high at smaller shifts and its magnitude decrease with the increase in shift values. Further, the joint shifts in the slope and intercept of the transformed model are incorporated, which indicates a significant improvement in the performance ability of linear profiling structures. Overall, findings of this study divulge that the ranked set sampling schemes improve the detection capability of control charts for the monitoring of linear profiles model. The scope of the current research can be extended and enhanced with the inclusion of Bayesian techniques, the run-rule schemes both in classical and Bayesian setup. Further the current simple linear random model can be extended for multivariate cases and of course extension in a nonlinear profiles model.



**NOMENCLATURE**

$R$ ;	Sampling techniques	$e_{i[j]k}$ ;	Residual terms ( $i^{th}$ value of $k^{th}$ cycle in $j^{th}$ profile)
$x$ ;	Random model	$MSE_j$ ;	Mean square error for $j^{th}$ profile
SRS;	Simple random sampling	$\mu_x$ ;	Mean of the explanatory variable (random) under simple random sampling
RSS;	Ranked set sampling	$\sigma_x^2$ ;	Variance of the explanatory variable (random) under simple random sampling
ERSS;	Extreme ranked set sampling	$\mu_{x[R]}$ ;	Mean of the explanatory variable (random) under ranked set sampling
MRSS;	Median ranked set sampling	$\sigma_{x[R]}^2$ ;	Variance of the explanatory variable (random) under ranked set sampling
DRSS;	Double ranked set sampling	$\sigma_0^2$ ;	Variance of error terms under simple random sampling
DERSS;	Double extreme ranked set sampling	$\sigma_{[0]}^2$ ;	Variance of error terms under ranked set sampling
DMRSS;	Double median ranked set sampling	$\theta$ ;	smoothing constant for EWMA statistic
$EWMA_{x[R]-3}$ ;	$EWMA_x-3$ chart using specified ranked set sampling	LCL;	Lower control limit
$EWMA_{x[SRS]-3}$ ;	$EWMA_x-3$ chart using simple random sampling	UCL;	Upper control limit
ARL;	Average run length	$L_{I[SRS]}$ ;	Control limit coefficient of EWMA for intercept using simple random sampling
$ARL_0$ ;	In-control ARL	$L_{S[SRS]}$ ;	Control limit coefficient of EWMA for slope using simple random sampling
$ARL_1$ ;	Out-of-control ARL	$L_{E[SRS]}$ ;	Control limit coefficient of EWMA for errors variance using simple random sampling
SPC;	Statistical process control	$L_{I[R]}$ ;	Control limit coefficient of EWMA for intercept using ranked set sampling
IC;	In-control: OOC; Out-of-control	$L_{S[R]}$ ;	Control limit coefficient of EWMA for slope using ranked set sampling
CUSUM chart;	Cumulative sum chart	$L_{E[R]}$ ;	Control limit coefficient of EWMA for errors variance using ranked set sampling
EWMA chart;	Exponentially weighted moving average chart	$\varphi_I$ ;	Shift in intercept; $\varphi_S$ ; Shift in slope
MCUSUM;	Multivariate cumulative sum	$\varphi_E$ ;	Shift in errors variance
MEWMA chart;	Multivariate exponentially weighted moving average chart	CCP;	Combined cycle power
$MEWMA_{x[R]-3}$ ;	$MEWMA_x-3$ chart using specified ranked set sampling	AP;	Atmospheric pressure
$MEWMA_{x[SRS]-3}$ ;	$MEWMA_x-3$ chart using simple random sampling	AT;	Ambient temperature; V; Vacuum
$n$ ; sample size: $l$ ;	a number of repetition	RH;	Relative humidity; PE; Electrical power output
$i$ ; $i^{th}$ sampled observation: $k$ ;	a number of cycles	AD;	Anderson-Darling test
$j$ ; $j^{th}$ profile: $Y_{i[j]k}$ ;	response variable ( $i^{th}$ sample of $k^{th}$ cycle in $j^{th}$ profile)	PP;	Probability plot
$X_{i[j]k}$ ;	explanatory variable (random)		
$\beta_0$ ;	Original intercept term		
$\beta_1$ ;	Original slope coefficient: $\varepsilon_{i[j]k}$ ; Error terms ( $i^{th}$ value of $k^{th}$ cycle in $j^{th}$ profile)		
$\rho$ ;	Correlation coefficient		
$A_{[0]}$ ;	Transformed intercept		
$A_{[1]}$ ;	Transformed slope coefficient		
$X_{i[j]k}^*$ ;	Transformed random explanatory variable ( $i^{th}$ value of $k^{th}$ cycle in $j^{th}$ profile)		

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