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Adaptive Distributed Control of a Flexible Manipulator Using an Iterative Learning Scheme

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ABSTRACT This paper presents an adaptive distributed control method based on an iterative learning scheme for a flexible manipulator to realize trajectory tracking and vibration reducing. The dynamical model of the flexible manipulator is captured by partial differential equations (PDEs). The control objective is to design a boundary controller and a distributed controller so that the motion of the flexible manipulator can track a desired position, and the deflection can be suppressed simultaneously subjecting to system parameters uncertainties and spatio-temporal distributed disturbances. A Lyapunov based stability analysis is carried out to achieve stability of the controlled system. Finally, simulation results evaluate the validity of the derived control scheme.

INDEX TERMS Distributed control, iterative learning control, adaptive control, flexible manipulator, distributed parameter system.

I. INTRODUCTION

Robotic manipulators can fulfill various mission requirements, especially flexible robotic manipulators with the characters of light weight, fast motion and low energy consumption, which can be broadly to aerospace and medical fields [1]–[4]. In the mathematical sense and considering the spatial structure, the flexible manipulator is a kind of distributed parameters system and should be described by PDEs since the states of the system depend on both time and spatial location, which complicates the control design [5]–[9], [11]–[15].

For promoting the wide application of flexible robotic manipulators, the development of various control methods is of great significance to achieve high-precision trajectory tracking and vibration reducing. Recently, the control design and stability analysis of flexible mechanical systems based on PDEs have achieved rapid development, especially the vibration control of flexible manipulators [16]–[19]. In [16], a boundary control law is proposed for a flexible manipulator for angle tracking and vibration suppressing in the presence of input backlash, and numerical simulations and physical experiments demonstrate the effectiveness of the control scheme. In [17], a boundary control is designed for a flexible robot manipulator to suppress the vibration. And neural network is used to eliminate the effects of dynamics uncertainties and input deadzone in the actuators. In [18], a PDE model and a boundary controller are presented for a three-dimensional flexible manipulator system. Compared with the boundary control, which only acts on the end of the system in [16]–[19], the distributed control can always achieve more satisfied control effect by applying the control force to every point of the system. However, the research results on the distributed control of flexible manipulators are still insufficient.

In practical applications and industrial environments, robot manipulators are always used to fulfill various repetitive tasks yielding plenty of control methods [20]. To achieve high precision control performance, iterative learning control (ILC) is widely used in robot domain. A lot of research works

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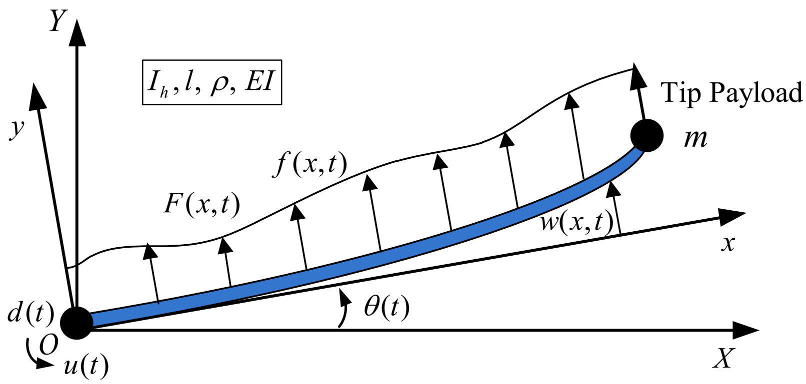


FIGURE 1. A typical flexible one-link manipulator.

have been documented for iterative learning control methods used in various repeatable control processes [5], [21]–[29]. These above works about different ILC methods are based on the ordinary differential equations (ODEs), which are not applicable for flexible mechanical systems descried by PDEs. For this reason, some researchers have tried to extend ILC to PDEs. In [30], an ILC feed-forward and a proportional and derivative control law are proposed for the unknown periodic motion on the right end for a class of axially moving material systems. In [31], [32], a P-type ILC scheme and a D-type ILC scheme are used to design boundary controller for nonlinear PDE systems. Boundary iterative learning control (BILC) schemes are proposed in [33] for flexible structures including a flexible string, an Euler-Bernoulli beam and a Timoshenko beam. The BILC schemes can suppress the vibration under varying disturbances. In [34], a hyperbolic tangent function is used to design a BILC scheme for an Euler-Bernoulli beam in the presence of input constraint. The above BILC schemes cannot achieve trajectory tracking and vibration suppressing of the flexible manipulators simultaneously. In [35], a BILC scheme is proposed for a flexible manipulator to track the desired trajectory and suppress the vibration. However, the distributed disturbances are not considered in this paper, which will seriously affect the control performance. Though most of the above researches force on the boundary control design, sometimes boundary control cannot achieve desirable control effect compared with distributed control. As far as we know, the distributed control of flexible mechanical systems using an iterative learning scheme is currently missing in the literature of control for PDEs.

In this paper, a state feedback structure and an adaptive term are proposed to address the problem of parameter uncertainty and spatially distributed disturbances. The Lyapunov direct method is used to prove the stability of the system. Contributions of this paper are given as follows: (1) An adaptive distributed control based on an iterative learning scheme is designed for a flexible manipulator for trajectory tracking and vibration reducing. Under the proposed controller, the position tracking error and the tail vibration can eventually converge to zero; (2) Parameter uncertainties as well as spatio-temporal distributed disturbances are both considered in the control design for practical consideration.

The paper is structured as follows. The problem formulation is given in Section II. In Section III, an adaptive distributed control based on the iterative learning scheme is designed for a flexible manipulator. Numerical simulations are carried out in Section IV, and the results demonstrate the validity. A conclusion is drawn in Section V.

II. PROBLEM STATEMENT

A. PDE DYNAMIC MODEL

As shown in Fig.1, $X-Y$ is the inertial reference coordinate system, and $x-y$ the body-fixed coordinate system attached to the flexible link. Let l be the total length of the beam, t be the time, x be the position, EI be the bending stiffness of the beam, I_h be the hub inertia, ρ be the mass of the unit length, m be the point mass tip payload, $\theta(t)$ be the angular position of the shoulder motor, θ_d be the constant ideal angular position, $f(x,t)$ be the distributed spatio-temporal disturbance and $d(t)$ be a vector containing the system parametric uncertainty and external disturbances. To control the flexible manipulator, $F(x,t)$ is the distributed control input which can be implemented by some piezoelectric actuators, and $u(t)$ is the control torque which can be realized by the shoulder motor. $w(x,t)$ is the elastic deflection, and the position vector $p(x,t)$ respective to the frame $X-Y$ is described by

$$p(x,t)=\begin{pmatrix}p_X(x,t)\\ p_Y(x,t)\end{pmatrix}=\begin{pmatrix}x\cos\theta-w(x,t)\sin\theta\\ x\sin\theta+w(x,t)\cos\theta\end{pmatrix}$$

Assuming the elastic deflection of the flexible manipulator is small, the total displacement $r(x,t)$ of a point $p(x,t)$ can be described as a function of both the angular position $\theta(t)$ and elastic deflection $w(x,t)$ [36]:

$$r(x,t)=x\theta(t)+w(x,t) \tag{1}$$

The kinetic energy of the flexible manipulator $E_{ki}(t)$ can be represented as

$$E_{ki}(t)=\frac{1}{2}I_h\dot{\theta}(t)^2+\frac{\rho}{2}\int_0^l\left(\dot{p}_X^2(x,t)+\dot{p}_Y^2(x,t)\right)dx \tag{2}$$

$$+\frac{1}{2}m\left(\dot{p}_X^2(l,t)+\dot{p}_Y^2(l,t)\right) \tag{3}$$

The potential energy E_{po} can be obtained from

$$E_{po}(t) = \frac{1}{2}\int_0^l EIw_{xx}^2(x,t)\,dx \tag{4}$$

The virtual work done on the system is given by

$$\delta W(t) = (u(t)+d(t))\,\delta\theta(t) \tag{5}$$
$$+\int_0^l (F(x,t)+f(x,t))\,\delta w(x,t)\,dx \tag{6}$$

We then use the Hamilton's principle

$$\int_{t_1}^{t_2}(\delta E_{ki}(t)-\delta E_{po}(t)+\delta W(t))dt = 0 \tag{7}$$

where $\delta(\cdot)$ represents the variation of $(\cdot)$, and obtain the governing equation

$$I_h\ddot{\theta}(t) - EIw_{xx}(0,t) = u(t)+d(t) \tag{8}$$
$$\rho(x\ddot{\theta}(t)+\ddot{w}(x,t)) + EIw_{xxxx}(x,t) = F(x,t)+f(x,t) \tag{9}$$

and the boundary conditions

$$m\ddot{w}(l,t)+ml\ddot{\theta}(t)-EIw_{xxx}(l,t)=0 \tag{10}$$
$$w(0,t)=w_x(0,t)=w_{xx}(l,t)=0 \tag{11}$$

Remark 1: For clarity, the following notations are introduced:
$(*)_x = \frac{\partial(*)}{\partial x}$, $(*)_{xx} = \frac{\partial^2(*)}{\partial x^2}$, $(*)_{xxx} = \frac{\partial^3(*)}{\partial x^3}$, $(*)_{xxxx} = \frac{\partial^4(*)}{\partial x^4}$, $(\dot{*}) = \frac{\partial(*)}{\partial t}$, $(\ddot{*}) = \frac{\partial^2(*)}{\partial t^2}$

Remark 2: The variables $\theta(t)$, $w(x,t)$, $u(t)$, $F(x,t)$, $d(t)$ and $f(x,t)$ are denoted as $\theta_k(t)$, $w_k(x,t)$, $u_k(t)$, $F_k(x,t)$, $d_k(t)$ and $f_k(x,t)$ respectively, where $k \in N$ is the iteration number.

B. PRELIMINARIES

Some necessary assumptions are given in this part:

Assumption 1 [6], [37]: The vector $d_k(t)$ is bounded, and there is a positive constant $\bar{d}$ meeting the condition $|d_k(t)| \leq \bar{d}\ \forall t \in [0,T]$.

Assumption 2 [34], [38]: The distributed spatio-temporal disturbance $f_k(x,t)$ are assumed to be bounded, and there exist a constant $\bar{f} \in R^+$ satisfying the condiation $\|f_k(x,t)\|_2 \leq \bar{f}, \forall(x,t) \in [0,l]\times[0,\infty)$.

Assumption 3 [35]: We assume that the initial conditions of the system can be reset for each iteration such that the initial conditions of the system are set to satisfy the following equalities: $\theta_d(0)-\theta_k(0) = 0$ and $w_k(x,0) = \dot{w}_k(x,0) = 0$ for all $k \in Z_+$.

Remark 3: In fact, the resetting condition in Assumption 3 can be relaxed to a certain extent if the alignment condition is satisfied [24], i.e., $V(0) = V_{k-1}(T)$. That is we can start the flexible beam from where it was stopped at the last operation instead of bringing it to the same initial position at each operation.

III. ADAPTIVE DISTRIBUTED ILC DESIGN AND ANALYSIS

This section aims to design a boundary control $u_k(t)$ and a distributed control $F_k(x,t)$ for angle tracking and vibration attenuation simultaneously under the circumstances of unknown system parameters and distributed disturbances. To achieve this, an iterative learning scheme is used to design the control method, and the Lyapunov's criterion is exploited to analyze and demonstrate the learning convergence.

Consider the kth iterative operation for system (8)-(11) with assumptions (1)-(3), the ILC laws are designed as

$$u_k(t) = -k_1e_k(t) - k_2\dot{e}_k(t) + k_3\int_0^l xw_k(x,t)dx - \hat{\delta}_k(t)\,\mathrm{sgn}(\dot{e}_k(t)) \tag{12}$$
$$F_k(x,t) = -k_3w_k(x,t) - k_4\dot{r}_k(x,t) - \hat{\sigma}_k(x,t)\,\mathrm{sgn}(\dot{r}_k(x,t)) \tag{13}$$

where $k_1, k_2, k_3, k_4 > 0$, and the adaptive laws

$$\hat{\delta}_k(t) = \hat{\delta}_{k-1}(t) + \gamma_1\dot{e}_k(t)\mathrm{sgn}(\dot{e}_k(t)) \tag{14}$$
$$\hat{\sigma}_k(x,t) = \hat{\sigma}_{k-1}(x,t) + \gamma_2\dot{r}_k(x,t)\,\mathrm{sgn}(\dot{r}_k(x,t)) \tag{15}$$

where $\gamma_1, \gamma_2 > 0$, $\hat{\delta}_{-1}(t) = 0$, $\hat{\sigma}_{-1}(x,t) = 0$ and $e_k(t) = \theta_k(t) - \theta_d$. Since θ_d is a constant, we have $\dot{e}_k(t) = \dot{\theta}_k(t) - \dot{\theta}_d = \dot{\theta}_k(t)$.

Theorem 1: Suppose the system (8)-(11) satisfies assumptions (1)-(3). Consider the flexible manipulator performing repetitive tasks with the designed distributed control scheme (12) - (15), then the following should hold.

1) $e_k(t)$, $\dot{e}_k(t)$, $\dot{r}_k(x,t)$, $w_k(x,t)$, $u_k(t)$ and $F_k(x,t)$ are all bounded for all $k \in Z_+$.

2) $\lim_{k\to\infty} e_k(t) = \lim_{k\to\infty} \dot{e}_k(t) = 0$ and $\lim_{k\to\infty} w_k(x,t) = \lim_{k\to\infty} \dot{w}_k(x,t) = 0, \forall(x,t) \in [0,l]\times[0,T]$.

Proof: The proof process can be found in Appendix 1. ■

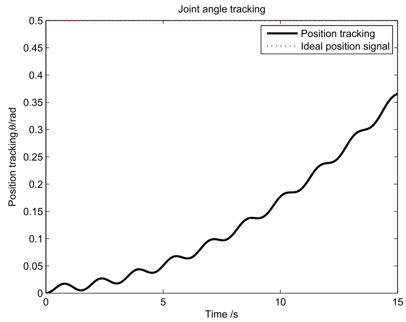
Therefore, the angle tracking and vibration attenuation can be achieved simultaneously under the proposed adaptive distributed ILC considering unknown system parameters and distributed disturbances.

Remark 4: In practice, we usually choose the hyperbolic tangent function $\tanh(t)$ instead of the signal function $\mathrm{sgn}(t)$ to obtain a smooth input without chattering phenomena. The smooth iterative learning control scheme is designed as follows

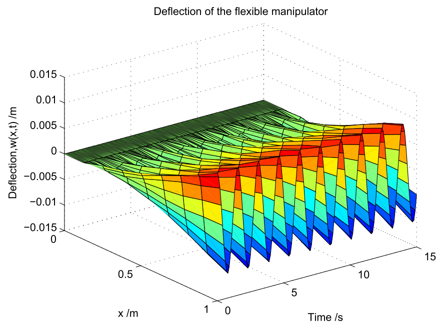
$$u_k(t) = -k_1e_k(t) - k_2\dot{e}_k(t) + k_3\int_0^l xw_k(x,t)dx - \hat{\delta}_k(t)\tanh(\dot{e}_k(t)) \tag{16}$$
$$F_k(x,t) = -k_3w_k(x,t) - k_4\dot{r}_k(x,t) -!\hat{\sigma}_k(x,t)\tanh(\dot{r}_k(x,t)) \tag{17}$$

and the adaptive laws

$$\hat{\delta}_k(t) = \hat{\delta}_{k-1}(t) + \gamma_1\dot{e}_k(t)\tanh(\dot{e}_k(t)) \tag{18}$$
$$\hat{\sigma}_k(x,t) = \hat{\sigma}_{k-1}(x,t) + \gamma_2\dot{r}_k(x,t)\tanh(\dot{r}_k(x,t)) \tag{19}$$

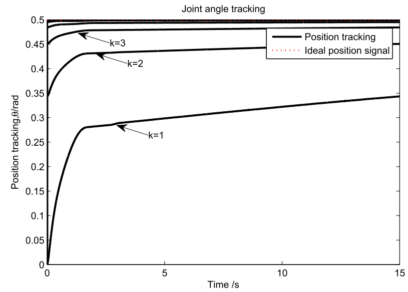


(a) The angular position of shoulder motor

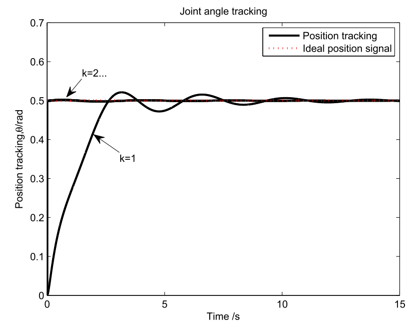


(b) The distributed deflection of the flexible manipulator

FIGURE 2. Without control input.



(a) With control laws (12) - (15)



(b) With control laws (16) - (19)

FIGURE 3. Angular position of shoulder motor in the k $(1-5)$th iteration.

It is worth noting that if use the hyperbolic tangent function in the proposed scheme, we will achieve better control performance, however, the stability of the smooth scheme needs further investigation and will be part of our future work.

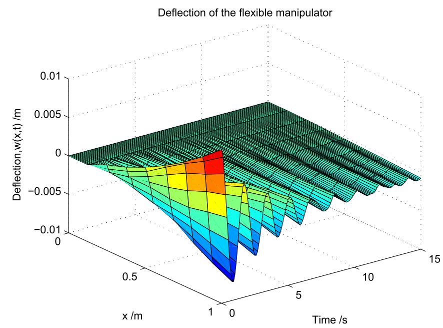
IV. NUMERICAL SIMULATIONS

Numerical simulations are used to test the control effect with the adaptive distributed iterative learning control laws (12) - (15). The desired angle $\theta_d = 0.5(rad)$. The disturbance $d_k(t)$ is given as $d_k(t) = 0.1 * \text{rand} * \sin(t)$, rand is a random function. The unknown distributed spatio-temporal disturbance along the flexible manipulator $f_k(x,t)$ is described as $f_k(x,t) = 0.05 + 0.01\sin(0.1\pi xt) + 0.03\sin(0.3\pi xt) + 0.05\sin(0.5\pi xt)$. The initial conditions are $\theta(0) = 0(rad)$, $w(x,0) = 0.01x^2$ and $\dot{w}(x,0) = 0$. The parameters are listed in Table 1.

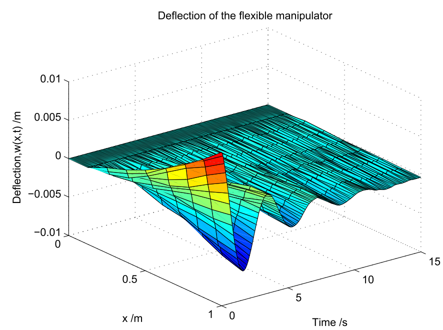
TABLE 1. Parameters of a flexible manipulator.

Parameter	Description	Value
l	Length of flexible link	1 m
EI	bending stiffness of flexible link	2 Nm^2
m	Point mass tip payload	0.5 kg
I_h	Hub inertia	0.5 kgm^2
ρ	Mass of unit length link	0.2 kg/m

For the sake of analysis, the maximal position tracking error $e_{\max}(k) = \sup_{t\in[0,T]} |e_k(t)|$, the maximal end-point deflection $w_{l\max}(k) = \sup_{t\in[0,T]} |w_k(l,t)|$ and the maximal distributed deflection $w_{\max}(x,k) = \sup_{t\in[0,T]} |w_k(x,t)|$ are firstly defined.



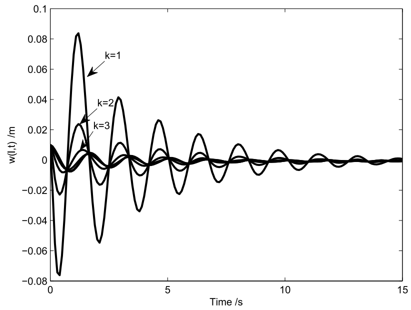
(a) With control laws (12) - (15)



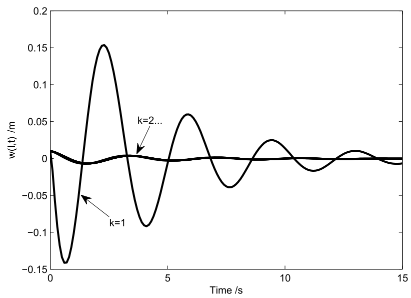
(b) With control laws (16) - (19)

FIGURE 4. Deflection of the flexible manipulator w(x,t) at the 5th iteration.

In order to prove the control effect, the following three response scenarios for the system are presented:

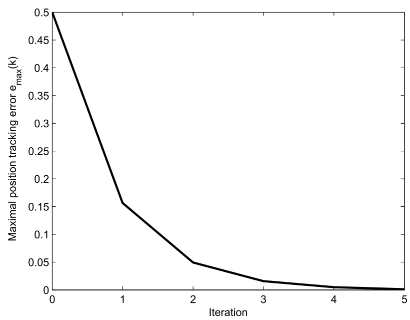


(a) With control laws (12) - (15)

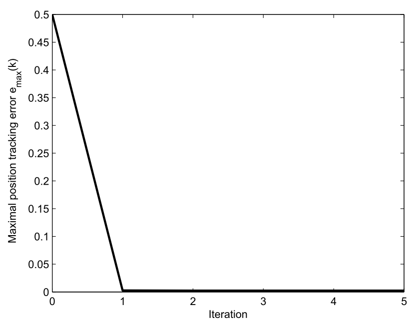


(b) With control laws (16) - (19)

FIGURE 5. The vibration of w(l,t) in the k $(1-5)$th iteration.



(a) With control laws (12) - (15)

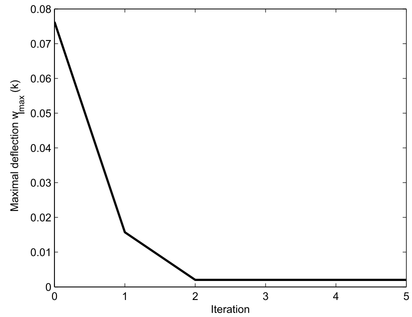


(b) With control laws (16) - (19)

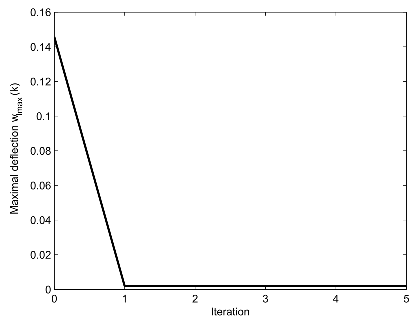
FIGURE 6. Maximal position tracking error $e_{\max}(k)$ versus the number of iterations.

Case 1: Without control input: $u_k(t) = 0$, $F_k(x, t) = 0$.

Case 2: With the proposed control (12) - (15): $k_1 = 10$, $k_2 = 10$, $k_3 = 10$, $k_4 = 10$, $\gamma_1 = 0.01$, $\gamma_2 = 0.01$ and iteration number $k = 5$.

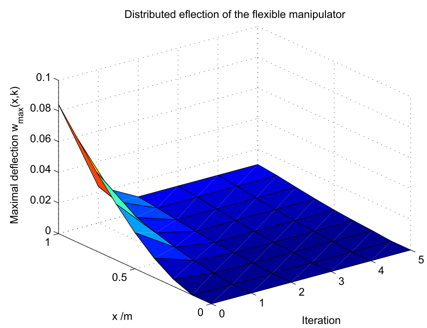


(a) With control laws (12) - (15)

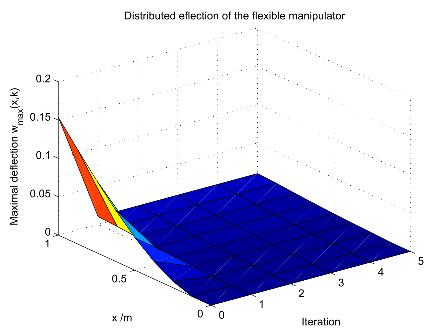


(b) With control laws (16) - (19)

FIGURE 7. Maximal deflection $w_{l\,\max}(k)$ versus the number of iterations.



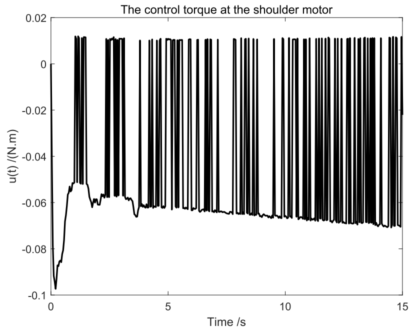
(a) With control laws (12) - (15)



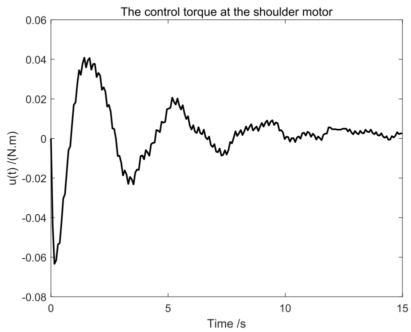
(b) With control laws (16) - (19)

FIGURE 8. Maximal deflection $w_{\max}(x, k)$ versus the number of iterations.

Case 3: With the proposed control (16) - (19): $k_1 = 10$, $k_2 = 10$, $k_3 = 10$, $k_4 = 10$, $\gamma_1 = 0.01$, $\gamma_2 = 0.01$ and iteration number $k = 5$.

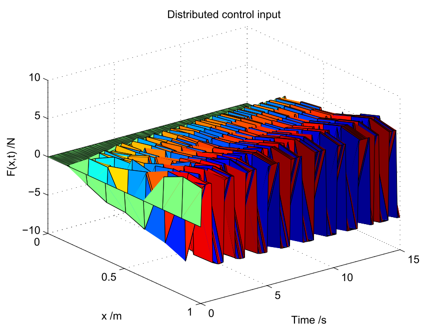


(a) With control laws (12) - (15)

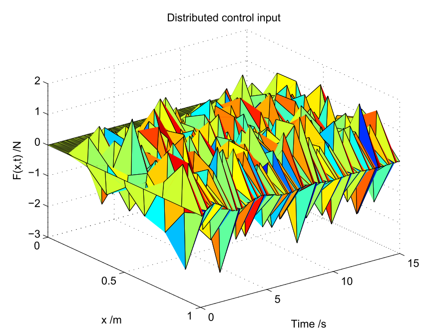


(b) With control laws (16) - (19)

FIGURE 9. Control torque $u(t)$ at the 5th iteration.



(a) With control laws (12) - (15)



(b) With control laws (16) - (19)

FIGURE 10. Distributed control $F(x, t)$ at the 5th iteration.

Simulation results without control are shown in the Fig.2. We can clearly see that the tracking error and distributed deflections are large.

The simulation results of cases 2 and 3 are shown in Figs. 3-10. Fig. 3 shows the angular position of the shoulder motor. The distributed deflection $w(x, t)$ at the 5th iteration are given in Fig. 4. Fig. 5 shows the end-point deflection $w(l, t)$ in the $k\,(1-5)$th iteration. And Fig. 6 - 8 shows the maximal errors $e_{\max}(k)$, $w_{l\max}(k)$ and $w_{\max}(x, k)$, respectively.

From Figs. 3-8, it is clear that the system is unstable in Case 1. With the proposed control in (11)-(14) and (15)-(18), the control effect can both be satisfactory at the 5th iteration. Besides, the tracking error can converge at a fast rate and the distributed deflection $w_k(x, t)$ has an obvious decreasing trend in both Cases 2 and 3.

As we can see from Figs. 9 and 10, the control inputs with the control scheme (16) - (19) are smoother than with the control scheme (12) - (15), illustrating the virtue of improved control in (15)-(18).

Form above analysis, the proposed distributed control scheme can realize the angle tracking and vibration abatement of the flexible manipulator, and the control effect is improved incrementally with the increase of the iteration number. In addition, choosing the function $\tanh(t)$ instead of $\mathrm{sgn}(t)$ can confine the undesired chattering have faster learning convergence. Therefore, the effectiveness of control in (15) - (18) and (11) - (14) are verified.

V. CONCLUSION

This study proposed a DILC scheme for a flexible manipulator with unknown parameters and spatio-temporal distributed disturbances. The designed adaptive DILC scheme combining a PD feedback structure and an adaptive term can guarantee that the motion of the flexible manipulator can track a desired reference position, and the elastic vibration is suppressed simultaneously. Comparing with the existing control methods, the advantages of this control scheme are that: (1) it can solve the problems of unknown parameters and distributed disturbances; and (2) it does not require a priori knowledge of the physical parameters. Finally, simulation results are presented to demonstrate the control performance. Future topics include exploiting iterative learning control of flexible manipulators with time-varying output constraints [39], and using other intelligent optimization methods, such as adaptive dynamic programming [40]–[43] and other intelligent learning algorithm [42], [44], [45].

APPENDIX A PROOF OF THEOREM 1

Define a Lyapunov candidate function as

$$\Gamma_k(t) = \Lambda_k(t) + \frac{1}{2}\int_0^t \gamma_1^{-1}\tilde{\delta}_k^2(\tau)d\tau + \frac{1}{2}\int_0^t\int_0^l \gamma_2^{-1}\tilde{\sigma}_k^2(x,\tau)\,dxd\tau \tag{20}$$

with $\tilde{\delta}_k(t) = \delta_k(t) - \hat{\delta}_k(t)$ and $\tilde{\sigma}_k(x,t) = \sigma_k(x,t) - \hat{\sigma}_k(x,t)$, where $\delta_k(t)$, $\sigma_k(x,t)$ are unknown vectors. $\hat{\delta}_k(t)$ and $\hat{\sigma}_k(x,t)$

are the estimated values of $\delta_k(t)$ and $\sigma_k(x,t)$. The unknown vectors $\delta_k(t)$ and $\sigma_k(x,t)$ are defined as $\delta_k(t) = \bar{d}$ and $\sigma_k(x,t) = \bar{f}$.

The term $\Lambda_k(t)$ in (20) is chosen as follows

$$\Lambda_k(t) = W_{1,k}(t) + W_{2,k}(t) \tag{21}$$

where

$$W_{1,k}(t) = frac12 \int_0^l \rho \dot{r}_k^2(x,t)dx + \frac{1}{2}EI\int_0^l w_{kxx}^2(x,t)dx \tag{22}$$

$$W_{2,k}(t) = \frac{1}{2}I_h\dot{e}_k^2(t) + \frac{1}{2}k_1e_k^2(t) + \frac{1}{2}m\dot{r}_k^2(l,t) + \frac{1}{2}k_3\int_0^l w_k^2(x,t)dx \tag{23}$$

Then, $\Lambda_k(t)$ can be rewritten as

$$\Lambda_k(t) = \Lambda_k(0) + \int_0^t \left(\dot{E}_{1,k}(\tau) + \dot{W}_{2,k}(\tau)\right)d\tau \tag{24}$$

where

$$\begin{aligned}
\dot{W}_{1,k}(t) &= \int_0^l \rho\dot{r}_k(x,t)\ddot{r}_k(x,t)dx \\
&\quad + EI\int_0^l w_{kxx}(x,t)\dot{w}_{kxx}(x,t)dx \\
&= \int_0^l (F_k(x,t) + f_k(x,t))\,\dot{r}_k(x,t)dx \\
&\quad - EIw_{kxxx}(l,t)l\dot{\theta}_k(t) - EIw_{kxx}(0,t)\dot{\theta}_k(t) \\
&\quad - EIw_{kxxx}(l,t)\dot{w}_k(l,t) \\
&= \int_0^l (F_k(x,t) + f_k(x,t))\,\dot{r}_k(x,t)dx \\
&\quad - EIw_{kxxx}(l,t)\dot{r}_k(l,t) - EIw_{kxx}(0,t)\dot{\theta}_k(t)
\end{aligned}$$

and

$$\begin{aligned}
&\dot{W}_{2,k}(t) \\
&= I_h\dot{e}_k(t)\ddot{e}_k(t) + m\dot{r}_k(l,t)\ddot{r}_k(l,t) \\
&\quad + k_1e_k(t)\dot{e}_k(t) + k_3\int_0^l w_k(x,t)\dot{w}_k(x,t)dx \\
&= I_h\dot{e}_k(t)\ddot{e}_k(t) + m\dot{r}_k(l,t)\ddot{r}_k(l,t) + k_1e_k(t)\dot{e}_k(t) \\
&\quad + k_3\int_0^l w_k(x,t)\dot{r}_k(x,t)dx - k_3\int_0^l x\dot{\theta}_k(t)w_k(x,t)dx
\end{aligned}$$

Since

$$\begin{aligned}
&\dot{W}_{1,k}(t) + \dot{W}_{2,k}(t) \\
&= \int_0^l (F_k(x,t) + f_k(x,t))\,\dot{r}_k(x,t)dx \\
&\quad - EIw_{kxxx}(l,t)\dot{r}_k(l,t) - EIw_{kxx}(0,t)\dot{\theta}_k(t) \\
&\quad + I_h\dot{e}_k(t)\ddot{e}_k(t) + m\dot{r}_k(l,t)\ddot{r}_k(l,t) + k_1e_k(t)\dot{e}_k(t) \\
&\quad + k_3\int_0^l w_k(x,t)\dot{r}_k(x,t)dx - k_3\int_0^l x\dot{\theta}_k(t)w_k(x,t)dx \\
&= \dot{e}_k(t)\left(u_k(t) + k_pe_k(t) + d_k(t) - k_3\int_0^l xw_k(x,t)dx\right) \\
&\quad + \int_0^l (F_k(x,t) + f_k(x,t))\,\dot{r}_k(x,t)dx \\
&\quad + k_3\int_0^l w_k(x,t)\dot{w}_k(x,t)dx
\end{aligned}$$

We can get

$$\begin{aligned}
&\Lambda_k(t) \\
&= \Lambda_k(0) + \int_0^t \dot{e}_k(\tau)\left(u_k(\tau) + k_pe_k(\tau)\right)d\tau \\
&\quad - \int_0^t \dot{e}_k(\tau)\left(k_3\int_0^l xw_k(x,\tau)dx + d_k(\tau)\right)d\tau \\
&\quad + \int_0^t\int_0^l (F_k(x,t) + f_k(x,t) + k_3w_k(x,t))\,\dot{r}_k(x,t)dxd\tau
\end{aligned} \tag{25}$$

Considering Assumptions 1 and 2, we obtain

$$\dot{e}_k(t)d_k(t) \le |\dot{e}_k(t)|\,\bar{d} = \bar{d}\dot{e}_k(t)\mathrm{sgn}\,(\dot{e}_k(t)) \tag{26}$$

and

$$f_k(x,t)\dot{r}_k(x,t) \le \bar{f}\,|\dot{r}_k(x,t)| = \bar{f}\dot{r}_k(x,t)\mathrm{sgn}\,(\dot{r}_k(x,t)) \tag{27}$$

Using the inequalities (26) and (27), (25) can be written as

$$\begin{aligned}
&\Lambda_k(t) \\
&\le \Lambda_k(0) + \int_0^t \dot{e}_k(\tau)\,(u_k(\tau) + k_1e_k(\tau))d\tau \\
&\quad + \int_0^t \dot{e}_k(\tau)\left(-k_3\int_0^l xw_k(x,\tau)dx + \bar{d}\mathrm{sgn}(\dot{e}_k(\tau))\right)d\tau \\
&\quad + \int_0^t\int_0^l (F(x,\tau) + k_3w_k(x,\tau))\,\dot{r}_k(x,\tau)dxd\tau \\
&\quad + \int_0^t\int_0^l \left(\bar{f}\mathrm{sgn}\,(\dot{r}_k(x,\tau))\right)\dot{r}_k(x,\tau)dxd\tau
\end{aligned} \tag{28}$$

Substituting (12) and (13) into (28) yields

$$\begin{aligned}
&\Lambda_k(t) \\
&\le \Lambda_k(0) - \int_0^t \dot{e}_k(\tau)\,(k_2\dot{e}_k(\tau))d\tau \\
&\quad - \int_0^t \dot{e}_k(\tau)\left(\hat{\delta}_k(\tau)\mathrm{sgn}\,(\dot{e}_k(\tau)) - \bar{d}\mathrm{sgn}(\dot{e}_k(\tau))\right)d\tau \\
&\quad - \int_0^t\int_0^l \left(k_4\dot{r}_k(x,\tau) + \bar{f}\mathrm{sgn}\,(\dot{r}_k(x,\tau))\right)\dot{r}_k(x,\tau)dxd\tau \\
&\quad - \int_0^t\int_0^l \left(\hat{\sigma}_k(x,\tau)\,\mathrm{sgn}\,(\dot{r}_k(x,\tau))\right)\dot{r}_k(x,\tau)dxd\tau
\end{aligned} \tag{29}$$

Considering (20) with $k = 0$, we get

$$\Gamma_0(t) = \Lambda_0(t) + \frac{1}{2}\int_0^t \gamma_1^{-1}\tilde{\delta}_0^2(\tau)d\tau + \frac{1}{2}\int_0^t\int_0^l \gamma_2^{-1}\tilde{\sigma}_0^2(x,\tau)\,dxd\tau \tag{30}$$

Since the vectors $\delta_k(t)$ and $\sigma_k(x,t)$ are defined as $\delta_k(t)=\bar{d}$ and $\sigma_k(x,t)=\bar{f}$. The derivative of (30) is

$$\begin{aligned}
&\dot{\Gamma}_0(t)\\
&=\dot{\Lambda}_0(t)+\frac{1}{2}\gamma_1^{-1}\tilde{\delta}_0^2(t)+\frac{1}{2}\int_0^l\gamma_2^{-1}\tilde{\sigma}_0^2(x,t)\,dx\\
&\le \dot{e}_0(t)\left(-k_2\dot{e}_0(t)+\tilde{\delta}_0(t)\,\mathrm{sgn}(\dot{e}_0(t))\right)\\
&\quad+\int_0^l\left(-k_4\dot{r}_0(x,t)+\tilde{\sigma}_0(x,t)\,\mathrm{sgn}(\dot{r}_0(x,t))\right)\dot{r}_0(x,t)dx\\
&\quad+\frac{1}{2}\gamma_1^{-1}\tilde{\delta}_0^2(t)+\frac{1}{2}\int_0^l\gamma_2^{-1}\tilde{\sigma}_0^2(x,t)\,dx
\end{aligned}\tag{31}$$

With $\hat{\delta}_{-1}(t)=0$, $\hat{\sigma}_{-1}(x,t)=0$, $\hat{\delta}_0(t)=\hat{\delta}_{-1}(t)+\gamma_1\dot{e}_0(t)\,\mathrm{sgn}(\dot{e}_0(t))$ and $\hat{\sigma}_0(x,t)=\hat{\sigma}_{-1}(x,t)+\gamma_2\dot{r}_0(x,t)\,\mathrm{sgn}(\dot{r}_0(x,t))$, we have

$$\begin{aligned}
\dot{\Gamma}_0(t)\le&-k_2\dot{e}_0^2(t)+\left(\hat{\delta}_0(t)+\frac{1}{2}\tilde{\delta}_0(t)\right)\gamma_1^{-1}\tilde{\delta}_0(t)\\
&+\int_0^l\left(\hat{\sigma}_0(x,t)+\frac{1}{2}\tilde{\sigma}_0(x,t)\right)\gamma_2^{-1}\tilde{\sigma}_0(x,t)\,dx\\
&-k_4\int_0^l\dot{r}_0^2(x,t)dx
\end{aligned}\tag{32}$$

Using $\hat{\delta}_0(t)=\delta_k(t)-\tilde{\delta}_0(t)$ and $\hat{\sigma}_0(x,t)=\sigma_k(x,t)-\tilde{\sigma}_0(x,t)$, Eq. (32) leads to

$$\begin{aligned}
\dot{\Gamma}_0(t)\le&-k_2\dot{e}_0^2(t)-\frac{1}{2}\tilde{\delta}_0(t)\gamma_1^{-1}\tilde{\delta}_0(t)+\delta_0(t)\gamma_1^{-1}\tilde{\delta}_0(t)\\
&-k_4\int_0^l\dot{r}_0^2(x,t)dx-\frac{1}{2}\int_0^l\tilde{\sigma}_0(x,t)\,\gamma_2^{-1}\tilde{\sigma}_0(x,t)\,dx\\
&+\int_0^l\sigma_0(x,t)\,\gamma_2^{-1}\tilde{\sigma}_0(x,t)\,dx
\end{aligned}\tag{33}$$

Using Young's inequality, we have

$$\delta_0(t)\gamma_1^{-1}\tilde{\delta}_0(t)\le\lambda_1\left(\gamma_1^{-1}\tilde{\delta}_0(t)\right)^2+\frac{1}{4\lambda_1}\delta_0{}^2(t)\tag{34}$$

and

$$\sigma_0(x,t)\,\gamma_2^{-1}\tilde{\sigma}_0(x,t)\le\lambda_2\left(\gamma_2^{-1}\tilde{\sigma}_0(x,t)\right)^2+\frac{1}{4\lambda_2}\sigma_0{}^2(x,t)\tag{35}$$

for any $\lambda_1,\lambda_2>0$.

Substituting (34) and (35) into (33), we have

$$\begin{aligned}
\dot{\Gamma}_0(t)\le&-k_2\dot{e}_0^2(t)-\frac{1}{2}\tilde{\delta}_0(t)\gamma_1^{-1}\tilde{\delta}_0(t)+\lambda_1\left(\gamma_1^{-1}\tilde{\delta}_0(t)\right)^2\\
&-k_4\int_0^l\dot{r}_0^2(x,t)dx-\frac{1}{2}\int_0^l\tilde{\sigma}_0(x,t)\,\gamma_2^{-1}\tilde{\sigma}_0(x,t)\,dx\\
&+\int_0^l\lambda_2\left(\gamma_2^{-1}\tilde{\sigma}_0(x,t)\right)^2dx+\frac{1}{4\lambda_1}\delta_0{}^2(t)\\
&+\int_0^l\frac{1}{4\lambda_2}\sigma_0{}^2(x,t)\,dx
\end{aligned}$$

Hence, we have

$$\begin{aligned}
\dot{\Gamma}_0(t)\le&-k_2\dot{e}_0^2(t)-\rho_1\tilde{\delta}_0^2(t)+\frac{1}{4\lambda_1}\delta_{\max}^2-k_4\int_0^l\dot{r}_0^2(x,t)dx\\
&-\rho_2\int_0^l\tilde{\sigma}_0^2(x,t)\,dx+\frac{1}{4\lambda_2}l\sigma_{\max}^2
\end{aligned}\tag{36}$$

with $\delta_{\max}=\mathrm{Sup}_{t\in[0,T]}\delta_0(t)$, $\sigma_{\max}=\mathrm{Sup}\sigma_0(x,t)$, $\forall(x,t)\in[0,l]\times[0,T]$, $\rho_1=\frac{1}{2}\gamma_1^{-1}-\lambda_1\gamma_1^{-2}$, $\rho_2=\frac{1}{2}\gamma_2^{-1}-\lambda_2\gamma_2^{-2}$ and $\lambda_1<\frac{1}{2}\gamma_1$, $\lambda_2<\frac{1}{2}\gamma_2$.

Then, we can conclude that

$$\dot{\Gamma}_0(t)\le\frac{1}{4\lambda_1}\delta_{\max}^2+\frac{1}{4\lambda_2}l\sigma_{\max}^2\tag{37}$$

which implies that $\Gamma_0(t)$ is uniformly continuous and thus bounded over $[0,T]$.

From the definition of $\Gamma_k(t)$, for the $j-1$th iteration we can get

$$\begin{aligned}
\Gamma_{k-1}(t)=&\Lambda_{k-1}(t)+\frac{1}{2}\int_0^t\gamma_1^{-1}\tilde{\delta}_{k-1}^2(\tau)d\tau\\
&+\frac{1}{2}\int_0^t\int_0^l\gamma_2^{-1}\tilde{\sigma}_{k-1}^2(x,\tau)\,dxd\tau
\end{aligned}\tag{38}$$

The difference of $\Gamma_k(t)$ is given by

$$\begin{aligned}
&\Delta\Gamma_k\\
&=\Gamma_k-\Gamma_{k-1}\\
&=\Lambda_k-\Lambda_{k-1}+\frac{1}{2}\int_0^t\gamma_1^{-1}\left(\tilde{\delta}_k^2(\tau)-\tilde{\delta}_{k-1}^2(\tau)\right)d\tau\\
&\quad+\frac{1}{2}\int_0^t\int_0^l\gamma_2^{-1}\left(\tilde{\sigma}_k^2(x,\tau)-\tilde{\sigma}_{k-1}^2(x,\tau)\right)dxd\tau\\
&=\Lambda_k-\Lambda_{k-1}-\frac{1}{2}\int_0^t\gamma_1^{-1}\left(\bar{\delta}_k^2(\tau)+2\tilde{\delta}_k(\tau)\,\bar{\delta}_k(\tau)\right)d\tau\\
&\quad-\frac{1}{2}\int_0^t\int_0^l\gamma_2^{-1}\left(\bar{\sigma}_k^2(x,\tau)+2\tilde{\sigma}_k(x,\tau)\,\bar{\sigma}_k(x,\tau)\right)dxd\tau
\end{aligned}\tag{39}$$

where $\bar{\delta}_k(t)=\hat{\delta}_k(t)-\hat{\delta}_{k-1}(t)$ and $\bar{\sigma}_k(x,t)=\hat{\sigma}_k(x,t)-\hat{\sigma}_{k-1}(x,t)$.

Substituting (14), (15) and (29) into (39), we obtain

$$\begin{aligned}
\Delta\Gamma_k(t)\le&-\Lambda_{k-1}(t)-\frac{1}{2}\int_0^t(\gamma_1^{-1}\bar{\delta}_k^2(\tau)+2k_2\dot{e}_k^2(\tau))d\tau\\
&-\frac{1}{2}\int_0^t\int_0^l\gamma_2^{-1}\bar{\sigma}_k^2(x,\tau)+2k_4\dot{r}_k^2(x,\tau)dxd\tau\le0
\end{aligned}\tag{40}$$

Therefore, $\Gamma_k(t)$ is a non-increasing sequence, that is $\Gamma_k(t)\le\Gamma_{k-1}(t)$. Hence, $\Gamma_k(t)$ is bounded for $\forall t\in[0,T]$.

So $W_{1,k}(t)$, $W_{2,k}(t)$, $\int_0^t\gamma_1^{-1}\tilde{\delta}_k^2(\tau)d\tau$ and $\int_0^t\int_0^l\gamma_2^{-1}\tilde{\sigma}_k^2(x,\tau)\,dxd\tau$ are bounded. Hence, $e_k(t)$, $\dot{e}_k(t)$, $r_k(x,t)$, $\dot{r}_k(x,t)$, $u_k(t)$, $F_k(x,t)$ are all bounded for all $k\in Z_+$ and $(x,t)\in[0,l]\times[0,T]$.

Then, we can write $\Gamma_k(t)$ as follows

$$\Gamma_k(t)=\Gamma_0(t)+\sum_{j=1}^k\Delta\Gamma_j(t)\tag{41}$$

and using (40) we have

$$\Gamma_k(t) \le \Gamma_0(t) - \sum_{j=1}^{k} \Lambda_{j-1}(t) \tag{42}$$

$$\le \Gamma_0(t) - \frac{1}{2}\sum_{j=1}^{k} \left(W_{1,j-1}(t) + W_{2,j-1}(t)\right) \tag{43}$$

which implies that

$$\sum_{j=1}^{k} \left(W_{1,j-1}(t) + W_{2,j-1}(t)\right) \le 2\left(\Gamma_0(t) - \Gamma_k(t)\right) \le 2\Gamma_0(t) \tag{44}$$

Hence, $\lim_{k\to\infty} W_{1,k}(t) = \lim_{k\to\infty} W_{2,k}(t) = 0$, $\forall t \in [0, T]$, since $\Gamma_k(t)$ is bounded for all $k \in Z_+$ and $t \in [0, T]$. Considering Eqs. (22) and (23), we obtain $\lim_{k\to\infty} e_k(t) = \lim_{k\to\infty} \dot{e}_k(t) = 0$, $\lim_{k\to\infty} w_k(x, t) = \lim_{k\to\infty} \dot{w}_k(x, t) = 0$ for all $k \in Z_+$ and $(x, t) \in [0, l] \times [0, T]$.

From the above analysis it can be seen that the proposed control laws can guarantee that angular position tracking error and the amplitude of elastic vibration converge arbitrarily close to zero with the increase of the iteration number.

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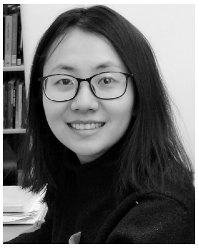

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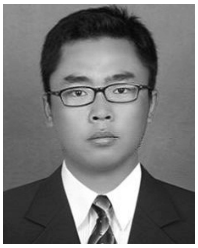

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